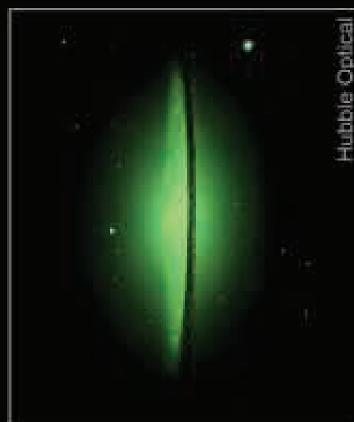
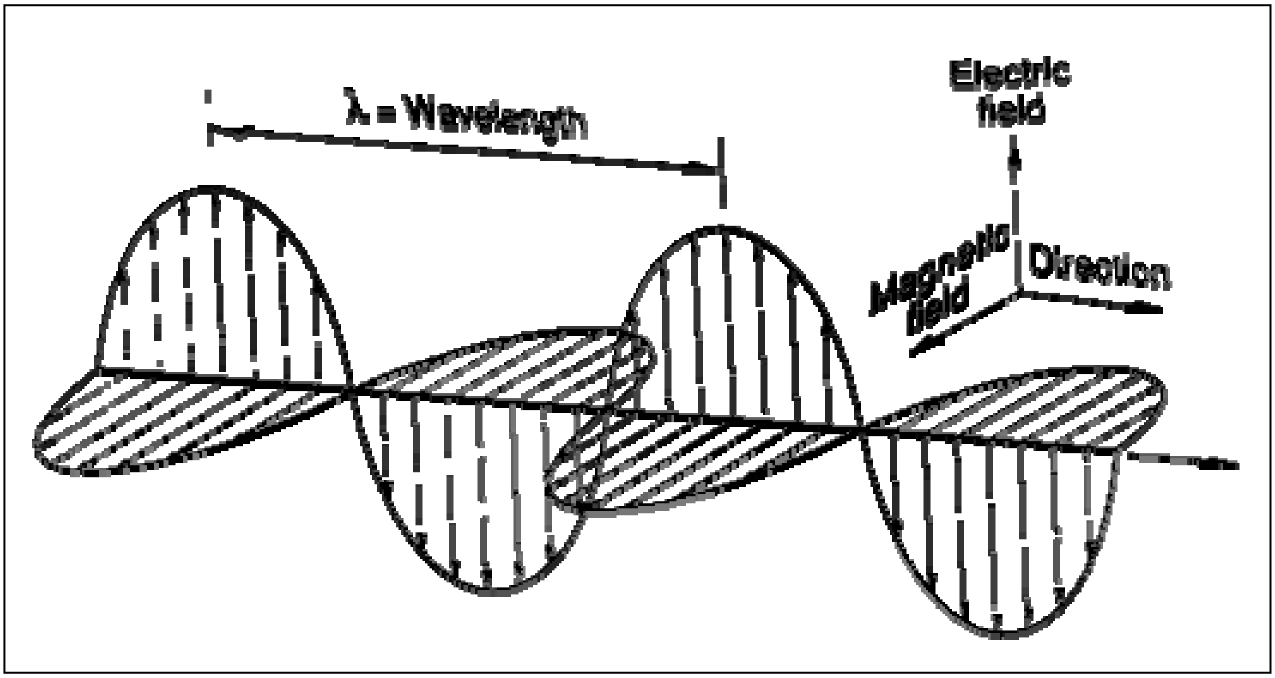
National Aeronautics and Space Administration

Electromagnetic Math



EM Math



A Brief Mathematical Guide
to
Electromagnetic
Radiation

Dr. Sten Odenwald
NASA

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2004-2010 school years. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 6 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

Acknowledgments:

I would like to thank Ms. Elaine Lewis for her careful reading of the manuscript and her essay 'How to Use This Book', written from a science curriculum developer's perspective.

Front and back cover credits: Multi-wavelength images of the Sombrero galaxy: X-ray: NASA/UMass/Q.D.Wang et al.; Optical: NASA/STScI/AURA/Hubble Heritage; Infrared: NASA/JPL-Caltech/Univ. AZ/R.Kennicutt/SINGS Team. Images of the sun: Gamma (NASA/CGRO); X-ray (Yohkoh/NASA); EUV(EIT/SOHO); UV(EIT/SOHO); H-alpha(NSO); 1083 nm Infrared(NSO); Optical(GONG); Millimeter(Atacama Millimeter Array); 5GHz radio(VLA/NRAO); 1.5GHz (VLA/NRAO); 150 MHz (Nancay); 327 MHz(VLA/NRAO)

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

To suggest math problem or science topic ideas, contact the Author, Dr. Sten Odenwald at

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Alignment with Standards

AAAS Project:2061 Benchmarks

(9-12) - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM:"Principles and Standards for School Mathematics"

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system;

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers		X	X	X	X		X																					X	X	X	
Numbers, patterns, percentages																															
Averages																															
Time, distance, speed	X					X			X	X	X	X	X	X	X	X	X	X	X			X	X								X
Areas and volumes																												X	X	X	
Scale drawings		X	X	X	X																							X	X	X	
Geometry																															
Probability, odds																															
Scientific Notation				X				X			X				X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Unit Conversions	X			X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
Fractions																															
Graph or Table Analysis													X	X				X	X	X	X	X	X	X	X		X				
Pie Graphs																															
Linear Equations																															
Rates & Slopes																															
Solving for X												X				X				X			X	X	X	X					X
Evaluating Fns																															
Modeling																															
Trigonometry																															
Logarithms																															
Calculus																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry	X																														
Technology, rulers			X	X	X	X				X																					
Numbers, patterns, percentages		X		X	X			X	X	X				X	X																
Averages																															
Time, distance, speed													X																		
Areas and volumes									X	X								X	X	X	X	X									
Scale drawings		X	X	X	X	X			X	X																					
Geometry		X	X	X	X	X		X	X	X								X	X	X	X	X	X	X	X	X					
Probability, odds																															
Scientific Notation							X						X			X	X	X	X	X	X		X	X	X		X	X			X
Unit Conversions			X	X	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X					
Fractions																															
Graph or Table Analysis							X				X	X																X	X	X	X
Pie Graphs																															
Linear Equations																															
Rates & Slopes														X				X	X				X		X	X	X				
Solving for X							X	X																							X
Evaluating Fns											X	X				X	X		X		X	X		X	X		X		X	X	X
Modeling																															
Trigonometry																															
Logarithms											X	X																X	X		
Calculus																															X

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																														
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84									
Inquiry		X					X				X			X		X		X			X										
Technology, rulers								X	X	X		X				X	X				X	X									
Numbers, patterns, percentages				X	X	X							X			X															
Averages																															
Time, distance, speed																															
Areas and volumes																															
Scale drawings								X	X	X						X	X				X	X									
Geometry																															
Probability, odds																															
Scientific Notation	X		X					X						X																	
Unit Conversions				X										X	X		X			X	X	X									
Fractions																															
Graph or Table Analysis	X		X					X	X	X		X						X	X												
Pie Graphs																															
Linear Equations																															
Rates & Slopes																															
Solving for X			X																												
Evaluating Fns			X				X	X						X	X			X	X	X	X	X									
Modeling			X																												
Trigonometry																															
Logarithms	X																														
Calculus																															

How to Use this Book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. This book covers a single topic **Electromagnetic Math**.

Electromagnetic Math is designed to be used as a supplement for teaching mathematical topics and in this supplement electromagnetism a science topic taught middle and high school. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery and also as a supplement in the science classroom, it is a good source as a complete study for electromagnetism and the mathematics applications.

In modeling phenomena, students should encounter a variety of common kinds of relationships depicted in graphs (direct proportions, inverses, accelerating and saturating curves, and maximums and minimums) and therefore develop the habit of entertaining these possibilities when considering how two quantities might be related. None of these terms need be used at first, however. "It is biggest here and less on either side" or "It keeps getting bigger, but not as quickly as before" are perfectly acceptable—especially when phenomena that behave like this can be described. (Benchmarks- the Mathematical World)

The effect of wavelength on how waves interact with matter can be developed through intrinsically interesting phenomena—such as the blueness of the sky and redness of sunsets resulting from light of short wavelengths being scattered most by the atmosphere, or the color of grass resulting from its absorbing light of both shorter and longer wavelengths while reflecting the intermediate green. Electromagnetic waves with different wavelengths have different effects on the human body. Some pass through the body with little effect, some tan or injure the skin, and some are absorbed in different amounts by internal organs (sometimes injuring cells). (Benchmarks-Physical Setting-Motion)

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Electromagnetic Math**. Read the scenario that follows:

Ms. Green, the High School Mathematics teacher and Mr. Brown the High School Physics teacher decided to use Electromagnetic Math as a supplement to teaching math applications within the science content- electromagnetism. They began by using the Bizarre Experiments 1-2-3 at the beginning of this book so that students could understand how motion, charge and magnetism are intertwined and the way these relationships can be mathematically modeled. The teachers each selected the activities that would enhance what they were teaching and worked together so that the sequence of activities was effective in the math and science classrooms.

Electromagnetic Math can be used as a classroom challenge activity, assessment tool, and enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

"Space Math has more up-to-date applications than are found in any textbook. Students enjoy real-world math problems for the math they have already learned. Doing Space Math problems has encouraged some of my students to take pre-calculus and calculus so they can solve the more advanced problems. I learned about Space Math through an email last year. I was very impressed with the problems. I assigned some of the problems to students in my Physics classes, printing them out to put in their interactive notebooks. I displayed other problems for group discussion, assigned some for homework and used some for group class work. I like the diversity, the color format and having the solutions. I expect to use them even more next year in our new space science class. We will have 50 students in two sections." (Alan. High School Science Teacher)

"It took time for them to make the connection between the math they learned in math class and applying it in the science classroom. Now I use an ELMO to project them. I have used them for class work and/or homework. The math activities were in conjunction with labs and science concepts that were being presented. The math helped "show" the science. Oftentimes students were encouraged to help and teach each other. Students began to see how math and science were connected. I knew the students were making the connections because they would comment about how much math they had to do in science. Their confidence in both classes increased as they were able practice the concepts they learned in math in my science class." (Brenda, Technology Resource Teacher)

Introduction to Electromagnetic Math

Like Siamese Twins, electricity and magnetism are ‘joined at the hip’ in all physical phenomena where charged particles are present. They are especially obvious whenever charged particles are in motion. This is why physicists usually refer to these phenomena as examples of ‘electromagnetism’. Running two separate words or names together is a common practice in science, especially when some deep connection is seen between separate things and only the combination of the two really has any meaning. By calling some phenomenon electromagnetism, physicists are emphasizing that the connection between electricity and magnetism is so deep and subtle that it makes no sense to treat the magnetic properties and electric properties separately.

The reason why electricity and magnetism are not separate kinds of phenomena is surprising, and a very bizarre introduction to how the physical world is actually put together! To understand this, let’s look at a simple experiment.

In most science classes, you learned that if you let a current of electricity flow in a copper wire, that a magnetic field will surround the wire, and you have created what is called an electromagnet. This does not happen if there is no current with the charged particles (electrons) just sitting motionless. Here’s where things get interesting!

Suppose your friend was in the back of a pick-up truck holding a charged particle in her hand. She would see a charged particle ‘just sitting there’ not moving, and producing its usual electrical field. If she brought another oppositely-charged particle close to the one in her hand, the electric fields would cause them to attract each other. But all she would be able to measure were the electric fields of these particles and their combination. You, however, would see something different!

From where you are standing, watching the truck move away, you would see that the charge in her hand was in motion. It was being carried away from you by the truck! Because moving charges produce magnetic fields, you would see her hand surrounded by a magnetic field. Amazingly, she would not see this magnetic field at all because the charged particle is not moving as she sees it from her perspective! This is an amazing example of what physicists call ‘relativity’. There is nothing spooky about it. This just the way that Nature works.

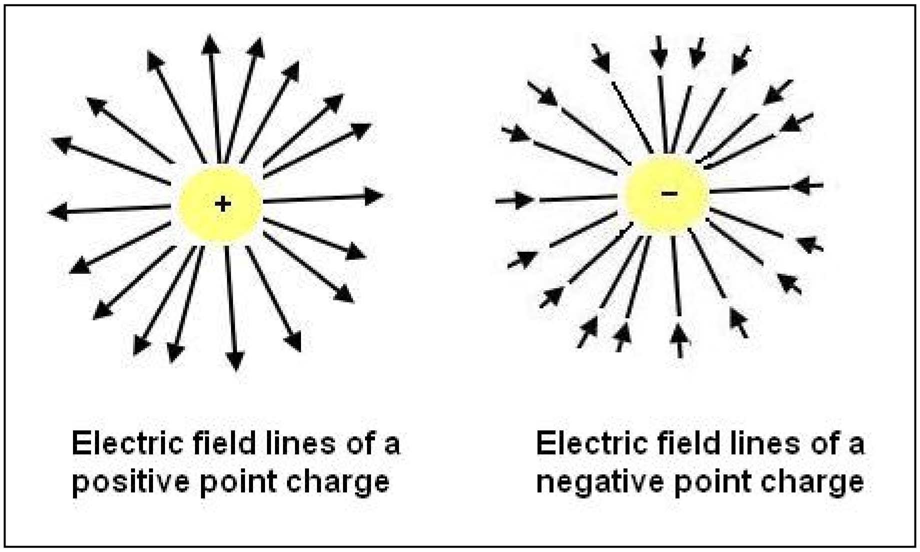
Matter can move in complex ways. What you see depends on what your ‘frame of reference’ is doing, and how it is moving relative to other bodies. Some of those bodies can include charged ones like atoms, and so magnetic fields can be seen in many different places in our environment whenever there are charged particles present. This presents certain difficulties.

If we wanted to describe the motion of those charged particles in the truck, we could do so by asking your partner what she was seeing. She would give a very simple description and say ‘I just see a charged particle motionless in my hand’. If you wanted to describe what was happening, you would have to include a complicated description of magnetic fields and the moving charge that produce them!

Because in most situations it is impossible to create a simple description of what charged particles are doing, we almost always have to include magnetic fields as ingredients to the description because it is impractical to find the simple reference frame where all of the charges are not moving. In most cases, it is actually impossible!

We speak of electromagnetism because it makes a lot of practical sense to do so, and more importantly, it is a reflection of the intimate way that motion, charge and magnetism are intertwined. Luckily for us, these relationships can be mathematically modeled with high precision! So, what does electromagnetism have to do with light waves and other forms of 'electromagnetic radiation'?

Bizarre Experiment 1: To understand this connection, let's have another look at that charged particle sitting in your partner's hand. Suppose, also, that she wasn't moving. If you had a sensitive-enough instrument, you could measure the strength of the electric field produced by the charged particle. Suppose that at a distance of 10 meters it had a strength of 1 Volt/meter. A mathematical feature of this electric field, is that it can be directed either directly towards the charged particle or directly away from the position of the charge. As the figure below shows, the direction depends on whether the charge is positive (towards) or negative (away).



If your partner were not moving, and the charge stayed exactly where it was, you could come back 10 minutes later and make the same measurement and still find that at 10 meters its strength was 1 Volt/meter and its direction would be identical to what it was before. If she had been holding a positive charge, the electric field would still be pointed exactly towards the charged particle from your location.

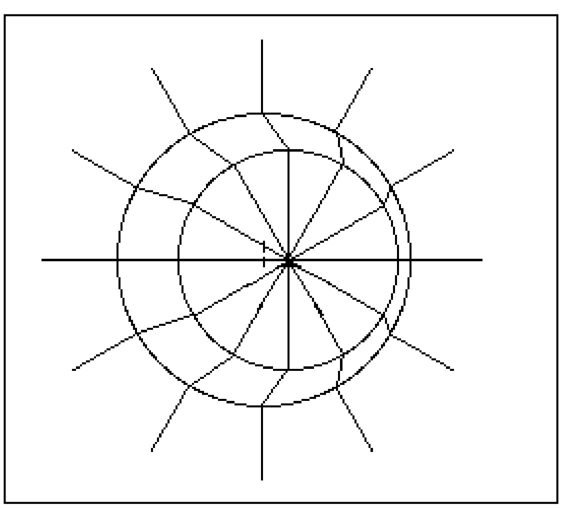
Suppose you had a super-duper instrument that could detect the electric field from this charged particle at a distance of 10 kilometers. At this distance it would have a strength of 1 microVolts/meter. Now suppose that your partner instantaneously neutralized the charge on this particle at Time=0. The electric field would vanish, but because the electric field travels at the speed of light, (300 million meters/sec) the instrument at 10 meters would not finally read 'zero' until T = 10 meters/300 million m/s = 0.000000033 sec after T=0, and the instrument at 10 kilometers would not read zero until T = +0.000033 sec!

Turning an electric charge on and off leads to some interesting results that have to do with the fact that fields move through space. In this case, when we measure the electric field, its intensity and direction at a particular point in space depends on how far away the charge is. When we turn the field off, a distant observer has to wait the appropriate 'light travel time' before they see the field 'wink out'. Now let's look at another possibility: The movement of the charged particle.

Bizarre Experiment 2: Suppose that, over the course of a second, your partner moves the charged particle 10 meters to the left of where her hand used to be, and the in the next second, she returns the charge to its original position. What will you observe and measure?

Your instrument at 10 meters from where she is sitting will, after the appropriate light travel time, measure the electric field strength change from its original 1 Volt/meter to a slightly lower value because the charge at the end of the movement is at a distance of about $d = (10^2 + 10^2)^{1/2} = 14$ meters from you. Its strength would be about 0.5 Volts/meter. After 2 seconds, this electric field change would stop, and your measurement would return to its previous 1 Volt/meter. After the appropriate light travel time has elapsed, a measurement at 10 km away would see the same 2-second change in the electric field occur and then stop. In addition to the intensity of the field, you would also measure a change in direction of the field because as she moves the charge, its direction changes from your vantage point. This also means that the direction of the electric field at 10 meters will change to follow the charge, but also with the appropriate light travel delay.

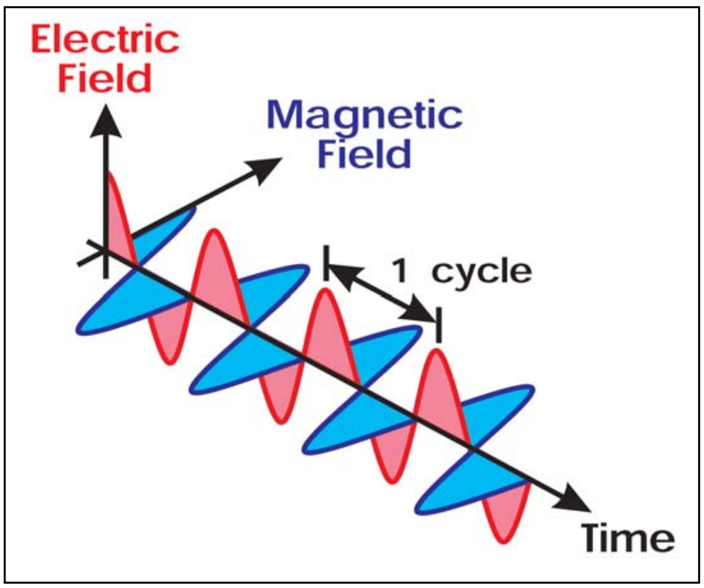
This 2-second 'kink' in the electric field travels outwards from the vicinity of the charged particle at the speed of light, and we can call this an electric-wave. But there is more to this disturbance that this. Just as a current of moving charges in a wire causes a magnetic field, a changing electric field causes a magnetic field. That means that the 2-second kink in the electric field also creates a 2-second magnetic field which travels outwards through space with the electric field. The kink, shown in the figure below, is now an **electromagnetic wave**, with a frequency of 1 cycle every 2 seconds (0.5 hertz)!



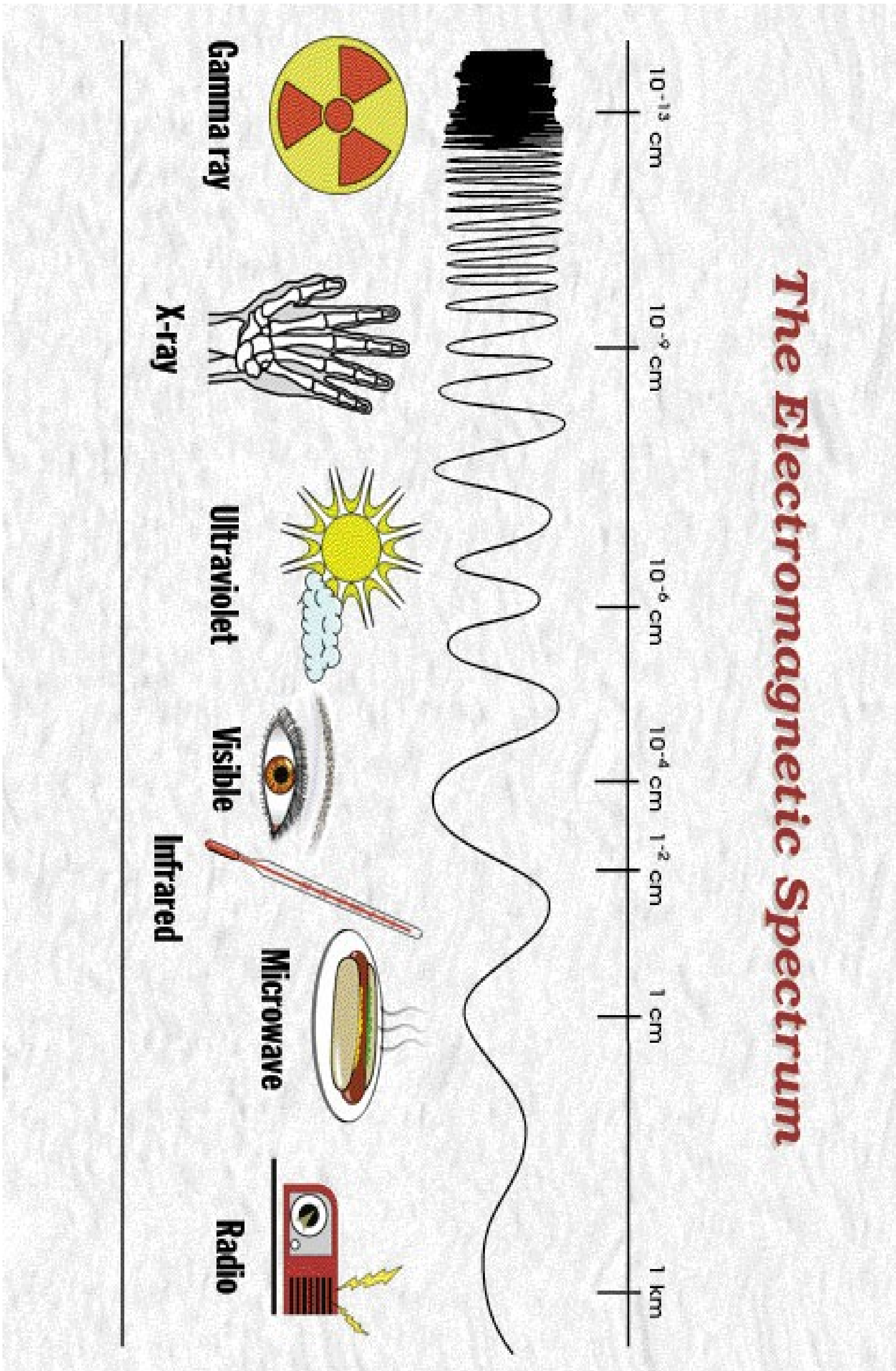
The kink is only produced when the charged particle is accelerating in its motion. Note that in the figure, as the particle moves to the right, close-by electric field lines point to where the particle is now, while distant lines point to where it was. Also, the width of the wave 'kink' changes in accordance with the Doppler effect.

The frequency, f, of the electromagnetic wave depends on how quickly the electric field changes as a charged particle is moving ‘back and forth’ in a periodic motion. The wavelength, w, depends on the frequency and then speed of light, c, so that c = f w.

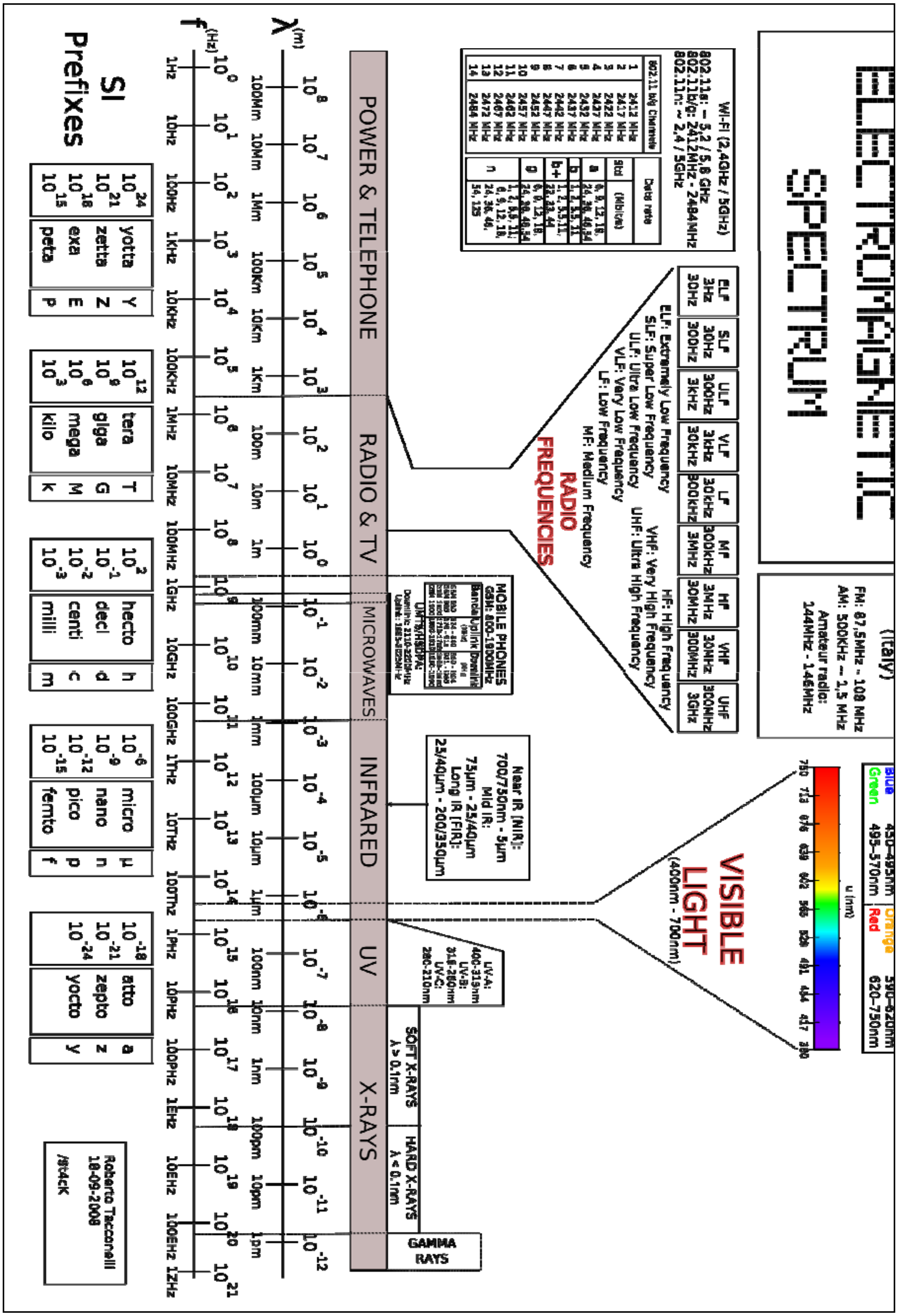
Because the movement (actually the acceleration) of charged particles can be very complex in time, moving charges seldom produce a pure, single frequency of electromagnetic radiation. Instead they produce a complicated spread of waves with many different frequencies and intensities called a spectrum. There are mathematical techniques that can predict the electromagnetic spectrum of many kinds of charged particles moving in complicated ways. These mathematical models can be used to investigate distant sources of electromagnetic radiation using the spectrum of the radiation as a fingerprint.



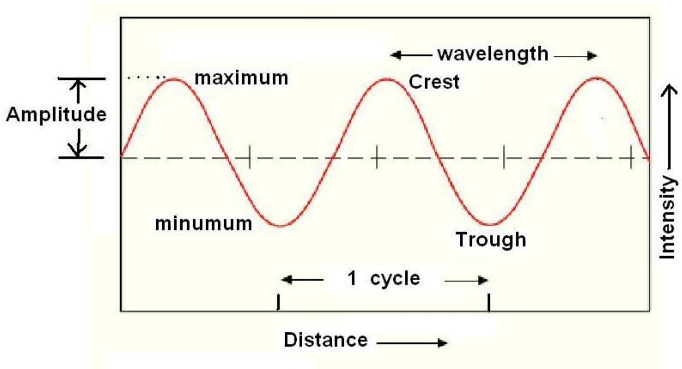
Bizarre experiment 3: Another feature of electromagnetic radiation is that, as the radiation leaves the vicinity of the accelerating charges, the energy is spread out over larger and larger areas as it travels. This means that, in addition to the total energy of the radiation and its direction, there is also another physical property we can use to describe it. This property depends on how far away from the source you are. We call this new property ‘brightness’ or ‘intensity’, and measure it in watts per square meter. Because in most situations, electromagnetic radiation is emitted in all directions equally, we can imagine the Source sitting at the center of a sphere. Brightness is just the energy emitted by the Source, divided by the surface area of the sphere whose radius equals the distance to the Observer which is just… **The Inverse-Square Law**!


The Electromagnetic Spectrum
10^{-13} cm
10^{-9} cm
10^{-6} cm
10^{-4} cm
1^{-2} cm
1 cm
1 km
Gamma ray
X-ray
Ultraviolet
Visible
Infrared
Microwave
Radio


Size reference
football field
baseball
paperclip thickness
paper thickness
cells
bacteria
viruses
water molecule
atom
subatomic particles →
1 ft
1 cm
1 mm
1 mil
1 μ
1 nm
1 Å
1 pm
wavelength λ (m)
wavenumber (cm⁻¹)
electron volt (eV)
frequency (Hz)
1 MHz
1 GHz
1 THz
1 PHz
1 EHz
1 ZHz
Bands
Radio Spectrum
Broadcast and Wireless
Microwave
Terahertz
Infrared
Far IR
Mid IR
Near IR
Visible wavelengths (nm)
700
625
575
540
470
440
Ultraviolet
Near UV
Extreme UV
X-ray
Soft X-ray
Hard X-ray
Gamma
electronics
optics
Sources and Uses of Frequency Bands
AM radio 600kHz-1.6MHz
Sound Waves ← 20Hz-10kHz
Ultrasound 1-20 MHz
FM radio 88-108 MHz
TV Broadcast 54-700 MHz
Mobile Phones 900MHz-2.4GHz
Wireless Data ~ 2.4 GHz
Microwave Oven
Radar 1-100 GHz
"mm wave"
"sub-mm"
Bio imaging 1-10 THz
Screening 0.2-4.0 THz
Fiber telecom 0.7-1.4 μ
Remotes 850 nm
Visible Light 425-750THz 700-400nm
Dental Curing 200-350nm
Suntan 400-290nm
Medical X-rays 10-0.1 Å
Baggage screen 10-1.0 Å
Crystallography 2.2-0.7 Å
Cosmic ray observations <<1 Å
PET imaging 0.1-0.01 Å


ELECTROMAGNETIC SPECTRUM
POWER & TELEPHONE
RADIO & TV
MICROWAVES
INFRARED
UV
X-RAYS
GAMMA RAYS
RADIO FREQUENCIES
VISIBLE LIGHT
(400nm - 700nm)
SOFT X-RAYS
HARD X-RAYS
SI Prefixes
Roberto Tacconelli
18-09-2008

The Parts of a Wave



One of the first things that humans like to do is to talk about 'things'. The only way we know how to do this meaningfully is to create words to describe 'things'.

When it comes to discussing waves, there is also a whole vocabulary that has been invented to help us speak sensibly about them.

Most kinds of waves that you know actually move through space, like water waves or sound waves. Let's take a look at ocean waves by the beach as a familiar example.

In the diagram above, we call the **crest** of the wave the point where the water in the wave is at its greatest distance above the undisturbed ocean level. The **trough** is the point where the water reaches its lowest distance below the undisturbed ocean level. In many other situations that involve waves, the terms crest and trough are often replaced by the more general terms '**maximum'** and **'minimum'** .

The **amplitude** of the wave is a measure of the intensity of the wave. When surfers talk about an exciting 30-foot wave that they just rode, they mean that the amplitude of the wave above the undisturbed ocean level was 30-feet to the crest.

The **wavelength** of a wave is just the distance between neighboring crests or troughs. For instance, for waves breaking on a beach, the distance between crests can be 50 meters. The wavelength of these 'breakers' would then be 50-meters.

Waves travel through space at a speed that depends on the medium that is 'waving'. For sound waves, the **wave speed** is 340 meters/sec (780 mph; also called Mach 1 by jet pilots). For water waves on a lake, the wave speed can be 2 meters/sec. For light waves, this speed is the speed of light: 300 million meters/sec.

A **cycle** is one complete change in the property of the wave (height, air pressure, etc), so if you watched 5 ocean waves break on the beach you saw 5 cycles of a wave.

The **frequency** of a wave is just the number of complete cycles that you can count in one second of time. For a water wave on a lake lapping on the shoreline, you might count 6 cycles in 3 seconds so the frequency is 6 cycles/3 seconds = 2 cycles per second. This is a compound unit, unlike time (seconds) or distance (meters) and it has its own unit called the Hertz. One Hertz equals exactly 1 cycle per second.

A very important relationship links together frequency, wavelength and wave speed, namely: wave speed = frequency x wavelength. This can be written mathematically as $\mathbf{C = f \times \lambda}$, where C is measured in meters/sec, f is in Hertz and λ is the wavelength in meters. Let's use this formula to investigate three different waves.

Problem 1 - A tsunami wave travels at 150 kilometers/hour as it approaches land. If the wavelength is 50 kilometers, what is the frequency of the wave in Hertz?

Problem 2 - A sound wave travels at 340 meters/sec, with a pitch (frequency) of 440 Hertz (Middle-C on the piano). What is its wavelength?

Problem 3 - A radio wave with a frequency of 800,000 Hertz in the middle of the AM band travels at the speed of light. What is its wavelength?

Answer Key

Problem 1 - A tsunami wave travels at 150 kilometers/hour as it approaches land. If the wavelength is 50 kilometers, what is the frequency of the wave in Hertz?

Answer: 160 kilometers/hour = 50 kilometers x f

$$F = \frac{160 \text{ kilometers}}{1 \text{ hour}} \times \frac{1 \text{ cycle}}{50 \text{ kilometers}} = 3 \text{ cycles per hour.}$$

Since 1 Hertz = 1 cycle per second ,we have to convert our answer to cycles/sec.

$$F = \frac{3 \text{ cycles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{3600 \text{ seconds}} = \mathbf{0.00083 \text{ Hertz.}}$$

Problem 2 - A sound wave travels at 340 meters/sec, with a pitch (frequency) of 440 Hertz (Middle-C on the piano). What is its wavelength?

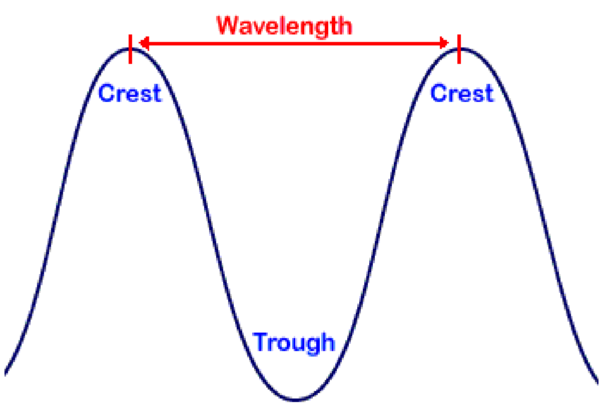
Answer: 340 meters/sec = 440 Hertz x λ

$$\lambda = \frac{340 \text{ meters}}{1 \text{ second}} \times \frac{1 \text{ second}}{440 \text{ cycles}} = \mathbf{0.77 \text{ meters}}$$

Problem 3 - A radio wave with a frequency of 800,000 Hertz in the middle of the AM band travels at the speed of light. What is its wavelength?

Answer: The speed of light is 300 million meters/sec so
300,000,000 meters/sec = 800,000 Hertz x λ

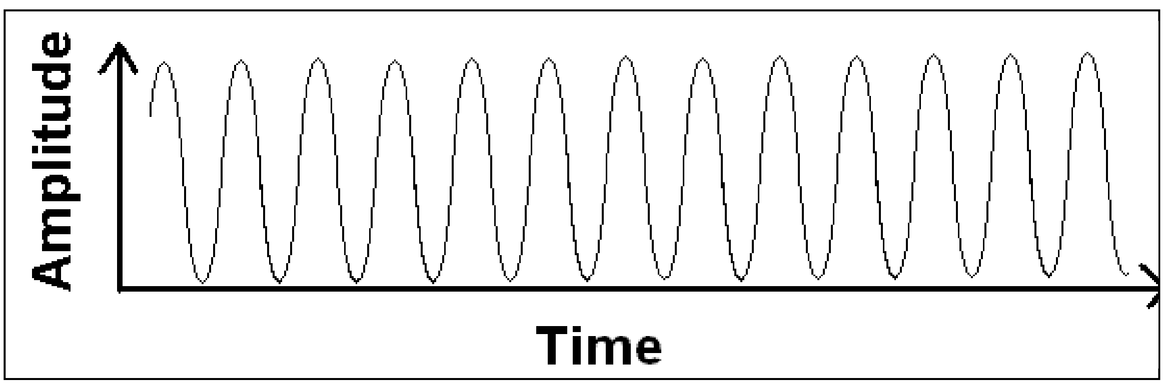
$$\lambda = \frac{300{,}000{,}000 \text{ meters}}{1 \text{ second}} \times \frac{1 \text{ second}}{800{,}000 \text{ cycles}} = \mathbf{375 \text{ meters}}.$$

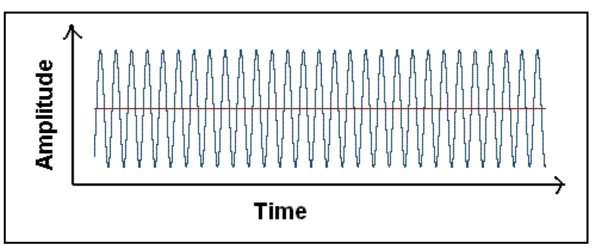


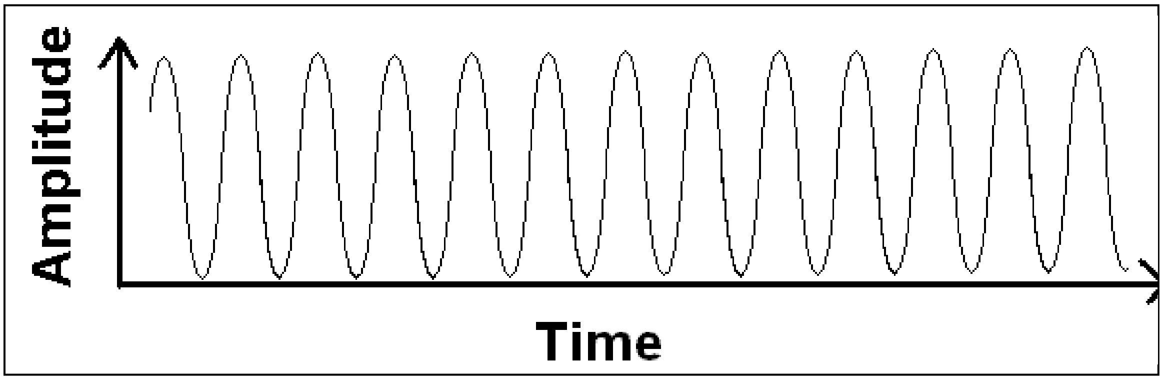
The basic parts of a wave shown on the left are its crests (peaks), its troughs (valleys) and the distance between any pairs of these parts: crest-to-crest or trough-to-trough. The distance between any two consecutive places on the wave is called the wavelength.

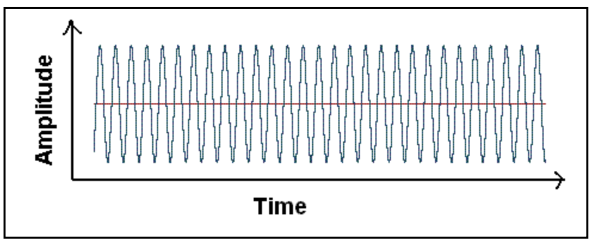
Usually, the wavelength is the same for all locations on the wave, but there are exceptions. A series of ocean waves approaching the shore, or sound waves traveling through different substances, can change their wavelengths as they travel.

With a millimeter ruler, measure the wavelengths of the examples shown below. You may need to use different methods such as measuring the total length, and dividing by the number of wavelengths in the wave.





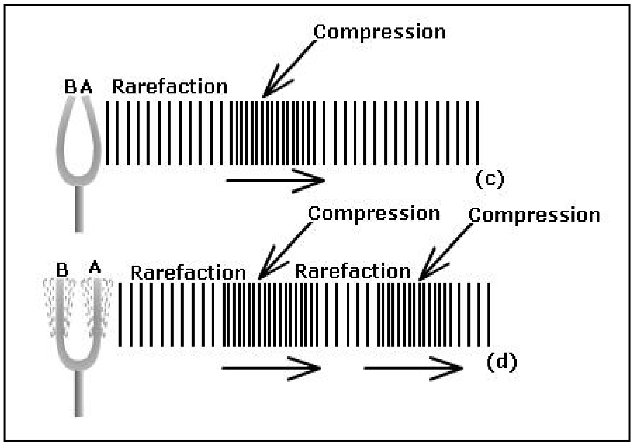


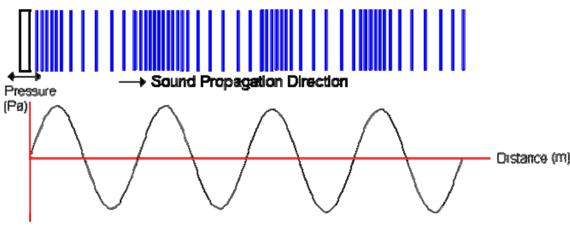


Top wave train - Measure the distance between two consecutive troughs on the front page to get about 10 millimeters. You can also note that the distance between the first peak at the start of the wave 'train' and the last peak is 122 millimeters and there are 12 complete pairs of peaks, so the wavelength is 122 millimeters/12 wave cycles = about 10 millimeters per wave cycle.

Bottom wave train - The length of the horizontal line is 59 millimeters, and there are 28 pairs of peaks along this distance, so the wavelength is 59 millimeters / 28 wave cycles = 2.2 millimeters per wave cycle. A direct measurement of consecutive peaks or troughs will give about 2 millimeters.

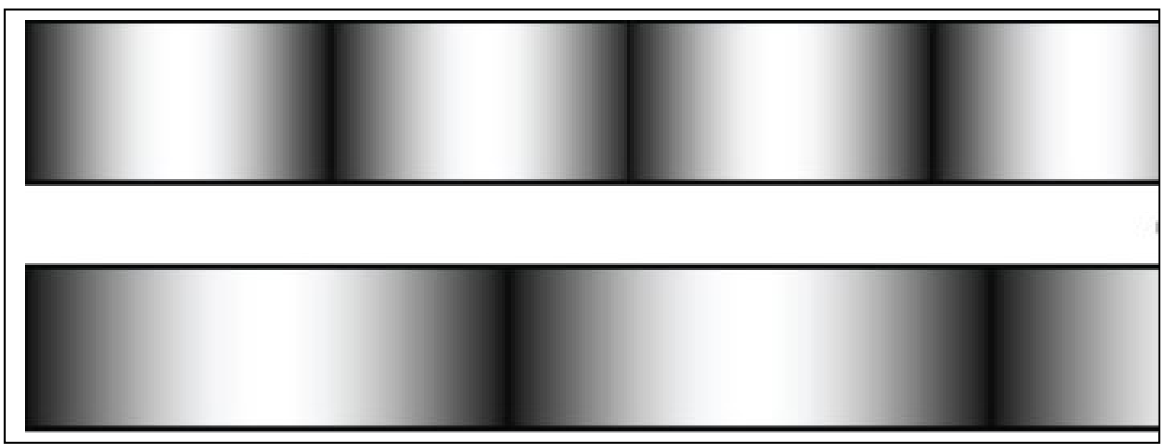
Exploring Sound Waves

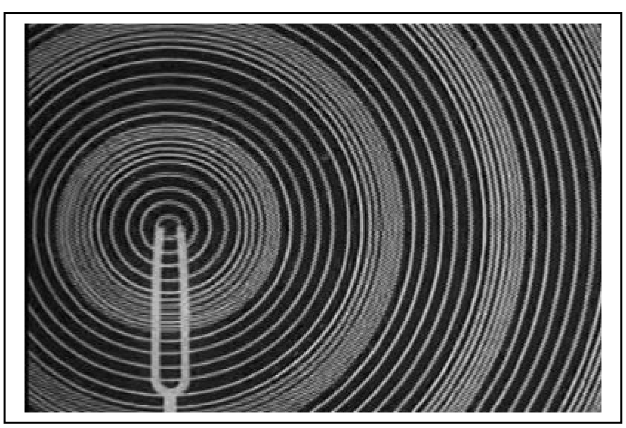




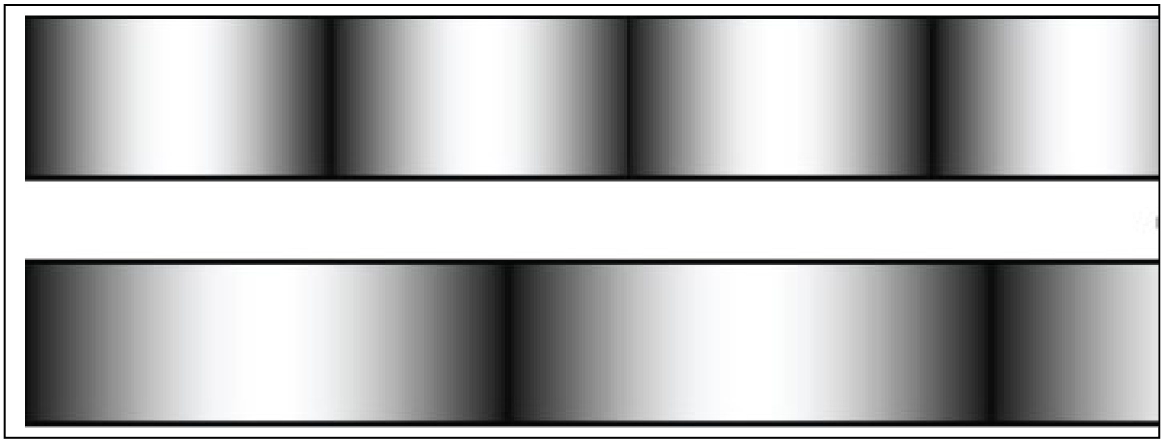
Sound waves are created when some object causes rhythmic changes in pressure in its surroundings. The object causes its surroundings to reach a high density (compression), followed by a period of low density (rarefaction). Our ear is sensitive to these density (pressure) changes in the air, which the brain interprets as sound. The distance between corresponding points in the sound wave defines the sound wave's wavelength.

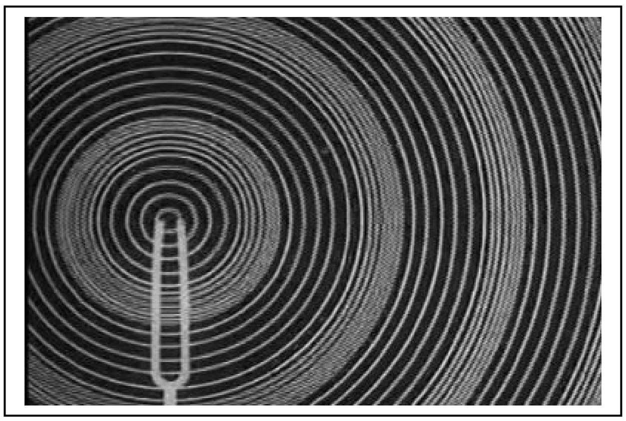
Problem 1 - Below are two ways in which sound waves are drawn. With a millimeter ruler, measure their wavelengths.





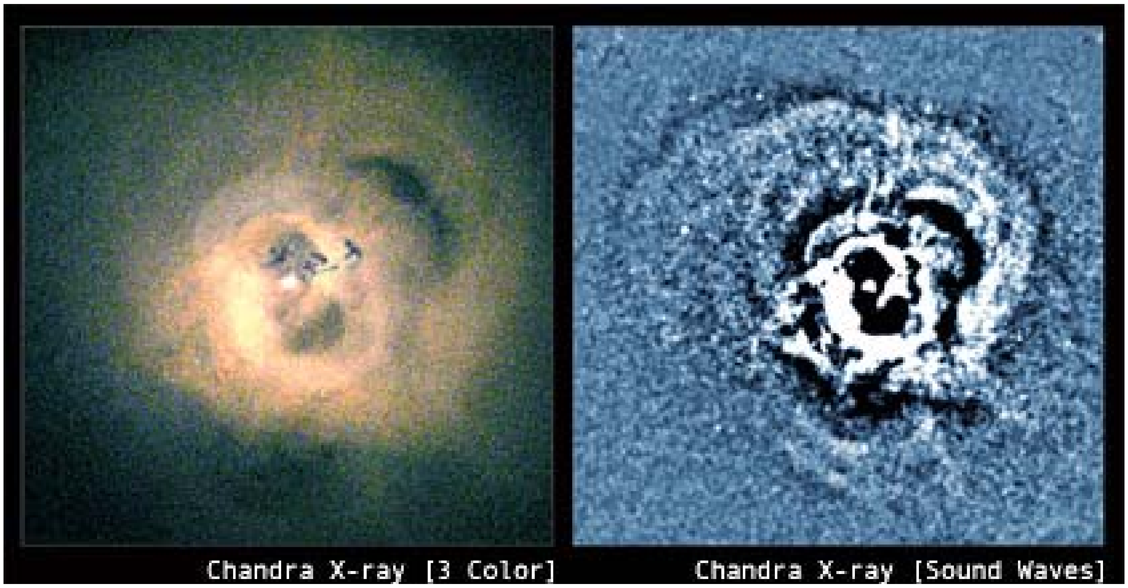
Locations of high density (pressure) are usually shaded darker, or drawn with more lines, that regions of lower density (pressure). The top figure might be used to illustrate pressure waves traveling through solid rock (seismic waves) while the figure to the left might be used to show sound waves created by some object (tuning fork).





Problem 1 - Answer: The top panel shows two pressure waves. The first one at the top has a wavelength of about **39 millimeters** between the dark bands. The bottom illustration is a longer-wavelength sound wave with a wavelength of about **62 millimeters.**

The bottom figure shows concentric rings meant to represent portions of pressure waves created by a tuning fork. The distance between the sections with the densest lines drawn is about **16 millimeters** which represents the wavelength of the pressure wave.



In September 2003, the Chandra Observatory took an x-ray image of a massive black hole in the Perseus Galaxy Cluster located 250 million light years from Earth. Although it could not see the black hole, it did detect the x-ray light from the million-degree gas in the core of the cluster. Instead of a featureless blob, the scientists detected a series of partial concentric rings which they interpreted as sound waves rushing out from the vicinity of the black hole as it swallowed gas in a series of explosions. The image above left shows the x-ray image, and to the right, an enhanced version that reveals the details more clearly.

Problem 1 - The image has a physical width of 350,000 light years. Using a millimeter ruler, what is the scale of the image in light years/millimeter?

Problem 2 - Examine the image on the right very carefully and estimate how far apart the consecutive crests of the sound wave are in millimeters. What is the wave length of the sound wave in light years?

Problem 3 - The wavelength of middle-C on a piano is 1.3 meters. If 1 light year = 9.5×10^{15} meters, and if 1 octave represents a change by a factor of 1/2 change in wavelength, how many octaves below middle-C is the sound wave detected by Chandra?

