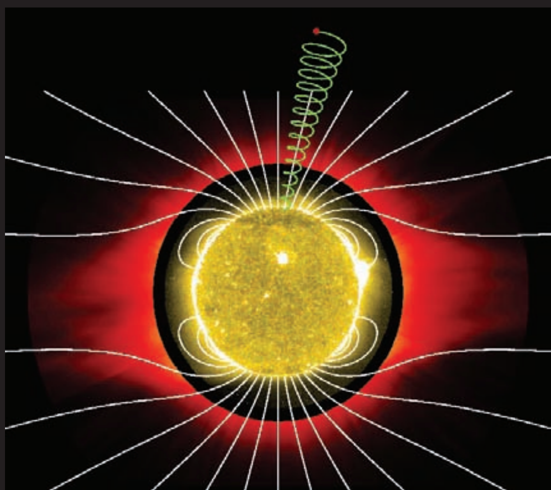
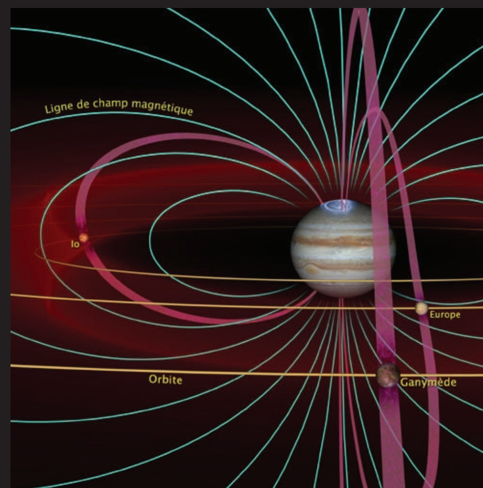
National Aeronautics and Space Administration

Magnetic Math





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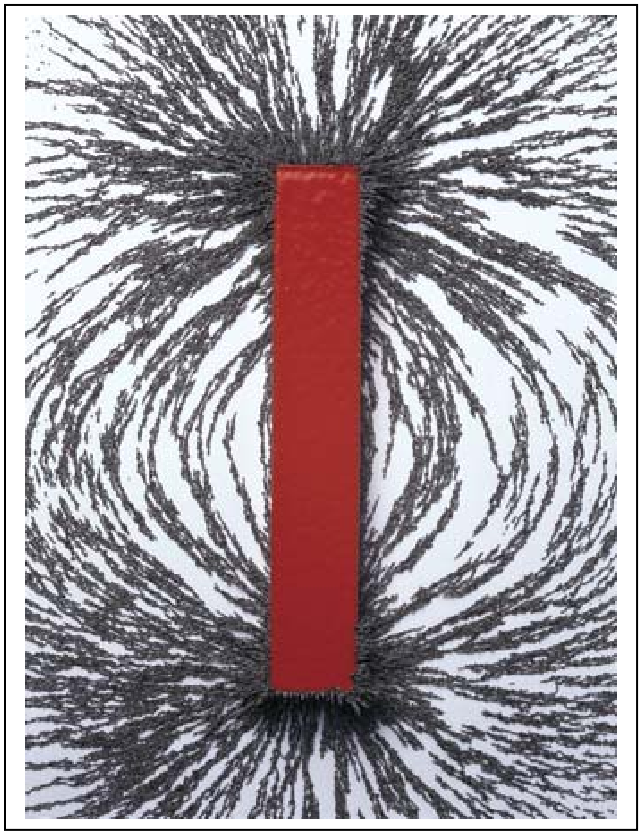

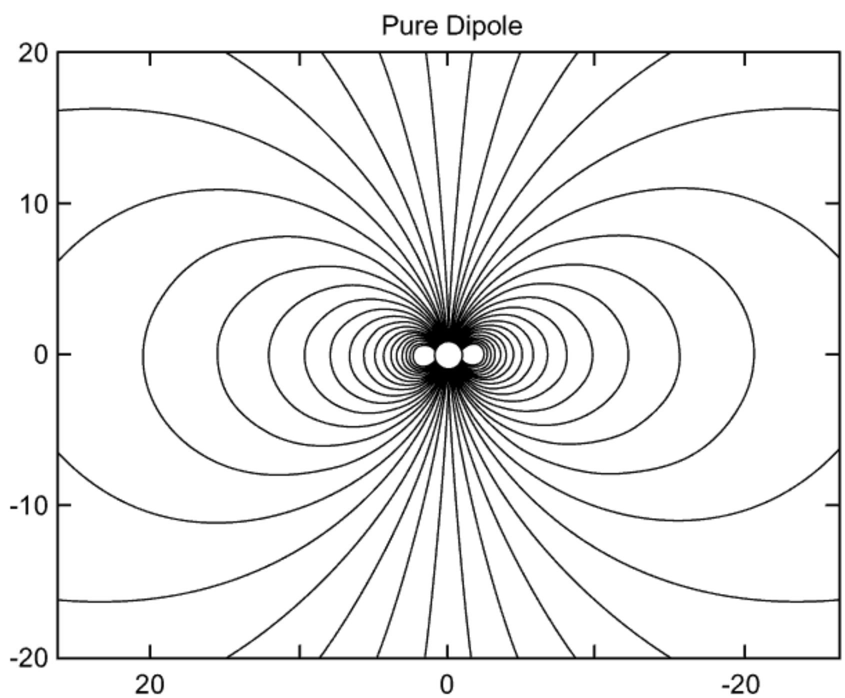

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2009-2010 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 9 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers. For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

To suggest math problem or science topic ideas, contact the Author, Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

Figure credits: Front Cover: Solar Magnetic Field Color representation of a three-dimensional model of the solar corona during August and September 1996 (SOHO: Ultraviolet Coronograph Spectrometer) Jovian magnetic field and inner satellite orbits (Courtesy John Spencer, Lowell Observatory). Solar prominence (NASA/TRACE); **Back Cover:** Diagram of magnetic field lines of Earth modeled by Dr. Gary Glatzmaier (Los Alamos) and Paul Roberts (UCLA).

This booklet was created under EPOESS-7 education grant, NNH08CD59C through the NASA Science Mission Directorate, and in collaboration with the following NASA Education and Public Outreach programs: IMAGE, Hinode, and THEMIS.

Table of Contents

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Note: An extensive, updated, and cumulative matrix of problem numbers, math topics and grade levels is available at

http://spacemanth.gsfc.nasa.gov/matrix.xls

Alignment with Standards (AAAS Project:2061 Benchmarks).

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1

(6-8) Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1

(9-12) - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry		X	X	X		X		X		X		X	X		X	X		X				X	X				X	X	X		X
Protractors, rulers, etc					X	X			X		X	X																			
Numbers, patterns, percentages	X											X		X	X	X						X									
Averages																															
Time, distance, speed											X		X																		X
Areas and volumes																									X						
Scale drawings			X		X	X	X			X						X						X									
Geometry					X	X	X	X	X	X	X						X	X	X					X							
Probability, odds																															
Scientific Notation																									X		X	X	X	X	X
Unit Conversions	X		X								X															X	X				
Fractions																															
Graph or Table Analysis					X						X	X	X	X	X																
Solving for X												X							X												X
Evaluating Fns		X	X	X															X	X	X			X		X	X	X	X	X	X
Modeling						X																									
Trigonometry							X	X			X																				
Pythagorean Theorem					X		X										X	X	X				X	X							
Vectors					X	X	X	X	X						X	X	X	X	X		X	X	X	X							
Gradients																							X								

Mathematics Topic Matrix (cont'd)

Topic	31	32	33	34	35	36	37			
Inquiry	X				X		X			
Protractors, rulers, etc							X			
Numbers, patterns, percentages										
Averages										
Time, distance, speed							X			
Areas and volumes		X				X				
Scale drawings							X			
Geometry										
Probability, odds										
Scientific Notation			X	X	X	X				
Unit Conversions										
Fractions										
Graph or Table Analysis										
Solving for X					X					
Evaluating Fns	X	X		X	X	X				
Modeling										
Trigonometry										
Pythagorean Theorem										
Vectors										
Gradients										

How to use this book

General Approach

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. NCTM Process Standards for grades k-12- Connections should enable all students to:

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

When instruction focuses on a small number of key areas of emphasis, students gain extended experience with core concepts and skills. Such experience can facilitate deep understanding, mathematical fluency, and an ability to generalize.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Magnetic Math**. Read the scenario that follows:

Ms. Green decided to enhance her magnetism activities this year by using the Magnetic Math book. She used most of the activities beginning with basic magnetism, even though her students seem to understand the basics of attracting and repelling of dipole magnets she decided that playing with objects and discovering the types of materials that are attracted to a magnet was a good review. She had the students use several magnets to map the magnetic influence and determine how the influence of several magnetic field interact. Then she took the students into the world of magnetic units of measure and watched the excitement of new discovery and the use of base 10.

Magnetic Math can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. The book was developed using related ideas, concepts, skills, and procedures that form the foundation for understanding and using mathematics and lasting learning.

Examples of Topic Strands

The problems in this book have been created to highlight interesting aspects of magnetism using mathematical exercises that are appropriate to the typical on-grade-level student taking science classes. This means that students in 8th grade will be expected to be familiar with topics in pre-algebra mathematics including working with decimals, fractions, scientific notation, statistics, basic triangle geometry and equations in one variable. Students in 9th and 10th grades will be expected to be familiar with concepts in Algebra I and Geometry, including trigonometry, working with equations with integer and fractional exponents, and unit conversions. Students in grades 11 and 12 will be expected to know Algebra I, Geometry and Algebra II basic content, and some additional advanced math applications not involving calculus.

Grade 6-8

Students will have had a basic exposure to the concept of magnetism through hands-on experiments such as those suggested in Labs 1-4, which allow students to map magnetic fields via compass or iron filings to show the standard polar geometry. To go beyond merely drawing magnetic fields, students must be introduced to their measurement, **Problem 1**, the two common units used (Gauss and Tesla) and their conversions. By organizing various phenomena in terms of their magnetic strengths, they appreciate how familiar things (e.g. Toy Magnets) stand in relation to other known magnetic systems much as a temperature scale or other physical scales. **Problems 5-18** and **Problems 22-23**, provide an introduction to graphing magnetic fields in 2-dimensions using nothing more complicated than a compass, protractor, rectangular grid paper and an application of the Pythagorean Theorem. These activities highlight the vector nature of the magnetic field, but do not mention the term 'vector' which can be an intimidating concept in this grade range. The proper treatment of vectors is customarily held off until Grade 9 Geometry. In **Problems 11-14**, students come into contact with the concept that Earth's magnetic field changes in time, and use a variety of graphical data to estimate the rate of change (slope) of this change, and use this to make future forecasts.

Grade 9-12

Students explore magnetic forces using more sophisticated mathematical tools. **Problems 3-4** and **Problems 20-21** assume that students can work with algebraic equations in more than one variable, and with both integer and fractional exponents to evaluate the strength of a magnetic field in terms of its mathematical description from a theoretical model. The mathematical description in terms of a dipole expresses algebraically, the Pythagorean Theorem in 3-dimensions, and the physical concept of magnetic pressure are used to define and explore various astronomical systems containing plasma and magnetic fields. **Problems 26-29** take a step-by-step approach in introducing magnetic pressure, first as an isolated property, then as a property combined with the gas pressure in a plasma, to study the equilibria of various systems such as sunspots and interstellar clouds.

508-Compliance

On August 7, 1998, President Clinton signed into law the Rehabilitation Act Amendments of 1998 which covers access to federally funded programs and services. The law applies to all Federal agencies when they develop, procure, maintain, or use electronic and information technology. Federal agencies must ensure that this technology is accessible to employees and members of the public with disabilities to the extent it does not pose an "undue burden." Section 508 speaks to various means for disseminating information, including computers, software, and electronic office equipment.

Each Federal agency implements this requirement differently. NASA's Education Program requires that all websites and PDF documents be readable using a text-to-audio interpreter. **Magnetic Math** technically complies with this requirement as a PDF file, but the user should be forewarned than no single document can authentically and realistically serve both the needs of sighted and sight-impaired students. This math guide incorporates many elements, including complex equations and colored images, that are difficult to convert into a useful experience for sight-impaired students. It is possible, however, for sight-impaired students to have an acceptable qualitative experience of this topic depending on their special needs and classroom resources.

SI, MKS and CGS units

Were applicable, we use the units that are commonly used in the various scientific specialties, and which make the concepts clear. For example, all astronomical applications of magnetism use the centimeter-gram-second (CGS) units because this is the unit standard adopted by this scientific discipline for the last century, and which is used in all of its extant literature. This may cause some concern to elementary students of physics who are more familiar with the meter-kilogram-second (MKS) system. Also, some topics such as magnetic pressure have a far more direct and simple formulation in the CGS system than in the MKS system, so that the former is preferred to help students more clearly see, and remember, the underlying relationships. For example:

If magnetic field B is expressed in Gauss (CGS) units: $Pm = \frac{B^2}{8\pi}$ dynes/cm^2

If magnetic field B is expressed in Tesla (MKS) units: $Pm = \frac{B^2}{2\mu_0}$ Newton/m^2

where μ_0 is a constant called the *Permeability of Free Space* and has a value of $4\pi\times10^{-7}$ Newtons·Ampere^{-2}.

An unreasonable adherence to MKS over CGS can lead to added complexity and confusion. Students entering science and engineering need to be facile in converting between the MKS and CGS systems to accommodate the deep historical range of the research literature in various fields. Moreover, 1 tesla is a HUGE field intensity, far larger (by x100 or x1000) than any you will measure in the classroom. **For teachers who find the use of the CGS system problematical in this guide, you may challenge your students to convert all CGS quantities to MKS.**

Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School, SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

"Space Math has more up-to-date applications than are found in any textbook. Students enjoy real-world math problems for the math they have already learned. Doing Space Math problems has encouraged some of my students to take pre-calculus and calculus so they can solve the more advanced problems. I learned about Space Math through an email last year. I was very impressed with the problems. I assigned some of the problems to students in my Physics classes, printing them out to put in their interactive notebooks. I displayed other problems for group discussion, assigned some for homework and used some for group class work. I like the diversity, the color format and having the solutions. I expect to use them even more next year in our new space science class. We will have 50 students in two sections." (Alan, High School Science Teacher)

"It took time for them to make the connection between the math they learned in math class and applying it in the science classroom. Now I use an ELMO to project them. I have used them for class work and/or homework. The math activities were in conjunction with labs and science concepts that were being presented. The math helped "show" the science. Oftentimes students were encouraged to help and teach each other. Students began to see how math and science were connected. I knew the students were making the connections because they would comment about how much math they had to do in science. Their confidence in both classes increased as they were able practice the concepts they learned in math in my science class." (Brenda, Technology Resource Teacher)

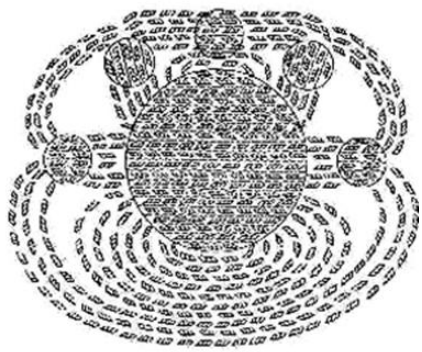
1.0 A Short Introduction to Magnetism

Many NASA resources describe the basics of magnetism, especially as it applies to earth's magnetic field and solar activity (see 'NASA Resources on Magnetism', page 40).

Magnetism is an ancient discovery. The earliest recorded description of magnetic forces occurred in China in 2637 B .C. when Emperor Hoang-ti's troops lost their way in heavy fog while in pursuit of Prince Tcheyeou. The Emperor constructed a chariot upon which stood a figure that always pointed south no matter how the chariot was pointed. The Greek philosopher Thales of Miletus (640-546 BC) is also credited with having conducted a careful study of lodestone and its magnetic properties, but this did not include a knowledge of magnetic polarity or its directive properties within earth's magnetic field - the basis for a true compass.

At the time of Columbus, magnetic compasses for navigation had been a standard technology for at least several centuries, but it was on Columbus's first voyage in 1492 that he discovered the needle didn't point to True North (Pole Star) in some locations. In fact, the deviation was as high as 10 degrees west of True North. To avoid an impending mutiny, it is claimed that Columbus altered the compass card to match the direction of the needle. This was very risky thing to do, because a nautical rule on the book stated that the penalty for tampering with a compass was that *"the hand which is most used would be fastened to the mast by a dagger thrust through it".* (Fleming, pp. 2)

Substantial work on magnetism, particularly terrestrial magnetism, was described in 1600 by Dr. William Gilbert. In an introduction to his book *De Magnete*, Gilbert debunks many of the older ideas of the causes and properties of magnetism. He attacked alchemists for their obscure language, and put many of the legendary claims for lodestone to direct experimental tests. One of these was that lodestone's power, dulled at night, could be restored by a bath in goat's blood. One of his most famous discoveries is that the earth is, itself, a magnet, which is why mariner's compasses work. He was the first to distinguish between magnetic and electrical attraction, and is credited with coining the term **electricity**. It was Descartes who ultimately made 'intangible and invisible' magnetic forces visible to the naked eye by inventing the iron filing method. He presented this technique in his *Principles of Philosophy* published in 1644, explaining that, *"The filings will arrange themselves in lines which display to view the curved paths of the filaments around the magnet...".*



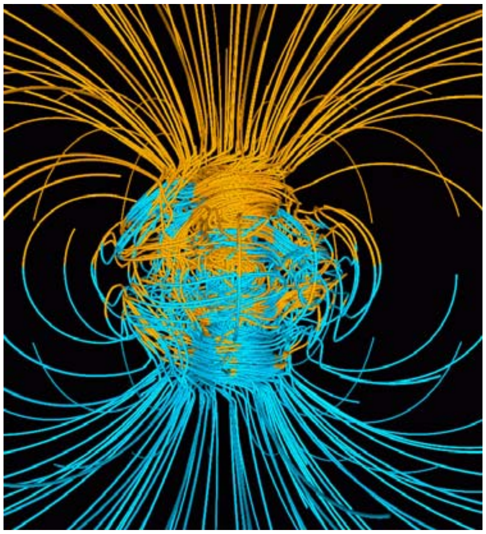
Descarte's drawing of magnetic lines of force

The pattern revealed by the iron filings vividly illustrated that something extremely well organized existed beyond the surface of the magnet, which was perhaps the origin of the magnetic force itself. A compass works the way it does because Earth has a magnetic field that looks a lot like the one in a magnet. The Earth's field is completely invisible, but it can be felt by a compass needle on Earth's surface, and it reaches thousands of miles out into space.

2.0 The Earth's Magnetic Field

In your classroom, you can make a magnetic field by letting a current flow through a piece of wire wrapped around a nail. When you attach the battery, the nail becomes an **electromagnet** and you can use it to lift paper clips.

Geologists are convinced that the core of the Earth is also an electromagnet. The powerful magnetic field generated by a dynamo process in the liquid outer core of Earth. The magnetic force passes out through Earth’s core, through the crust that we stand upon with our compasses, and enters space. This picture created by a computer from a mathematical model, shows the lines of force colored gold for a south-type polarity and blue for a north-type. By the time the field has reached the surface of Earth, it has weakened a lot, but it is still strong enough to keep your compass needles pointed towards one of its poles. As you recall from grade-school science classes, magnets have two POLES: a North Pole and a South Pole. Scientists call this a **dipolar field**.



A mathematical model of Earth's magnetic field near the core. (Courtesy: Gary Glatzmaier)

There is another thing that we know about magnets and magnetism: When you put like poles together (South facing South or North facing North) they repel each other. You can feel this force of repulsion yourself! When you put unlike poles together (South facing North) you can feel magnetic attraction. In the Northern Hemisphere, your compass needle points North, but if you think about it for a moment, you will discover that the magnetic pole in the Earth's Northern Hemisphere has to be of a South polarity. This is so, because the North-type magnetism of the compass needle has to be attracted by a South-type magnetism in the Earth in order to 'seek out' the North Pole.

A common way to refer to magnetic fields is by using the term '**magnetic lines of force**', which comes from seeing iron filings linked together to form such lines of magnetism. In fact, there are actually no such lines 'painted onto space' in the region surrounding a magnet. At each point in space, a magnetized object will act like a compass needle with one end becoming a North and one a South pole. A close-by object will also orient itself that way, so that if these objects were individual iron needles they would appear to form a line in space. In places where the field is strongest, these magnetized objects will be crowded together giving the appearance of lines crowded together. This is such a useful, and intuitive, way to model the intensity and orientation of the magnetic field in space that physicists use lines of force as a helpful way to mathematically describe magnetic fields. It can, however, lead to a severe misconception if you do not remind yourself that the lines are not actually real.

3.0 Magnetic fields in the Universe

For thousands of years, mariners have used the Earth's magnetic field as a compass to find their way to safe harbor. The Earth's field looks very much like the magnetic field of a common bar magnet. The axis of the field is tilted by about 11.5 degrees to the axis of rotation of Earth. It is a mystery why this is so.

No one knows why, but these kinds of offsets between magnetic and rotational axis are found among the magnetic fields of some of the other planets shown in the table on the next page.

Planet	Tilt (degrees)	Dipole Moment	Fluid
Mercury	~14	6×10^{12}	Fe-Ni
Earth	11.5	8×10^{15}	Fe-Ni
Jupiter	10	1.66×10^{20}	Metallic H
Saturn	Co-axial	4.6×10^{18}	Metallic H
Uranus	59	3.9×10^{17}	Unknown
Neptune	47	2.16×10^{17}	Unknown

Note: The Dipole Moment is in units of Tesla meter^3

The sun and planets in our solar system are not the only bodies known to have magnetic fields. Astronomers have been able to determine that some dark, interstellar clouds several light years across may be partially supported against gravitational collapse by internal magnetic fields. These fields are a thousand times weaker than Earth’s magnetic field, but fill up a volume of space many cubic light years in size.

Astronomers have also detected magnetic fields within clouds of plasma ejected by massive black holes in the cores of some galaxies. The image below of the giant radio galaxy Cygnus A spans 1 million light years from edge-to-edge and shows a pair of gas clouds supplied by magnetically-focused beams from the core of the galaxy, seen only as a spot of light at the center of the image.

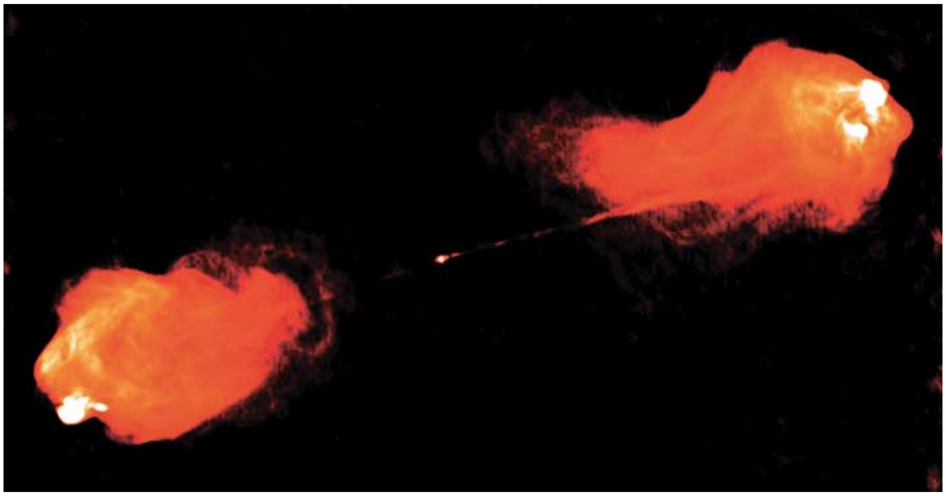


Image of the radio galaxy Cygnus A revealing magnetized plasma

4.0 Magnetic Storms and Auroral Activity

Near the poles of Earth, observers have often seen glowing clouds shaped like curtains, tapestries, snakes, or even spectacular radiating beams. Northern Hemisphere observers call them the Northern Lights or the Aurora Borealis. Southern Hemisphere observers call them the Southern Lights or Aurora Australis. Because most people, and land masses, are found north of the equator, we have a longer record of observing them in northern regions such as Alaska ,Canada, Scandinavia, but sometimes as far south as the Mediterranean Sea or Mexico!

In the 1740's, George Graham (1674-1751) in London, and Anders Celsius (1701-1744) in Uppsala, Sweden began taking detailed hourly measurements of changes in the Earth's magnetic declination. The fact that this quantity varied at all was known as early as 1634 by Gellibrand's observation of the 'variation of the (magnetic) variation' (Fleming, 1939). It didn't take very long before Celsius and his assistant Olof Hiorter uncovered in the 6638 hourly readings, a correlation between these disturbances and local auroral activity. Moreover, comparing the records between Uppsala and London, it became quite apparent that the magnetic disturbances occurred at the same times at both locations. By 1805, the independently wealthy, scientific traveler, Baron von Humbolt (1769-1859) had also noted these magnetic disturbances and called them magnetic storms' since they caused the same gyrations of his compass needles as local lightning storms would do. During a 13 month period, Humbolt and his assistant also made thousands of half-hourly readings of a compass needle just as Celsius and Hiorter nearly 100 years earlier. They would peer into a microscope at a needle on a graduated scale, little more than an ordinary compass. At half-hourly intervals, day and night, the position of the needle would be noted. By the 1850's networks of observatories amassed millions of these observations.

These 'magnetic storms' are spawned by major Coronal Mass Ejections (CMEs) from the sun. If Earth has the misfortune of being in the 'right' place in its orbit, within a few days, these million kilometer/hour plasma clouds reach the Earth and impact its magnetic field. The momentary compression of the field caused an increase in the field strength at the Earth's surface. Many physical processes are then precipitated as the CME particles and magnetic fields invade geospace causing minute-to-minute changes in the geomagnetic field near ground level. Magnetometers then notice complex field changes which last until the CME plasma passes the Earth and geospace conditions return to normal. Major magnetic storm events also lead to spectacular auroral displays even at low geographic latitudes.

What is Magnetism?

We have all had the experience of using simple magnets to hold notes on surfaces such as refrigerator doors. Magnetism is the force produced by magnets which does all of the "holding". Magnetism is also a very important force in nature which can move hot gases in stars, and in the space around the earth. In this laboratory activity, students will investigate magnetism and magnetic forces. The students will explore the attracting and repelling properties of magnets through hands on experiences.

Materials:

Magnets (enough for class); Paper clips; String; Books; Ruler; Learning Log

Objectives:

- The students will investigate that magnets are attracted to items that contain metals such as iron.
- The students will experience that a magnetic force is an invisible force.
- The students will explore magnets attracting and repelling properties.

Procedure:

- Give each student a magnet. Have the students explore the objects that the magnet would be attracted to. The students should look at the objects and find common characteristics. The students should record their findings in a Learning Log.
- Tape one end of a piece of string to a desk; tie the other end onto a paper clip. Take a second piece of string and suspend the magnet from a ruler anchored with books. Adjust the level of books so that the distance between the magnet and the paper clip allows the clip to stand up without touching the magnet. The students should see that a magnetic force could be invisible. You can place pieces of paper or cloth between the clip and the magnet to show the strength of the magnetic force. Can the students find materials that block magnetic forces?
- With the string still attached, have the students try to raise the paper clip from the desk with a magnet. They should try to accomplish this without letting the magnet and paper clip touch. The students should keep a log of how they were able to accomplish this; what methods and strategies were used.
- Allow the students time to explore the attracting and repelling properties of magnets. They should be able to demonstrate that a magnet has two ends or poles that will attract or repel from other poles. Have the students observe what happens when two magnets are repelling from each other. The students should find a partner and discuss what they have seen and whether their classmate was able to discover the same properties.
- Have students complete the form on the reverse to test their understanding of how common magnets and magnetism are at home and elsewhere.

Conclusions:

1) The students will learn the elementary characteristics of magnetism. 2) The students will demonstrate the attracting and repelling properties of magnets.

Key Terms:

Magnet - a metal that can attract certain other metals.

Magnetic Properties - refers to an item that can attract or repel items like a magnet.

Poles - refers to the two areas of a magnet where the magnetic effects are the strongest. ***Polarity*** - The poles are generally termed the north and south poles. Poles that are alike (both north or both south) will repel each other, while poles that are different (one north, one south) will attract each other.

A Magnetic Treasure Hunt!

Magnets and magnetic forces are very common, though sometimes not very obvious, in the world around you. How many different kinds of magnetic devices, or phenomena can you identify in your home, school, city, country and in nature? Create a list, and compare it with your classmates!

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What are Magnetic Fields?

In physical science, a "field of force" is a region or space in which an object can cause a push or pull. Magnetic forces are felt around the entire magnet. The region in which the magnetic forces can act is called the magnetic field. Magnetic lines of force define the magnetic field. The students will explore the lines of force of magnets and compare it to the lines of force on the sun and the earth**.**

Materials:

Strong Magnets (enough for class or small groups); Plastic wrap; Iron filings; Paper (white); Plastic teaspoon; Plastic tray; Compass; Photograph of sunspot magnetic loops; Learning Log.

Objectives:

- The students will explore the magnetic field lines of a magnet.
- The students will investigate the magnetic field lines between two attracting and two repelling magnetic poles.
- The students will learn that the earth and the sun have magnetic properties.

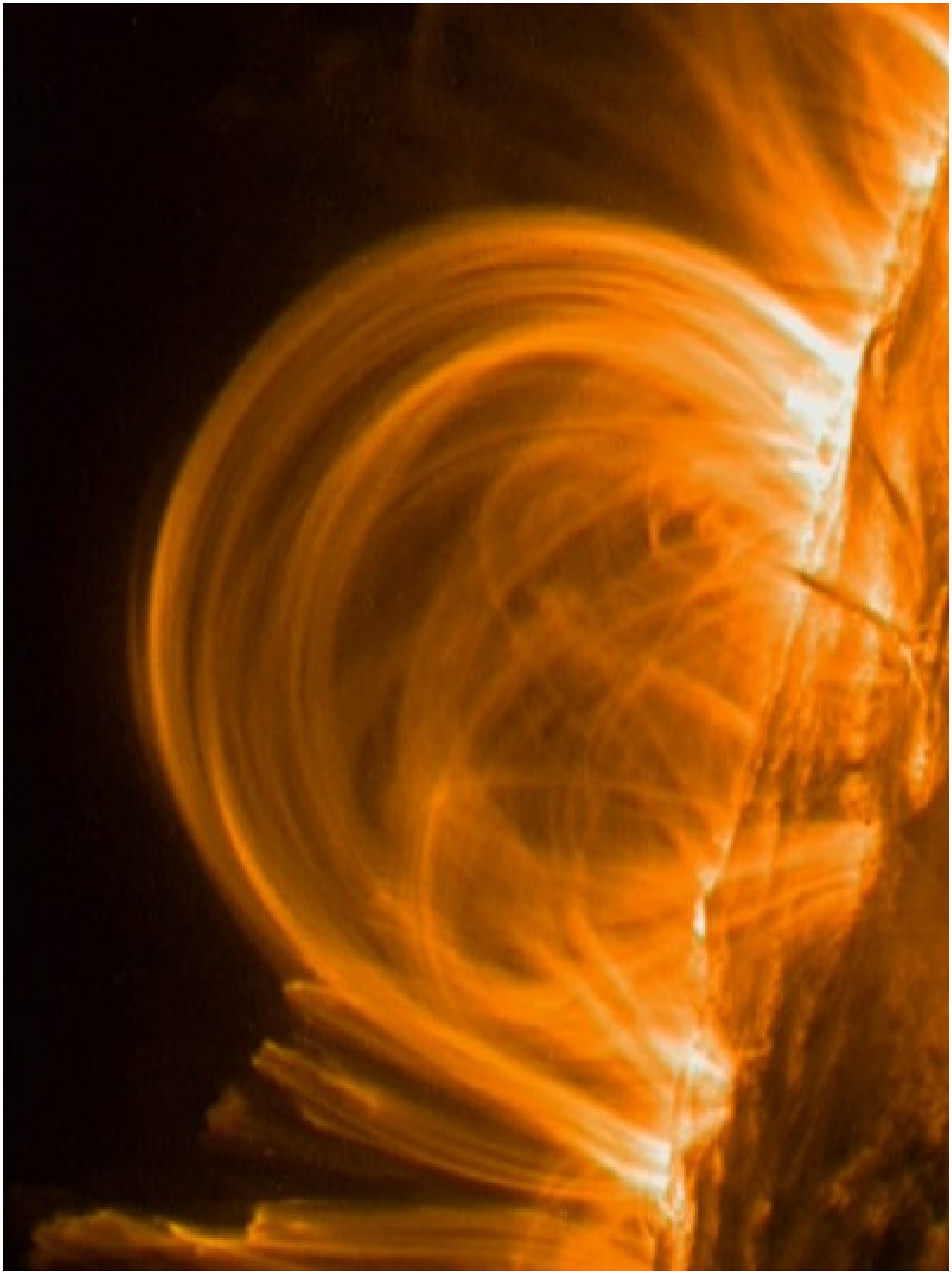
Procedures:

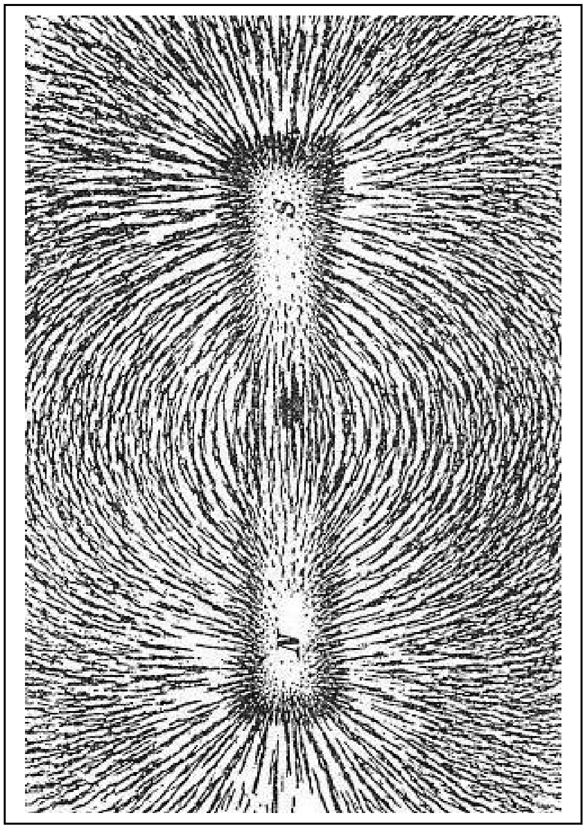
****Caution the students that the iron filings should not be eaten or blown into eyes. ****

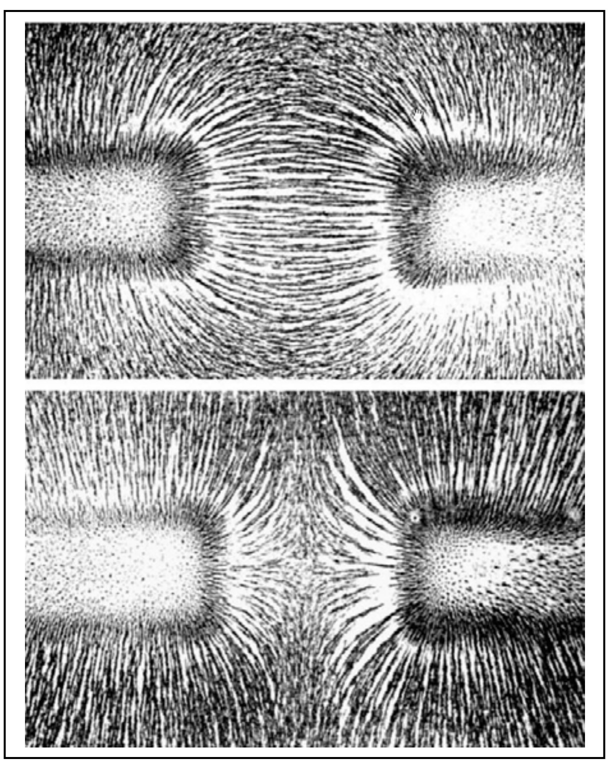
- Cover the magnets with plastic wrap to keep the iron filings off them. Place the covered magnet in the plastic tray and place the paper on top. The students should carefully use the spoon to sprinkle a small amount of the iron filings on the paper. The iron filings will stay in a pattern that indicates the lines of force of that magnet. The students should draw their observations in their learning logs. After the students have completed their observations, the iron filings can be poured off the paper and the tray back into the container.
- Give each group of students a pair of covered magnets. Place the covered magnets about 3 cm apart in the plastic tray and place the paper on top. The students should carefully sprinkle a small amount of the iron filings on the paper. The iron filings will stay in a pattern that indicates the lines of force between the magnets. The students should look at the lines of force and determine whether the magnetic poles are alike or different. Have the students record their observations in their Learning Logs.
- Have the students repeat the activity of finding lines of force, but this time one of the magnets must be reversed so that its opposite pole is about 3 cm away from the other magnet. The students should look at the lines of force and determine whether the magnetic poles are alike or different. The students should record their observations in their learning logs.
- Display the photograph of magnetic loops on the Sun's surface without informing the students of the source. Question the students about what they observe in the photograph. The image should resemble the magnetic lines of force the students saw in the previous activity. The students, as scientists, should understand that they are seeing magnetic properties on the Sun. Discuss with the students what other property the shapes on the Sun need to share with a magnetic field if they are in fact, magnetic. Answer - They should display a definite North and South polarity as well as loops. Scientists have in fact confirmed this using other observations.
- Discuss the student's observations and update the K-W-L chart with new questions and information.
- Display a compass to the students. Explain that in the Northern Hemisphere the needle of the compass will point to the magnetic north because it is magnetized. When a compass is held on Earth, the Earth's magnetic field exerts a force on the needle. This should help the students understand that Earth also has magnetic properties. If the "north" part of a compass is attracted to the magnetic north pole of the Earth, what is the polarity of the Earth's north magnetic pole? Answer - South, because only magnetic opposites attract one another!