Problem 1 - The image has a physical width of 350,000 light years. Using a millimeter ruler, what is the scale of the image in light years/millimeter?

Answer: The width is approximately 70 millimeters wide, so the scale is 350,000 light years / 70 millimeters = **5,000 light years/millimeter**.

Problem 2 - Examine the image on the right very carefully and estimate how far apart the consecutive crests of the sound wave are in millimeters. What is the wave length of the sound wave in light years?

Answer: Depending on where the student makes the measurement, such as the set of two bright parallel features on the lower part (7-o'clock position) of the image, the separations will be about 6 millimeters, so the wavelength is 6 mm x (5,000 light years/ 1 mm) = **30,000 light years!**

Problem 3 - The wavelength of middle-C on a piano is 1.3 meters. If 1 light year = 9.5×10^{15} meters, and if 1 octave represents a change by a factor of 1/2 change in wavelength ,how many octaves below middle-C is the sound wave detected by Chandra?

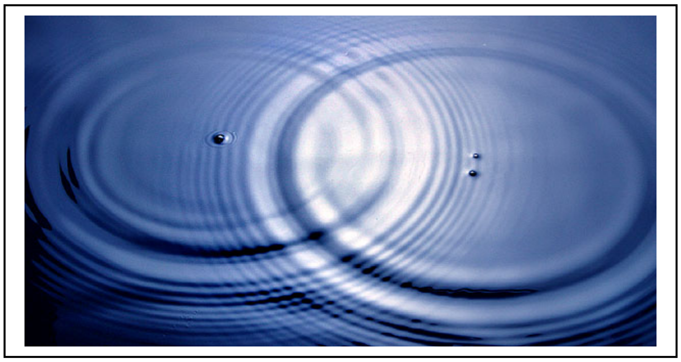
Answer: The sound wave has a wavelength of 30,000 light years x (9.5×10^{15} meters / 1 light year) = 2.9×10^{20} meters. Octaves are determined in terms of powers of two changes in sound waves so that a wavelength change from 32 meters to 16 meters = 1/2, from 32 meters to 8 meters = 1/2 x 1/2 = 1/4 or 2^{-2}, and so on. Middle-C has a wavelength of 1.3 meters, so the Perseus Cluster sound wave differs from middle-C by a factor of 2.9×10^{20} meters /1.3 meters = 2.2×10^{20} times. We have to find N such that $2^N = 2.2 \times 10^{20}$. Using a calculator and repetitive multiplications (or visit the power-of-two table at http://web.njit.edu/~walsh/powers/) we get the table below:

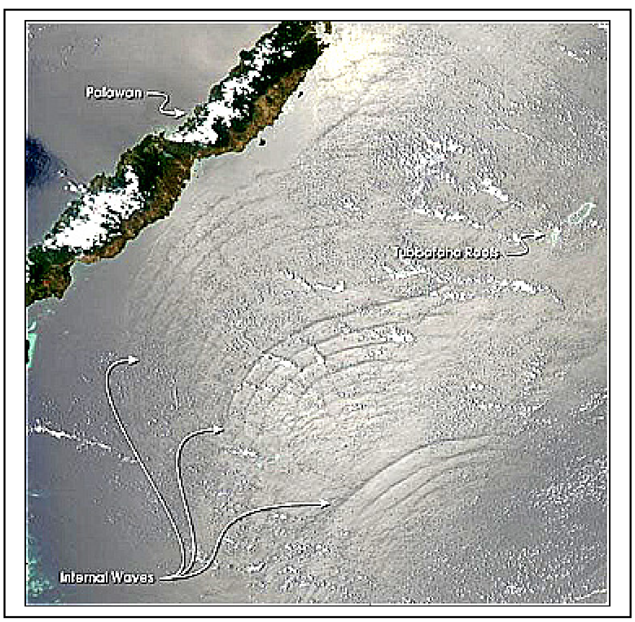
N	factor
10	1,024
30	1.07×10^{9}
40	1.1×10^{12}
57	1.4×10^{17}
68	2.9×10^{20}

So the answer is approximately **68 octaves below middle-C**.

Note to Teacher: A tremendous amount of energy is needed to generate the cavities, as much as the combined energy from 100 million supernovae. For more information, visit the Chandra web page at http://chandra.harvard.edu/press/03_releases/press_090903.html

Exploring Water Waves





Water waves are the most familiar to us because you can actually SEE them, whether they are the beautiful concentric rings produced from a pebble dropped into a pond, or the delightful and sometimes terrifying 'breakers' by the beach.

The wavelength of a water wave is measured as the distance between corresponding points on two consecutive waves such as their peaks or troughs.

Use a millimeter ruler, and clues to the scale of each picture, to determine the wavelengths shown in each of the images.

Top - Pond waves from a pebble. The width of the image is 1 meter.

Middle - Image taken by astronauts onboard the International Space Station of ocean waves in the Gulf of California. The image width is 100 kilometers.

Bottom - This image was taken by MODIS experiment on NASA's Aqua satellite of ocean waves near Palawan Island in the Sulu Sea. The island width is 50 kilometers.

Answer Key

Top - Pond waves from a pebble. The width of the image is 1 meter.

Answer: A ruler placed along the lower edge of the photo measures about 80 millimeters, so the scale is 1 meter/80 mm = 0.0125 meters/mm.
The distance between the crests of the ripples is about 2 millimeters, so this equals a real wavelength of 2 mm x (0.0125 meters/mm) = 0.025 meters or **25 millimeters** on the pond surface.

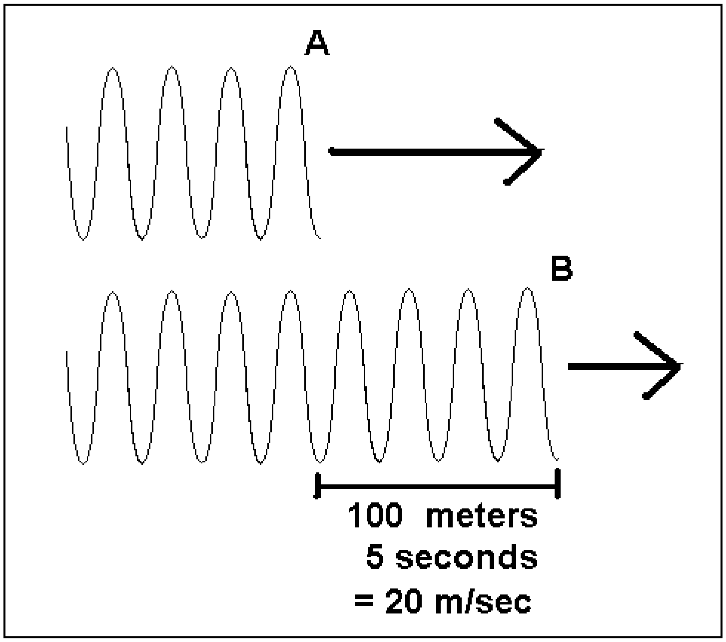
Middle - Image taken by astronauts onboard the International Space Station of ocean waves in the Gulf of California. The image width is 100 kilometers.

Answer: The lower edge measures 78 millimeters so the scale is 100 kilometers/78 millimeters = 1.3 kilometers/mm. The waves have a spacing of about 1 or 2 millimeters, so the wavelengths are between **1.3 and 2.6 kilometers**.

Bottom - This image was taken by MODIS experiment on NASA's Aqua satellite of ocean waves near Palawan Island in the Sulu Sea. The island width is 50 kilometers.

Answer: The lower edge measures 75 millimeters, so the scale is 50 kilometers/75 mm = 0.67 kilometers/mm. The waves are about 2.5 to 3 millimeters apart, so on the ocean this equals 2.5 x 0.67 = **1.7 kilometers** to 3 x 0.67 = **2 kilometers**.

Exploring Wave Speed



If you pick a single wave and follow it as it moves, its speed can depend on many properties of the medium that is 'waving' such as its density and composition. This is most easily seen with sound waves, which are compression waves that travel through a medium such as gases, liquids or even solid matter.

The figure shows how wave speed is measured when a wave travels the distance between two points, A and B, separated by 100 meters. It takes the wave 5 seconds to make the trip, so the wave speed is 20 meters/second.

Problem 1 - A seismic 'P' wave takes 24 minutes to travel a distance 7,300 kilometers through the Earth from the location of the earthquake epicenter in Japan to the seismograph station at U.C. Berkeley. What was the average speed of this seismic wave as it traveled through the Earth?

Problem 2 - A sunbather on Rehoboth Beach is passing her time by counting waves as they break on the beach. She watches as one distant wave takes 10 seconds to travel about 100 meters to the shore. What was the speed of this wave?

Problem 3 - One end of a 20-meter long steal rod is struck by a hammer and sends a pressure wave to the far end. If it takes 1/250 second (i.e. 0.0004 sec) for the pressure pulse to arrive, what was the wave speed of the sound wave through the steal rod?

Problem 4 - A radio wave message is sent from Earth to the Cassini spacecraft orbiting Saturn at a distance of 1.2 billion kilometers. If it took the signal 66.7 minutes to arrive, what was the speed of the light signal in kilometers/sec?

Answer Key

Problem 1 - A seismic 'P' wave takes 24 minutes to travel a distance 7,300 kilometers through the Earth from the location of the earthquake epicenter in Japan to the seismograph station at U.C. Berkeley. What was the average speed of this seismic wave as it traveled through the Earth? Answer: (7,300 km)/(24 minutes) = 292 km/minute. Then 292 km/minute x (1 minute/60 seconds) = **4.8 km/sec**

Problem 2 - A sunbather on Rehoboth Beach is whiling away her time by counting waves as they break on the beach. She watches as one distant wave takes 10 seconds to travel about 100 meters to the shore. What was the speed of this wave? Answer: Speed = 100 meters/10 seconds = **10 meters/sec**.

Problem 3 - One end of a 20-meter long steal rod is struck by a hammer and sends a pressure wave to the far end. If it takes 1/250 second (i.e. 0.0004 sec) for the pressure pulse to arrive, what was the wave speed of the sound wave through the steal rod? Answer = V = 20 meters/0.0004 sec = **5000 meters/sec**.

Problem 4 - A radio wave message is sent from Earth to the Cassini spacecraft orbiting Saturn at a distance of 1.2 billion kilometers. If it took the signal 66.7 minutes to arrive, what was the speed of the light signal in kilometers/sec? Answer:
V = 1.2×10^9 km/66.7 minutes = 18 million km/min .Converting to km/sec V = 18 million km/min x (1 min/60 sec) = **300,000 km/sec**.

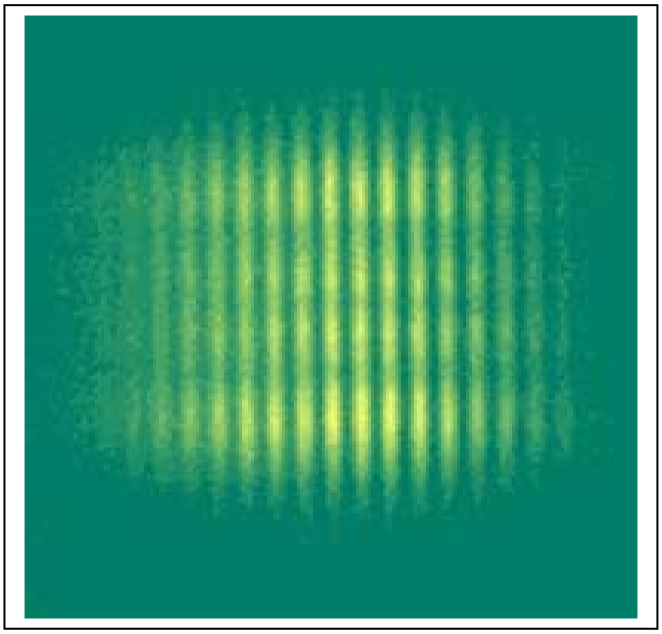
Exploring Waves that Don't Move!

There are many kinds of data that have regularly-spaced features that are described using the 'wavelength' concept, but that are not in motion. Here are a few examples. Can you think of others?

Transverse Aeolian Ridges (TAR) in sand dunes on mars near the Schiaparelli Crater. Image width 800 meters.

Clouds forming on the lee-side of a mountain. Image width 5 kilometers.

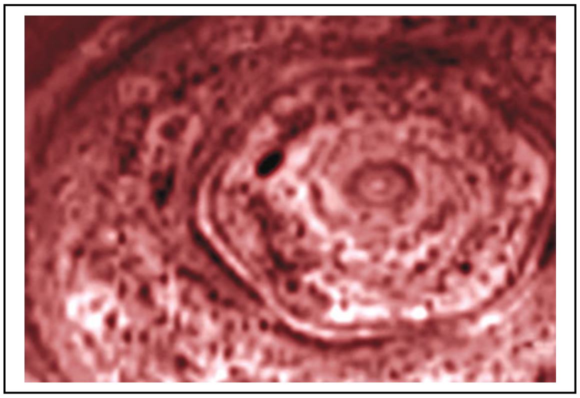
Diffraction pattern of light produced by two interfering laser beams. Image width 1 centimeter.



Hexagonal stationary wave on the North Pole of Saturn. Image width 1,000 kilometers.

Problem 1 - Use a millimeter ruler and the information provided for each image to determine the scale of each image.

Problem 2 - Identify the features that resemble regularly-spaced peaks and valleys and measure the length of the wave.



Problem 1 - Use a millimeter ruler and the information provided for each image to determine the scale of each image.

Answer: Mars TAR. Scale = 800 meters/ 77 millimeters = **10.4 meters/mm**
Cloud Wave Scale = 5000 meters / 78 mm = **64 meters/mm**
Diffraction pattern Scale = 1 centimeter / 77 mm = **0.13 millimeters/mm**
Saturn Storm Scale = 1,000 km / 67 mm = **15 kilometers/mm**

Problem 2 - Identify the features that resemble regularly-spaced peaks and valleys and measure the length of the wave.

Answer:
Mars TAR Wavelength = 4 millimeters ; 4 mm x 10.4 m/mm = **42 meters**
Cloud Wave Wavelength = 5 millimeters ; 5 mm x 64 m/mm = **320 meters.**
Diffraction pattern Wavelength = 4 millimeters; 4 mm x 0.13 mm/mm = **0.5 mm**
Saturn Storm Wavelength = 1 centimeter; 10 mm x 15 km/mm = **150 kilometers**

Note to Teacher:

TAR - The wavelength of these sand dunes is related to the speed of the surface winds on mars, and the sizes (mass) of the grains. Very roughly, for the average prevailing winds and dust grain mass, there is a typical distance that they can be carried aloft. That carrying distance is related to the distance between sand dunes.

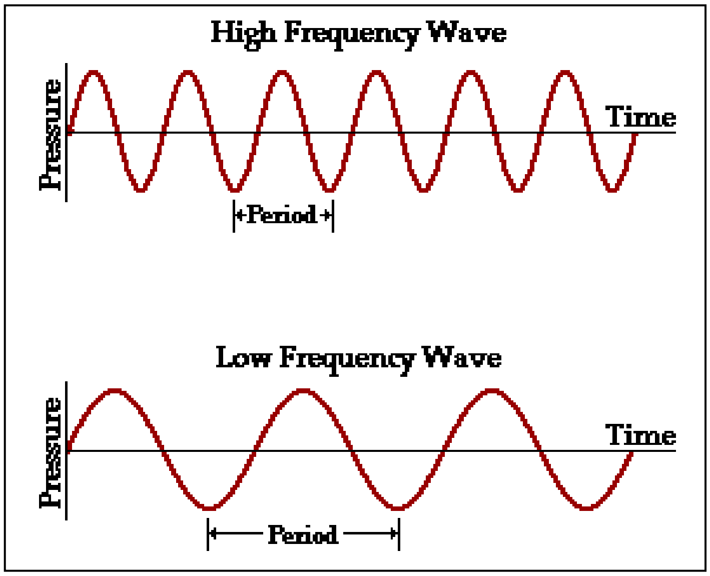
Cloud Wave - This is a standing wave pattern set up on the lee-side of a mountain. As an air mass travels through the wave, it undergoes repeated uplift and descent. If there is enough moisture in the atmosphere, clouds will form at the crests of these waves.

Diffraction Pattern - Light has a wavelike property. When two light waves pass through each other, crests and troughs from one ray can be superimposed on the crests and troughs of the other light ray. Crests add to make the light more intense, but when a crest and trough meet, they cancel leaving a dark, light-free zone.

Saturn Storm - The Earth has a circulating airmass over its South Pole called the Polar Vortex. The gentle rotation of Earth leads to a simple concentric pattern, but for Saturn with its much higher wind speeds, the Polar Vortex has a more complicated pattern as it interacts with airmasses at lower latitudes. The result is a series of concentric, circulating clouds that move in longitude, but are relatively stable in their latitude locations. The differing temperature, turbulence and chemical compositions allow clouds to form in some regions but not in others, leading to the visual appearance.

The 'take away' message from this activity is that periodic phenomena in space can be physically defined by a 'spatial wavelength' but the phenomena are stationary in space and do not move in a direction perpendicular to the 'wave'.

Measuring Wave Frequency



In some situations it is more convenient to discuss a wave in terms of its frequency rather than its wavelength. For example, sound waves created by a musical instrument, or radio waves received by an AM or FM radio are examples where frequency is the favored term.

Frequency refers to the number of cycles of a wave that pass by a specific point in space in a given amount of time. Usually, the unit of 'cycle per second' is used with '1 second' being the time unit.

Problem 1 - A tuning fork is measured to check that it actually vibrates at the correct frequency in order to tune a piano. With some clever equipment, the engineer measures 26400 cycles of the sound wave in one minute in order to get a precise number. What is the frequency of the tuning fork in Hertz?

Problem 2 - The **period** of a wave is the amount of time that has to elapse for the wave to complete one full cycle, from crest-to-crest, or trough-to-trough. What is the period of the sound wave measured by the engineer in Problem 1 in A) seconds? B) milliseconds? C) microseconds?

Problem 3 - A physicist measures the period of a signal to be 100 trillionths of a second. What is the frequency of this phenomenon?

Answer Key

Problem 1 - A tuning fork is measured to check that it actually vibrates at the correct frequency in order to tune a piano. With some clever equipment, the engineer measures 26400 cycles of the sound wave in 1 minute in order to get a precise number. What is the frequency of the tuning fork in Hertz? Answer; F = 26400 cycles / 60 seconds = 440 cycles/sec = **440 Hertz.**

Problem 2 - The **period** of a wave is the amount of time that has to elapse for the wave to complete one full cycle, from crest-to-crest, or trough-to-trough. What is the period of the sound wave measured by the engineer in Problem 1 in A) seconds? B) milliseconds? C) microseconds?

Answer: There were 440 cycles in 1 second, so 1 cycle lasted

A) 1 second/440 Hertz = **0.0023 seconds**.

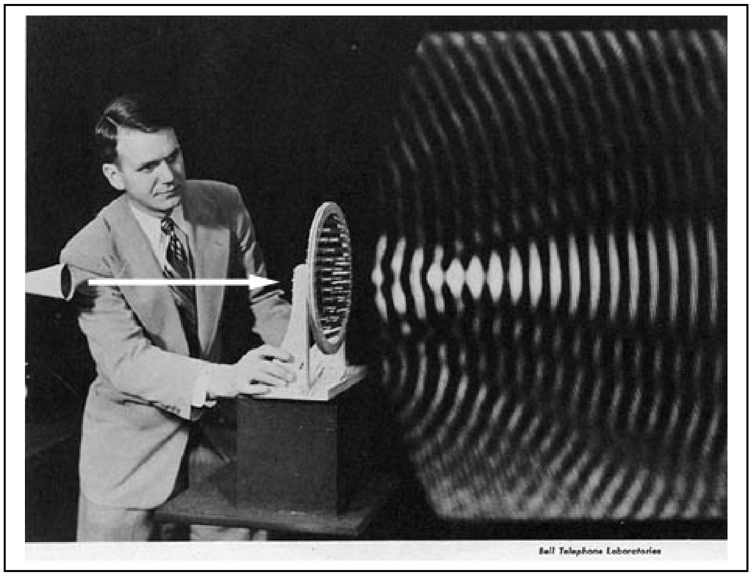
B) 1 millisecond = 0.001 seconds so the period of the sound wave was
0.0023 seconds x (1 millisecond/ 0.001 seconds) = **2.3 milliseconds**.

C) 1 microsecond = 0.000001 seconds, so the period was
0.0023 seconds x (1 microsecond/0.000001 seconds) = **230 microseconds**.

Problem 3 - A physicist measures the period of a signal to be 100 trillionths of a second. What is the frequency of this phenomenon?

Answer. Frequency = 1/Time so for T = 1.0×10^{-10} seconds,
Frequency = $1/1.0 \times 10^{-10}$ seconds
= **10^{10} cycles/second or 10 billion Hertz**.

Sound Wave Frequency



Frequency is measured in 'cycles per second' and indicates how many crests (or troughs) of a wave pass a given point in space each second. Although cycles-per-second or CPS is a very easy unit to remember, it has been replaced by the unit 'hertz' (hz) as the modern measure of frequency.

$$\text{Frequency} = \frac{\text{Number of cycles}}{\text{Time in seconds}}$$

Problem 1 – A pressure gauge measures air pressure on a sensitive diaphragm much like the membrane in your eardrum. In a 2 second time period, it detects 500 pressure changes passing in a series of cycles. What is the frequency of the pressure wave?

Problem 2 – A tuning fork beats at a frequency of 'A above middle-C' at 440 hz. How many seconds elapsed between each pair of pressure peaks in the wave in milliseconds?

Problem 3 – A sound wave was detected at a frequency of 1,000 hz and lasted 0.5 seconds. How many cycles of the wave were detected?

Answer Key

Photo: A visible pattern of sound waves. This technique, developed by Bell Laboratories in ca 1953, demonstrates the focusing effect of an acoustical lens on sound waves issuing from the horn at extreme left. Wave pattern is produced by a scanning technique' From the book The First Book of Sound: A Basic Guide to the Science of Acoustics by David C. Knight, Franklin Watts, Inc. New York (1960). p. 80

Problem 1 – A pressure gauge measures air pressure on a sensitive diaphragm much like the membrane in your eardrum. In a 2 second time period, it detects 500 pressure changes passing in a series of cycles. What is the frequency of the pressure wave?

Answer: 500 cycles/2 seconds = **250 hz**.

Problem 2 – A tuning fork beats at a frequency of 'A above middle-C' at 440 hz. How many seconds elapsed between each pair of pressure peaks in the wave in milliseconds?

Answer: Time = 1 second/440 hz = 0.0023 seconds. 1 millisecond = 0.001 second so the time between peaks is **2.3 milliseconds**.

Problem 3 – A sound wave was detected at a frequency of 1,000 hz and lasted 0.5 seconds. How many cycles of the wave were detected?

Answer: Number of cycles = Frequency x time so Cycles = 1,000 cycles/sec x 0.5 seconds = **500 cycles**.

Water Wave Frequency

Spending time at the beach you might eventually count the time between breakers so that you can surf the 'perfect wave' at the right time.

The time between crests of a water wave is related to the wave frequency by

$$Frequency = \frac{1}{Time}$$

Problem 1 – The time between the breakers at Rehoboth Beach in Virginia was measured to be 10 seconds. What is the period of the water waves breaking on the beach?

Problem 2 - During the course of 15 minutes, 100 waves were counted on the beach. What is the frequency of the water waves?

Answer Key

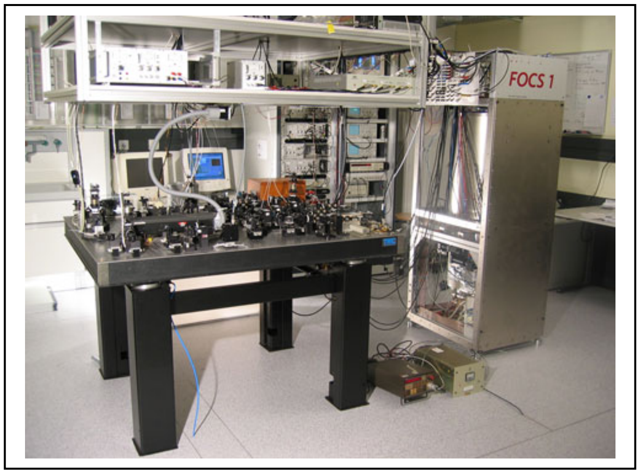
Problem 1 – The time between the breakers at Rehoboth Beach in Virginia was measured to be 10 seconds. What is the period of the water waves breaking on the beach?

Answer: Frequency = 1/10 sec so
Frequency = **10 hz**.

Problem 2 - During the course of 15 minutes, 100 waves were counted on the beach. What is the frequency of the water waves?

Answer: There are 900 seconds in 15 minutes, so

frequency = 100 waves/900 seconds
= 1/9 hz
= **0.11 hz**



Many different systems have a natural vibration frequency, and it is sometimes important to determine the time between vibrations. For instance, the time between crests of a water wave is more important for a surfer to know that its frequency.

Time and frequency are inversely related by

$$Frequency = \frac{1}{Time}$$

The photo above shows the primary frequency standard device, FOCS-1, one of the most accurate devices for measuring time in the world. It stands in a laboratory of the Swiss Federal Office of Metrology METAS in Bern.

Problem 1 – Atomic Time is based on the vibrations of atoms of Cesium-133 in an atomic clock. If the frequency of the vibration is 9,192,631,770 Hertz, about how many seconds elapse between each cycle of the vibration to two significant figures A) in microseconds? B) in nanoseconds?

Problem 2 - During one vibration of the Cesium-133 atom ,how far would a beam of light travel if the speed of light is 3.0×10^{10} centimeters/sec?

Answer Key

Problem 1 – Atomic Time is based on the vibrations of atoms of Cesium-133 in an atomic clock. If the frequency of the vibration is 9,192,631,770 Hertz, about how many seconds elapse between each cycle of the vibration to two significant figures A) in microseconds? B) in nanoseconds?

Answer: Time = 1 / 9,192,631,770
so T = 1.1×10^{-10} seconds

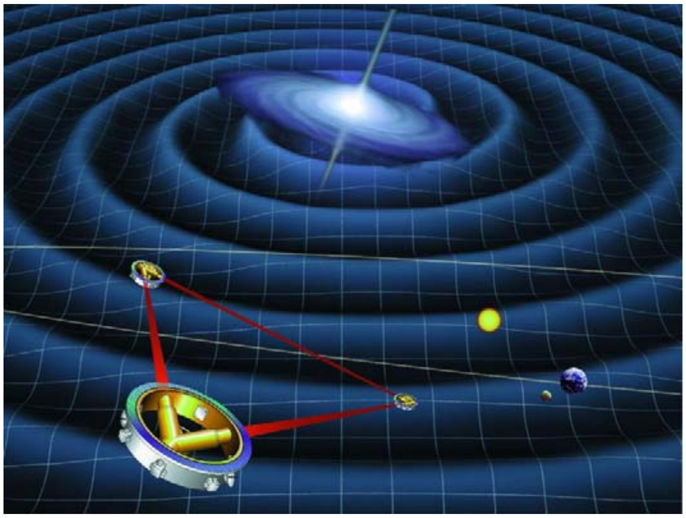
A) 1 microsecond = 10^{-6} seconds so
T = 1.1×10^{-10} seconds x (1 microsecond/10^{-6} seconds)
= **0.00011 microseconds**

B) 1 nanosecond = 10^{-9} seconds so
T = 1.1×10^{-10} seconds x (1 nanosecond/10^{-9} seconds)
= **0.11 nanoseconds**

Problem 2 - During one vibration of the Cesium-133 atom ,how far would a beam of light travel if the speed of light is 3.0×10^{10} centimeters/sec?

Answer: D = 1.1×10^{-10} seconds x 3.0×10^{10} centimeters/sec
so D = **3.3 centimeters**

Frequency, Wavelength and Speed



Wavelength is measured in meters per cycle while frequency is measured in cycles per second, so the product of frequency time wavelength is the speed of the wave in meters per second. A simple formula relates frequency, f, wavelength, w, and speed, s, according to

$$S = f \times w$$

Problem 1 – What are the two related formulas that define frequency and wavelength in terms of the other two wave quantities?

Problem 2 – At sea level, sound waves travel at a speed of 330 meters/sec. If the frequency of middle-C is 440 Hertz, what is the wavelength of the sound wave in centimeters?

Problem 3 – At what sound frequency will the wavelength equal the distance between a pair of human ears (10 cm)? (Note: The ability to locate the direction of a sound wave becomes worse as the wavelength shortens below 10 cm)

Problem 4 – A gravity wave produced by two orbiting neutrons stars travels at the speed of light, c = 300,000 km/sec. If the period of the orbital motion is 10 seconds, what is the distance between crests of the gravity wave?

Answer Key

Problem 1 – What are the two related formulas that define frequency and wavelength in terms of the other two wave quantities?

Answer: $f = S / w$ $w = S / f$

Problem 2 – At sea level, sound waves travel at a speed of 330 meters/sec. If the frequency of 'A above middle-C' is 440 Hertz, what is the wavelength of the sound wave in centimeters?

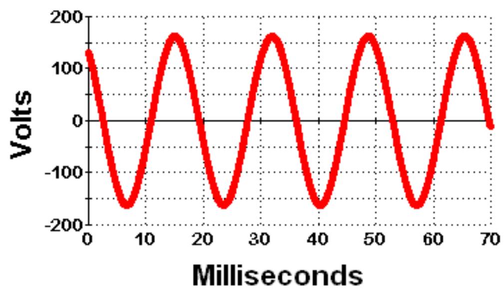
Answer: W = 330/440 = 75 cm.

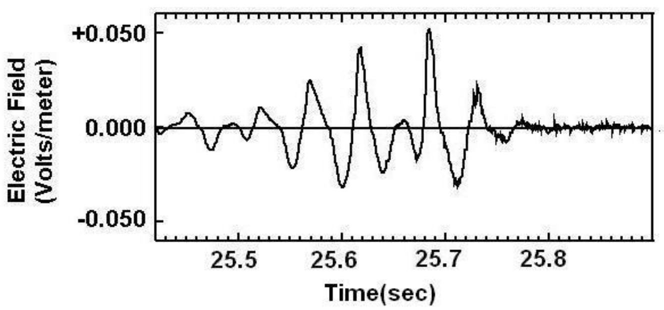
Problem 3 – At what sound frequency will the wavelength equal the distance between a pair of human ears (10 cm)? (Note: The ability to locate the direction of a sound wave becomes worse as the wavelength shortens below 10 cm)

Answer: f = 330 / 0.10 = **3300 hz**.
Note: 3,300 hz is 3 octaves higher than A above middle-C. This is also the frequency of the average human whistle.

Problem 4 – A gravity wave produced by two orbiting neutrons stars travels at the speed of light, c = 300,000 km/sec. If the period of the orbital motion is 10 seconds, what is the distance between crests of the gravity wave?

Answer: The distance is just the wavelength so
d = 300,000 km/sec x 10 sec
= **3 million km**.





Electricity that we use to run our homes and electronic gadgets comes in two flavors: Direct Current (DC) and Alternating Current (AC). In the United States, AC electricity produces an electric voltage that oscillates at 60 cycles-per-second within a range of about +120 Volts to -120 Volts (Top figure). Other physical systems also produce electric waves.

Any physical system in which electric charges are made to move 'in step' produce electric waves. The image to the left shows an electric field measured by NASA's Polar satellite over 35,000 kilometers from Earth. It was produced by a solar storm event on May 5, 1996 between 02:05 and 02:06 UT. The wave traveled at speed of 80 kilometers/sec.

Problem 1 - From start to finish, about how long did this storm event last?

Problem 2 - About how many complete cycles of the electric wave were detected as it passed-by the Polar satellite?

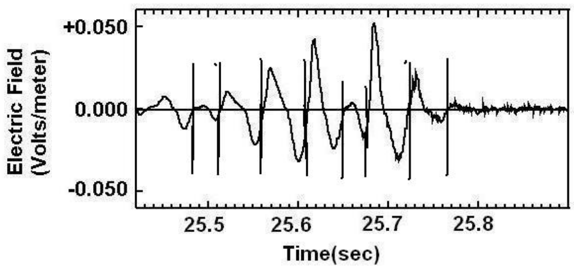
Problem 3 - What was the frequency of this wave in cycles per second?

Problem 4 - What was the wavelength of this electric wave in kilometers, if the wave speed was 80 kilometers/sec?

Answer Key

Problem 1 - From start to finish, about how long did this storm event last? Answer: It began at 25.4 sec and ended around 25.8 sec so it lasted about **0.4 seconds.**

Problem 2 - About how many complete cycles of the electric wave were detected as it passed-by the Polar satellite? Answer: **About 8 cycles**.



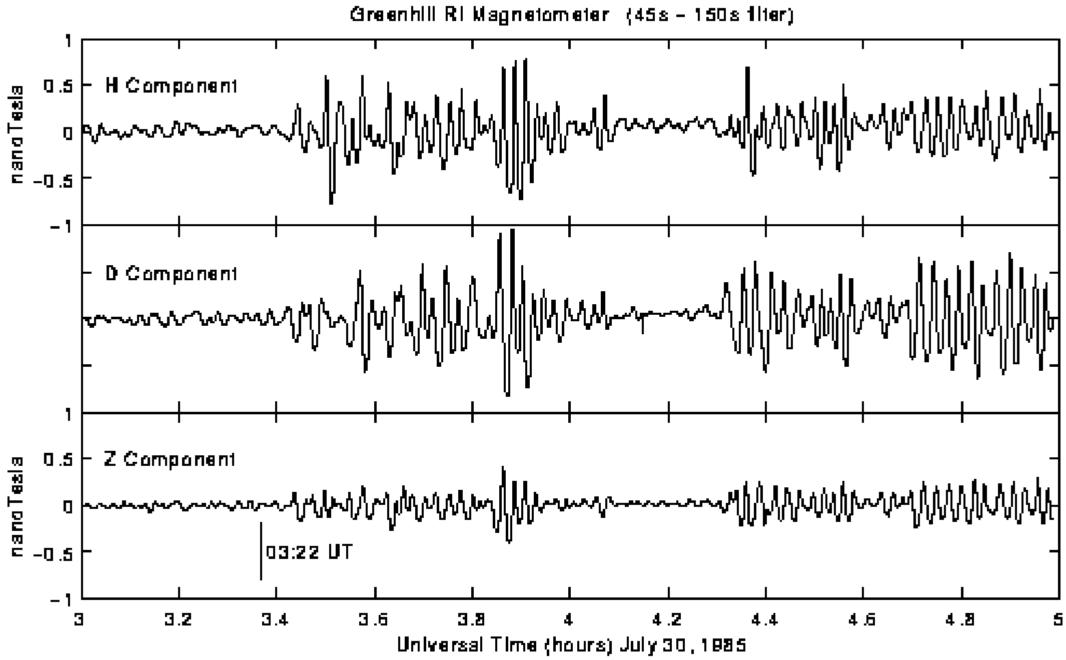
Problem 3 - What was the frequency of this wave in cycles per second? Answer: There were 8 cycles spanning a duration of 0.4 seconds so the frequency was 8 cycles/0.4 seconds = **20 cycles per second (20 Hertz)**

Problem 4 - What was the wavelength of this electric wave in kilometers, if the wave speed was 80 kilometers/sec? Answer: There were 20 cycles in one second ,so each cycle lasted 1sec/20 = 0.05 seconds. At a speed of 80 kilometers/sec, one wavelength would occupy 80 km/sec x 0.05 sec = **4.0 kilometers.**

Electric Field data from: C. Cattell et al, 1997, "Observations of large amplitude parallel electric field wave packets at the plasma sheet boundary" (Journal of Geophysical Research)

"We describe the first observations of large amplitude electric fields parallel to the Earth's magnetic field in this region. This discovery was enabled by the electric field instrument on the Polar spacecraft which is the first to make very high time resolution measurements of the full 3 components of the electric field in this region of the earth's magnetosphere**.** The figure shows a large amplitude (up to 50 mV/m), short duration (~0.2s) wave packet parallel to the magnetic field with a dominant frequency of ~**20 Hz**. The propagation velocity of the parallel wave was determined to be ~80 km/s, resulting in a small net parallel potential. The parallel wavelength is ~**4 km** and the scale size for the packet is ~16 km."

Magnetic Waves



In addition to waves of 'electricity' physicists can measure wave of magnetic energy. These waves, called Alfven waves, are caused by changes in a magnetic field that travel along the lines of magnetic force. These are commonly detected by specially-designed instruments called magnetometers that measure the intensity and direction of Earth's magnetic field in space.

The graph above shows a series of magnetic field measurements made between 03:00 and 05:00 UT on July 30, 1985 at the Haystack Observatory in Massachusetts.

Problem 1 – Select a portion of the magnetic data that shows a clear series of magnetic wave cycles. What is the duration, in minutes, of the wave series that you selected?

Problem 2 – How many cycles of the magnetic wave can you count in the time period you selected?

Problem 3 – What is the frequency of the Alfven wave that you measured?

Problem 4 – If the speed of the Alfven wave is 1,200 km/sec, what is the wavelength of the Alfven wave seen in this data?

Answer Key

Problem 1 – Select a portion of the magnetic data that shows a clear series of magnetic wave cycles. What is the duration, in minutes, of the wave series that you selected?

Answer: The series on the right-hand edge of the graph in the middle panel is the clearest, and lasts from 4.3 to 5.0 UT and has a duration of 0.7 hours or **42 minutes**.

Problem 2 – How many cycles of the magnetic wave can you count in the time period you selected?

Answer: During the 42 minutes, there were **about 26 cycles**.

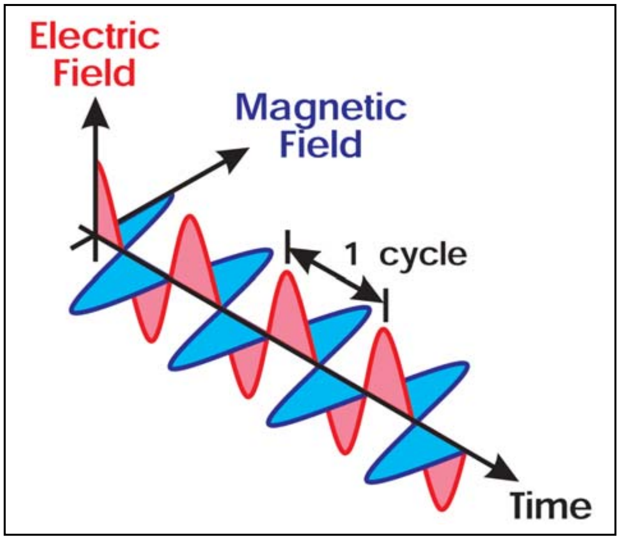
Problem 3 – What is the frequency of the Alfven wave that you measured?

Answer: F = 26 cycles/42 minutes = 0.62 cycles/minute. In terms of hz, 1 minute=60 seconds so f = 0.62 cycles/min x (1 min/60 sec) = **0.01 hz**.

Problem 4 – If the speed of the Alfven wave is 1,200 km/sec, what is the wavelength of the Alfven wave seen in this data?

Answer: d = S / f so d = 1,200 km/sec / (0.01 hz) = **120,000 km**.

Electromagnetic Waves



All other familiar waves that travel through space involve only one quantity (water height, gas pressure, etc) that varies in a cyclic manner. They also travel through a physical medium (water, air, rock, etc).

As the figure to the left shows, electromagnetic waves involve two quantities that vary in step with each other: an electric field (red) and a magnetic field (blue). Electromagnetic waves do not require a medium to travel through because the fields that make them up are their own medium!

As with other wave phenomena, the speed, frequency and wavelength of electromagnetic waves are related by the simple formula c = f x w where c is the speed of light (299,792.4 km/sec).

Problem 1 – The first electromagnetic wave ever created artificially was created in 1887 by the German physicist Heinrich Hertz in his laboratory. The wavelength was 4 meters. At what frequency did his transmitter have to oscillate to create these waves?

Problem 2 – A physicist uses a piece of electrical equipment to generate an electric current that changes at a frequency of 1 trillion hertz. If he measures a wavelength of 0.0003 meters, what was the speed of the 'EM' radiation he was able to experimentally derive in this way?

Problem 1 – The first electromagnetic wave ever created artificially was created in 1887 by the German physicist Heinrich Hertz in his laboratory. The wavelength was 4 meters. At what frequency did his transmitter have to oscillate to create these waves?

Answer: Frequency = c/w so
= 300,000,000 m/s / 4 meters
= 75,000,000 hz or 75 megahertz.

Problem 2 – A physicist uses a piece of electrical equipment to generate an electric current that changes at a frequency of 1 trillion hertz. If he measures a wavelength of 0.0003 meters, what was the speed of the 'EM' radiation he was able to experimentally derive in this way?

Answer: Speed = frequency x wavelength
= $1.0\text{x}10^{12}$ hz x 0.0003 meters
= 3×10^{8} meters/sec

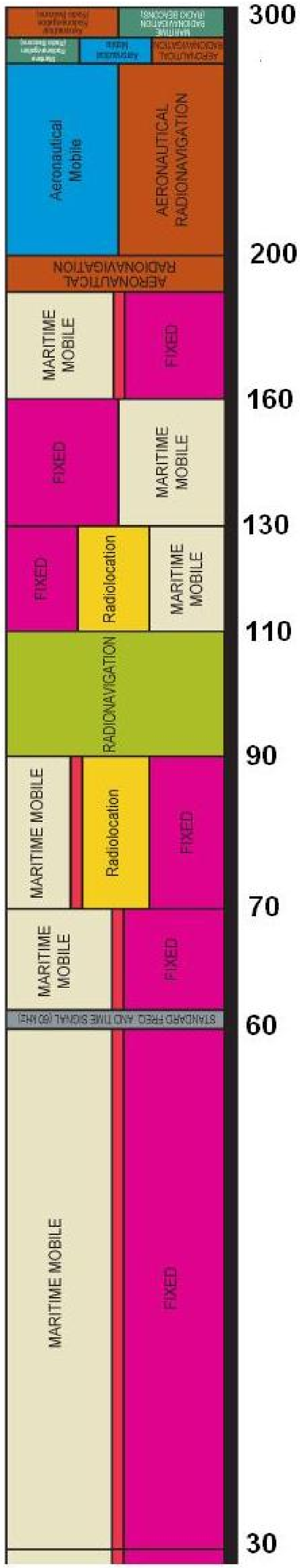
Note to Teacher:

Hertz did not realize the practical importance of his experiments. He stated that,

> "*It's of no use whatsoever*[...] *this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there.*"

Asked about the ramifications of his discoveries, Hertz replied,

> "*Nothing, I guess*."



Below the AM band are several very low frequency bands that span the range from about 1 kHz to 300 kHz. This band is divided into:

Low-Frequency (LF)	30 kHz – 300 kHz
Very-Low-Frequency (VLF),	3 kHz – 30 kHz
Extremely Low Frequency (ELF)	300 Hz – 3 kHz
Ultra-low frequency (ULF)	below 300 Hz.

At these frequencies, radio waves can 'skip' off of Earth's ionosphere and travel long distances around the world. These radio signals can be man-made or can be caused by lightning bursts, the northern lights, and other natural phenomena.

Problem 1 - What is the wavelength of the A) high-frequency edge of the ULF band? B) High frequency edge of the LF band?

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do the radio wave quanta carry on each end of the LF-ULF band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)

Problem 3 – At these low frequencies, especially below 20 kHz it is difficult or even impossible to transmit voice signals, so these bands are used for submarine 'Morse Code' or 'digital code' transmission at slow rates. A binary '1' or '0' requires 1 cycle of electromagnetic energy to transmit, so an 800 bite (100 byte) signal requires a frequency of 800 Hertz to transmit in 1 second. How long would it take you top transmit a 15 megabyte picture from your digital camera using a 15 kiloHertz VLF signal between two submarines at sea?

Answer Key

Problem 1 - Answer A) Wavelength = (300,000,000 meters/sec)/frequency. so at 300 Hz the wavelength is 300,000,000/300 = **1 million meters or 1,000 kilometers.** B) At 300 kHz, the wavelength is 300,000,000/300,000 = **1000 meters or 1 kilometer**.

Problem 2 - Answer: At 300 Hertz the energy of a photon is
$E = 6.63\times10^{-34}$ (300) = **2.0×10^{-31} Joules or 1.3×10^{-12} eV** .

At 300 kiloHertz the energy of a photon is
$E = 6.63\times10^{-34}$ (300,000) = **2.0×10^{-28} Joules or 1.3×10^{-9} eV**.

Problem 3 – At these low frequencies, especially below 20 kHz it is difficult or even impossible to transmit voice signals, so these bands are used for submarine 'Morse Code' or 'digital code' transmission at slow rates. A binary '1' or '0' requires 1 cycle of electromagnetic energy to transmit, so an 800 bite (100 byte) signal requires a frequency of 800 Hertz to transmit in 1 second. How long would it take you top transmit a 15 megabyte picture from your digital camera using a 15 kiloHertz VLF signal between two submarines at sea?

Answer: If it takes 1 second to transmit 800 bits at a frequency of 800 Hertz, then by proportions at 15,000 Hertz it will take 800 Hz/15,000 Hz x 1 second = 0.053 seconds to transmit the same 800 bits at the higher frequency. The 15 megabyte picture contains 15,000,000 bytes x (8 bits/1 byte) = 120,000,000 bits, so by proportions:

$$\frac{T}{0.053 \text{ sec}} = \frac{120{,}000{,}000 \text{ bits}}{800 \text{ bits}}$$

so T = (150,000) (0.053 sec) = **7,950 sec or 2.2 hours**

Frequency (kiloHertz)	Call Sign
970	WAMD
1010	WOLB
1030	WWGB
1050	WZAA
1070	WCRW
1090	WBAL
1160	WMET
1190	WBIS
1230	WRBS
1240	WCEM
1270	WCBC

The 'AM Band' was the first radio wave system developed for public use in the early-20th century.

Like all electromagnetic radiation, AM radio waves travel at the speed of light from transmitter to receiver.

The AM Band is located between frequencies of 530 kiloHertz and 1700 kiloHertz, which is why your radio dial marks can range from '530' to '1700'.

Each station in a given area is licensed to broadcast on a specific ‘center’ frequency, within a band that is about 10 kiloHertz wide, centered on the station's main frequency.

Problem 1 - In the above table showing some of the AM stations you can hear from the Washington, DC area. What frequency range does the station WCRW occupy?

Problem 2 - What is the range of frequencies, in kiloHertz, spanned by the entire AM band?

Problem 3 - What is the wavelength of the A) low-frequency edge of the AM band? B) High frequency edge of the AM band?

Problem 4 - Two radio stations, WAAM in Ann Arbor, Michigan and WFIS in Fountain Inn, South Carolina are both licensed to transmit on the same frequency of 1600 kHz. Explain why this will not be a problem for listeners in each city?

Problem 5 - There are 4761 AM radio stations in the United States. About how many stations are licensed to broadcast in a typical 10 kiloHertz-wide band in the AM spectrum?

Problem 6 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do the AM radio wave quanta carry on each end of the AM band? (Note: The amounts of energy are so small, physicists use the ‘electron-Volt’ (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)

Answer Key

Problem 1 - In the above table, what frequency range does the station WCRW occupy? Answer: Its center frequency is 1070 kiloHertz. It is allocated a slot that is 10 kiloHertz wide, which means 5 kiloHertz to either side of 1070 kHz, so its frequency range is from **1065 to 1075 kiloHertz.**

Problem 2 - What is the range of frequencies, in kilohertz, spanned by the entire AM band? Answer: 1700 - 530 = **1,170 kHz.**

Problem 3 - What is the wavelength of the A) low-frequency edge of the AM band? B) High frequency edge of the AM band? Answer A) Wavelength = 300,000 km/frequency So at 530 kiloHertz the wavelength is 300,000,000/530,000 = **566 meters.** B) At 1,700 kilohertz, the wavelength is 300,000,000/1700000 = **176 meters**.

Problem 4 - Two radio stations, WAAM in Ann Arbor, Michigan and WFIS in Fountain Inn, South Carolina are both licensed to transmit on the same frequency of 1600 kHz. Explain why this will not be a problem for listeners in each city? **Answer: These stations are so far apart that a radio in Ann Arbor would only hear the strong signal from WAAM and not the very weak signal from WFIS…and vice versa.**

Problem 5 - There are 4761 AM radio stations in the United States. About how many stations are licensed to broadcast in a typical 10 kiloHertz-wide band in the AM spectrum? Answer: There are 1,170 khz/10 khz = 117 consecutive slots. To accommodate 4671 stations there would be an average of 4671/117 = **40 stations per band!**

Problem 6 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula $E = 6.6\times10^{-34}$ F where f is the frequency of the EM wave. How much energy, in eV, do the AM radio wave quanta carry on each end of the AM band? (Note: 1 eV = 1.6×10^{-19} Joules)

Answer: At 530 kiloHertz the energy of a photon is
$E = 6.63\times10^{-34}$ (530,000) = **3.5×10^{-28} Joules or 2.2×10^{-9} eV** .

At 1,700 kiloHertz the energy of a photon is
$E = 6.63\times10^{-34}$ (1,700,000) = **1.1×10^{-27} Joules or 7.0×10^{-9} eV**.

Station broadcast times

kHz	Time UT	City
5140	**02:00**	**Havana**
5869	**06:00**	**Stockholm**
5940	**10:00**	**Paris**
5948	**22:00**	**Taiwan**
9653	**01:00**	**Berlin**
8642	**13:00**	**Beijing**
10755	**21:00**	**Saudi Arabia**
12900	**20:00**	**Madrid**

The short-wave (SW) or 'Ham' band is, historically, the band that continues to be favored for long-distance communication, especially between countries around the world. For generations, it was the favored means for obtaining news about distant wars and international news.

The SW Band is located between the AM and FM bands at frequencies of 2.0 MHz and 30 MHz, and you need a special; radio receiver to pick up these stations. (Note 1 MHz = 1 megaHertz = 1 million cycles per second.)

At these frequencies, radio waves can 'skip' off of Earth's ionosphere so that, unlike FM reception, transmitters and receivers can be separated by thousands of kilometers and still 'come in'. Amateur radio operators call this DXing.

Problem 1 - What is the range of frequencies, in MHz, spanned by the entire SW band?

Problem 2 - What is the wavelength of the A) low-frequency edge of the SW band? B) High frequency edge of the SW band?

Problem 3 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do the SW radio wave quanta carry on each end of the SW band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)

Problem 4 – Unlike AM and FM stations that only broadcast to local listeners, SW stations are more expensive to run and so programs change by the hour. The table above shows a typical schedule. The frequencies are customarily given in kiloHertz so '5247' means 5.247 kiloHertz. Times are also given in Universal Time because programs are heard well-beyond a local 'time zone' boundary. Suppose you are listening from your home in New York City and want to create a listening schedule so that you can hear all the stations in the above table. If your time zone is Eastern Daylight Time, which is 4 hours **behind** Universal Time, what will be your local times for tuning into these stations in your schedule?

Answer Key

Problem 1 - Answer: 30 MHz – 2 MHz = **28 MHz**.

Problem 2 - Answer A) Wavelength = (300,000,000 meters/sec)/frequency. so at 2.0 MHz the wavelength is 300,000,000/2000000 = **150 meters.** B) At 30 MHz, the wavelength is 300,000,000/30000000 = **10 meters**.

Problem 3 - Answer: At 2 megaHertz the energy of a photon is
E = 6.63×10^{-34} (2,000,000) = **1.3×10^{-27} Joules or 8.1×10^{-9} eV** .

At 30 megaHertz the energy of a photon is
E = 6.63×10^{-34} (30,000,000) = **2.0×10^{-26} Joules or 1.3×10^{-7} eV**.