Conclusions:

1) The students will gain an understanding of the presence of magnetic fields around magnets, the Sun and Earth. 2) The students will learn that the magnetic poles attract when they are different and repel when they are the same.







Before students can take the next steps in understanding magnetism, they need to master the 'artistic' technique of rendering magnetic field lines with reasonable accuracy. The upper photograph shows the magnetic field lines rendered using iron filings for an ordinary bar magnet with the South (S) and North (N) polarities indicated. The lower photograph shows the magnetic filings near two magnetic poles that are (top) opposite polarity (N and S) and (bottom) like polarity (S and S).

Materials:

A copy of the photographs to the left; Pencil; Learning Log

Objectives:

- The students will learn how to draw accurate magnetic field line patterns for bar magnets in simple orientations.

Procedure:

- Provide each student with their own copy of the photographs to use as a guide.
- Draw two circles about 1 cm diameter and labeled 'N' and 'S' separated by 5 cm.
- Connect the circles with realistic field lines.
- Draw other pairs of circles with like-polarities and draw the accompanying field lines.
- Challenge: Place 6 circles with 3 'N' and 3 'S' labels in random locations and have students predict what the iron filing picture field will look like.

Conclusions:

The students will learn that fields line drawings can help predict what magnetic fields look like when rendered accurately.

Key Terms:

Magnet Field Line - an imaginary line that describes the shape of the magnetic field.

A Few Notes to the Teacher!

1 - Students should be reminded that the first step to understanding Nature is to make sure you have carefully observed it! A sloppy drawing that is not faithful to what Nature is presenting to your senses only leads to a poor starting point for creating an accurate interpretation. This can cause you to waste a LOT of time.

2 - Magnetic field shapes, revealed by iron filings, are not jagged lines, but smooth, flowing arcs that even the course jagged iron filings cannot do justice.

3 - Students should be encouraged to look at the photographs VERY CAREFULLY and not assume that they 'got the idea' of what the patterns of lines look like just by a quick glance at the photos.

4 - Student sketches should show individual lines as smooth, nearly circular, arcs that leave one point on a bar magnet, and symmetrically connect with a mirror point on the opposite end of the bar magnet. All of the lines must begin and end only on the pole of a magnet, but not the same pole.

5 - Students should notice that field lines do not cross each other or form kinks.

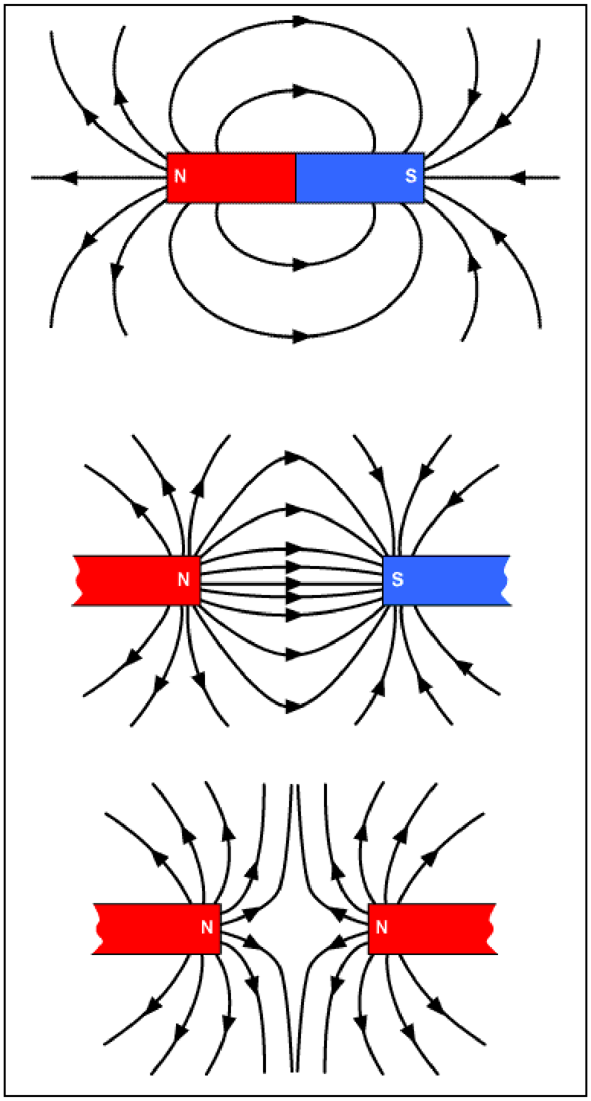
6 - Students studying the bar magnet should notice that if they placed a mirror exactly perpendicular to the mid-point on the bar magnet, that the field lines will look symmetric about this 'mid plane'.

7 - For opposite poles face each other, a field line on one pole will connect with a matching point on the other magnet pole.

8 - When like poles face each other, a line forms mid-way between the two poles where the field lines do NOT cross to the other magnet pole. Instead, the lines bunch up (but do not cross) each other. The lines leave this region by bending back, gracefully, to the opposite pole of the same magnet.

Note also that the photograph of the bar magnet iron filings was from the book 'Practical Physics' published in 1914 by Macmillan and Company. The photograph of the like and opposing poles was from 'A textbook of Physics' by Alexander Duff, published in 1916 by Blakiston's Son and Company, Philadelphia. Since the publication dates are prior to 1928, they are copyright-free and may be reproduced as needed, especially for educational purposes!

How to Draw Magnetic Fields - II



The next step in rendering an accurate model of a magnetic field is to represent the polarity of the field. Every magnetic field line has one end that is the South Polarity and one end that is the North Polarity. No field lines have the same polarity at both ends. By convention, physicists represent magnetic polarity by using arrows that point along the field line in the direction of the South Polarity. The figure to the left shows some common polarity situations.

Materials:

Pencil; Learning Log

Objectives:

- The students will learn how to draw accurate magnetic field line patterns, including polarity, for bar magnets in simple orientations.

Procedure:

- Provide each student with their own copy of the diagram on the left to use as a guide.
- Draw two circles about 1 cm diameter and labeled 'N' and 'S' separated by 5 cm.
- Connect the circles with realistic field lines that have arrows pointed in the correct direction.
- Draw other pairs of circles with like-polarities and draw the accompanying field lines.
- Challenge: Place 6 circles with 3 'N' and 3 'S' labels in random locations and have students predict what polarities are present in different region of the map.

Conclusions:

The students will learn that fields line drawings can help predict what magnetic fields and their polarities, look like when rendered accurately.

Key Terms:

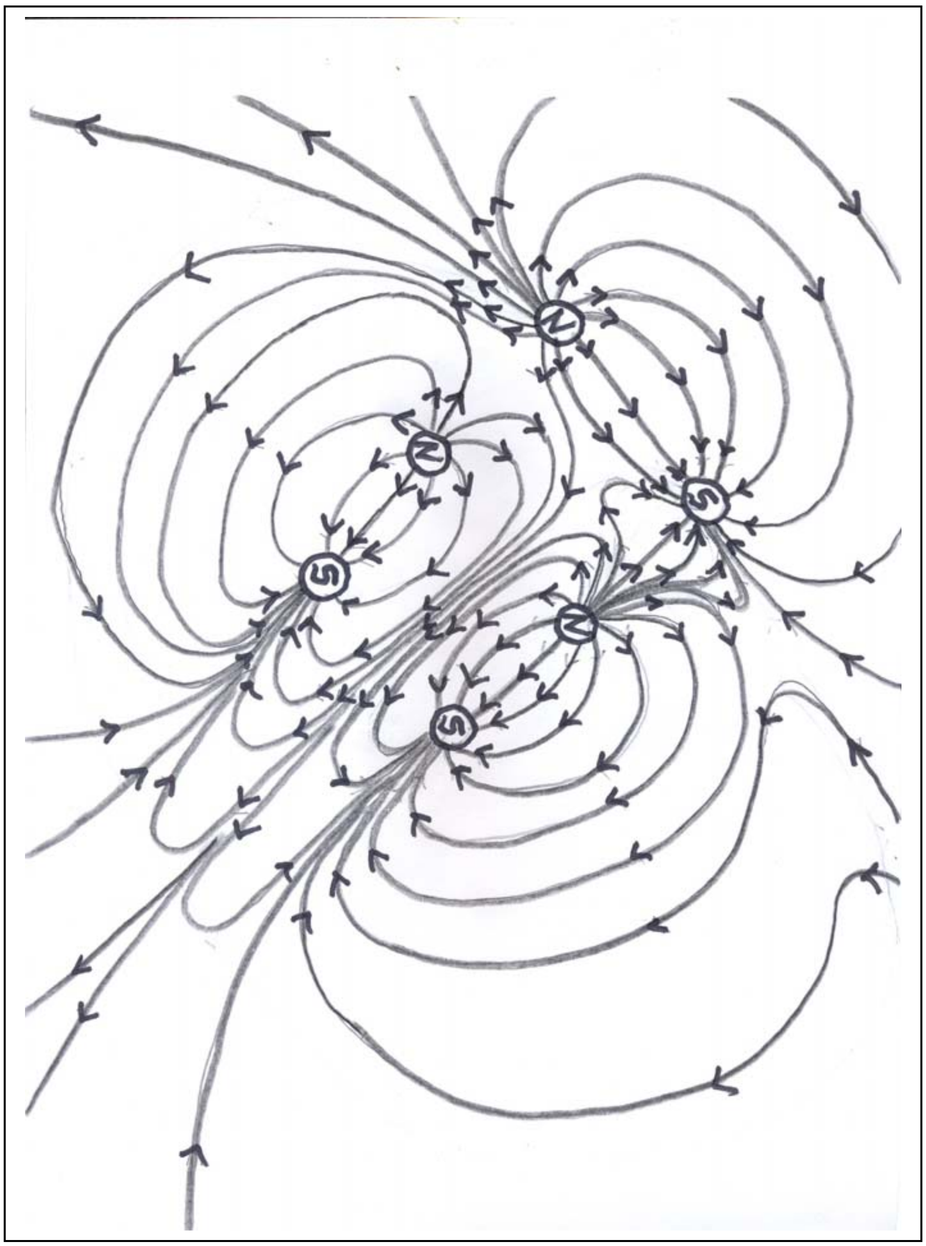
Magnetic Polarity - One of two possible conditions for a region of magnetic field that, by convention, are called North and South.

Notes to Teacher

1 - Because this is a follow-up to Lab Activity 3, students should be able to render a reasonably convincing field line model that represents a hypothetical 'iron filing' picture.

2 - Students should first draw the field lines, then add the arrows in the correct directions afterwards. Students should pick one field line from a magnetic pole, and follow it to its South Pole with the arrow consistently pointing 'South' along the field line.

3 - The figure below shows one such hypothetical field line and polarity diagram based on 6 magnetic poles in the diagram.



A magnetic field diagram (Lab 1-4) can be used to estimate the actual strength and direction of the over-all magnetic field of a system. The strength of the magnetic force is proportional to the density of the magnetic field lines in 3-dimensions. The more magnetic field lines that are present in a given volume (area) of space, the more intense will be the expected magnetic forces. In this activity, students will estimate on a 2-dimensional plane, whether they would predict strong, medium or weak magnetic forces at various points in their diagram.

As an option, students may also use crayons with three different colors to fill-in the space in their diagram to represent the three magnetic force intensities.

Materials:

Pencil; Optional crayons or colored pencils; Learning Log

Objectives:

- The students will learn how to translate simple field line patterns into a qualitative measure of magnetic force strength.

Procedure:

- Have each student construct a simple field line drawing based on four poles.
- Students will identify all of the regions with a high density of lines and marks these as 'Strong' regions.
- Students will then identify all of the 'Weak' areas with few magnetic field lines.
- Students will then decide at what level their field line patterns represent 'Medium Strength' forces.

Conclusions:

The students will learn that fields line drawings can help predict what intensity of magnetic forces will be experienced in different regions of space.

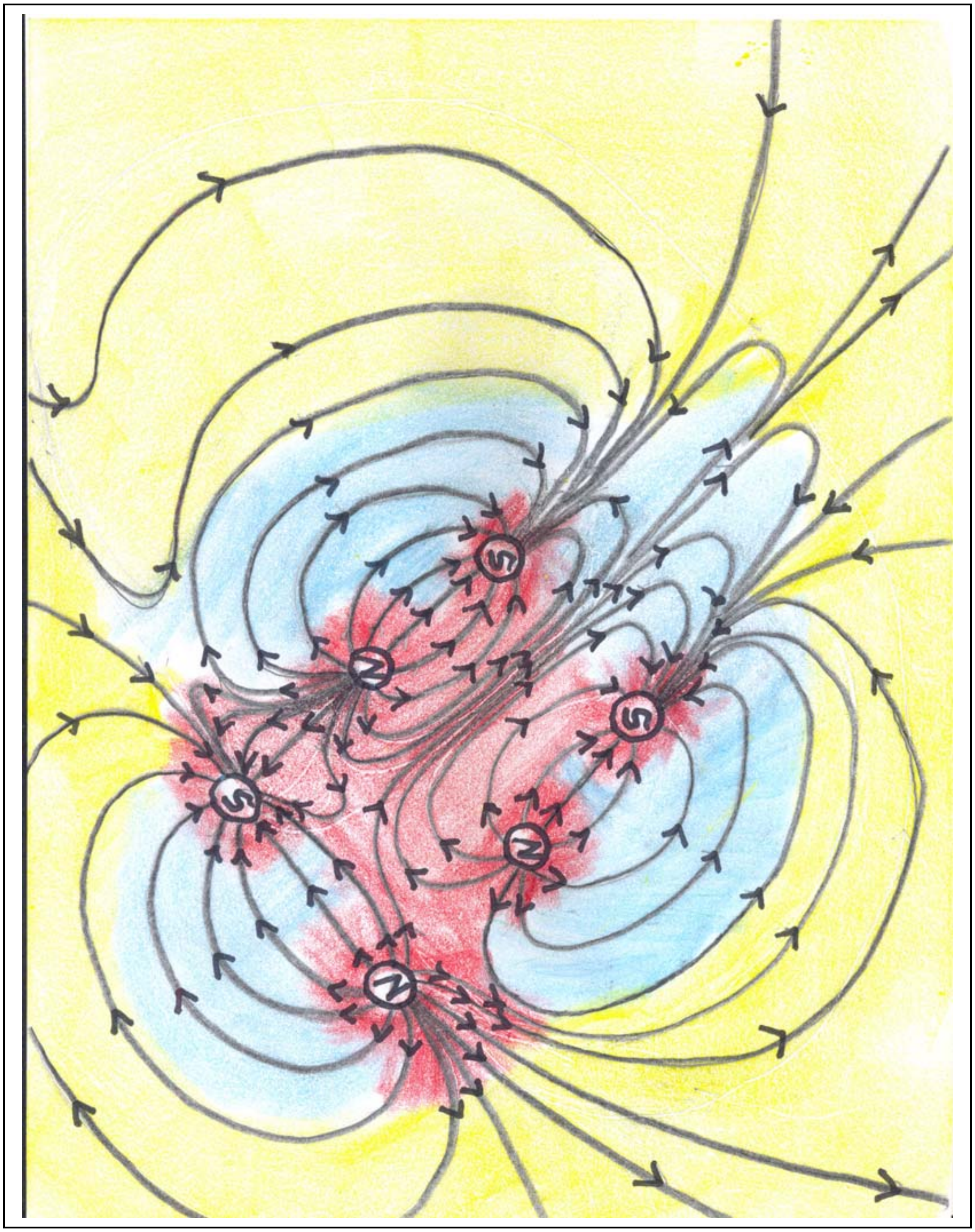
Key Terms:

Magnetic Field Line - An imaginary mathematical line that indicates the direction and polarity of a portion of a magnetic field.

Magnetic Field Strength - The number of field lines that pass across a given surface area.

Example of a Magnetic Field Strength Map

The figure below is an example of a magnetic field strength (magnetic force) map rendered as directed.



What is the Magnetosphere?

Scientists call the region surrounding the Earth where its magnetic field is located, the **magnetosphere**. When the solar wind sends it streams of hot gases (plasma) towards Earth, the magnetosphere deflects most of this gas. Students will use hands-on experiences to learn about the magnetosphere (the magnetic field surrounding Earth). They will learn how the solar wind (the stream of electrically conducting plasma emitted by the sun) interacts with the magnetosphere.

Materials:

Magnets– strong polarity bar magnet (enough for groups if possible); Plastic wrap; Iron filings; Plastic salad tray or aluminum tray; Straws

Objectives:

- The students will use models to learn about Earth's magnetosphere.
- The students will use models to learn how the solar wind interacts with the magnetosphere.

Procedure:

What protects the Earth?

- The Earth has a protective cover called the magnetosphere. It works as skin does on your body to keep out harmful things. Students can observe a model of the magnetosphere using magnets and iron filings. To keep your bar magnet clean, wrap it in plastic wrap with tape around it, or put contact paper around it. Place a bar magnet under a plastic salad tray or aluminum tray. Sprinkle some iron filings onto the tray from a distance of about 10 inches. Observe the pattern made by the iron filings held in place by the forces between the opposite poles of the magnets. Earth's magnetosphere can be modeled by blowing softly through a straw towards the magnetic field lines. A squishing of the field lines on one side of the model shows how Earth's magnetosphere looks. Have the students draw the model of Earth's magnetosphere in their learning logs.

What happens when the solar wind approaches Earth's magnetosphere?

- Students can observe the way water flows around a stone as a pattern of the solar wind as it flows around the earth.
- Place the bar magnet under a plastic tray or aluminum tray. Place a small button directly above the center of the magnet to model the earth. Sprinkle the iron filings along the edge of one side of the tray covering the magnet. Softly blow the filings toward the button through a straw. Caution the students to blow carefully so that no iron filings get into eyes or mouth! Depending on the force used in blowing, the filings will be trapped in the magnetic lines of force. Compare this to the trapping of the solar particles by the Earth's magnetosphere. Have the students draw the model of the effects of the solar wind on the earth's magnetosphere.

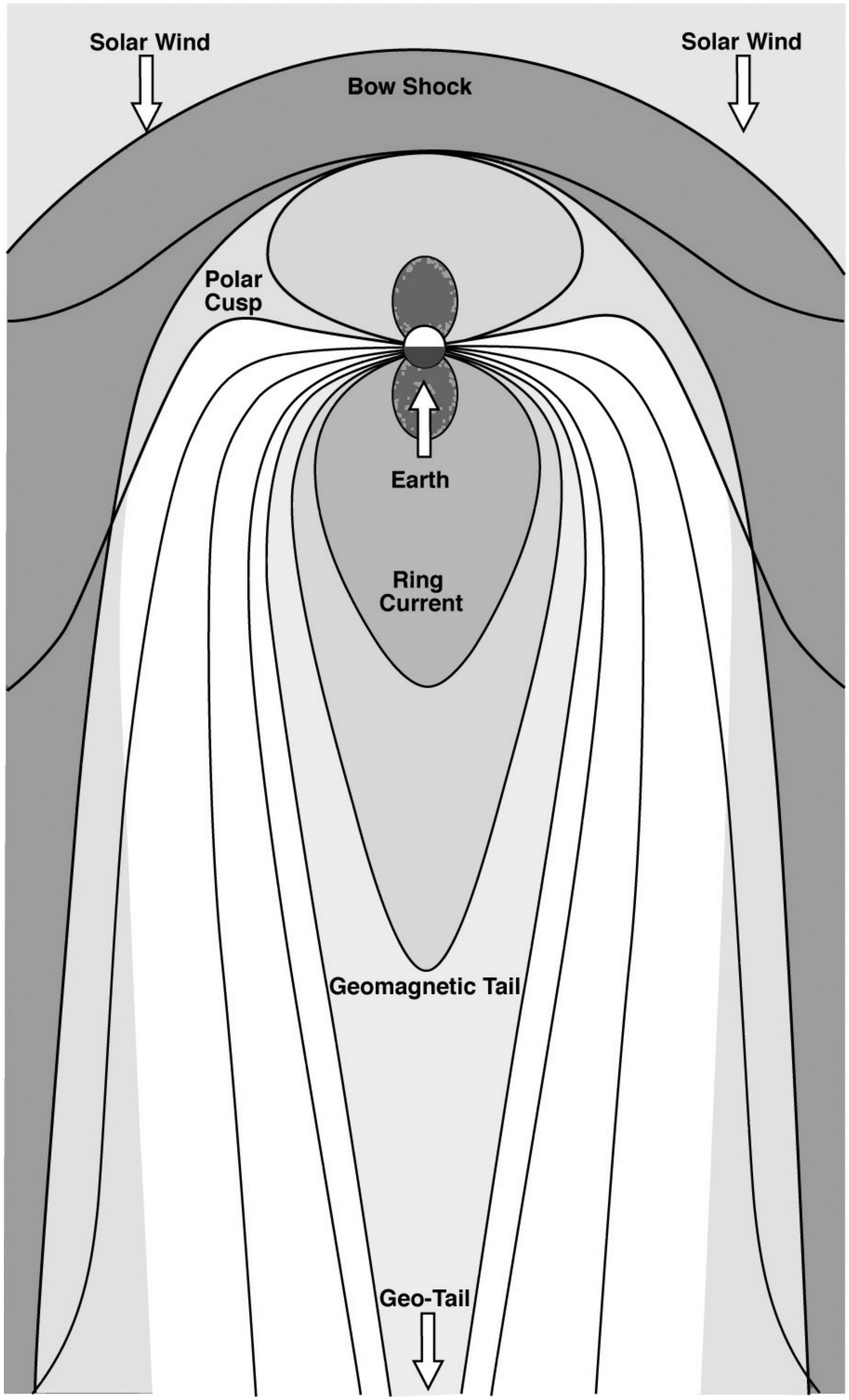
Conclusions:

The students will gain an understanding of Earth's protective magnetic field, called the magnetosphere. The students will gain an understanding of how Earth's magnetosphere interacts with the charged plasma sent from the sun in solar wind and CMEs.

Key Terms:

Magnetosphere – magnetic cavity carved out by the solar wind by virtue of the magnetic field surrounding Earth.

A Reference Figure Showing the Magnetosphere



Magnetic Units of Measure

Object	Strength
Electron Microscope	10,000 G
Cosmic Field	1 picoG
Sunspot	5 kiloG
Fluorescent Lamp	0.1 G
NHMFL magnet	45 T
Earth	700 milliG
Computer Monitor	0.25 mG
LHC accelerator	65,000 G
Toy Magnet	0.3 milliT
Hair Dryer	400 mG
Medical MRI	3 T
Sunlight	3 microT
Magnetar	1,000 terraG
Interstellar Cloud	10 milliG
Solar Surface	5 G
Milky Way	0.000005 G
Neodymium Magnet	12 kiloG
Lodestone	0.001T
Jupiter	0.1 T
Tokamak Reactor	25 G
Pulsar	1 gigaG
White Dwarf	300 T
Super Nova	100 G
Research magnets	40 T
Research magnets	850 T
Solar Wind	20 nanoT
Brain Neuron	.05 picoT

Helpful prefixes:

Terra	10^{+12}	1,000,000,000,000
Giga	10^{+9}	1,000,000,000
Mega	10^{+6}	1,000,000
Kilo	10^{+3}	1,000
Milli	10^{-3}	0.001
Micro	10^{-6}	0.000001
Nano	10^{-9}	0.000000001
Pico	10^{-12}	0.000000000001

Magnetic fields can be measured just like other physical quantities such as mass (grams, kilograms), length (centimeters, meters), force (dynes, Newtons) and energy (ergs, joules). Magnetic field strengths are measured in units called Gauss or Teslas, but the choice of which units to use depends on the scientific discipline, so scientists are facile in converting from one unit to the other.

A magnetic field with a strength of 10,000 Gauss also has a strength of 1 Tesla. We can also convert into decade units. For example, 100 kiloGauss equals 10 Tesla, and 1 microTesla equals 10 milliGauss. It is also much easier to remember that Earth's magnetic field has a strength of 0.5 Gauss than 0.00005 Teslas.

The table to the left shows a variety of different objects along with their magnetic field strengths, but not ordered according to either increasing or decreasing field strength. The strength is provided either as Gauss (G) or Teslas (T). Typically, each area of physics or engineering adopts the unit of magnetism most convenient to the objects being studied.

Problem 1 - Choose either the Tesla or the Gauss scale, and convert all of the numbers into the correct decimal value.

Problem 2 - Order the objects from strongest to weakest magnetic field strength.

Problem 3 - What is the ratio of the most intense to the weakest magnetic field in the list; A) written as a decimal? B) Written in scientific notation?

Problem 4 - What is the range of natural magnetic fields compared to human-created magnetic fields?

Answer Key

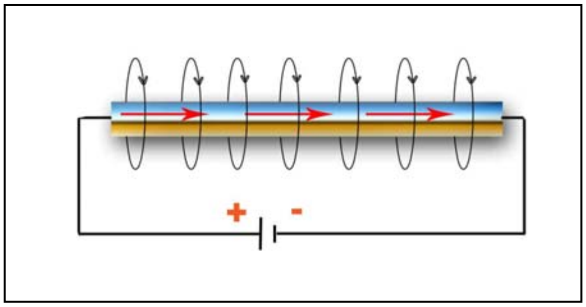
Object	Strength	Gauss units	Tesla units	Rank
Electron Microscope	10,000 G	10,000	1	10
Cosmic Field	1 picoG	0.000000000001	0.0000000000000001	25
Sunspot	5 kiloG	5,000	0.5	11
Fluorescent Lamp	0.1 G	0.1	0.00001	20
NHMFL magnet	45 T	450,000	45	5
Earth	700 milliG	0.7	0.00007	18
Computer Monitor	0.25 mG	0.00025	0.000000025	23
LHC accelerator	65,000 G	65,000	6.5	7
Toy Magnet	0.3 milliT	3	0.0003	17
Hair Dryer	400 mG	0.4	0.00004	19
Medical MRI	3 T	30,000	3	8
Sunlight	3 microT	0.03	0.000003	21
Magnetar	1,000 terraG	1,000,000,000,000,000	100,000,000,000	1
Interstellar Cloud	10 milliG	0.01	0.000001	22
Solar Surface	5 G	5	0.0005	16
Milky Way	0.000005 G	0.000005	0.0000000005	24
Neodymium Magnet	12 kiloG	12,000	1.2	9
Lodestone	0.001T	10	0.001	15
Jupiter	0.1 T	1,000	0.1	12
Tokamak Reactor	25 G	25	0.0025	14
Pulsar	1 gigaG	1,000,000,000	100,000	2
White Dwarf	300 T	3,000,000	300	4
Super Nova	100 G	100	0.01	13
Research magnets	40 T	400,000	40	6
Research magnets	850 T	8,500,000	850	3
Solar Wind	20 nanoT	0.0002	0.00000002	24
Brain Neuron	.05 picoT	0.0000000005	0.00000000000005	26

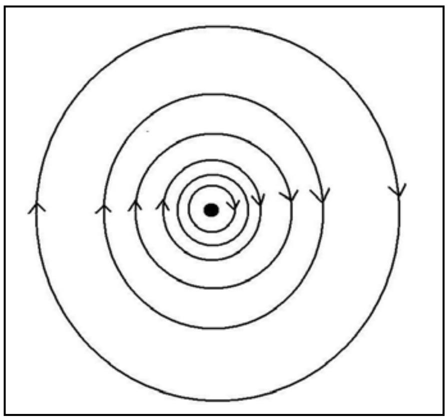
Problem 1 - Answer: See above table column 3 in Gauss units or column 4 in Teslas. Example: 0.05 picoTeslas = 0.05 x 10^{-12} Teslas x (10,000 Gauss/1 Tesla) = 5 x 10^{-10} G

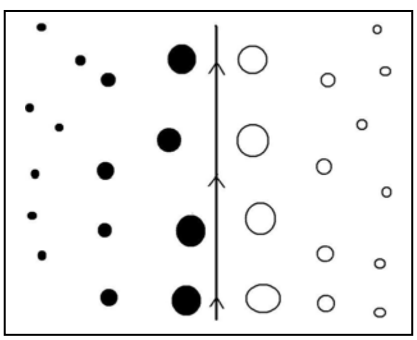
Problem 2 - Answer: See table, column 5.

Problem 3 - Answer: A) Magnetar compared to Cosmic magnetic fields: 1,000,000,000,000,000 Gauss / 0.000000000001 Gauss = 1,000,000,000,000,000,000,000,000,000 times
B) 10^{+15} G / 1x10^{-12} G = 1 x 10^{27} times.

Problem 4 - What is the range of natural magnetic fields compared to human-created magnetic fields? Answer; Natural range = Magnetar to Cosmic = 1 x 10^{27} times. Human Technology: Pulse Magnets compared to Computer Monitor or 8,500,000/0.00025 = 3.4 x 10^{9} = 34 billion times







$$B = \frac{4\pi(1.0x10^{-7})I}{2\pi D}$$

If you connect a battery to a wire, the current flowing through the wire will create a magnetic field around the wire that looks like the figure to the left. If you take your right hand, palm up, with your thumb pointed in the direction of the current, your fingers will curl in the direction of the magnetic field arrows. This is called the Right-Hand Rule.

Notice that the magnetic field looks different depending on how you draw it in 2-dimensions. If you place the wire perpendicular to the paper with the current flowing into the paper, the magnetic field looks like the middle sketch. If you place the wire along the length of the paper with the current flowing from bottom to top, it would look like the bottom sketch, where the arrows on the left side (filled circles) are coming out of the page, and the arrows on the right side (open circles) are going into the page. In the lower figure, the size of the circle represents the strength of the field. In the upper figure, the number of circles in a given space (density) represents the strength.

The formula, below, gives the strength of the magnetic field from the wire, in Teslas units, as a function of distance, D, in meters from the wire, and the current, I, flowing in the wire, in amperes. Note that a current of +10 Amperes is flowing in the opposite direction than a current of -10 amperes.

Problem 1 - If the current increases by a factor of 4, and the distance increases by a factor of 2, by what factor does B change?

Problem 2 - For a +10 Ampere current, what is the value for B at a distance of 0.5 meters: A) In Teslas? B) In Gauss?

Problem 3 - Suppose you had two parallel wires. In Wire-A the current was +10 Amperes, and in Wire-B the current was -10 Amperes. What would be the value for B in Teslas, at a distance of 0.5 meters?

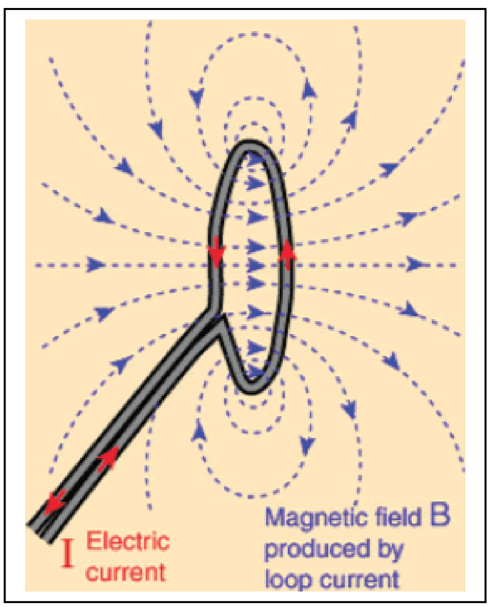
Problem 4 - Sketch the magnetic field pattern if the wire were bent around into a perfect circle. Where would the magnetic field be the most intense, and what B would result if I = +10 amperes and the circle had a radius of 0.5 meters?

Problem 1 - If the current increases by a factor of 4, and the distance increases by a factor of 2, by what factor does B change? Answer: From the formula, if I and D are the initial values for B-initial, and 4I and 2D are the final values, then B-final = $4 \pi (1.0 \times 10^{-7})$ 4I/(2π 2D) = B-initial x 4/2 = 2 x B-initial. So, B-final = 2 x B-initial.

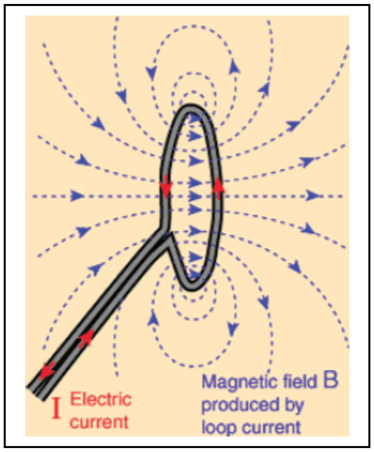
Problem 2 - For a +10 Ampere current, what is the value for B at a distance of 0.5 meters: A) In Teslas? B) In Gauss? Answer: A) $4 \pi (1.0 \times 10^{-7}) (+10)/(2 \pi \times 0.5)$ = +0.000004 Teslas. B) Since 1 tesla = 10,000 Gauss, we have 0.000004 Teslas x (10,000 Gauss/1Tesla) = +0.04 Gauss.

Problem 3 - Suppose you had two parallel wires. In Wire-A the current was +10 Amperes, and in Wire-B the current was -10 Amperes. What would be the value for B, in Teslas, at a distance of 0.5 meters?
Answer: From the answer to Problem 2, for I = +10 Amperes, B = +0.000004 Teslas, and for I = -10 Amperes, B = -0.000004 Teslas, so the combined magnetic field would be B = +0.000004 - 0.000004 = 0 Teslas

Problem 4 - Sketch the magnetic field pattern if the wire were bent around into a perfect circle. Where would the magnetic field be the most intense, and what B would result if I = +10 amperes and the circle had a radius of 0.5 meters?
Answer: Students should be able to draw a diagram resembling the one below. The most intense field will be at the geometric center of the loop. Calculating the actual intensity is a bit tricky. At the geometric center, half of the field comes from each side of the loop. From the wire on one side of the loop, B = +0.000004 Teslas at the center where D = 0.5 meters. From the wire on the other side of the loop, B = + 0.000004 Teslas at the center where D = 0.5 meters, so the combined magnetic strength is the sum of these, or +0.000008 Teslas.



The Magnetic Field of a Solenoid



$$B = \frac{4\pi(1.0x10^{-7})I}{2\pi D}$$

Imagine a hollow tube with a length of insulated wire smoothly wrapped in one layer around its circumference so that each turn is adjacent to the next one with no gaps. This is a basic electrical coil that can be used in many ways. For example, if you pass a current through the coil, an electromagnet can be created.

Often times, the wires is wrapped around a nail or a pencil, and it can be used to pick up paperclips or tacks. In this set of problems, we are going to investigate what the magnetic field looks like, and use a formula to determine the strength of the field.

Problem 1 - The formula above gives the strength, B, in Teslas, of a magnetic field from a wire carrying a current of I amperes at a distance of D meters. Suppose a +1 Ampere current flows through a wire. What is the value for B at a distance of 0.5 centimeters?

Problem 2 - Draw two loops of wire with the current circulating in the same direction, and separated by 1/2 their diameter. To make the field line diagram simple to interpret, show the loops of wire as though viewed edge-on. Draw four magnetic field lines of the first loop. Draw four magnetic field lines produced by the second loop. Allow the second set of magnetic field lines to be drawn on top of the field lines of the first loop. Include the correct arrows giving the polarity of the magnetic field line.

Problem 3 - Simplify the field line sketch by starting with the first overlapping pair of corresponding field line circles. Replace the pair with a new field line that combines the two, then erase the original lines. Continue until all 16 of the original field lines have been replaced by four new, smoothed, field lines. Note: the final field lines must not cross each other or have kinks. Based on the pattern for two loops, what do you think the field will look like for a long string of loops placed side-by-side and spaced close together?

Problem 4 - The intensity of the magnetic field, B in Teslas, inside a solenoidal coil is given by the formula B = cN I / L where $c = 4 \pi \times 10^{-7}$, N is the number of turns, L is the length in meters and I is the current in amperes. A student makes an electromagnet by wrapping 100 turns of copper wire on a nail with a length of 3 centimeters. If a current of 1.5 amperes is applied to the coil. If 10,000 Gauss equals 1 Tesla, what is the strength of the electromagnet in A) Teslas? B) Gauss? C) How does the strength of this electromagnet compare to Earth's magnetic field at the ground (0.7 Gauss) and the strength of a refrigerator magnet (100 Gauss)?

Problem 1 - Answer: 4 (3.14)(1.0 x 10^{-7}) (+1.0)/(2 x 3.14 x 0.005) = 0.00004 Teslas.

Problem 2 - Answer; See sketch below (left).

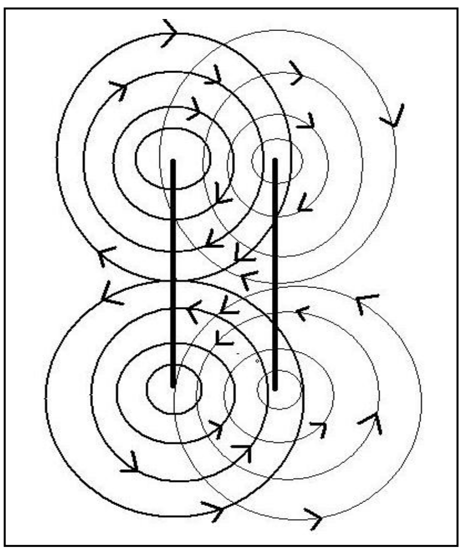
Problem 3 - Answer: See sketch below (right). By deduction, students should be able to sketch a hypothetical field similar to the bottom figure. (Courtesy NDT Education Resource Center, University of Iowa)

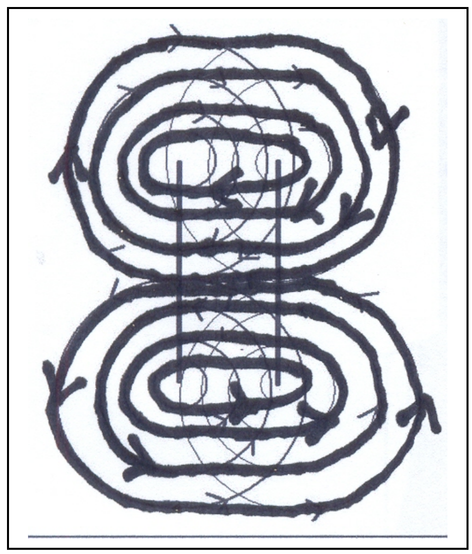
Problem 4 - Answer: First we have to convert all length units to meters so L = 3 cm becomes 0.03 meters.

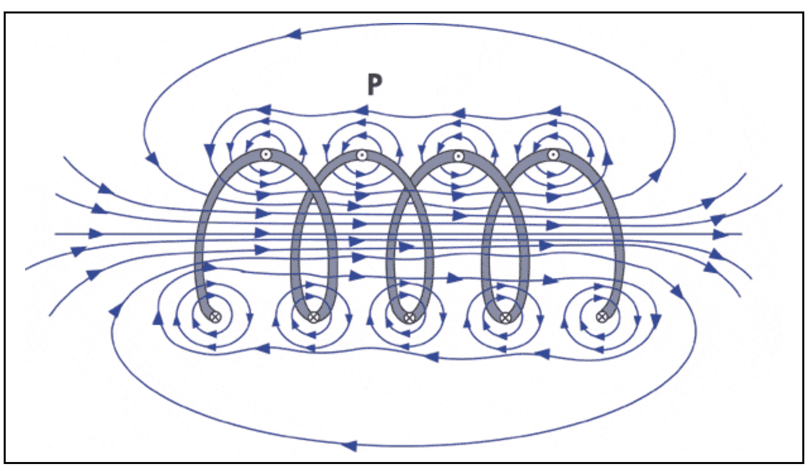
A) From the formula, B = 4 π x 10^{-7} (100 turns) x (1.5 Amps)/0.03 meters = 0.0063 Teslas.

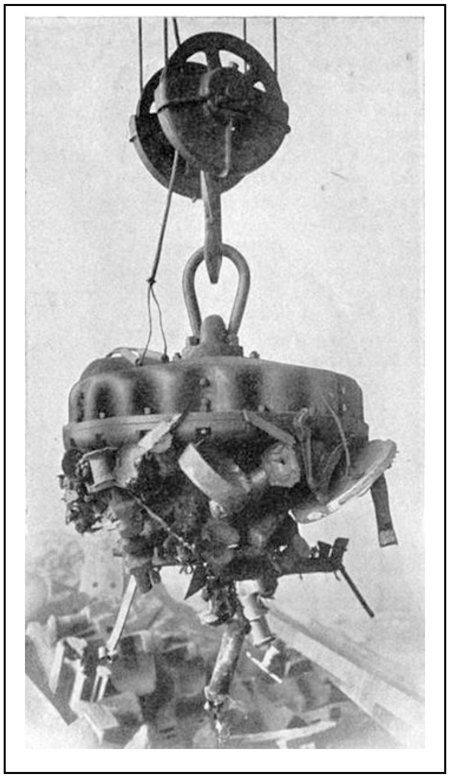
B) 0.0063 Teslas x (10,000 Gauss/1 Tesla) = 63 Gauss.

C) Compared to Earth's ground-level field, this is 63 Gauss/0.7 Gauss = 90 times stronger, and compared to the refrigerator magnet it is 63 Gauss/100 Gauss = 0.63 as strong.









A student wants to make an electromagnet that will lift 0.5 kilograms of iron nails. He knows that the gravitational force acting on the steel is F = mg, which is just F = 0.5 kg x 9.8 m/sec^2 = 4.9 Newtons. He also knows that the formula for the force **F** in Newtons produced by an electromagnet is the one shown below, where **L** is the length of the coil in meters, **N** is the number of turns of wire wrapped around a steal core, **I** is the current in amperes through the wire, and **A** is the surface area in square-meters of the lifting face of the magnet. The quantity $\mu = 4\pi (1\times10^{-7})$ Newton/Ampere^2 is called the *permeability of free space*, and the magnetic permeability for steel is given by the constant **C** = 700.

$$F = \frac{\mu C^2 N^2 I^2 A}{2L^2}$$

Problem 1 - How many turns of wire, N, will be needed to create an electromagnet capable of lifting 0.5 kilograms of iron nails if I = 0.5 Amps, A = 0.0016 m^2, L = 0.1 meters?

Problem 2 - An industrial 'junk yard' electromagnet lifts cars weighing up to 1,500 kilograms. The diameter of the lifting plate is 1 meter, and the thickness of the coil is 0.4 meters. If the coil consists of 10 windings of heavy-gauge wire, how many amperes must be used to create the required lift?

Problem 3 - A student wants to increase the quantities N, I, A and L by a factor of 2 to make a larger electromagnet. By what factor will the lifting mass increase?

Electromagnets have been used in junk yards to lift scrap metal for over 100 years. The photograph is from the 1914 book 'Elementary Magnetism and Electricity' by C. Jansky published by McGraw-Hill.

Problem 1 - How many turns of wire, N, will be needed to create an electromagnet capable of lifting the 0.5 kilograms of iron nails if I = 0.5, A = 16 cm^2, Amperes, L = 10 centimeters? Answer; For F = 4.9 Newtons solve for N^2 to get
$N^2 = (4.9)(2)\ (0.1)^2/(4 \times 3.14 \times 10^{-7})(700)^2\ (0.5)^2\ (0.0016) =$ 398 so that after taking the square-root of both sides we get N=19.95 which rounds to 20 turns of wire.

Problem 2 - An industrial 'junk yard' electromagnet lifts cars weighing up to 1,500 kilograms. The diameter of the lifting plate is 1 meter, and the thickness of the coil is 0.4 meters. If the coil consists of 10 windings of heavy-gauge wire, how many amperes must be used to create the required lift?
Answer: First you have to calculate how many Newtons of force ,F, are required. For F = m g this is just F = (1,500 kg) x (9.8 meters/sec^2) = 14,700 Newtons of force.
Next, you also need to convert the diameter of the plate to a surface area for the electromagnet of A = π(1 meter/2)^2 = 0.78 meters^2.

You need to find out what current is needed so solve the equation for I to get

$$I = (2\ F\ L^2)/(\mu\ C^2 N^2 A)^{1/2}$$

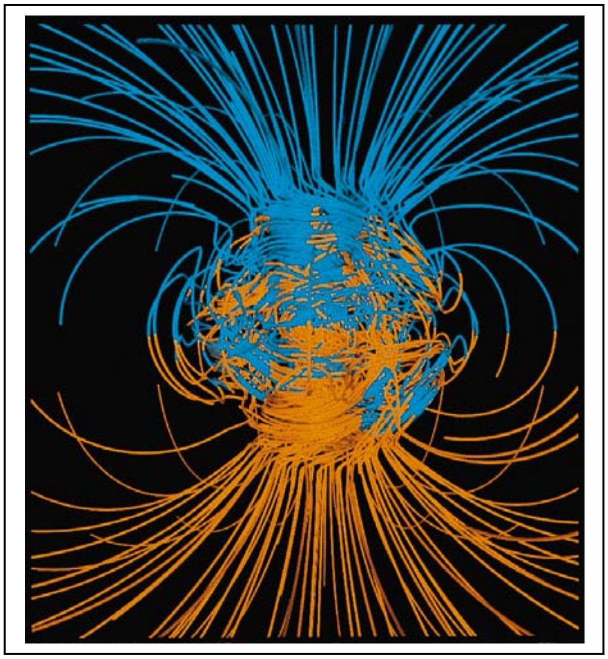
then substitute the stated values to get

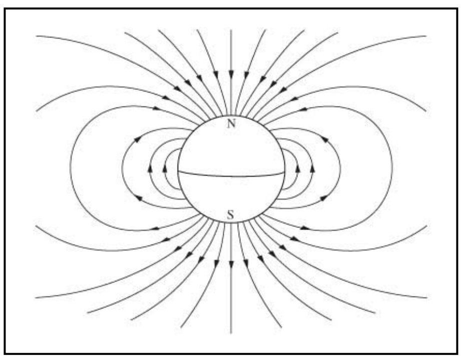
$$I = (\ \ (2\ (14{,}700)\ (0.4)^2)\ /\ ((4 \times 3.141 \times 10^{-7})\ (700)^2 (10)^2 (0.78))\ )^{1/2}$$
$$= (4.7/\ 4.8)^{1/2}$$
= 10 Amperes.

Problem 3 - A student wants to increase all of the quantities N, I, A and L by a factor of 2 to make a larger electromagnet. By what factor will the lifting mass increase?

Answer: N becomes 2N, I becomes 2I, A becomes 2A and L becomes 2L, so substituting into the formula $F = \mu\ C^2 N^2 A\ /\ 2\ F\ L^2$ we get $F(new) = \mu\ (2C)^2 (2N)^2\ 2A\ /\ 2\ \ (2L)^2$ which simplifies to $F(new) = \mu\ (2C)^2 (2N)^2\ 2A\ /\ 2\ \ (2L)^2$ and so
F(new) = 8xF(old). The new electromagnet with be 8 times stronger, and so since F = m g, and the acceleration of gravity,g is a constant, it will be able to lift 8 times more mass.

Magnetic Fields in Two Dimensions





The magnetic field from a bar magnet actually surrounds the complete magnet. When we sprinkle iron filings on a sheet of paper to reveal the magnetic lines of force, we are only seeing what the field looks like in two of the three dimensions to space. For example, the top picture is a perspective model of Earth's magnetic field in 3-dimensions, while the lower picture is a 2-dimensional version of a similar magnetic field.

The strength of a magnetic field at a particular point in two-dimensional space actually consists of two distinct numbers that define the strength of the magnetic field along the X and Y axis.

These two 'components' form the two sides of a right-triangle and we can define them as Bx along the X-axis and By along the Y-axis. If we use the Pythagorean Theorem, the total strength of the magnetic field is just the length of the hypotenuse of this 'magnetic' triangle. The angle between the X-axis and the hypotenuse measures the direction angle of the magnetic field in space.

Problem 1 - On a graph paper, and at a convenient scale of 1 Tesla per division, draw the triangles representing the following magnetic fields with the components given as the ordered pairs (Bx, By) and with the strength measured in Teslas: A) (3.0, 4.0); B) (10.0, 10.0); C) (6.0, 8.0) D) (13.0, 10.0)

Problem 2 - Using a protractor, measure the direction angles for the four magnetic fields drawn in Problem 1.

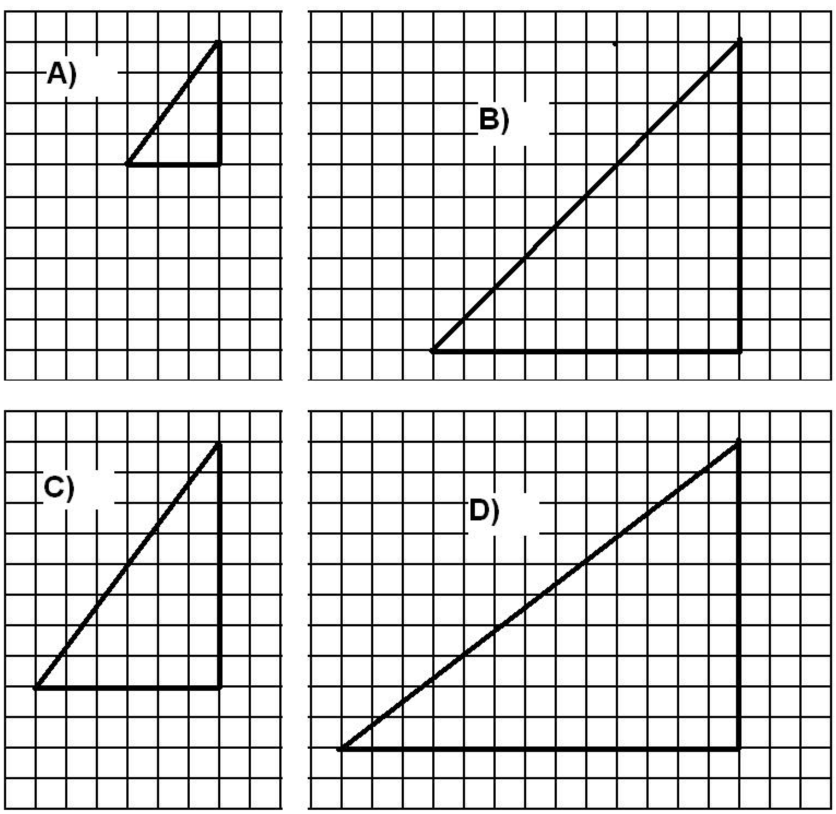
Problem 3 - Using the Pythagorean Theorem, calculate the total strength in two-dimensions of the magnetic fields in Problem 1 and 2. Give the answers in Teslas to one decimal place accuracy.

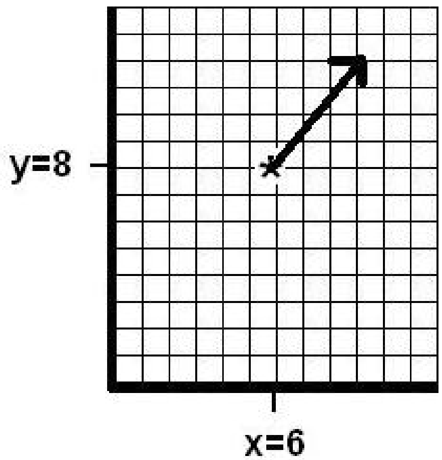
Problem 1 - On a graph paper, and at a convenient scale of 1 Tesla per division, draw the triangles representing the following magnetic fields with the components given as the ordered pairs (Bx, By) and with the strength measured in Teslas: A) (3.0, 4.0); B) (10.0, 10.0); C) (6.0, 8.0) D) (13.0, 10.0) Answer: See figure below.

Problem 2 - Using a protractor, measure the direction angles for the four magnetic fields drawn in Problem 1. Answer: Students may also verify their measurements by using the properties of 30:60:90, 45:45:90 triangles and simple trigonometry. A 60 degrees, B) 45 degrees. C) 53 degrees (this angle is the arctangent of 8/6); D) 38 degrees (this angle is the arctangent of 10/13)

Problem 3 - Using the Pythagorean Theorem, calculate the total strength in two-dimensions of the magnetic fields in Problem 1 and 2. Give the answers in Teslas to one decimal place accuracy.

Answer: A) $B = (3^2 + 4^2)^{1/2}$ = 5.0 Teslas B) $B = (10^2 + 10^2)^{1/2}$ = 14.1 Teslas C) $B = (6^2 + 8^2)^{1/2}$ = 10.0 Teslas D) $B = (13^2 + 10^2)^{1/2}$ = 16.4 Teslas.





Plotting a magnetic field on a regular Cartesian coordinate grid is actually more complicated than it may sound, because you are plotting both position in space and the intensity and direction of the field at that point. The example to the left shows the magnetic field at one point in space (X= 6.0, Y=8.0), with the direction of the magnetic field at that point given by an arrow inclined at a specific angle; in this case 60 degrees. A compass placed at that location will point 'North' in the direction of the arrow.

Problem 1 - Using a protractor, graph the magnetic field in the First and Fourth Quadrants defined by the following points where the units for X and Y are centimeters, and the direction is given in degrees. For convenience, use graph paper with 1-cm grids, and draw the arrows 1-cm long.

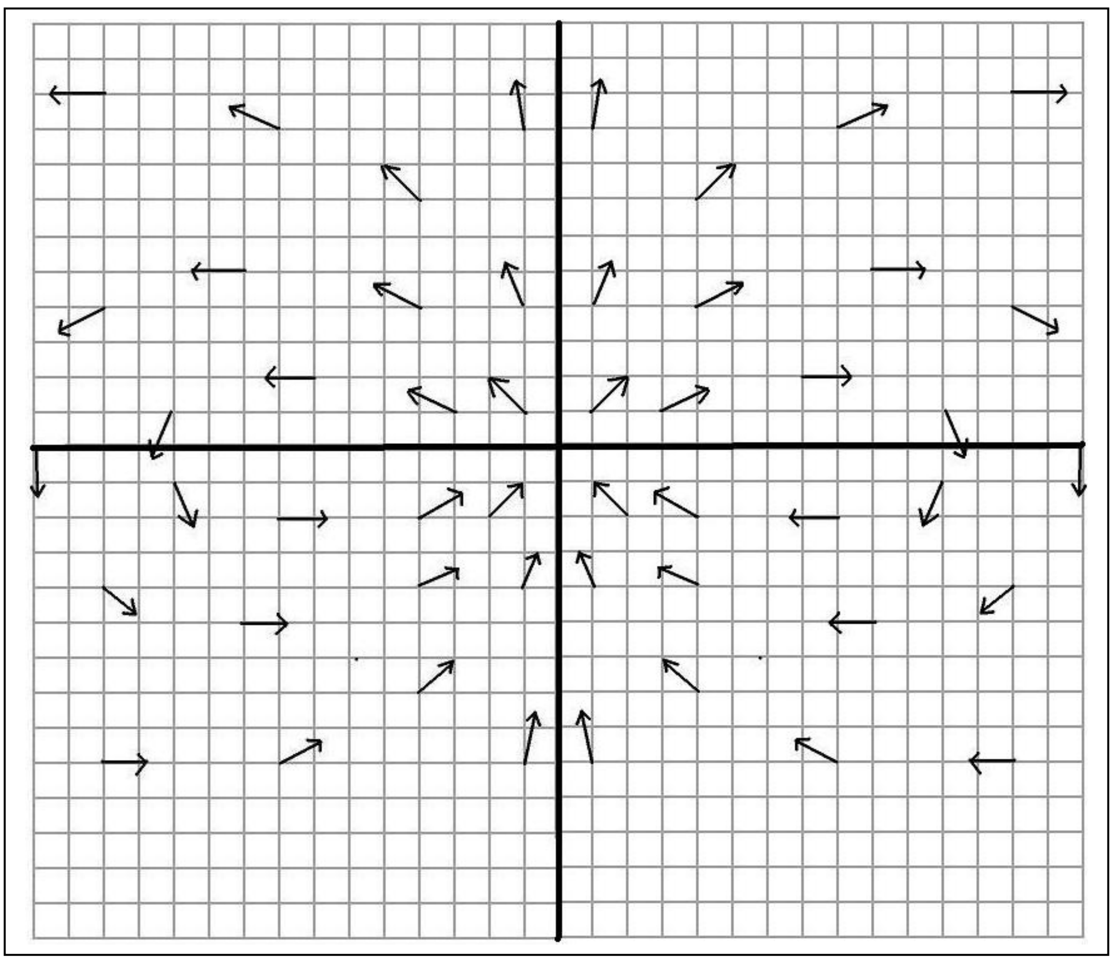
Point	X (cm)	Y (cm)	Angle
A	1	1	45
B	3	1	30
C	1	4	60
D	4	4	30
E	4	7	45
F	7	2	0
G	9	5	0
H	11	1	300
I	13	4	330
J	1	9	80
K	8	9	30
L	13	10	0
M	15	0	270
N	2	-2	135
O	1	-4	120
P	4	-4	150
Q	8	-2	180
R	11	-1	240
S	13	-4	225
T	9	-5	180
U	4	-7	135
V	1	-9	100
W	8	-9	150
X	13	-9	0
Y	4	-2	150

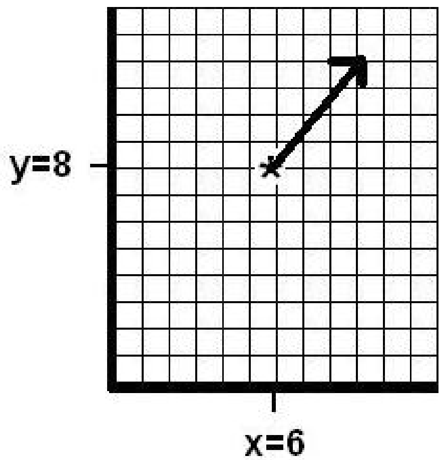
Problem 2 - Use reflection symmetry to draw the magnetic field in the other two quadrants. What kind of object has a similar type of magnetic field pattern?

Answer Key

Problem 1 - Graph the magnetic field in the First and Fourth Quadrants defined by the following points where the units for X and Y are centimeters. Answer: See figure below. Note: As usual in geometry, angles are measured counterclockwise relative to the horizontal axis.

Problem 2 - Use reflection symmetry to draw the magnetic field in the other two quadrants. What kind of object has this magnetic field pattern? Answer: See figure below. The figure resembles a bar magnet with the North Pole at the top and the South Pole at the bottom, oriented vertically along the Y axis.





Plotting a magnetic field on a regular Cartesian coordinate grid is actually more complicated than it may sound. The example to the left shows the magnetic field (Bx=3.0 Teslas, By=4.0 Teslas) at one point in space (X= 6.0, Y=8.0), with the length of the arrow representing the strength of the field pointed in the proper polarity direction. Here's how it was done.

Step 1 - Plot the point (6.0, 8.0) on the Cartesian grid for its location in space.

Step 2 - Determine the strength of the magnetic field at that point by using the Pythagorean Theorem $B = (Bx^2 + By^2)^{1/2}$ to get $B = (3.0^2 + 4.0^2)^{1/2} = 5.0$ Teslas.

Step 3 - Determine the direction of B by finding the angle α = arccos (Bx/B) so α = arccos(3.0/5.0) and so α = 53 degrees.

Step 4 - Starting at the point (6.0,8.0), draw an arrow that represents 5.0 Tesla in length that points at an angle of 53 degrees with the X-axis. Use this same length to scale other magnetic field strengths in the rest of the map.

Problem 1 - Graph the magnetic field in the First and Fourth Quadrants defined by the following points where the units for X and Y are in meters, and Bx and By are in Teslas. (Note: the values for Bx and By have been selected to make the plotting easier and do not accurately indicate how the intensity changes with distance in an actual magnet.)

Point	X (meters)	Y (meters)	Bx (Tesla)	By (Tesla)
A	1.0	1.0	40	40
B	4.0	4.0	40	23
C	9.0	5.0	20	0
D	13.0	4.0	20	-12
E	15.0	0	0	-10
F	13.0	-4.0	-10	-10
G	9.0	-5.0	-10	0
H	4.0	-4.0	-12	20
I	2.0	-2.0	-40	40

Problem 2 - Use reflection symmetry to draw the magnetic field in the other two quadrants. What kind of object has this magnetic field pattern?

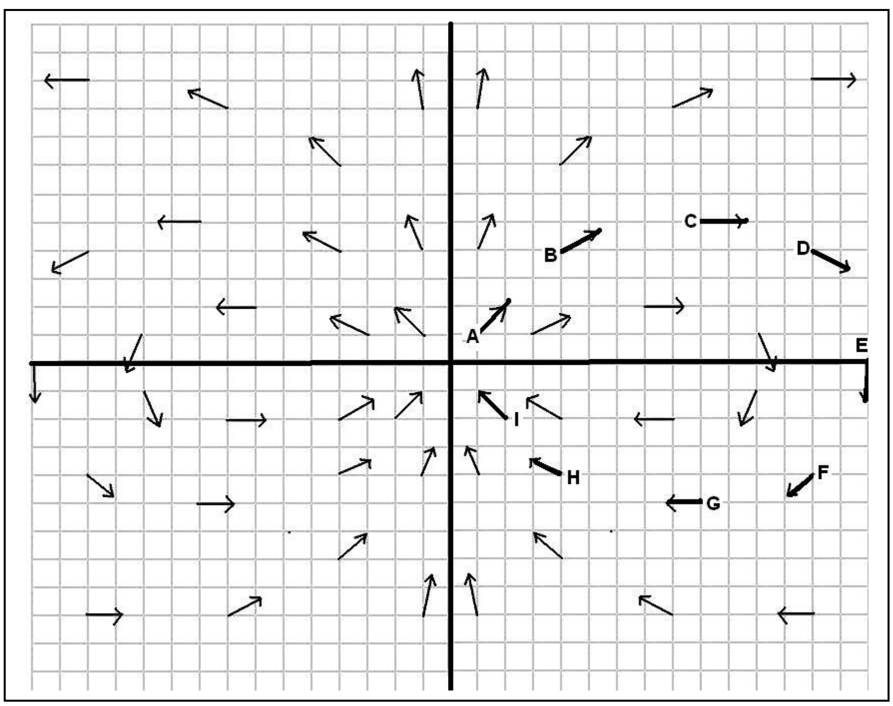
Answer Key

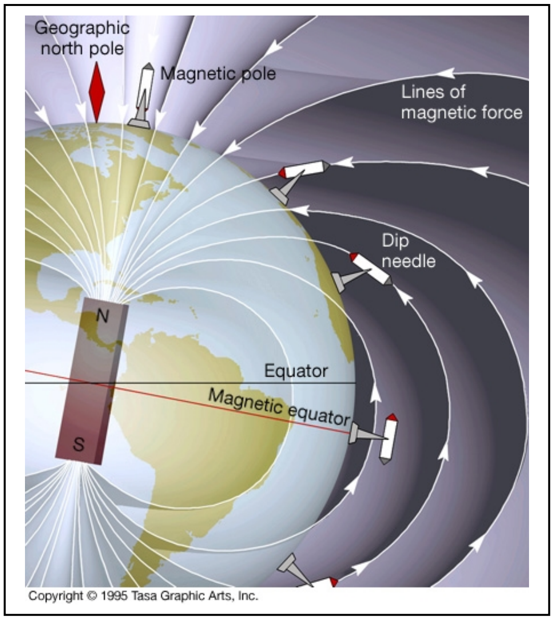
Problem 1 - Graph the magnetic field in the First and Fourth Quadrants defined by the following points where the units for X and Y are centimeters, and Bx and By are in Gauss. Answer: See figure below (with angles drawn approximately). Note: Positive angles are measured counterclockwise relative to the horizontal axis. Students need to be careful calculating the angle from the cosine so that the actual angle matches the correct quadrant determined by the signs (e.g. + and -) of Bx and By.

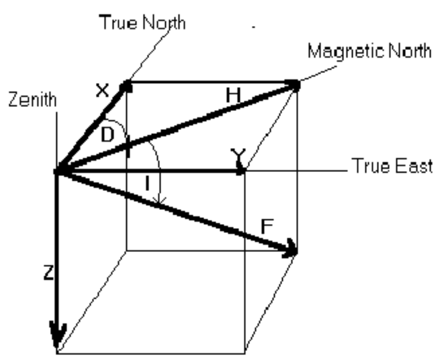
Example: for Point-F, Bx is negative and By is negative so the angle is in the Third Quadrant, and Bx/B = -0.71 and cos(-0.71) = -135 which is the same as 360-135 = 225 in the standard angle naming convention.

Point	X (meter)	Y (meter)	Bx (Teslas)	By (Teslas)	B (Teslas)	Bx/B	Angle
A	1.0	1.0	40	40	57	0.71	45
B	4.0	4.0	40	23	46	0.87	30
C	9.0	5.0	20	0	20	1.00	0
D	13.0	4.0	20	-12	23	0.86	330
E	15.0	0	0	-10	10	0.0	270
F	13.0	-4.0	-10	-10	14	-0.71	225
G	9.0	-5.0	-10	0	10	-1.00	180
H	4.0	-4.0	-12	20	23	-0.50	120
I	2.0	-2.0	-40	40	57	-0.71	135

Problem 2 - Use reflection symmetry to draw the magnetic field in the other two quadrants. What kind of object has this magnetic field pattern? Answer: See figure below. Although it may be a challenge to interpret the figure, which is only the data for one pair of magnetic field lines, the figure resembles a bar magnet with the North Pole at the top and the South Pole at the bottom, oriented vertically along the Y axis.







Earth's magnetic field is a 3-dimensional object in space. At each point in space defined by the simple Cartesian coordinates (x,y,z) a magnetic field can have three 'components, each with differing intensity, which we define by the triplet of numbers (Bx, By, Bz). The combination of these three numbers using the Pythagorian Theorem gives the total intensity of the magnetic field at that point in space.

Imagine a sheet of paper exactly tangent to a specific geographic location (Latitude, Longitude). Draw a Cartesian grid on this paper so that X increases towards North, and the Y-axis increases towards the East. The Z-axis will be perpendicular to the sheet of paper, and increases towards the center of Earth.

When you use a magnetic compass, the needle points to Magnetic North. This is a slightly different direction than True Geographic North, which as you remember is along the X-axis of our coordinate system. The angle difference between True North and Magnetic North is called magnetic Declination. It is positive if the compass needle points to the east of True North, and negative if it points to the west of True North. This is a very important number to keep track of for navigation, and for hundreds of years, seafarers kept detailed logs of its value so that they could navigate the high seas accurately, and reach the intended harbor thousands of miles away.

Magnetic Declination is trigonometrically defined in terms of the values for Bx and By as D = arctan (By/Bx). For example, at the location of the city of Chicago, Bx = 18,934 nT and By = -1120 nT, so By/Bx = -0.0591 and D = arctan(-0.0591), and so D = -3.38 degrees. This means that True North is 3.38 degrees to the east of the direction that your needle is pointing. This can also be calculated by plotting Bx and By as the legs of a right-triangle, and measuring the angle from the X-axis with a protractor. This is less accurate because the angles are usually small in most continental locations.

Problem 1 - An explorer sets out from Anchorage, Alaska to travel to the North Pole. If Bx= +14,455 nT and By= +5,061 nT, what is her Magnetic Declination: A) Determined by using a protractor; B) Determined by using arctangents?

Problem 2 - The US Geological Survey provides websites where the magnetic Declination can be calculated for any geographic location:

http://www.ngdc.noaa.gov/geomagmodels/Declination.jsp or

http://www.ngdc.noaa.gov/geomagmodels/IGRFWMM.jsp

Where in the United States will your Declination be close to zero?

Problem 1 - An explorer sets out from Anchorage, Alaska to travel to the North Pole. If Bx= +14,455 nT and By= +5,061 nT, what is her Magnetic Declination: A) Determined by using a protractor; B) Determined by using arctangents?

Answer: A) See scaled figure below, where each division represents 1,000 nT. A protractor yields an angle measure of about 19 degrees. B) D = Arctan (5061/14455) = +19.3 degrees, so Magnetic North is 19.3 degrees to the East of True North,

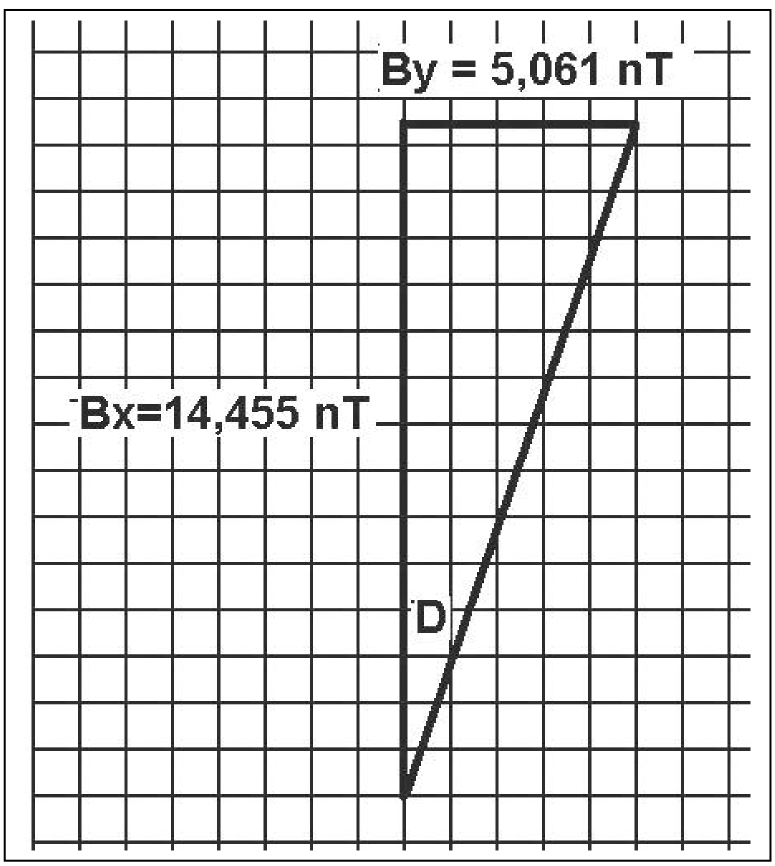
Note to Teacher: Because the magnetic declination of Anchorage is positive, the needle is pointing to the east of True North, so the explorer needs to follow a path that is 19.3 degrees to the west of where her compass needle is pointing.

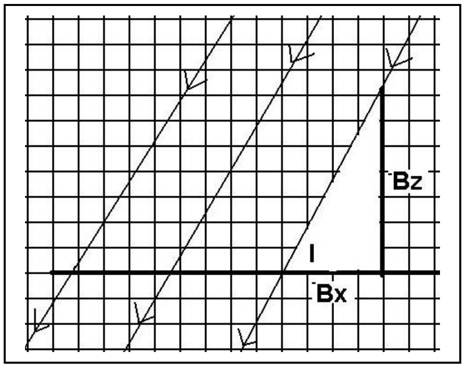
Problem 2 - The US Geological Survey provides a website where the magnetic Declination can be calculated for any geographic location:

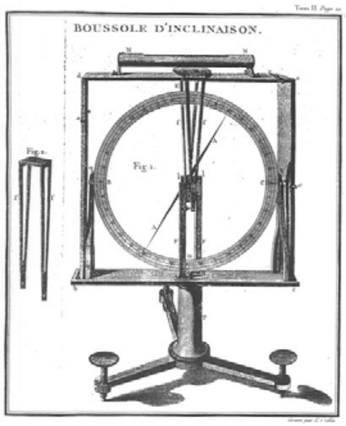
http://www.ngdc.noaa.gov/geomagmodels/Declination.jsp or
http://www.ngdc.noaa.gov/geomagmodels/IGRFWMM.jsp

Where in the United States will your Declination be close to zero?

Answer: Students should examine a string of US cities that span the continental US, then narrow their search until they find a declination approximately between -0.1 and +0.1 degrees. By trial-and-error, cities that are close to this range include, for example, New Orleans (-0.1 degree); Saint Louis (-0.7 degrees).







The 3-dimensional properties of Earth's magnetic field lead to some interesting features when we explore it from various 2-dimensional perspectives. In the Bx-By plane, we examined the Magnetic declination angle, which is important to compass navigation. The Bz-Bx plane also provides an interesting new ingredient as shown in the top figure.

The magnetic Inclination angle, **I**, also called the Magnetic Dip angle, is a measure of how steeply a magnetic field line passes into the surface of Earth. If you were to hold a compass perfectly horizontal, it would 'point North' but its tip would also dip vertically to the ground. This feature was first discovered in 1581 by the English mariner and compass builder, Robert Norman. The lower figure shows an instrument used in 1808 to measure this angle.