Problem 4 Answer: First convert each of the UT times to EDT by **subtracting** 4 hours from each listed time.

	kHz	Time UT	Local Time (EDT)	City
1	**5140**	**02:00**	**10:00 PM**	**Havana**
2	**5869**	**06:00**	**2:00 AM**	**Stockholm**
3	**5940**	**10:00**	**6:00 AM**	**Paris**
4	**5948**	**22:00**	**6:00 PM**	**Tiawan**
5	**9653**	**01:00**	**9:00 PM**	**Berlin**
6	**8642**	**13:00**	**11:00 AM**	**Beiging**
7	**10755**	**21:00**	**7:00 PM**	**Saudi Arabia**
8	**12900**	**20:00**	**4:00 PM**	**Madrid**

Your schedule of listening might look like this:

	AM												PM										
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11
1																						X	
2		X																					
3						X																	
4																		X					
5																					X		
6											X												
7																			X				
8																X							

Radio Waves: Exploring the FM band

FM stations in Washington DC

MHz	Call Sign	City
88.5	WAMU	Washington, DC
90.5	WETA	Arlington, VA
97.9	WIYY	Baltimore, MD
99.9	WFRE	Fredrick, MD
100.5	WZEZ	Richmond, VA
103.1	WRNR	Annapolis, MD
104.9	WAXI	Rockville, MD
106.7	WJFK	Manassas, VA

The 'FM Band' is one of the most popular bands in use by common radio receivers, and has evolved since the 1970s into the preferred 'channel' for popular music and other radio programs. By the 1980's, music stations had abandoned the noisier AM band in favor of the quieter FM band.

The FM Band is located between frequencies of 87.5 MHz and 108 MHz, which is why your radio dial marks, or digital tuner, can range from '87.5' to '108'. (Note 1 MHz = 1 megaHertz = 1 million cycles per second.)

Because of the higher frequencies available for licensing by the FCC to a radio station, much more information can be transmitted, including high-quality stereo programming.

Problem 1 - What is the range of frequencies, in MHz, spanned by the entire FM band?

Problem 2 – If the AM band is 1,200 kiloHertz wide and can accommodate 4,200 stations, how many AM bands, and AM stations, could fit inside one single FM band?

Problem 3 - What is the wavelength of the A) low-frequency edge of the FM band? B) High frequency edge of the FM band?

Problem 4 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do the FM radio wave quanta carry on each end of the FM band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)

Problem 5 – Lower-frequency AM signals can 'skip' off Earth's ionosphere so that AM stations can be heard over long distances. FM signals, however, can rarely be heard more than about 50 kilometers from their origin because at their high frequencies, the ionosphere is nearly transparent and skipping does not occur. In the table above, which FM stations can probably be heard from Washington, DC?

Answer Key

Problem 1 - Answer: 108 MHz – 87.5 MHz = **20.5 MHz**.

Problem 2 –Answer: The FM band is 20.5 megaHertz wide, while the AM band is 1,200 kiloHertz wide. Converting to megahertz, the AM band is 1.2 megaHertz wide, wo the number of AM bands would be 20.5 MHz/1.2 MHz = 1**7 AM bands**. The number of AM stations would be 17 x 4,200 = **71,400 stations!**

Problem 3 - Answer A) Wavelength = (300,000,000 meters/sec)/frequency. so at 97.5 MHz the wavelength is 300,000,000/97500000 = **3.0 meters or 30 cm.** B) At 108 MHz, the wavelength is 300,000,000/108000000 = **2.7 meters or 27 cm**.

Problem 4 - Answer: At 87.5 megaHertz the energy of a photon is
E = 6.63×10^{-34} (87,500,000) = **5.8×10^{-26} Joules or 3.6×10^{-7} eV** .

At 108 megaHertz the energy of a photon is
E = 6.63×10^{-34} (108,000,000) = **7.1×10^{-26} Joules or 4.4×10^{-7} eV**.

Problem 5 –In the table above, which FM stations can probably be heard from Washington, DC? Answer: You need to use geography to determine the distances from Washington Dc to each of the cities in the table. Online resources such as **http://www.geobytes.com/citydistancetool.htm** are very helpful. Here is the table with the estimated distances:

MHz	Call Sign	City	Distance (km)
88.5	WAMU	Washington, DC	**0.0**
90.5	WETA	Arlington, VA	**6.0**
97.9	WIYY	Baltimore, MD	**54.0**
99.9	WFRE	Fredrick, MD	**64.0**
100.5	WZEZ	Richmond, VA	**158.0**
103.1	WRNR	Annapolis, MD	**46.0**
104.9	WAXI	Rockville, MD	**21.0**
106.7	WJFK	Manassas, VA	**41.0**

The only stations that are within about 50 kilometers of Washington DC are WAMU, WETA, WRNR, WAXI and WJFK. Note: This ’50 Kilometer Rule’ is not mathematically exact, and only approximate. Under some conditions of local ‘interference with buildings or trees, even stations 10 km away may have poor reception.

Some common TV channels

Channels	Frequency (MHz)
2 to 4	54-72
5 and 6	76-88
7 to 13	174-216
14 to 22	121-174
23 to 36	216 to 300
37 to 62	300 to 456
63 to 158	457 to 1002

Over-the-Air TV programs are carried in select portions of the radio band between 54 and 696 MHz. A typical TV broadcast band is 6 MHz wide to handle the analog information needed to create the picture and the sound portion of a program.

Because radio waves at these frequencies pass through the ionosphere without 'skipping', TV stations cannot be more than about 50 km from the receiving TV set. National stations (CBS, NBC, ABC) have to be 'affiliated' with local stations in order for their content to appear at your home.

Problem 1 - What is the wavelength, in centimeters, of A) Channel 2? B) UHF Channel 158?

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do Channel 2 and Channel 158 radio wave photons carry at the ends of the TV band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)

Problem 3 – The transparency of the ionosphere is determined by the density of the electrons present in the layer. The critical frequency for reflection is given by the simple formula $fc(kHz) = 9\sqrt{N}$ where N is the density of the electrons in electrons/cm^3. For frequencies below fc, the ionosphere will be either translucent or opaque, and signals will skip long distances. For frequencies higher than fc, the ionosphere will become mostly transparent and the signal will continue into space. At the lower level (90 km) called the E-region, N= 30,000 electrons/cm^3 and in the upper F-region (120 km) N = 2,000,000 electrons/cm^3. A) What is fc in the E and F Layers? B) Explain what will happen to a radio wave with a frequency of 9 MHz

Answer Key

Problem 1 - What is the wavelength, in centimeters, of A) Channel 2? B) UHF Channel 158?
Answer: Channel 2 is at 54 MHz and Channel 158 is at 1002 MHz.
A) Wavelength = (300,000,000 meters/sec)/frequency. so at 54 MHz the wavelength is 300,000,000/54,000,000 = **5.5 meters or 550 cm.** B) At 1002 MHz, the wavelength is 300,000,000/1,002,000,000 = **0.3 meters or 30 cm**.

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do Channel 2 and Channel 158 radio wave photons carry at the ends of the TV band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)
Answer: Channel 2: At 54 megaHertz the energy of a photon is
E = 6.63×10^{-34} (54,000,000) = **3.6×10^{-26} Joules or 2.2×10^{-7} eV** .

For Channel 158 at 1002 megaHertz the energy of a photon is
E = 6.63×10^{-34} (1,002,000,000) = **6.6×10^{-25} Joules or 4.1×10^{-6} eV**.

Problem 3 – The transparency of the ionosphere is determined by the density of the electrons present in the layer. The critical frequency for reflection is given by the simple formula $fc(kHz) = 9\sqrt{N}$ where N is the density of the electrons in electrons/cm3. For frequencies below fc, the ionosphere will be either translucent or opaque, and signals will skip long distances. For frequencies higher than fc, the ionosphere will become mostly transparent and the signal will continue into space. At the lower level (90 km) called the E-region, N= 30,000 electrons/cm^3 and in the upper F-region (120 km) N = 2,000,000 electrons/cm^3. A) What is fc in the E and F Layers? B) Explain what will happen to a radio wave with a frequency of 9 MHz

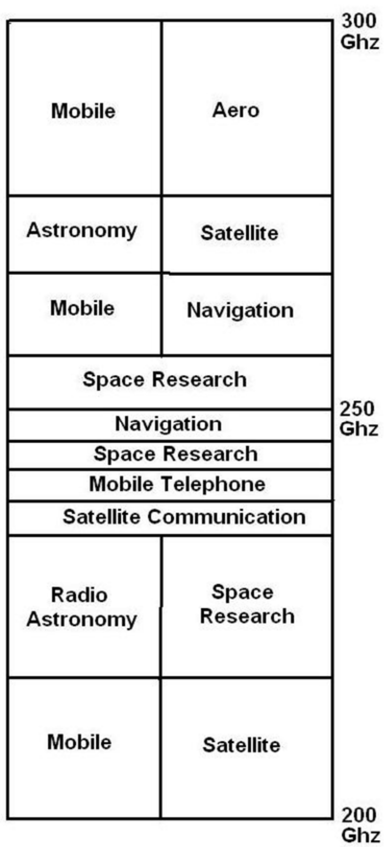
Answer: A) For the E-region, fc = 9 $(30000)^{1/2}$ = **1600 kHz which equals 1.6 MHz**. For the F-region, fc=9 $(2,000,000)^{1/2}$ = **13,000 kHz which equals 13 MHz**.

B) Because 9 MHz is more than fc for the E-layer it will **Not** be reflected by the E-layer. Because 9 MHz is less than fc for the F-Layer, it **will be reflected** by the F-layer and return to Earth.

Microwave band designations

Band Name	GHz
P-Band	0.3 to 1
L-Band	1 to 2
S-band	2 to 4
C-Band	4 to 8
X-Band	8 to 12.5
Ku-band	12.5 to 18
K-Band	18 to 26
Ka-Band	26 to 40
U-Band	40 to 60

Frequency allocation



This radio band extends from about 1,000 MHz to 300,000 MHz. We now use the new frequency prefix 'giga' and use 'GHz' to indicate the units, which now become 1 GHz and 300 GHz. The lower figure shows a portion of the FCC allocations between 150 and 300 GHz.

Microwaves carry so much energy per photon that they can be used, when concentrated, to heat food. Their very high frequencies allow them to carry enormous amounts of digital information very quickly. It is also the main band for RADAR operations.

Problem 1 - What is the wavelength range, in centimeters, that is spanned by the microwave band from 1 to 300 GHz?

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do the photons carry at the ends of the microwave band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)

Problem 3 - If one data bit of information (a '1' or '0') can be carried for each cycle of the radio wave, A) How many bytes/sec of information can be carried by a microwave with a frequency of 300 GHz? B) How long would it take to download the contents of the Library of Congress (10 terabytes) on a microwave link between two computers at this frequency? (1 terabyte = 1,000 gigabytes)

Answer Key

Problem 1 - Answer: 1 GHz.
A) Wavelength = (300,000,000 meters/sec)/frequency. so at 1 GHz the wavelength is 300,000,000/1,000,000,000 = **0.3 meters or 30 cm.** B) At 300 GHz, the wavelength is 300,000,000/300,000,000,000 = **0.001 meters or 0.1 centimeters (1 millimeter)**.

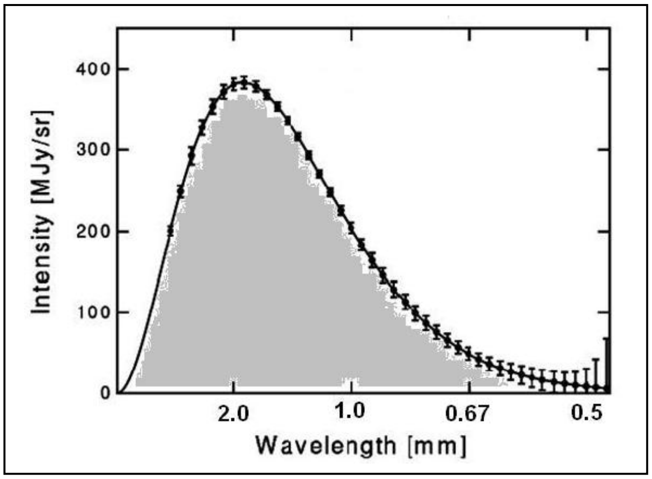
Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do radio wave photons carry at the ends of the microwave band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)
Answer: At 1 gigaHertz the energy of a photon is
E = 6.63×10^{-34} (1,000,000,000) = **6.6×10^{-25} Joules or 4.1×10^{-6} eV** .

At 300 gigaHertz the energy of a photon is
E = 6.63×10^{-34} (300,000,000,000) = **2.0×10^{-22} Joules or 0.001 eV**.

Problem 3 - If one data bit of information (a '1' or '0') can be carried for each cycle of the radio wave, A) How many bytes/sec of information can be carried by a microwave with a frequency of 300 GHz? B) How long would it take to download the contents of the Library of Congress (10 terabytes) on a microwave link between two computers at this frequency?

Answer: A) The microwave can carry 300 gigabits of information, and since there are 8 bits to one byte, this equals 300/8 = **37.5 gigabytes of data every second**.

B) The Library of Congress contains 10 terabytes of data. One terabyte equals 1,000 gigabytes, so it would take 10,000 gigabytes / (37.5 gigabytes/sec) = **267 seconds**.



The sub-millimeter radio band represents the change-over in how we conveniently describe the kind of radiation in question. Although 'gigaHertz' is reasonably familiar ,the next unit up, the 'teraHertz', is not, so it becomes simpler to speak in terms of wavelength units.

Currently, there are no technology applications for radiation in this band, which starts at 300 GHz and run up to 3,000 GHz. Receivers in use for this radiation are almost exclusively for astronomical research since many astronomical systems emit radiation at these wavelengths.

Problem 1 - What is the wavelength range, in millimeters, that is spanned by the sub-millimeter band from 300 to 3000 GHz?

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do the photons carry at the ends of the sub-millimeter band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule. 1 eV = 1.6×10^{-19} Joules)

Problem 3 - In 1989, NASA's Cosmic Background Explorer (COBE) satellite was launched to measure the brightness of the Cosmic Background Radiation (CMB); the fireball light left over from the Big Bang over 13 billion years ago. Within a year of observation, astronomers were able to measure its brightness across the millimeter and sub-millimeter bands with incredible accuracy. The graph above shows this brightness curve. The horizontal axis is the wavelength in millimeters. The vertical axis is the brightness across the sky in units of megaJanskys per steradian.

A) At what wavelength and frequency does the CMB radiation reach its peak?

B) Over what wavelength range does the CMB have a brightness of more than 1/2 its peak value?

Answer Key

Problem 1 - Answer:
A) Wavelength = (300,000,000 meters/sec)/frequency. so at 300 GHz the wavelength is 300,000,000/300,000,000,000 = **0.001 meters or 1 millimeter.**

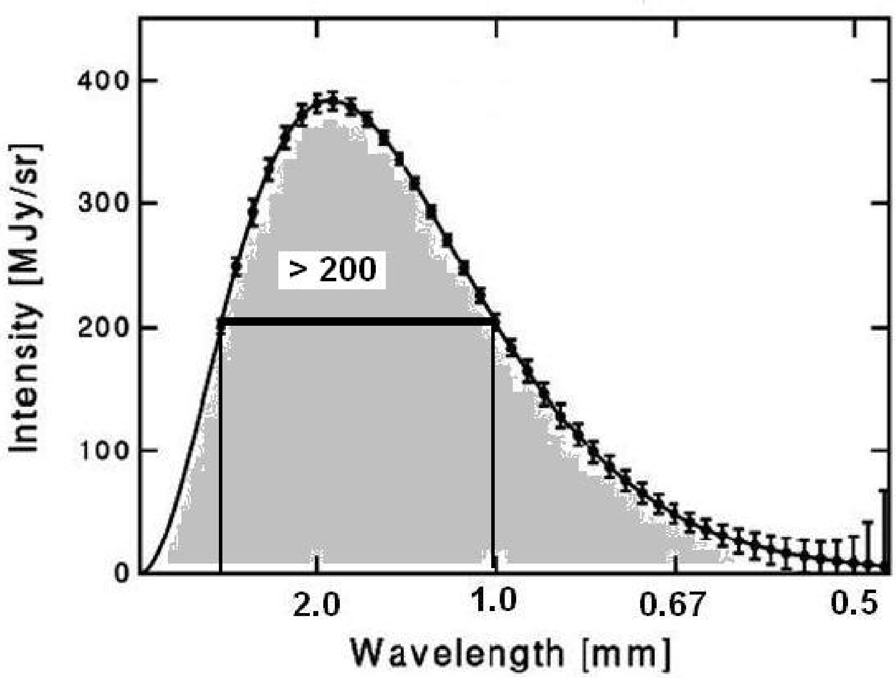
B) At 3000 GHz, the wavelength is 300,000,000/3,000,000,000,000 = **0.1 millimeters (100 microns)**.

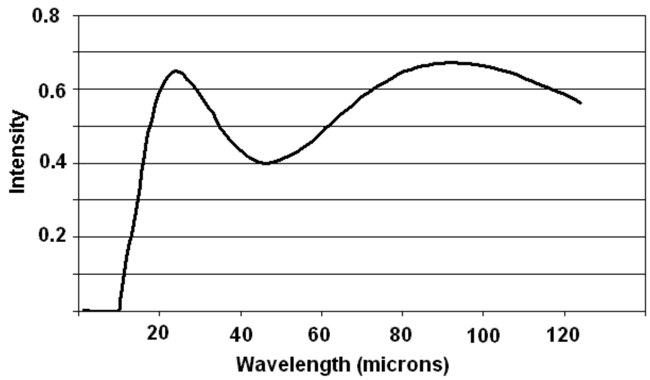
Problem 2 - Answer: At 300 gigaHertz the energy of a photon is
E = 6.63×10^{-34} (300,000,000,000) = **2.0×10^{-22} Joules or 0.001 eV** .

At 3000 gigaHertz the energy of a photon is
E = 6.63×10^{-34} (3,000,000,000,000) = **2.0×10^{-21} Joules or 0.01 eV**.

Problem 3 - A) At what wavelength and frequency does the CMB radiation reach its peak? Answer: The peak of the curve occurs near a wavelength of **2.0 millimeters**. The exact location is at 1.9 millimeters.

B) Over what wavelength range does the CMB have a brightness of more than 1/2 its peak value? Answer: The peak of the curve is about 400 megaJanskys per steradian, so the 1/2 peak value is 200 MJy per steradian. This brightness level occurs between **1.0 and 2.5 millimeters.**





Beyond the sub-millimeter Band at wavelengths between 0.1 millimeter (100 microns) and 0.001 millimeter (1 micron) we are in a complicated electromagnetic band in which warm bodies emit most of their heat energy. The 'IR' band is also the band in which many common molecules emit specific frequencies of light as their constituent atoms rotate and vibrate.

Problem 1 - What is the frequency range, in teraHertz, that is spanned by the IR band from 0.1 millimeters to 0.001 millimeters?

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do these infrared photons carry at the ends of the infrared band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule.
1 eV = 1.6×10^{-19} Joules)

Problem 3 - At these wavelengths, warm bodies produce infrared light across the IR band. The temperature of a body can be determined by the wavelength at which the maximum brightness occurs according to the formula T = 2897/L where L is the wavelength in microns, and T is the temperature in Kelvin degrees. For example, a human with a temperature of 98 F (309 K) will have a 'black body' curve that peaks at a wavelength of 9.4 microns.

A) The above graph shows the brightness of the infrared light from two bodies at two different temperatures. What are the two wavelengths for which the combined spectrum has its two maximum intensities?

B) From the formula, what are the temperatures for each of the two bodies?

Answer Key

Problem 1 - Answer:
A) Frequency = (300,000,000 meters/sec)/wavelength. so at 0.1 millimeters (0.0001 meters) the frequency is 300,000,000/0.0001 = **3×10^{12} Hertz or 3 teraHertz.**

B) Frequency = (300,000,000 meters/sec)/wavelength. so at 0.001 millimeters (0.000001 meters) the frequency is 300,000,000/0.000001 = **300×10^{12} Hertz or 300 teraHertz.**

Problem 2 - Answer: At 3 teraHertz the energy of a photon is
$E = 6.63\times10^{-34} (3\times10^{12})$ = **2.0×10^{-21} Joules or 0.01 eV** .

At 300 teraHertz the energy of a photon is
$E = 6.63\times10^{-34} (3\times10^{14})$ = **2.0×10^{-19} Joules or 1.0 eV**.

Problem 3 - At these wavelengths, warm bodies produce infrared light across the IR band. The temperature of a body can be determined by the wavelength at which the maximum brightness occurs according to the formula T = 2897/L where L is the wavelength in microns, and T is the temperature in Kelvin degrees. For example, a human with a temperature of 98 F (309 K) will have a 'black body' curve that peaks at a wavelength of 9.4 microns.

A) The above graph shows the brightness of the infrared light from two bodies at two different temperatures. What are the two wavelengths for which the combined spectrum has its two maximum intensities? Answer: **The peaks are at 20 microns and 80 microns.** Students may use estimates for the peak locations between 20-25 K and 80-100K.

B) From the formula, what are the temperatures for each of the two bodies?
Answer; For the 20-micron peak, and peak wavelength of 20 microns, the temperature is T = 2897/20 = **144 K**. For the 80 micron peak, the temperature is T = 2897/80 = **36 K**.

Visible band frequencies and wavelengths

Color	Wavelength (nm)	Frequency (THz)
Violet	380-450	668-789
Blue	450-495	606-668
Green	495-570	526-606
Yellow	570-590	508-526
Orange	590-620	484-508
Red	620-750	400-484

At wavelengths between 380 to 750 nanometers (nm) we have the Visual Band, or spectrum, to which our eyes have been tuned to work optimally.

Objects heated to temperature of 1,000 K or more shine brightly in this band. Atoms emit an enormous number of 'atomic spectral lines' in this frequency range.

Problem 1 - What is the frequency range that is spanned by the Visable Spectrum from 380 nm to 750 nm in A) teraHertz? B) petaHertz?

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-34} f where f is the frequency of the EM wave in Hertz. How much energy, in eV, do these visible light photons carry at the ends of the visible band between 380 and 750 nm? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule.
1 eV = 1.6×10^{-19} Joules)

Problem 3 - A simple linear relationship exists between the temperature of a body, and the dominant wavelength of the light that it produces given by the formula T = 2897000/L where T is the temperature of the body in Kelvin degrees, and L is the wavelength where most of the light is emitted in nanometers. Using the table above, explain what you will see as a body is heated from a temperature of 4,000 K to 7,000 K?

Answer Key

Problem 1 - Answer:
A) Frequency = (300,000,000 meters/sec)/wavelength. so at 380 nanometers ($3.8x10^{-7}$ meters) the frequency is $300,000,000/3.8x10^{-7}$ = **$789x10^{12}$ Hertz or 789 teraHertz.** At 750 nanometers ($7.5x10^{-7}$ meters) the frequency is $300,000,000/7.5x10^{-7}$ = **$400x10^{12}$ Hertz or 400 teraHertz.**

B) One petaHertz (PHz) = 1,000 teraHertz so the frequency range in PHz is from **0.4 to 0.789 PHz**.

Problem 2 - Answer: At 789 teraHertz the energy of a photon is
$E = 6.63x10^{-34} (7.89x10^{14})$ = **5.2×10^{-19} Joules or 3.3 eV** .

At 400 teraHertz the energy of a photon is
$E = 6.63x10^{-34} (4.00x10^{14})$ = **2.7×10^{-19} Joules or 1.7 eV**.

Problem 3 - A simple linear relationship exists between the temperature of a body, and the dominant wavelength of the light that it produces given by the formula T = 2897000/L where T is the temperature of the body in Kelvin degrees, and L is the wavelength where most of the light is emitted in nanometers. Using the table above, explain what you will see as a body is heated from a temperature of 4,000 K to 7,000 K?

Answer: At T=4,000 K the peak wavelength is 2897000/4000 = 724 nm, so the peak emission occurs in the 'red' band and the object will appear to be red-hot. At T=7,000 K, the peak emission occurs at a wavelength of 2897000/7000 = 413 nm, so the body will appear 'violet-hot'. (Note: for objects hotter than the sun, T=5770 K, the actual color will be blue-white or 'white-hot') **So, as the body is being heated, its color will appear red and then gradually increase to orange, yellow and then violet 'or white hot'.**

Ultraviolet band designations

Name	Wavelength (nm)	Energy (eV)
UV-A	400-315	3.10 - 3.94
Near	400-300	3.10 - 4.13
UV-B	315-280	3.94 - 4.43
Middle	300-280	4.13 - 6.20
UV-C	280-100	4.43 - 12.4
Far	200-122	6.20 - 10.2
Vacuum	200-10	6.20 - 124
Low	100-88	12.4 - 14.1
Super	150-10	8.28 - 124
Extreme	121-10	10.2 - 124

The 'UV' band occupies wavelengths from 10 nm to 400 nm. Earth's atmosphere absorbs more than 98% of the UV-A to UV-C radiation from the sun, and all of the radiation with wavelengths shorter than 200 nm. Only observations in the 'vacuum' of space can detect the UV radiation beyond this limit, hence the name: Vacuum Ultraviolet region.

Problem 1 - What is the frequency range that is spanned by the ultraviolet band from 10 nm to 400 nm in A) petaHertz? B) exaHertz? (note 1000 petaHertz = 1 exaHertz)

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(Joules) = 6.63×10^{-22} f where f is the frequency of the EM wave in teraHertz. How much energy, in eV, do these UV photons carry at the ends of the UV band between 10 and 400 nm? (Note: The amounts of energy are so small, physicists use the ‘electron-Volt’ (eV) instead of the Joule.
1 eV = 1.6×10^{-19} Joules)

Problem 3 - A simple inverse relationship exists between the temperature of a body, and the dominant wavelength of the light that it produces given by the formula T = 2897000/L where T is the temperature of the body in Kelvin degrees, and L is the wavelength where most of the light is emitted in nanometer. In what UV bands would you expect to find the peak emission for:

A) The bright star Rigel which has a surface temperature of 11,000 K;

B) The young white dwarf star KPD0005+5106, which has an estimated temperature of 200,000 K?

Answer Key

Problem 1 - Answer: The wavelength range is from 10 to 400 nm.

A) Frequency = (300,000,000 meters/sec)/wavelength. so at 400 nanometers (4.0×10^{-7} meters) the frequency is 300,000,000/4.0×10^{-7} = **750×10^{12} Hertz or 750 teraHertz.** At 10 nanometers (1.0×10^{-8} meters) the frequency is 300,000,000/1.0×10^{-8} = **3.0×10^{16} Hertz or 30000 teraHertz.**

B) One petaHertz (PHz) = 1,000 teraHertz so the frequency range in PHz is from **0.75 to 30 PHz**.

Problem 2 - Answer: From the previous answer, the frequency range is 750 to 30,000 teraHertz. At 750 teraHertz the energy of a photon is
E = 6.63×10^{-22} (750) = **5.0×10^{-19} Joules or 3.1 eV** .

At 30,000 teraHertz the energy of a photon is
E = 6.63×10^{-22} (30,000) = **2.0×10^{-17} Joules or 124 eV**.

Problem 3 - A simple inverse relationship exists between the temperature of a body, and the dominant wavelength of the light that it produces given by the formula T = 2897000/L where T is the temperature of the body in Kelvin degrees, and L is the wavelength where most of the light is emitted in nanometer. In what UV bands would you expect to find the peak emission for:

A) The bright star Rigel which has a surface temperature of 11,000 K;
Answer: L = 2897000/11,000 = **263 nm, which is in the UV-C band**

B) The young white dwarf star KPD0005+5106, which has an estimated temperature of 200,000 K?
Answer: L = 2897000/200,000 = **15 nm, which is in the Vacuum UV band (alternately the Super or Extreme UV bands)**

X-ray band designations

Name	Frequency (petaHertz)
Soft X-rays	30 to 30,000
Hard X-rays	30,000 to 300,000

We mostly know about X-rays from visits to the dentist or hospital. Since the discovery of 'Roentgen Rays' back in the late-1800's many uses for this penetrating radiation have been invented.

Very hot gas (called a plasma) at temperatures of 100,000 K or higher are intense sources of X-ray light in the universe.

Problem 1 - What is the wavelength range, in picometers, that is spanned by the X-ray band from 30 petaHertz to 300,000 petaHertz? (1 picometer = 1.0×10^{-12} meters)

Problem 2 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(eV) = 4.56 f where f is the frequency of the EM wave in petaHertz. How much energy, in keV, do these x-ray photons carry at the ends of the x-ray band? (Note: The amounts of energy are so small, physicists use the 'electron-Volt' (eV) instead of the Joule.
1 eV = 1.6×10^{-19} Joules; and 1 keV = 1,000 eV)

Problem 3 - A simple linear relationship exists between the temperature of a body and the dominant frequency of the light that it produces given by the formula $f = 4.2 \times 10^{8} / T$ where T is the temperature of the body in Kelvin degrees, and f is the frequency, in petaHertz, where most of the light is emitted. In what X-ray bands would you expect to find the peak emission for the following situations:

A) The Hinode satellite observes the corona of the sun. The corona of the sun at a temperature of 2 million K.

B) In 2001, the Chandra Observatory determined that the cosmic x-ray background is produced mainly by distant galaxies in which massive black holes are consuming matter and producing hot gases. What is the temperature of the plasma that produced the x-ray light seen at an energy of 10 keV?

Problem 1 - What is the wavelength range, in picometers, that is spanned by the X-ray band from 30 petaHertz to 300,000 petaHertz?

Answer: The frequency range is from 30 petaHertz to 300,000 petaHertz.

Wavelength = (300,000,000 meters/sec)/frequency. so at 30 petaHertz ($3.0 \times 10^{+16}$ Hertz) the wavelength is $300,000,000/3.0 \times 10^{+16} = 1.0 \times 10^{-8}$ meters or **10,000 picometers**. At 300,000 petaHertz (3.0×10^{20} Hertz) the wavelength is $300,000,000/3.0 \times 10^{20} = 1.0 \times 10^{-12}$ meters or **1.0 picometers.**

Problem 2 - Answer: From the previous answer, the frequency range is 30 to 300,000 petaHertz. At 30 petaHertz the energy of a photon is
E = 4.56 (30) = 137 eV or **0.137 keV**.

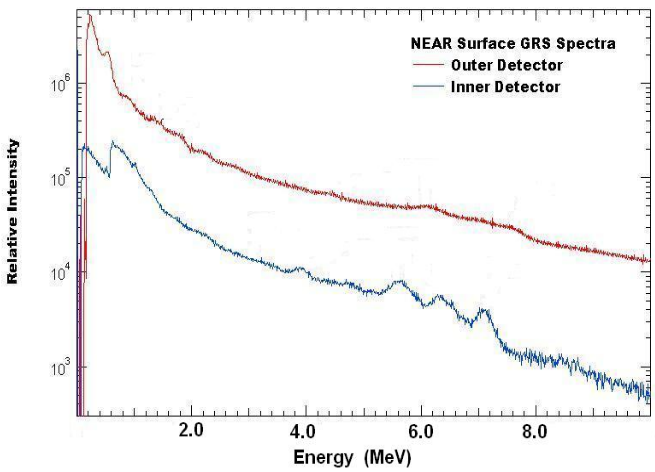
At 30,000 petaHertz the energy of a photon is
E = 4.56 (300,000) = 136,700 eV or **136.7 keV**.

Problem 3 - A simple linear relationship exists between the temperature of a body and the dominant frequency of the light that it produces given by the formula $f = 4.2 \times 10^8 / T$ where T is the temperature of the body in Kelvin degrees, and f is the frequency, in petaHertz, where most of the light is emitted. In what X-ray bands would you expect to find the peak emission for the following situations:

A) The Hinode satellite observes the corona of the sun. The corona of the sun at a temperature of 2 million K.
Answer: $f = 4.2 \times 10^8/2,000,000 = 210$, so the peak is at a frequency of 210 petaHertz. This is in the **soft-Xray band**.

B) In 2001, the Chandra Observatory determined that the cosmic x-ray background is produced mainly by distant galaxies in which massive black holes are consuming matter and producing hot gases. What is the temperature of the plasma that produced the x-ray light seen at an energy of 10 keV?

Answer: From E(eV)= 4.56 f and E(eV) = 10,000 eV we have a frequency of f = 10,000/4.56 = 2,194 petaHertz. This is in the **soft-Xray band**.



The most energetic forms of light in the universe are the gamma-rays. Gamma rays typically have frequencies above 10,000 petaHertz, and energies above 50 keV with wavelengths smaller than an atom (10 picometers) or even an atomic nucleus (0.0001 picometer)!

Commonly, gamma-rays are classified according to the energy they carry in units of MeV (1 million eV), GeV (1 billion eV) or TeV (1 trillion eV). Gamma-rays are produced during nuclear reactions including fission, fusion and radioactive decay.

Problem 1 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(eV) = 4.56 f where f is the frequency of the EM wave in petaHertz. What is the frequency, in exaHertz, of a gamma ray photon with an energy of A) 1 MeV, B) 1 GeV, and C) 1 TeV? (Note: 1 exaHertz = 1,000 petaHertz, or 10^{18} Hertz)

Problem 2 - The gamma-ray spectrum shown above was obtained by the NASA, NEAR spacecraft as it encountered the asteroid Eros. It shows the gamma-ray light from the sun reflected off of the surface of the asteroid. It also shows various 'bumps' produced as the solar gamma-rays interact with the nuclei of specific elements in the compounds on the surface. From a table of known element features in the gamma-ray spectrum shown below, identify which elements were present on the surface of Eros.

Major line energies for common elements

Element	Energy (MeV)
Iron	7.0
Oxygen	6.4
Oxygen	3.8
Oxygen	5.4
Silicon	1.8

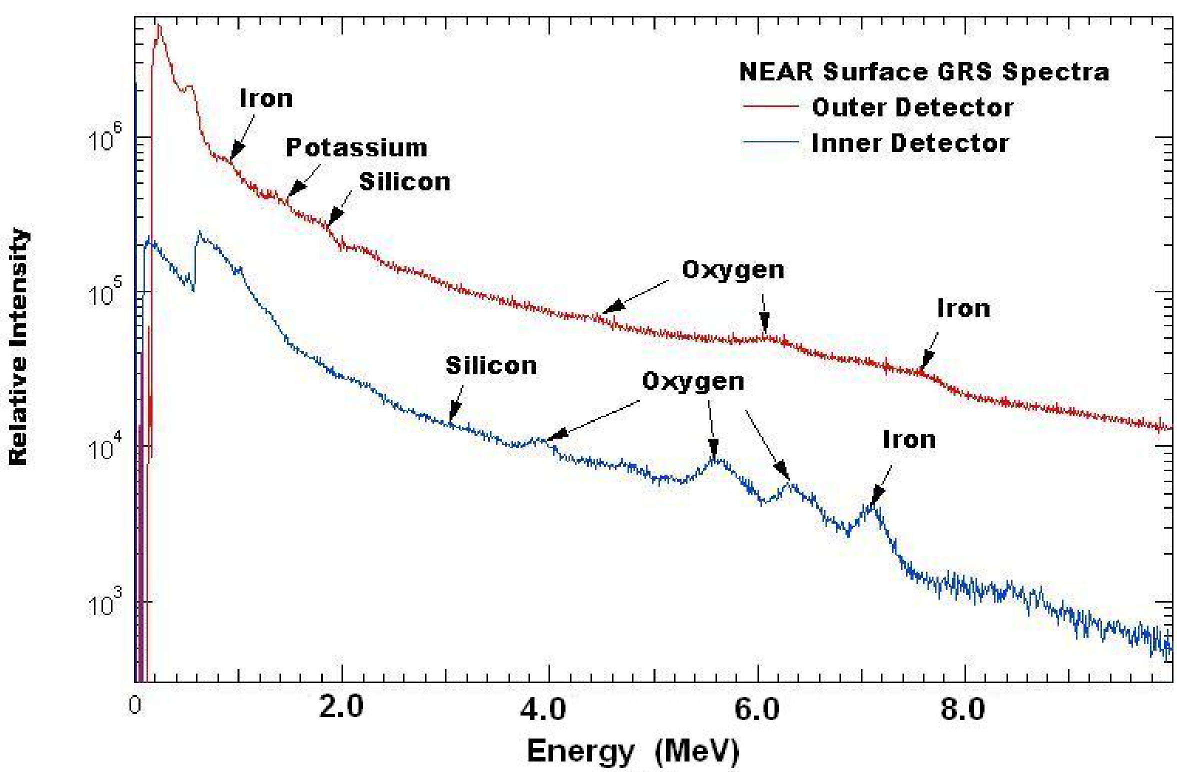
Problem 1 - The energy of a quantum of electromagnetic energy (called a photon) is given by the formula E(eV) = 4.56 f where f is the frequency of the EM wave in petaHertz. What is the frequency, in exaHertz, of a gamma ray photon with an energy of A) 1 MeV? ,B) 1 GeV?, C) 1 TeV? (Note: 1 exaHertz = 1,000 petaHertz or 10^{18} Hertz)

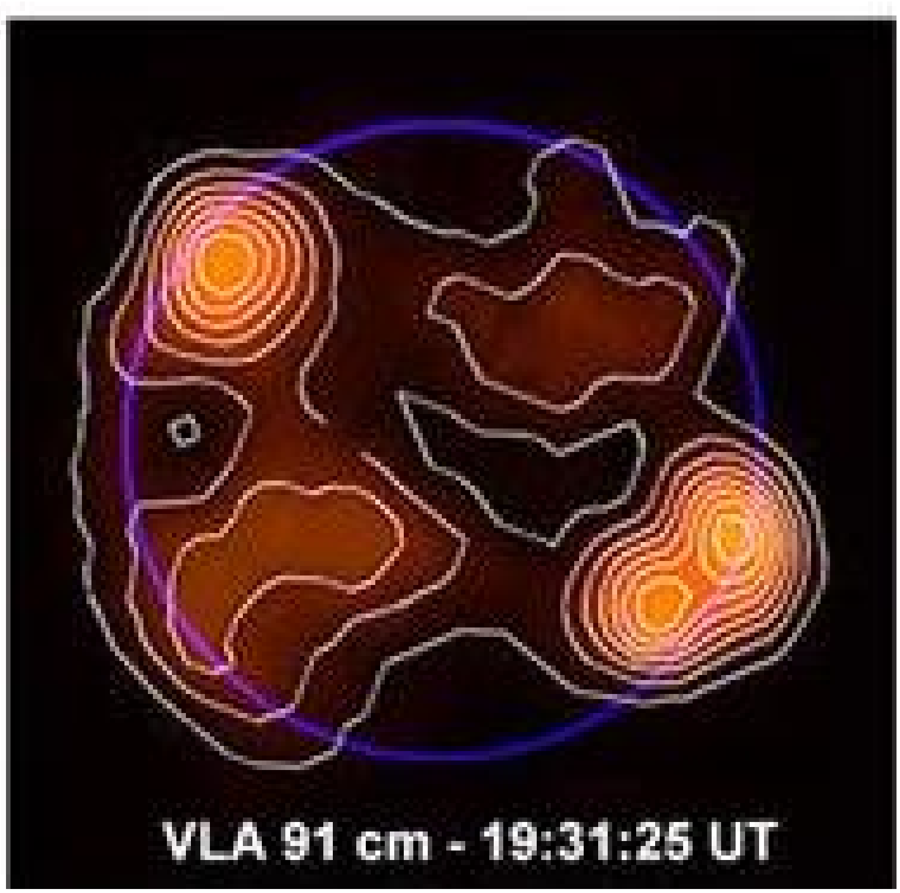
Answer: A) E = 4.56 (f) = 1 million eV
so f = 1,000,000/4.56 = 219,000 petaHertz or **219 exaHertz**

B) E = 4.56 (f) = 1 billion eV
so f = 1,000,000,000/4.56 = 219,000,000 petaHertz or **219,000 exaHertz**

C) E = 4.56 (f) = 1 trillion eV
so f = 1,000,000,000,000/4.56 = 219,000,000,000 petaHertz
or **219,000,000 exaHertz**

Problem 2 - Answer: The figure below shows the indicated lines and a few additional ones not in the catalog.





This radio map of the sun at a wavelength of 91 centimeters (0.92 meters), or a frequency of 327 megaHertz (3.27×10^{6} Hz), was obtained with the Very Large Array (VLA) during a solar flare episode on February 22, 2004. This is the longest wavelength at which this high-resolution array can operate, revealing features larger than about 56 arcseconds. High resolution VLA observations of the Sun at 20 and 91 cm have made it possible to locate and resolve sites of impulsive energy release at two different heights in the solar corona. Investigations of microwave burst emission have also yielded insights to the pre-flare heating of coronal loops, the successive interaction of adjacent loops . The VLA map shows intense Type I noise storm emission above the active region at the limb. This time-variable radio emission followed, by a few minutes, a solar flare and coronal mass ejection detected by SOHO and TRACE, This suggests that these evolving EUV sources may have played a role in accelerating electrons that were eventually seen in the noise storm at greater heights. (Courtesy; Tufts Solar Physics Group and NRAO/VLA)

Problem - The volume of Earth is 1 trillion km^3. From the scale of this image and the size of the radio burst in the upper left corner of the solar image, about how many Earth's could fit inside the plasma volume of the radio burst? (Sun

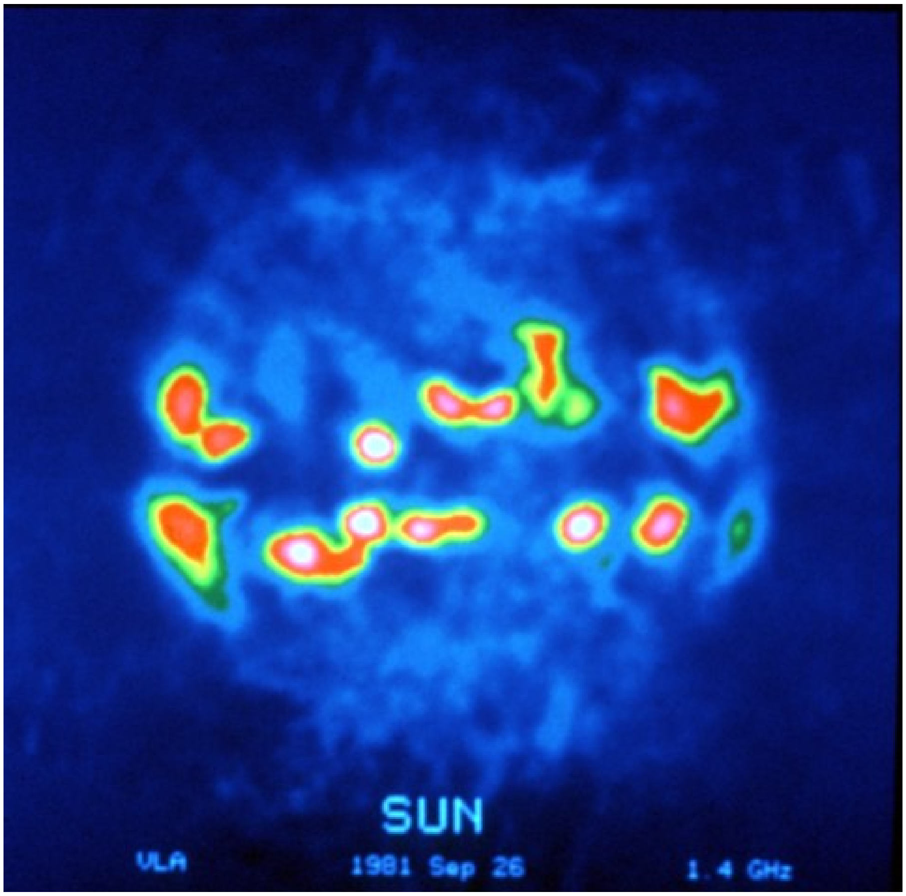
Answer Key

Problem - The volume of Earth is 1 trillion km^3. From the scale of this image and the size of the radio burst in the upper left corner of the solar image, about how many Earth's could fit inside the plasma volume of the radio burst? (Sun diameter = 1,300,000 km)

Answer: The disk of the sun has a diameter of about 85 millimeters, so the scale of the image is 1,300,000 km / 85 mm = 15,000 km/mm .The approximate diameter of the radio burst region is 30 mm, so its physical size is about 30 mm x 15,000 km/mm = 450,000 km. The volume of a sphere with this diameter is just
$V = 4/3\ (3.141)\ (450000/2)^3 = 4.8 \times 10^{16}\ km^3$.

The number of equivalent Earth volumes is just

$$N = \frac{4.8 \times 10^{16}\ km^3}{1.0 \times 10^{12}\ km^3} = \textbf{48,000 Earths.}$$



False-color image of the sun obtained at a wavelength of 20 centimeters (0.2 meters) or a frequency of 1.5 gigaHertz ($1.5x10^9$ Hz) with the Very Large Array (VLA) radio telescope in New Mexico on September 26, 1981.

The image shows a dozen intense sources of radio emission forming two belts of activity parallel to the solar equator. The radio emission comes from electrons at temperatures of over 2 million degrees K. The fainter 'blue' regions are produced by plasma at lower temperatures near 50,000 K. The corona appears as a patchwork of cloudy regions and 'holes' depending on whether magnetic fields are confining the plasma (bright) or whether the plasma is free to escape into interplanetary space (dark).

Problem - If you approximated each active region as a sphere, about what is the total volume occupied by the radio-emitting hot plasma compared to the volume of Earth (1 trillion km^3).

Problem - If you approximated each active region as a sphere, about what is the total volume occupied by the radio-emitting hot plasma compared to the volume of Earth (1 trillion km^3).

Answer: The diameter of the solar disk is about 90 millimeters, so the scale is 1,300,000 km/90 mm = 14,500 km/mm. The active regions are about 5 mm in diameter or 5 mm x 14,500 km/mm = 73,000 km. Assuming a spherical volume, each active region is about V = 4/3 (3.14) $(73{,}000/2)^3$ = 2.0×10^{14} km^3 so for 12 regions we have a total volume of 2.4×10^{15} km^3. The number of equivalent Earth volumes is just

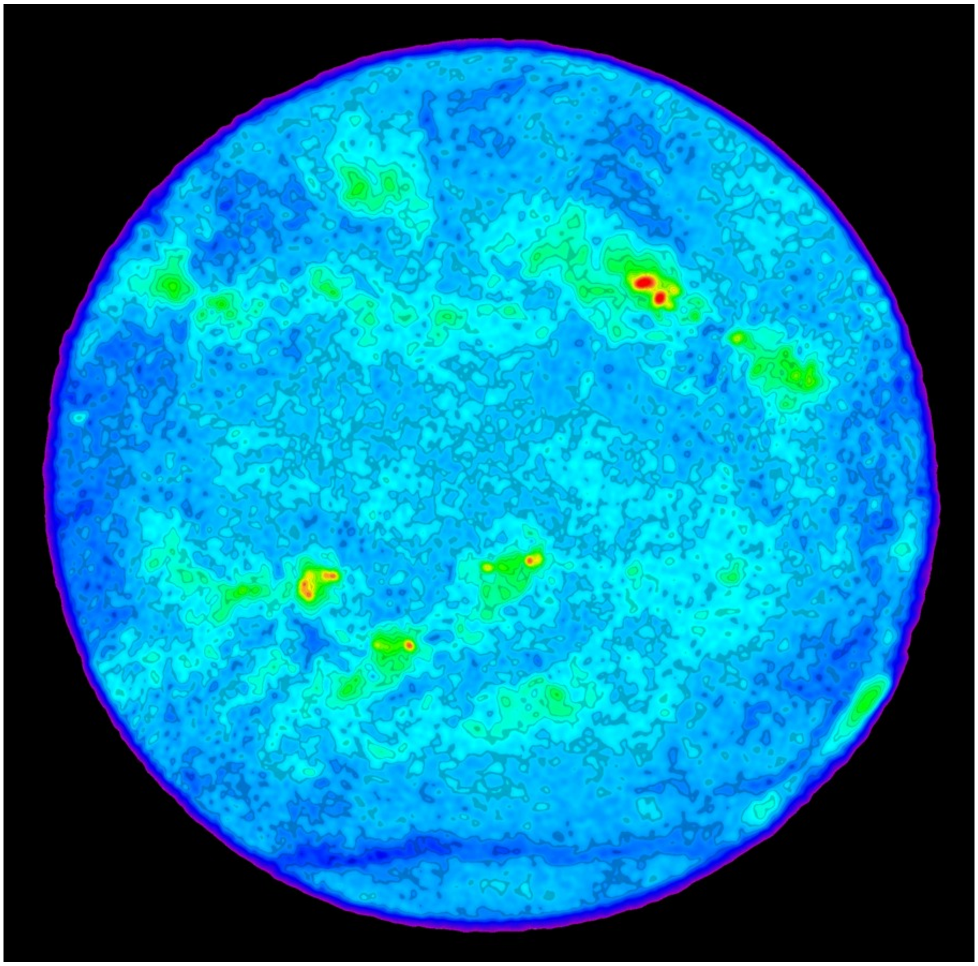
$$N = \frac{2.4 \times 10^{15} \text{ km}^3}{1.0 \times 10^{12} \text{ km}^3} = \textbf{2,400 Earths}$$

The brightness of the Sun at these wavelengths is about

$$F = 6.0 \times 10^{-21} \text{ watts/meter}^2$$

But during disturbed conditions near sunspot maximum, the brightness is about

$$F = 8.0 \times 10^{-19} \text{ watts/meter}^2$$



High resolution false-color image obtained at a frequency of 4.7 GHz (0.06 meters) or 5.0 gigaHertz (5.0×10^9 Hz) by the Very Large Array radio telescope (VLA) of the 'quiet sun' at a resolution of 12 arcseconds, from plasma emitting at 30,000 K. The brightest features (red) in this false-color image have temperatures of about 100,000 degrees K and coincide with sunspots. The green features are cooler and show where the Sun's atmosphere is very dense. At this frequency the radio-emitting surface of the Sun has an average temperature of 30,000 K, and the dark blue features are cooler yet. (Courtesy: Stephen White, University of Maryland, and NRAO/AUI).

Problem - From the scale of this image, what is the size of the smallest feature compared to the diameter of Earth (12,500 km)? (Sun diameter = 1,300,000 km)

Problem - From the scale of this image, what is the size of the smallest feature compared to the diameter of Earth (12,500 km)?

Answer: The sun disk is about 115 millimeters in diameter so the scale is 1,300,000 km/115 mm = 11,300 km/mm. The smallest features are the dark blue 'freckles' which hare about 1-2 millimeters across, corresponding to a physical size of 11,000 to 23,000 kilometers. This is about **1 to 2 times the size of Earth.**

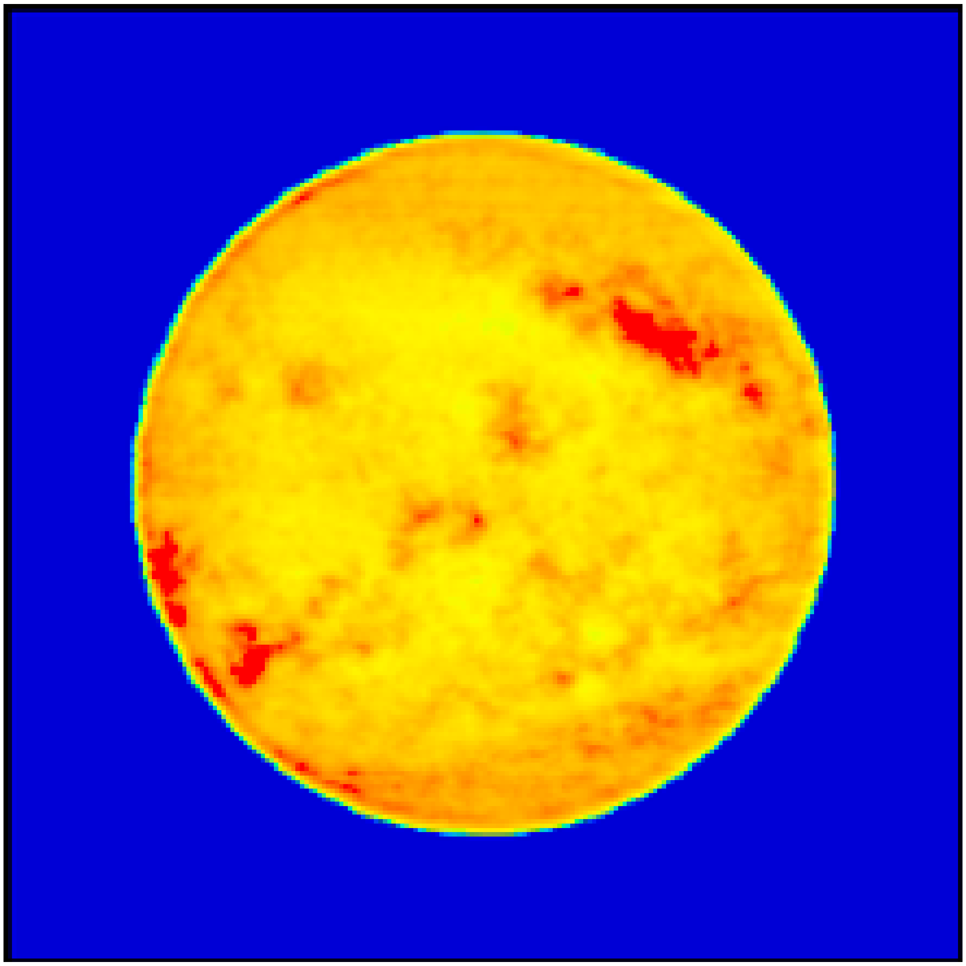
The brightness of the sun at this wavelength varies from

$$F = 2 \times 10^{-21} \text{ watts/meter}^2$$

to

$$F = 1 \times 10^{-19} \text{ watts/meter}^2$$

depending on the level of solar 'sunspot' activity.