Problem 1 - The table below shows the magnetic field values (in units of nanoTeslas: nT) for several major cities. Calculate for each, using either trigonometry, or a scaled drawing and using a protractor, what the magnetic field inclination angle is in each instance. The data were obtained from the US Geological Survey, and are available at the website:
http://www.ngdc.noaa.gov/geomagmodels/IGRFWMM.jsp

City/Country	Bz (nT)	Bx (nT)	I (degrees)
Anchorage	53,720	14,455	74.9
Boston	48,991	19,143	
Miami	37,081	25,208	
San Diego	39,940	24,419	
Honolulu	21,617	27,304	
Equador	10,492	27,431	
Santiago, Chile	-13,433	20,247	
Buenos Ares, Argentina	-14,282	18,133	
Cairo, Egypt	30,279	30,940	
Paris, France	43,188	20,733	
Rome, Italy	39,243	24,419	
Panama City, Panama	20,342	27,677	
Cayenne, French Guiana	8,292	26,525	
La Paz, Bolivia	-3,713	23,438	
Churchill, Canada	58,692	9,253	

Answer Key

City/Country	Bz (nT)	Bx (nT)	I (degrees)
Anchorage	53,720	14,455	74.9
Boston	48,991	19,143	68.7
Miami	37,081	25,208	55.8
San Diego	39,940	24,419	58.6
Honolulu	21,617	27,304	38.4
Equador	10,492	27,431	20.9
Santiago, Chile	-13,433	20,247	-33.6
Buenos Ares, Argentina	-14,282	18,133	-38.2
Cairo, Egypt	30,279	30,940	44.4
Paris, France	43,188	20,733	64.4
Rome, Italy	39,243	24,419	58.1
Panama City, Panama	20,342	27,677	36.3
Cayenne, French Guiana	8,292	26,525	17.4
La Paz, Bolivia	-3,713	23,438	-9.0
Churchill, Canada	58,692	9,253	81.1

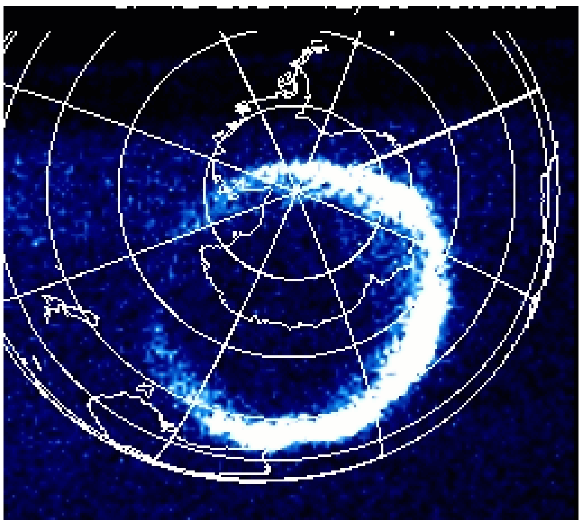
Problem 1: Solve using trigonometry I = arctan(Bz/Bx):

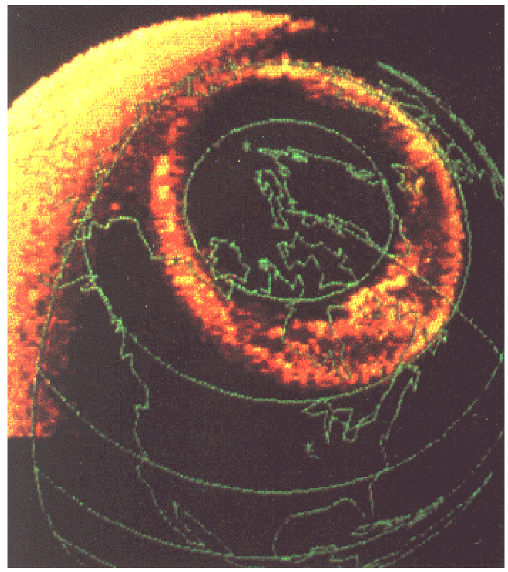
Example Anchorage: I = arctan(Bz/Bx) = arctan(53720/14455) = 74.9 degrees

The North and South Magnetic Poles

The aurora form a glowing halo of light above Earth's North and South Polar Regions. Because aurora are caused by charged particles that are affected by Earth's magnetic field, the Auroral Ovals are centered in Earth's magnetic poles, not its geographic poles about which the planet rotates.

The photos below, were taken of the two polar aurora by the IMAGE FUV (Left) and the Polar (right) instruments. The data has been colorized to bring out details of interest to scientists.

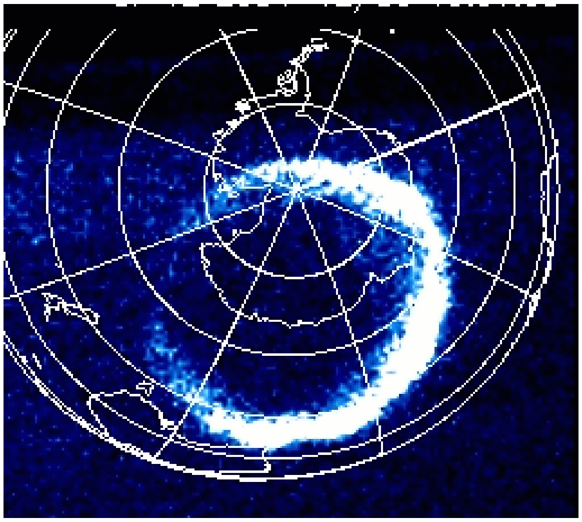


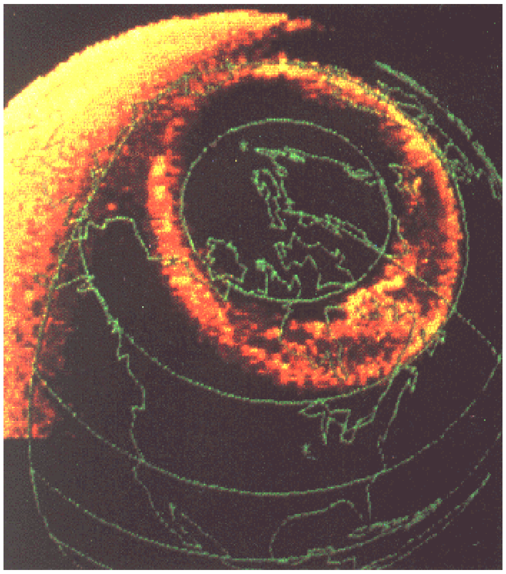


Problem 1 - The South Magnetic Pole is located in the Northern Hemisphere. From the appropriate image above, locate this magnetic pole on a map.

Problem 2 -- The North Magnetic Pole is located in the Southern Hemisphere. From the appropriate image above, locate this magnetic pole on a map.

Problem 3: -- From the geographic clues in the map, estimate the diameter of the auroral oval in kilometers. (Hint: The radius of Earth is 6,378 kilometers)





Problem 1 - The South Magnetic Pole is located in the Northern Hemisphere. From the appropriate image above, locate this magnetic pole on a map.

Answer: *The right-hand image from the Polar satellite shows the Arctic Region and the contours of Greenland and North America/Canada. From a world map, students can estimate that the center of the auroral oval is near longitude 105 West and latitude 83 North)*

Problem 2 -- The North Magnetic Pole is located in the Southern Hemisphere. From the appropriate image above, locate this magnetic pole on a map.

Answer: *The left-hand image from the IMAGE satellite shows Antarctica. The center of the auroral oval is near longitude 110 West and latitude 72 South.*

Problem 3: -- From the clues in the map, estimate the diameter of the auroral oval in kilometers. (Hint: The radius of Earth is 6,378 kilometers)

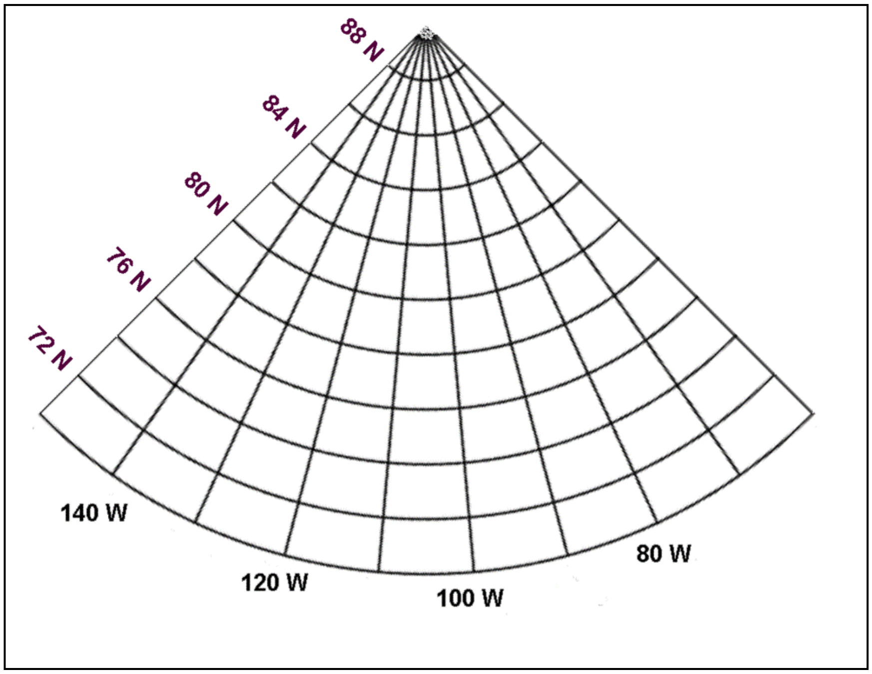
Answer: *The longitude and latitude coordinate grids shown in each image cover the Earth, so the maximum diameter of the grid is the diameter of Earth. Using a millimeter ruler, the diameters are (South: Left) = 78 mm and (North; right) = 77 mm.*

Calculate the scale of each image (kilometers per millimeter) : North = 6378 km/77mm = 83 km/mm; South = 6378 km/78 mm = 82 km/mm

Multiply by the diameter of each auroral oval in millimeters. For the north polar aurora, its diameter is about 6400 kilometers. For the south polar aurora, the diameter is about 6,000 kilometers.

The Speed of Magnetic Pole Wander

Several centuries ago, mapmakers noticed that the compass bearings on their navigation charts slowly changed in time. This forced mapmakers to re-draw their maps every few decades, and even more frequently for selected locations. The reason for this change, called secular variation, is that the entire magnetic field of Earth is slowly shifting so that its poles 'wander' across the surface. Using modern instruments, we can measure this movement from year to year. The following table shows the position of the magnetic North Pole over the past 100 years.



	Year (AD)	Lat.	Long.
A	1900	+70.5N	96.2W
B	1910	+70.8N	96.7W
C	1920	+71.3N	97.4W
D	1930	+72.3N	98.7W
E	1940	+73.3N	99.9W
F	1950	+74.6N	100.8W
G	1970	+75.9N	101.0W
H	1980	+76.9N	101.7W
I	1990	+78.1N	103.7W
J	2000	+81.0N	109.7W

In the following problems, assume that on this portion of the polar grid that 1 degree of latitude equals 110 kilometers.

Problem 1 - Plot the pole locations for the tabulated years.

Problem 2 - About how far did the magnetic North Pole move between; A) 1900 and 1920? B) 1990 and 2000?

Problem 3 - How far did the magnetic North Pole move in meters in one year between; A) 1900 and 1920? B) 1990 and 2000?

Problem 4 - Approximately how far did the magnetic North Pole move per day between; A) 1900 and 1920? B) 1990 and 2000?

Answer Key

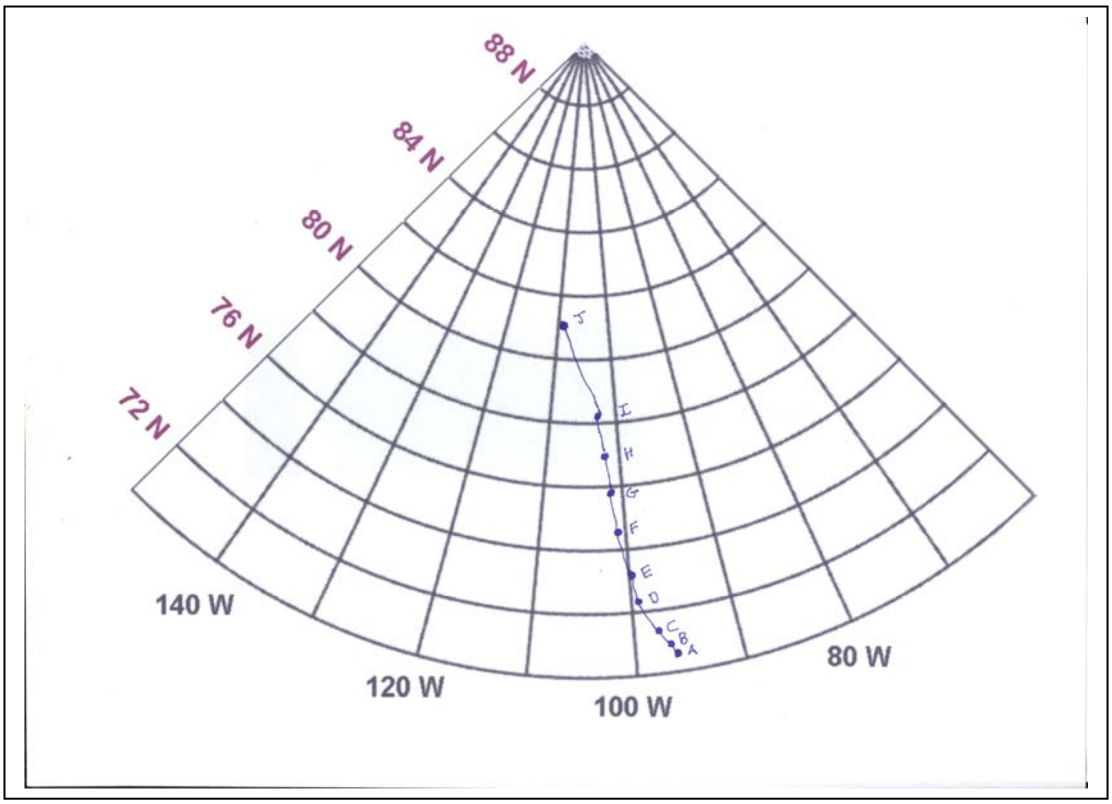
The data used in the table comes from the International Geomagnetic Reference Field version 10

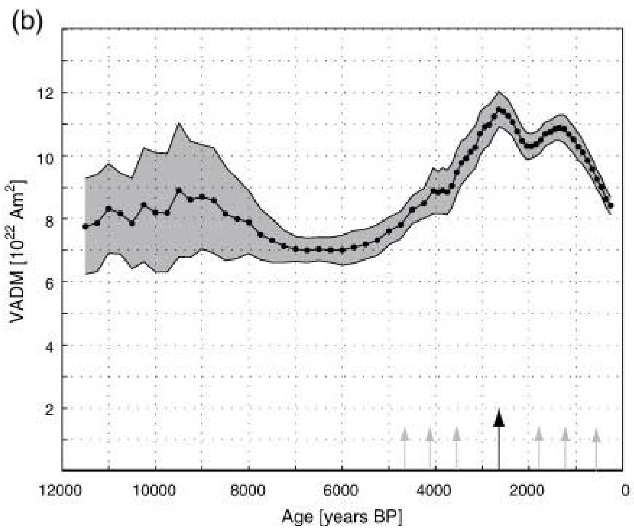
Problem 1 - Plot the pole locations for the tabulated years. Answer: See figure below.

Problem 2 - About how far did the magnetic North Pole move between; A) 1900 and 1920? B) 1990 and 2000? Answer: A) Scaling the interval from the plotted graph, the distance is about 110 km. B) The distance is about 340 km.

Problem 3 - How far did the magnetic North Pole move in meters in one year between; A) 1900 and 1920? B) 1990 and 2000? Answer: A) speed = 110 km/10 yrs = 11 km/year or 11000 meters/year B) speed = 340 km/10 years = 34 km/yr = 34,000 meters/year

Problem 4 - Approximately how far did the magnetic North Pole move per day between; A) 1900 and 1920? B) 1990 and 2000? Answer: A) 11000 meters/year x (1 year/365 days) = 30 meters/day. B) 34000 meters/year x (1 year/365 days) = 93 meters/day

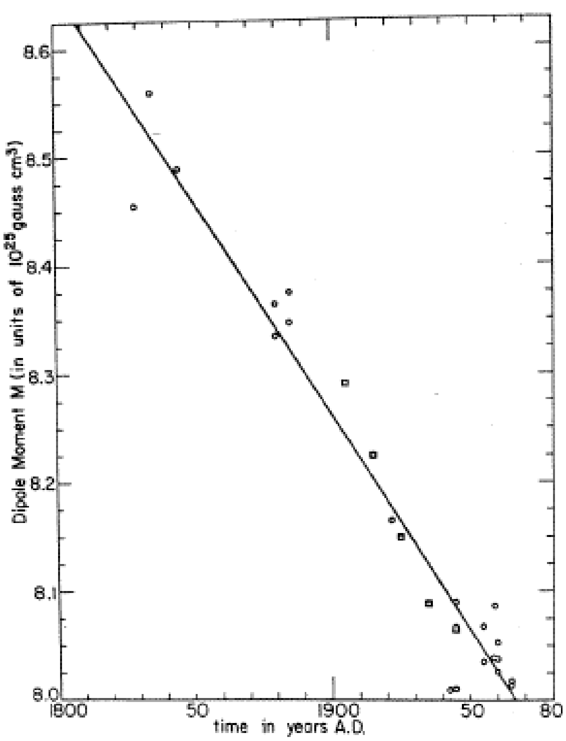




Not only do the magnetic poles of Earth drift over time, but the entire strength of Earth's magnetic field increases and decreases. The strength of this field is commonly measured in terms of a quantity called VADM with the units of Ampere meter^2 (Am^2). For example, a 1 Ampere current circulating in a closed circle with an area of 1 meter^2 has a VADM of 1 Am^2.

The top figure shows the variations in Earth's VADM since end of the last Ice Age about 12,000 years ago. The gray area represents the range of measurement uncertainty. The current era is to the far-right of the plot.

The lower figure shows the most recent changes since 1800 using a slightly different unit scale.



Problem 1 - During the last 12,000 years, what has been the range in the VADM dipole strength as indicated by the black line?

Problem 2 - In about how many years from the present would you predict that the VADM will reach the lower end of its range in the last 12,000 years?

Problem 3 - Based on the slope of the line in the lower figure, what is the current rate-of-change of the magnetic field in terms of percent per century?

Problem 4 - If the decline continues at this pace, by what year will the strength of Earth's main dipole field be near-zero?

Problem 5 - Comparing the trends displayed by the upper plot with the lower graph, do you think the current rate-of-change exceptional?

Data from "Variations in the geomagnetic dipole moment during the Holocene and the past 50 kyr" by Mads Faurschou Knudsen, Peter Riisager, Fabio Donadini, Ian Snowball, Raimund Muscheler, Kimmo Korhonen, and Lauri J. Pesonen in the journal 'Earth and Planetary Science Letters' ,Vol. 272, pp 319-329.

Teacher note: VADM is an acronym for "Virtual Axial Dipole Moment"

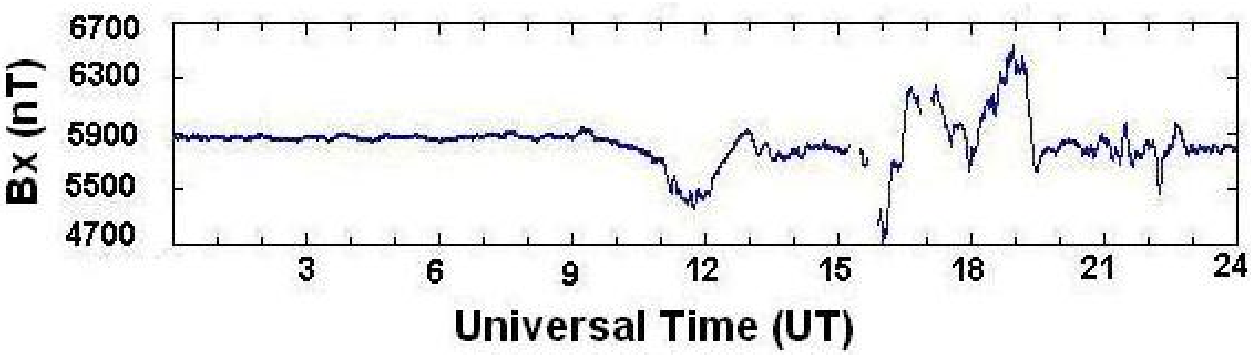
Problem 1 - During the last 12,000 years, what has been the range in the VADM dipole strength as indicated by the black line? Answer: Estimating from the lowest and highest values reached by the black line we get a range from 7.0 to 11.5 x 10^{22} Am^2.

Problem 2 - In about how many years from the present would you predict that the VADM will reach the lower end of its range in the last 12,000 years? Answer: The 'current era' are the years to the far-right in the top graph. The trend shows a slope of 10.5 to 8.5 from about 1000 years ago to 250 years ago. The slope is then (8.5 - 10.5)/(250 - 1000) = 0.003/year. The lower limit of the range is at 7.0 which is 1.5 below the last plotted point that occurred 250 years ago, so -250 + 1.5/0.003 = 500 years from now. Students may also solve this problem graphically with a ruler by extending the line for 'VADM=7.0' to where it meets up with the trend line from the last 1000 years of data.

Problem 3 - Based on the slope of the line in the lower figure, what is the current rate-of-change of the magnetic field in terms of percent per century? Answer: The decrease was from 8.6 to 8.0 over 170 years. This is a percentage change of 0.6/8.6 x 100% = 6.9% over 1.7 centuries and so the rate of decrease has been 6.9%/1.7C = 4% per century.

Problem 4 - If the decline continues at this pace, by what year will the strength of Earth's main dipole field be near-zero? Answer: To go from 8.0 to 0.0 at a rate of 4%/100 years will take 100% / 4% = 25 centuries or 2500 years. Adding this to the current year, 2009 gives us the year 4509 AD. Students answer will vary depending on the actual current year.

Problem 5 - Comparing the trends displayed by the upper plot with the lower graph, do you think the current rate-of-change exceptional? Answer: This question asks whether there have been other times in the last 12,000 years when the SLOPE of the data has been at least as steep as the current slope (i.e. rate of change). Some of the line slopes around 9000 years ago seem, at least for a limited time, to be as rapid as the current era. The period between 2000 and 3000 years ago also shows a similar rapid decline. The current era does seem unique in terms of the duration of this decline which has lasted for 1,500 years.



In addition to the very slow changes that take years or centuries, Earth's magnetic field also changes over times as short as seconds to hours. Most of these changes are very sudden and infrequent, so scientists have come to call them 'magnetic storms'. They are discovered by measuring the strength of Earth's magnetic field at ground-level, with an instrument called a magnetometer. The Earth's field has an average strength of about 50,000 nano-Teslas (nT), but magnetic storms causes changes as small as 1 nT to occur.

The figure above shows a 24-hour long magnetogram that presents the measurements made every minute of the day of Earth's Bx magnetic field component. Recall that Earth's magnetic field can be described by three numbers (called components) measured along a north-south line (Bx), along an east-west line (By), and along the vertical direction (Bz). The horizontal axis in the magnetogram is marked every 3 hours in Universal Time (UT). The vertical axis is a measurement of Bx in units of nanoTeslas. For example, at 12:00 UT the value for Bx was about 5500 nT.

Problem 1 - During what period of the day was Earth's magnetic field relatively quiet and undisturbed, and want was the average vale for Bx during this time?

Problem 2 - What was the Bx value near 24:00 UT in units of milliGauss? (Note 1 Tesla = 10,000 Gauss).

Problem 3 - A geophysicists looks at this magnetogram and sees that there were three distinct storm events during this day. These are often called magnetic sub-storms. During what time intervals did they occur, and what was the highest or lowest value for Bx that was reached?

Problem 4 - Earth's magnetic field can become dilated or compressed for brief periods of time. Which of the three storm events might correspond to compressions or dilations?

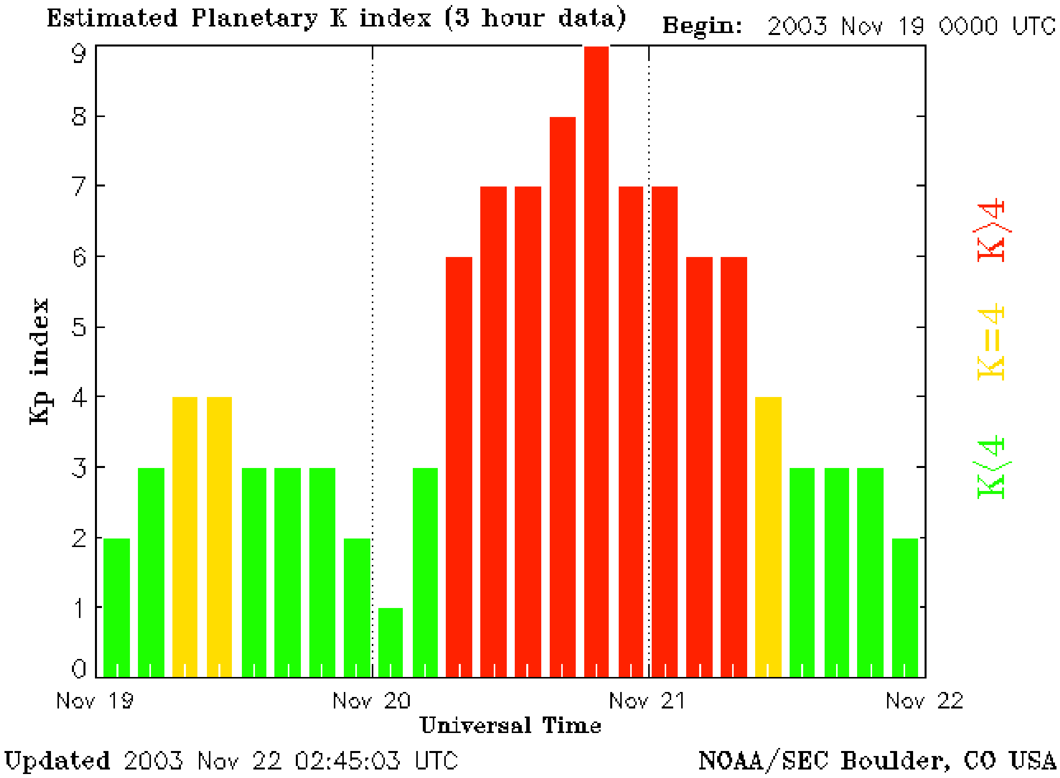
Problem 1 - During what period of the day was Earth's magnetic field relatively quiet and undisturbed, and want was the average vale for Bx during this time? Answer: The graph is mostly flat between 00:00 and 09:00 UT with an average value close to about 5900 nT.

Problem 2 - What was the Bx value near 24:00 UT in units of milliGauss? (Note 1 Tesla = 10,000 Gauss). Answer: The graphed value is near 5800 nT then 5800 nT x (1.0 T/1,000,000,000 nT) x (10,000 G/1 T) = 0.058 Gauss. Then 0.058 Gauss x (1000 milliGauss/1 Gauss) = 58 milliGauss. Depending on estimation methods used, students answers should be close to this value. The important thing is the unit conversion.

Problem 3 - A geophysicists looks at this magnetogram and sees that there were three distinct storm events during this day. These are often called magnetic sub-storms. During what time intervals did they occur, and what was the highest or lowest value for Bx that was reached? Answer; The three largest variations occurred approximately between 10:00 - 13:00 UT, 15:00 - 18:00 UT and 18:00 - 19:30 UT.

Problem 4 - Earth's magnetic field can become dilated or compressed for brief periods of time. Which of the three storm events might correspond to compressions or dilations? Answer: Students should be able to figure out that, when a field is 'dilated' it becomes weaker, and when it is compressed it becomes stronger. The possible interpretations of the three sub-storms is that the field was dilated between 10:00-13:00 UT, and compressed in two separate episodes between 15:00-18:00 UT and 18:00-19:30 UT.

Magnetic Storms - II



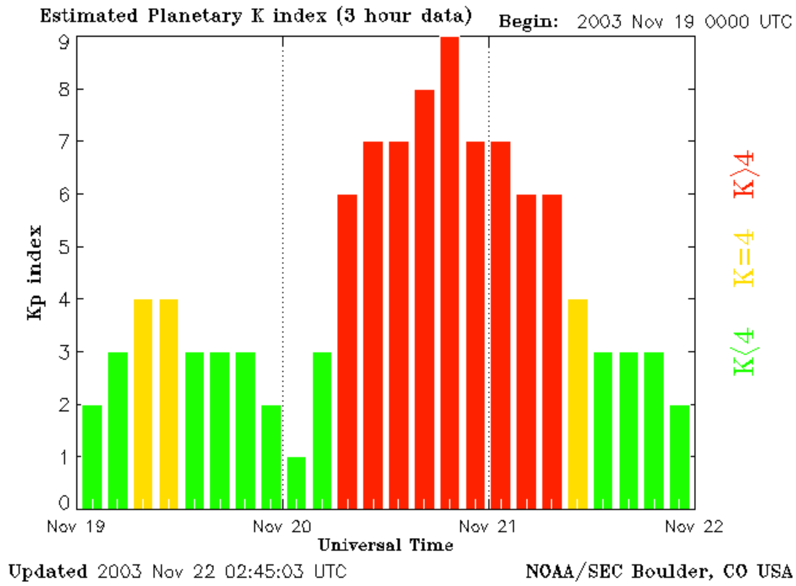
Magnetic observatories generate a huge amount of data – far too much for anyone to digest easily. To help scientists take a quick measure of Earth's magnetic storminess, the magnitude of the disturbance in Earth's magnetic field is measured at 13 observatories and then averaged together. This average value is then reported every three hours as the **Kp Index**, which is then converted to a range from 0 to 9.

The bar graph above shows the changes in this index during the time of a major magnetic storm on November 20, 2003. Prior to the storm, Earth's magnetic field was in a typically disturbed state with variations in Kp from 2 to 4. But after a solar disturbance collided with Earth's magnetic field, the variations jumped to a Kp of 7 and higher within a few hours. This particular storm caused spectacular Northern Lights seen all across North America and Northern Europe.

Problem 1 – If each bar is 3-hours wide, how long did the storm last above a level of Kp = 4?

Problem 2 – At what time did the storm reach its maximum Kp value?

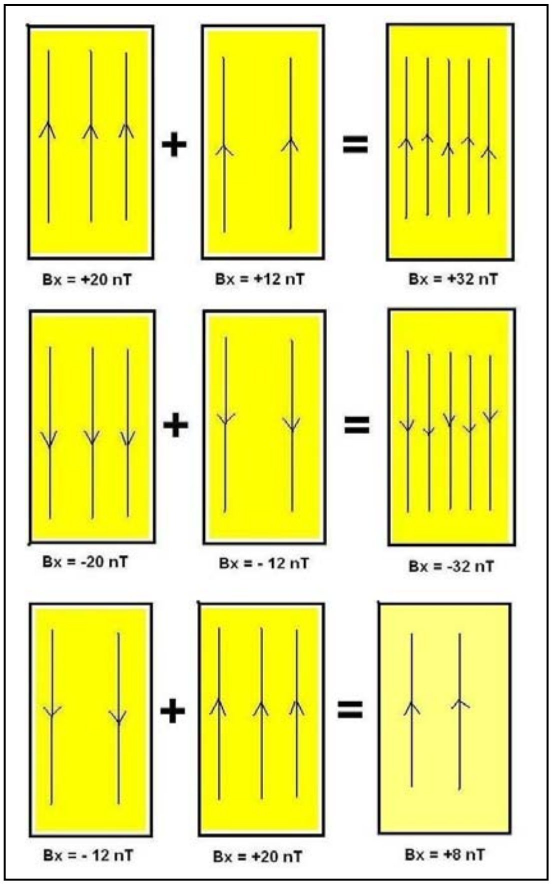
Problem 3 - If New York City is 4 hours behind Universal Time, what time was it in New York during the height of the storm?



Question 1 – If each bar is 3-hours wide, how long did the storm last?
Answer: The red portion of the bar graph which covers the most intense phase of the storm extends 9 bars or 9 x 3h = 27 hours!

Question 2 – At what time did the storm reach its maximum Kp value?
Answer: This occurred at the bar which spans the times 19:00 to 21:00 UT so you can take the start time as 19:00 UT, or the end time 21:00 UT or the mid-point time of the bar of 20:30 UT.

Question 3 - If New York City is 4 hours behind Universal Time, what time was it in New York during the height of the storm?
Answer: Taking the mid-time of 20:30 UT, the Eastern Standard Time in New York would be 20:30 – 4:00 = 16:30 EST or 4:30 PM.



Like any other physical object, you can add and subtract magnetic fields in much the same way you add and subtract money, water, or acorns! The difference is that a magnetic field doesn't exist at one point in space like a dollar-bill, or an acorn!

When you place two bar magnets close together, the magnetic fields automatically add and subtract at each point in space to create the new, combined, magnetic field.

The figure to the left shows an example of magnetic addition for North and South polarities. Notice that the sign of the magnetic intensity, Bx, indicates the polarity. Also notice that the larger the magnitude of Bx, the more magnetic lines of force we draw in the box. Each box represents a region of space that can be 1 centimeter across (bar magnet) , or 100 million kilometers across (the sun). It all depends on how big the magnetic field is in space!

Problem 1 - Add the following magnetic fields, and state the polarities of the two magnetic fields, and the final polarity of the resulting magnetic field:

A)	Bx = +13 Gauss	Bx = - 7 Gauss
B)	Bx = -45 Gauss	Bx = -15 Gauss
C)	Bx = +0.0035 Gauss	Bx = +0.0070 Gauss
D)	Bx = -21 Gauss	Bx = +21 Gauss
E)	Bx = -1,457,981 Gauss	Bx = +1,457,900 Gauss

Problem 2 - A student decides to combine two bar magnets to make a stronger magnetic field at the location of a compass. At the location of the compass, Magnet-A has a strength of Bx = -25.2 Gauss and Magnet B has a strength of Bx= -25.2 Gauss.

A) Are the magnets oriented the same way?

B) What is the new intensity of the combined field at the location of the compass?

C) What is the polarity of the combined field?

D) Suppose one of the magnets was flipped in its North-South orientation. What would be the strength of the magnetic field at the compass location, and would the compass continue to operate?

Problem 1 - Add the following magnetic fields, and state the polarities of the two magnetic fields, and the final polarity of the resulting magnetic field:

A) Bx = +13 Gauss Bx = - 7 Gauss

13 Gauss North plus 7 Gauss South equals + 13 - 7 = +6 Gauss which is 6 Gauss North polarity

B) Bx = -45 Gauss Bx = -15 Gauss

45 Gauss South plus 15 Gauss South equals -45 + -15 = -60 Gauss or 60 Gauss South polarity

C) Bx = +0.0035 Gauss Bx = +0.0070 Gauss

0.0035 Gauss North plus 0.0070 Gauss North equals +0.0105 Gauss or 0.0105 Gauss North polarity

D) Bx = -21 Gauss Bx = = +21 Gauss

21 Gauss South plus 21 Gauss North equals -21 + 21 = 0 Gauss or 0 Gauss with no polarity

E) Bx = -1,457,981 Gauss Bx = +1,457,900 Gauss

1,457,981 Gauss South plus 1,457,900 Gauss North equals 81 Gauss South polarity

Problem 2 - A student decides to combine two bar magnets to make a stronger magnetic field at the location of a compass. At the location of the compass, Magnet-A has a strength of Bx = -25.2 Gauss and Magnet B has a strength of Bx= -25.2 Gauss.

A) Are the magnets oriented the same way?

Answer: Yes because the sign of the Bx is the same at the same location in space.

B) What is the new intensity of the combined field at the location of the compass?

Answer: Bx = -25.2 + (-25.2) = -50.4 Gauss.

C) What is the polarity of the combined field? Answer: The sign of Bx is negative so the polarity is South.

D) Suppose one of the magnets was flipped in its North-South orientation. What would be the strength of the magnetic field at the compass location, and would the compass continue to operate?

Answer: At the location of the compass, the polarity of one of the magnets would be reversed from South to North. The values for Bx would be Bx = -25.2 Gauss and Bx = +25.2 Gauss. Adding these two together would give Bx = -25.2 + (+25.2) = 0 Gauss, so at the compass, the magnetic fields would cancel with no net polarity. The compass would not sense any magnetic field and so would not operate.

Magnetic Arithmetic - II

$$A = (a, b, c)$$

$$B = (d, e, f)$$

$$S = A+B = (a+d, b+e, c+f)$$

$$S_x = a + d$$
$$S_y = b + e$$
$$S_z = c + f$$

A real magnetic field exists in 3-dimensions, and this makes adding them a bit more challenging. Instead of just one number to keep track of, you have three: One for each of the three directions in space.

Although physicists can draw the magnetic field at each location in space, and then add them geometrically, it is much easier to just work with their 'magnetic coordinates'. These are written, at each point in space (X,Y,Z) as a set of three numbers called the magnetic components: (Bx, By, Bz). For example, (+3.0 G, -2.0 G, -10.5 G) are the components of the magnetic field, and Bx = +3.0 means that along the x-direction, it has a strength of 3.0 Gauss pointed Northwards.

The figure to the left shows how to add two 3-dimensional magnetic fields, A and B, using their intensity components.

Problem 1 - Add the following magnetic fields for a particular point in space:

A) (-2.0, + 3.0, -5.0) and (+5.0, -3.0, -4.0)
B) (+5.0, -4.0, + 7.0) and (-3.0, +2.0, -6.0)
C) (+13.0, -3.5 ,-9.6) and (-6.5 ,-3.0, +3.1)
D) (-1054, +1203, -4529) and (+235, -1123, -471)

Problem 2 - Which of the two combinations (A + B) has the strongest combined field?

1) A = (-3.0, +4.0, -5.0) B = (+5.0, +1.0, +10.0)
2) A = (+145, - 350, -1100) B = (-120, +375, +1125)

Problem 3 - What might you suppose the rules are for subtracting magnetic fields? Create an example of subtracting two magnetic fields A and B.

Problem 1 - Add the following magnetic fields for a particular point in space:
A) (-2.0, + 3.0, -5.0) (+5.0, -3.0, -4.0)
Answer: Sx = -2.0 + (+5.0) = +3.0 Sy = +3.0 + (-3.0) = 0.0 Sz = -5.0 + (-4.0) = -9.0
So S = (+3.0, 0.0, -9.0)

B) (+5.0, -4.0, + 7.0) (-3.0, +2.0, -6.0) Answer: S = (+2.0, -2.0, +1.0)
C) (+13.0, -3.5 ,-9.6) (-6.5 ,-3.0, +3.1) Answer: S = (+6.5, -6.5, -6.5)
D) (-1054, +1203, -4529) (+235, -1123, -471) Answer: S = (-819, +80, -5,000)

Problem 2 - Which of the two combinations (A + B) has the strongest combined field?
1) A = (-3.0, +4.0, -5.0) B = (+5.0, +1.0, +10.0) S = (+2.0, +5.0, +5.0)
2) A = (+145, - 350, -1100) B = (-120, +375, +1125) S = (+25.0, +25.0, +25.0)
Answer: The second combination produces a resulting field in which each component is larger than the corresponding components of the first combination, so the second combination leads to the stronger field.

Problem 3 - What might you suppose the rules are for subtracting magnetic fields? Create an example of subtracting two magnetic fields A and B.
Answer: In the case of subtraction,

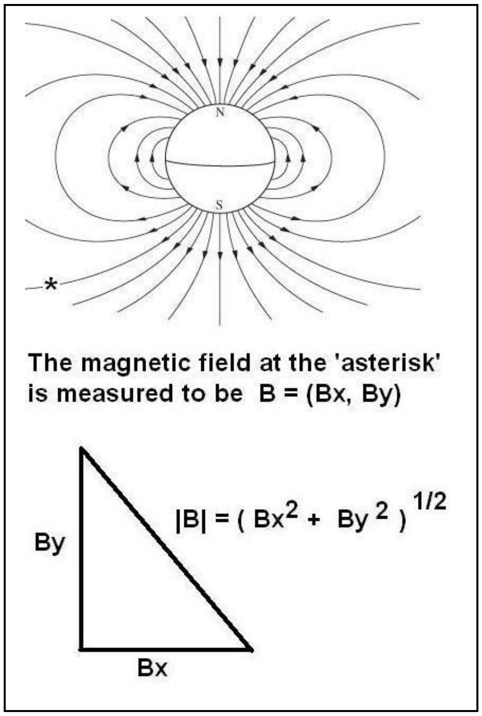
If A = (a, b, c) and B = (d, e, f)

Then S = A-B = (a-d, b-e, c-f)
And so,
Sx = a - d
Sy = b - e
Sz = c - f

An example would be A = (+3.0, -4.0, +5.0) and B = (-6.0, +8.0, +10.0)

So Sx = a - d = +3.0 - (-6.0) = +3.0 + 6.0 = +9.0
Sy = b - e = -4.0 - (+8.0) = -4.0 - 8.0 = -12.0
Sz = c - f = +5.0 - (+10.0) = +5.0 - 10.0 = -5.0

And so the resulting magnetic field would have a strength of S = (+9.0, -12.0, -5.0) at a particular location in space.



Imagine that you have traced out the magnetic field from a bar magnet on a 2-dimensional sheet of paper. How do you figure out how strong the field is at a particular point in space?

Once you know the components to the field at that point, (Bx, By), it's a piece of cake! That's because Bx and By are actually the legs of a right-triangle! The total strength of the field is just the hypotenuse of that triangle, whose length you can figure out, either by constructing a scaled drawing, or using the Pythagorean Theorem.

Problem 1 - Draw a scaled model for the following problems and determine the magnitude (i.e. the absolute magnitude) of the total field defined by the two components to an accuracy of one decimal point. (Hint: use a scale of 1 Gauss = 1 Centimeter).

A) Bx= +3.0 Gauss and By = +4.0 Gauss
B) Bx = +1.0 Gauss and By = +1.0 Gauss
C) Bx = -3.0 Gauss and By = +4.0 Gauss

Problem 2 - The Pythagorean Theorem states that the length of the hypotenuse of a right triangle equals the square-root of the sum of the squares of the other two sides. Use the 'PT' to calculate the magnitude of the magnetic field, B, indicated by the following components to an accuracy of one decimal point:

A) Bx= +3.0 Gauss and By = +4.0 Gauss
B) Bx = +1.0 Gauss and By = +1.0 Gauss
C) Bx = -3.0 Gauss and By = +4.0 Gauss
D) B = (-2.5, +3.8)
E) B = (+12.5, -2.45)

Problem 3 - What is the magnitude of the sum, S, of the following pairs of magnetic fields?
A) A = (-2.0, +5.9) and B = (+5.0, +6.0)
B) A = (+123.0, +114.0) and B = (+27.0, -100.0)

Problem 1 - Draw a scaled model for the following problems and determine the magnitude of the total field defined by the two components to an accuracy of one decimal point. (Hint: use a scale of 1 Gauss = 1 Centimeter). Answer: Students may use graph paper and draw the Bx and By axis at a scale of 1 cm = 1 Gauss per division. They will draw the triangle with the measured sides, then with a ruler, measure the length of the hypotenuse in centimeters. This will then be converted into Gauss units (1 cm = 1 Gauss) to obtain the answer.

A) Bx= +3.0 Gauss and By = +4.0 Gauss Answer: 5.0 Gauss.
B) Bx = +1.0 Gauss and By = +1.0 Gauss Answer: 1.4 Gauss.
C) Bx = -3.0 Gauss and By = +4.0 Gauss Answer: 5.0 Gauss.

Note that the problem is asking for the magnitude of the field so the sign does not matter.

Problem 2 - The Pythagorean Theorem states that the length of the hypotenuse of a right triangle equals the square-root of the sum of the squares of the other two sides. Use the 'PT' to calculate the magnitude of the magnetic field, B, indicated by the following components to an accuracy of one decimal point:

A) Bx= +3.0 Gauss and By = +4.0 Gauss Answer: $B = (3.0^2 + 4.0^2)^{1/2} = 5.0$ Gauss
B) Bx = +1.0 Gauss and By = +1.0 Gauss Answer: $B = (2)^{1/2} = 1.4$ Gauss
C) Bx = -3.0 Gauss and By = +4.0 Gauss Answer: $B = ((-3.0)^2 + 4.0^2)^{1/2} = 5.0$ Gauss
D) B = (-2.5, +3.8) Answer: $B = ((-2.5)^2 + 3.8^2)^{1/2} = 4.5$ Gauss
E) B = (+12.5, -2.45) Answer: $B = ((12.5)^2 + (-2.45)^2)^{1/2} = 12.7$ Gauss

Problem 3 - What is the magnitude of the sum, S, of the following pairs of magnetic fields?
A) A = (-2.0, +5.9) and B = (+5.0, +6.0)
Answer S = A + B = (+3.0, +11.9) so the magnitude of $S = ((3.0)^2 + (11.9)^2)^{1/2} = 12.3$ Gauss
B) A = (+123.0, +114.0) and B = (+27.0, -100.0)
Answer S = A + B = (+150.0, +14.0) so the magnitude of $S = ((150)^2 + (14)^2)^{1/2} = 150.7$ Gauss

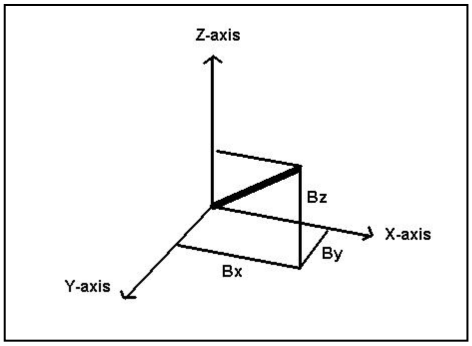
Unlike temperature, magnetism requires three numbers to define the strength of its field in space. Scientists call magnetism a **Vector** quantity because it is defined by both its magnitude at a point in space, and its direction at that point, given by the coordinate (X, Y, Z). The Pythagorean Theorem is used to calculate the magnitude (or total strength) of the magnetic field from the separate Bx, By and Bz quantities that make up its description as a field in 3-dimensional space. To find the Bx, By and Bz components of Earth's magnetic field (in units of nanoTeslas, nT) where you live, visit the International Geomagnetic Reference Field Model (Part 2 Form)

http://nssdc.gsfc.nasa.gov/space/model/models/igrf.html

Enter the year (2004) and the requested geographic latitude, longitude (in degrees, minutes and seconds – D M S entries in table) and elevation (Use 0.0 km for table). You can find the geographic coordinates for a specific location at

http://geonames.usgs.gov

Follow 'Query GNIS" to the input form. Select 'Civil' for a town name.



The Pythagorean Theorem in 3-dimensions is

$$D = \sqrt{x^2 + y^2 + z^2}$$

Use the Pythagorean Theorem to fill-in the last column of the table

City	Longitude D M S	Latitude D M S	Bx (nT)	By (nT)	Bz (nT)	Total B (nT)
Chicago	87 54 55	41 50 05	26600	1234	48620	**55434**
Boston	71 05 00	42 18 00	25251	2234	46676	
Miami	80 32 00	25 37 00	36274	0.2	28396	
Hollywood	118 20 00	34 01 00	32161	-2684	39236	
Bangor	68 47 15	44 49 56	23437	2600	48244	
Kansas City	94 43 37	39 07 06	28846	365	46535	
Sioux Falls	96 43 48	43 32 48	25602	283	50988	
Spokane	117 22 00	47 37 00	22977	-3263	53054	
Provo	103 52 06	43 10 02	26045	-875	50767	
Anchorage	149 15 02	61 10 00	16377	-3572	53739	
Honolulu	154 53 24	19 33 15	32644	1402	14594	
Sedona	111 47 35	34 50 38	31818	-1978	41379	

Problem 1 - What cities have the highest and lowest magnetic field (**B**) strengths?

Problem 2 - What is the average **B** value of Earth's magnetic field for all locations?

Problem 3 – Some adults think that Sedona Arizona has special 'powers'. How does the magnetism at this location compare to other locations in the table?

Problem 4: Plot the **By** values on a map. What pattern do you see?

City	Longitude D M S	Latitude D M S	Bx (nT)	By (nT)	Bz (nT)	Total B (nT)
Chicago	87 54 55	41 50 05	26600	1234	48620	**55434**
Boston	71 05 00	42 18 00	25251	2234	46676	75116
Miami	80 32 00	25 37 00	36274	0.2	28396	65148
Hollywood	118 20 00	34 01 00	32161	-2684	39236	71847
Bangor	68 47 15	44 49 56	23437	2600	48244	75941
Kansas City	94 43 37	39 07 06	28846	365	46535	77430
Sioux Falls	96 43 48	43 32 48	25602	283	50988	80688
Spokane	117 22 00	47 37 00	22977	-3263	53054	81894
Provo	103 52 06	43 10 02	26045	-875	50767	80701
Anchorage	149 15 02	61 10 00	16377	-3572	53739	79609
Honolulu	154 53 24	19 33 15	32644	1402	14594	50607
Sedona	111 47 35	34 50 38	31818	-1978	41379	73871

Problem 1 - What cities have the highest and lowest magnetic field (**B**) strengths?
Answer: The city with the highest total magnetic field strength is Spokane, Washington (81894 nT). The city with the smallest total magnetic field strength is Honolulu, Hawaii (50607 nT)

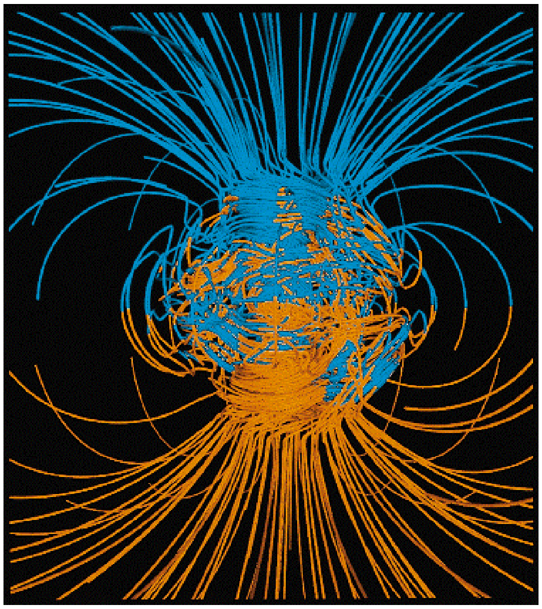
Problem 2 - What is the average **B** value of Earth's magnetic field for all locations?
Answer : (55434 + 75116 + 65148 + 71847 + 75941 + 77430 + 80688 + 81894 + 80701 + 79609 + 50607 + 73871) / 12 = 868286/12 = **72357 nT**

Remember to have the students give the answer in the correct physical units.

Problem 3 – Some adults think that Sedona Arizona has special 'powers'. How does the magnetism at this location compare to other locations in the table?
Answer: There are several things the student can note. 1) It has only the 8^{th} strongest magnetic field out of 12 cities; 2) It has the third lowest Bz value (41379 nT); and 3) It has the fourth-lowest By value (-1978). None of these are as remarkable as what we find among the other large cities in this random sample.

Problem 4: Plot the **By** values on a map. What pattern do you see?
Answer: The most obvious thing the students should notice is that:

1) The By magnetic values are always much smaller than for the Bx and Bz magnetic components. In fact they are typically only about 10% of the other two components;
2) The values to the east of latitude 100 to 105 degrees are positive. The values to the west are negative. **Note, the reason for this is that the longitude of the magnetic pole is 105 degrees, so this is the 'axis of symmetry' for these values.**


This is a mathematical model describing Earth's magnetic field based upon functions computed for specific points in space.
(Courtesy Gary Glatzmaier, Los Alamos)

A magnetic field is defined at each point in space (x,y,z) in terms of its strengths in Teslas or Gauss units, but it can be a nuisance to have to carry around large tables that give specific values at each point. In fact such a table would have to be infinite in size!

Luckily, magnetic fields can be described mathematically by using specific equations; one equation for each of its three components, (Bx, By, Bz), in 3-dimensional space. In the function notation:

$$Bx = f(x,y,z)$$
$$By = g(x,y,z)$$
$$Bz = h(x,y,z)$$

Now all we need to do is to create the particular functions to give us the magnetic field values at each point. This can be done by 'fitting' various functions to the data to fine the best match, or by using physics to actually calculate what the functions should look like along each axis. Both techniques are very common.

Problem 1 - Create a short table evaluated at 5 points of your choosing, of the values for Bx, By, Bz and the magnitude of B for the indicated functional forms. Round all values to the nearest 0.1 place:

A) $Bx = x$; $By = 0$; $Bz = y$

B) $Bx = yz$, $By = xz$ $Bz = 3xy$

C) $B = (x+5,\ xy-z,\ y^2+3)$

Problem 2 - If a magnetic field is given by $B = (x - 3, y + 2, z - 8)$, find all of the points in space (x,y,z) where the field vanishes (i.e all three components are simultaneously zero).

Problem 1 - Create a short table evaluated at 5 points of your choosing, of the values for Bx, By, Bz and the magnitude of B for the indicated functional forms:

A) Bx = x; By = 0; Bz = y

X	Y	Z	Bx	By	Bz	B
0	0	2	0	0	0	0
1	-1	5	1	0	-1	$(2)^{1/2}$ =1.4
3	2	-2	3	0	2	$(13)^{1/2}$ = 3.6
-5	4	3	-5	0	4	$(41)^{1/2}$ = 6.4
-3	-3	2	-3	0	-3	$3(2)^{1/2}$ = 4.2

Note: $B = (bx^2 + by^2 + Bz^2)^{1/2}$

B) Bx = yz, By = xz Bz = 3xy

X	Y	Z	Bx	By	Bz	B
0	0	2	0	0	0	0
1	-1	5	-5	5	-3	$(34)^{1/2}$ = 5.8
3	2	-2	-4	-6	18	$(376)^{1/2}$ = 19.4
-5	4	3	12	-15	-60	$(3969)^{1/2}$ = 63.0
-3	-3	2	-6	-6	27	$(801)^{1/2}$ = 28.3

C) $B = (x+5, xy-z, y^2+3)$

X	Y	Z	Bx	By	Bz	B
0	0	2	5	-2	3	$(38)^{1/2}$ = 6.2
1	-1	5	6	-6	4	$(88)^{1/2}$ = 9.4
3	2	-2	8	8	7	$(177)^{1/2}$ = 13.3
-5	4	3	0	-23	19	$(890)^{1/2}$ = 29.8
-3	-3	2	2	7	12	$(197)^{1/2}$ = 14.0

Note to Teacher; The computed magnetic values are in magnetic units such as Gauss or Teslas, not in terms of the physical coordinate units such as centimeters or meters.

Problem 2 - If a magnetic field is given by B = (x - 3, y + 2, z - 8), find all of the points in space (x,y,z) where the field vanishes (i.e all three components are simultaneously zero).

Answer: (+3, -2, +8) since if x=3, y=-2 and z=8 B = (0,0,0) and so B vanishes

Parametric Functions and Substitution

Image courtesy Dick Hutchinson

Our sun is an active star that ejects a constant stream of particles into space called the 'solar wind'. From time to time, magnetic activity on its surface also launches fast-moving clouds of plasma into space called 'coronal mass ejections' or CMEs.

When some of these clouds directed at Earth arrive after traveling 93 million miles (150 million km), they cause intense disturbances in Earth's magnetic field. Since the 1800's, these disturbances have been called 'magnetic storms', because instruments on Earth can measure the strength of these disturbances, and they resemble storms in an otherwise very calm magnetic field.

Scientists measure the strength of these magnetic storms in terms of the size of the change they make in the Earth's magnetic field. The strength of Earth's field at the ground is about 0.7 Gauss or 70,000 nanoTeslas. The most intense magnetic storms can change the ground-level field by several percent.

According to research by V. Yurchyshyn, H. Wang and V. Abramenko, which was published in 2004 in the journal Space Weather (vol. 2) the relationship between the magnetic field disturbance, Dst and the Z-component of the interplanetary magnetic field, B_z, is given by:

$$(1) \qquad Dst = -2.846 + 6.54 B_z - 0.118B_z^2 - 0.002B_z^3$$

where Dst and Bz are measured in nanoTeslas (nT).

In 2004, W. D. Gonzales and his colleagues published a paper in the Journal of Atmospheric and Solar Terrestrial Physics, in which they determined a relation between the speed of a solar coronal mass ejection V, in km/sec, and the strength of Dst in nT according to

$$(2) \qquad Dst = 0.00052 \times (0.22 V + 340)^2$$

The relationship between the travel time to Earth from the sun and the speed of the CME was determined from catalogs of CME events by M. J. Owens and P. J. Cargill in research published in 2002 in the Journal of Geophysical Research (vol. 107, p. 1050) in terms of the transit time in days, T, for these coronal mass ejections and their speed, V, in km./sec by

$$(3) \qquad T = -0.0042 \times V + 5.14$$

They also found that the maximum interplanetary magnetic field strength of the CME was given by

$$(4) \qquad B_T = 0.047 V + 0.644$$

1) From the equation 2 and 3 above, find a function that gives Dst in terms of the transit time of the CME. Write the result in expanded form as a quadratic equation.

2) Assuming that $B_Z = B_T / (2)^{1/2}$ use equations 1 and 4 to find a function that gives Dst in terms of V.

3) From equations 2 and 4, find a function that gives Dst in terms of B_T.

Answer Key:

(1) $Dst = -2.846 + 6.54\ B_z - 0.118B_z^2 - 0.002B_z^3$

(2) $Dst = 0.00052 \times (0.22\ V + 340)^2$

(3) $T = -0.0042 \times V + 5.14$

(4) $B_T = 0.047\ V + 0.644$

Problem 1: From the equation 2 and 3: Dst in terms of the transit time of the CME.

Eqn 3: solve for V. $V = (T - 5.14)/(-0.0042) = -238.1\ T + 1223.8$
Eqn 2: Substitute for V in terms of T:

$$Dst = 0.00052 \times (0.22 (-238.1\ T + 1223.8) + 340)^2$$
$$= 0.00052 \times (609.2 - 52.4\ T)^2$$

In expanded form: $Dst = 1.4\ T^2 - 33.2\ T + 193.0$ in nT units

Problem 2: Assuming that $B_Z = B_T / (2)^{1/2}$ use equations 1 and 4 and find Dst in terms of V.

Eqn 4: $Bz = B_T / (2)^{1/2} = (0.047\ V + 0.644)/(2)^{1/2}$
$= 0.033\ V + 0.46$

Substituting into Eq 1:

$$Dst = -2.846 + 6.45 (0.033\ V + 0.46) - 0.118 (0.033\ V + 0.46)^2 - 0.002 (0.033\ V + 0.46)^3$$

$$= (-2.846 + 0.46*6.45 - 0.118*0.46^2 - 0.002* 0.46^3) +$$
$$(6.45*0.033 -0.118*2*0.46*0.033 -0.002*3.0* 0.46^2*0.033)\ V +$$
$$(-0.002*3.0*0.46*0.033^2)\ V^2 -0.002 * 0.033^3\ V^3$$

$$Dst = 0.096 + 0.21\ V - 3.0 \times 10^{-6}\ V^2 - 7.2 \times 10^{-8}\ V^3$$

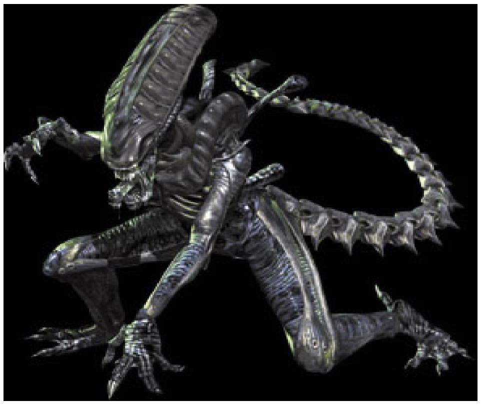
Problem 3: From equations 2 and 4, find a function that gives Dst in terms of B_T.
Eq 4: Solve for V

$$V = (B_T - 0.644)/0.047 = 21.3\ B_T - 13.7$$

Substitute into Eqn 1 : $Dst = 0.00052 \times (0.22 (21.3\ B_T - 13.7) + 340)^2$
$= 0.00052 (4.7\ B_T + 337)^2$

Expanded: $Dst = 0.011\ B_T^2 + 1.65\ B_T + 59.1$ in nT units

Monster Functions in Space Science I



Forget about the wimpy formulas you have played with before. Here is a reasonably complex formula that you will have to evaluate, and which will involve all the skills you have previously learned in algebra…and a mastery of scientific notation too!

Be careful, but don't be shy!

Keep track of your decimal points and exponents!!

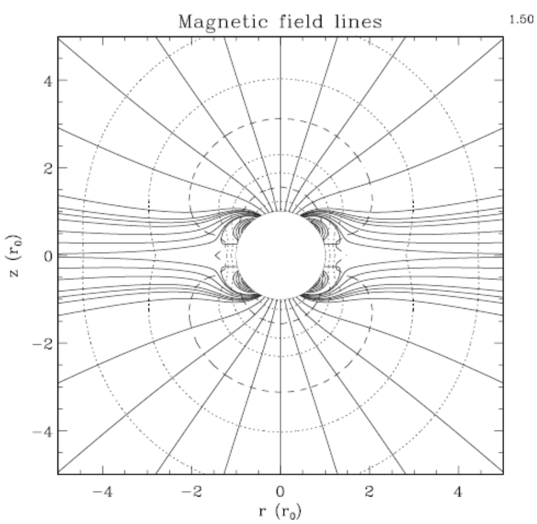
And, oh yes….Watch your back!!!

Abstract. We describe a simple analytic model for the magnetic field in the solar corona and interplanetary space which is appropriate to solar minimum conditions. The model combines an azimuthal current sheet in the equatorial plane with an axisymmetric multipole field representing the internal magnetic field of the Sun. The radial component of the field filling

From '*An Analytic Solar Magnetic Field Model*" by Banaszkiewicz, Axford and McKenzie (Astronomy and Astrophysics, vol. 337, p. 940-944.

$$\frac{B_\rho}{M} = \frac{3\rho z}{r^5} + \frac{15Q}{8}\frac{\rho z}{r^7}\frac{(4z^2-3\rho^2)}{r^2} + \frac{K}{a_1}\frac{\rho}{[(|z|+a_1)^2+\rho^2]^{3/2}}, \quad (1)$$

$$\frac{B_z}{M} = \frac{2z^2-\rho^2}{r^5} + \frac{3Q}{8}\frac{(8z^4+3\rho^4-24\rho^2 z^2)}{r^9} + \frac{K}{a_1}\frac{|z|+a_1}{[(|z|+a_1)^2+\rho^2]^{3/2}}, \quad (2)$$



These formulas give the two components of the solar magnetic field, in units of Gauss, where **B** = $\mathbf{B}_\rho\, \rho + \mathbf{B}_z\, z$ where ρ and z are the unit vectors along these two directions.

<u>Problem 1:</u> Evaluate to the nearest tenth (B_ρ) and (B_z) for the following conditions appropriate to a distance from the sun equal to Earth's orbit using the following information:

$r^2 = \rho^2 + z^2$ K = 1.0

M = $6.03 \times 10^{+17}$ kilometers^3 Q = 1.5

$\alpha_1 = 1.07 \times 10^{+6}$ kilometers

where $z = -3.48 \times 10^7$ kilometers
$\rho = 1.46 \times 10^8$ kilometers.

<u>Problem 2:</u> Find the magnitude of the magnetic field strength using the values of the two computed components from Problem 1.

Answer Key:

$$\frac{B_\rho}{M} = \frac{3\rho z}{r^5} + \frac{15Q}{8}\frac{\rho z}{r^7}\frac{(4z^2 - 3\rho^2)}{r^2} + \frac{K}{a_1}\frac{\rho}{[(|z| + a_1)^2 + \rho^2]^{3/2}}, \quad (1)$$

$$\frac{B_z}{M} = \frac{2z^2 - \rho^2}{r^5} + \frac{3Q}{8}\frac{(8z^4 + 3\rho^4 - 24\rho^2 z^2)}{r^9} + \frac{K}{a_1}\frac{|z| + a_1}{[(|z| + a_1)^2 + \rho^2]^{3/2}}, \quad (2)$$

For $z = -3.48 \times 10^7$ kilometers $\rho = 1.46 \times 10^8$ kilometers.
Then $r = 1.5 \times 10^8$ kilometers………this equals the Earth-Sun orbital distance!

$$B\rho/M = \frac{3\,(1.46 \times 10^8)(-3.48 \times 10^7)}{(1.5 \times 10^8)^5} + \frac{15\,(1.5)}{8}\,\frac{(1.46 \times 10^8)(-3.48 \times 10^7)}{(1.5 \times 10^8)^7}\,\frac{(4\,(-3.48 \times 10^7)^2 - 3\,(1.46 \times 10^8)^2)}{(1.5 \times 10^8)^2}$$

$$+ \frac{1.0}{1.07 \times 10^{+6}}\,\frac{1.46 \times 10^8}{[\,(3.48 \times 10^7 + 1.07 \times 10^{+6})^2 + (1.46 \times 10^8)^2\,]^{3/2}}$$

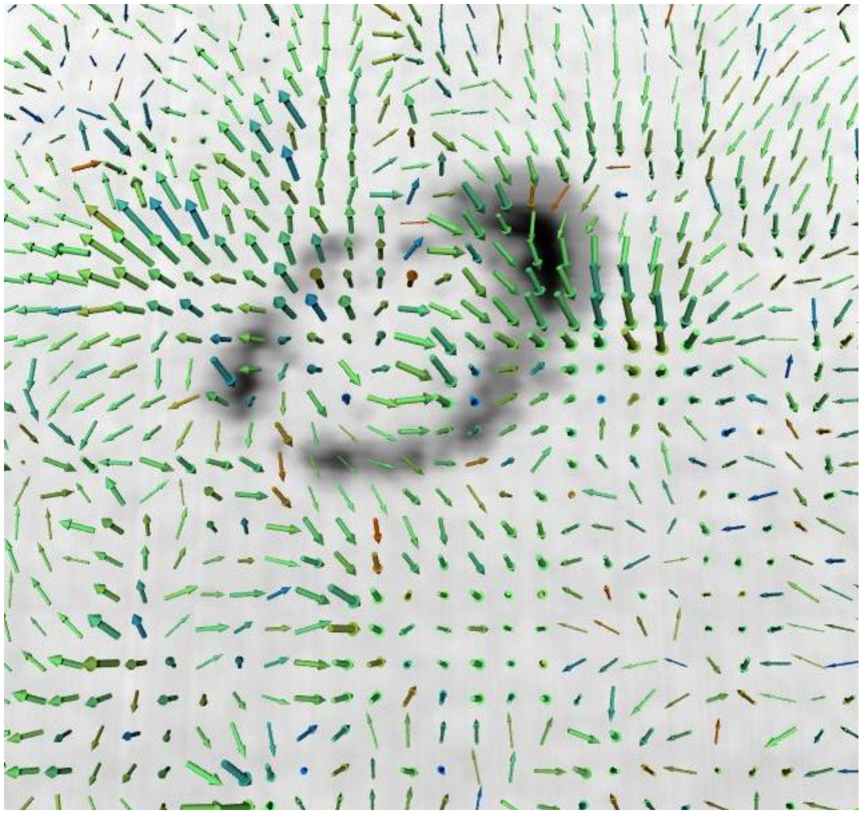
$B_\rho = (6.03 \times 10^{+17})\,(-2.0 \times 10^{-25} + 2.3 \times 10^{-41} + 4.0 \times 10^{-23}) = 2.4 \times 10^{-5}$ Gauss

$$Bz/M = \frac{2(-3.48 \times 10^7)^2 - (1.46 \times 10^8)^2}{(1.5 \times 10^8)^5} + \frac{3(1.5)}{8}\,\frac{[\,8(-3.48 \times 10^7)^4 + 3(1.5 \times 10^8)^4 - 24(1.46 \times 10^8)^2\,(-3.48 \times 10^7)^2]}{(1.5 \times 10^8)^9}$$

$$+ \frac{1.0}{(1.07 \times 10^{+6})}\,\frac{(3.48 \times 10^7 + 1.07 \times 10^6)}{(\,(3.48 \times 10^7 + 1.07 \times 10^6)^2 + (1.46 \times 10^8)^2\,)^{3/2}}$$

$B_Z = (6.03 \times 10^{+17})\,(-2.5 \times 10^{-25} + 1.1 \times 10^{-41} + 9.8 \times 10^{-24}) = 5.8 \times 10^{-6}$ Gauss

Problem 2: Use the Pythagorean Theorem to find B. $B = (\,(2.4 \times 10^{-8})^2 + (5.8 \times 10^{-6})^2)^{1/2} = 2.5 \times 10^{-5}$ Gauss.



The above figure shows a map of the magnetic field directions near a sunspot (shaded area). (Courtesy: Christoph Keller, SOLIS Vector SpectroMagnetograph) The image is about 10,000 kilometers across.

Problem 1 - Circle those areas in which the magnetic field polarity is mostly Southwards.

Problem 2 - Circle those areas in which the magnetic field polarity is mostly Northwards.

Problem 3 - A simple bar magnet has exactly one South and one North pole. What can you conclude about the sources of sunspot magnetism in the above figure?

Problem 4 - If you examine the magnetic field in the immediate vicinity of the sunspot, what can you conclude about the magnetic field of a sunspot?

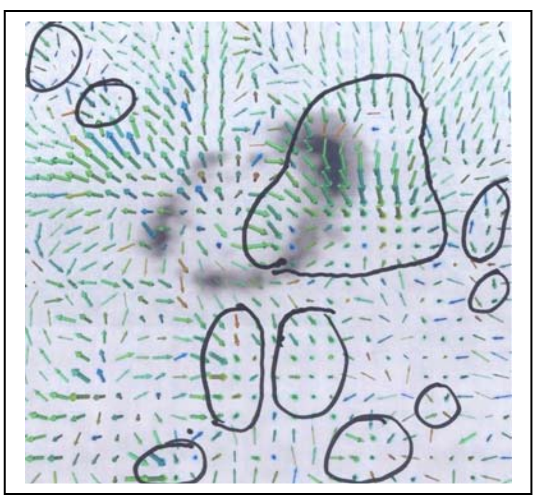
Problem 1 - Circle those areas in which the magnetic field polarity is mostly Southwards.
Answer: See figure below-left. Students should recall that the convention for representing magnetic polarity is that the arrows on field lines point into the surface for a South-type polarity. Students should look at the map and encircle all of the regions where the arrows are mostly pointing INTO the page.

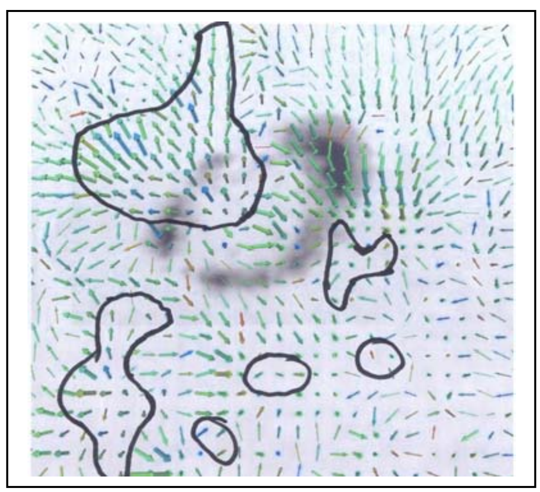
Problem 2 - Circle those areas in which the magnetic field polarity is mostly Northwards.
Answer: See figure below-right. Students should look at the map and encircle all of the regions where the arrows are mostly pointing OUT of the page.

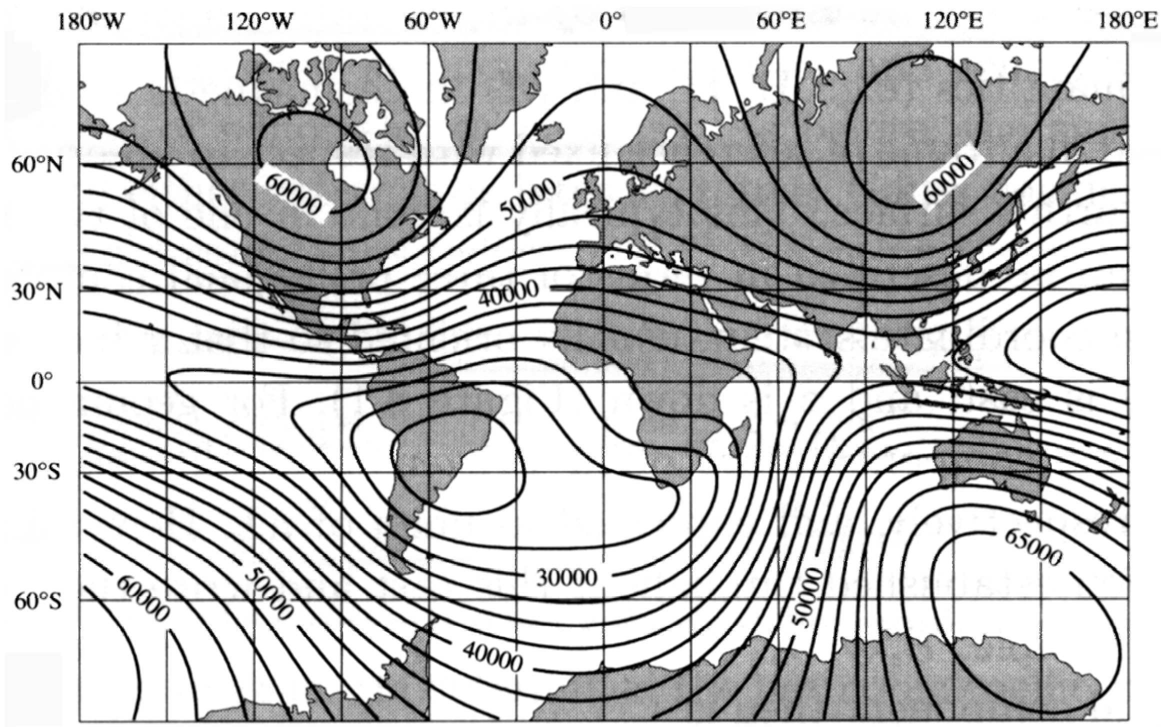
Problem 3 - A simple bar magnet has exactly one South and one North pole. What can you conclude about the sources of sunspot magnetism in the above figure?
Answer: Although the surface is magnetically complicated, there are about as many regions circled on the South polarity map as on the North polarity map, so the solar surface near a sunspot does resemble a number of 'bar magnet' regions combined together.