This solar image at a wavelength of 1 millimeter (0.001 meters) or a frequency of 300 gigaHertz (3.0×10^{11} Hz) was obtained by the Atacama Large Millimeter Array (ALMA). The telescope array consists of 64 antennas each 12 meters in diameter. The antennas may all be grouped together in an area 150 meters in diameter to provide about 1.0 arcsecond angular resolution, or they may be distributed over an area 14 kilometers in extent to provide an angular resolution as high as 0.01 arcseconds (10 milliarcsec). Millimeter radio emission is extremely sensitive to dynamic processes in the chromosphere; a layer of the sun located just above the photosphere but below the corona. The image above shows features that coincide with large sunspot groups in the photosphere.

Problem - At these frequencies, electrons are emitting radio-wave energy by spiraling in magnetic fields. This causes them to lose energy very rapidly. The lifetime of an electron before it can no longer emit much energy is given by the formula $T = 422 / (B^2E)$ where T is in seconds, B is the magnetic field strength in Gauss, and E is the electron energy in electron-Volts (eV). If B = 100 Gauss and E = 0.001 eV, how long do these electrons continue to radiate radio waves?

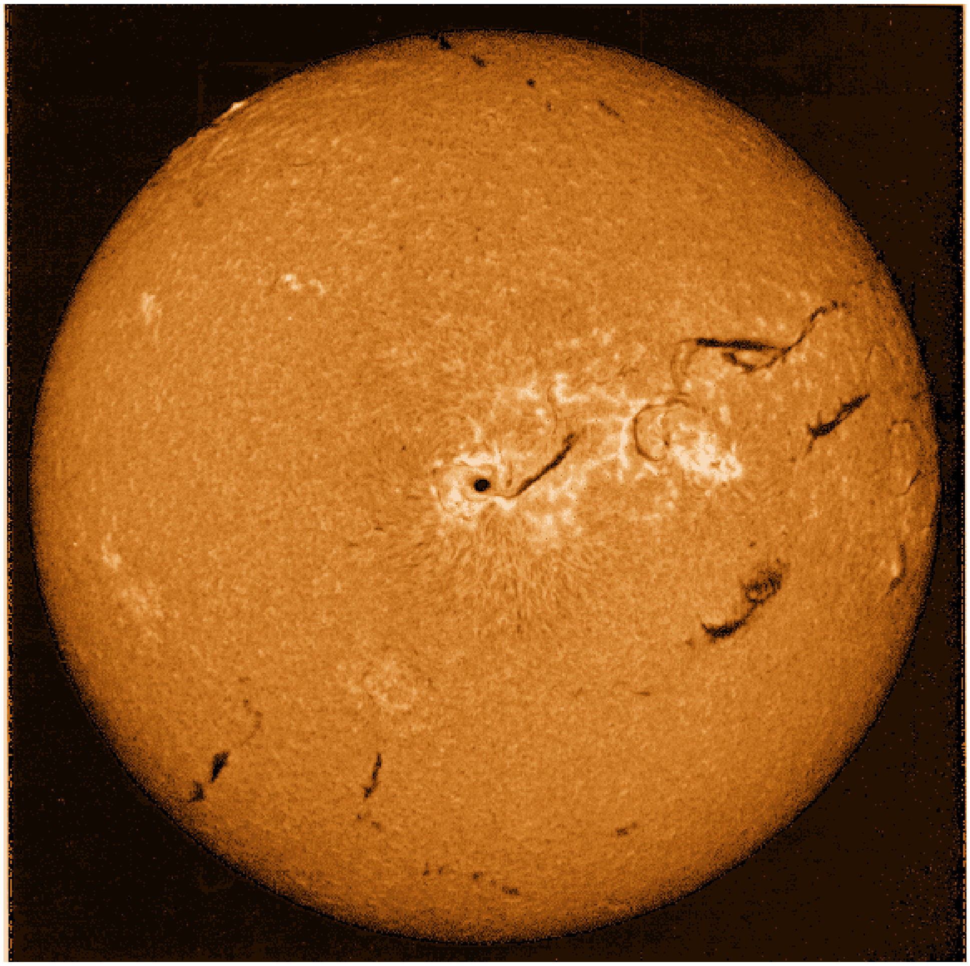
Answer Key

Problem - At these frequencies, electrons are emitting radio-wave energy by spiraling in magnetic fields. This causes them to lose energy very rapidly. The lifetime of an electron before it can no longer emit much energy is given by the formula T = 422 / (B2E) where T is in seconds, B is the magnetic field strength in Gauss, and E is the electron energy in eV. If B = 100 gauss and E = 0.001 eV, how long doe these electrons continue to radiate radio waves?

Answer: $T = \frac{422}{(100)^2 (0.001)}$ = **42 seconds.**

Note: At the typical speed of the electrons of about 500 km/sec, they can travel a distance of about 42 sec x 500 km/sec = 21,000 kilometers. At the scale of the radio image (1,300,000 km/90mm) of 14,500 km/mm, this distance is about 1 - 2 millimeters.

The Sun in Hydrogen Light



This is an image taken by the National Solar Observatory on Sacramento Peak using light from hydrogen atoms (H-alpha). It was obtained at a wavelength of 635 nm, or a frequency of 460 teraHertz (4.6×10^{14} Hz), and reveals hydrogen gas at temperatures between 10,000 and 30,000 K,. These gases are found in prominences and near sunspots. The image shows considerable detail in the photosphere and chromosphere of the sun that cannot be as well seen in broad-band optical light. Once reserved only for the most advanced observatories, these H-alpha images can routinely be produced by amateur astronomers using equipment costing less than $1,000.

Problem - A cloud of gas in front of a light source will absorb light at the specific wavelengths of its atoms, and then re-emit this light in different directions at exactly the same wavelengths. From one vantage point, the cloud will appear dark, while from other vantage point the cloud can appear to be emitting light. This is an example of Kirchoff's Law in spectroscopy. Can you explain how this principle accounts for the details seen in the H-alpha image of the sun above?

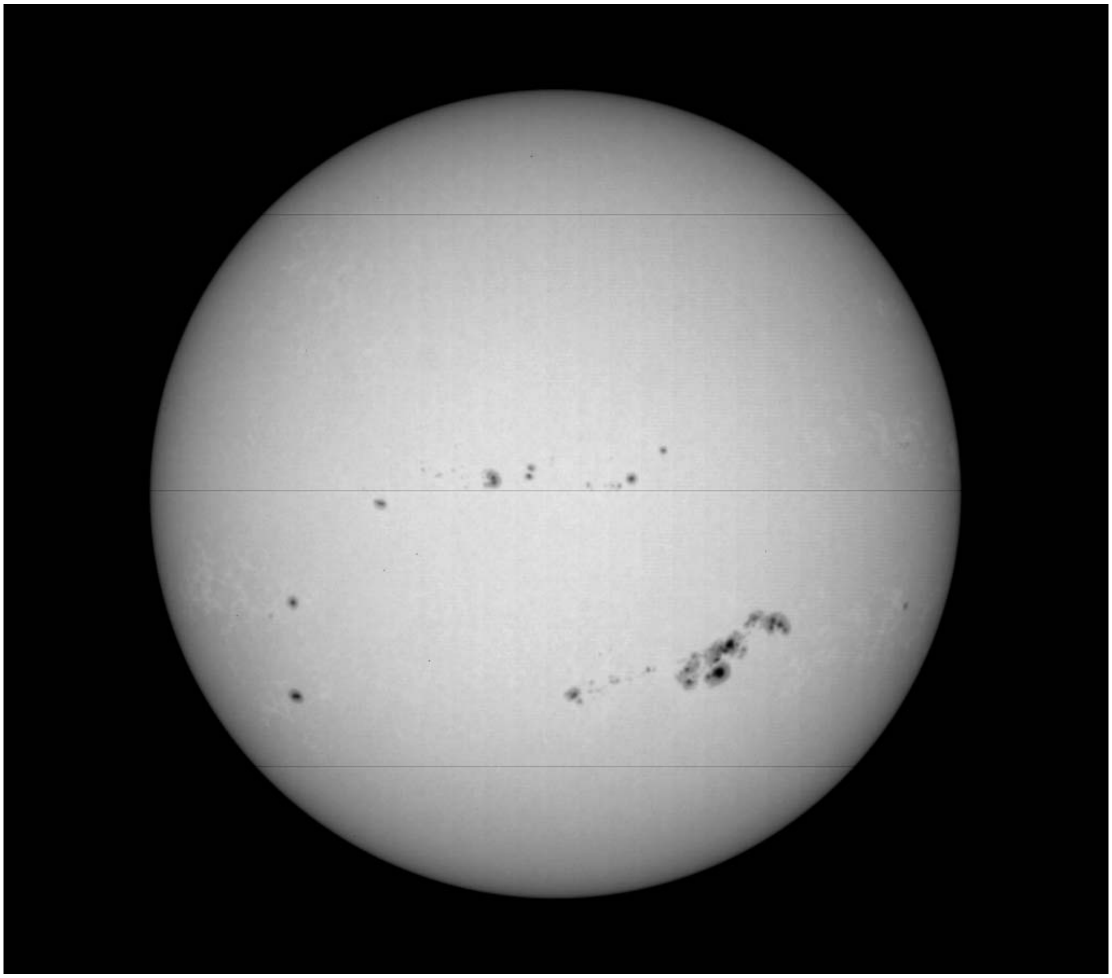
Problem - A cloud of gas in front of a light source will absorb light at the specific wavelengths of its atoms, and then re-emit this light in different directions at exactly the same wavelengths. From one vantage point the cloud will appear dark, while from other vantage points the cloud can appear to be emitting light. This is an example of Kirchoff's Law in spectroscopy. Can you explain how this accounts for the details seen in the H-alpha image of the sun above?

Answer: A dense hydrogen gas cloud (prominence) seen against the hot solar surface absorbs and re-emits light at the specific wavelengths of hydrogen such as the H-alpha spectral line. When we see a cloud on the face of the sun, most of the scattered light does not travel towards the observer, so the cloud appears to dim the light from the sun at this wavelength. When seen against the dark sky at the limb of the sun, the same hydrogen cloud appears to glow with a reddish color.

At this wavelength, the sun has a brightness of about

$$2.6 \times 10^{-21} \text{ watts/meter}^2$$

The Sun in Visible Light



This picture shows the Sun in white light as seen by the Global Oscillation Network Group (GONG) instrument in Big Bear, California on March 30, 2001. The image was obtained at a wavelength of 500 nm, or a frequency of 600 teraHertz (6.0×10^{14} Hz). The large sunspot is more than 140,000 kilometers across; about 22 times the diameter of Earth. (Image courtesy: NSO/AURA/NSF).

Visible light images of the sun have been used for hundreds of years to study its sunspots and other features, since the advent of the telescope by Galileo in 1609. Ancient Chinese astrologers viewed the sun near sunset and often saw large sunspots. Recently, the intense study of sunspots, their cycles of activity, and other features such as prominences and coronal mass ejections, have led to a comprehensive understanding of solar activity and 'space weather'.

Problem - If the solar diameter is 0.5 degrees and the human eye can see features about 2 arcminutes across, how large would a naked-eye sunspot be on the scale of the image above? (Note: 1 degree = 60 arcminutes)

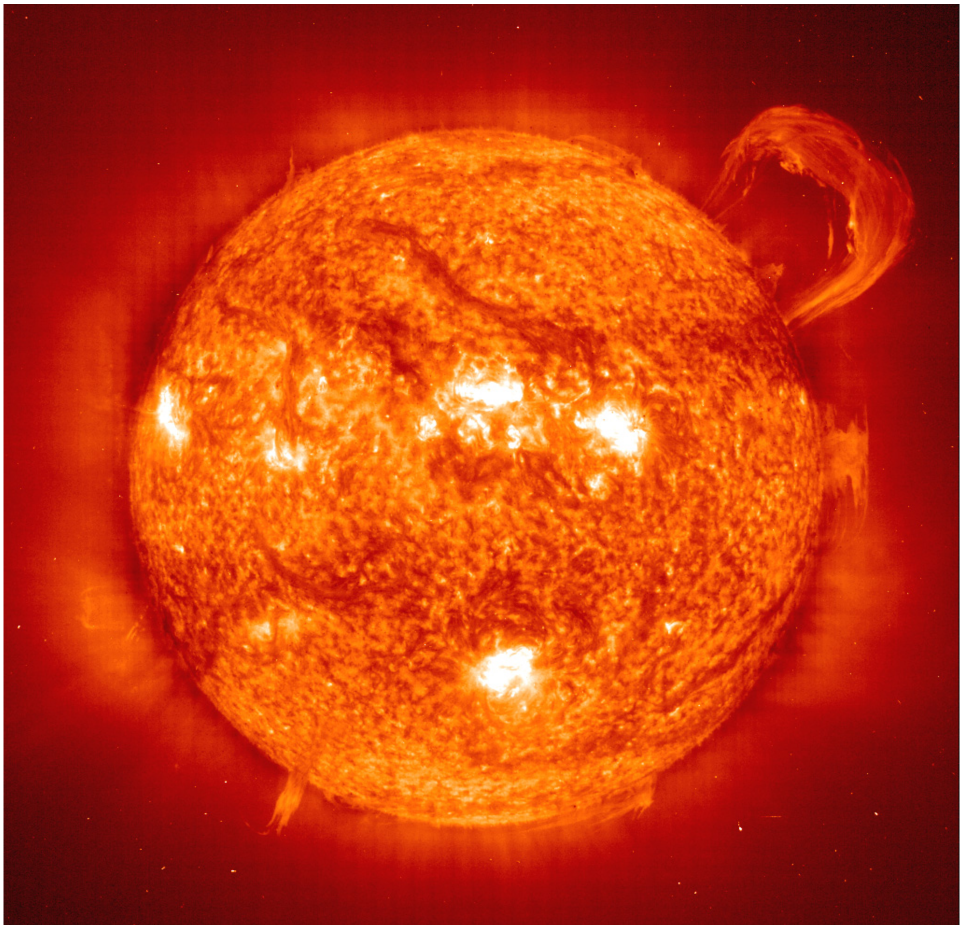
Problem - If the solar diameter is 0.5 degrees and the human eye can see features about 2 arcminutes across, how large would a naked-eye sunspot be on the scale of the image above?

Answer: Since 1 degree = 60 arc minutes, 0.5 degrees is 30 arcminutes. For a sun disk diameter of about 90 mm, the proportion would be: 90 mm x (2 arcminutes/30 arcminutes) = **6 millimeters**. That is about as large as the main clump of spots in the largest group in the image to the lower right.

The brightness of the sun at visible wavelengths is about

$$F = 2 \times 10^{-21} \text{ watts/meter}^2$$

An Extreme Ultraviolet Image of the Sun



False-color image taken by the Solar and Heliospheric Observatory (SOH)) satellite's Extreme Ultraviolet Imaging Telescope (EIT) using ultraviolet light emitted by helium ions at temperatures between 60,000 and 80,000 K.

The image, obtained at a wavelength of 30 nm and a frequency of 10 petaHertz (1.0×10^{16} Hz) shows large active regions associated with sunspot groups (yellow-white); prominences and other filamentary features caused by matter being ejected from the sun into the corona; dark swaths of cooler hydrogen plasma that are prominences seen from above projected onto the solar disk; and the speckled surface caused by solar granulation cells.

Problem – Use a millimeter ruler to determine the scale of the image. What was the height and diameter of the huge prominence seen at the upper right edge of the sun? (Sun diameter = 1,300,000 km)

Problem - What was the height and diameter of the huge prominence seen at the upper right edge of the sun? (Sun diameter = 1,300,000 km)

Answer: The sun disk has a diameter of about 95 millimeters, so the scale is 1,300,000 km/95 mm = 14,000 km/mm. The maximum height of the center of the prominence is about 25 mm above the disk edge, for a height of 25 mm x 14,000 km/mm = **350,000 kilometers**. The diameter of one of its 'legs' is about 4 mm near the photosphere or about **56,000 kilometers**. At the top of the loop, the diameter is about 10 mm or **140,000 kilometers**.

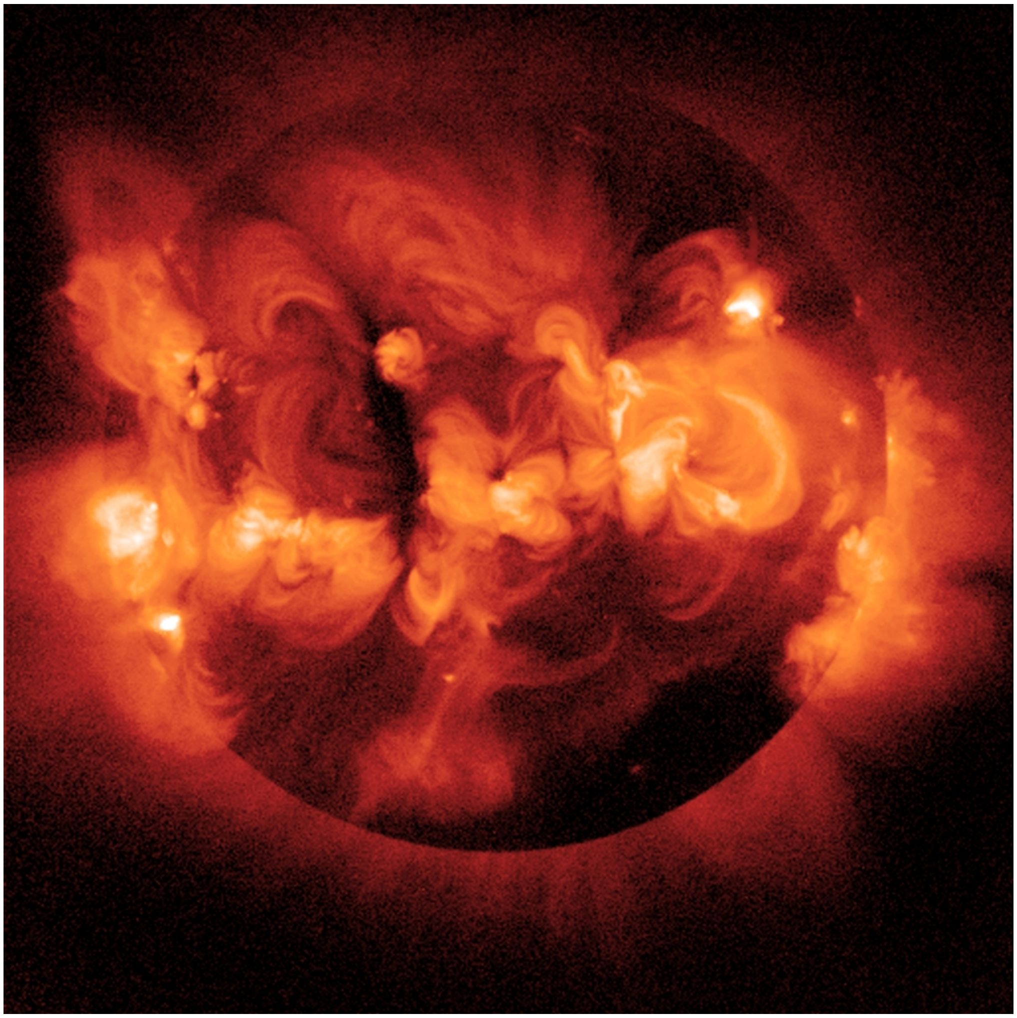
The brightness of the sun at this wavelength varies depending on how many active regions are present. Near sunspot minimum the brightness is about

$$F = 3 \times 10^{-29} \text{ watts/meter}^2$$

But near sunspot maximum it can be higher than

$$F = 3 \times 10^{-27} \text{ watts/meter}^2$$

X-Ray Image of the Sun



This image was taken by the Yohkoh X-ray Satellite using X-rays emitted by the Sun at energies between 0.25 - 4 keV. The equivalent wavelength is 5.0 nm corresponding to a frequency of 60 petaHertz (6.0×10^{16} Hz). The Yohkoh satellite had a resolution of 2.5 arcseconds, which is similar to the newer, more sensitive, x-ray imager on the Hinode satellite.

The image shows a very active sun during the maximum activity period of sunspot cycle 23 in 2000. The billowy clouds are plasma heated to several million degrees above intense sunspot groups on the surface. The magnetically-confined plasma can be detected through the light produced by heavily ionized iron atoms at these temperatures. Most of this structure is located in the Sun's inner corona.

Problem 1 - What features of these clouds in this image suggest that magnetic fields may be confining them?

Problem 2 - How far above the solar photosphere does the coronal structure extend? (Sun diameter = 1,300,000 km).

Answer Key

Problem 1 - What features of these clouds in this image suggest that magnetic fields may be confining them?

Answer: There appear to be faint filaments within the cloud shapes that resemble the magnetic fields you see when you sprinkle iron filings on a bar magnet.

Problem 2 - How far above the solar photosphere does the coronal structure extend? (Sun diameter = 1,300,000 km).

Answer: The diameter of the sun disk is about 115 mm, so the scale of the image is 1,300,000 km/115mm = 11,000 km/mm. Measuring the height of the clouds on the edge of the Sun, they seem to be about 20 mm above the disk edge ,for a height of 20 mm x 11,000 km/mm = **220,000 kilometers**.

X-Ray Image of the Sun

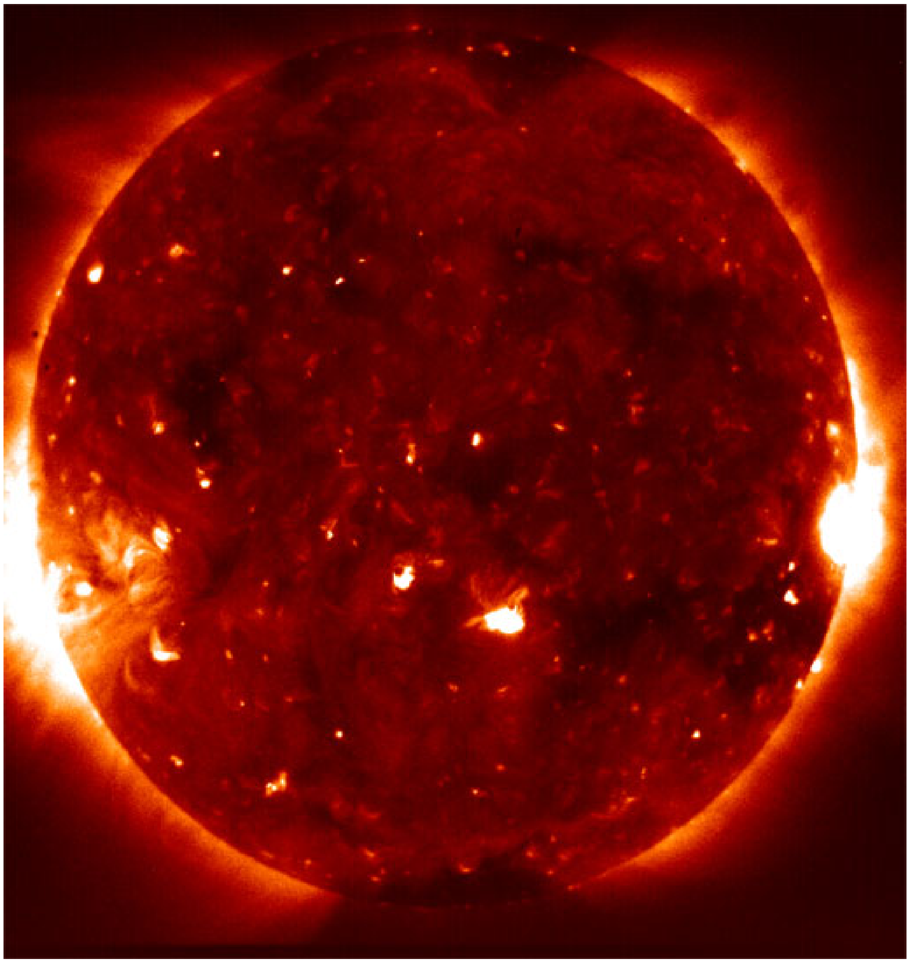


Image taken by the Hinode satellite's X-ray Telescope (XRT) using x-rays emitted by the sun at energies between 1,000 to 10,000 electron volts (1 to 10 keV). The equivalent wavelength is 0.2 nm or a frequency of 1.5 petaHertz (1.5×10^{18} Hz). The resolution is 2 arcseconds. At these energies, only plasma heated to over 100,000 degrees K produce enough electromagnetic energy to be visible. The solar surface, called the photosphere, at a temperature of 6,000 K is too cold to produce x-ray light, and so in X-ray pictures it appears black.

The Hinode image shows for the first time that the typically dark areas of the sun can contain numerous bright 'micro-flares' that speckle the surface, releasing energy into the corona.

Problem 1 - How big are the micro-flares compared to Earth? (Sun diameter = 1,300,000 km; Earth diameter = 12,500 km).

Problem 2 - About how many micro-flares can you see on this hemisphere of the Sun, and how many would you estimate exist at this time for the entire solar surface?

Answer Key

Problem 1 - How big are the micro-flares compared to Earth? (Sun diameter = 1,300,000 km; Earth diameter = 12,500 km).
Answer: The disk of the Sun measures about 110 millimeters in diameter, so the scale is 1,300,000 km/110 mm = 12,000 km/mm. The micro-flares are just over 1 millimeter in diameter or 12,000 kilometers, which is **similar to the diameter of Earth.**

Problem 2 - About how many micro-flares can you see on this hemisphere of the Sun, and how many would you estimate exist at this time for the entire solar surface?

Answer: A careful count should find **about 200** of these 'spots' some bright and some faint. Over the entire solar surface there would be **about 400.**

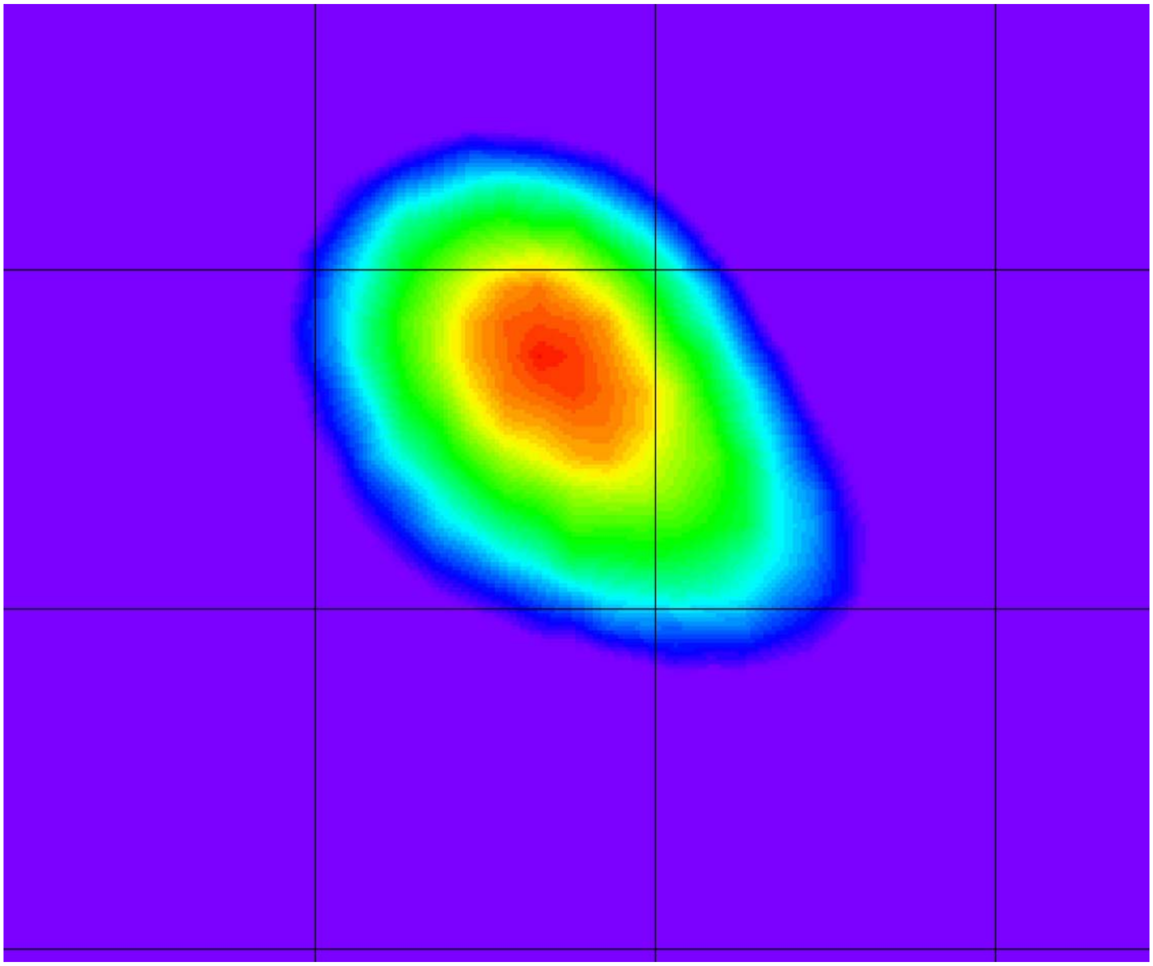
The brightness of the sun at x-ray wavelengths varies depending on the level of solar activity from

$$F = 3 \times 10^{-29} \text{ watts/meter}^2$$

to

$$F = 3 \times 10^{-27} \text{ watts/meter}^2$$

A Gamma-Ray Image of the Sun



This is a colorized plot of the 40 MeV gamma-ray light from the sun during a major solar flare event on June 15, 1991. The equivalent wavelength of the light is 0.00003 nm and a frequency of 1.0×10^{22} Hertz. The image was created by NASA's Compton Gamma Ray Observatory (CGRO). The squares in the grid measure 10 degrees on a side, so the sun (0.5 degrees) cannot be resolved.

This image provided the first evidence that the sun can accelerate particles for several hours. This phenomenon was not observed before CGRO and represents a new understanding of solar flares. (Courtesy: COMPTEL team, University of New Hampshire).

Problem - How big would the disk of the sun be at the scale of this gamma-ray image?

Problem - How big would the disk of the sun be at the scale of this gamma-ray image?

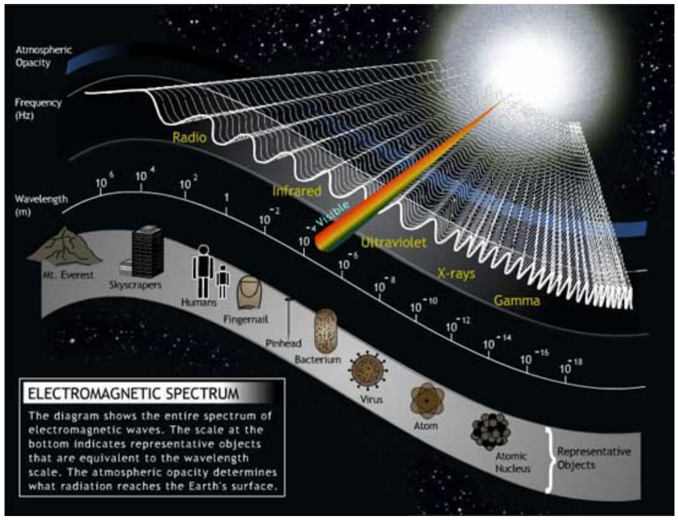
Answer: The grid interval measures 45 millimeters, so the scale is 0.22 degrees/mm. The sun disk would be **2 millimeters** in diameter at this scale.

The brightness of the sun at gamma-ray energies is generally

$$F = 0.0 \text{ watts/meter}^2$$

But during intense flares it can brighten to

$$F = 10^{-35} \text{ watts/meter}^2$$



The sun emits electromagnetic energy at all wavelengths from gamma-ray to radio. Astronomers study the sun at each wavelength band in order to examine different aspects of how solar phenomena work.

To see all of this information at one time, astronomers use a Log-Log plot to graph the emission at each wavelength (or frequency).

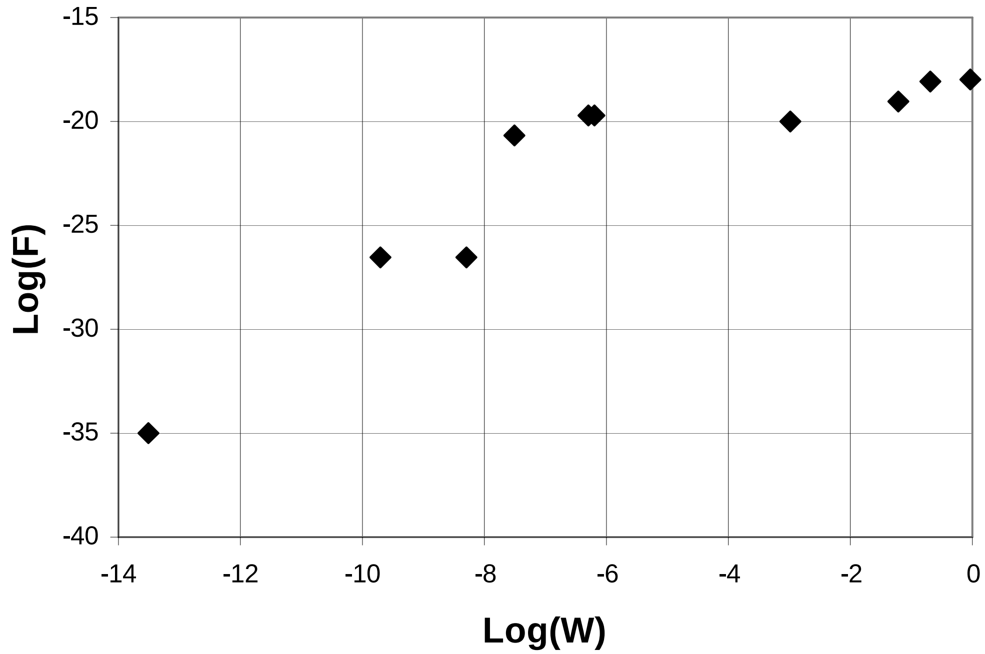
In a Log-Log graph, instead of using the units of 'x' and 'y', we use the units $Log_{10}(x)$ and $Log_{10}(y)$. For example, the point (100, 100000) would be graphed at (2.0, 5.0). The x-axis scale would be marked in uniform intervals of '1.0, 2.0, 3.0…' as before, but these would represent magnitudes of '10, 100 and 1000…' An x-value of '2.5' represents $10^{2.5}$ which is 316 on the usual 'linear' scale.

Problem - In this book, the problems showing the face of the sun at a variety of wavelengths also indicate the brightness of the sun at these wavelengths. The numbers are indicated in the table below. From the brightness data, create a Log(w) vs Log(F) grid, and plot the wavelength and brightness of the sun, F in watts/meter^2, in each electromagnetic wavelength, w, in meters.

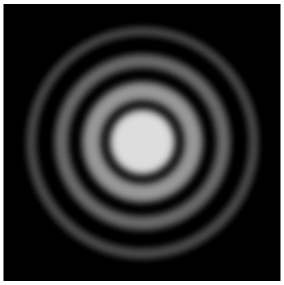
Example: For the 'Gamma ray' point, F = $\log_{10}(1.0 \times 10^{-35}) = -35.0$; and W = $\log_{10}(1.0 \times 10^{22}) = +22.0$, so plot this point at (+22,-35) on your graph.

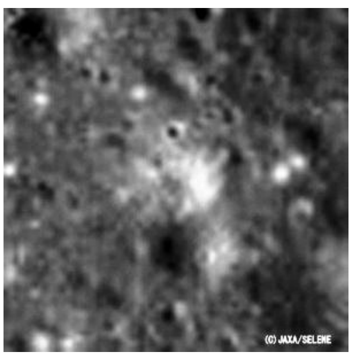
Band	Frequency (Hertz)	Brightness (watts/meter^2)
Gamma ray	1.0×10^{22}	1.0×10^{-35}
Hard X-ray	1.5×10^{18}	3.0×10^{-27}
Soft X-ray	6.0×10^{16}	5.0×10^{-27}
Extreme Ultraviolet	1.0×10^{16}	3.0×10^{-26}
Visible	6.0×10^{14}	2.0×10^{-21}
Millimeter	3.0×10^{11}	5.0×10^{-20}
Microwave	5.0×10^{9}	1.0×10^{-19}
Microwave	1.5×10^{9}	8.0×10^{-19}
Radio	3.3×10^{6}	1.0×10^{-19}

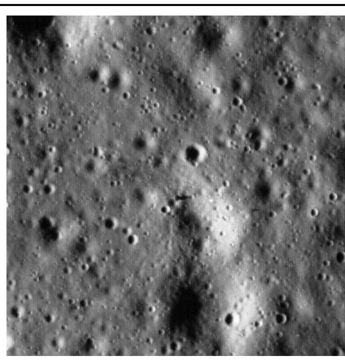
Problem 1 - Answer: In each of the 10 solar images, a notation has been made as to the brightness of the Sun at the indicated wavelength. Students should calculate Log(W) and Log(F) for each image, and plot this point on a Log-Log graph. The result should resemble the graph below:



$$R = 1.22\frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (0.5 meters/pixel at a 50km orbit elevation: aperture =). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = **0.14 degrees.** And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = **8.4 arcminutes.**

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is **0.069 microradians.** B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = **14 milliarcseconds.**

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

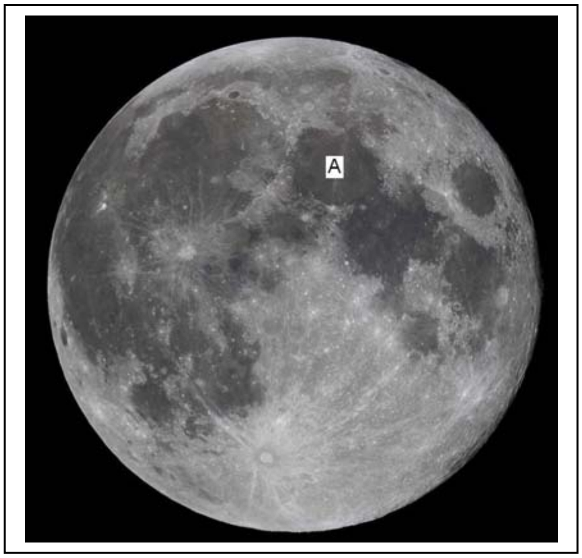
Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

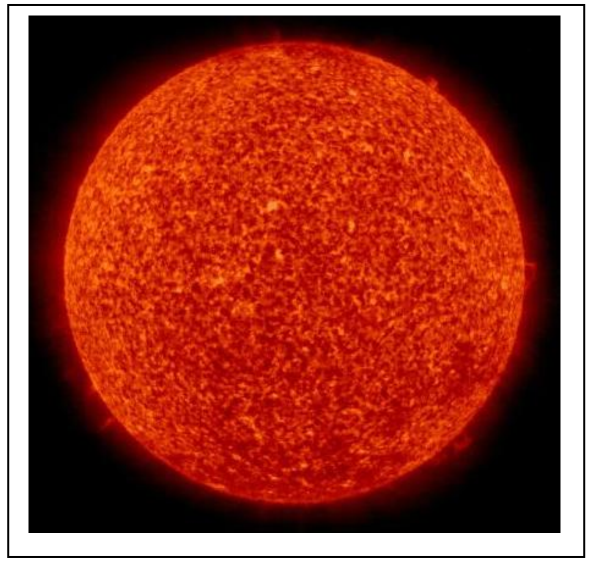
R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/(0.0000048 radians) = **5.1 meters.**

Getting an Angle on the Sun and Moon





The Sun (Diameter = 696,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon?

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

Problem 5 - What is the area of Mare Serenitatis in square kilometers?

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter? Answer: Moon diameter = 65 mm and sun diameter = 61 mm so the lunar image scale is 1,865 asec/65mm = 28.7 asec/mm and the solar scale is 1865 asec/61 mm = 30.6 asec/mm.

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon? Answer: the smallest feature is about 0.5 mm or 0.5 x 28.7 asec/mm = 14.4 asec for the Moon and 0.5 x 30.6 asec/mm = 15.3 asec for the Sun.

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon? Answer: The diameter of the mare is 1 centimeter, so the radius is 5 mm or 5 mm x 28.7 asec/mm = 143.5 asec. Assuming a circle, the area is $A = \pi \times (143.5 \text{ asec})^2 = 64{,}700 \text{ asec}^2$.

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle? Answer: The angular scale at the sun would correspond to 400 x 1.9 km = 760 kilometers per arcsecond.

Problem 5 - What is the area of Mare Serenitatis in square kilometers? Answer: We have to convert from square arcseconds to square kilometers using a two-step unit conversion 'ladder'.

$$64{,}700 \text{ asec}^2 \times (1.9 \text{ km/asec}) \times (1.9 \text{ km/asec}) = 233{,}600 \text{ km}^2.$$

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun? Answer: The angular area is 400-times further away, so we have to use the scaling of 760 kilometers/asec deduced in Problem 4. The unit conversion for the solar area becomes:

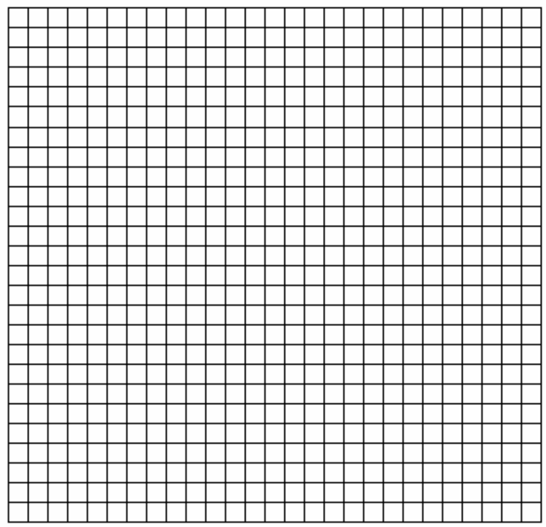
$$64{,}700 \text{ asec}^2 \times (760 \text{ km/asec}) \times (760 \text{ km/asec}) = 37{,}400{,}000{,}000 \text{ km}^2.$$

Resolving the Moon

Although a pair of binoculars or a telescope can see amazing details on the Moon, the human eye is not so gifted!

The lens of the eye is so small, only 2 to 5 millimeters across, that the sky is 'pixelized' into cells that are about one arcminute across. We call this the resolution limit of the eye, or the eye's visual acuity.

One degree of angle measure can be divided into 60 minutes of arc. For an object like the full moon, which is 1/2-degree in diameter, it also measures 30 arcminutes in diameter. This means that, compared to the human eye, the moon can be divided into an image that is 30-pixels in diameter.



Problem 1 - Convert the following degree measures into their equivalent measure in arcminutes; A) 5 degrees; B) 2/3 degree; C) 15.5 degrees; D) 0.25 degrees

Problem 2 - Convert the following arcminute measures into their equivalent measure in degrees: A) 15 amin; B) 1/2 amin; C) 120.5 amin; D) 3600 amin.

Problem 3 - Convert the following area measures in square-degrees into their equivalent measures in square arcminutes ($amin^2$): A) 1.0 deg^2; B) 0.25 deg^2

Problem 4 - The figure to the above-left is a telescopic photo of the full moon showing its many details including craters and dark mare. Construct a simulated image of the moon in the grid to the right to represent what the moon would look like at the resolution of the human eye. First sketch the moon on the grid. Then use the three shades; black, light-gray and dark-gray, and fill-in each square with one of the three shades using your sketch as a guide.

Problem 5 - Why can't the human eye see any craters on the Moon?

Answer Key

Problem 1 - Convert the following degree measures into their equivalent measure in arcminutes; A) 5 degrees; B) 2/3 degree; C) 15.5 degrees; D) 0.25 degrees

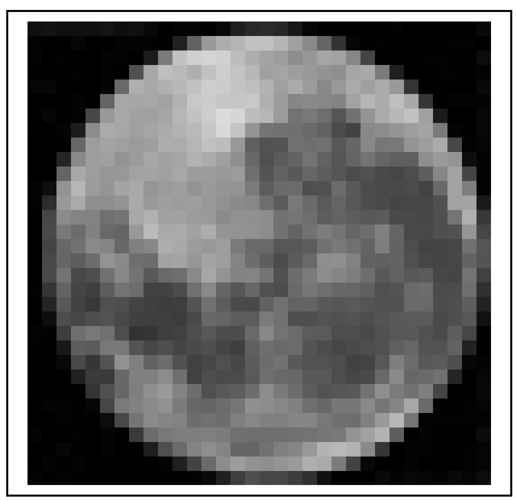
Answer: A) 5 degrees x (60 amin/1 deg) = 300 amin. B) 2/3 degree x (60 amin/1 deg) = 120/3 = 40 amin. C) 15.5 degrees x (60 amin/1deg) = 930 amin; D) 0.25 deg x (60 amin/1 deg) = 15 amin.

Problem 2 - Convert the following arcminute measures into their equivalent measure in degrees: A) 15 amin; B) 1/2 amin; C) 120.5 amin; D) 3600 amin.
Answer: A) 15 amin x (1 deg/60 amin) = 0.25 deg. B) 1/2 amin x (1 deg / 60 amin) = 1/120 deg. C) 120.5 amin x (1 deg/60 amin) = 2.008 deg. D) 360 amin x (1 deg/60 amin) = 60 deg.

Problem 3 - Convert the following area measures in square-degrees into their equivalent measures in square arcminutes ($amin^2$): A) 1.0 deg^2; B) 0.25 deg^2
Answer; A) 1.0 deg^2 x (60 amin/1 deg) x (60 amin/1 deg) = 3600 $amin^2$. B) 0.25 deg^2 x (60 amin/1 deg) x (60 amin/1 deg) = 0.25 x 3600 = 900 $amin^2$.

Problem 4 - See the image below which has been pixelized to the grid resolution. How well did your version match the image on the right?

Problem 5 - Why can't the human eye see any craters on the Moon? Answer: The human eye can only see details 1 arcminute across and this is too low a resolution to see even the largest craters.



Resolving the Universe: The Visual Band

The ability of a telescope to see minute details depends, not only on the wavelength at which you are observing, but also the diameter of the instrument (lens or mirror) being used. At optical wavelengths, the resolution R, in arcseconds, given the diameter, D, in meters, is determined by:

$$R = \frac{0.103}{D}$$

Problem 1 - The basic unit of resolution in astronomy is the arcsecond. It indicates the angular size of an object seen at a distance from the observer. There are 60 arcseconds in an arcminute, and 60 arcminutes in a degree. A second unit often used is the radian. One radian is equivalent to an angular measure of 57.3 degrees. A golf ball viewed at a distance of 10 kilometers an angular diameter of about 0.000005 radians (also called 5 microradians). How many arcseconds does this equal?

Problem 2 - The smallest detail discernible to the human eye is about 1 arcminute in diameter; about the size of a very large crater on the moon. A) How many arcseconds is this? B) How many milliradians is this?

Problem 3 - Fill-in the table below using the resolution formula. Create a vertical 'temperature' scale with the highest resolution (smallest arcseconds) at the bottom, and the lowest resolution at the top. Create the scale based on the Log10 of the resolution over the range [-4.0, +2.0]. If an optical interferometer were built in space with D equal to the diameter of Earth (12,700 kilometers) what would its resolution be, and where would it fall on your scale?

Optical telescopes and their resolving ability

Instrument	Built	Optical Element	Diameter (meters)	Resolution (arcseconds)
Amateur Telescope		Mirror	0.30	
Yerkes Observatory	1893	Lens	1.0	
Keck II telescope	1996	Mirror	10.0	0.04
Keck Interferometer	1996	Interferometer	85.0	0.001
Hubble Space Telescope	1990	Mirror	2.4	0.05
MROI Interferometer	2013	Interferometer	350.0	
Inexpensive camera		Lens	0.01	
High-end camera		Lens	0.05	
Mount Palomar Observatory	1949	Mirror	5.0	
Mayall - Kitt Peak	1960	Mirror	3.8	
Gran Telescopio Canaris	2007	36 Mirrors	10.4	0.014
OWL telescope	2015+	3048-Mirrors	100	
Very Large Telescope	1998	Interferometer	200	
Subaru telescope	1998	Mirror	8.3	
Human Eye		Lens	0.005	

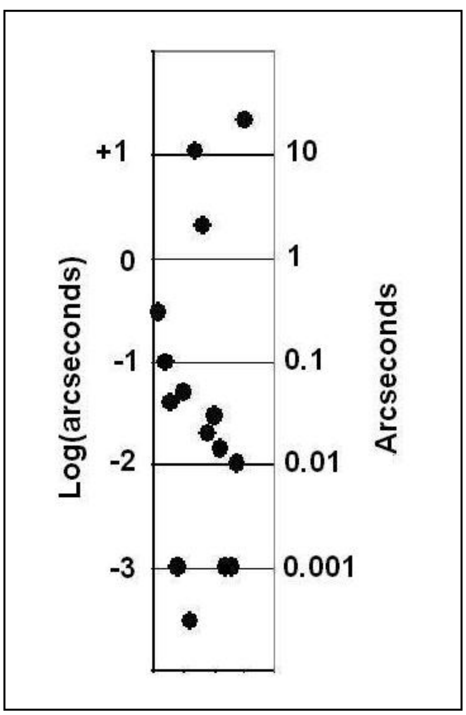
Answer Key

Problem 1 - Answer: 0.000005 radians x (57.3 degrees/radian) x (3600 arcsec/1 degree) = **1 arcseconds**.

Problem 2 - Answer: A) 60 arsceconds. B) 1 radian = 57.3 degrees or 57.3 x 60 = 3438 arcminutes, so 1 arcminute = 1/3438 radians or **0.3 milliradians**.

Problem 3 - Answer: R = 0.103/(12,700,000 meters) = **8.0×10^{-9} arcseconds** (also written as 8 nanoarcseconds). See graph below.

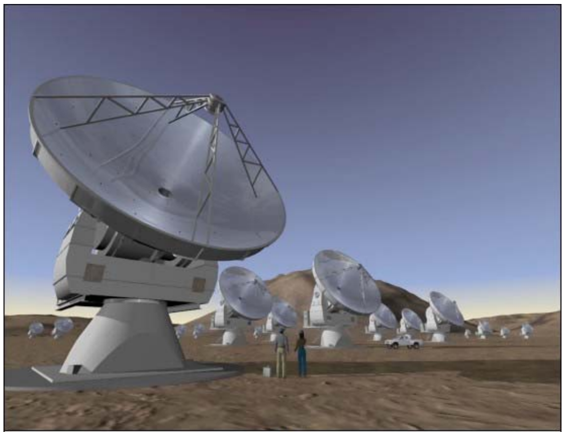
Instrument	Built	Optical Element	Diameter (meters)	Resolution (arcseconds)
Amateur Telescope		Mirror	0.30	**0.3**
Yerkes Observatory	1893	Lens	1.0	**0.1**
Keck II telescope	1996	Mirror	10.0	0.04
Keck Interferometer	1996	Interferometer	85.0	0.001
Hubble Space Telescope	1990	Mirror	2.4	**0.05**
MROI Interferometer	2013	Interferometer	350.0	**0.0003**
Inexpensive camera		Lens	0.01	**11.0**
High-end camera		Lens	0.05	**2.0**
Mount Palomar Observatory	1949	Mirror	5.0	**0.02**
Mayall - Kitt Peak	1960	Mirror	3.8	**0.03**
Gran Telescopio Canaris	2007	36 Mirrors	10.4	0.014
OWL telescope	2015+	3048-Mirrors	100	**0.001**
Very Large Telescope	1998	Interferometer	200	**0.001**
Subaru telescope	1998	Mirror	8.3	**0.01**
Human Eye		Lens	0.005	**21.0**



The bar graph shows both the regular resolution in arcseconds, and the log10 of the resolution.

The top point on the scale is for the resolution of the human eye. The lowest point on the scale is for the MROI optical interferometer, which is used to measure the diameters of distant stars.

For a resolution of 8 nanoarcseconds ,the point would be located at $\log(8 \times 10^{-9})$ = **-8.1 on the scale, and below the lowest-plotted point.**


ALMA *

The ability of a telescope to see minute details depends, not only on the wavelength at which you are observing, but also the diameter of the instrument (lens or mirror) being used. At a particular wavelength λ given in meters, the resolution R, in arcseconds, given the diameter, D, in meters, is determined by:

$$R = 252,000\frac{\lambda}{D}$$

Problem 1 - The resolution of a system varies across the entire electromagnetic spectrum, as does the technology needed to capture and 'count' the photons themselves. Radio waves require a different technology (radio dishes) than optical telescopes (mirrors and lenses) or gamma rays (scintillation counters). The table below gives a list of some of the highest resolutions that have been achieved so far. Calculate the maximum resolution for each system in arcseconds. (Note: The first three telescopes are particle, not wave, detectors and so their resolution limits do not follow the above resolution formula.)

Problem 2 - Create a graph that displays on the horizontal 'X' axis $\log_{10}$ (wavelength), and on the vertical 'Y' axis $\log_{10}$ (arcseconds). Select the following domain X:[-20.0,+3.0] and range Y:[-15.0, +6.0] for the graph, with intervals of 1.0]. Plot each of the items in the table below.

Problem 3 - There are physical limits to how far we can 'push' our ability to resolve details in each band. For radio systems, D will probably be the diameter of the solar system (10 trillion meters), while for UV, optical and infrared systems, D will probably be limited to the diameter of Earth (10 million meters). For gamma-ray systems it will be about 1 arcsecond, for x-rays systems about 0.001 arcseconds. Graph R=0.02 λ for earth-diameter limits, and R =10^{-8} λ for solar system-diameter observations (R in arcseconds and λ in meters) . Use these limits to exclude all regions below them in resolution. In which bands are we getting 'close' to the practical resolution limits?

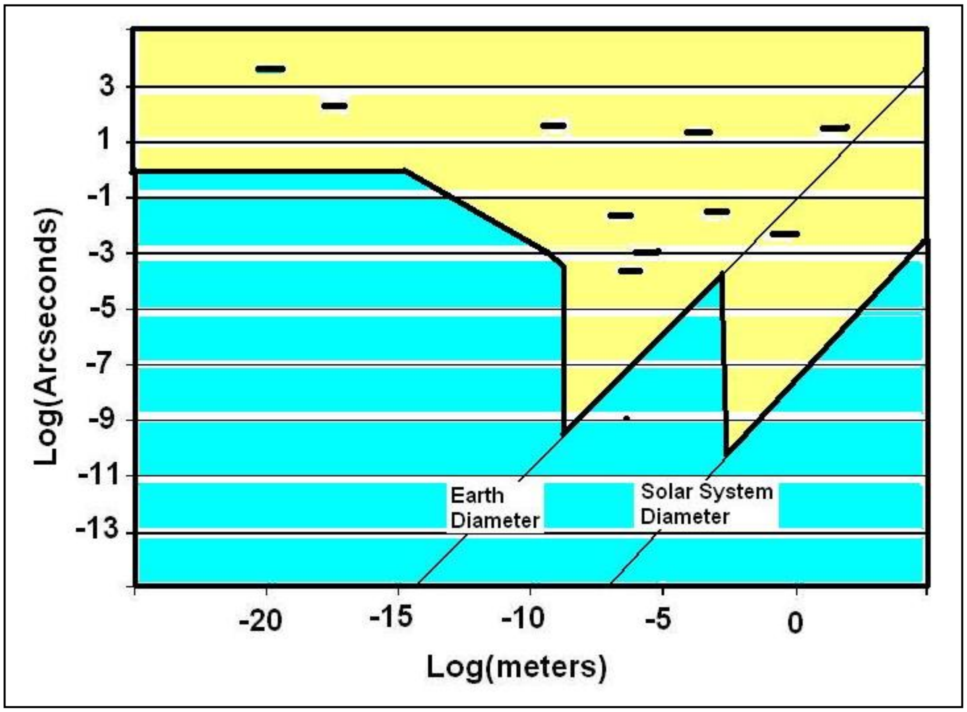
Spectrum Band and System	Wavelength (meters)	Technology for Highest Resolution	Size (meters)	Resolution (arcsecs)
Gamma ray-HESS (10 TeV)	1.0×10^{-20}	Atmospheric Cherenkov	100	4000
Gamma ray-Fermi (300 GeV)	4.0×10^{-18}	Scintillator	3.0	200
X-ray-Chandra (2KeV)	6.0×10^{-10}	Nested Mirrors	1.2	30
Ultraviolet - HST (0.2 nm)	2.0×10^{-7}	Mirror	2.4	
Optical - SUSI - (0.5 nm)	5.0×10^{-7}	Interferometer	640	
Near Infrared - MRO (2 microns)	2.0×10^{-6}	Mirror	400	
Far-Infrared (200 microns)	2.0×10^{-4}	Mirror	2.5	
Millimeter - ALMA (300 GHz)	1.0×10^{-3}	Radio Dish Array	10 km	
Radio-VSOP (1GHz)	3.0×10^{-1}	Space Interferometer	20,000 km	
Radio-LFSA (10MHz)	15	Space Interferometer	300 km	

Problem 1 - Answer:

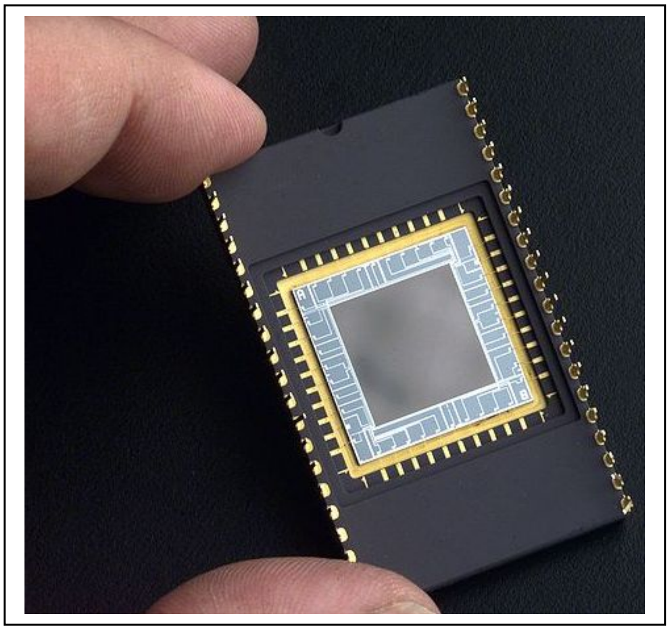
Spectrum Band and System	Wavelength (meters)	Size (meters)	Resolution (arcsecs)
Gamma ray-HESS (10 TeV)	$1.0x10^{-20}$	100	**4000**
Gamma ray-Fermi (300 GeV)	$4.0x10^{-18}$	3.0	**200**
X-ray-Chandra (2KeV)	$6.0x10^{-10}$	1.2	**30**
Ultraviolet - HST (0.2 nm)	$2.0x10^{-7}$	2.4	**0.02**
Optical - SUSI - (0.5 nm)	$5.0x10^{-7}$	640	**0.0002**
Near Infrared - MRO (2 microns)	$2.0x10^{-6}$	400	**0.001**
Far-Infrared (200 microns)	$2.0x10^{-4}$	2.5	**20.0**
Millimeter - ALMA (300 GHz)	$1.0x10^{-3}$	10 km	**0.03**
Radio-VSOP (1GHz)	$3.0x10^{-1}$	20,000 km	**0.004**
Radio-LFSA (10MHz)	15	300 km	**12.6**

Problem 2 - See horizontal points in the graph below.

Problem 3 - Answer: Students should be able to create a graph that resembles the one below. Horizontal points are the resolutions and wavelengths of the various systems in the Table. The two diagonal lines are the 'LogLog' curves for R = 0.02 λ (Earth) and 10^{-8} λ (Solar System) baseline resolutions. The dark line represents the boundary across the spectrum determined by the limits in Problem 2. The shaded (blue) region below it is the resolution-wavelength region where observations are not likely to be performed. The light-shaded region above the line represents resolutions that could be achievable by existing or advanced technology in the future. **Note: An interesting caveat is that eventually we resolve a source so completely that there are no longer any photons left from the surface to be detected by the telescope! Objects only produce a limited number of photons from their surfaces, so this fundamental limit (not plotted) will make the 'excluded' zone significantly more shallow in every spectral region, and exclude most of the 'solar-system-diameter' measurements.**



Your Digital Camera: Counting Photons



One of the benefits of 'going digital' in our technology is that the quantum nature of some phenomena becomes much more obvious. Your every-day digital camera is a perfect example!

Inside your camera is a sensitive array of light detectors called a Charge-Coupled Device (CCD). A '4 megapixel' camera has a square CCD 'chip that has a size of 2048 x 2048 pixels.

Each pixel responds to individual incoming photons of visible light and generates electrons, which are stored in each pixel until the pixel is 'read out' at the end of the exposure.

Resolution - The ability of a camera to see fine details depends on the size of the CCD array and the magnification provided by the optical system in front of the CCD chip. Resolution is measured in angular size. The human eye, with its 10-millimeter lens, can see details about 1 arcminute across. (Note: 1 degree = 60 arcminutes, and 1 arcminute = 60 arcseconds). The way that resolution, R, is determined is by measuring the size of an individual pixel, L, and multiplying this by the 'image scale', S, provided by the optical system, usually stated in arcseconds per millimeter: R = L x S.

Problem 1 - A) A common digital camera has a chip with 16 megapixels that occupy an area that is 1.0 cm across. If a typical photograph covers a field of view that is 100 degrees wide, what is the resolution of a single pixel in arcminutes, and how large are the pixels in microns? B) The Hubble Space Telescope Wide-Field Planetary Camera III (WFPC3) camera has a 4096x4096 pixel CCD array for which pixels are 15 microns across, and the 2.4-meter mirror provides an image scale of 2.7 arcseconds/mm. What is the finest detail visible in digital images from HST?

Sensitivity - Also called Quantum Efficiency, E, measures just how efficient an average pixel is in detecting the incoming photons and producing one electron, which can be counted by the electronics to 'build up' an image. E = 5 means that 5 photons are required to register 1 electron for counting.

Problem 2 - A new CCD chip has been designed with a Quantum Efficiency of 4.0. If the CCD is designed to image a typical scene with a brightness of 10^7 photons/sec/pixel, A) how many photo-electrons will be produced in each pixel 'well'? B) The electrons are counted after each exposure, and converted into digital numbers that are 2-bytes large. What is the maximum exposure that this CCD can accommodate before the number of counted electrons exceeds 2-bytes?

Answer Key

Problem 1 - A) A common digital camera has a chip with 16 megapixels that occupy an area that is 1.0 cm across. If a typical photograph covers a field of view that is 100 degrees wide, what is the resolution of a single pixel in arcminutes, and how large are the pixels in microns?
B) The Hubble Space Telescope Wide-Field Planetary Camera III (WFPC3) camera has a 4096x4096 pixel CCD array for which pixels are 15 microns across, and the 2.4-meter mirror provides an image scale of 2.7 arcseconds/mm. What is the finest detail visible in digital images from HST?

Answer: A) If the total number of pixels is 16 megapixels, so the array is 4096x4096. The field is 100 degrees wide, which is 100 degrees x (60 minutes/1 degree) x (60 seconds/1 minute) = 360,000 arcseconds, so each pixel has an angular resolution of 360,000 arcseconds/4096 = **88 arcseconds or 1.5 arcminutes**. (This is similar to a human eye). The size of a pixel is 1 cm/4096 = 0.00024 cm x (1 meter/100 cm) x (1 million microns)/1 meter) = **2.4 microns**.
B) First determine the size of a pixel in millimeters: 15 microns x (1 meter/1 million microns) x (1000 mm/1 meter) = 0.015 mm. Then R = 2.7 arcseconds/mm x (0.015 mm) = **0.04 arcseconds**.

Problem 2 - A new CCD chip has been designed with a Quantum Efficiency of 4.0. If the CCD is designed to image a typical scene with a brightness of 10^7 photons/sec/pixel,

A) How many photo-electrons will be produced in each pixel 'well'?
Answer: The Quantum Efficiency states that 4.0 photons are required in order for the pixel to generate one electron, so the 10^7 photons each second will correspond to $10^7/4$ = electrons/second per pixel. 2.5×10^6 electrons/sec.

B) The electrons are counted after each exposure, and converted into digital numbers that are 2-bytes large. What is the maximum exposure that this CCD can accommodate before the number of counted electrons exceeds 2-bytes?
Answer: Since 1-byte = 8-bits, and 8-bits is equal to 2^8 = 256, 2 bytes is 16-bits which is equal to 2^{16} = 256x256 = 65,536. A 2-byte data 'word' can be used to represent any number up to 65,536. So the maximum number of electrons that can be counted in a pixel for each exposure is 65,536. In Part A we calculated that in one second the pixel would receive 2.5×10^6 electrons. We can only count 65,536 of these at a time, so that means in one second we have to read-out the pixel about 2.5×10^6 / 65,536 = 38 times. That means that the exposure for each image is 1/38 seconds or **0.026 seconds**. This is also referred to as a read rate of 26 milliseconds or 1/0.026 = 38 Hertz.

It is a challenge to design cameras with large CCD arrays (8 megapixel, 10 megapixel, etc) because all of the pixels in the entire array have to be read out before the pixels reach their limit of storing electrons .This limit is called the saturation limit for the array. Advanced cameras use a 16-bit read out and a speed of 2 megaHertz per pixel (0.0000005 seconds/pixel. Combining these numbers with a quantum efficiency of 2.0, this camera can operate with a scene brightness of

$$B = \frac{(1/2) \times 2^{16}}{0.0000005 \text{ sec}} = 6.5 \times 10^{10} \text{ photons/sec/pixel.}$$

The ability of a satellite to resolve details on a planet's surface of a particular size depends on the satellite's distance from the planet's surface, the wavelength of operation, and the diameter of the camera lens or mirror. When scientists design satellite 'imaging systems' all three factors have to be considered. Because the altitude of a satellite depends on the size of the rocket, and the resolution of the system depends on the camera size, the desired system can be expensive, so compromises often have to be made. The image above of Las Vegas was taken by the WorldView-2 satellite from an altitude of 770 km and has a resolution of 1.8-meters.

Problem 1 - Suppose survey of ground cover requires a minimum resolution of 1 meter in order to discriminate between the various types of biomes covering a planet's land area. If the orbit of the satellite is 250 kilometers, what is the angular resolution, R, required in arcseconds if R = 206265 L/H where L is the resolution size in meters and H is the altitude of the satellite in meters?

Problem 2 – The survey will be conducted at a wavelength, W, where chlorophyll has the strongest reflectivity in the visible spectrum. If the maximum resolution in arcseconds of the optical system, R, is related to the wavelength of operation W in meters, and the diameter of the camera mirror, D, in meters by R = 252000 W/D, how large will the camera mirror have to be, in centimeters, in order to obtain the resolution required from your answer to Problem 1 if W = 650 nanometers? (1 nanometer = 10^{-9} meters)

Problem 3 – The engineers designing this satellite system have determined that for the amount of money available, there is only enough to fabricate a mirror with a diameter of 10 centimeters, and that the camera will have to be piggy-backed on another satellite that has an orbit altitude of 350 kilometers. What will be the resolution of the compromise satellite system in meters?

Problem 1 - Suppose survey of ground cover requires a minimum resolution of 1 meter in order to discriminate between the various types of biomes covering a planet's land area. If the orbit of the satellite is 250 kilometers, what is the angular resolution, R, required in arcseconds if R = 206265 L/H where L is the resolution size in meters and H is the altitude of the satellite in meters?

Answer: H = 250 km x (1000 m/1km) = 250000 meters so R = 206265 x 1 meter/250000 meters and so **R= 0.8 arcseconds**.

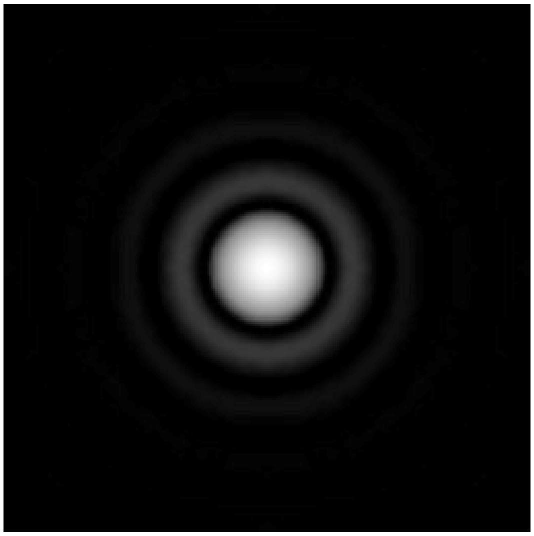
Problem 2 – The survey will be conducted at a wavelength, W, where chlorophyll has the strongest reflectivity in the visible spectrum. If the maximum resolution in arcseconds of the optical system, R, is related to the wavelength of operation W in meters, and the diameter of the camera mirror, D, in meters by R = 252000 W/D, how large will the camera mirror have to be, in centimeters, in order to obtain the resolution required from your answer to Problem 1 if W = 650 nanometers?

Answer: 1 nm = $1.0\text{x}10^{-9}$ meters so W = $6.5\text{x}10^{-7}$ meters and since R = 0.8 arcseconds, solving for D one obtains
D = 252000 ($6.5\text{x}10^{-7}$)/0.8 and so
D = 0.2 meters or 20 centimeters.

Problem 3 – The engineers designing this satellite system have determined that for the amount of money available, there is only enough to fabricate a mirror with a diameter of 10 centimeters, and that the camera will have to be piggy-backed on another satellite that has an orbit altitude of 350 kilometers. What will be the resolution of the compromise satellite system in meters?

Answer: By scaling from the equations for angular size and resolution, making the mirror smaller will reduce the resolution on the ground to R = 1 meter x (20 cm/10 cm) = 2.0 meters at the altitude of the older design (250 km). At the altitude of the new satellite, (350 km) this distance change will result in a resolution on the surface that is further degraded to R = 2.0 meters x (350 km/250 km) so **R = 2.8 meters** at the higher altitude.

The scientists will not be able to meet their science goals (1-meter resolution on the ground) unless they can lower the orbit (costs money) and increase the size for the mirror (costs more money!).



The bright spot is where most of the light energy falls, but it is surrounded by a large number of rings of light, called the diffraction pattern. The angle between the center of the main spot, and the first ring is given by the Airy Disk formula for θ.

Because light is a wave-like phenomenon, it causes interference when it is reflected and concentrated in an optical system. This pattern of interference makes it impossible to clearly see details that are smaller than this interference pattern.

There is a geometric relationship between the resolution of an imaging system and the wavelength at which it operates given by

$$\theta = 1.22\frac{\lambda}{D}$$

where θ is the resolution in units of radians, λ is the wavelength of the radiation in meters, and **D** is the diameter of the camera or telescope lens or mirror in meters.

Problem 1 - If 1 radian = 206265 arcseconds, what is the resolution formula in terms of arcseconds?

Problem 2 - A biologist wants to study deforestation with a satellite camera that has a pixel resolution of 10-meters/pixel, which at the orbit of the satellite corresponds to an angular resolution of 6 arcseconds. To measure the loss of plant matter, she detects the reflection by the ground of chlorophyll, which is the most intense at a wavelength of 700 nanometers (1 nanometer = 10^{-9} meters). What is the diameter of the camera lens that will insure this resolution at the orbit of the satellite?

Problem 3 – Construct a graph that shows the diameter of lens or mirror that is needed to obtain a resolution of 1 arcsecond from far-ultraviolet wavelengths of 200 nanometers to infrared wavelengths of 10 micrometers. From orbit, a human subtends an angle of 1 arcseconds, and emits infrared energy at a wavelength of 10 microns. How large would the camera have to be to resolve a human by his heat emission?

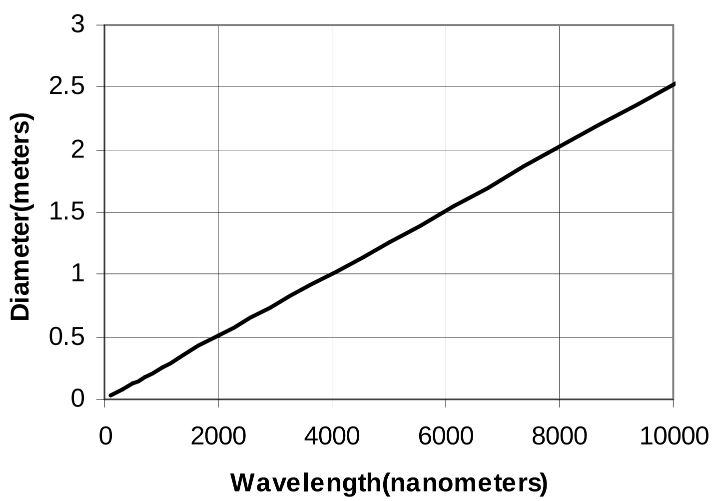
Problem 1 - If 1 radian = 206265 arcseconds, what is the resolution formula in terms of arcseconds?

Answer: $\theta = (1.22 \times 206265)\ \lambda/D$ so $\theta = 251{,}643\ \lambda/D$

Problem 2 - A biologist wants to study deforestation with a satellite camera that has a pixel resolution of 10-meters/pixel, which at the orbit of the satellite corresponds to an angular resolution of 6 arcseconds. To measure the loss of plant matter, she detects the reflection by the ground of chlorophyll, which is the most intense at a wavelength of 700 nanometers (1 nanometer = 10^{-9} meters). What is the diameter of the camera lens that will insure this resolution at the orbit of the satellite?

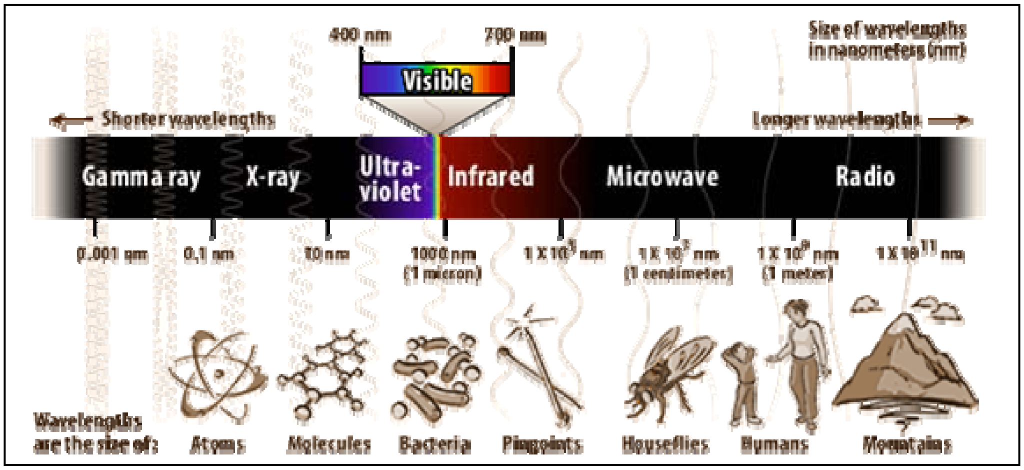
Answer: We want θ = 6 arcseconds. Then for $\lambda = 7.0\times10^{-7}$ meters we have
$D = 251{,}643 \times 7.0\times10^{-7}/6.0$
D = 0.03 meters or 3 centimeters.

Problem 3 – Construct a graph that shows the diameter of lens or mirror that is needed to obtain a resolution of 1 arcsecond from far-ultraviolet wavelengths of 100 nanometers to infrared wavelengths of 10 micrometers (10000 nanometers). From orbit, a human subtends an angle of 1 arcseconds, and emits infrared energy at a wavelength of 10 microns. How large would the camera have to be to resolve a human by his heat emission at a wavelength of 10 microns?



Answer: The graph suggests a mirror diameter of 2.5 meters!
Algebraically: $1.0 = 251643 \times (10^{4} \times 10^{-9}\ \text{meters})/D$ so D = 2.5 meters.

The Energy of Light



Light is defined as electromagnetic radiation, but 'EM radiation' comes in many other forms too. The X-rays that doctors use to probe your body carry enough energy to penetrate skin and bones, while infrared radiation does nothing more than warm you on a summer's day. EM radiation can be conveniently classified according to its frequency (or wavelength). But there is another way to classify it that helps physicists study its interaction with matter: the energy that it carries.

Light consists of particles called photons that travel at the speed of light - 300,000 km/s. Each of these particles carry a different amount of energy depending on their frequency. The formula that relates the energy in Joules (E) of a photon to its frequency in Hertz (F) is just

$$E = 6.6 \times 10^{-34}\ F \quad \text{Joules.}$$

Another form of this formula takes advantage of the fact that, for an electron, 1 'electron Volt' (eV) equals 1.6×10^{-19} Joules so

$$E = 4.1 \times 10^{-15}\ F \quad \text{electron-Volts}$$

Problem 1 - A photon of visible light has a frequency of 6×10^{14} Hertz, so how much energy does it carry in A)Joules? B) Electron volts.

Problem 2 - A radio station broadcasts at 100 megaHertz, how many eVs of energy do these radio-wave photons carry in the FM band?

Problem 3 - A dental X-ray machine uses X-rays that carry 150 keV of energy. A) What is the frequency of these X-rays? B) How many times more energy do they carry than visible light?

Problem 4 - If Frequency(Hertz) x Wavelength(meters) = 3×10^{8} meters/sec, re-write the second equation in terms of the wavelength of light given in units of nanometers (10^{-9} meters). Check your equation with your answer for the visible light photons in Problem 1.

Answer Key

Problem 1 - A photon of visible light has a frequency of 6×10^{14} Hertz, so how much energy does it carry in

A) Joules: $E = 6.6 \times 10^{-34} \times 6 \times 10^{14} =$ **4.0×10^{-19} Joules.**

B) Electron volts: $E = 4.1 \times 10^{-15} \times 6 \times 10^{14} =$ **2.5 electron Volts**

Problem 2 - A radio station broadcasts at 100 megaHertz, how many eVs of energy do these radio-wave photons carry in the FM band?

Answer: $E = 4.1 \times 10^{-15} \times 1 \times 10^{8} =$ **4.1×10^{-7} electron Volts**

Problem 3 - A dental X-ray machine uses X-rays that carry 150 keV of energy. A) What is the frequency of these X-rays? B) How many times more energy do they carry than visible light?

Answer: A) Use the energy formula in electron volts and solve for F to get $F = 2.4 \times 10^{14}$ E(volts) and so $F = 2.4 \times 10^{14}$ (150,000) = **3.6×10^{19} Hertz**.

B) From Problem 1, E = 2.6 eV, so the X-rays carry 150,000 eV/2.5 eV = **60,000 times more energy per photon.**

Problem 4 - If Frequency(Hertz) x wavelength(meters) = 3×10^{8} meters/sec. Re-write the second equation in terms of the wavelength of light given in nanometers (1.0×10^{-9} meters).

Answer: Let W be the wavelength in meters, then

$F = 3 \times 10^{8} / W$

But 1 meter = 10^{9} nanometers so, $F = 3 \times 10^{17} / W$

Then upon substitution for F,

$E = 4.41 \times 10^{-15} \times (3 \times 10^{17} / W)$ and we get

$$\mathbf{E = \frac{1230}{W(nm)} \ eV}$$

Visible light $F = 6 \times 10^{14}$ Hertz ; $W = 3 \times 10^{17} / F = 500$ nm, then E = 2.5 eV - check!

$$E = 6.63 \times 10^{-34} \ f \quad \text{Joules}$$

$$E = 4.1 \times 10^{-15} \ f \quad \text{eV}$$

where F = frequency in Hertz

The electric and magnetic properties of electromagnetic waves allow them to carry energy from place to place. The quantum character of this radiation allows this energy to be carried in packets called photons. A simple relationship connects the frequency of an EM photon with a specific amount of energy being delivered by each photon in the EM wave.

The two equations above make it convenient to compute the energy in two different energy units in common usage in the study of EM radiation, once the frequency of the EM wave is given in Hertz. The 'Joule' is a common measure of energy used in the so-called MKS or 'SI' system of units, however it is very cumbersome when measuring the minute energies carried by atomic systems. The 'electron Volt' is widely used by physicists, chemists and astronomers when referring to atomic systems measured under laboratory conditions or in the universe at-large.

For example, the frequency of visible 'yellow' light is about 520 trillion Hertz, so the energy carried by one 'yellow' photon is $E = 6.63 \times 10^{-34} (5.2 \times 10^{14}) = 3.4 \times 10^{-19}$ Joules, or from the second equation we would get 2.1 eV.

Problem 1 - Although these formula refer to the frequency of EM radiation, we can also re-write them to refer to the wavelength of EM radiation using the familiar rule that f = c/L where c is the speed of light in meters/sec and L is the wavelength in meters. If c = 300 million meters/sec, what are the two new energy laws expressed in terms of wavelength in meters?

Problem 2 - How much energy, in eV, is carried by each photon in the FM radio band if the frequency of your station is 101.5 MHz?

Problem 3 - How much energy, in eV, is carried by a single photon in the infrared band if the wavelength of the photon is 25 microns (1 micron = 1.0×10^{-6} meters)?

Problem 4 - How much energy is carried by a single gamma-ray photon if its wavelength is the same as the diameter of an atomic nucleus (1×10^{-14} meters) computed in A) Joules? B) eV?

Answer Key

Problem 1 - Although these formula refer to the frequency of EM radiation, we can also re-write them to refer to the wavelength of EM radiation using the familiar rule that f = c/L where c is the speed of light in meters/sec and L is the wavelength in meters. If c = 300 million meters/sec, what are the two new energy laws expressed in terms of wavelength in meters?

Answer: From $E = 6.63 \times 10^{-34} f$ we have, with substitution,

$E = 6.63\times10^{-34} (3 \times 10^{8} / L)$ or

$\mathbf{E = 2.0 \times 10^{-25} / L}$ **Joules.**

From $E = 4.1 \times 10^{-15} f$ we have, with substitution

$E = 4.1\times10^{-15} (3 \times 10^{8} / L)$ or

$\mathbf{E = 1.23 \times 10^{-6} / L}$ **eV**.

Problem 2 - How much energy, in eV, is carried by each photon in the FM radio band if the frequency of your station is 101.5 MHz?
Answer: The information is stated in terms of frequency, so use the fact that 1MHz = 1 million Hertz and so $E = 4.1\times10^{-15} (101.5 \times 1 \times 10^{6}) = \mathbf{4.2 \times 10^{-7}}$ **eV**.

Problem 3 - How much energy, in eV, is carried by a single photon in the infrared band if the wavelength of the photon is 25 microns (1 micron = 1.0×10^{-6} meters)?
Answer: Use the relationship for E in terms of wavelength to get
$E = 1.32 \times 10^{-6} /(25.0 \times 1.0\times10^{-6}) =$ **0.053 eV**.

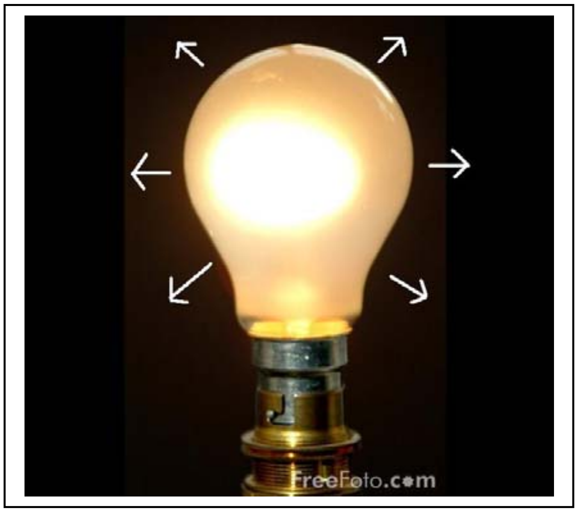
Problem 4 - How much energy is carried by a single gamma-ray photon if its wavelength is the same as the diameter of an atomic nucleus (1×10^{-15} meters) computed in A) Joules? B) eV?

Answer: A) $E = 2.0\times10^{-25} /1.0\times10^{-15} = \mathbf{2 \times 10^{-10}}$ **Joules**.

B) $E = 1.32 \times 10^{-6}/(1.0\times10^{-15}) = \mathbf{1.32 \times 10^{9}}$ **eV**

or 1.32 giga eV (also abbreviated as 1.32 GeV)

The Flow of Light Energy: Power



Because electromagnetic waves carry energy from place to place, there are a number of different ways to express how rapidly, and how intensely, this energy is being moved for a given source. The first of these has to do with the transmitted power, measured in watts.

One watt is equal to one Joule of energy being transmitted, or received, each second. The watt is the same unit that you use when you select light bulbs at the store, or measure the amount of work that something is doing. (A 350 horse-power car engine delivers 260,000 watts!).

Problem 1 - At a frequency of 100 million Hertz (100 MHz) a single photon of electromagnetic energy carries about 6×10^{-26} Joules of energy. If 10 trillion of these '100 MHz' photons are transmitted every 3 seconds, how many watts of electromagnetic energy are being moved?

Problem 2 - On your FM dial, the radio signal for station WXYZ is 101.5 MHz. The weakest signal that your radio can detect at this frequency is 10^{-12} watts. How many photons per second can you detect at this sensitivity level if the photon energy, in Joules, is given by $E = 6.63 \times 10^{-34} f$, where f is the frequency in Joules?

Problem 3 - The faintest star you can see in the sky has an apparent magnitude of +6.0 in the visual band. The light from such a star carries just enough power for your retinal 'rods' to detect the star. If the frequency of visible light is 520 trillion Hertz, and the minimum power you can detect is about 3×10^{-17} watts, how many photons are being detected by your retina each second at this light threshold?

Problem 4 - Staring at the sun can have harmful consequences, including blindness! If the sun delivers 0.01 watts of energy to your retina, and the frequency of a visible light photon is 520 trillion Hertz, how many photons per second reach your eye?

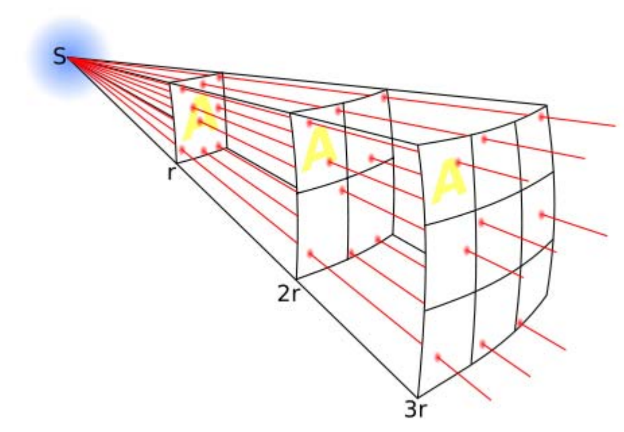
Problem 1 - At a frequency of 100 million Hertz (100 MHz) a single photon of electromagnetic energy carries about 6×10^{-26} Joules of energy. If 10 trillion of these '100 MHz' photons are transmitted every 3 seconds, how many watts of electromagnetic energy are being moved?
Answer: We know how much energy is carried by a single photon, so multiplying this by the number of photons gives the total energy: $E = 6 \times 10^{-26}$ Joules/photon x (10×10^{12} photons) = 6×10^{-13} Joules every 3 seconds. Since the definition of a watt is 1 Joule/1 second, we have the power transmitted as $P = 6 \times 10^{-13}$ Joules / 3 seconds = **2×10^{-13} Watts**.

Problem 2 - On your FM dial, the radio signal for station WXYZ is 101.5 MHz. The weakest signal that your radio can detect at this frequency is 10^{-12} watts. How many photons per second can you detect at this sensitivity level if the photon energy, in Joules, is given by $E = 6.63 \times 10^{-34} f$, where f is the frequency in Joules?
Answer: Each photon carries $E = 6.63 \times 10^{-34} (101.5 \times 10^{6}) = 6.7 \times 10^{-26}$ Joules/photon. Since the minimum detected power is 10^{-12} watts, we re-write this as 10^{-12} Joules/second. Then we do the conversion: $N = 10^{-12}$ Joules/second x (1 photon/6.7×10^{-26} Joules) = 15 trillion photons/second.

Problem 3 - The faintest star you can see in the sky has an apparent magnitude of +6.0 in the visual band. The light from such a star carries just enough power for your retinal 'rods' to detect the star. If the frequency of visible light is 520 trillion Hertz, and the minimum power you can detect is about 3×10^{-17} watts, how many photons are being detected by your retina each second at this light threshold?
Answer: A single photon carries $E = 6.63 \times 10^{-34}$ (520 trillion) = 3.4×10^{-19} Joules.
The threshold power is 3×10^{-17} watts, or 3×10^{-17} Joules/sec, so
$N = 3 \times 10^{-17}$ Joules/sec x (1 photon/3.4×10^{-19} Joules) = **88 photons/sec**.

Problem 4 - Staring at the sun can have harmful consequences, including blindness! If the sun delivers 0.01 watts of energy to your retina, and the frequency of a visible light photon is 520 trillion Hertz, what is the threshold level for retinal damage in photons/sec?
Answer: A single photon carries $E = 6.63 \times 10^{-34}$ (520 trillion) = 3.4×10^{-19} Joules.
The threshold power is 0.01 watts, or 0.01 Joules/sec, so
N = 0.01 Joules/sec x (1 photon/3.4×10^{-19} Joules) = **2.9×10^{16} photons/sec**.

Note: The range of the human eye is then a factor of $2.9 \times 10^{16}/88$ = 329 trillion!



As light is emitted (S), it fans-out into space in every direction, so that its intensity is diluted. The Inverse Square Law describes how the intensity changes with increasing

The power produced by a source of electromagnetic energy is measured in watts. A second important quantity that indicates how much power is flowing through a surface is called the flux. The units for this quantity are Watts per square meter (W/m^2).

Why do we need TWO different quantities to describe light energy? Because although 'power' tells us how much energy the source is producing each second, flux tells us how bright the source will be at some distance away. A 100-watt bulb looks very bright up close, but 100 meters away it is very dim!

Problem 1 - Flux is calculated by dividing the power of a light source in watts, by the area that the light passes through in square meters. Example, 100 watts flows through an area of 10 square-meters has a flux, F, of 100 watts/10 m^2 = 10 watts/m^2. Because most light sources emit the power in equal directions, the area, A, that it passes through is just the surface area of a sphere located at the distance, R, from the source. If $A = 4\pi R^2$, and P is the power of the electromagnetic radiation from the source,

A) What is the formula for the flux at a distance R where R is in meters and P is in watts?

B) Explain how this implies the Inverse-Square Law for radiation?

Problem 2 - The sun emits about 4.0×10^{26} watts of electromagnetic energy. If its distance from Earth is 150 million kilometers,

A) What is the flux of EM energy at Earth?

B) A home owner wants to install a solar-electric system on his roof. How many square meters of solar panels will have to be installed to generate 1000 watts of electricity if the system is 100% efficient?

Problem 1 - Flux is calculated by dividing the power of a light source in watts, by the area that the light passes through in square meters. Example, 100 watts flows through an area of 10 square-meters has a flux, F, of 100 watts/10 m^2 = 10 watts/m^2. Because most light sources emit the power in equal directions, the area, A, that it passes through is just the surface area of a sphere located at the distance, R, from the source. If $A = 4\pi R^2$, and P is the power of the electromagnetic radiation from the source, A) what is the formula for the flux at a distance R where R is in meters and P is in watts? B) Explain how this implies the Inverse-Square Law for radiation?

Answer: A) Flux = Power/Area so:

$$F = \frac{P}{4\pi R^2} \quad \text{and so} \quad F = 0.079\frac{P}{R^2}$$

B) As you double the distance from the source, the flux of energy from the source decreases by a factor of $2^2 = 4$, which is the Inverse-Square Law.

Problem 2 - The sun emits about 4.0 x 10^{26} watts of electromagnetic energy. If its distance from Earth is 150 million kilometers, A) what is the flux of EM energy at Earth? B) A home owner wants to install a solar-electric system on his roof. How many square meters of solar panels will have to be installed to generate 1000 watts of electricity if the system is 100% efficient?

Answer: A) 150 million kilometers equals 1.5 x 10^{11} meters, so the surface area is A = $4\pi(1.5 \times 10^{11})^2$ = 2.83 x 10^{23} meters^2. Then F = 4.0 x10^{26} watts/2.83x10^{23} m^2 = 1,415 watts/m^2 at Earth.

B) If it is 100% efficient, that means that 1 watt of electromagnetic energy from the sun that falls on the roof, will be converted into exactly 1 watt of electrical power. The sun produces 1,415 watts/meter^2 of flux at the roof, so to generate 1,000 watts of electricity, the home owner will need an area of 1,000 watts x (1 meter^2/1,415 watts) = **0.7 square meters** of solar panels.

There are many situations that require knowing the flux of electromagnetic radiation. Simply put, you buy a light bulb according to its 'power' but it is the flux from this bulb that determines if you have enough light to read-by!

The following problems are rather common applications of how electromagnetic flux calculations are used in a variety of different ways. Engineers and scientists have to doe these calculations when designing spacecraft, or new telescopes with which to study distant, faint, objects in the universe.

Problem 1 - The geostationary satellite, DirecTV-10, is located 35,000 km from Earth and broadcasts TV programs on Transponder 11 at a power of 240 watts to a ground station on Earth. A homeowner uses a satellite dish with a diameter of 18-inches to 'capture' some of the signal at a Ka-band frequency of 35 GHz. How many watts of signal will his satellite dish be able to deliver to the electronics that detect the signal and create a TV image?

Problem 2 - The Hubble Space Telescope has a mirror diameter of 2.4 meters. It can detect stars that generate about 10 photons per second in the digital imager. What is the minimum flux of visible light from the star at a wavelength of 500 nanometers that can be detected in this way?

Problem 3 - The Voyager 1 spacecraft is currently located about 16.6 billion kilometers from Earth. Its 20-watt radio transmitter operates at X-band at a frequency of 8.4 Ghz and its signals are received on Earth by the 70-meter dish of the Goldstone Deep Space Network. The data is sent as a binary string at a rate of 160 bits/sec.

A) What is the power received from this transmitter by the Goldstone dish?

B) How many photons does one 'bit' occupy?

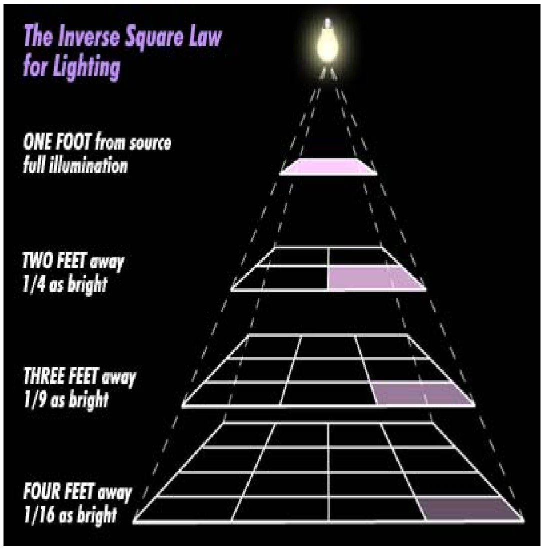
Problem 1 - Answer: First we have to calculate how much flux is arriving at Earth at that distance.
Area = $4\pi R^2$ = 4 (3.141)(35,000 x 1000)^2 = 1.5x10^{16} meters^2.
Then F = 240 watts/1.5x10^{16} meters^2 = 1.6 x 10^{-14} watts/meter^2.
Now we calculate how much of this flux is captured by the satellite dish. The radius of the dish in meters is 1/2 (18 inches)x(0.0254 m/in) = 0.23 meters, so the dish area is about A = π (0.23)^2 = 0.17 m^2.
Then the intercepted satellite power in this channel is
P = 1.6 x 10^{-14} watts/meter^2 x 0.17 m^2 so **P = 2.7 x 10^{-15} watts**
(Note: Because the signal is 'beamed' into an area that is only 1/1000 the area of the sky, the actual received power is 1000 times larger, or P = 2.7 x 10^{-12} watts)

Problem 2 - Answer: In this case we have to work backwards! The photon rate is 10 photons/second. To find what power this corresponds to we multiply by the energy carried by each photon using E = 2.0x10^{-25}/L where L is the wavelength in meters. Since L = 500 x 10^{-9} meters, we have E = 4.0 x 10^{-19} Joules. So the critical detection power is just P = 10 photons/sec x (4.0x10^{-19} Joules/ 1 photon) = 4.0 x 10^{-18} watts. The last step is to calculate the flux by dividing the power by the area of the Hubble Space Telescope mirror. The area is A = π (2.4/2)^2 = 4.5 meter^2 so the critical flux of radiation from the star is just F = 4.0 x 10^{-18} watts /4.5 meter^2 so
F = 8.9 x 10^{-19} watts / meter^2

Problem 3 -
Answer: A) The surface area of a sphere 16.6 billion kilometers (1.66 x 10^{12} meters) in radius is A = $4\pi R^2$ = 4 (3.141)(1.66 x 10^{12})^2 = 3.4x10^{25} meters^2. The flux at Earth is then F = P/A = 20 watts/3.4x10^{25} meters^2 so F = 5.9 x10^{-25} watts/meters^2. The power 'captured' by the Goldstone dish is then P = F x A so
P = 5.9 x10^{-25} watts/meters^2 x π (70/2)^2 or **P = 2.3 x10^{-21} watts**. (Note: the signal from the spacecraft is actually 'beamed' to Earth so that it is concentrated into an area on the sky only 1/100,000 the full sky area. That means at Earth the signal is 100,000 greater than the P calculated above or about P = 2.3 x10^{-16} watts)

B) The energy per photon at this frequency is E = 6.63 x 10^{-34} (8,400,000,000) = 5.5 x 10^{-24} Joules. The received signal is P = 2.3 x10^{-21} Joules/sec, so the number of photons detected is N = 2.3 x10^{-21} Joules/sec x (1 photon / 5.5x10^{-24} Joules) = 420 photons/sec. The telemetry rate is 160 bits/sec, so there are
n = 420 photons/sec x (1 sec/160 bits) and so **n is about 3 photons per bit**.
(Note: The number is actually higher than this by about 100,000 times because the energy from Voyager 1 is 'beamed' as described in A).

The Inverse-Square Law - I



Suppose a source of light emits 100 watts of light power in all directions. The brightness of the light source is determined by the amount of light power that passes through the surface of a sphere centered on the light source at a distance of d meters according to the formula

$$B = \frac{P}{4\pi d^2}$$

For example, if you were standing at a distance of
d = 2.82 meters from a light source with
P = 100 watts, you can easily show that
B = 1.0 watts/meter^2.

Problem 1 – Which light source is the brightest: Source-A with P = 1000 watts at a distance of 3.5 meters or Source-B with P = 3 x 10^{26} watts at a distance of d = 1.5 x 10^{11} meters?

Problem 2 – An astronomer wants to create a light source in her lab that mimics the brightness of the star Sirius, which has a brightness of 1.1 x 10^{-7} watts/meter^2. How far will she have to place a 5-watt flashlight lamp from her measuring instrument so that its brightness equals Sirius?

Problem 3 - An amateur astronomer wants to find a 1000-watt street light somewhere in the city below the mountain he is observing from, that has the same brightness as the star Sirius, which he wants to photograph with his camera. If Sirius has a brightness of 1.1 x 10^{-7} watts/ meter^2, how far from the camera does the street light have to be to mimic the faint star in the same photograph?

Problem 1 – Which light source is the brightest: Source-A with P = 1000 watts at a distance of 3.5 meters or Source-B with P = 3 x 10^{26} watts at a distance of d = 1.5 x 10^{11} meters?

Answer:
Source-A : B = $1000/4\pi(3.5)^2$ = 6.5 watts/m^2
Source-B : B= 4×10^{26} watts/$4\pi(1.5 \times 10^{11})^2$ = 1400 watts/$meter^2$

so **Source-B is brig**htest!

Problem 2 – An astronomer wants to create a light source in her lab that mimics the brightness of the star Sirius, which has a brightness of 1.1 x 10^{-7} Watts/$meter^2$. How far will she have to place a 5-watt flashlight lamp from her measuring instrument so that its brightness equals Sirius?

Answer:
P^* = 1.1 x 10^{-7} w/m^2.
But P^* = 5 watts/$4\pi d^2$

So **d = 1,900 meters**.

Problem 3 - An amateur astronomer wants to find a 1000-watt street light somewhere in the city below the mountain he is observing from, that has the same brightness as the star Sirius, which he wants to photograph with his camera. If Sirius has a brightness of 1.1 x 10^{-7} watts/$meter^2$ how far from the camera does the street light have to be to mimic the faint star in the same photograph?

$1.1 \times 10^{-7} = 1000/4\pi d^2$

so **d = 27 kilometers**.

One of the most common uses for the inverse-square law is in the placement of flood lights used in a photography studio.

To avoid shadows in the wrong places, photographers mostly figure out the lamp placement by trial and error, but we can easily estimate where lamps ought to be placed using a scale-model of the studio.

A photographer has three flood lights in his studio, which has a floor marked as a Cartesian 'X-Y' coordinate grid with the Subject sitting at the Origin. The flood lights are located as follows: Flood light A emits 100 watts at a location of (-1 meters, +2 meters); Flood light B emits 75 watts at (+2 meters, +3 meters) and Flood light C emits 150 watts at (+1 meters, +4 meters).

Problem 1 – Using the distance formula $d^2 = x^2 + y^2$, how far are the three flood lights from the Subject?

Problem 2 - Using the Inverse-Square Law, how bright are the three flood lights at the location of the Subject?

Problem 3 – What is the total brightness of the three flood lights at the location of the Subject?

Problem 4 – If the three flood lights were replaced by a single flood light located at (+1 meters, +2 meters), what would the power rating of the new light have to be to equal the brightness of the other three flood lights at the Subject's location?

Problem 1 – Using the distance formula $d^2 = x^2 + y^2$, how far are the three flood lights from the Subject?

Answer: **A: 2.2 meters; B = 3.6 meters; C = 4.1 meters**

Problem 2 - Using the Inverse-Square Law, how bright are the three flood lights at the location of the Subject?