Problem 4 - If you examine the magnetic field in the immediate vicinity of the sunspot, what can you conclude about the magnetic field of a sunspot? Answer: Students should be able to tell that the strongest magnetic field (longest arrows) are found directly near the sunspot region, and that there is a distinct separation between the North and South polarity field components.

Note: For more about Vector SpectroMagnetograms visit the SOLIS website at http://solis.nso.edu for additional information and an archive of published research papers. These often include images such as the one used in this math problem.







The map above shows the intensity of Earth's magnetic field in 1990 in units of nanoTeslas (nT). It is called an isodynamic map, and the contour interval is 2,500 nT. The intensity at each point, B, is computed from the individual magnetic field components (Bx, By, Bz) by using the Pythagorean Theorem.

Problem 1 - In which geographic regions is the surface field: A) The weakest? B) The strongest?

A gradient is a measure of how rapidly a quantity changes its value across a span of distance. A 'steep' gradient, such as 1000 nT/kilometer, indicates a very rapid change in the quantity across a distance, while a 'shallow' gradient such as 5 nT/kilometer indicates a very slow change of the quantity with distance. A gradient is computed like a slope by using the formula $G = (y2 - y1)/(x2 - x1)$.

Problem 2 - Calculate the following magnetic gradients for the indicated pairs of points;
A) Panama City to Mexico City: P1 = (35,000 nT, 0.0) P2 = (42,500 nT, 1,200 km)
B) Buenos Aires to Rio de Jeniero: P1 = (30,000 nT, 0.0) P2 = (23,000 nT, 2,200 km)

Problem 3 - A geologist makes a series of 6 magnetic intensity measurements equally spaced along two different tracks. Each track is 1 kilometers in length. A) What is the average magnetic intensity for each path? B) What are the largest and smallest magnitudes for gradients (nT/meter) detected along each path, and C) Which track suggests something interesting that is probably worthy of further investigation? (Units are in nT)

Path 1)	50,000	50,100	49,900	50,300	50,500	50,800
Path 2)	49,800	49,500	50,200	51,300	49,800	49,500

Problem 1 - In which geographic regions is the surface field:
A) The weakest? Answer: The contour levels over Brazil are near 25,000 nT. This is called the South Atlantic Anomaly because this is where the van Allen Belts are slightly closer to Earth's surface and are responsible for radiation effects when astronauts in orbit and commercial jet flights pass through this region.
B) The strongest? Answer: Near the magnetic poles where the contours are near 60,000 nT (North Magnetic Pole) and 65,000 nT near the South Magnetic Pole.

Problem 2 - Calculate the following magnetic gradients for the indicated pairs of points;
A) Panama City to Mexico City: P1 = (35,000 nT, 0.0) P2 = (42,500 nT, 1,200 km)
B) Buenos Aires to Rio de Jeniero: P1 = (30,000 nT, 0.0) P2 = (23,000 nT, 2,200 km)

Answer: A) G = (42,500 - 35,000)/(1,200 - 0.0) = + 6.25 nT/kilometer. This means as you travel northward in the northern hemisphere, the magnetic field increases in strength as your latitude increases. This is because you are approaching the North magnetic Pole.

Answer B) G = (23,000 - 30,000)/(2,200 - 0.0) = -3.2 nT/kilometer. This means that as you are traveling northward in the southern hemisphere, the magnetic field is decreasing in strength as you are moving away from the South Magnetic Pole.

Problem 3 - A geologist makes a series of 6 magnetic intensity measurements equally spaced along two different tracks. Each track is 1 kilometers in length. A) What is the average magnetic intensity for each path? B) What are the largest and smallest magnitudes for gradients (nT/meter) detected along each path, and C) Which track suggests something interesting that is probably worthy of further investigation? (Units are in nT)

Path 1)	50,000	50,100	49,900	50,300	50,500	50,800
Path 2)	49,800	49,500	50,200	51,300	49,800	49,500

Answer: See the calculations in the following Table:

Position (meters)	Track 1 (nT)	Track 2 (nT)	Gradient 1 (nT/m)	Gradient 2 (nT/m)
0	50000	49800		
200	50100	49500	0.5	-1.5
400	49900	50200	-1.0	3.5
600	50300	51300	2.0	5.5
800	50500	49800	1.0	-7.5
1000	50800	49500	1.5	-1.5

A) Average for Track 1 = 50,267 nT Track 2 = 50,016 nT
B) Track 1: smallest = 0.5 largest = 2.0 Track 2: smallest = 1.5 largest = 7.5
C) Track 2 has the largest gradient change between +5.5 and -7.5 in only 200 meters.
Note that although Track 1 has the highest average intensity, Track 2 has the largest changes in gradient which suggests that something is changing the overall magnetic field locally.

Magnetic Force in Three Dimensions

A magnetic field is more complicated in shape than a gravitational field because magnetic fields have a property called 'polarity'. All magnets have a North and South magnetic pole. Depending on where you are in the space near a magnet, the force you feel will be different. The strength of the magnetic field along each of the three directions can be thought of in terms of the axis of a Cartesian coordinate system **x**, **y** and **z** so that, for example, **Bx** is its strength along the x-axis. The three magnitudes for the magnetic strength are given by the formulas in the box below.

$$B_x = \frac{3\,x\,z\,M}{r^5}$$

$$B_y = \frac{3\,y\,z\,M}{r^5}$$

$$B_z = \frac{(3\,z^2 - r^2)\,M}{r^5}$$

x, y and z represent the coordinates of a point in space in multiples of the radius of Earth where 1.0 Re = 6,378 km. For example, 'x = 2.4' means a physical distance of 2.4 x 6378 km = 15,307 kilometers. Any point in space near Earth can be described by its address (x, y, z).

r is the distance from (x,y,z) to the center of Earth found by using the Pythagorean Theorem:

$$r = (Bx^2 + By^2 + Bz^2)^{1/2}$$

M is a constant equal to 31,000 nT Re^3.

Bx, By and Bz computed from these formulae will be in units of nanoTeslas (nT).

Problem 1 - Evaluate these three equations at the orbit of communications satellites for the case where x = 7.0, y = 0.0, z = 0.0 and r = 7.0

Problem 2 - Evaluate these three equations in the Van Allen Belts for the case where x = 0.38, y = 0.19, z = 1.73 and r = 3.0

Problem 3 - Evaluate these three equations near the Moon for the case where x = 0.0, y = 48.0, z = 36 and r = 60.0

Problem 4 - Use the Pythagorean Theorem in 3-dimensions to determine the total strength of Earth's magnetic field for problems 1, 2 and 3.

Answer Key

Problem 1 - For x = 7.0, y = 0.0, z = 0.0 and r = 7.0

$Bx = 3 (7.0) (0.0) (31,000)/(7.0)^5$ = **0.0 nT**

$By = 3 (0.0) (0.0) (31,000) / (7.0)^5$ = **0.0 nT**

$Bz = [3(0.0)^2 – (7.0)^2](31,000) / (7.0)^5$
$= - (31,000)(7.0)^2/(7.0)^5$
$= - 1,519,000 / 16807$
= **- 90 nT**

Problem 2 - For x = 0.38, y = 0.19, z = 1.73 and r = 3.0

$Bx = 3 (0.38) (1.73) (31,000)/(3.0)^5$ = **+251 nT**

$By = 3 (0.19) (1.73) (31,000) / (3.0)^5$ = **+126 nT**

$Bz = [3(1.73)^2 – (3.0)^2) (31,000)/ (3.0)^5$
$= (-0.021)(31000)/243$
= **- 2.7 nT**

Problem 3 - For x = 0.0, y = 48.0, z = 36 and r = 60.0

$Bx = 3 (0.0) (36) (31,000)/(60)^5$ = **0.0 nT**

$By = 3 (48.0) (36) (31,000) / (60)^5$ = **0.21 nT**

$Bz = [3(36)^2 – (60)^2] (31,000) / (60)^5$
$= (288)(31,000)/(7,776,000,000)$
= **0.0011 nT**

Problem 4 - Use the Pythagorean Theorem in 3-dimensions to determine the total strength of Earth's magnetic field for problems 1, 2 and 3.

1) $B = (Bx^2 + By^2 + Bz^2)^{1/2} = ((-90)^2)^{1/2}$ = 90 nT at communications satellite orbit.

2) $B = ((251)^2 +(126)^2 +(-2.7)^2)^{1/2}$ = 281 nT at Van Allen belts

3) $B = ((0.0)^2 +(0.21)^2 +(0.0011)^2)^{1/2}$ = 90 nT at the Moon

The solar surface is not only a hot, convecting ocean of gas, but is laced with magnetism. The sun's magnetic field can be concentrated into sunspots and complex 'loopy' patterns that the magnetic fields make. The above image was taken by NASA's TRACE satellite and shows one of these magnetic loops rising above the surface near two sunspots. The horseshoe shape of the magnetic field is anchored at its two 'feet' in the dark sunspot regions. The heated gases become trapped by the magnetic forces in sunspot loops, which act like magnetic bottles. The gases are free to flow along the lines of magnetic force, but not across them. The above image only tells scientists where the gases are, and the shape of the magnetic field, which isn't enough information for scientists to fully understand the physical conditions within these magnetic loops. Satellites such as Hinode carry instruments like the EUV Imaging Spectrometer, which lets scientists measure the temperatures of the gases and their densities as well.

Problem 1 - The Hinode satellite studied a coronal loop on January 20, 2007 associated with Active Region AR 10938, which was shaped like a semi-circle with a radius of 20,000 kilometers, forming a cylindrical tube with a base radius of 1000 kilometers. What was the total volume of this magnetic loop in cubic centimeters assuming that it is shaped like a cylinder?

Problem 2 - The Hinode EUV Imaging Spectrometer was able to determine that the density of the gas within this magnetic loop was about 2 billion hydrogen atoms per cubic centimeter. If a hydrogen atom has a mass of 1.6×10^{-24} grams, A) what was the total mass of the gas trapped within this cylindrical loop in metric tons? B) An oil tanker carrying 700,000 barrels of oil has a mass of about 100,000 tons. How many tankers equal the mass of one coronal loop?

Answer Key:

Problem 1: The Hinode satellite studied a coronal loop on January 20, 2007 associated with Active Region AR 10938, which was shaped like a semi-circle with a radius of 20,000 kilometers, forming a cylindrical tube with a base radius of 1000 kilometers. What was the total volume of this magnetic loop in cubic centimeters assuming that it is shaped like a cylinder?

Answer: The length (h) of the cylinder is 1/2 the circumference of the circle with a radius of 20,000 km or $h = 1/2 (2\pi R) = 3.14 \times 20,000$ km = 62,800 km

The volume of a cylinder is $V = \pi R^2 h$ so that the volume of the loop is

$V = \pi (1000 \text{ km})^2 \times 62,800$ km

$= 2.0 \times 10^{11}$ cubic kilometers.

1 cubic kilometer = 10^{15} cubic centimeters so

= **2.0×10^{26} cubic centimeters**

Problem 2: The Hinode EUV Imaging Spectrometer was able to determine that the density of the gas within this magnetic loop was about 2 billion hydrogen atoms per cubic centimeter. If a hydrogen atom has a mass of 1.6×10^{-24} grams, A) what was the total mass of the gas trapped within this cylindrical loop in metric tons? B) An oil tanker carrying 700,000 barrels of oil has a mass of about 100,000 tons. How many tankers equal the mass of one coronal loop?

Answer: A) The total mass is the product of the density times the volume, so

Density = 2×10^9 particles/cc x (1.6×10^{-24} grams/particle) = 3.2×10^{-15} grams/cm^3

The approximate volume of the magnetic loop in cubic centimeters is

$V = (2.0 \times 10^{11} \text{ km}^3) \times (1.0 \times 10^{15} \text{ cm}^3/\text{km}^3)$

$= 2.0 \times 10^{26} \text{ cm}^3$

Mass = Density x Volume = $(3.2 \times 10^{-15} \text{ grams/cm}^3) \times (2.0 \times 10^{26} \text{ cm}^3) = 6.4 \times 10^{26-15}$ = 6.4×10^{11} grams or 6.4×10^8 kilograms or **640,000 metric tons.**

B) Loop = 640,000 tons/100,000 tons = **6.4 oil tankers**.

Magnetic Pressure - I

Toy magnets provide an excellent introduction to a very 'tactile' property of magnetic fields. When you try to push like poles together, you can feel an invisible pressure pushing back at your effort. Weak magnets can be easily overcome so that you can actually get the poles to physically touch. For very strong magnetic fields, this is humanly impossible.

The amount of pressure, in dynes per square centimeter, can be easily calculated from the strength of the field according to the formula on the right.

$$Pm = \frac{B^2}{8\pi}$$

P is the magnetic pressure
B is the magnetic field strength in Gauss

For example, at a particular location in space, the magnetic field components of a toy bar magnet are given by B = (Bx, By, Bz) and are measured to have the values, in Gauss units, of (+25, +15, +38). The total strength of the magnetic field at that location, using the Pythagorean Theorem, is $B = (25^2 + 15^2 + 38^2)^{1/2}$ = 48 Gauss. Then Pm = $(48)^2$ / (8x3.141) = 92 dynes/cm^2 . If we forced two magnets together with this same field strength and polarity, they will produce a total pressure of twice this amount or 184 dynes/cm^2.

Problem 1 - Earth's magnetic field at the surface has a strength of 0.7 Gauss. What is the magnetic pressure in A) dynes/cm^2 B) Newtons/m^2 C) Pounds/$inch^2$ D) microPascals? (Note: 1 Tesla = 10,000 Gauss; 1 Newton = 10^7 dynes; 1 Pound = 4.45 Newtons; 1 Pascal = 1 Newton/ m^2).

Problem 2 - A sunspot magnetic field has an average strength of 4,000 Gauss. What is the magnetic pressure in Pascals?

Problem 3 - Two magnetic fields are in close contact. The left-hand field has an average field of BL = (+3.5, -2.2, +15.0) while the right-hand field has a strength of BR=(-1.0, +5.2, -13.4). If the units for the fields are in Gauss, which field has the largest pressure in Pascals?

Problem 4 - A 50 kg man hangs on a rope attached to a metal plate that is 0.25 meters square, and exerts a pressure of 1,960 Pascals. What must be the strength of the magnetic field to support this load; A) In Gauss? B) In Teslas?

Problem 1 - Earth's magnetic field at the surface has a strength of 0.7 Gauss. What is the magnetic pressure in A) dynes/cm^2 B) Newtons/m^2 C) Pounds/inch^2 D) microPascals? (Note: 1 Tesla = 10,000 Gauss; 1 Newton = 10^5 dynes; 1 Pound = 4.45 Newtons; 1 Pascal = 1 Newton/ m^2).

A) Pm = $(0.7)^2/(8 \times 3.14)$ = 0.02 dynes/cm^2.

B) Pm = 0.02 dynes/cm^2 x (1 Newton/100000 dynes) x (10000 cm^2/1 m^2) = 0.002 Newton/m^2.

C) Pm = 0.002 Newton/m^2 x (1 Pound/4.45 Newtons) x (1 meter/39.37 inches) x (1 meter/39.37 inches) = 0.00000029 pounds/inch^2

D) Pm = 0.002 Newton/m^2 x (1,000,000 / 1) = 2000 microPascals.

Note: At sea-level 1 Atmosphere = 14 pounds/inch^2 so magnetic pressure is irrelevant.

Problem 2 - A sunspot magnetic field has an average strength of 4,000 Gauss. What is the magnetic pressure in Pascals?

Answer: Pm = $(4{,}000)^2/(8 \times 3.141)$ = 640,000 dynes/cm^2.

640000 dynes/cm^2 x (1 Newton/10^5 dynes) x (10000 cm^2/1m^2) = 64000 Newtons/m^2 and since 1 Newton/m^2 = 1 Pascal, we have 64,000 Pascals.

Problem 3 - Two magnetic fields are in close contact. The left-hand field has an average field of BL = (+3.5, -2.2, +15.0) while the right-hand field has a strength of BR=(-1.0, +5.2, -13.4). If the units for the fields are in Gauss, which field has the largest pressure in Pascals? Answer: BL = $(3.5^2 + 2.2^2 + 15^2)^{1/2}$ = 15.5 Gauss. BR = $(1^2 + 5.2^2 + 13.4^2)^{1/2}$ = 14.4 Gauss. Because BL > BR, the BL field produces the most pressure. Pm = $(15.5)^2/(8 \times 3.141)$ = 9.6 dynes/cm^2. Then 9.6 x (1 Newton/100000dynes) x (10000 cm^2/1m^2) = 0.96 Newtons/m^2 = 0.96 Pascals.

Problem 4 - A 50 kg man hangs on a rope attached to a metal plate that is 0.25 meters square, and exerts a pressure of 1,960 Pascals. What must be the strength of the magnetic field to support this load; A) In Gauss? B) In Teslas?

Answer: First convert 1,960 Pascals to dynes/cm2 to get the pressure units used in the equation. Then Pm = 1960 Pascal x (100000 dynes/Newton) x (1 m^2/10000cm^2) = 19,600 dynes/cm^2. Solving the equation for B to get B = $(8 \pi P)^{1/2}$ we obtain B = $(8 \pi 19600)^{1/2}$ = A) 702 Gauss. and B) 702 Gauss x (1 Tesla/10000 Gauss) = 0.0702 Teslas.

Magnetic pressure, described by the top equation on the right, can interact with gases that consist of charged particles (called plasmas). Plasmas are very common in Nature, and behave like gases. They produce pressure through the collision of individual atoms and ions in the plasma with the atoms or ions in other materials.

$$Pm = \frac{B^2}{8\pi}$$

P is the magnetic pressure
B is the magnetic field strength in Gauss

A simple equation for gas pressure is provided to the lower right. Because we are dealing with plasmas that can feel the effects of magnetic fields, we can compare the gas pressure of plasma with the pressure produced by a magnetic field acting upon it to investigate some basic phenomena in Nature.

$$Pg = \frac{3}{2}NkT$$

N gas density in particles/ m^2
T is the temperature in K
k Boltzman's Constant $=1.38 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$

For example, suppose we have a magnetic field in a particular region of space that produces a magnetic pressure of Pm = 10 Newtons/m^2. If the density of the plasma is 10^{20} particles/m^3, at what temperature will the magnetic field and plasma be in pressure equilibrium so that Pm = Pg? We see that since

Pm = Pg,

$10 = 1.5 \times (1.0\times10^{20}) \times (1.38 \times 10^{-23})\ T$ and so we get

$T = 10/(1.5 \times (1.0\times10^{20}) \times (1.38 \times 10^{-23}))$
$= 4{,}800^{o}$ Kelvin.

Problem 1 - The tensile strength of steel is 750 million Pascals. What is the strongest magnetic field (in Teslas) that can be stored in a steel magnet before the steel explodes?

Problem 2 - The solar surface has a density of 2.0×10^{23} particles/m^3 and a temperature of 6,000 K. What strength of magnetic field, in Gauss, would be in equilibrium with the plasma under these conditions?

Problem 3 - At the time the solar system formed, the proto-planetary accretion disk had a density of 2.0×10^{20} particles/m^3 and a temperature of 2,000 K just inside the orbit of Mercury. What is the strongest magnetic field that can be in pressure equilibrium with the gas in this disk?

Problem 1 - The tensile strength of steel is 750 million Pascals. What is the strongest magnetic field (in Teslas) that can be stored in a steel magnet before the steel explodes?

Answer: Pm = 750 million Pascals which is $7.5 \times 10^8 \times (10 \text{ dynes/cm}^2) / (1 \text{ Pascal}) = 7.5 \times 10^9 \text{ dynes/cm}^2 = B^2/8\pi$ so $B = (8 \pi\ 7.5 \times 10^9)^{1/2}$ = 430,000 Gauss. Since 10,000 Gauss = 1 Tesla, we have the desired answer of 43 Teslas.

Problem 2 - The solar surface has a density of 2.0×10^{17} particles/m^3 and a temperature of 6,000 K. What strength of magnetic field, in Gauss, would be in equilibrium with the plasma under these conditions?

Answer: The plasma pressure is

$$Pg = 1.5 \times (2.0 \times 10^{17}) \times (1.38 \times 10^{-23}) \times 6{,}000$$

$$Pg = 0.025 \text{ Pascals.}$$

Since 10 dyne/cm^2 = 1 Pascal, we have Pg = 0.25 dyne/cm^2.

Then from $Pm = B^2/(8\pi) = 0.25$ we solve for B and get B = 2.5 Gauss.

Note: This is typical of the average solar surface field deduced from other data.

Problem 3 - At the time the solar system formed, the proto-planetary accretion disk had a density of 2.0×10^{20} particles/m^3 and a temperature of 2,000 K just inside the orbit of Mercury. What is the strongest magnetic field that can be in pressure equilibrium with the gas in this disk?

Answer: We have $N = 2.0 \times 10^{20}$ particles/m^3

T = 2,000 K

$$\text{So } Pg = 1.5\ (2.0 \times 10^{20}) \times (1.38 \times 10^{-23}) \times 2000$$

$$Pg = 8.3 \text{ Pascals}$$

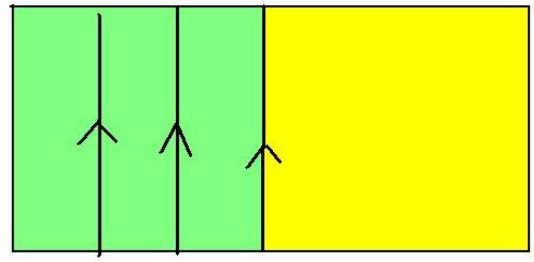
Since 10 dyne/cm^2 = 1 Pascal, we have Pg = 83 dyne/cm^2.

Then from $Pm = B^2/(8\pi) = 83$ we solve for B and get B = 46 Gauss.

Magnetic Pressure - III

Magnetic field pressure, and gas pressure, can combine in many different ways in a plasma. Because plasmas are electrically-charged, magnetic fields can be generated by them through current flows, or can be dragged around by plasmas whose charged particles become 'anchored' to the field.

The total pressure of a plasma is a combination of magnetic and gas pressure. This allows two different plasmas to be in pressure balance, but with very different physical properties. The figure to the right shows such a situation. Examples of such systems include sunspots, and Earth's complex magnetic field environment.



$$\frac{3}{2}NkT_a + \frac{B^2}{8\pi} = \frac{3}{2}NkT_b$$

N is the gas density (particles/cm^3) in each region, B is the magnetic field strength (Gauss) in the left-hand region, and T_a and T_b are the gas temperatures in the left and right-hand regions.

Problem 1 - Suppose we have a 10,000 Gauss magnetic field embedded in a plasma with room temperature and density (Ta=300 K and N = 3.0 x 10^{21} particles/cm^3). It is in contact with a second plasma with the same density. What will be its temperature (Tb) so that the pressures are balanced between the two regions? (Note: Boltzmann's Constant: k = 1.38 x 10^{-16} ergs/ degree K)

Problem 2 - Early models of sunspots proposed that they were in pressure balance between the inner region containing a strong magnetic field, and the surrounding solar surface, which contained a weak magnetic field. If the properties of the plasma inside the sunspot are given by, B = 5000 Gauss, N = 3.0 x 10^{18} ions/cm^3, and the region outside the sunspot has T = 5770 K and N = 3.0 x 10^{18} ions/cm^3, what would be the minimum temperature of the plasma inside the sunspot in order to allow pressure balance?

Problem 3 - An interstellar cloud is observed to be in pressure balance with its surroundings. The outside gas temperature and density are T = 100,000 K N = 1.0 atoms/cm^3, and the cloud's temperature and density are T = 50 K, N= 2,000 atoms/cm^3. An astronomer measures a magnetic field of B = 0.001 Gauss inside the cloud. Is this cloud in pressure equilibrium?

Problem 1 - Suppose we have a 10,000 Gauss magnetic field embedded in a plasma with room temperature and density (Ta=300 K and N = 3.0 x 10^{21} particles/cm^3). It is in contact with a second plasma with the same density. What will be its temperature (Tb) so that the pressures are balanced between the two regions?
(Note: Boltzmann's Constant, k = 1.38 x 10^{-16})

Answer; For the left-hand region:

$Pm = B^2/8\pi = (10,000)^2/(8 \times 3.141) = 3.9 \times 10^6$ dynes/cm^2

$Pg = 1.5 \times (3.0 \times 10^{21}) \times (1.38 \times 10^{-16}) \times (300) = 1.86 \times 10^8$ dynes/cm^2

So P(total) = Pm + Pg = 1.9 x 10^8 dynes/cm^2

So balancing the left and right-hand pressures we get

1.9×10^8 dynes/cm^2 = $1.5 \times (3.0 \times 10^{21}) \times (1.38 \times 10^{-16}) \times T$

And solving for T we get T = 306 K.

Problem 2 - Early models of sunspots proposed that they were in pressure balance between the inner region containing a strong magnetic field, and the surrounding solar surface, which contained a weak magnetic field. If the properties of the plasma inside the sunspot are given by, B = 5000 Gauss, N = 3.0 x 10^{18} ions/cm^3, and the region outside the sunspot has T = 5770 K and N = 3.0 x 10^{18} ions/cm^3, what would be the minimum temperature of the plasma inside the sunspot in order to allow pressure balance?

Answer; For the left-hand region inside the sunspot has:

$Pm = B^2/8\pi = (5,000)^2/(8 \times 3.141) = 9.95 \times 10^5$ dynes/cm^2

$Pg = 1.5 \times (3.0 \times 10^{18}) \times (1.38 \times 10^{-16}) \times (T) = 621\ T$ dynes/cm^2

So P(total) = Pm + Pg = $(9.95 \times 10^5 + 621\ T)$ dynes/cm^2

The outside region has

$Pg = 1.5 \times (3.0 \times 10^{18}) \times (1.38 \times 10^{-16}) \times (5770) = 3.58 \times 10^6$ dynes/cm^2

So balancing the left and right-hand pressures we get

$(9.95 \times 10^5 + 621\ T) = 3.58 \times 10^6$

And solving for T we get T = 4,160 K.

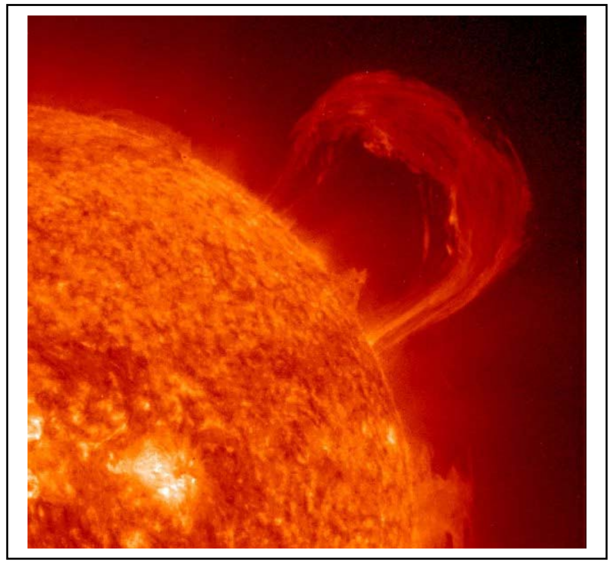
Note: This simple pressure-balance predicts that the cooler plasma inside the sunspot would emit less light at the lower temperature, and so appear darker than the surrounding solar surface.

Problem 3 - An interstellar cloud is observed to be in pressure balance with its surroundings. The outside gas temperature and density are T = 100,000 K N = 1.0 atoms/cm^3, and the cloud's temperature and density are T = 50 K, N= 2,000 atoms/cm^3. An astronomer measures a magnetic field of B = 0.001 Gauss inside the cloud. Is this cloud in pressure equilibrium?

Answer; Inside pressure pushing outwards is

$Pg + Pm = 2.1 \times 10^{-11} + 4.0 \times 10^{-8} = 4.0 \times 10^{-8}$ dynes/cm^3.

The outside pressure pushing inwards is Pg = 2.1 x 10^{-11} dynes/cm^3, so the inside pressure of the cloud is nearly 1000 times higher and the cloud is not in pressure balance. Note: In a real interstellar cloud, the gravity of the cloud's mass is an additional internal 'pressure' that helps support the cloud and balance the outward thermal and magnetic pressures.



Gravity is an important force in most astronomical systems, and often plays an important role in how magnetic fields and plasmas will behave. An important crucible for observing these interactions is the solar surface.

Hot plasmas with entangled magnetic fields produce many spectacular phenomena including breath-taking solar prominences like the one in the image on the right taken on September 14, 1999 by the SOHO satellite. Although most prominences are immediately hurled out into space as eruptive prominences, another category called quiescent prominences seem to linger motionless for days at a time.

In this problem, we will explore how quiescent prominences may manage this delicate balancing act!

Problem 1 - If the prominence has a density of 5.6×10^{-13} grams/cm^3 and is 70,000 kilometers in diameter, how many grams of prominence gas are present in each square-centimeter of area?

Problem 2 - At the top of the arch of a quiescent prominence, the force of the sun's gravity is about 275 Newtons per kilogram of matter being suspended. From your answer to Problem 1, what is the gravitational force in dynes, for each gram of prominence material? (Note: 1 Newton = 100,000 dynes)

Problem 3 - The magnetic field in the prominence has a strength of B = 50 gauss. What is the magnetic pressure of the prominence in dynes/cm^2? ($Pm = B^2/8\pi$)

Problem 4 - The plasma has a density of 3.5×10^{-11} atoms/cm^3 and suppose that observations suggest that it is at a temperature of 100,000 K because the gas is emitting x-rays. What is its gas pressure in dynes/cm^2? (Note: $Pg = 3/2\ N\ k\ T$ where $k = 1.38 \times 10^{-16}$ erg/deg Kelvin)

Problem 5 - Compare the total pressure of the prominence material (Pm + Pg) with the gravitational pressure. Are they in pressure balance?

Problem 6 - What happens to your answer to Problem 5 if you change any of the parameters slightly?

Problem 7 - Explain how a quiescent prominence might suddenly be transformed into an eruptive prominence?

Problem 1 - If the prominence has a density of 5.6×10^{-13} grams/cm^3 and is 70,000 kilometers in diameter, how many grams of prominence gas are present in each square-centimeter of area? Answer: D = 5.6×10^{-13} grams/cm^3 x 70,000 km x (100000 cm/1 km) = 0.0039 grams/cm^2

Problem 2 - At the top of the arch of a quiescent prominence, the force of the sun's gravity is about 275 Newtons per kilogram of matter being suspended. From your answer to Problem 1, what is the gravitational force in dynes, for each gram of prominence material? (1 Newton = 100,000 dynes) Answer: You need to convert 275 Newton/kg to dynes/cm. Fg = 275 x (100000 dynes/Newton) x (1 kg/1000 grams) = 27500 dynes per gram. Since the mass is 0.0039 grams for each square-centimeter, the gravitational force Fg=27500x0.0039 = 107 dynes for each cm^2

Problem 3 - The magnetic field in the prominence has a strength of B = 50 gauss. What is the magnetic pressure of the prominence in dynes/cm^2 ? ($Pm = B^2/8\pi$)
Answer: $B = B^2/(8 \pi) = (50)^2/(8 \times 3.141)$ = 99.5 dynes/cm^2

Problem 4 - The plasma has a density of 3.5×10^{-11} atoms/cm^3 and suppose that observations suggest that it is at a temperature of 100,000 K because the gas is emitting x-rays. What is its gas pressure in dynes/cm^2? (Note: Pg = 3/2 N k T where k = 1.38×10^{-16}) Answer: Pg = 1.5 x (3.5×10^{11}) x 1.38×10^{-16} x (100,000) = 7.2 dynes/cm^2.

Problem 5 - Compare the total pressure of the prominence material (gas pressure plus magnetic pressure) with the gravitational pressure. Are they in pressure balance?
Answer; Pgravity = 107 dynes/cm^2 while for the prominence, Pg + Pb = 7.2 + 99.5 = 107 dynes/cm^2. The pressures are equal, so the system is in pressure balance. This means, mechanically, that the prominence is balanced against gravity through the tension in its magnetic field acting upon the charged plasma.

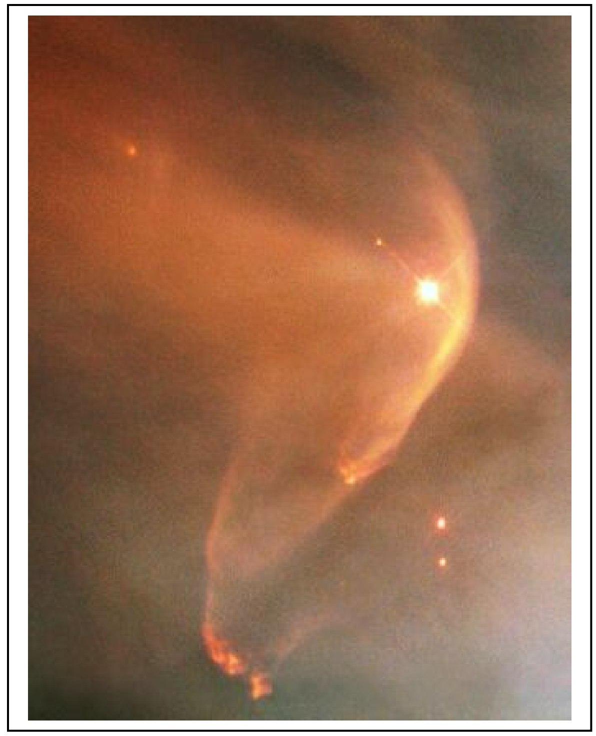
Problem 6 - What happens to your answer to Problem 5 if you change any of the parameters slightly? Answer; Changing the gas density or magnetic field strength by as little as 1% will change the pressures acting on the prominence and make them unequal. This means that the pressure-balance equilibrium is unstable and may be easily lost as the system changes, the magnetic field weakens, or the plasma cools.

Problem 7 - Explain how a quiescent prominence might suddenly be transformed into an eruptive prominence? Answer: The solar surface is constantly changing, and if the magnetic field, gas density, or gas temperature in the prominence were to change even slightly, the prominence would no longer be stable and would start to move and 'erupt'.

A common situation is to have one cloud of plasma run into another cloud, or into a magnetic field system. The collective action of the plasma mass traveling at a given speed is to produce a collision pressure known as ram pressure.

Mathematically, it is a simple product of the density of the gas and its velocity-squared. For example, if the density of the cloud is given in grams/cm^3 and the velocity is expressed in centimeters/sec, the pressure will be in units of dynes/cm^2.

$$P_{ram} = \rho V^2$$



There are many examples of ram pressure in action. For instance, the photo shows the curved, parabola-shaped, 'bow shock' produced just ahead of the young star LL Orionis (Courtesy NASA; Hubble Space Telescope) as its emitted gas plows through a dense cloud of gas through which the star is traveling. Other examples of ram pressure effects include Earth's magnetosphere and its interaction with the solar wind, or the sun's heliopause beyond the orbit of Pluto, where the solar wind interacts with the interstellar medium.

Problem 1 - The solar wind has a density of 100 atoms/cm^3 and a speed of 450 km/sec. if the mass of the average atom (hydrogen) is 1.6 x 10^{-24} grams, what is the ram pressure provided by the solar wind?

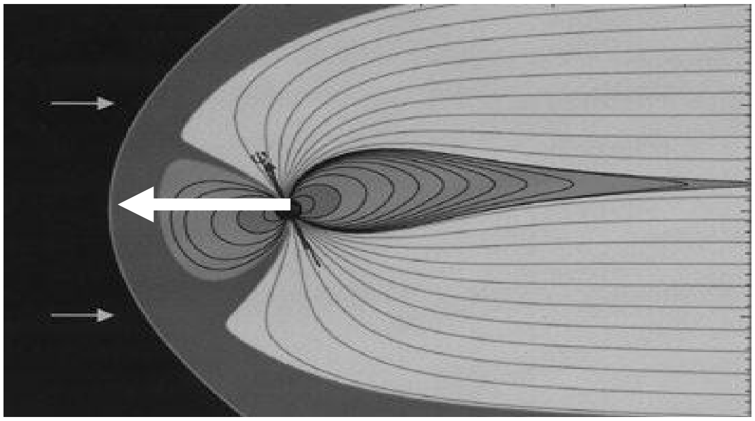
Problem 2 - The magnetic field pressure within a sunspot is 6,300 dynes/cm^2. If the density of the solar gas is 2 x 10^{15} atoms/cm^3 and each atom has a mass of m = 1.6 x 10^{-24} grams, what is the minimum speed of the gas, V in km/s, that will keep the sunspot magnetic field confined?