Answer: A = $100 / 4\pi(2.2)^2$ = **1.6 watts/m^2**

B = $75/4\pi(3.6)^2$ = **0.46 watts/m^2**

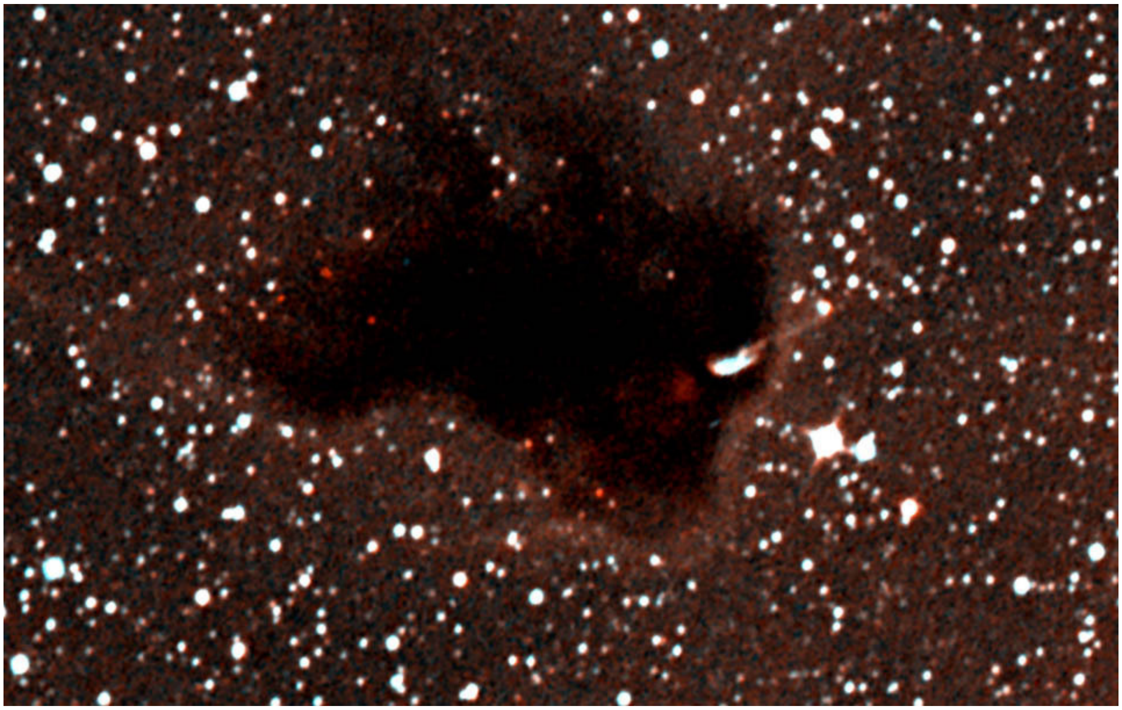
C = $150 / 4\pi(4.1)^2$ = **0.71 watts/m^2**

Problem 3 – What is the total brightness of the three flood lights at the location of the Subject? Answer: 1.6 + 0.46 + 0.71 = **2.8 watts/m^2**

Problem 4 – If the three flood lights were replaced by a single flood light located at (+1 meters, +2 meters), what would the power rating of the new light have to be to equal the brightness of the other three flood lights at the Subject's location?

Answer: The distance of this flood light from the subject is $d = (1^2+2^2)^{1/2} = 2.2$ meters. Then 2.8 watts/m^2 = $P/4\pi(2.2)^2$ so solving for P we have **P = 170 watts**.

The Inverse-Square Law - III



Interstellar dust clouds, like Bok Globule HH 46 shown in the above Spitzer Space telescope image, dim light from background stars and make them appear to be fainter and further away than they actually are. This led to a problem when early astronomers tried to map out the extent of the Milky Way galaxy. Dark clouds were mistaken for regions where stars were absent. The dust also made nearby stars appear farther away. Let's see how this works!

An astronomer compares the brightness of two identical stars. Star-A emits 4×10^{26} watts and is located 1,000 light years from our Sun where 1 light year = 9.6×10^{12} kilometers. Star-B emits 4×10^{26} watts of light and is also located 1,000 light years from the Sun, but there is an invisible dust cloud in the direction of Star-B. The dust reduces the brightness of Star-B by a factor of 1/25.

Problem – How far does Star-B appear to be from Earth?

Problem – How far does Star-B appear to be from Earth?

Answer: Although the stars emit the same light energy, the Inverse Square Law says that if a body appears 1/25 as bright, it is 5 times further away, so Star B will appear to be at a distance of 5,000 light years rather than its true distance of 1,000 light years. Another way to show this is as follows:

1,00 light years equals 9.6×10^{15} meters.

P(Star-A) = 4×10^{26} watts and the brightness of Star-A is

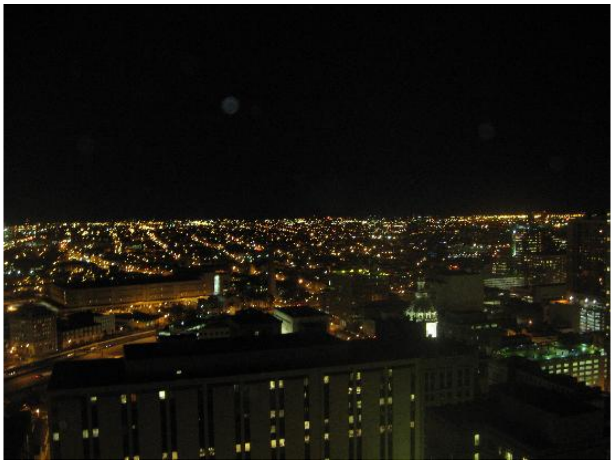
Brightness = 4×10^{26} watts / $4 \pi (9.6 \times 10^{15}$ meters$)^2$ = 3.5×10^{-7} watts/m^2.

But the identical star, Star-B appears to be 1/25 as bright so B = 1.4×10^{-8} w/m^2. Since $B = P/4\pi d^2$, we have

$1.4 \times 10^{-8} = 4 \times 10^{26}$ watts / $4\pi d^2$ and solving for the distance d we get

d = **4.8×10^{16} meters** or **5,000 light years**. (rounding needed for 1 sig fig accuracy)

Star-B appears to be nearly 5 times farther away than Star-A because of the dust absorption effect!


Although each street lamp has the same power, measured in watts, their apparent brightness to the eye differs depending on their distance.

The inverse-square law can be used 'backwards' to determine how powerful, P, a light source is, based on its distance, D, and apparent brightness, B.

$$P = 4\pi d^2 B$$

This principle is very important in astronomy when studying unknown distant objects to determine whether they are normal stars or other exotic objects.

Problem 1 - A light source in the laboratory has a brightness of B = 0.001 watts/meter^2 at a distance of 10 meters. What is the total power, P, of this light in watts?

Problem 2 – Two light sources are observed on the summits of two different mountains. Source-A is located 10 kilometers from the Observer and has a brightness of B=8.0 x 10^{-8} watts/m^2 while Source-B is located 35 kilometers from the Observer and has a measured brightness of B=3.3 x 10^{-9} watts/m^2. Which light source is the more powerful?

Problem 1 - A light source in the laboratory has a brightness of 0.001 watts/meter^2 at a distance of 10 meters. What is the total power of this light in watts?

Answer: $P = 4\ (3.14)\ (10)^2\ 0.001$ so **P = 1.3 watts**.

Problem 2 – Two light sources are observed on the summits of two different mountains. Source-A is located 10 kilometers from the Observer and has a brightness of 8.0×10^{-8} watts/m^2 while Source-B is located 35 kilometers from the Observer and has a measured brightness of 3.3×10^{-9} watts/m^2. Which light source is the more powerful?

Answer: Source-A: $P = 4\pi\ (10{,}000 \text{ meters})^2\ (8.0 \times 10^{-8}) =$ 100 watts.
Source-B: $P = 4\pi\ (35{,}000 \text{ meters})^2\ (3.3 \times 10^{-9}) =$ 50 watts.
Source-A is the most powerful (e.g. most luminous).

Light can be amplified much like other 'flowing' substances in nature. A good analogy is the flow of raindrops from a storm cloud, which can be amplified by using a funnel.

Although stars are very faint, the brightness of these stars can be concentrated using a variety of telescopes that have large collecting areas. The basic formula is

$$P = B \times A$$

Where P is the collected star power in watts, B is the star's brightness in watts per square meter, and A is the area, in meters, of the telescope's mirror, lens or dish.

The pictures how the 64-meter Parkes Radio Telescope in Australia and the 2.4-meter Hubble Space Telescope mirror.

Problem 1 – NASA's Deep Space Network of radio telescopes includes a 70-meter dish to detect the faint signals from spacecraft beyond the orbit of Jupiter. The Voyager-1 spacecraft uses a P=20-watt transmitter at a distance of D=17 billion kilometers from Earth to transmit its measurements.

A) Using the inverse-square law, what is the brightness, B, of this signal at Earth in watts/meter^2?

B) How many watts of power will the 70-meter dish be able to collect at Earth?

Problem 2 – An astronomer wants to do a SETI survey to search for any signs of advanced civilizations in the Milky Way or beyond that emit 1% of our sun's power, P, as pure radio energy. If P = 4×10^{24} watts, and the SETI Allen Telescope Array has a surface area of 10,000 meters^2 and a sensitivity of B = 10^{-12} watts/meter^2 ,what is the farthest distance, in light years, that such a civilization could be in order for it to be detected by SETI? (1 light year = 9.6×10^{15} meters)

Problem 1 – NASA's Deep Space Network of radio telescopes includes a 70-meter dish to detect the faint signals from spacecraft beyond the orbit of Jupiter. The Voyager-1 spacecraft uses a P=20-watt transmitter at a distance of D=17 billion kilometers from Earth to transmit its measurements. A) Using the inverse-square law, what is the brightness, B, of this signal at Earth in watts/meter^2? B) How many watts of power will the 70-meter dish be able to collect at Earth?

Answer: A) $B = 20 / 4(3.14)(1.7\times10^{13})^2 = 5.5 \times 10^{-27}$ watts/meter^2
B) $P = B \times A$ so $P = 5.5 \times 10^{-27}$ watts/meter^2 x $(3.14)(35)^2$ meters^2
P = 2.3×10^{-23} watts.

Problem 2 – An astronomer wants to do a SETI survey to search for any signs of advanced civilizations in the Milky Way or beyond that emit 1% of our sun's power, P, as pure radio energy. If $P = 4\times10^{24}$ watts, and the SETI Allen Telescope Array has a surface area of A=10,000 meters^2 and a sensitivity of $Ps = 1.0\times10^{-8}$ watts, what is the farthest distance, in light years, that such a civilization could be in order for it to be detected by SETI? (1 light year = 9.6×10^{15} meters)

Answer: First we calculate the minimum detectable brightness, Bm, of the Allen Array which is just Bm = P / A so
$Bm = (1.0\times10^{-8})/(10,000) = 1.0\times10^{-12}$ watts/meter^2.

Now we calculate the distance at which the alien civilization has this brightness at Earth. Since from the Inverse-square Law we have $B = P/4\pi d^2$
Solving for d and inserting the stated values for P and Bm we have

$d = [4\times10^{24} \text{ watts}/4(3.14)(1.0\times10^{-12})]^{1/2}$ so
$d = 5.6\times10^{17}$ meters.

In light years,
$d = 5.6 \times 10^{17}$ meters x (1 light year / 9.6×10^{15} meters)
= **58 light years!**

Solar cells, also called photovoltaic (PV) cells, generate electricity as the photons from the sunlight interact with electrons in the PV cells to generate a current of electricity.

The amount of electricity produced depends on the amount of sunlight that falls on the panel during each day, from sunrise to sunset, and during the year as the seasons vary.

Depending on how the PV cell is tilted, the amount of solar flux, measured in watts/meter^2, varies as well.

Commercially-bought solar panels are about 20% efficient in converting solar electromagnetic energy into electrical energy. Solar panels used in NASA satellites have much higher efficiencies of about 40% using advanced, and very expensive, technology.

The cost for a commercial solar 'photovoltaic' panel is about $10.00 per watt. This can be lowered to $8.00 per watt if the home owner takes advantage of various rebates offered by State and Federal government 'green' programs.

Problem 1 - A homeowner wants to go 'off grid' by installing a solar panel array that tracks the sun, and delivers 2 kilowatts of electricity. If the solar electromagnetic flux at Earth's surface on a clear day is about 1,000 watts/meter^2. How many square meters of solar panel will he need to meet his energy needs if the conversion efficiency is 20%, and how much will it cost, if all rebates are used?

One of the most important factors that affects how much solar energy falls on a panel's area is the tilt of the panel with respect to the sun's direction. Another way to think about this is the 'slant' of the sunlight. As you know, we have cooler temperatures in the winter because the sun is 'low' on the horizon and the rays of the sun fall on a given area of the Earth with a very high slant. This means that a given amount of sunlight falls on more ground area, and so less solar power is available for heating. This phenomenon applies to seasonal changes, but also applies to changes in the latitude. At the equator, sunlight falls more directly on a surface than at higher latitudes. These relationships also apply to solar panels. We can mathematically express this with the formula $I = I0 \cos(\theta)$ where I0 is the maximum possible 'clear day' solar power, and θ is your latitude. During the June solstice at the equator, I0 = 1000 watts/meter^2.

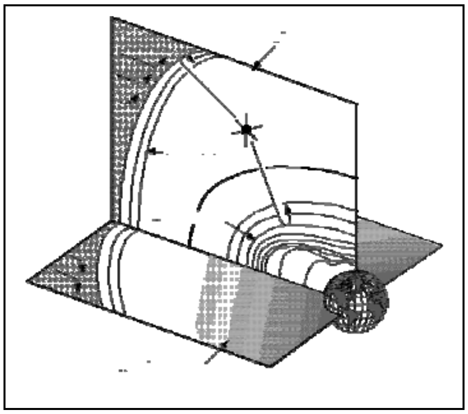
Problem 2 - A solar panel is located at the latitude of Denver, Colorado (39^d 44'). A) What is the maximum solar energy flux at this latitude? B) How much roof area will the solar system in Problem 1 occupy in Denver taking into account the latitude of Denver?

Problem 1 - A homeowner wants to go 'off grid' by installing a solar panel array that tracks the sun, and delivers 2 kilowatts of electricity. If the solar electromagnetic flux at Earth's surface on a clear day is about 1,000 watts/meter^2. How many square meters of solar panel will he need to meet his energy needs if the conversion efficiency is 20%, and how much will it cost if all rebates are used?
Answer: the fact that the panels will 'track' the sun means that the sunlight will always strike the panels exactly vertically, which will result in the maximum possible solar energy passing through the panel's area. Area = 2 kilowatts x (1 meter^2/1,000 watts) = 2 meters^2, but the panel is only 20% efficient, so he needs 5x this area or **10 meters^2**. The cost will be 2,000 watts x ($8.00/watt) = **$16,000**.

Problem 2 - A solar panel is located at the latitude of Denver, Colorado (39^d 44'). A) What is the maximum solar energy flux at this latitude? B) How much roof area will the solar system in Problem 1 occupy in Denver taking into account the latitude of Denver?

Answer A) I = 1,000 cos(39.7^o) = 1,000 (0.77) = 770 watts/m^2.
B) The estimate for the solar panel area needed to generate 2 kilowatts of electricity was based upon capturing 1,000 watts/m^2 of sunlight. Because of the effect of latitude, which causes the solar rays to be slanted as they strike the solar panel, the homeowner will have to use a larger surface area of panels to capture the same amount of solar power because the amount of sunlight only produces 770 watts/m^2 at this latitude, not 1,000 watts/m^2. So, he will need an area 1,000/770 = 1.3 larger. Instead of an area 2 meters^2, he will need to cover a roof area of **2.6 meters^2**.

Note: 'Insolation' is a common term used in sizing solar electric PV systems. It is a measure of the average daily flux of solar energy at a particular location of Earth, taking into account the latitude effect, and the length of day. In Arizona the insolation is about 7.0 kilowattHours per meter^2 per day. Since 1 day is 24 hours long, this implies a usable solar flux of about 7,000/24 = 291 watts/meter^2 averaged over the entire year to allow for seasonal changes. In Maine, the insolation is about 4.0 kilowattHours/meter^2/day so we get about 166 watts/meter^2. The insolation number is what is used to calculate how many solar panels are needed to generate the desired wattage. Example, for a 2 kilowatt system in Arizona with 20% efficient solar cells, you will need 2,000 /(0.2 x 291) = 34 meters^2 of solar panels! A typical panel produces 130 watts over an area of 1 meter^2 and costs about $1,000, so a 34 meter^2 system will cost about $34,000.



The IMAGE spacecraft (shown as the 'star' in the figure) contains an instrument called the Radio Plasma Imager (RPI). This instrument sends out a powerful pulse of radio energy (the fish hook-shaped lines) at frequencies from 3,000 to 3 million cycles per second (Hertz). When the echos from these pulses are later received by the instrument, they can be analyzed to find the location and density of the plasma that reflected them.

Clouds of plasma in space have an interesting property: When radio waves reflect off of them, the radio frequency of the reflected signal depends on the density of the cloud! The formula that relates the reflection frequency, F, to the density, N, is given by

$$F = 9000\sqrt{N}$$

The unit of frequency is Hertz (cycles per second) and the unit for the density of the cloud is electrons per cubic centimeter.

In this problem, you will use some of the same methods and equations that IMAGE scientists use, to study the properties of plasma clouds near Earth. Although the properties of these clouds, and their locations, have been 'made-up' for this problem, your analysis of them will be similar to the methods employed by IMAGE scientists using real data.

With the formula above, solve for the density, N, and complete the table entries below.

Table showing radio properties of plasma clouds

Location	Direction (degrees)	Distance in Earth Radii (Re)	Reflection Frequency (Hertz)	Density (electrons per cc)
1	300	1.0	284,000	995
2	315	2.5	201,000	
3	350	6.5	12,600	
4	45	4.5	20,100	
5	60	3.9	25,500	
6	90	4.1	28,500	
7	120	4.0	25,500	
8	135	5.5	20,100	
9	215	7.2	12,600	
10	230	3.5	220,000	
11	270	1.2	348,000	

The equation solved for N is:

$$N = (F/9000)^2$$

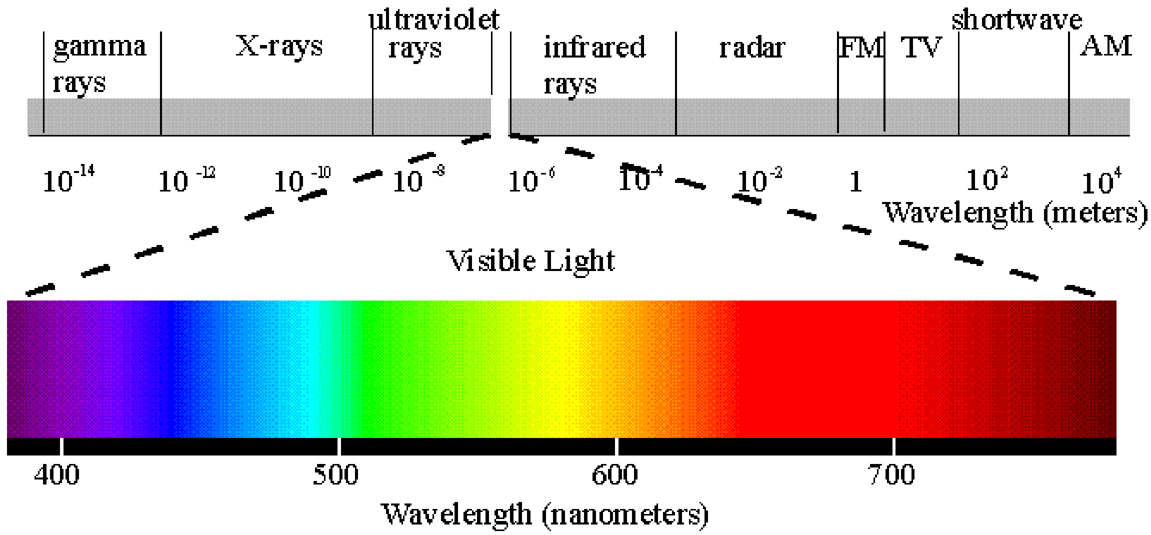
Lets' calculate the answer for N for the first location in Line one of the table. Divide the reflection frequency in Column 4 by 9,000. For example, at Location 1, 284,000/9,000 = 31.55. Find the square of this number: 31.55 x 31.55 = 995.4. Round this number to the nearest whole number, therefore the density of the cloud, N, at Location 1 is 995 electrons per cubic centimeter. Students will enter this answer in Column 5.

Complete the rest of the entries to Column 5 in similar fashion.

Location	Direction (degrees)	Distance in Earth Radii (Re)	Reflection Frequency (Hertz)	Density (electrons per cc)
1	300	1.0	284,000	995
2	315	2.5	201,000	**498**
3	350	6.5	12,600	**2**
4	45	4.5	20,100	**5**
5	60	3.9	25,500	**8**
6	90	4.1	28,500	**10**
7	120	4.0	25,500	**8**
8	135	5.5	20,100	**5**
9	215	7.2	12,600	**2**
10	230	3.5	220,000	**597**
11	270	1.2	348,000	**1495**

For extra credit, students can use a protractor, compass and a 4-quadrant graph paper with axis marked in intervals of 1.0 Re, to plot each of the 11 points. A red and blue crayon can be used to code each point as a high-density region (red = 400 to 1500 electrons/cc) or a low-density region (blue = 1 to 10 electrons/cc).

The low-density regions are farther from Earth and represent the plasma, which fills Earth magnetic field. The high-density regions is closer to Earth and the satellite, and corresponds to the plasmasphere region of the upper atmosphere.



We have looked at individual parts of the electromagnetic spectrum. Now let's look at the entire spectrum all at once! Because the wavelength ranges are so vast, we can no longer use a linear scale to encompass the entire spectrum from radio to gamma rays. Instead we use a 'Log' scale (i.e. Log, base 10) in which we step by powers-of-ten from one spot to the next along the wavelength scale.

Problem 1 - If Log(100) = +2.0 and Log(0.000001) = -6.0 what is Log(1,000,000,000,000,000,000)?

Problem 2 - Using a calculator, evaluate the following to two decimal places: a) Log(3.5 x 10^{-25}); b) Log(4.7 x 10^{+13}); c) Log(2.145 x 10^{-15}); d) Log(3.14159)

Problem 3 - Plot the data in the table below on a LogE versus LogF graph with Log(eV) on the vertical axis and Log(Hertz) on the horizontal axis. What spectral region includes the point at Log(Hz) = +23.0, and what is the energy of these photons based on your graph?

Regions in the electromagnetic spectrum

Name	Frequency (Named)	Frequency (Hertz)	Energy (eV)
Radio	5 kiloHertz	5.0×10^{3}	2.0×10^{-11}
Microwave	30 gigaHertz	3.0×10^{9}	1.2×10^{-4}
Sub-Millimeter	300 gigaHertz	3.0×10^{11}	1.2×10^{-3}
Infrared	30 teraHertz	3.0×10^{13}	1.2×10^{-1}
Visible(Green)	500 teraHertz	5.0×10^{14}	2.1
Ultraviolet	3 petaHertz	3.0×10^{15}	12.4
X-ray	3 exaHertz	3.0×10^{18}	12,000
Gamma ray (1 MeV)	250 exaHertz	2.5×10^{20}	1.0×10^{6}
Gamma ray (1 TeV)	250 yottaHertz	2.5×10^{26}	1.0×10^{12}

Problem 1 - If Log(100) = +2.0 and Log(0.000001) = -6.0 what is Log(1,000,000,000,000,000,000)? Answer: Count the number of factors of ten: **+18.0**

Problem 2 - Using a calculator, evaluate the following to two decimal places:

a) Log(3.5×10^{-25}); Answer = **-24.45**

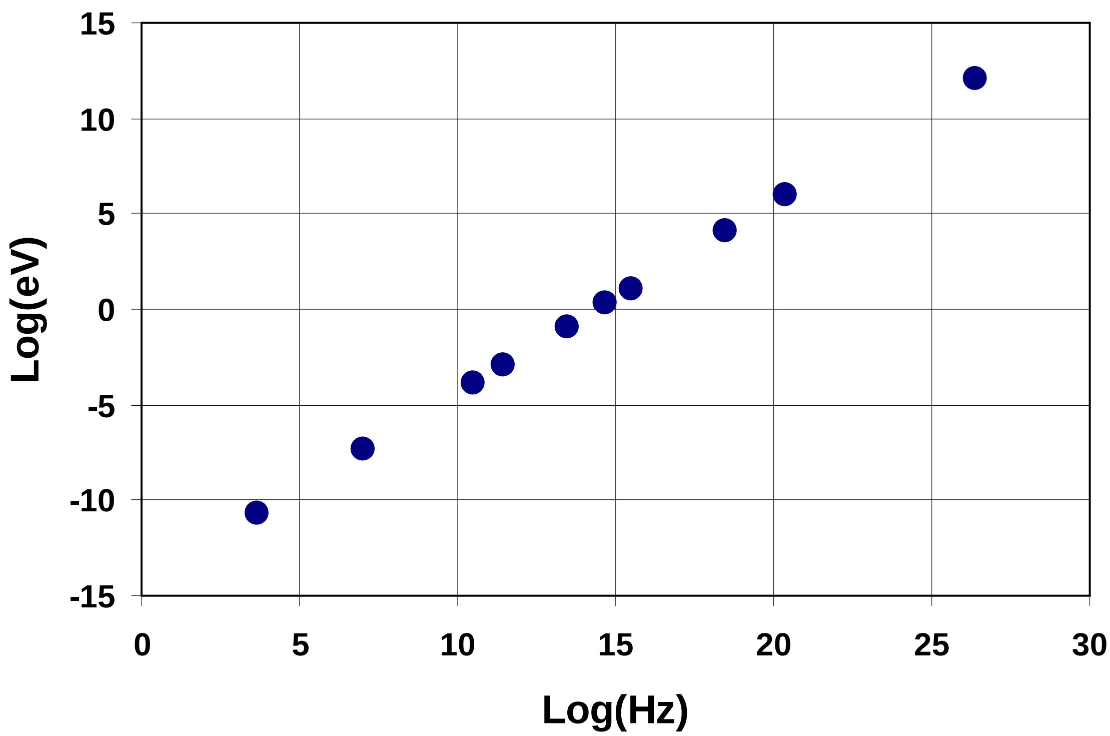
b) Log($4.7 \times 10^{+13}$); Answer = **+13.67**

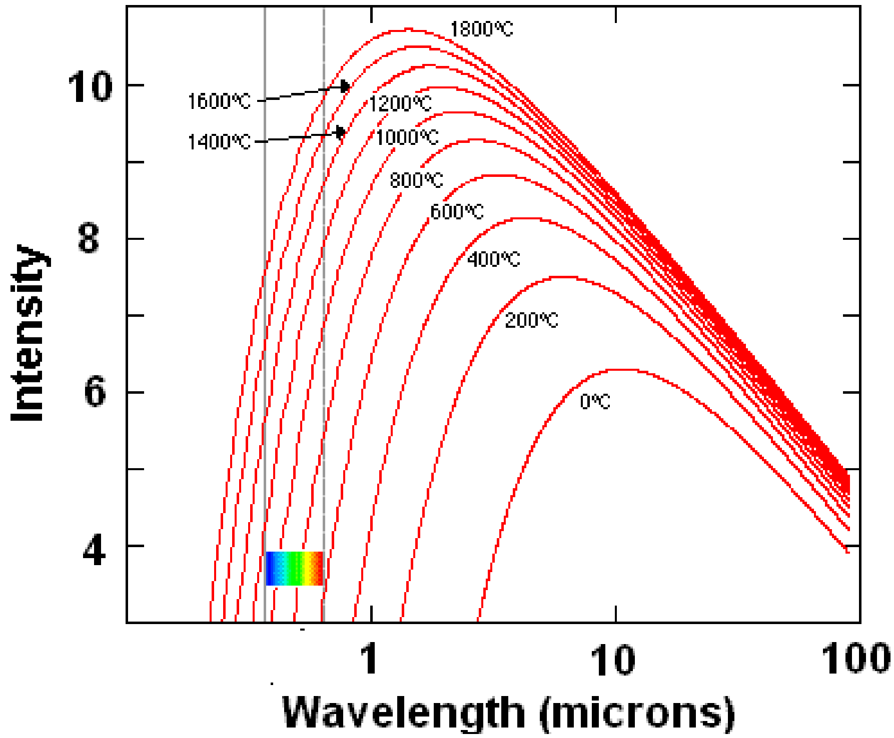
c) Log(2.145×10^{-15}); Answer = **-14.67**

d) Log(3.14159); Answer = **+0.50**

Problem 3 - See graph below. What spectral region includes the point at Log(Hz) = +23.0 and what is the energy of these photons based on your graph??

Answer: The frequency would be 10^{23} Hertz. Since the closest match to this in the table is in the gamma-ray region (10^{20} to 10^{26} Hertz) this is **gamma-ray radiation** with an energy of **10^{10} eV or 10 billion eV (10 giga eV or 10 GeV)**.





Heated bodies, such as the sun or a stove-top, emit electromagnetic radiation at many different wavelengths. When you graph the intensity of the light with wavelength, a very simple curve results, called a black body curve. The graph above shows the black body curves for a series of bodies with different temperatures. Because of the huge range of wavelengths and intensities for black bodies, physicists plot the intensity and wavelength information on Log-Log graphs. Instead of using linear scales for plotting x vs y, they use Base-10, Log(x) vs Log(y). The Intensity units are written as '4' for 10^4, while the wavelength units are given in ordinary values so that '100' just means 100 microns.

Problem 1 - As the temperature increases, what happens to the wavelength where most of the light is emitted by the body?

Problem 2 - An important property of all black body curves is the Wein Displacement Law, which states that the wavelength of maximum emission depends on the temperature of the body emitting the radiation. The formula relating the wavelength of peak emission to the temperature of the body is given by

$$\lambda = \frac{2897768}{T}$$

where **T** is the temperature in Kelvins (i.e. 274 K = 0 Centigrade), and λ is the wavelength in nanometers. Suppose an astronomer wants to search for planets beyond the orbit of Neptune where temperatures are between 20 K and 40K. What is the wavelength range, in micrometers (microns), where the planets will be emitting most of their black body radiation?

Answer Key

Problem 1 - As the temperature increases, what happens to the wavelength where most of the light is emitted by the body? Answer: the graph shows that, for example, a body with a temperature of 200 C corresponds to a curve with a peak near a wavelength of 10 microns (10^{-5} meters), while a body with a temperature of 1800 C has a peak emission near 1 micron (10^{-6} meters), so **as the temperature increases, the wavelength of peak emission becomes shorter**.

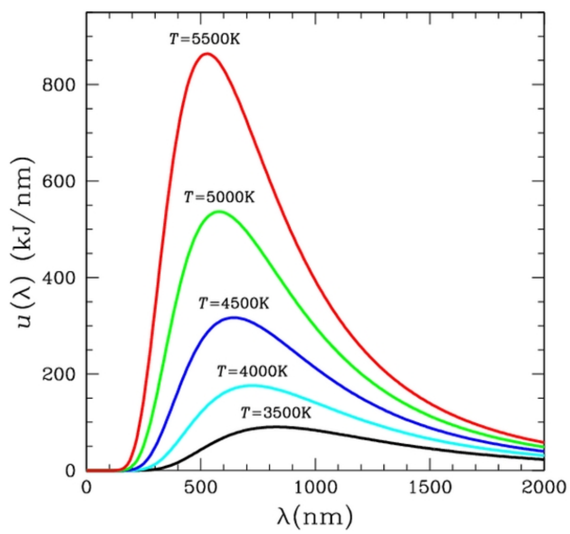
Problem 2 - An astronomer wants to search for planets beyond the orbit of Neptune where temperatures are between 20 K and 40K. What is the wavelength range in micrometers (microns) where the planets will be emitting most of their black body radiation?

Answer: From the formula, T is in the range 20 K to 40 K so evaluating the formula we have that λ is in the range from wavelengths of 144,888 to 72,444 nanometers. Since 1 micrometer = 1000 nanometers, we have a wavelength range from **72 to 144 micrometers**.

Measuring Star Temperatures

Careful measurements of a star's light spectrum gives astronomers clues about its temperature. For example, incandescent bodies that have a red glow are 'cool' while bodies with a yellow or blue color are 'hot'. This can be made more precise by measuring very carefully exactly how much light a star produces at many different wavelengths.

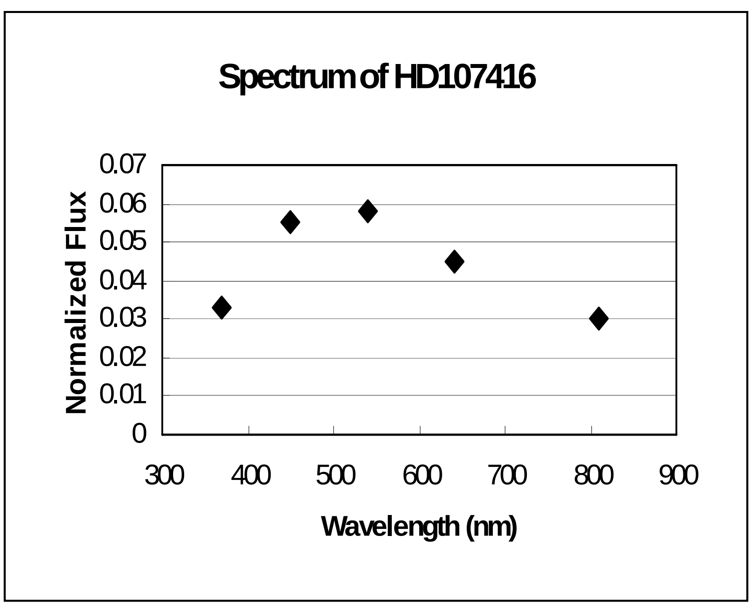
In 1900, physicist Max Planck worked out the mathematical details for how to exactly predict a body's spectrum once its temperature is known. The curve is therefore called a Planck 'black body' curve. It represents the brightness at different wavelengths of the light emitted from a perfectly absorbing 'black' body at a particular temperature.



From the mathematical properties of the Planck Curve, it is possible to determine a relationship between the temperature of the body and the wavelength where most of its light occurs - the peak in the curve. This relationship is called the Wein Displacement Law and looks like this:

$$Temperature = \frac{2897000}{Wavelength}$$

Where the temperature will be in units of Kelvins, and the wavelength will be in units of nanometers.



The lower plot shows measurements of the spectrum of the star HD107146. The horizontal axis is in units of nanometers (nm).

Problem 1 - Based on the overall shape of the curve, and the wavelength where most of the light is being emitted, use the Wein Displacement Law to determine the temperature of HD107146.

Problem 2 - What would be the peak wavelengths of the following stars in nanometers.

A) Antares .... 3,100 K
B) Zeta Orionis……..…… 30,000 K
C) Vega ………………….. 9,300 K
D) Regulus……………..….. 13,000 K
E) Canopus………………. 7,300 K
F) OTS-44 brown dwarf… 2,300 K
G) Sun…………………….. 5,770 K

Answer Key:

Problem 1 - The peak of the curve is **near 500** nanometers. The temperature is 2897000 / 500 = **5,794 K**.

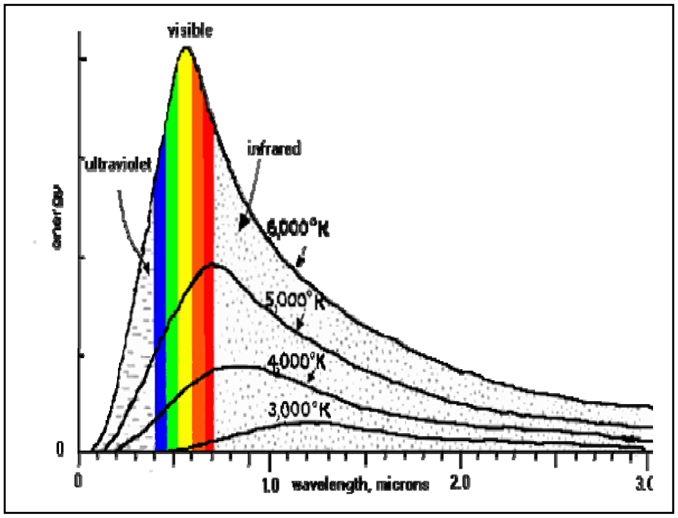
Problem 2 - What would be the peak wavelengths of the following stars in Angstroms:

Answer:

A)	Antares occurs at…….	2897000/3100 K	=	934 nanometers
B)	Zeta Orionis is at……..	2897000/30,000 K	=	97 nanometers
C)	Vega …………………..	2897000/9,300 K	=	311 nanometers
D)	Regulus…………..…..	2897000/13,000 K	=	223 nanometers
E)	Canopus……………….	2897000/7,300 K	=	397 nanometers
F)	OTS-44 brown dwarf…	2897000/2,300 K	=	1260 nanometers
G)	Sun…………………….	2897000/5770Ak	=	502 nanometers

Figure Credits:

The spectrum of HD 107146 is adapted from a paper by Williams et al. published in the Astrophysical Journal, 2004 vol. 604 page 414. The graph of Planck curves is from Wikimedia and is copyright-free.



When radiation is produced by a heated body, the intensity of electromagnetic radiation depends on frequency (wavelength) in a manner defined by the Planck Function. There is a simple law, called the Wein Displacement Law, which relates the temperature of a body to the frequency where the Planck curve has its maximum value. In this exercise, we will use two different methods to derive this law.

$$I(\lambda,T) = \frac{A}{\lambda^5\left(e^{\frac{14394}{\lambda T}} - 1\right)}$$

where: A = 3.747×10^{14} watts microns^4/m^2/str

Peak wavelengths and temperatures

Temperature (K)	Peak Wavelength (microns)
10,000	**0.2898**
9,000	**0.322**
8,000	**0.362**
7,000	**0.414**
6,000	**0.483**
5,000	**0.579**
4,000	**0.724**
3,000	**0.966**
2,000	**1.449**
1,000	**2.828**
500	**5.796**
300	**9.660**

Algebra Problem:

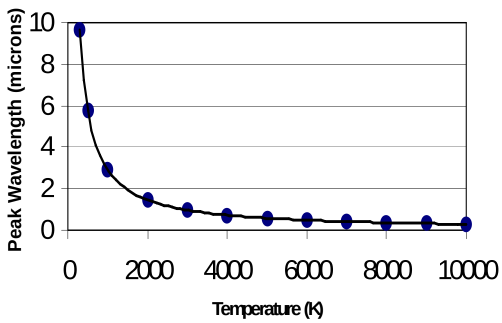
A) From the data in the table, use a calculator to find a formula that fits the data. Some possibilities might include a linear equation, $\lambda = a\,T$, power laws such as $\lambda = b\,T^{-1}$, $\lambda = c\,T^{-2}$ or $\lambda = d\,T^{2}$, $\lambda = e\,T^{3}$ or exponential functions such as $\lambda = f\,e^{(gT)}$ where a,b,c,d,e,f,g are constants determined by the fitting process. B) Which function fits the tabulated data the best, and what is the value of the constant?

Calculus Problem:

A) Find Equation 2 for the maximum of the function $I(\lambda,T)$ by differentiating with respect to the wavelength, λ, and setting the derivative equal to zero.

B) Find the solution to Equation 2, which you found in Part A, using the technique of 'successive approximation', or 'trial and error'. (Note, ignore trivial solutions involving zero! From this iterated solution, find the form of the function for the maximum wavelength as a function of temperature.)

Answer Key:



Algebra Problem: The figure above shows the data plotted in the table, and the best fit curve. This was done by copying the table into an Excel spread sheet, plotting the data as an XY scatter plot, and using 'Add Trend line'. Students may experiment with various choices for the fitting function using an HP-83 graphing calculator, or the Excel spread sheet trend line options, but should find that the best fit has the form: $\lambda = b\ T^{-1}$ where b = 2898.0 micronsKelvins. Students may puzzle over the units 'micronsKelvins' but it is often the case in physics that units have complex forms that are not immediately intuitive.

Calculus Problem: A) Below is a recommended strategy:

Let $U = A\,\lambda^{-5}$ then $dU/d\lambda = -5\,A\,\lambda^{-6}$

Let $V = e^{(14329/(\lambda\ T))} - 1$ then $dV/d\lambda = -14329 / (\lambda^{2}\ T)\ e^{14329/(\lambda t)}$

Then use the quotient rule: $d/d\lambda\ (U/V) = 1/V\ dU/d\lambda - U/V^{2}\ dV/d\lambda$

To get $dU/d\lambda - U/V\ dV/d\lambda = 0$

Then by substitution and a little algebra

$5\ l\ T\ (e^{14329/(\lambda T)} - 1) - 14329\ e^{14329/\lambda T} = 0$

Let $X = 14329/ (\lambda\ T)$ then we get a simpler equation to solve:

$5(e^{x} - 1) - x\ e^{x} = 0$ (Equation 2)

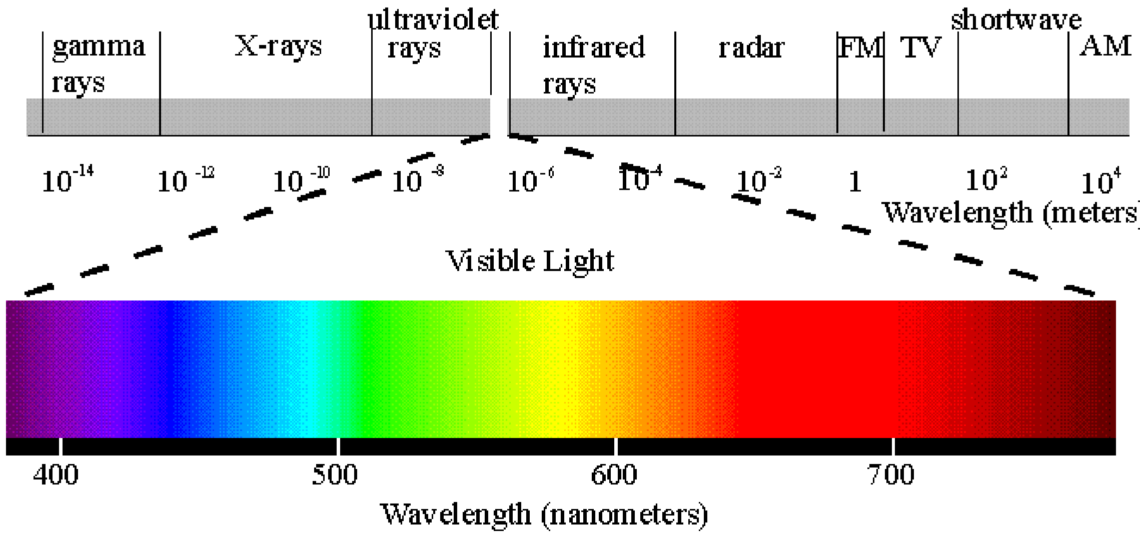
Equation 3, when solved, will give the location of the extrema for the Planck Function, however, it cannot be solved exactly. You will need to program a graphing calculator or use an Excel spreadsheet to find the value for X that gives, in this case, the maximum value of the Planck Function.

Calculus Problem: B) The table shows some trial-and-error results for Equation 3, and a convergence to approximately X = 4.965 The formula for the peak wavelength is then

$4.965 = 14394 / (\lambda\ T)$ or **$\lambda = 2899 / T$**

This is similar to the function derived by fitting the tabulated data.

X	Eq. 3
1	5.873127
2	17.16717
3	35.17107
4	49.59815
4.5	40.00857
4.9	8.428978
4.95	2.058748
4.96	0.703752
4.965	0.015799
4.966	-0.12263



The vast majority of electromagnetic radiation in the universe is produced by large collections of charged particles in motion. When these particles absorb and emit this radiation the energy of the photons in the radiation begins to mimic the typical energies of the particles in the heated gas. Physicists call this black body radiation. An important relationship exists between the temperature of the matter emitting black body radiation and the wavelength (frequency) where most of the light is emitted.

According to the Wein Displacement Law, the temperature of a black body, T, is related to the frequency of its peak emission, f, by the formula $T = 1.7 \times 10^{-11} f$. For example, for the Sun the peak occurs at a frequency of about $f = 3.4 \times 10^{14}$ Hertz, so the temperature T = 5,770 K.

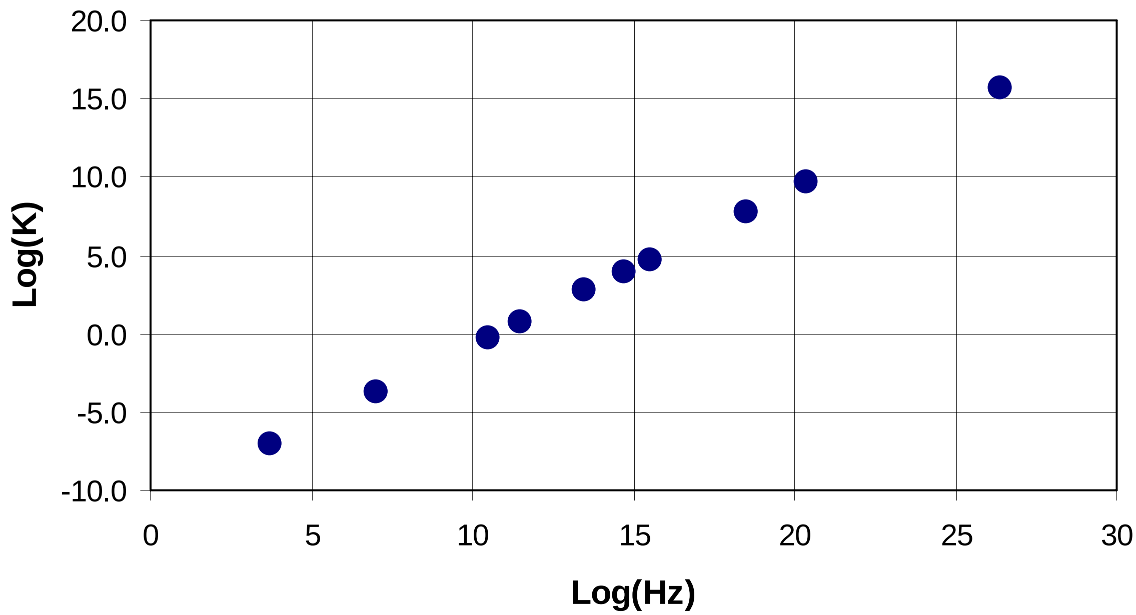
Problem 1 - Plot the data in the table below on a LogT - LogF graph with Log(K) on the vertical axis and Log(Hertz) on the horizontal axis. What spectral region includes the point at Log(Hz) = +23.0, and what is the temperature of these photons based on your graph?

Regions of the electromagnetic spectrum

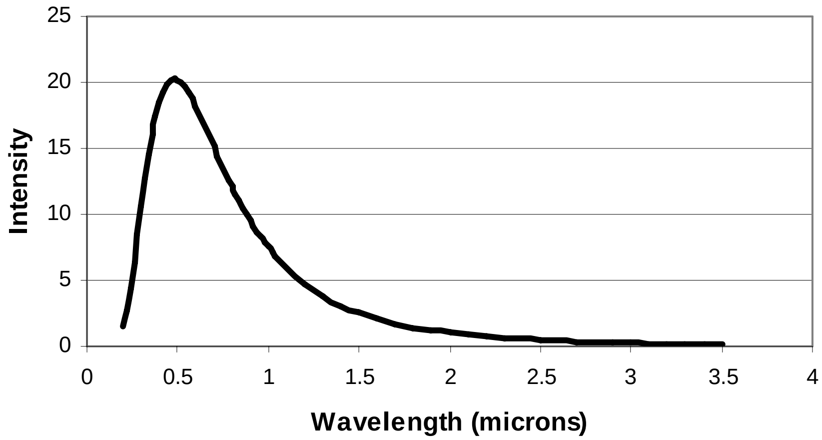
Name	Frequency (Named)	Frequency (Hertz)	Temperature (K)
Radio	5 kiloHertz	5.0×10^{3}	8.5×10^{-8}
Microwave	30 gigaHertz	3.0×10^{9}	0.51
Sub-Millimeter	300 gigaHertz	3.0×10^{11}	5.1
Infrared	30 teraHertz	3.0×10^{13}	510
Visible(Green)	500 teraHertz	5.0×10^{14}	8,500
Ultraviolet	3 petaHertz	3.0×10^{15}	5.1×10^{4}
X-ray	3 exaHertz	3.0×10^{18}	5.1×10^{7}
Gamma ray (1 MeV)	250 exaHertz	2.5×10^{20}	4.2×10^{9}
Gamma ray (1 TeV)	250 yottaHertz	2.5×10^{26}	4.2×10^{15}

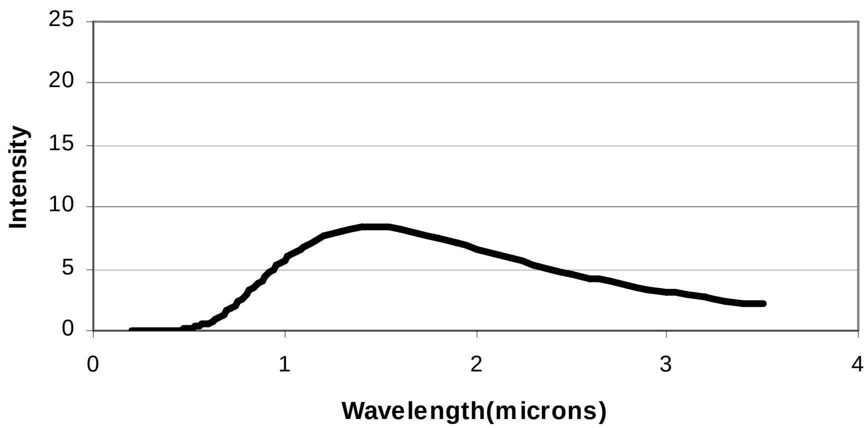
Problem 1 - See graph below. What spectral region includes the point at Log(Hz) = +23.0, and what is the temperature of these photons based on your graph?

Answer: The frequency would be 10^{23} Hertz. Since the closest match to this in the table is in the gamma-ray region (10^{20} to 10^{26} Hertz) this is **gamma-ray radiation** with an equivalent black body temperature of $T = 1.7 \times 10^{-11}$ (1.0×10^{23}) = **1.7 trillion K**



The Black Body Curve





The brightness of heated bodies that emit electromagnetic radiation follows a specific curve called a Black Body Curve. This curve determines the maximum amount of radiation that the body can emit at every wavelength. The two curves above show the curves for a hot body (top) and a cold body (bottom). The wavelengths are given in units of the micron (1 micron = 10^{-6} meters). Light at wavelengths of 0.5-1.0 microns is in the visible spectrum, while light at wavelengths longer than 1 micron are in the infrared spectrum.

Problem 1 - Can the colder body be seen in the ultraviolet region (0.5 microns)?

Problem 2 - Can the hotter body be seen at a wavelength of 3.0 microns?

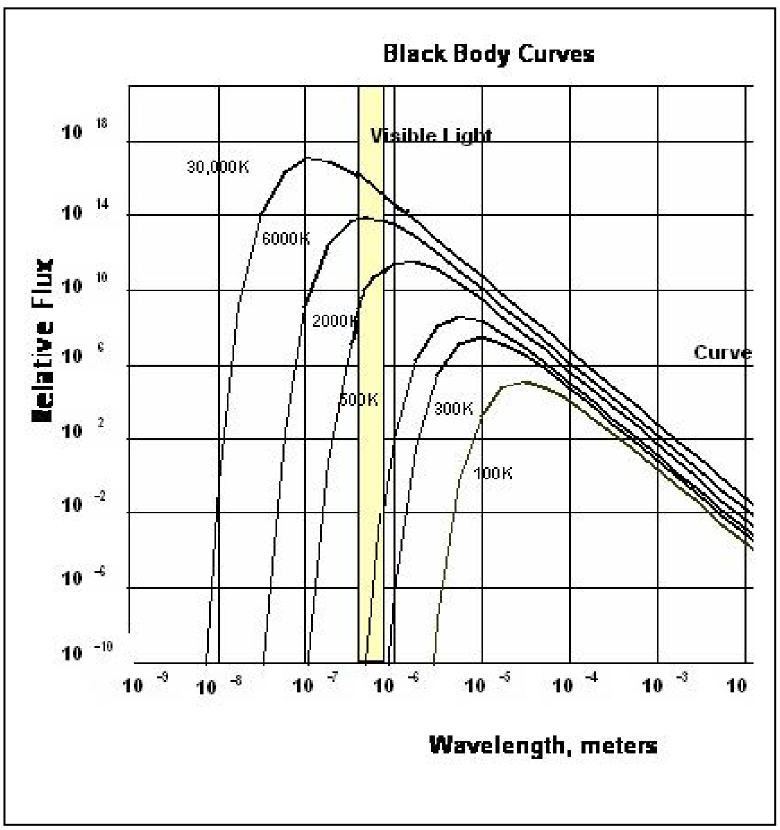
Problem 3 - At about what wavelength are the two bodies equally bright?

Problem 1 - Can the colder body be seen in the ultraviolet spectrum?
Answer: **No.** The intensity of the colder body shown in this example is nearly zero while the hotter body is very bright (about 20 Intensity units) at these wavelengths.

Problem 2 - Can the hotter body be seen at a wavelength of 3.0 microns?
Answer: **No.** The hotter body in this example has a brightness of nearly zero at this wavelength.