Answer Key

Problem 1 - The solar wind has a density of 100 atoms/cm^3 and a speed of 450 km/sec. if the mass of the average atom (hydrogen) is 1.6 x 10^{-24} grams, what is the ram pressure provided by the solar wind? Answer: First calculate the gas density:
ρ = 100 atoms/cm^3 x 1.6 x 10^{-24} gm/atom = 1.6 x 10^{-22} grams/cm^3. Next convert km/sec to cm/sec: 450 km/sec x 100000 cm/km = 4.5 x 10^7 cm/sec.
Then Pram = (1.6 x 10^{-22}) x (4.5 x 10^7 $)^2$ = 3.2 x 10^{-7} dynes/cm^2

Problem 2 - The magnetic field pressure within a sunspot is 6,300 dynes/cm^2. If the density of the solar gas is 2 x 10^{15} atoms/cm^3 and each atom has a mass of m=1.6 x 10^{-24} grams, what is the minimum speed of the gas, V in km/s, that will keep the sunspot magnetic field confined?
Answer; First convert the number density into grams/cm^3 to get
r = 2 x 10^{15} atoms/cm^3 x 1.6 x 10^{-24} grams/atom = 3.2 x 10^{-9} greams/cm^3.

We need 6,300 dynes/cm^2 = ρV^2 and solving for V we get
V = (6300/3.2 x 10^{-9} $)^{1/2}$ = 1,400,000 cm/sec or 14 kilometers/sec.



When the solar wind flows past Earth, it pushes on Earth's magnetic field and compresses it. There is a region in space called the magnetopause where the pressure of the solar wind balances the outward pressure of Earth's magnetic field. The distance from the Earth, R, (white arrow in drawing) where these two pressures are balanced is given by the equation:

$$R^6 = \frac{0.72}{8\pi D V^2}$$

In this equation, D is the density in grams per cubic centimeter (cc) of the gas (solar wind, etc) that collides with Earth's magnetic field, and V is the speed of this gas in centimeters per second. Let's do an example to see how this equation works!

Step 1 - The solar wind has a typical speed of 450 km/s or equivalently V = 4.5×10^7 cm/s. To find the density of the solar wind in grams/cc we have to do a two-step calculation. The wind usually has a particle density of about 5 particles/cc, and since these particles are typically protons (each with a mass of 1.6×10^{-24} gm) the density is then $5 \times (1.6 \times 10^{-24}$ gm)/cc so that D = 1.28×10^{-23} gm/cc. Next we use a calculator!

Step 2 - We substitute D and V into the equation and get R^6 = 1105242. so that R = $(1105242.6)^{1/6}$. To solve this, we use the calculator with a key labeled **Y^x** First type '1105242.6' and hit the '**Enter**' key. Then type '0.1666' (which equals 1/6) and press the **Y^X** key. In this case the answer will be '10.16' and it represents the value of R in multiples of the radius of Earth (6378 kilometers). Scientists simplify the mathematical calculation by using the radius of Earth as their unit of distance, but if you want to convert 10.16 Earth radii to kilometers, just multiply it by '6378 km' which is the radius of Earth to get 64,800 kilometers. That is the distance from the center of Earth to the magnetopause where the magnetic pressure is equal to the solar wind pressure for the selected speed and density. Now let's apply this example to finding the magnetopause distance for some of the storms that have encountered Earth between 2000 and 2003 years.

Problem 1 - Complete the table below, rounding the answer to three significant figures:

Storm	Date	Day Of Year	Density (particle/cc)	Speed (km/s)	R (km)
1	11/20/2003	324	49.1	630	
2	10/29/2003	302	10.6	2125	
3	11/06/2001	310	15.5	670	
4	3/31/2001	90	70.6	783	
5	7/15/2000	197	4.5	958	

Problem 2 - The fastest speed for a solar storm 'cloud' is 1500 km/s. What must the density be in order that the magnetopause is pushed into the orbits of the geosynchronous communication satellites at 6.6 Re?

Problem 1 - The values calculated from the equation for R are shown in the table below. Note: This is a good opportunity to go over the concept of 'significant figures', and why calculator digits have to be interpreted very carefully in the context of the actual accuracy of the input numbers. The particle densities below establish the maximum number of significant figures that students should quote in the final answers.

Storm	Date	Day Of Year	Density (particle/cc)	Speed (km/s)	Max Sig. Figs	R (km)
1	11/20/2003	324	49.1	630	3	42,700
2	10/29/2003	302	10.6	2125	3	37,000
3	11/06/2001	310	15.5	670	3	51,000
4	3/31/2001	90	70.6	783	3	37,600
5	7/15/2000	197	4.5	958	2	55,000

Problem 2 - The fastest speed for a solar storm 'cloud' is 3000 km/s. What must the density be in order that the magnetopause is pushed into the orbits of the geosynchronous communication satellites at 6.6 Re (42,000 km)?
Answer: Solve the equation for D to get:

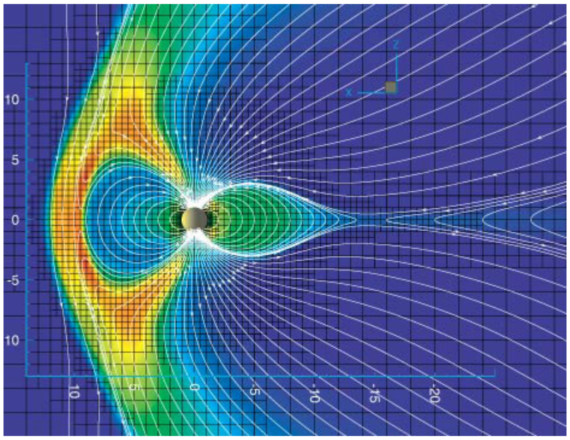
$$D = \frac{0.72}{8\pi R^6 V^2}$$

For 1500 km/s $V = 1.5 \times 10^8$ cm/s, and for R = 6.6, we have

$$D = 0.72/(8 \times 3.14 \times 6.6^6 \times (1.5 \times 10^8)^2) = 1.52 \times 10^{-23} \text{ gm/cc}$$

Since a proton has a mass of 1.6×10^{-24} grams, this value for the density, D, is equal to $(1.52 \times 10^{-23} / 1.6 \times 10^{-24}) = 9.5$ protons/cc.

Extra Credit: Have students compute the density if the solar storm pushed the magnetopause to the orbit of the Space Station (about R = 1.01 RE).
Answer: D = 3×10^{-19} gm/cc or 187,000 protons/cc. A storm with this density has never been detected, and would be catastrophic!



$$R = \left(\frac{1.8x10^{12}}{NV^2} \right)^{\frac{1}{6}}$$

$$Pr = 1.6x10^{-8} NV^2$$

$$Pm = 4.0x10^{-6} B^2$$

The ACE satellite measures the density and speed of the solar wind as it approaches Earth, and also measures the strength of its magnetic field. Both the magnetic field, and the kinetic energy of the particles, cause a build-up of pressure acting upon Earth's magnetic field. This forces Earth's magnetic field closer to the planet's surface, and can expose satellites orbiting Earth to the potentially harmful effects of cosmic rays and other high-energy particles. Based on actual data from the ACE satellite, in this problem you will calculate the particle and magnetic pressure and determine the distance from Earth of the pressure equilibrium region of the magnetic field, called the magnetopause.

The equations to the left give the magnetopause distance, R, in multiples of Earth's radius (6,378 km), and the magnetic (Pm) and ram (Pr) pressures in units of microErgs/cm^3, once the sped of the cloud (V in km/sec), density of the cloud (N in particles/cm^3) and the strength of the cloud's magnetic field (B in nanoTeslas) are specified.

Date	Flare	N (particle/cc)	V (km/s)	B (nT)	Pr	Pm	Distance (Re)
9-7-2005	X-17	50	2500	50	5.0	0.01	4.2
7-13-2005	X-14	30	2000	20			
1-16-2005	X-2.8	70	3700	70			
10-28-2003	X-17	100	2700	70			
11-4-2003	X-28	80	2300	49	6.8	0.01	4.0
4-21-2002	X-1.5	20	2421	10			
7-23-2002	X-4.8	40	1200	15			
4-6-2001	X-5.6	20	1184	20			
7-14-2000	X-5.7	30	2300	60			
11-24-2000	X-1.8	50	2000	10	3.2	0.0004	4.6
8-24-1998	X-1	15	1500	10			

Problem 1: Use the formulae and the values cited in the table to complete the last three columns. A few cases have been computed as examples.

Problem 2: A geosynchronous communications satellite is orbiting at a distance of 6.6 Re (1 Re = 1 Earth radius= 6,378 km). For which storms will the satellite be directly affected by the solar storm particles?

Date	Flare	N (particle/cc)	V (km/s)	B (nT)	Pr	Pm	Distance (Re)
9-7-2005	X-17	50	2500	50	5.0	0.01	4.2
7-13-2005	X-14	30	2000	20	**1.9**	**0.002**	**5.0**
1-16-2005	X-2.8	70	3700	70	**15.3**	**0.02**	**3.5**
10-28-2003	X-17	100	2700	70	**11.7**	**0.02**	**3.7**
11-4-2003	X-28	80	2300	49	6.8	0.01	4.0
4-21-2002	X-1.5	20	2421	10	**1.9**	**0.0004**	**5.0**
7-23-2002	X-4.8	40	1200	15	**0.9**	**0.0009**	**5.6**
4-6-2001	X-5.6	20	1184	20	**0.4**	**0.002**	**6.3**
7-14-2000	X-5.7	30	2300	60	**2.5**	**0.01**	**4.7**
11-24-2000	X-1.8	50	2000	10	3.2	0.0004	4.6
8-24-1998	X-1	15	1500	10	0.5	0.0004	6.1

Note: Density and magnetic field strength are estimates for purposes of this calculation only.

Problem 1: Use the formulae and the values cited in the table to complete the last three columns.

Answer: See above shaded table entries. This is a good opportunity to use an Excel spreadsheet to set up the calculations. This also lets students change the entries to see how the relationships change, as an aid to answering the remaining questions.

Problem 2: A geosynchronous communications satellite is orbiting at a distance of 6.6 Re. For which storms will the satellite be directly affected by the solar storm particles?

Answer: If the equilibrium radius is less than 6.6 Re, the satellite will be outside Earth's protective magnetosphere and within the region of space directly affected by the storm particles and fields. This condition is satisfied for all of the storms except for the ones on April 6, 2001 and August 24, 1998

Note to Teacher: Ram pressure is the pressure produced by a cloud of particles traveling at a particular speed with a particular density. We call this a ‘ram’ pressure because it is also the pressure that you feel as you ‘ram’ your way through the air when you are in motion. Because only the relative speed is important, you will feel the same pressure if you are ‘stationary’ and a gas is moving past you at a particular speed, or if the gas is ‘stationary’ and you are trying to move through it at the same speed. Technically, ram pressure is the product of the gas density and the square of this relative speed.

The values for the ram pressure (Pr) are all substantially larger than the values for the magnetic pressure (Pm), so we conclude that ram pressure is stronger than the cloud’s magnetic pressure. This means that when the cloud impacts another object such as Earth, it is mostly the ram pressure of the cloud that determines the outcome of the interaction.

The image used in this problem shows a computer-calculated model of Earth's magnetic field during compression. The yellow coloration indicates regions of maximum pressure. Image courtesy the University of Michigan http://www.tecplot.com/showcase/studies/2001/michigan.htm

Exploring Magnetic Energy

Most science courses discuss the various familiar forms of energy such as kinetic, potential, chemical, thermal and electrical. Most courses also discuss magnetism, and electromagnetism. Combining these two ideas, we can imagine that magnetic fields also store and release energy – **magnetic energy**. This energy plays an important role in creating many of the most dramatic events scientists study, from solar flares and the Northern lights, to pulsars.

Most objects that astronomers study have shapes that can be approximated as spheres, or cylinders. Here are the formulas for the volumes, **V**, of these shapes. In these formulae to the right, the variables are as follows:

R = radius of sphere or cylinder
h = height of cylinder

The amount of magnetic energy, **Em**, for a field strength, **B**, given in Gauss units and a volume, **V**, given in cubic centimeters can be calculated from this formula:

$$E_m = \frac{B^2}{8\pi} V$$

Sphere $\frac{4}{3}\pi R^3$

Cylinder $h \pi R^2$

For example, a cubical region of space near a toy bar magnet 1000 centimeters on a side has a magnetic field with a strength of B = 150 Gauss. The volume $V = (1000)^3 = 1.0 \times 10^9$ cm^3, and so $Em = B^2 V/8\pi = (150)^2 (1.0 \times 10^9)/ (8 \times 3.141) = 8.9 \times 10^{11}$ ergs.

Problem 1 - Using the formulas above, calculate the total magnetic energy, Em, of each system in the table below, based on the assumed shape indicated. You will need to use scientific notation and a calculator!

Object	Shape	B (Gauss)	R (cm)	h (cm)	Em (ergs)
Earth	Sphere	0.5	6.4×10^8		
Geotail	Cylinder	0.002	5×10^9	1.5×10^{10}	
Sun	Sphere	5.0	6.9×10^{10}		
Solar Prominence	Cylinder	100	5×10^9	2.0×10^9	

Answer Key

<u>Worked Answers:</u>

Earth: Volume of sphere = 1.333 x 3.141 x $(6.4 \times 10^{8}\text{ cm})^{3}$ = $1094.8 \times 10^{24}\text{ cm}^{3}$ = $1.1 \times 10^{24}\text{ cm}^{3}$

Em = 0.0398 x $(.5)^{2}$ x 1.1×10^{24} = 1.1×10^{22} ergs.

Geotail: Volume of cylinder = 3.141 x $(5 \times 10^{9}\text{ cm})^{2}$ x 1.5×10^{10} cm = $117.8 \times 10^{28}\text{ cm}^{3}$ = $1.2 \times 10^{30}\text{ cm}^{3}$

Em = 0.0398 x $(0.002)^{2}$ x 1.2×10^{30} = 1.9×10^{23} ergs

Sun: Volume of sun = $1.38 \times 10^{33}\text{ cm}^{3}$

Em = 1.4×10^{33} ergs

Solar Prominence: Volume of cylinder = $1.6 \times 10^{29}\text{ cm}^{3}$

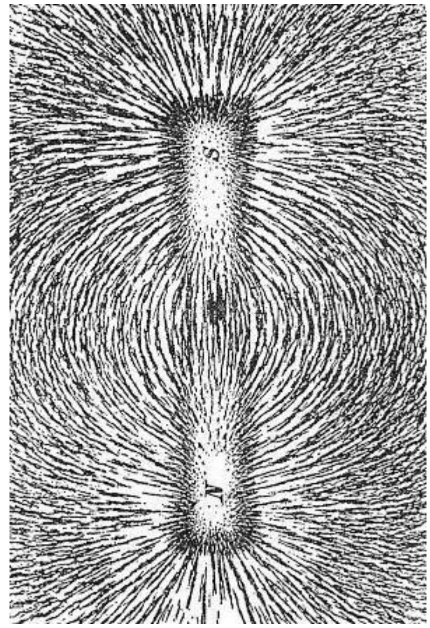
Em = 6.4×10^{31} ergs

<u>Definitions:</u>

The **Geotail** is the extension of Earth's magnetosphere into a comet-like tail in the opposite direction of the Sun. The magnetic energy released in the geotail causes charged particles to flow along the magnetic lines of force into Earth's Polar Regions, causing the Aurora Borealis and Aurora Australis.

Solar prominences are loops of magnetic force above large sunspots. The magnetic energy released by prominences can lead to solar flares and expulsions of gas called Coronal Mass Ejections.

Object	Shape	B (Gauss)	R (cm)	H (cm)	Em (ergs)
Earth	Sphere	0.5	6.4×10^{8}	[redacted]	1.1×10^{22}
Geotail	Cylinder	0.002	5×10^{9}	1.5×10^{10}	1.9×10^{23}
Sun	Sphere	5.0	6.9×10^{10}	[redacted]	1.4×10^{33}
Solar Prominence	Cylinder	100	5×10^{9}	2.0×10^{9}	6.4×10^{31}



When we play with toy magnets, we experience what we call a magnetic force, but we do not do so directly. If we did, your fingers would be able to actually feel magnetic forces just like the iron filings do!

An important aspect of magnetism is that it can only be directly experienced by charged matter. Unlike gravity, we do not experience magnetism directly.

One way to think of this is that we 'feel' the magnetic forces in bar magnets because the force is first communicated to the charged particles (electrons, protons and atomic dipoles) in the iron magnets. The particles then 'communicate' their reactions to the magnetic field to our hands that are gripping the magnet.

At its most elementary level, a magnetic field only produces a force on moving, charged particles. This is called the Lorentz Force, and its magnitude is given by the equation to the right. Here are some interesting facts about magnetic forces:

1) The charged particle (q) has to be moving in the magnetic field (B) to react to this force (V can't be zero).

2) If the particle's speed is exactly in the same direction as B, it actually feels no force at all.

3) The direction of the force will always be exactly perpendicular to the plane that contain B and the plane that contains the motion of the particle: V.

$$F=qVB$$

q = electric charge (Coulombs)
V = speed of particle (meters/sec)
B = Magnetic field (Teslas)

F will be in units of Newtons

Problem 1 - A charged particle in Earth's magnetic field with q = $+1.6 \times 10^{-19}$ Coulombs travels at +2,500 km/sec along the positive X-axis in a magnetic field with a strength of B = +15 nanoTeslas along the positive Y-axis. What will be the magnitude and direction of the Lorentz force acting on this particle?

Problem 2 - A magnet produces a field along the Z-axis of +45.2 Teslas. A proton with a charge of $+1.6 \times 10^{-19}$ Coulombs is introduced into the chamber with a speed of 100,000 kilometers/sec along the -X-axis direction. What is the Lorentz force acting on this particle, and in which direction?

Problem 1 - A charged particle in Earth's magnetic field with q = $+1.6 \times 10^{-19}$ Coulombs travels at 1,000 km/sec along the positive X-axis in a magnetic field with a strength of B = 15 nanoTeslas along the positive Y-axis. What will be the magnitude and direction of the Lorentz force acting on this particle?

Answer: Convert 2,500 km/sec to meters/sec to get $V = +2.5 \times 10^{8}$ m/s. Convert 15 nanoTeslas to Teslas to get $B=+1.5 \times 10^{-8}$ Teslas. Then

$F = q V B$

$F = (+1.6 \times 10^{-19}) \times (+2.5 \times 10^{8}) \times (+1.5 \times 10^{-8})$

$F = +6.0 \times 10^{-19}$ Newtons. Because F is perpendicular to both V and B it must be along the only remaining axis; the Z-axis. Because of the positive sign of F, the Lorentz force on the particle is in the positive direction along the Z-axis.

Problem 2 - A magnet produces a field along the Z-axis of +45.2 Teslas. A proton with a charge of $+1.6 \times 10^{-19}$ Coulombs is introduced into the chamber with a speed of 100,000 kilometers/sec along the -X-axis direction. What is the Lorentz force acting on this particle, and in which direction?

Answer: Convert 100,000 km/sec to meters/sec to get $V = -1.0 \times 10^{10}$ m/s. Then

$F = q V B$

$F = (+1.6 \times 10^{-19}) \times (-1.0 \times 10^{10}) \times (+45.2)$

$F = -7.2 \times 10^{-8}$ Newtons. Because F is perpendicular to both V (along the X axis) and B (along the Z axis) it must be along the only remaining axis; the Y-axis. Because of the negative sign of F, the Lorentz force on the particle is in the negative direction along the Y-axis.

Note: Huge instruments such as the Stanford Linear Accelerator (SLAC) use the Lorentz force provided by powerful quadrupole magnets like the one shown below to guide streams of particles into high-energy beams.

Electromagnets, like this quadrupole ("four-pole") magnet, focus particle beams in the accelerator. There are four steel pole tips, two opposing magnetic north poles and two opposing magnetic south poles. The steel is magnetized by a large electric current that flow in the coils of tubing wrapped around the poles.

Steering and focusing magnets such as these are rated at about 2 Teslas. More intense fields exceeding 8 Teslas can be generated using superconducting technology. This allows the accelerators to be made smaller in size.

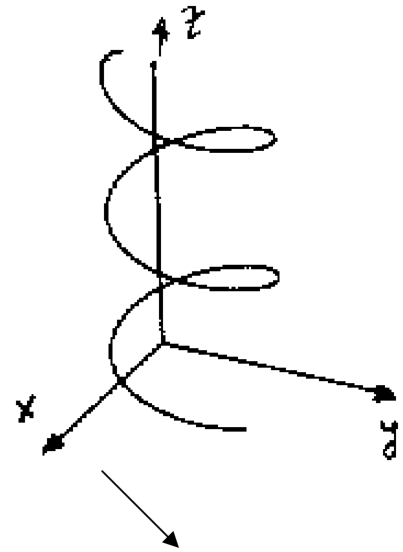
Magnetic forces and Particle Motion.

Magnetic forces are more complicated than gravity in several important ways. Unlike gravity, magnetic forces depend on the degree to which a particle is charged. Also unlike gravity, magnetic forces possess a quality called 'polarity'. All magnets have both a north and a south 'pole'. Because of the property of polarity, the motion of charged particles in a magnetic field is more complicated than the motion under gravitational forces alone.

One common motion is for a charged particle to move in a spiral path along a line of magnetic force. As the particle moves along the field, it also executes a circular 'orbit' around the line of force, so its path resembles a helix. The spiral path can be thought of as a circular path with a radius, R, which moves at a constant speed along the line of force. Adding up the 'circular' and 'linear' motions of the particle gives you a spiral path like an unwound spring or 'Slinky' toy.

To 'orbit' the magnetic field line, the Lorentz force F = qVB must be equal to the centrifugal force acting on the particle given by $F = mV^2/R$. The formula that reflects this balance is just:

$$qVB = \frac{mV^2}{R}$$



where q is the charge on the particle (in Coulombs), V is the speed of the particle as it orbits perpendicular to the line of force (in meters/s), B is the magnetic field strength (in Teslas), m is the mass of the particle (in kilograms), and R is the radius of the particle's orbit (in meters).

Problem 1 – What is the relationship for R after solving and simplifying this equation?

Problem 2 – An electron with a charge **q** = 1.6×10^{-19} Coulombs and a mass **m** = 9.1×10^{-31} kilograms is traveling at **V** = 1.0×10^{9} meters/sec in a magnetic field with a strength **B** = 0.00005 Teslas. What is the radius of its spiral orbit in meters?

Problem 3 - If an oxygen ion has twice the electron's charge, and 29400 times an electron's mass, what will its spiral radius be for the same values of **B** and **V** in Problem 1?

Problem 1 - What is the relationship for R after solving and simplifying this equation?
Answer: After a little algebra:

$$q V B = \frac{m V^2}{R} \quad \text{becomes} \quad \boxed{R = \frac{m V}{q B}}$$

Problem 2 – An electron with a charge **q** = 1.6 x 10^{-19} Coulombs and a mass **m** = 9.1 x 10^{-31} kilograms is traveling at **V** = 1.0 x 10^{9} meters/sec in a magnetic field with a strength **B** = 0.00005 Teslas. What is the radius of its spiral orbit in meters?

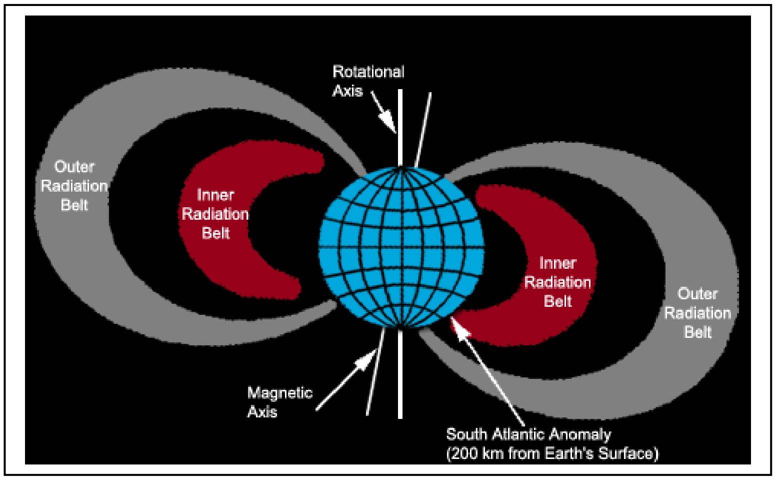
Answer: From the equation that we just solved for **R** in Question 1,
$R = (9.1 \times 10^{-31}) \times (1.0 \times 10^{9}) / (1.6 \times 10^{-19} \times 0.00005)$

$R = (9.1 \times 1.0 / (1.6 \times 5)) \times 10^{(-31 + 9 +19 + 5)}$
$R = 113$ meters.

Problem 3 - If an oxygen ion has twice the electron's charge, and 29400 times an electron's mass, what will its spiral radius be for the same values of **B** and **V** in Problem 1?

Answer: The formula says that if you double the charge, the radius is decreased by ½. If you increase the mass by 29400 times, then the radius will also increase by the same amount. So, the net change in R is (29,400/2) = 14700 times the electron's radius or 14700 x 113 meters = 1.66 million meters or 1,660 kilometers. Students can solve it this way, or simply substitute into the equation for R, B = 0.00005, V = 1.0 x 10^{9}, q = 2 x (1.6 x 10^{-19}) , m = 29400 x 9.1 x 10^{-31}

The Mass of the Van Allen Radiation Belts



The van Allen Radiation Belts were discovered in the late-1950's at the dawn of the Space Age. They are high-energy particles trapped by Earth's magnetic field into donut-shaped clouds.

Earth's inner magnetic field has a 'bar magnet' shape that follows the formula

$$R(\lambda) = L \cos^2\lambda$$

where the angle, λ, is the magnetic latitude of the magnetic field line emerging from Earth's surface, and L is the distance to where that field line passes through the magnetic equatorial plane of the field. The distance, L, is conveniently expressed in multiples of Earth's radius (1 Re = 6378 kilometers) so that L=2 Re indicates a field line that intersects Earth’s magnetic equatorial plane at a physical distance of 2 x 6378 km = 12,756 km from Earth's center.

To draw a particular field line, you select L, and then plot R for different values of λ. Because the van Allen particles follow paths along these field lines, the shape of the radiation belts is closely related to the shape of the magnetic field lines.

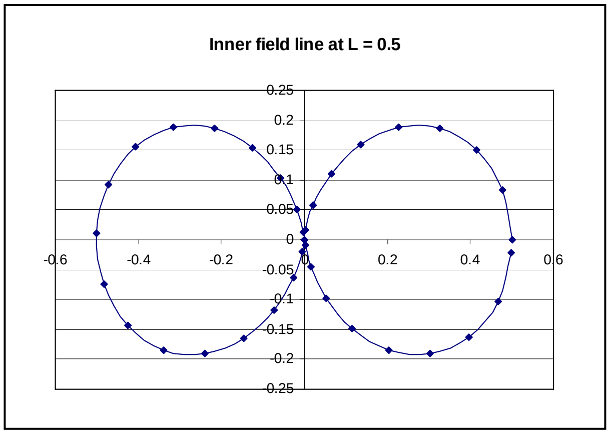
Problem 1 - Using the field line equation, plot in polar coordinates a field line at the outer boundary of the van Allen Belts for which L = 6 Re, and on the same plot, a field line at the inner boundary where L=0.5 Re. Shade-in the region bounded by these two field lines.

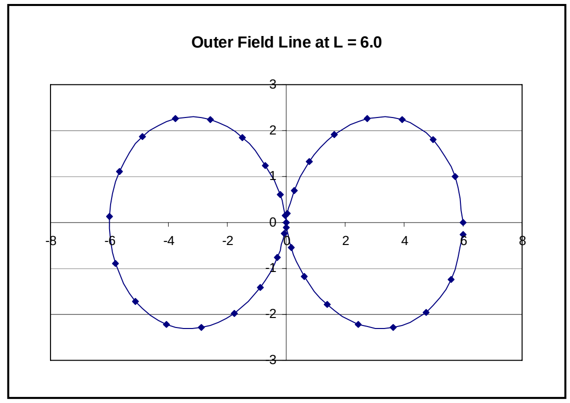
Problem 2 – If you rotate the shaded region in Problem 1 you get a 3-d figure which looks a lot like two nested toroids. Approximate the volume of the shaded region by using the equation for the volume of a torus given by $V = 2 \pi^2 r R^2$ where R is the internal radius of the circular cross-section of the torus, and r is the distance from the Origin (Earth) to the central axis of the torus. (Think of the volume as turning the torus into a cylinder with a cross section of πR^2 and a height of $2 \pi r$).

Problem 3 - Assuming that the maximum, average density of the van Allen Belts is about 100 protons/cm^3, and that the mass of a proton is 1.6×10^{-24} grams, what is the total mass of the van Allen Belts in A) kilograms? B) grams?

Answer Key

Problem 1 - This may be done, either using an HP-83 graphing calculator, or an Excel spreadsheet. The later example is shown below. (Note the scale change). For Cartesian plots in Excel, (X-Y) you will need to compute X and Y parametrically as follows: (Polar to Cartesian coordinates) $X = R\cos(\lambda)$, $y = R\sin(\lambda)$, then since $R = L\cos^2(\lambda)$ we get $X = L\cos^3(\lambda)$ and $Y = L\cos^2(\lambda)\sin(\lambda)$.





Problem 2 – Answer: The outer torus of the van Allen Belt model has an internal radius of R = (6Re – 0.5 Re)/2 = 2.75 Re or 17540 km. The radius of the Belts, r = 2.75 Re or 17,540 km. This makes the total volume Vouter = $2\ (3.141)^2\ (17540\text{ km x}1000\text{ m/km})^3 = 1.1 \times 10^{23}$ meters^3. The volume of the inner torus is defined by R = 0.25Re = 1595 km and r = 0.25 Re = 1595 km, so its volume is Vinner = $2\ (3.141)^2\ (1595\text{ km x}1000\text{ m/km})^3 = 8.0 \times 10^{19}$ meters^3 The approximate volume of the shaded region in Problem 1 is then the difference between Vouter and Vinner or 1.1×10^{23} meters^3 - 8.0×10^{19} meters^3 = $11000 \times 10^{19} - 8.0 \times 10^{19}$ = 1.1×10^{23} meters^3 because although it is technically correct to subtract the inner volume (containing no Belt particles) from the outer volume, practically speaking, it makes no difference numerically. This would not be the case if we had selected a much larger inner boundary zone for the problem!

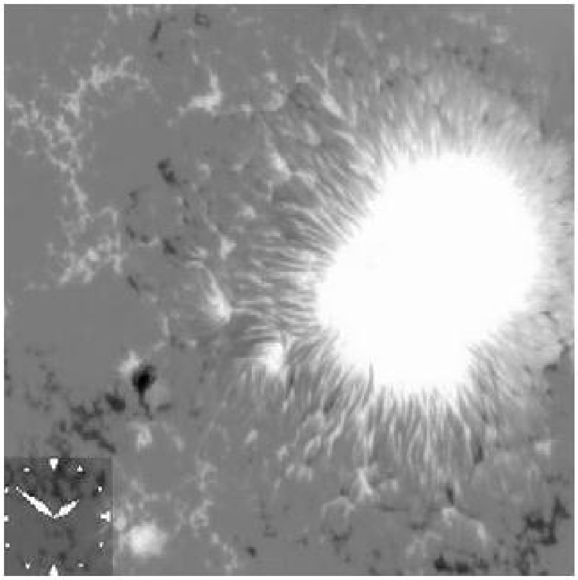
Problem 3 - Assuming that the maximum, average density of the van Allen Belts is about 100 protons/cm^3, and that the mass of a proton is 1.6×10^{-24} grams, what is the total mass of the van Allen Belts in A) kilograms? B) grams?

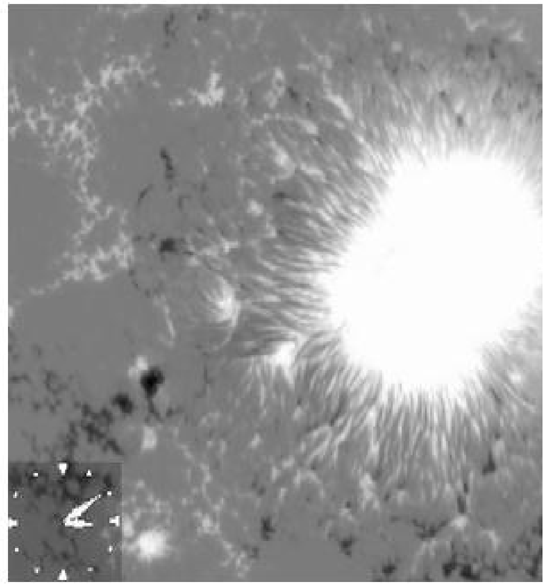
Answer: Mass = density x Volume, V = 1.1×10^{23} meters^3 .

D = 100 protons/cm^3 x 1.6×10^{-24} grams/proton = 1.6×10^{-22} grams/cm^3 which, when converted into MKS units gives 1.6×10^{-22} g/cm^3 x (1 kg/1000 gm) x (100 cm/1 meter)^3 = 1.6×10^{-25} kg/m^3. So the total mass is about M = 1.6×10^{-25} kg/m^3 x 1.1×10^{23} meters^3 and so A) M = 0.018 kilograms.

B) M = 18 grams.

Moving Magnetic Filaments Near Sunspots





These two images were taken by the Hinode solar observatory on October 30, 2006. The size of each image is 34,300 km on a side. The clock face shows the time when each image was taken, and represents the face of an ordinary 12-hour clock.

Problem 1 - What is the scale of each image in kilometers per millimeter?

Problem 2 - What is the elapsed time between each image in; A) hours and minutes? B) decimal hours? C) seconds?

Carefully study each image and look for at least 5 features that have changed their location between the two images. (Hint, use the nearest edge of the image as a reference).

Problem 3 - What direction are they moving relative to the sunspot?

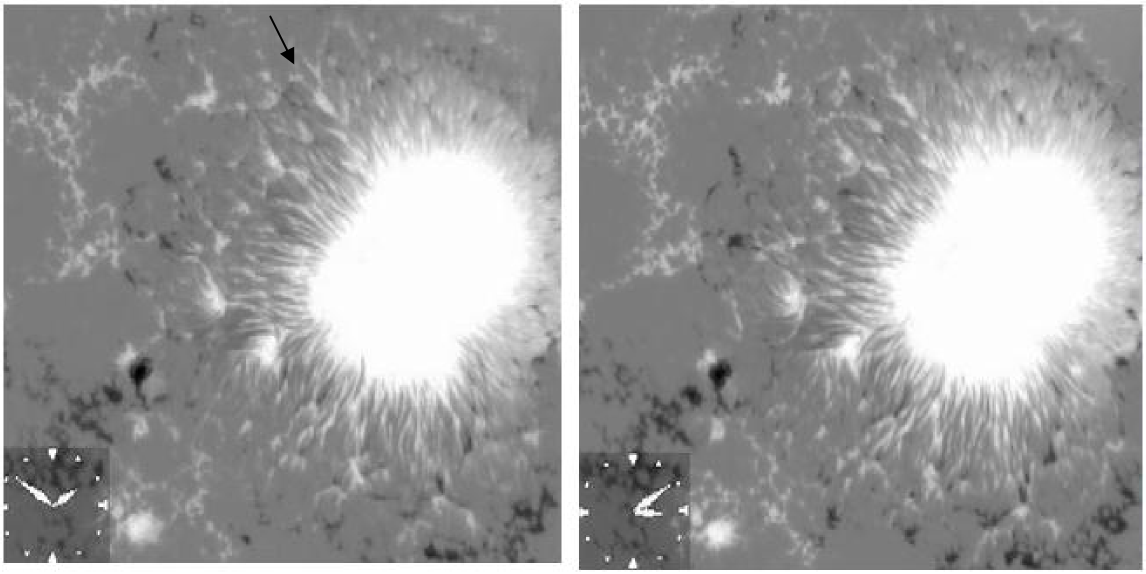
Problem 4 - How far, in millimeters have they traveled on the image?

Problem 5 - From your answers to questions 1, 2 and 4, calculate their speed in kilometers per second, and kilometers per hour.

Problem 6 - A fast passenger jet plane travels at 600 miles per hour. The Space Shuttle travels 28,000 miles per hour. If 1.0 kilometer = 0.64 miles, how fast do these two craft travel in kilometers per second?

Problem 7 - Can the Space Shuttle out-race any of the features you identified in the sunspot image?

Answer Key:



Problem 1 - What is the scale of each image in kilometers per millimeter? **Answer:** The pictures are 75 mm on a side, so the scale is 34,300 km /75mm = **457 km/mm**

Problem 2 - What is the elapsed time between each image in;

A) hours and minutes? About 1 hour and 20 minutes.
B) decimal hours? About 1.3 hours
C) seconds? About 1.3 hours x 3600 seconds/hour = 4700 seconds

Carefully study each image and look for at least 5 features that have changed their location between the two images. (Hint, use the nearest edge of the image as a reference). Students may also use transparent paper or film, overlay the paper on each image, and mark the locations carefully.
The above picture shows one feature as an example.

Problem 3 - What direction are they moving relative to the sunspot?
Answer: Most of the features seem to be moving away from the sunspot.

Problem 4 - How far, in millimeters have they traveled on the image? **Answer:** The feature in the above image has moved about 2 millimeters.

Problem 5 - From your answers to questions 1, 2 and 4, calculate their speed in kilometers per second, and kilometers per hour. **Answer:** 2 mm x 457 km/mm = 914 kilometers in 4700 seconds = 0.2 kilometers/sec or 703 kilometers/hour.