Problem 3 - At about what wavelength are the two bodies equally bright?
Answer: Look for a location where the intensities are nearly equal. **This occurs near a wavelength of about 1.0 microns.**

Note: *For two identical surfaces observed at the same distance, the black body curves predict that the hotter body will **always** be brighter than the colder body at every wavelength. The two examples given in the problem were not identical surfaces but only used to illustrate a few basic principles. When the curves are plotted on a Log(Intensity)-Log(wavelength) graph, their shapes can be seen to be identical though shifted in wavelength in accordance with their temperatures. This similarity and temperature shift is shown in the example below*.



Cold Planets and Hot Stars

$$B(\lambda) = \frac{7.4x10^{26}}{\lambda^5\left(e^{\frac{2383}{\lambda}} - 1\right)}$$

$$B(\lambda) = \frac{7.4x10^{26}}{\lambda^5\left(e^{\frac{9533}{\lambda}} - 1\right)}$$

The brightness of a star is measured at many different wavelengths, and forms a characteristic curve, B(λ), called a Planck function or 'black body' curve. The wavelength where this function is a maximum determines the color of the star. Red stars are cooler than yellow stars, and their Planck function peaks at a longer wavelength. By measuring the intensity of light from a star or other body, astronomers can determine the temperature of the body.

The two Planck functions to the left represent a hot star (top) with a temperature of 6,000 K (like our sun), and a cooler companion body (bottom) with a temperature of 1,500 K (like a brown dwarf star). The wavelength λ is given in nanometers.

Problem 1 - Use a graphing calculator or Excel spreadsheet to program and plot the hot-star Planck function over the range from 200 to 3500 nanometers. At about what wavelength, λ, does the function have a maximum?

Problem 2 - Use a graphing calculator or Excel spreadsheet to program and plot the brown dwarf star Planck function over the range from 200 to 3500 nanometers. At about what wavelength, λ , does the function have a maximum?

Problem 3 - Combine the two Planck functions to create a new function, S(λ), and plot this over the range from 200 to 3500 nanometers.

Problem 4 - What is the end-behavior of S(λ) for large values of λ? (Astronomers call this the Rayleigh-Jeans spectrum)

Problem 5 - What is the end-behavior of S(λ) for small values of λ? (Astronomers call this Wein's Law).

Problem 6 - Over what wavelength range would an astronomer have the best chance of detecting the brown dwarf star if it were in orbit around the hotter star?

Problem 1 - Answer: **At about 500 nanometers**. This is '0.5 microns' in the visible light portion of the electromagnetic spectrum. The temperature is similar to that of the sun.

Problem 2 - Answer: **At about 2000 nanometers**. This is '20 microns' in the infrared portion of the electromagnetic spectrum.

Problem 3 - Answer: **See graph below.**

Problem 4 - Answer: The exponential term can be approximated by the power series 1 +(2382/λ) + 1/2 $(2382/\lambda)^2$ + … so for large λ only the first two terms are significant and so for the two black bodies, individually, we get:

$$B(\lambda) = \frac{3.1x10^{23}}{\lambda^4} \quad \text{and} \quad B(\lambda) = \frac{7.8x10^{25}}{\lambda^4} \quad \text{so} \quad S(\lambda) = \frac{7.83x10^{25}}{\lambda^4}$$

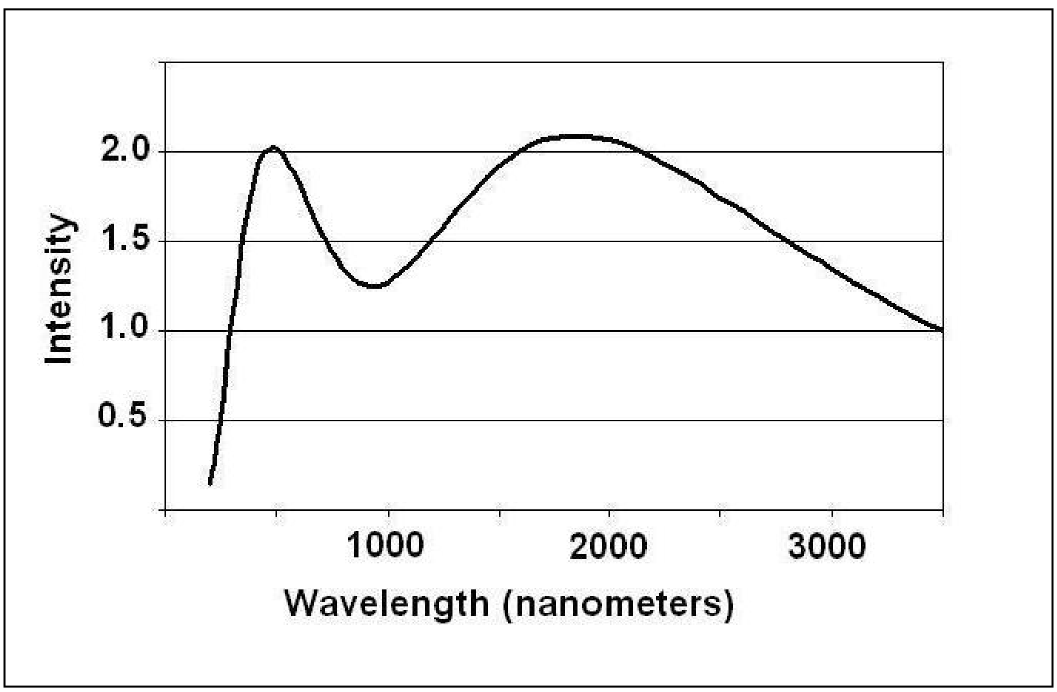
Note: S(λ) at long wavelengths is dominated by the cold black body!

Problem 5 - Answer: The exponential of the hottest black body always 'wins' so we get:

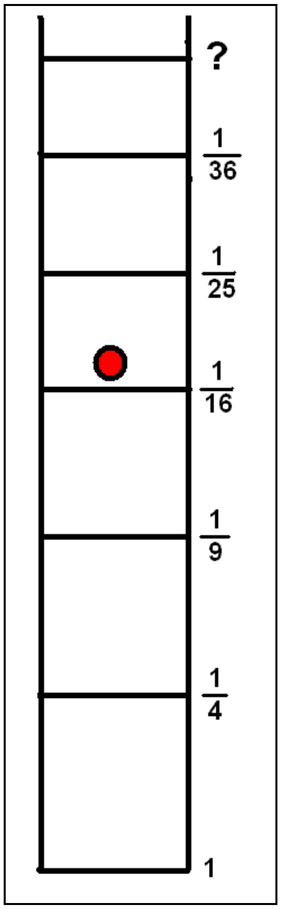
$$B(\lambda) = \frac{7.4x10^{26} e^{-\frac{2383}{\lambda}}}{\lambda^5}$$

For very small λ, the end behavior becomes increasingly more like $B(\lambda) \propto e^{-\frac{2383}{\lambda}}$

Problem 6 - Answer: The astronomer would directly measure S(λ), so the curve below shows that the region near the peak of the curve between 1,500 and 2,000 nm (15 to 20 microns) would be the best bet for detecting the brown dwarf.



Atomic Fractions I

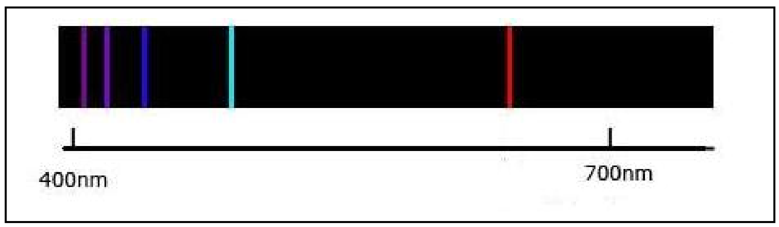


The single electron inside a hydrogen atom can exist in many different energy states. The lowest energy an electron can have is called the Ground State: this is the bottom rung on the ladder marked with an energy of '1'.

The electron must obey the Ladder Rule. This rule says that the electron can gain or lose only the exact amount of energy defined by the various ladder intervals.

For example, if it is located on the third rung of the ladder marked with an energy of '1/9', and it loses enough energy to reach the Ground State, it has to lose exactly 1 - 1/9 = 8/9 units of energy.

The energy that the electron loses is exactly equal to the energy of the light that it emits. This causes the spectrum of the atom to have a very interesting 'bar code' pattern when it is sorted by wavelength like a rainbow. The 'red line' is at a wavelength of 656 nanometers and is caused by an electron jumping from Energy Level 3 to Energy Level 2 on the ladder.



To answer these questions, use the Energy Fractions in the above ladder, and write your answer as the simplest fraction. Do not use a calculator or work with decimals because these answers will be less-exact than leaving them in fraction form!

Problem 1 - To make the red line in the spectrum, how much energy did the electron have to lose on the energy ladder?

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5
B) Level-3 to Level-1
C) Level-6 to Level-4
D) Level-4 to Level-6
E) Level-2 to Level-4
F) Level-5 to Level-1
G) Level-6 to Level-5

Problem 3 - From the energy of the rungs in the hydrogen ladder, use the pattern of the energy levels (1, 1/4, 1/9, 1/16, 1/25, …) to predict the energy of the electron jumping from A) the 10th rung to the 7th rung; B) the rung M to the lower rung N.

Problem 4 - If an energy difference of '1' on the ladder equals an energy of 14 electron-Volts, in simplest fractional form, how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer Key

Problem 1 - Answer: The information in the figure says that the electron jumped from Level-3 to Level-2. From the energy ladder, this equals a difference of 1/9 - 1/4. The common denominator is '36' so the fractions become 4/36 - 9/36 and the difference is -5/36 .because the answer is negative, the electron has to **lose 5/36** of a unit of energy to make the jump.

Problem 2 - How much energy will the electron have to gain (+) or lose (-) in making the jumps between the indicated rungs:
A) Level-2 to Level-5 = 1/4 - 1/25 = (25 - 4)/100 = **+21/100** so it has to GAIN energy.
B) Level-3 to Level-1 = 1/9 - 1 = 1/9 - 9/9 = **-8/9** so it has to LOSE energy.

C) Level-6 to Level-4 = 1/36 - 1/16 = **-5/144** so it has to LOSE energy
Two ways to solve:
First: (16 - 36) / (16x36) = -20 / 576 then simplify to get - 5 / 144
Second: Find Least Common Multiple
36: 36, 72, 108, **144**, 180, …
16: 16, 32, 48, 64, 80, 96, 112, 128, **144**, 160, …
LCM = 144, then
1/36 - 1/16 = 4/144 - 9/144 = **-5/144**

D) Level-4 to Level-6 = 1/16 - 1/36 = **+5/144** so it has to Gain energy.
E) Level-2 to Level-4 = 1/4 - 1/16 = 4/16 - 1/16 = **+3/16** so it has to GAIN energy
F) Level-5 to Level-1 = 1/25 - 1 = 1/25 - 25/25 = **-24/25** so it has to LOSE energy
G) Level-6 to Level-5 = 1/36 - 1/25 = (25 - 36)/ 900 = **- 11/900** so it has to LOSE energy

Problem 3 - Answer: Students should be able to see the pattern from the series progression such that the energy is the reciprocal of the square of the ladder rung number.

Level 2 Energy = $1/(2)^2$ = 1/4
Level 5 Energy = $1/(5)^2$ = 1/25

A) the 10th rung to the 7th rung: Energy = 1/100 - 1/49 = (49 - 100)/4900 = **-51/4900**.

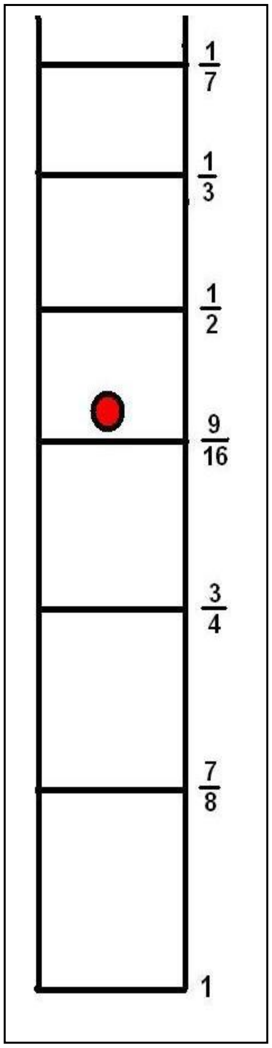
B) the rung M to the lower rung N. Energy = $1/M^2 - 1/N^2$

Problem 4 - If an energy difference of '1' on the ladder equals an energy of 13.6 electron-Volts, in simplest fractional form how many electron-Volts does the electron lose in jumping from Level-6 to Level-4?

Answer; The energy difference would be 1/36 - 1/16 = **-5/144** energy units.

Since an energy difference of 1.0 equals 14 electron-Volts, by setting up a ratio we have:

$$\frac{5/144 \text{ Units}}{1 \text{ Unit}} = \frac{X}{14 \text{ eV}} \quad \text{so} \quad X = 14 \times (5/144) = \frac{5 \times 2 \times 7}{2 \times 72} = \mathbf{\frac{35}{72}\ eV}$$



The single electron inside an atom can exist in many different energy states. The lowest energy an electron can have is called the Ground State: this is the bottom rung on the ladder marked with an energy of '1' in the figure to the left.

The electron must obey the Ladder Rule. This rule says that the electron can gain or lose only the exact amount of energy defined by the various ladder intervals.

For example, if the electron jumps from the fourth rung of the energy ladder marked with an energy of '9/16', to the Ground State, the energy change is E = 9/16 - 1 = -7/16 units of energy. This difference is negative, which means that the electron has LOST 7/16 energy units.

In the problems below, leave all answers in the simplest fractions.

Problem 1 - The electron gets a boost of energy and jumps from level 3 to Level 6. How much did it gain?

Problem 2 - An electron falls from Level 6 to Level 2. How much did it lose?

Problem 3 - An electron changes from Level 7 to Level 3. How much did it gain or lose?

Problem 4 - An electron is excited from Level 2 to Level 7. How much energy was gained?

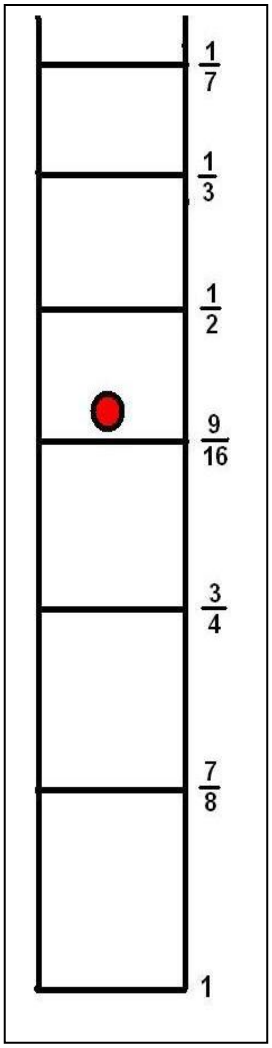
Problem 5 - The atom collides with another atom and the electron jumps from Level 3 to Level 6. How much energy did the other atom lose in the collision?

Problem 6 - An electron jumps from Level 3 to Level 2 and give of a particle of light. What energy is carried off by the light?

Problem 7 - An electron jumps from Level 7 to Level 4, then from Level 4 to Level 2. How much energy was lost with each jump, and what was the total energy lost after the two jumps?

Problem 8 - An electron jumps from the Ground State to Level 5, then is deexcited to Level 3, and after a while it is excited to Level 6, and then loses energy in a jump to Level 2. What is the total energy change of the electron between the start and end of this process?

Answer Key



Problem 1 - Answer: 3/4 - 1/3 = (9 - 4)/12 = +5/12 energy unit. This is positive, so **the electron GAINED 5/12 energy units**

Problem 2 - Answer: 1/3 - 7/8 = (8 - 21)/24 = -13/24 energy unit. This is negative, so **the electron LOST 13/24 energy units.**

Problem 3 - Answer: 1/7 - 3/4 = (4 - 21)/28 = -17/28 energy unit. So because this is negative **the electron LOST 17/28 energy unit.**

Problem 4 - Answer: 7/8 - 1/7 = (49 - 8)/56 = 41/56 energy unit. This is positive so **the electron had GAINED 41/56 energy units.**

Problem 5 - Answer: 3/4 - 1/3 = (9 - 4)/12 = 5/12 energy unit. This is positive, **so the electron has GAINED 5/12 energy units.**

Problem 6 - Answer: 3/4 - 7/8 = (24 - 28)/32 = -4/32 = -1/8 energy unit. This is negative ,so the electron LOST 1/8 energy unit, and so there was **1/8 energy unit carried away by the light particle.**

Problem 7 - Answer: The sequence is broken into two parts, which can be represented by bracketed quantities:

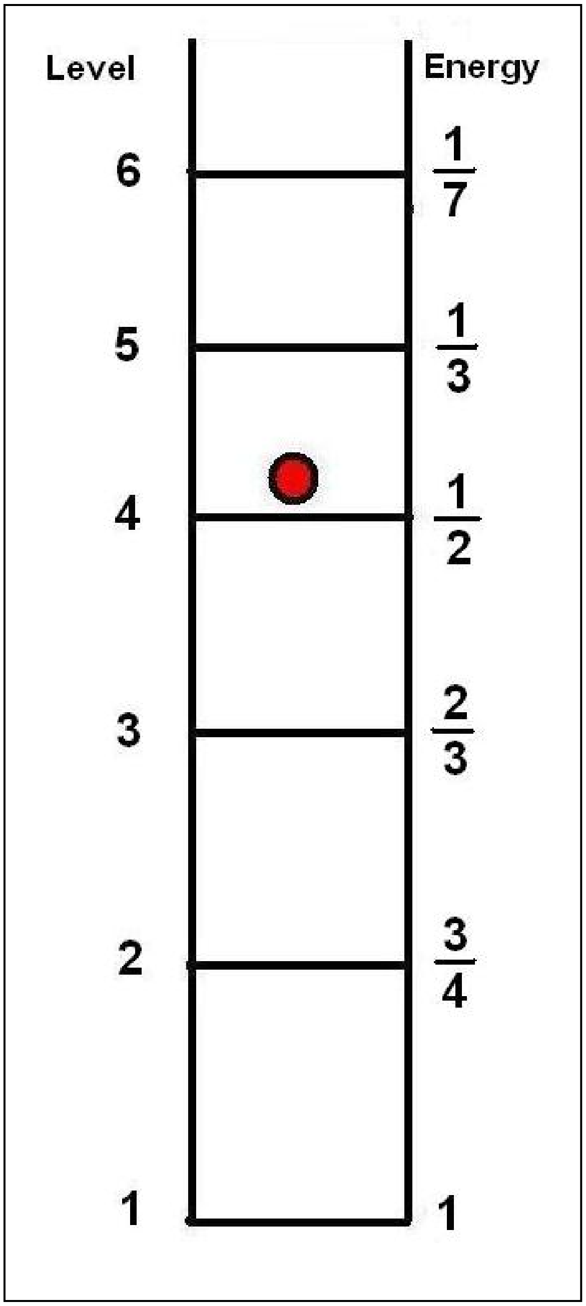
(1/7 - 9/16) + (9/16 - 7/8)
= 1/7 - 9/16 + 9/16 - 7/8
= 1/7 - 7/8
= (8 - 49)/56
= -41/56

so the electron **lost 41/56 of an energy unit. Students may note that this problem is of the form: (A - B) + (B - C) = A - C**

Problem 8 - Answer: (1 - 1/2) + (1/2 - 3/4) + (3/4 - 1/3) + (1/3 - 7/8) =
1 - 1/2 + 1/2 -3/4 + 3/4 - 1/3 + 1/3 - 7/8 =
1 + 0 + 0 + 0 - 7/8 =
1 - 7/8 = **1/8.**

This net energy is positive, so there was a net gain of 1/89 energy unit by the electron. Note, the sequence can be represented by (A - B) + (B - C) + (C - D) + (D - E) = A - E

Atomic Fractions - III

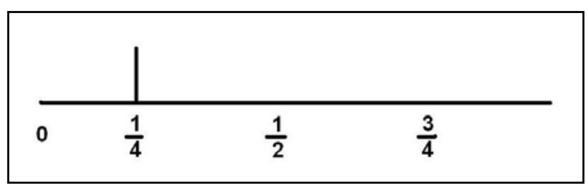


The electron inside an atom exists in one of many possible energy levels. These levels are like the rungs of a ladder. When it jumps from one level (rung) to the next, it gains or loses a specific amount of energy.

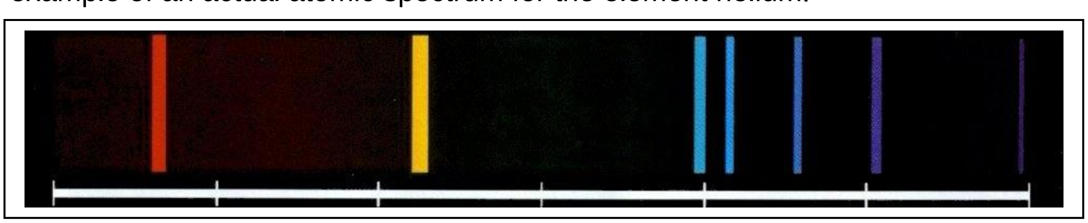
For example, if the energy of one rung is 3/4 and the energy of the next level is 1/4, the electron will lose 3/4 - 1/4 = 1/2 a unit if it jumps from the higher to the lower energy level.

Problem 1 - Suppose the energy level ladder of an imaginary element looked like the one to the left. What are all of the possible energies that an electron could lose as it jumped from a higher level to a lower one based on this ladder? (Leave your answers as simple fractions)

Problem 2 - On a number line, order the list of possible energy differences you tabulated in Problem 1, from lowest (left) to highest (right). For example, the energy difference between Level 2 and Level 1 is 1 - 3/4 = **1/4**, so draw a single vertical line at the location '1/4' on the number line. If a second energy difference is found to have the same value of '1/4' , draw the vertical line twice as tall, and so on. The graph is called a histogram.

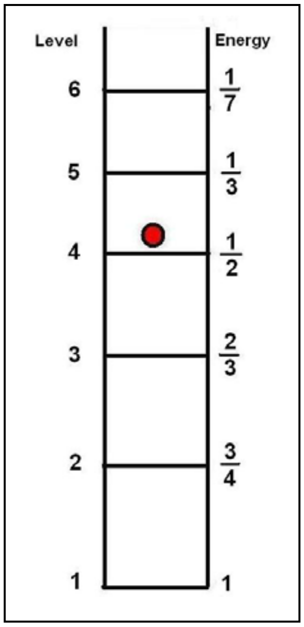


When you are finished with Problem 1 and 2, your number line will represent all of the possible ways that that atom can emit light. Every atom has its own pattern of atomic 'lines' which are based on each atom's unique energy ladder. This pattern is called a spectrum, and it is the unique fingerprint that allows scientists to identify each atom. Here is an example of an actual atomic spectrum for the element helium.



Answer Key

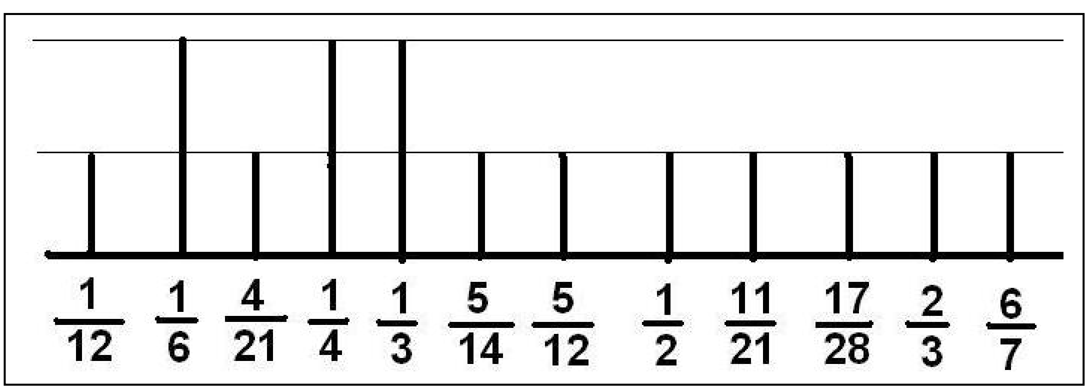
Problem 1 - Suppose the energy level ladder of an imaginary element looked like the one to the left. What are all of the possible energies that an electron could lose as it jumped from a higher level to a lower one based on this ladder? (Leave your answers as simple fractions)

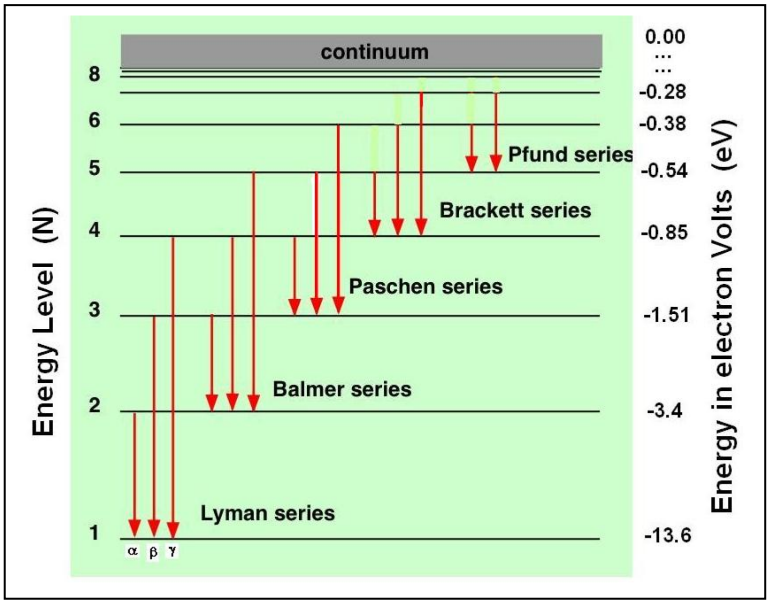


Levels	Energy Difference	Energy Value
2 to 1	1 - 3/4	= **1/4**
3 to 1	1 - 2/3	= **1/3**
4 to 1	1 - 1/2	= **1/2**
5 to 1	1 - 1/3	= **2/3**
6 to 1	1 - 1/7	= **6/7**
3 to 2	3/4 - 2/3 = 9/12 - 8/12	= **1/12**
4 to 2	3/4 - 1/2 = 3/4 - 2/4	= **1/4**
5 to 2	3/4 - 1/3 = 9/12 - 4/12	= **5/12**
6 to 2	3/4 - 1/7 = 21/28 - 4/28	= **17/28**
4 to 3	2/3 - 1/2 = 4/6 - 3/6	= **1/6**
5 to 3	2/3 - 1/3	= **1/3**
6 to 3	2/3 - 1/7 = 14/21 - 3/21	= **11/21**
5 to 4	1/2 - 1/3 = 3/6 - 2/6	= **1/6**
6 to 4	1/2 - 1/7 = 7/14 - 2/14	= **5/14**
6 to 5	1/3 - 1/7 = 7/21 - 3/21	= **4/21**

For 6 energy levels, there are 15 possible differences.

Problem 2 - The energies calculated in Problem 2 are displayed below. Note that in this diagram the scale is not linear but is merely used to illustrate the relative placements of the lines and their tallies. Students may use a more accurate number line to give a better impression of the spectrum and the non-uniform placement of the lines horizontally.





This figure shows the first 8 energy levels, N, of a hydrogen atom. The vertical lines represent the specific 'jumps' that electrons make in order to produce the specific spectral lines identified by name. Only the first three lines in each named family are shown. The lines are named 'alpha' if the jump is one energy level, 'beta' if two levels are jumped, and so on. The energy level, in electron Volts (eV) is negative because the electrons form a 'bound' system. The way to read this diagram is like this:

An electron falls from Level N=3 to Level N=1. It emits the spectral line called 'Lyman-Beta'. The difference in energy between the two levels is $E = E_3 - E_1 = (-1.51) - (-13.6) = 12.09$ eV. This produces a photon of light that carries off exactly 12.09 eV of energy. The relationship between the energy of a photon and its wavelength is given by

$$E = \frac{1243}{\lambda}$$ where E is in units of eV and the wavelength, λ, is in nanometers.

Problem 1 - In the above example, what is the wavelength of the Lyman-Beta spectral line for hydrogen?

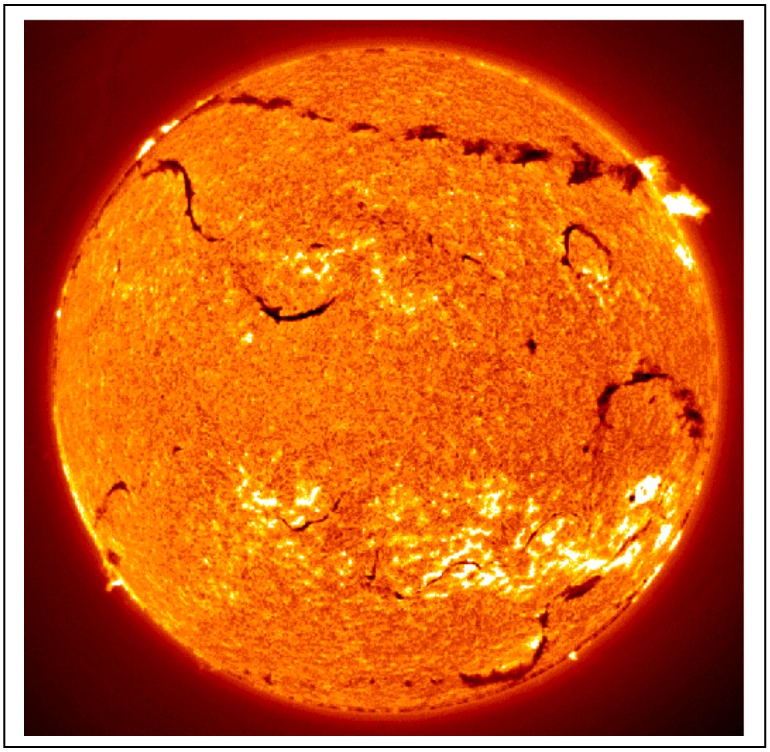
Problem 2 - An astronomer is using a filter to study details on the surface of the sun. It only allows light with a wavelength of 656 nm to be photographed. Which of the hydrogen lines is the one being studied using this 'H-alpha' filter?

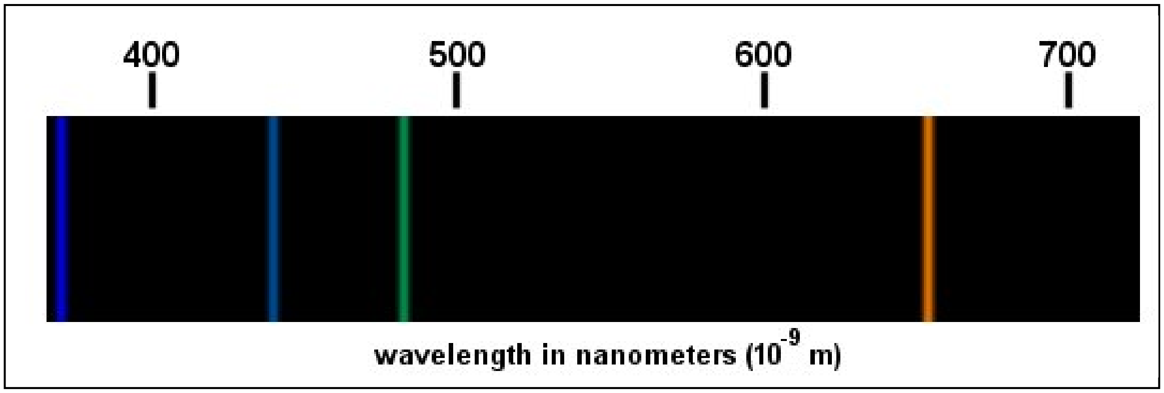
Answer Key

Problem 1 - In the above example, what is the wavelength of the Lyman-Beta spectral line for hydrogen? Answer: E = 12.09 eV so from the formula we get λ **= 109 nm**.

Problem 2 - An astronomer is using a filter to study details on the surface of the sun. It only allows light with a wavelength of 656 nm to be photographed. Which of the hydrogen lines is the one being studied using this 'H-alpha' filter?
Answer: First convert the wavelength to energy units because the diagram is based upon energy units. E = 1243/656 = 1.89 eV. Next we look at the energy differences between the various levels to find a difference that equals 1.89 eV. Since the filter is called 'H-alpha', we see if this is a clue suggesting that it is an 'alpha' line in one of the line series. Lyman-alpha has an energy of E = 13.6 - 3.4 = 10.2 eV so that is too large. The Paschen-alpha line has E = 1.51 - 0.85 = 0.66 eV which is too small. Next we try Balmer-alpha which has E = 3.4 - 1.51 = 1.89 eV. This matches the energy of the photons that are passed by the filter. **So, the 'H-alpha' filter passes only the Balmer-alpha spectral line.**

Note: Below is an H-alpha photo of the sun. It represents only those areas of the sun that are producing only this spectral line of hydrogen. (Courtesy: Institute fur Astrofysik, Gottingen. URL: http://www.astro.physik.uni-goettingen.de/research/solphys/ehalpha.html)





The specific spectral 'lines' that an element produces are unique to that element, but the lines are not random! The particular pattern is determined by specific physical laws that work for atomic systems. One of the most important ones is that energy, including light energy, comes in packets called **quanta**. Also, electrons inside atoms change their energy by emitting or absorbing quanta, which means that atoms have **energy levels**, much like the rungs on a ladder. When an electron drops from a higher energy level to a lower one, it emits a quantum of light called a photon. The wavelength of that photon precisely corresponds to the energy difference between the two energy levels. The result of this is shown in the visible light spectrum above. A large collection of hydrogen atoms have electrons 'jumping' from one energy level down to lower ones. The pattern of spectral lines represents the only possible photon wavelengths that can result from all of these 'transitions'. The relationship between photon wavelength and energy is given by the formula:

$$E = \frac{1243}{\lambda}$$

where λ is the photon wavelength in nanometers and E is the photon energy in electron Volts. (1 eV = 1.6×10^{-19} Joules). From this information, find the answers to the questions below.

Problem 1 - Using a millimeter ruler, what is the scale of the spectrum above in nanometers per millimeter?

Problem 2 - What are the wavelengths of the four hydrogen spectral lines?

Problem 3 - What is the energy of the photons that produced each spectral line in; A) electron Volts? B) Joules?

Problem 4 - Why do you think that the spectral lines have their particular color?

Answer Key

Problem 1 - Using a millimeter ruler, what is the scale of the spectrum above in nanometers per millimeter? Answer: Between '400' and '700' nm, the interval measures 118 mm, so the scale is (700-400)nm /118mm = **2.5 nm/millimeter**.

Problem 2 - What are the wavelengths of the four hydrogen spectral lines?
Answer: Starting from the marker for '700' nm, you can measure to the left to get Line 1 at 130mm, Line 2 at 102 mm, Line 3 at 85 mm and Line 4 at 18 mm. The wavelengths are
Line 1 = 700 - (130*2.5) = **375 nm**
Line 2 = 700 - (102*2.5) = **445 nm**
Line 3 = 700 - (85*2.5) = **487 nm**
Line 4 = 700 - (18*2.5) = **655 nm**.

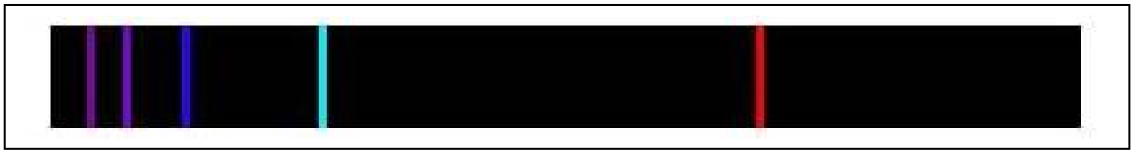
Problem 3 - What is the energy of the photons that produced each spectral line in; A) electron Volts? B) Joules?
Answer: A) Line 1 E = 1243/375 = **3.3 eV**
Line 2 E = 1243/445 = **2.8 eV**
Line 3 E = 1243/487 = **2.6 eV**
Line 4 E = 1243/655 = **1.9 eV**

B) Line 1 E = 3.3 eV x (1.6×10^{-19} Joules/eV) = **5.3×10^{-19} Joules**
Line 2 E = 2.8 eV x (1.6×10^{-19} Joules/eV) = **4.5×10^{-19} Joules**
Line 3 E = 2.6 eV x (1.6×10^{-19} Joules/eV) = **4.2×10^{-19} Joules**
Line 4 E = 1.9 eV x (1.6×10^{-19} Joules/eV) = **3.0×10^{-19} Joules**

Problem 4 - Why do you think that the spectral lines have their particular color?
Answer: The lines appear in the visible light spectrum whose colors run from deep-blue at the shorter wavelengths to deep red at the longer wavelengths. The color of the line in this image corresponds to the color of the spectrum at this wavelength,

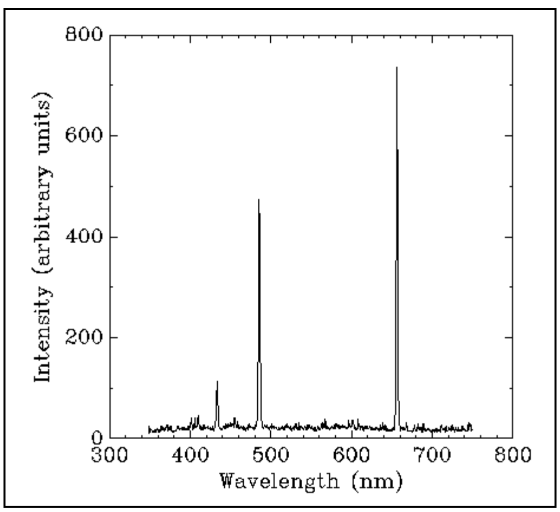
Note: These four lines are part of the Balmer line series for hydrogen.

Spectral line scaling



Problem 1: Using a simple spectrograph attached to a telescope, an astronomer recognizes the lines in a nebula from the element hydrogen as shown in the figure above. He knows that the bright 'Balmer-alpha' red line occurs at a wavelength of 656.3 nanometers (nm), and that the turquoise Balmer-beta line next to it on the left occurs at a wavelength of 486.1 nm. Using this information and a millimeter ruler, what is the horizontal wavelength scale of the spectrum in the figure in nm per millimeter (nm/mm)?

Problem 2: About what are the wavelengths, in nm, of the three other hydrogen lines, Balmer-gamma, delta and epsilon, in the blue part of the spectrum?



With modern day instruments, astronomers usually work with spectra in which the intensity of the line is indicated on the vertical axis, and the wavelength is along the horizontal axis like the one on the left. This spectral plot shows a portion of an atomic spectrum detected with an instrument called a spectrophotometer, that measures the intensity of the spectrum at each wavelength point from 300 to 600 nm.

Problem 3: From the spectrogram above, what is the wavelength scale in nm/mm?

Problem 4: What are the wavelengths of the three vertical 'lines' shown in the plot in nm?

Problem 5: Can you identify which element this spectrogram represents?

Answer Key

Problem 1 - Students should measure with a millimeter ruler, the distance in the picture between the first two lines. This is about 58 millimeters. The wavelength difference is 656.3 - 486.1 = 170.2 nm, so the wavelength scale is 170.2 nm /58 mm = **2.93 nm/millimeter.**

Problem 2 - For example, measure with a ruler the distance, right to left, from the main line at 656.3 nm to the third line on the right (dark blue) to get 76 mm, then 76 x 2.93 = 222.68 nm. Because the wavelength decrease from right to left, subtract 222.68 nm from 656.3 nm to get **433.6 nm**. Repeat this for the fourth line (purple) to get 84 mm, 246.1 nm and **410.2 nm**; and for the fifth line (violet) to get 89 mm ,260.77 nm and **395.5 nm.**

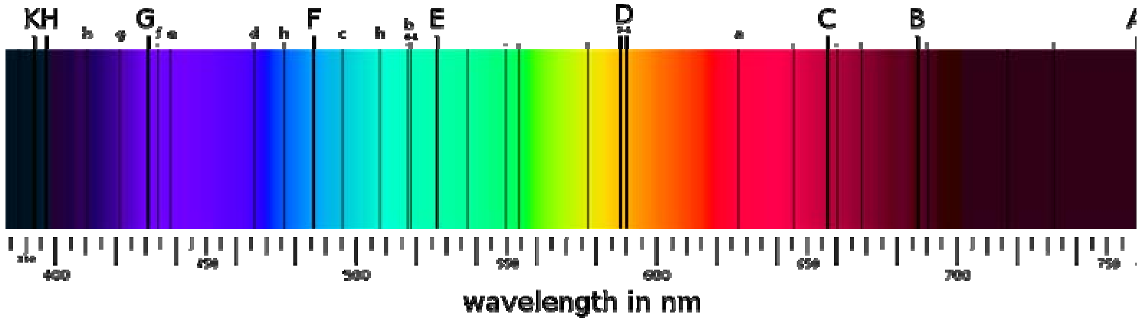
Problem 3 - The wavelength difference is 800 nm - 300 nm = 500 nm, and the distance between the marks is about 53 millimeters, so the scale is **9.43 nm/millimeters.**

Problem 4 - 800 nm - 15mm x 9.43 nm/mm = **658.6 nm**
800 nm - 33mm x 9.43 nm/mm = **488.8 nm**
800 nm - 39 mm x 9.43 nm/mm = **432.2 nm**

Problem 5 - To within the measurement accuracy of the millimeter ruler, the wavelengths measured in Problem 4 are the same as the ones measured for hydrogen.

Balmer - Alpha	656.3 nm	vs	658.6 nm
Balmer - Beta	486.1 nm	vs	488.8 nm
Balmer - Gamma	433.6 nm	vs	432.2 nm

Identifying Elements in the Sun



Iron:
- 527
- 516
- 495
- 466
- 438
- 430

Sodium:
- 589

Calcium:
- 396
- 393

Hydrogen:
- 656
- 486
- 410

Helium:
- 587

The solar spectrum was observed at high resolution for the first time over 200 years ago. With the help of a new instrument called the spectroscope, physicists were eventually able to see thin dark lines cross the familiar 'rainbow' spectrum. These lines were found to be the fingerprints of a number of common elements.

The image above shows the old-style lettered names of many of these 'Fraunhofer Lines'. The table to the right gives the wavelengths of many of the brightest lines in the solar spectrum between 400 and 750 nanometers (nm).

Problem 1 - Identify the letters corresponding to the lines in the table.

Problem 2 - Which pair of spectral lines are the closest together in the spectrum?

Problem 3 - If you designed a spectroscope to study the spectrum between 460 to 500 nm, which lines would you see, and to the nearest integer, what would be their wavelengths?

Problem 1 - Identify the letters corresponding to the lines in the table.
Answer:

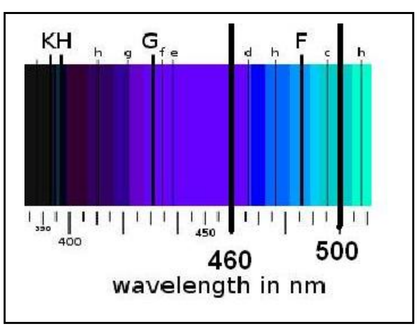
Iron:		Sodium:		Hydrogen:	
527	E	589	D	656	C
516	b			486	F
495	c	Calcium:		410	h
466	d	396	H	Helium:	
438	e	393	K	587	D
430	G				

Problem 2 - Which pair of spectral lines are the closest together in the spectrum?
Answer: The **Sodium-D and Helium-D lines** are only 589-587 = 2 nm apart and hard to discern, so they have the same letter in this figure. Actually, a spectroscope can easily 'resolve' these two lines into three lines: Sodium-D1 at 589.592 nm; Sodium-D2 at 588.995 nm; and Helium-D3 at 587.562 nm.

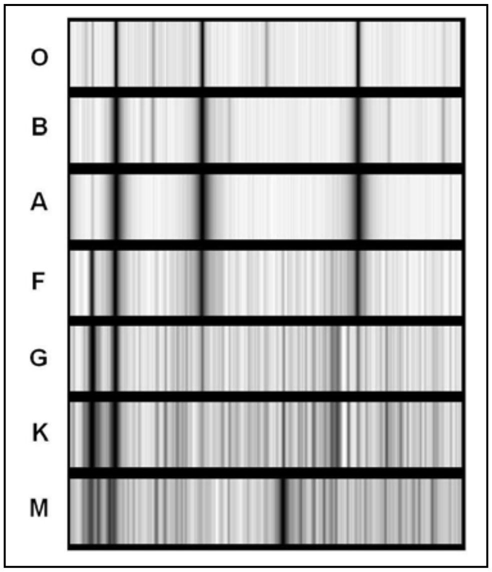
Problem 3 - If you designed a spectroscope to study the spectrum between 460 to 500 nm, which lines would you see, and to the nearest integer, what would be their wavelengths?

Answer: The region would be the one shown below. Using a millimeter ruler on the original image, the distance between '400' and '700' nanometers is 116 millimeters. This represents 300 nm, so the scale is 300 nm/ 116 millimeters or 2.6 nm/millimeter. Between 460 to 520 nm, we have the following lines: d, h, F, and c. Starting from the marker for 460 nm, we measure right-wards to each line:

d = 2mm h= 6mm F=10mm and c=14mm. Then we convert these to nm using the scaling factor. d = 2mm x 2.6 nm/mm = 5.2 nm h = 6 x 2.6 = 15.6 nm; F = 10 x 2.6 = 26 nm and c = 14x2.6 = 36.4 nm .Then we add these to the 460 nm which was our reference mark and round the answer to the nearest integer:
d = 465 nm;
h = 476 nm;
F = 486 nm;
C = 496 nm.

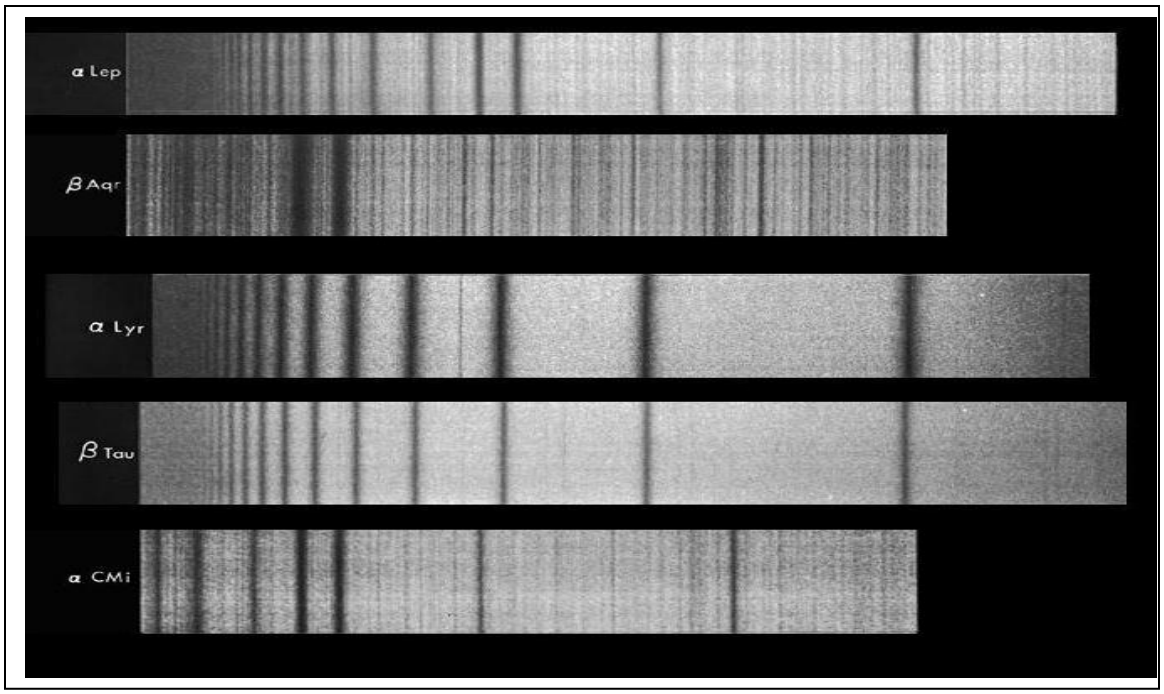


Spectral Classification of Stars



The advent of the spectroscope in the 1800's allowed astronomers to study the temperatures and compositions of stars, and to classify stars according to their spectral similarities. At first, 26 classes were defined; one for each letter in the alphabet. But only 7 are actually major classes, and these survive today as the series 'O, B, A, F, G, K, M'. This series follows decreasing star temperatures from 30,000 K (O-type) to 3,000 K (M-type).

Images courtesy: Helmut Abt (NOAA).



Problem 1 - Sort the five stellar spectra according to their closest matches with the standard spectra at the top of the page. (Note, the spectra may not be to the same scale, aligned vertically, and may even be stretched!)

Problem 2 - The star α Lyr (Alpha Lyra) has a temperature of 10,000 K and β Aqr (Beta Aquarii) has a temperature of 5,000 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

Problem 1 - This is designed to be a challenge! Student strategies should include looking for general similarities first. One obvious way to group the spectra in terms of the increasing (or decreasing!) number of spectral lines.

Alpha Lyr (Alpha Lyra) and Beta Tau (Beta Tauri) can be grouped together because of the strong lines that appear virtually alone in the spectra (these are hydrogen lines). The second grouping, Group 2, would include Alpha Lep (Alpha Leporis) and Alpha CMi (Alpha Canis Minoris) because they have a different pattern of strong lines than Group 1 but also the hint of many more faint lines in between. The last 'group' would be for Beta Aqr (Beta Aquarii) because it has two very strong lines close together (the two lines of the element calcium), but many more lines that fill up the spectrum and are stronger than in Group 2.

Group 1:
Vega (Alpha Lyra) - A type star
Alnath (Beta Tauri) - B type star

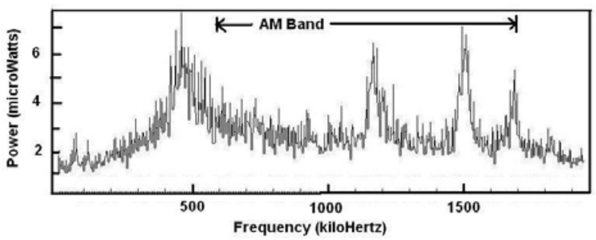
Group 2:
Arneb (Alpha Leporis) - F type star
Procyon (Alpha Canis Minoris) - F type star

Group 3:
Sadalsuud (Beta Aquarii) - G type star

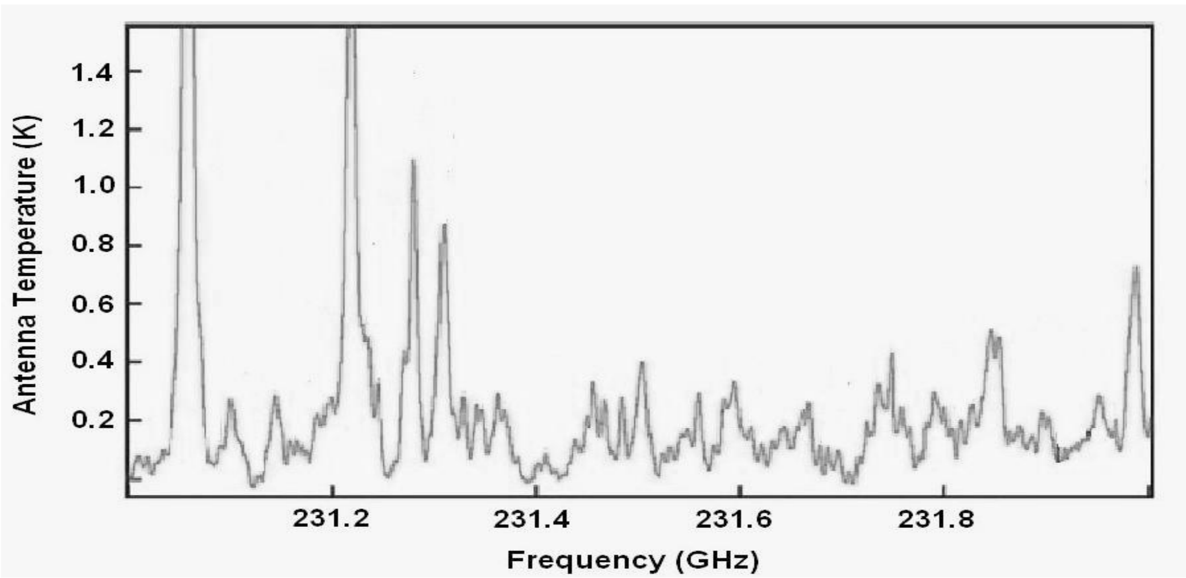
Comparing Group 1, 2 and 3 with the standard chart, Group 1 consists of the B and A type stars, in particular Alpha Lyra has the thick lines of an A-type star, and Beta Tauri has the thinner lines of a B-type star; Group 2 is similar to the spectra of the F-type stars, and Group 3 is similar to the G-type stars.