Problem 6 - A fast passenger jet plane travels at 600 miles per hour. The Space Shuttle travels 28,000 miles per hour. If 1.0 kilometer = 0.64 miles, how fast do these two craft travel in kilometers per second? Jet speed = 600 miles/hr x (1 / 3600 sec/hr) x (1 km/0.64 miles) = **0.26 km/sec** Shuttle = 28,000 x (1/3600) x (1/0.64) = **12.2 km/sec.**

Problem 7 - Can the Space Shuttle out-race any of the features you identified in the sunspot image? **Answer:** Yes, in fact a passenger plane can probably keep up with the feature in the example above!

9.0 Earth's Magnetism

An ordinary compass works because the Earth is itself a giant magnet with a north and a south pole. Navigators have known about the pole-seeking ability of magnetized compass needles and lodestone for thousands of years. During the last two centuries, much more has been learned about the geomagnetic field, and how it shapes the environment of the Earth in space.

The geomagnetic field is believed to be generated by a **magnetic dynamo** process near the core of the Earth through the action of currents in its outer liquid region. Geologic evidence shows that it reverses its polarity every 250,000 to 500,000 years. In fact, the geomagnetic field is decreasing in strength by 5% per century, suggesting that in a few thousand years it may temporarily vanish as the next field reversal begins. Although the geomagnetic field deflects high-energy cosmic rays, past magnetic reversals have not caused obvious biological impacts traceable in the fossil record. Earth's atmosphere, by itself, is very effective in shielding the surface from cosmic rays able to do biological damage. The location of the magnetic poles at the surface also wanders over time at about 10 kilometers per year. Mapmakers periodically update their maps to accommodate this drift.

The domain of space controlled by Earth's magnetic field is called the **magnetosphere**. The geomagnetic field resembles the field of a bar magnet, however there are important differences due to its interaction with the **solar wind**: an interplanetary flow of plasma from the Sun. The magnetosphere is shaped like a comet with Earth at its head. The field on the dayside is compressed inwards by the pressure of the solar wind. A boundary called the **magnetopause** forms about 60,000 kilometers from Earth as the solar wind and geomagnetic field reach an approximate pressure balance. The field on the night side of Earth is stretched into a long **geomagnetic tail** extending millions of kilometers from Earth. Above the polar regions, magnetic field lines from Earth can connect with field lines from the solar wind forming a **magnetospheric cusp** where plasma and energy from the solar wind may enter. Ionized gases from Earth's upper atmosphere can escape into the magnetosphere through the cusp in gas outflows called **polar fountains**. The magnetosphere is a complex system of circulating currents and changing magnetic conditions often affected by distant events on the Sun called "space weather." The conveyor belt for the worst of these influences is the ever-changing solar wind itself. Space weather "storms" can trigger changes in the magnetospheric environment, cause spectacular aurora in the polar regions, and lead to satellite damage and even electrical power outages.

9.1 Trapped Particles and other Plasmas

Within the magnetosphere there are several distinct populations of neutral particles and plasmas. The **Van Allen Radiation Belts** were discovered in 1958 during the early days of the Space Age. The inner belts extend from altitudes between 700 to 15,000 km and contain very high-energy protons trapped in the geomagnetic field. The outer belt extends from 15,000 to 30,000 km and mostly consists of high-energy electrons. Geosynchronous satellites orbit Earth just outside the outer belt. Human space activity is confined to the zone within the inner edge of the inner belt. Space suited astronauts exposed to the energetic particles in the Van Allen Belts would receive potentially lethal doses of radiation. The particles that make up the Van Allen Belts bounce along the north and south-directed magnetic field lines to which they are trapped like water flowing in a pipe. At the same time, there is a slow drift of these particles to the west if they are positively charged, or east if they are negatively charged. There are also three additional systems of particles that share much the same space as the Van Allen Belts, but have much lower energies: the geocorona; the plasmasphere; and the ring current.

Extending thousands of kilometers above Earth is the continuation of its tenuous outer atmosphere called the **geocorona**. It is a comparatively cold, uncharged gas of hydrogen and helium atoms whose particles carry little energy. In the geocoronal region, there is a low-energy population of charged particles called the **plasmasphere** which are a high-altitude extension of the ionosphere. Unlike the geocorona, the plasmasphere is a complex, ever-changing system controlled by electrical currents within the magnetosphere. These changes can cause this region to fill up with particles, and empty, over the course of hours or days.

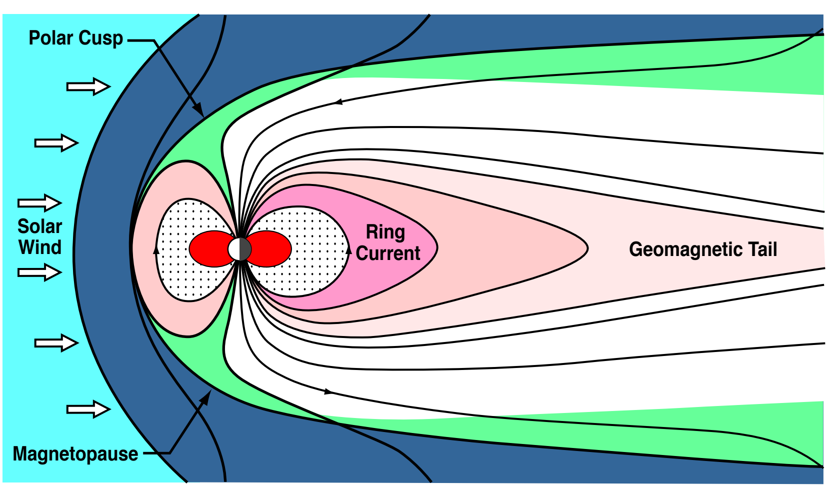


Figure 5-1 Earth's Magnetic Field. The geomagnetic field resembles the field of an ordinary bar magnet. The north magnetic pole of Earth is located near the south geographic pole while the south magnetic pole of Earth is located near the north geographic pole. The figure also shows the major regions of Earth's magnetosphere. The dotted region contains the Van Allen Radiation Belts. The red region is the plasmasphere.

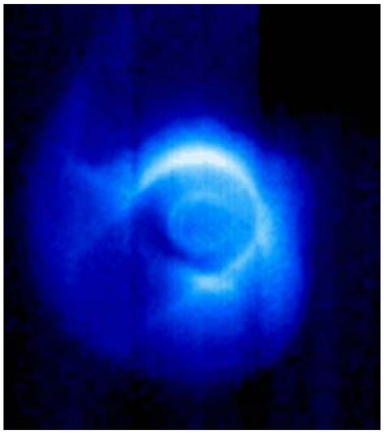


Figure 5-2 The Plasmasphere.
A view from above the North Pole of the plasmasphere illuminated by ultraviolet light from the Sun. The Sun is located beyond the upper right corner.

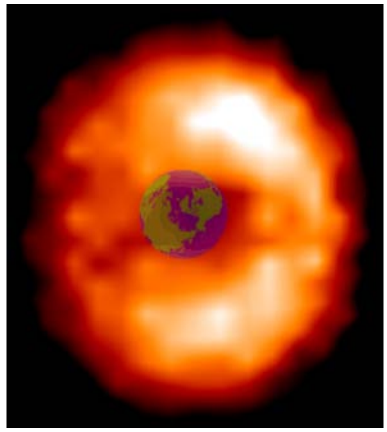


Figure 5-3 The Ring Current.
From above the North Pole, the current is seen flowing around the equatorial regions of the Earth.

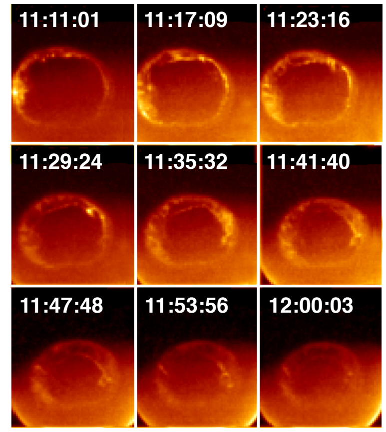


Figure 5-4 The Auroral Oval.
From space, the aurora borealis appears as a ring of light that changes its appearance from minute to minute.

During severe storms, compasses display incorrect bearings as the surface geomagnetic field changes its direction. In the equatorial regions, an actual decrease in the strength of the geomagnetic field can often be measured. This is generally attributed to the existence of a temporary river of charged particles flowing between 6,000 to 25,000 kilometers above ground: the **ring current**. These particles have energies between those within the plasmasphere and those in the Van Allen Belts. They appear to originate within the geomagetic tail as charged particles that are injected deep into the magnetosphere. Most of the time there are few particles in the ring current, but during severe storms, it fills up with a current of millions of amperes which spread into an invisible ring encircling Earth. Just as a flow of current through a wire creates its own magnetic field, the ring current generates a local magnetic field that can reduce some of Earth's surface field by up to 2% over the equatorial regions.

In addition to these families of particles, there are also powerful currents of particles that appear during especially stormy conditions, and lead to visually dramatic phenomena called the **aurora borealis** and the **aurora australis:** the Northern and Southern lights.

9.2 The Aurora

For thousands of years humans have been able to look up at the northern sky and see strange, colorful, glows of light. By the early 1900's, spectroscopic studies had shown that auroral light was actually caused by excited oxygen and nitrogen atoms emitting light at only a few specific wavelengths. The source of the excitation was eventually traced to currents of electrons and protons flowing down the geomagnetic field lines into the polar regions where they collide with the atmospheric atoms. However, aurora are not produced directly by solar flares. Radio communications blackouts on the dayside of Earth are triggered by solar flares as these high-energy particles disturb the ionosphere. When directed toward Earth, expulsions of matter by the Sun called **coronal mass ejections** contribute to the conditions that cause some of the strongest aurora to light up the skies. At other times, a simple change in magnetic polarity of the solar wind from north-directed to south-directed seems to be enough to trigger aurora without any obvious solar disturbance.

Because of the existence of the magnetospheric cusp on the dayside of Earth, solar wind particles can, under some conditions, flow down this entryway into the polar regions. This causes daytime aurora, or the diffuse red glows of nightime auroras. This is, virtually, the only instance where solar wind particles can directly cause aurora. It is not, however, the cause of the spectacular nightime polar aurora that are so commonly photographed. To understand how these aurora are produced, it is helpful to imagine yourself living inside a television picture tube. We don't see the currents of electrons guided by magnetic forces, but we do see them paint serpentine pictures on the atmosphere, which we then see as the aurora. The origin of these currents is in the distant geomagnetic tail region, not in the direct inflow of solar wind plasma.

When the polarity of the solar wind's magnetic field turns southward, its lines of force encounter the north-directed lines in Earth's equatorial regions on the dayside. The solar wind field lines then connect with Earth's field in a complex event that transfers particles and energy into Earth's magnetosphere. While this is happening near Earth, in the distant geomagnetic tail, other changes are causing the geomagnetic field to stretch like rubber bands, and snap into new magnetic shapes. This causes billions of watts of energy to be transferred into the particles already trapped in the magnetosphere out in these distant regions. These particles, boosted in energy by thousands of volts, then flow down the field lines into the polar regions to cause the aurora, like the electrons in a television picture tube that paint a pattern on the phosphor screen.

NASA Resources on Magnetism

Despite the fact that, next to gravity, magnetism is the most popularly-known force of nature, there are surprisingly few resources at NASA that express any of its interesting quantitative aspects. Let alone any mathematical expression of its many qualities. Below I have collected together the resources that NASA has identified as recommended resources for educators, based upon the item's having been submitted to an independent quality review panel and passed muster for their scientific and educational accuracy and content. The ones in this list included some mathematical or quantitative aspect to them. Usually this is directed at elementary or middle school students below grade 8.

It is still fairly common for science courses in grades 8 or lower not to attempt very much mathematical exposition, or quantitative discussion, of magnetism. The items below are usually more advanced than what most science teachers would prefer to cover in middle school science classes, although high school teachers (physics) often find these resources fertile ground for hands-on laboratory experiments in magnetism to get their students familiar with the basic concepts.

Solar Storms and You! (IMAGE Mission; Grades 7-9)
http://spacemath.gsfc.nasa.gov/NASADocs/nasa2.pdf - This was the first series of books on the magnetic interaction between the sun and earth, and was one of 5 guides featuring 3-5 math problems suitable for middle school students. The problems cover many aspects of graph analysis, using graphing calculators, and doing simple forecasting.

Exploring Earth's Magnetic Field (IMAGE Mission; Grades 7-9)
http://spacemath.gsfc.nasa.gov/NASADocs/magbook2002.pdf - This is a collection of 23 math problems spanning Grades 6-12 that cover magnetic fields, earth's magnetic field, solar storms and how to build and use a simple 'jam jar' magnetometer to detect magnetic storms.

A Guide to Earth's Magnetic Personality (THEMIS Mission; Grades 9-14)
http://cse.ssl.berkeley.edu/SEGwayed/lessons/exploring%5Fmagnetism/earths%5Fmagnetic%5Fpersonality/ - An historical introduction to the discovery of terrestrial magnetism, why we have solar storms, and the role of ground and space-based magnetometers in studying the geomagnetic field and its changes. A few brief math problems related to the 3-D shape of the magnetic field.

Exploring Magnetism on Earth (THEMIS Mission; Grades 8-12)
http://cse.ssl.berkeley.edu/SegWayEd/lessons/exploring_magnetism/magnetism_on_earth/explore_mag_on_earth.pdf - All about compass navigation, polar wander and reversals with some math-related and hands-on problems and lab activities.

Exploring Magnetism - A Teacher's Guide (THEMIS Mission: Grades 6-9 and 8-12) http://cse.ssl.berkeley.edu/impact/magnetism/MagGuide.htm - This is a web portal to 6 topic guides that cover solar flares, the solar wind, space weather, geomagnetism and electromagnetism. Each guide has up to 5 individual math-related lab activities that explore magnetism in detail, with helpful teacher information for how to conduct the various hands-on activities.

Exploring Magnetism in Solar Flares (RHESSI Mission; Grades 6-9) http://cse.ssl.berkeley.edu/SEGwayed/lessons/exploring_magnetism/in_Solar_Flares/ - This is a detailed exploration of solar magnetism, sunspots, sunspot cycles, with a number of math-based exercises in determining scale and speed of solar plasmas.

Mapping Magnetic Influence (Space Weather Action Center: Grades 6-9) http://sunearthday.nasa.gov/swac/materials/Mapping_Magnetic_Influence.pdf - This is one of the best step-by-step guides to revealing the magnetic field lines around bar magnets using various techniques including the familiar 'iron filing' method, and the more accurate 'Magnaprobe' method. Students chart the extent and polarity of magnetic fields under sheets of paper and sketch fieldlines in all their complexity, without using messy iron filings ,which are banned in many classrooms because of safety issues.

The Exploration of the Earth's Magnetosphere (Grades 9-14) http://www.phy6.org/Education/Intro.html - This is an extensive, largely non-mathematical, online textbook that covers virtually all aspects of magnetism.

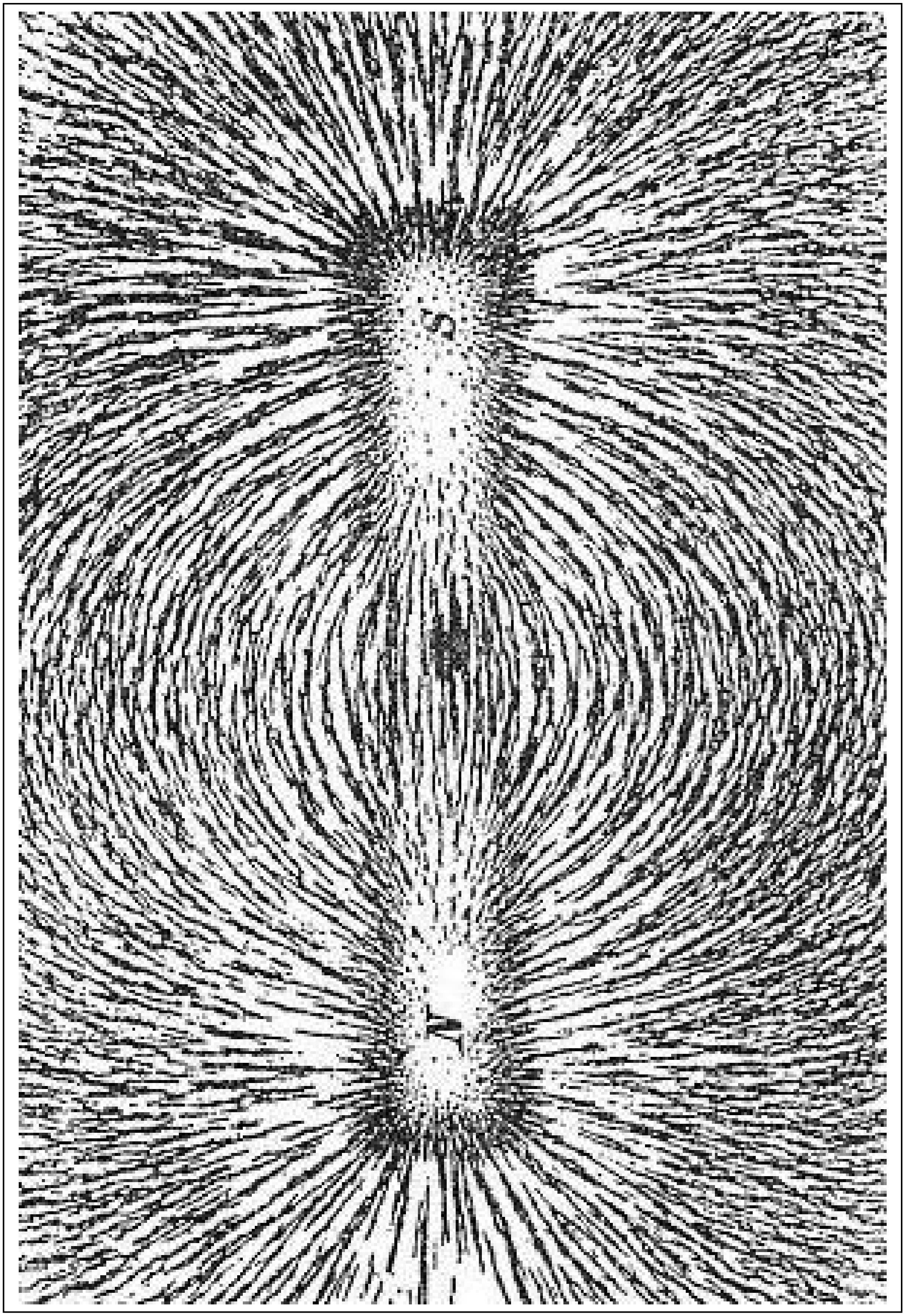
The next few pages provide you with some basic, and very handy, diagrams of magnetic fields represented by field lines.

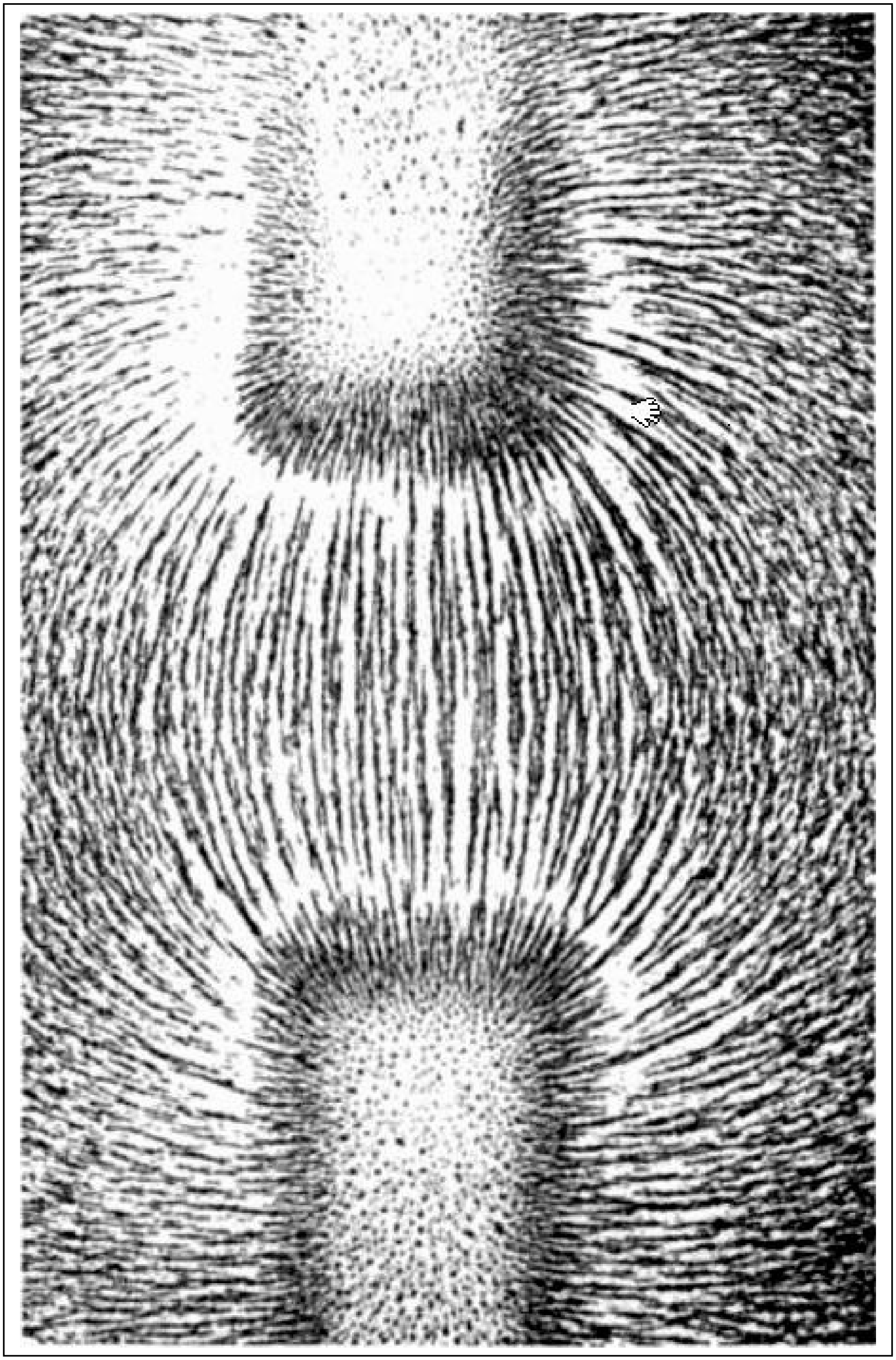
Image 1 - The classical bar magnet with iron filings from an old 1906 textbook.

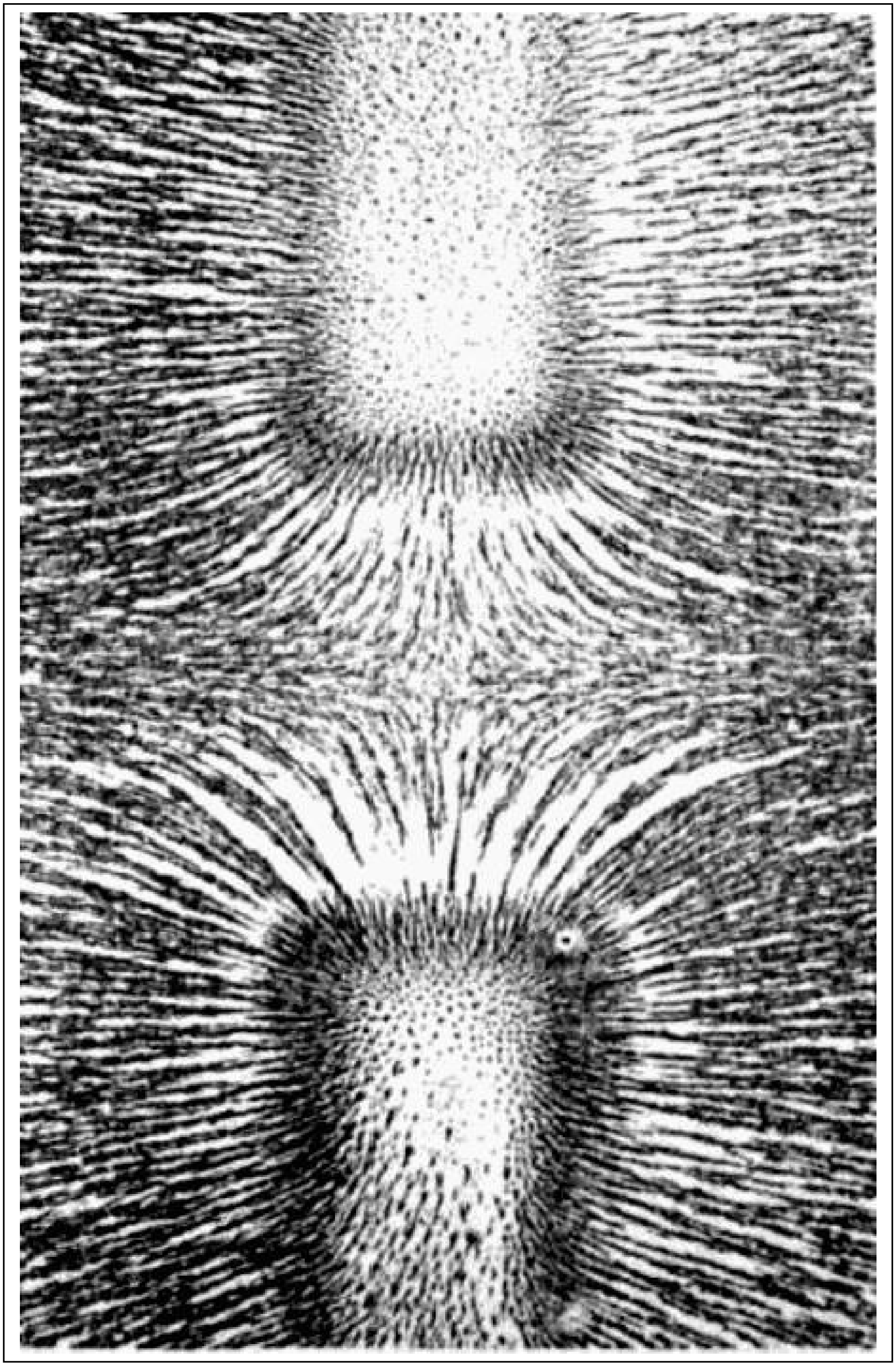
Image 2 - Opposite-polarity field lines for two magnets (North + South poles)

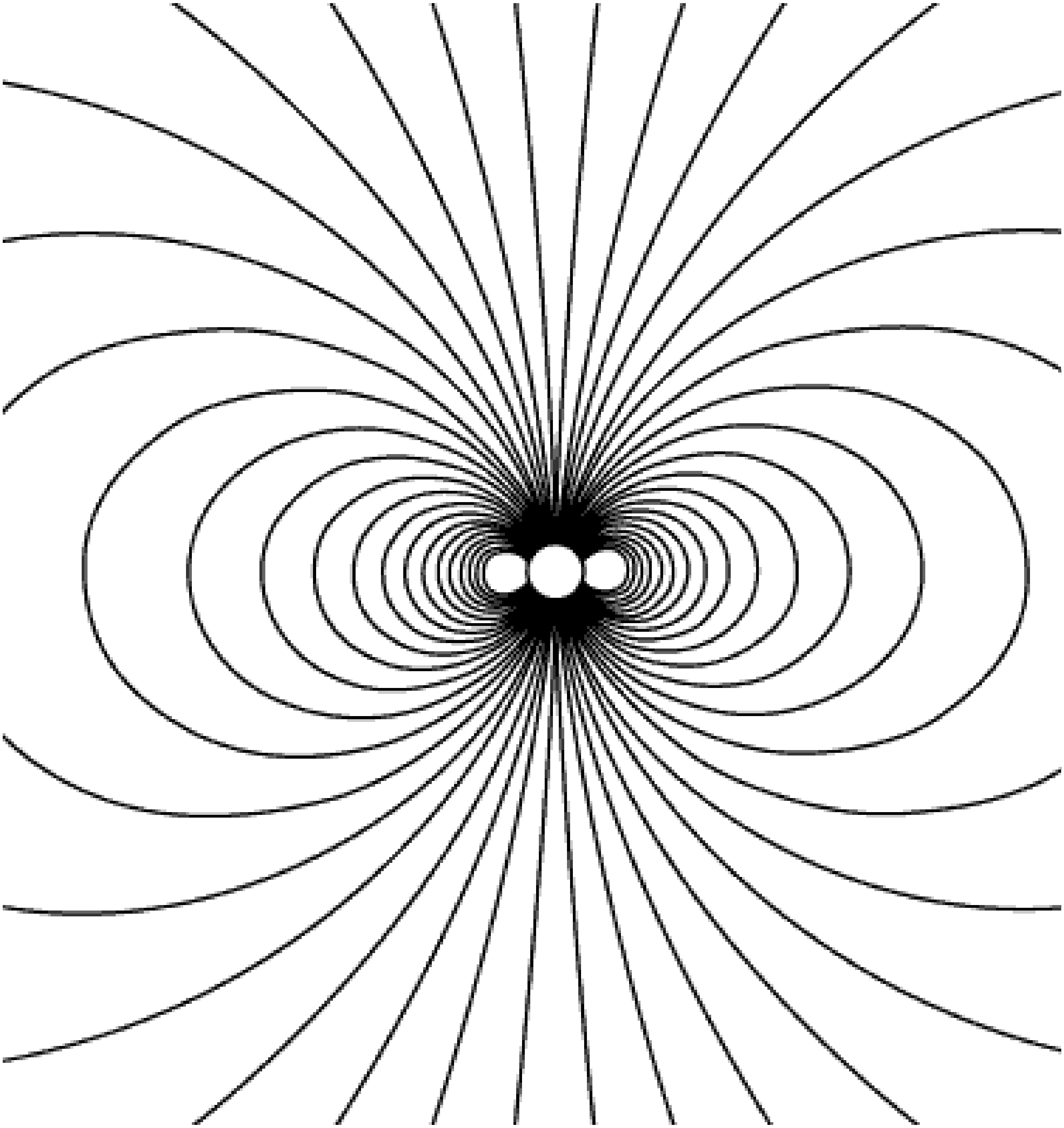
Image 3 - Like-polarity field line pattern for two magnets. (North + North poles)

Image 4 - Computed magnetic field lines for a small magnetized sphere at center.









Additional Resources and Links

Here are some resources that will give you a quick introduction to basic geophysics and magnetism:

USGS:
http://geomag.usgs.gov

NASA/GSFC – Exploring the Magnetosphere:
http://www-istp.gsfc.nasa.gov/Education/Intro.html

NASA/IMAGE:
http://image.gsfc.nasa.gov/poetry/magnetism/magnetism.html

Magnetic Reversals and Polar Wander
http://www.geolab.nrcan.gc.ca/geomag/reversals_e.shtml

More Magnetic Reversal Info:
http://www.geomag.bgs.ac.uk/reversals.html

NASA/IMAGE:
http:**/image.gsfc.nasa.gov/poetry**

NOAA/SEC: **http://www.sec.noaa.gov/SWN**

NASA/NSSDC International Geophysical Reference Field:
http://modelweb.gsfc.nasa.gov/models/cgm/cgm.html
http://ccmc.gsfc.nasa.gov/modelweb/models/igrf.html

A Note from the Author

August 18, 2009

Dear Teacher and Student,

Magnetism is such a neat force! It was probably the first one we all played with when we were young. You don't after all 'play' with gravity, and with magnetism you have a keener sense of invisible things going on in the world that you can control at least just a bit. It seems like such a simple force, with the exception that it seems to have two 'polarities' to keep track of. Gravity just has the one polarity, 'neutral', and is always attractive. Iron filings show that it has a very distinct geometer revealed by the beautiful lines that the filings make out in otherwise empty space. But the beauty underscores a very important fact. It took over 4000 years since the Chinese first mentioned it, for humans to finally understand how magnetism works, and why Nature chose to create such an odd force to do its business.

This booklet is a hodge-podge of math-related problems having to do with magnetism. I designed them as the next logical step beyond what students explore in their middle-school Earth Science textbooks. The Lab exercises bridge the experiential gap in learning about magnetism so that students can work the problems with a bit more intuitive insight as to what is going on.

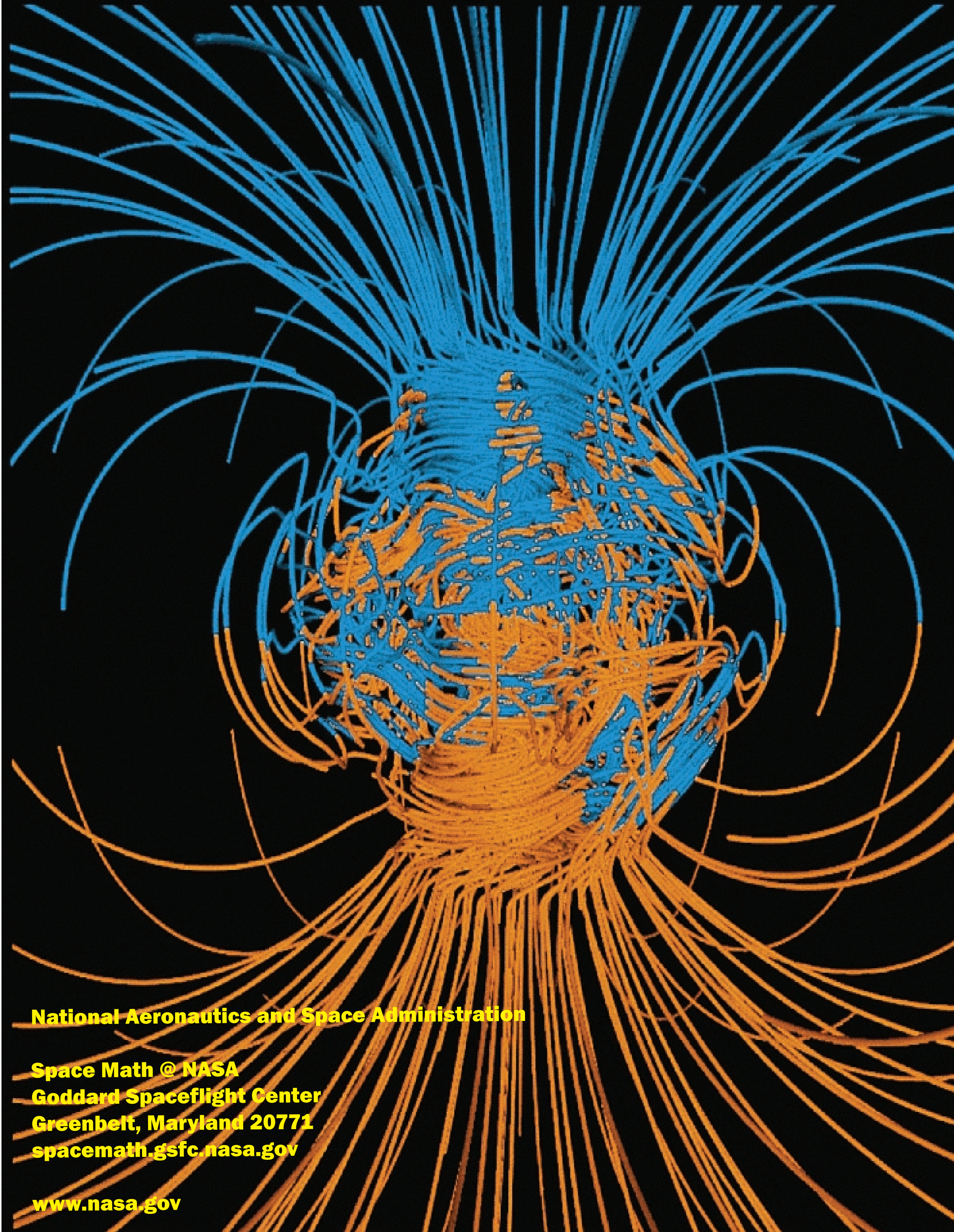
One thing that is important to realize is that, no matter how visually compelling the 'magnetic lines of force' model seems to be after working with iron filings, there are actually NO individual lines of magnetism enscribed into the space around the magnet. You should think of these 'lines' as just the pattern made by tiny rows of compass needles attached end-to-end. Mathematically and for physicists ,lines of force are a handy model to use to do calculations of magnet intensity. You can easily draw on a piece of paper the essential features of a magnetic field in a region of space, just by drawing the magnetic field lines that pass through it. This becomes a short-hand notion for describing in a drawing the intensity and orientation of a magnetic field in space. In many ways, magnetic field lines are like the contour curves on a topographic map. They don't exist either, but they sure are handy when you are looking at a map and trying to decide the best way to hike into a valley!

Anyway, have fun with these problems, and enjoy exploring invisible forces. They are an important lesson in physics in which we can learn about invisible things in the world just by playing with them, examining their consequences upon other visible things, and making a few measurements!

In the scientific exploration of our physical world, it doesn't get any better than this!!!

Sincerely,

Dr. Sten Odenwald



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