Problem 2 - Vega has a temperature of 10,000 K and Beta Aquari has a temperature of 5,500 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

Answer: What you should notice is that, as the temperature of the star gets cooler, the number of atomic lines in this part of the spectrum (visible light) increases.



The graph to the left shows the strength of the broadcast signals from four radio stations in the AM radio band between 500 kiloHertz and 1700 kiloHertz as shown on the scale.



The graph above shows the radio spectrum measured for a dense interstellar cloud called Sagittarius B2 located in the nucleus of the Milky Way, 26,000 light years from Earth. It shows individual radio stations called 'spectral lines', produced by specific molecules found in the gas within this cloud. The frequency scale is in gigahertz, where1 gigaHertz = 1 billion Hertz. (Courtesy the Atacama Large Millimeter Array: Arizona Radio Observatory)

Problem 1 - At what frequencies are the AM Band stations broadcasting most of their radio emission? (These correspond to the strongest stations that would be heard in your reception area)

Problem 2 - Over what range of frequencies, in gigaHertz, does the Sagittarius B2 interstellar cloud broadcast in this spectrum?

Problem 3 - If you were to draw the Sagittarius B2 spectrum to the same frequency scale as the printed AM Band spectrum in the above figure, how wide would the Sagittarius B2 spectrum have to be in meters?

Problem 4 - Under laboratory conditions, a chemist determines the locations of the radio lines for the following molecules: CH_3CHO at 231.280 GHz, $CH3NH_2$ at 231.850 GHz, OCS at 231.050 GHz and CS at 231.210 GHz. Where are these lines located in the spectrum for the interstellar molecular cloud Sagittarius B2?

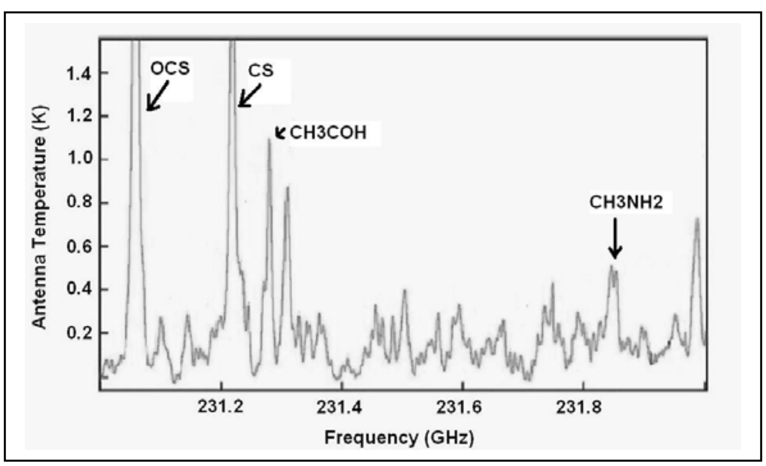
Problem 1 - At what frequencies are the AM Band stations broadcasting most of their radio emission? Answer: Draw a vertical line down from the peak of each curve to the horizontal frequency axis. **From left to right, the peaks are near frequencies of 450 kiloHertz, 1160 kiloHertz, 1500 kiloHertz and 1670 kiloHertz.**

Problem 2 - Over what range of frequencies does the Sagittarius B2 interstellar cloud broadcast in this spectrum? Answer: **The range is from 231 to 232 GHz or a width of 1 GHz.**

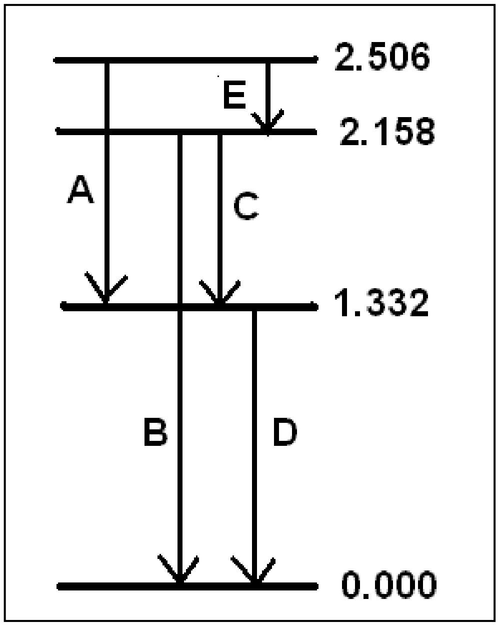
Problem 3 - If you were to draw the Sagittarius B2 spectrum to the same frequency scale as the AM Band spectrum, how wide would the Sagittarius B2 spectrum have to be in meters? Answer: Using a millimeter ruler, the horizontal scale of the AM band graph is (1500 - 100 kHz)/60 mm = 23.3 kHz/mm. The scale of the Sagittarius B2 graph is (231.4 - 231.2)/26 mm = 0.0077 GHz/mm or since 1 GHz = 1,000,000 kHz this is 7700 kHz/mm. The ratio of these two scales is 7700/23.3 = 330. So the AM station scale is 330 times larger than the Sagittarius B2 scale, so the new scale would be 330 x (width of Sagittarius B2 spectrum) = 330 x 13 cm, or **43 meters**.

Problem 4 - Under laboratory conditions, a chemist determines the locations of the radio lines for the following molecules: CH_3CHO at 231.28 GHz, CH3NH2 at 231.85 GHz, OCS at 231.05 GHz and CS at 231.210 GHz. Where are these lines located in the spectrum for the interstellar molecular cloud Sagittarius B2?

Answer: Using the scale for the spectrum of 0.0077 GHz/mm, the first line is just to the right of the marker '231.2'. The frequency difference is (231.28 - 231.2) = 0.08 GHz, and on the scale this corresponds to 0.08/0.0077 = 10.4 mm to the right of the marker. Similar calculations are performed for the other lines. The figure below shows the result.



Nuclear Spectral Lines



Just as electrons can shift between energy levels to create spectral lines, protons within the atomic nucleus also have specific energy levels that they occupy. However, when protons move from a higher to a lower energy level, they emit photons that carry much more energy than those produced by the electrons.

Atomic electrons can produce spectral lines that span frequency range from x-rays to radio waves. Protons within the atomic nucleus produce gamma rays with far-higher frequencies than even x-rays.

By shining gamma-ray light on a sample of material, some of the photons cause protons in the nuclei of atoms to jump to higher energy levels. After a period of time, the protons fall to lower energy levels and emit specific gamma ray photons whose energies and patterns uniquely identify the atoms present.

This technique, called gamma-ray fluorescence spectroscopy, is used, not only to study microscopic samples for clues to their composition, but also planetary surfaces. Gamma rays from the sun strike the surface of the planet, and with the right equipment, astronomers can survey what rocks and elements are present from a distant satellite.

The figure above shows the first four 'rungs' of energy level 'ladder' for the nucleus of the isotope nickel-60, including the ground state at '0.000'. The energy units on the right are in million electron-Volts (MeV). The lines with the arrows indicate all of the possible ways in which an excited proton can jump to lower energy levels.

Problem 1 - When a proton jumps to a lower energy level, it emits a gamma-ray photon whose energy equals the difference in the energy level energies. What are the gamma-ray energies of the photons that correspond to the five transitions shown above?

Problem 2 - A proton starts out in the excited energy level at 2.506 MeV. How many different sequences of jumps can it take to reach the ground-state energy at 0 MeV?

Problem 3 - Suppose this nucleus has one excited proton occupying each of the energy levels at 1.332 MeV, 2.158 MeV and 2.506 MeV. How many different ways can the three protons take to reach ground state, and what is the tally for the gamma-ray lines that result from all of the possibilities?

Problem 4 - Using your answer to Problem 3 as a clue, which of the gamma-ray spectral lines would you expect to see as the most intense line in such a sample of the isotope nickel-60?

Problem 1 - When a proton jumps to a lower energy level, it emits a gamma-ray photon whose energy equals the difference in the energy level energies. What are the gamma-ray energies of the photons that correspond to the five transitions shown above?

Answer: A) 2.506 - 1.332 = **1.174 MeV**
B) 2.158 - 0 = **2.158 MeV**
C) 2.158 - 1.332 = **0.826 MeV**
D) 1.332 - 0 = **1.332 MeV**
E) 2.506 - 2.158 = **0.348 MeV**

Problem 2 - A proton starts out in the excited energy level at 2.506 MeV. How many different sequences of jumps can it take to reach the ground-state energy at 0 MeV?

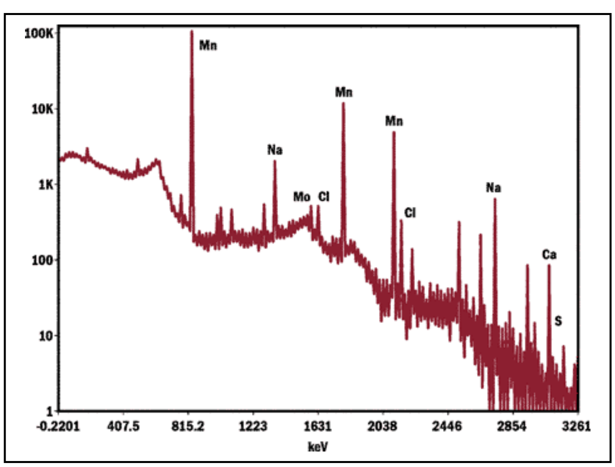
Answer: The possibilities are
E to C to D
E to B
A to D

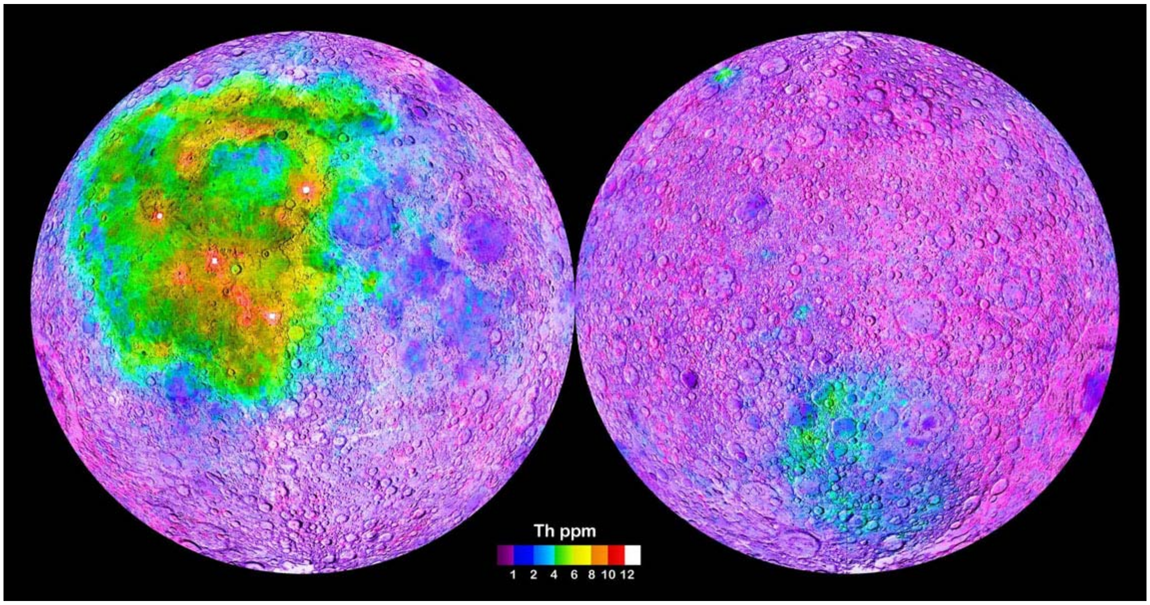
Problem 3 - Suppose this nucleus has one excited proton occupying each of the energy levels at 1.332 MeV, 2.158 MeV and 2.506 MeV. How many different ways can the three protons take to reach ground state, and what is the tally for the gamma-ray lines that result from all of the possibilities?

Answer: Starting at 2.506: A to D, E to B and E to C to D
Starting at 2.158: C to D and B
Starting at 1.332: D only

Tally: A=1, B=2, C=2, D=4, E=1

Problem 4 - Using your answer to Problem 3 as a clue, which of the gamma-ray spectral lines would you expect to see as the most intense line in such a sample of the isotope nickel-60?
Answer: The gamma-ray 'D line' with an energy of 1.332 MeV is the most common transition so it would produce a line that is more intense because more photons are present. Below is a gamma-ray spectrum showing several lines from various elements including sodium (Na) and manganese (Mn). The energy units are in kilo-electron-Volts (keV). (1 MeV = 1000keV)





Thorium nuclei are unstable. The excited nature of the protons in these radio-active nuclei creates a unique series of gamma-ray lines that can be used as a fingerprint to identify this element. In particular, when these excited protons return to 'ground state' many of them emit gamma-ray photons with an energy of 2.6 MeV.

The Gamma Ray Spectrometer on NASA's Lunar Prospector mission was able to detect this high-energy light from the lunar surface and create a map, shown above, that indicates where the concentration of thorium is the highest within the lunar surface rocks. The left-hand image is the side of the Moon facing Earth. The right-hand image is the far side of the Moon never seen from Earth.

Problem 1 - From the energy of this thorium line, what is the wavelength in meters, and frequency in Hertz, of the corresponding photons?

Problem 2 - From the appearance of the surface regions shown in the map above, what can you conclude about the kinds of terrain in which thorium seems to be the most abundant?

Problem 3 - The color on the map indicates the concentration of thorium in parts-per-million (ppm) of the surface rock. The average lunar basalt found in the mare regions contains about 10^{23} atoms per cubic centimeter of silicon dioxide and aluminum oxide, with an combined average molar mass of 100 grams/cc. The density of the lunar surface basalts is about 3 grams/cc. If the molar mass of thorium is 232 grams/mole, how many liters of lunar rock would you have to process in order to get 1 gram of thorium? (Note: 1 mole = 6.02×10^{23} atoms).

Problem 1 - Answer: E = 2.6 MeV . Converting to Joules we have

$E = 2.6 \times 10^{6} \times (1.6 \times 10^{-19}$ Joules/ 1 eV) $= 4.16 \times 10^{-13}$ Joules.

Then since E(Joules) $= 6.63 \times 10^{-34}$ x frequency, we have

Frequency = 4.16×10^{-13} Joules / (6.63×10^{-34}) = $\mathbf{6.3 \times 10^{20}}$ **Hertz**.

The wavelength is then

$\lambda = c/f$ c = speed of light = 3×10^{8} meters/sec.

$= 3\times10^{8}/6.3\times10^{20}$

$=$ $\mathbf{4.8 \times 10^{-13}}$ **meters**.

Problem 2 - From the appearance of the surface regions shown in the map above, what can you conclude about the kinds of terrain in which thorium seems to be the most abundant?
Answer: The thorium deposits seem to avoid heavily-cratered regions and favor the mare regions that were produced by lava flows, which carried rock from the lunar interior to the surface.

Problem 3 - The color on the map indicates the concentration of thorium in parts-per-million (ppm) of the surface rock. The average lunar basalt found in the mare regions contains about 10^{23} atoms per cubic centimeter of silicon dioxide and aluminum oxide, with an combined average molar mass of 100 grams/cc. The density of the lunar surface basalts is about 3 grams/cc. If the molar mass of thorium is 232 grams/mole, how many liters of lunar rock would you have to process in order to get 1 gram of thorium? (Note: 1 mole = 6.02×10^{23} atoms).

Answer: The thorium concentration of 12 ppm means that for every 1 million atoms of lunar basalt rock, there will be 12 atoms of thorium. The information in the problem states that the molar mass of the rock is 100 grams/mole, and that the density of the rock is 3 grams/cc, and that 1 mole = 6.02×10^{23} atoms. So the number of basalt atoms in a cubic centimeter is just:

$$N = \frac{3 \text{ grams}}{1 \text{ cc}} \times \frac{1 \text{ mole}}{100 \text{ grams}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mole}} = \frac{1.8 \times 10^{22} \text{ atoms}}{1 \text{ cc}}$$

The number of thorium atoms is then:

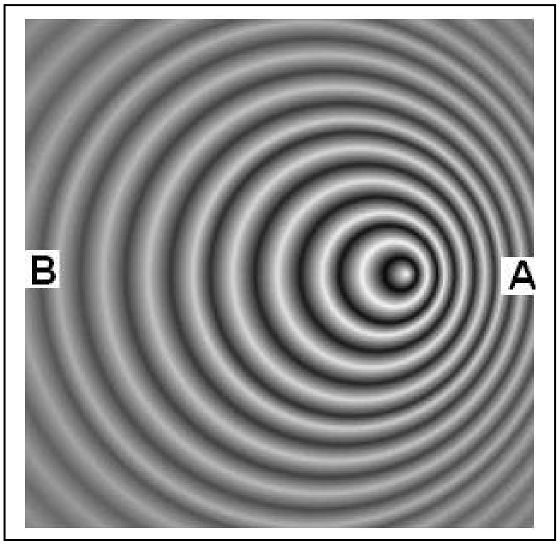
$$N(th) = \frac{12 \text{ thorium atoms}}{1 \text{ million basalt atoms}} \times \frac{1.8 \times 10^{22} \text{ basalt atoms}}{1 \text{ cc}} = \frac{2.1 \times 10^{17} \text{ thorium atoms}}{1 \text{ cc}}$$

1 cc of rock produces

2.1×10^{17} thorium atoms x 1 mole/6.02×10^{23} atoms x 232 grams/mole

$= 8.1 \times 10^{-5}$ grams of thorium.

So to get 1 gram of thorium we have to process $1.0 / 8.1 \times 10^{-5} =$ 12,300 cubic centimeters of lunar rock, which equals **12.3 liters.**



The pressure waves from a source of sound will be slightly compressed in the direction of motion (Point A) and stretched out in the opposite direction (Point B). Someone standing at Point A will hear a shorter-wavelength, higher-pitched, sound, than someone standing at Point B. This is the familiar 'siren effect' you hear when a fire truck speeds by.

This relationship between pitch and speed can be summarized by the formula;

$$V = S\frac{(f - F)}{F}$$

where **F** is the frequency of the sound inside the fire truck, **f** is the frequency at Point A.

For sound waves at sea level, the speed of the wave **S** = 340 meters/sec (758 mph = 'Mach 1.0'). For example, suppose the pitch of the siren was 440 Hertz, but standing at Point A you heard the pitch at a frequency of 500 Hertz. The fire truck would be traveling at a speed of (340) (470-440)/440 = -23 meters/sec (51 mph!). The negative sign means that that the truck is approaching you.

Problem 1 - You hear the fire truck siren at a frequency of 410 Hertz. What is the fire truck's speed and direction in relation to where you are standing?

Problem 2 - An acoustic engineer wants to re-write the formula in terms of the wavelength change in the sound waves. If S = f x λ, where λ is the wavelength of the sound wave, show the engineer what the formula will look like.

Problem 3 - A bat emits a chirp with a frequency of 45,000 Hertz. A biologist wants to determine how fast bats fly in the dark, so she sets up a sound frequency spectrometer and records the frequency shifts as the bats fly by. If the frequency shift is 1,000 Hertz, what is the bat's speed in A) meters/sec/ B) miles per hour?

Answer Key

Problem 1 - You hear the fire truck siren at a frequency of 410 Hertz. What is the fire truck's speed and direction in relation to where you are standing?
Answer: V = 340 (410 - 440)/440 = **+23 meters/sec (+51 mph)** so because the sign is positive, **the truck is moving away from you**.

Problem 2 - An acoustic engineer wants to re-write the formula in terms of the wavelength change in the sound waves. If S = f x λ, where λ is the wavelength of the sound wave, show the engineer what the formula will look like.
Answer:

$$V = S\left(\frac{\frac{S}{\lambda} - \frac{S}{\lambda_0}}{\frac{S}{\lambda_0}}\right) \quad \text{becomes} \quad V = S\frac{\lambda_0}{S}\left(\frac{\lambda_0 S - \lambda S}{\lambda_0 \lambda}\right) \quad \text{so} \quad V = S\left(\frac{\lambda_0 - \lambda}{\lambda}\right)$$

where λ_0 is the wavelength, in meters, of the original sound wave, and λ is the wavelength that it appears to be from the vantage point of an observer not moving with the source.

Problem 3 - A bat emits a chirp with a frequency of 45,000 Hertz. A biologist wants to determine how fast bats fly in the dark, so she sets up a sound frequency spectrometer and records the frequency shifts as the bats fly by. If the frequency shift is 1,000 Hertz, what is the bat's speed in A) meters/sec/ B) miles per hour?
Answer:

$$V = (340)\ \frac{1{,}000 \text{ Hertz}}{45{,}000 \text{ Hertz}} = \mathbf{7.6 \text{ meters/sec}}.$$

Since 1 kilometer = 0.64 mph,

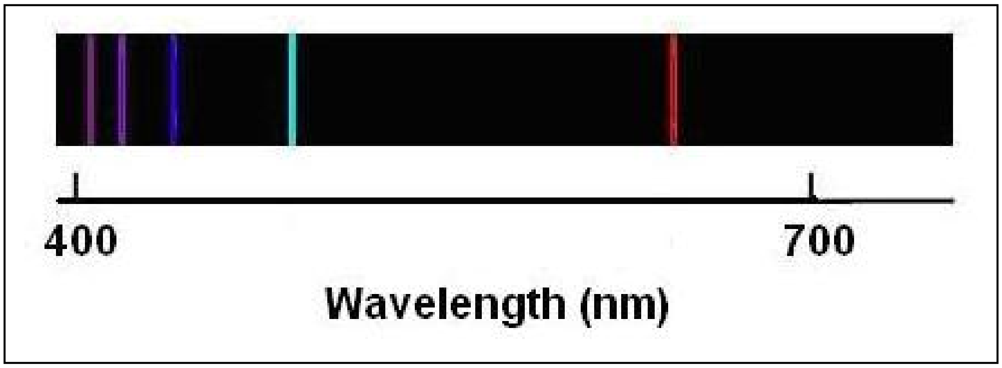
$$V = \frac{7.6 \text{ meters}}{\text{Sec}} \times \frac{1 \text{ kilometer}}{1000 \text{ meters}} \times \frac{0.64 \text{ miles}}{1 \text{ kilometer}} = 0.0048 \text{ miles/sec}$$

Then converting the time units to hours,

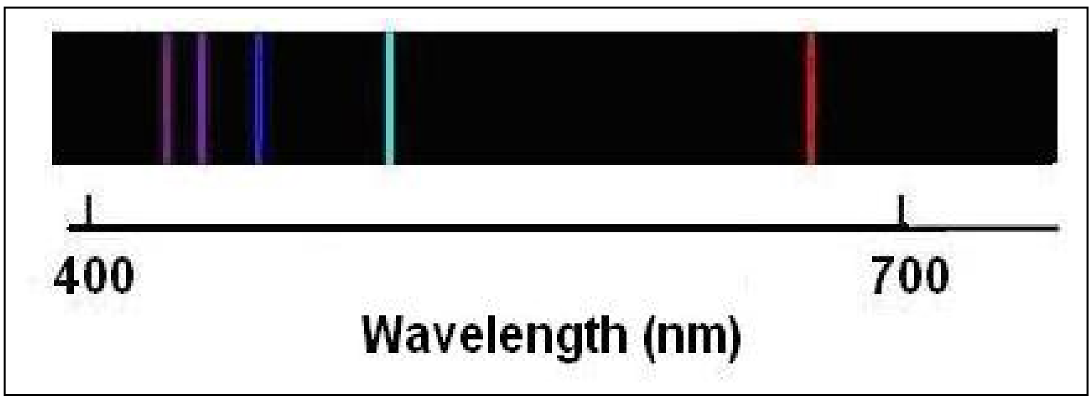
V = 0.0048 miles/sec x (3600 sec/ 1 hour) = **17 miles/hour**.

Note the use of 'significant figures' to report the answers only to two significant figures after the final division and multiplication steps.

Spectral Line Doppler Shifts



The two bright lines (red and blue) in the above laboratory spectrum of the element hydrogen are from Balmer-Alpha at 656.1 nm and Balmer-Beta at 486.1 nm. From this information, we can create a wavelength scale. Once we know the wavelength scale, we can use it to calculate the wavelengths of the other lines in the spectrum. Suppose an astronomer use a spectrometer to measure the spectrum of a distant cloud of hydrogen gas and found the pattern shown below:



Problem 1: Using a millimeter ruler, what are the wavelengths of the Balmer-Alpha and Balmer-Beta spectral lines in the second image of the hydrogen gas in the distant interstellar cloud?

Problem 2: What is the wavelength difference between the spectral lines seen in the interstellar hydrogen gas and in the laboratory spectrum of hydrogen?

Problem 3: Compared to the laboratory hydrogen wavelengths for the Balmer-Alpha and Beta lines, by what percentage did the wavelengths change for each line? What was the average percentage change?

Problem 4: According to the Doppler Effect, was the cloud approaching Earth or moving away from Earth?

Problem 1 - The wavelength scale of the bottom image is 300 nm/107 millimeters = 2.8 nm/millimeter.

The Balmer-Alpha line is located at 700 nm - 12.0 mm x 2.8 nm/mm = **692.1 nm.** The Balmer-beta line is located at 700 nm - 67 mm x 2.8 nm/mm = **512.4 nm.**

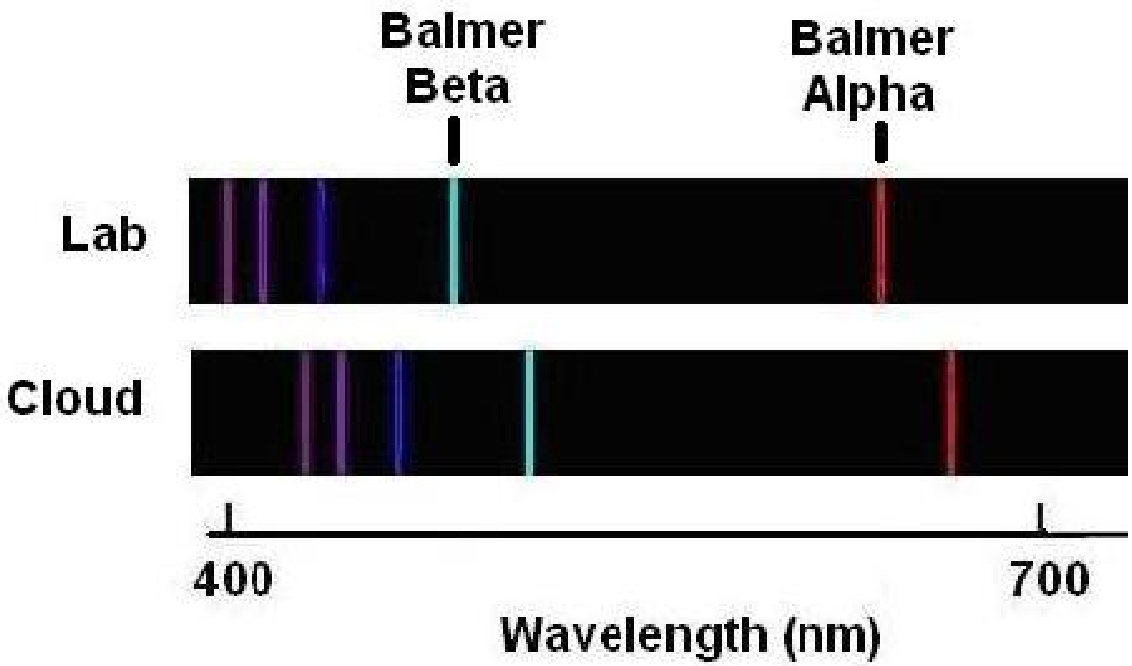
Problem 2 - Balmer-Alpha : 692.1 nm - 656.1 nm = **+36 nm.**
Balmer-Beta : 512.4 nm - 486.1 nm = **+ 26.3 nm**

Problem 3 - Balmer-Alpha : 100% x (36 nm/656.1 nm) = **5.49 %**
Balmer-Beta : 100% x (26.3 nm/486.1 nm) = **5.41 %**

The average change was (5.49 + 5.41)/2 = **5.45 %**

Problem 4 - According to the Doppler Effect, when a source of light or sound moves away from the observer, the wavelengths of the sound or light waves are stretched and lengthened. The wavelengths of the Balmer-Alpha and Balmer-beta spectral lines were increased by 5.45 % so the cloud was **moving away from Earth**.

The Doppler Shift I



The top spectrum is of the element hydrogen seen under laboratory conditions with a spectroscope. An astronomer measures a distant cloud in space and sees the spectrum marked 'Cloud'. The spectral lines form an identical pattern to the Lab spectrum for hydrogen, but the lines are shifted slightly in wavelength. The astronomer concludes that the cloud also consists of hydrogen gas, but that the cloud is moving. We know that the Doppler Effect causes the wavelength of atomic spectral lines to be shifted to longer wavelengths if the source is moving away from you, or shorter wavelengths if it is moving towards you.

Problem 1: Is the interstellar cloud moving towards Earth or away from Earth?

$$V = 300{,}000 \; \frac{W(\text{source}) - W(\text{Lab})}{W(\text{Lab})}$$

W(source) is the wavelength of the source spectral line in nanometers (nm). **W(Lab)** is the wavelength of the same spectral line seen in the laboratory, also given in units of nanometers (nm).

If you use the same wavelength units for W(source) and W(lab), the formula to the left gives the speed of the source in km/s.

Problem 2: From the data in the two spectra, and the Doppler formula, what is the speed of the gas cloud?

Problem 1 – The spectral lines in the 'Cloud' spectrum are located at longer wavelengths than for the 'lab' spectrum, so the source is moving away from Earth.

Problem 2 - First determine the wavelength scale for each of the two spectra. Use a millimeter ruler, and measure the distance between the 400 and 700 nm marks on the scale. The student should get approximately 108 mm. In both cases, the wavelength span is 700 - 400 = 300 nm, so the scales are Top: 300/108 = **2.77 nm/mm**

Now we need to find the wavelengths for one of the common lines in the two spectra. From the Lab spectrum, we will select the red line on the far right. It is located 21.5 mm to the left of the '700 nm' axis mark, so its wavelength is 700 nm - 21.5x2.77 = **640 nm.** For the bottom spectrum, this line is located 12 mm to the left of the '700 nm' tick mark, so its wavelength will be 700 - 12.0 x 2.77 = **667 nm**.

From the Doppler formula

W(lab) = 640 nm, W(source) = 667 nm so

V = 300,000 x (667 - 640)/640
= **+ 12,656 km/sec.**

Note: Students numbers may vary if they use excessive 'accuracy'. If we use the proper significant figures, the millimeter measurements are made to 3-SF accuracy, so the wavelengths should be quoted to the nearest nm with 3-SF, and the speed would then be **+12,700 km/s** to 3-SF.

The Doppler Shift II

The Doppler Effect causes the wavelength of light and sound waves to be shifted to longer wavelengths if the source is moving away from you, or shorter wavelengths if it is moving towards you. If the spectral line shifts are caused by the Doppler Effect, there is a simple formula that lets you calculate the speed, V, of the source. It looks like this:

$$V = 300,000 \frac{W(source) - W(Lab)}{W(Lab)}$$

In this formula, W(source) is the wavelength (in nanometers) of the source spectral line and W(Lab) is the wavelength (in nanometers) of the same spectral line seen in the laboratory. If you use the same wavelength units for W(source) and W(lab), the formula gives the speed of the source, V, in km/s.

Problem 1 - The following table lists some interesting astronomical objects for which Doppler shifts have been recorded. From the data in the table, and the Doppler formula, determine the missing table entries. Show velocity values to one decimal place accuracy.

Doppler shifts for spectral lines of astronomical objects

Object	Type	Motion	Spectral Line	Lab	Source	Speed (km/s)
Sun	Star	Rotation	Nickel I	676.7800 nm	676.7755 nm	
Zeta UMa	Binary Star	Orbital	Sodium K	393.7000 nm	393.6234 nm	
DR-21(OH)	Nebula	radial	CO molecule	1.30130 mm	1.30129 mm	
A20264	Young star	radial	CO molecule	1.30130 mm	1.30112 mm	
NGC-520	Galaxy	radial	Formaldehyde	6.2125 cm	6.2595 cm	
NGC-4594	Galaxy	rotation	Hydrogen	656.3000 nm	657.0657 nm	
Milky Way	Galaxy	rotation	Hydrogen	21.12670 cm	21.13944 cm	
Crab nebula	Supernova	expansion	Oxygen II	372.7000 nm	370.8365 nm	
Solar Flare	Sun Storm	Gas flow	Iron XXV	1.86610 A	1.86734 A	
SS-433	Gas jet	Outflow	Hydrogen	656.300 nm	820.375 nm	

Problem 2 - Why do astronomers call certain Doppler shifts 'Red Shifts' and others 'blue shifts"? Explain what these terms mean in terms of how the source is moving and why the spectrum is affected the way it is.

Problem 3 - A) Draw a sketch, not to scale, of the solar flare on the sun, the location of Earth, and the direction of motion of the gas described in the table. B) What would the observer on Earth see in the spectrum if the direction of motion were reversed?

Problem 4 - An astronomer detects two distinct spectral lines in the source SS-433 source; one with a large blue shift with V=-75,000 km/s and one with a large redshift with V = + 75,000 km/s. The astronomer cannot resolve SS-433 to see what is going on in a photograph. What can you deduce from the spectral data about SS-433?

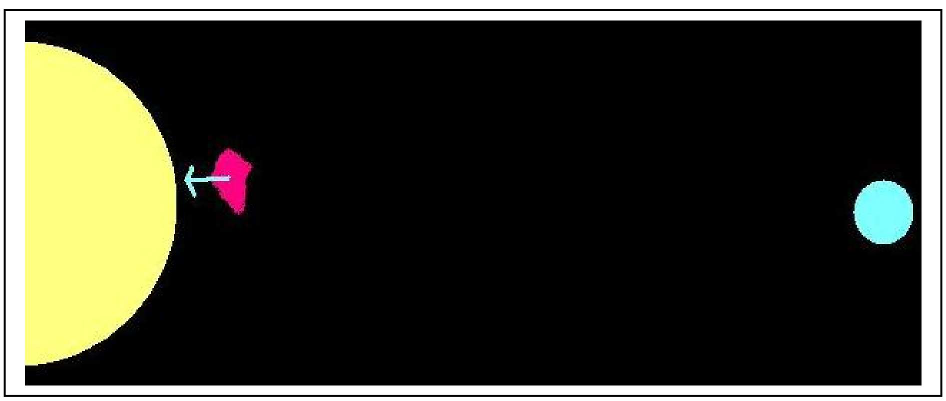
Problem 1

Object	Type	Motion	Spectral Line	Lab	Source	Speed (km/s)
Sun	Star	Rotation	Nickel I	676.7800 nm	676.7755 nm	-2.0
Zeta UMa	Binary Star	Orbital	Sodium K	393.7000 nm	393.6234 nm	-58.4
DR-21(OH)	Nebula	radial	CO molecule	1.30130 mm	1.30129 mm	-2.3
A20264	Young star	radial	CO molecule	1.30130 mm	1.30112 mm	-41.5
NGC-520	Galaxy	radial	Formaldehyde	6.2125 cm	6.2595 cm	+2269.5
NGC-4594	Galaxy	rotation	Hydrogen	656.300 nm	657.0657 nm	+350.0
Milky Way	Galaxy	rotation	Hydrogen	21.12670 cm	21.13944 cm	+180.9
Crab nebula	Supernova	expansion	Oxygen II	372.7000 nm	370.8365 nm	-1500.0
Solar Flare	Sun Storm	Gas flow	Iron XXV	1.86610 A	1.86734 A	+199.3
SS-433	Gas jet	Outflow	Hydrogen	656.300 nm	820.375 nm	+75,000

Calculation example: A20264 .W(source) = 1.30112 millimeters and W(lab) = 1.3013 millimeters so V = 300,000 x (1.30112 - 1.30130)/ 1.30130 = 300,000 x (-0.00014) = -41.5 km/sec

Problem 2: Students should answer something like this: The blue end of the spectrum is light at shorter wavelengths than the red end of the spectrum, so when spectral lines are shifted, they are either shifted blue-wards of where they should be if the Doppler Effect is making the wavelengths shorter, or red-wards if the Doppler Effect is making the wavelengths longer. This also means that a source moving towards you has its spectrum shifted blue-wards, and a source moving away from you has its spectrum shifted red-wards.

Problem 3: A) See below. The Doppler shift is to longer wavelengths so the gas is moving away from Earth, and therefore towards the surface of the sun as shown by the arrow.



B) Answer; the spectral line would be shifted to shorter wavelengths towards the blue end of the spectrum as seen from Earth, because the gas is moving towards the observer rather than away.

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Problem 4 - There must be two distinct gas clouds within SS-433 that are moving in opposite directions from the center of the source. One of these clouds is moving towards Earth while the other is moving away from Earth.

Doppler Shifts - III

$$V = 300{,}000 \; \frac{W(\text{source}) - W(\text{Lab})}{W(\text{Lab})}$$

The Doppler Shift formula is a very basic formula used in many different situations in astronomy. Imagine a collection of atoms in a box. If you were observing the spectral lines from these atoms, they would be Doppler shifted by different amounts depending on how fast they were moving relative to your laboratory frame of reference.

Imagine that you were observing the Sodium K spectral line at a wavelength of 676.78 nm emitted by one of the atoms in the box. If the atom were moving towards you the spectral line would be 'blue shifted' to shorter wavelengths and observed at, say, 670.00 nm. If the atom were moving away from you, the hydrogen line would be 'red shifted' to longer wavelengths and observed by your instrument at say 680.00 nm.

Problem 1 - Suppose a distant star cluster consisted of 100 stars. You take the spectrum of each star and identify the Sodium K line, and measure the actual wavelengths of that line in each of the star's spectrum. The table below gives your tally of the number of stars that shared each line wavelength. Plot a histogram of the number of stars detected at each wavelength.

Doppler shifts of galaxies in a hypothetical cluster

Number of Stars	Wavelength (nm)	Shift (nm)	Velocity (km/s)
1	677.42		
3	677.43		
10	677.44		
19	677.45		
34	677.46		
19	677.47		
10	677.48		
3	677.49		
1	677.50		

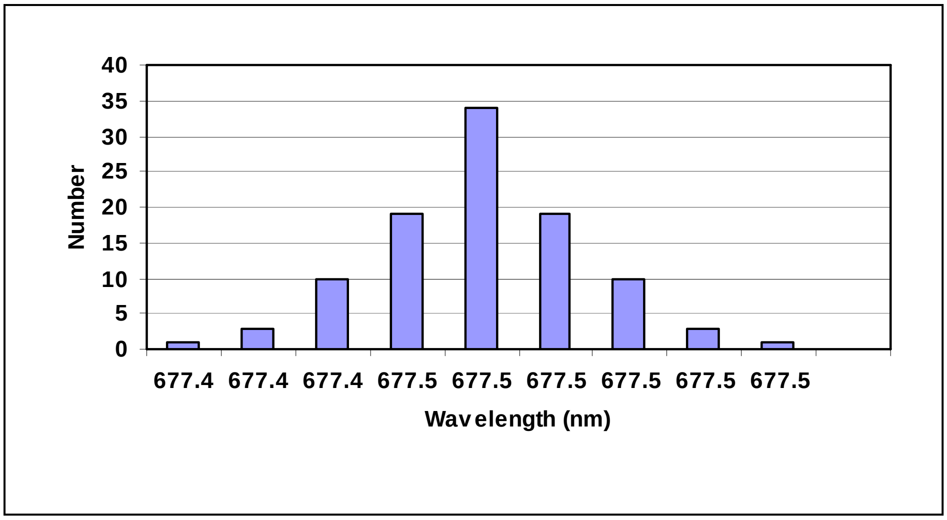
Problem 2 - Using the Doppler Shift formula, fill-in the missing table entries. (Don't forget to indicate whether the star is approaching or receding by noting the sign of the speed: Approaching = -V; receding = +V)

Problem 3 - Create a bar graph of the velocities of the stars.

Problem 4 - What is the average speed of the cluster of stars? Is the cluster moving towards or away from Earth? What is the maximum speed of the stars relative to the average speed of the cluster?

Answer Key

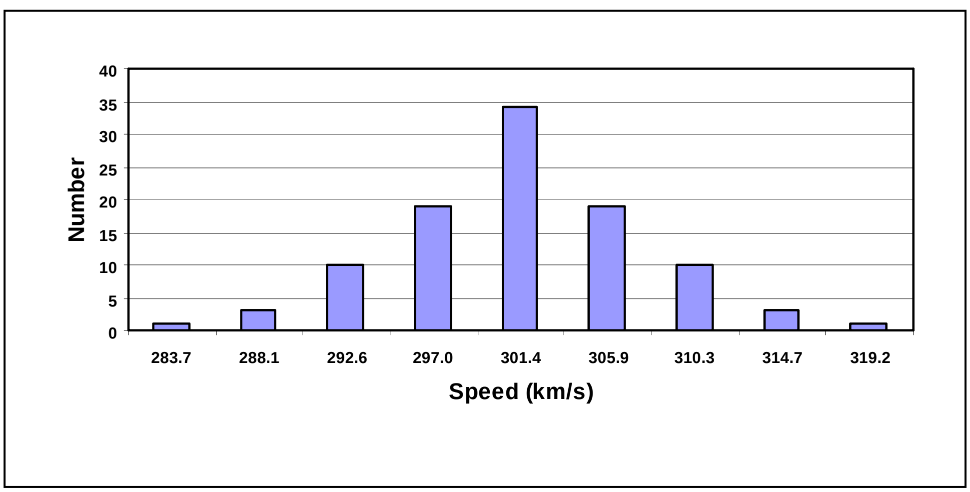
Problem 1



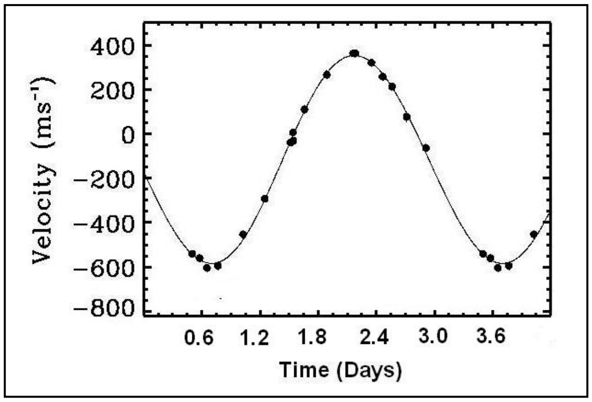
Problem 2 -

Number of Stars	Wavelength (nm)	Shift (nm)	Velocity (km/s)
1	677.42	+0.64	283.7
3	677.43	+0.65	288.1
10	677.44	+0.66	292.6
19	677.45	+0.67	297.0
34	677.46	+0.68	301.4
19	677.47	+0.69	305.9
10	677.48	+0.70	310.3
3	677.49	+0.71	314.7
1	677.50	+0.72	319.2

Problem 3



Problem 4 - The average speed of the cluster is +301.4 km/sec. Because the Doppler Shift is positive, the cluster is moving away from Earth at this average speed, while individual stars in the cluster are moving slightly faster or slower than this average speed. The largest star speeds are +319.2 km/sec and -283.7 km/sec which differ from the average speed of 301.4 of the cluster by (301.4 - 319.2) = 17.8 km/sec and (301.4 - 283.7) = 17.8 km/sec. So the maximum star speed is 17.8 km/sec relative to the cluster.



A common application for the Doppler Shift in astronomy is in measuring the speed of a planet, star or galaxy. Although this application only measures the speed either toward or away from the observer on Earth, it can be combined with other methods to determine the full speed and direction of an object.

The basic formula is the same as for sound waves, except that we now use the speed of light, c = 300,000 km/sec, not the speed of sound, 340 meters/sec, as the speed of the waves:

$$V = 300,000\left(\frac{\lambda_0 - \lambda}{\lambda}\right)$$

The graph above shows the changing speed of the star Tau Bootes measured by Dr. Paul Butler at San Francisco State University and his colleagues. As a planet orbits its star each 'year' it causes the star to move towards and away from the observers on Earth. The observations were made by carefully measuring spectral lines in the optical spectrum from the star, and comparing the star's spectral lines with identical lines on Earth. The Doppler Shift was used to compare the wavelengths of the Earth spectral lines (λ_0) with the measured wavelengths of the spectral lines from the star (λ).

Problem 1 - From the graph, about what is the period of the planet's orbit around its star?

Problem 2 - What is the range of speeds for the planet that were measured by the telescope on Earth?

Problem 3 - How would you interpret the sign of the speed being negative or positive?

Problem 4 - If the wavelength of the spectral line on Earth was 500 nm, what wavelength difference does the range of speeds correspond to?

Problem 1 - From the graph, about what is the period of the planet's orbit around its star?
Answer: Students can conveniently measure the time interval between the two minima of the wave and estimate a period of **about 3 days**.

Problem 2 - What is the range of speeds for the planet that were measured by the telescope on Earth?
Answer; **The range in meters/sec is [-600, +400]**

Problem 3 - How would you interpret the sign of the speed being negative or positive?
Answer; **From the Doppler formula, a negative speed means that the object is moving towards you, and a positive speed means it is moving away from you**.

Problem 4 - If the wavelength of the spectral line on Earth was 500 nm, what wavelength difference does the range of speeds correspond to?

Answer: The total speed change is 1 kilometer/sec. So from the formula:

$$1.0 = 300,000\left(\frac{\lambda_0 - \lambda}{500}\right) \quad \text{so}$$

$$\lambda_0 - \lambda = 500\frac{1.0}{300,000}$$

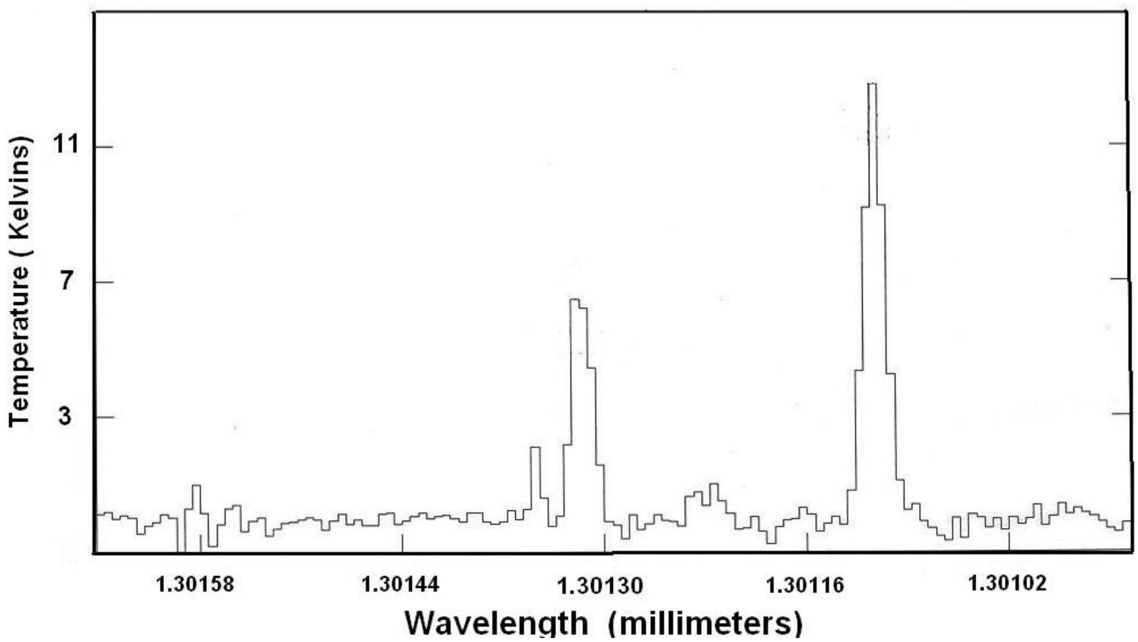
And so the total wavelength shift is $\lambda_0 - \lambda = 0.0017$ nm.

Note: Because 0.0017 this is only 0.0017/500 = 0.0000034 or 3.4 parts per million of the actual wavelength of a spectral line in the optical spectrum, the technique is to measure the wavelengths of hundreds of lines in the spectrum and combine the statistics to reduce the wavelength measurement uncertainty to about 1 part per million or better.

Current techniques can reliably measure speed differences as small as 3 meters/sec which is a measurement on wavelength of about 10 parts per billion (0.00000001) of the original line wavelength at rest.

A Doppler Study of Interstellar Clouds

On December 30, 1990 Dr. Sten Odenwald used the radio telescope at the Kitt Peak Radio Observatory in Tucson, Arizona to study young star-forming regions in the constellation Cygnus. The particular molecule he was using to explore these objects, carbon monoxide (CO) is found in interstellar clouds, and can be easily detected by one of its strong spectral line at a frequency of 230.538 gigahertz. Occasionally, with the radio telescope pointed in a single direction, you can detect several different clouds that are emitting at the same CO (230.538) spectral line, but they display Doppler shifts. The star-forming region A20264+4042 is one such region. The spectrum below shows the actual radio spectrum detected in the direction of this stellar nursery.



Problem 1 - What is the wavelength scale of this spectrum plot in wavelength units(millimeters) per millimeter? At what wavelengths were the two intense spectral lines detected?

Problem 2 - What is the wavelength of the CO line in millimeters, if its frequency is 230,538,000,000 Hertz? Identify this 'rest' wavelength on the spectrum plot. (Let the speed of light = 30 billion cm/sec)

Problem 3 - If we assume that both of the lines are produced by the CO molecule in the cloud emitting at a rest frequency of 230.538 gigaHertz, use the Doppler Shift principle to determine which clouds are moving towards Earth and which clouds are moving away from Earth.

Problem 4 - use the Doppler Formula S = 300000 (W(observed) - W(rest))/W(rest) to determine the speeds of the two clouds in kilometers/sec.

Problem 5 - Draw a possible sketch of the locations of these clouds and how they are moving. Can you create an interpretation of what the gas clouds are doing in this star-forming region?

Answer Key

Problem 1 - What is the wavelength scale of this spectrum plot in wavelength units(millimeters) per millimeter? At what wavelengths were the two intense spectral lines detected?

Answer: The distance between the tic marks for '1.30130' and '1.30116' is 27 mm, so the scale is (1.301300 - 1.30116)/27mm = 0.00014/27 = 0.0000052 mm/mm. using 1.30130 as a reference on the wavelength axis, the first line, Cloud A, is located 3mm to the left or at 1.30130 + 0.0000052 x 3 = **1.30132 mm**. The second line, Cloud B, is located 9mm to the right of 1.30116 or at 1.30116 + 0.0000052 x 9 = **1.30111 mm.**

Problem 2 - What is the wavelength of the CO line in millimeters, if its frequency is 230,538,000,000 Hertz? Identify this 'rest' wavelength on the spectrum plot.

Answer. Wavelength = speed of light/frequency so 30 billion cm/s / 230538000000 = 0.01301390 cm or **1.30130 mm.**

Problem 3 - If we assume that both of the lines are produced by the CO molecule in the cloud emitting at a rest frequency of 230.538 gigaHertz, use the Doppler Shift principle to determine which clouds are moving towards Earth and which clouds are moving away from Earth.

Answer: Cloud A is detected at a longer wavelength than the rest wavelength for CO (1.30130 mm) so it is moving away from Earth. Cloud B is detected at a shorter wavelength than the rest wavelength for CO, so it is moving towards Earth.

Problem 4 - use the Doppler Formula S = 300000 (W(observed)-W(rest))/W(rest) to determine the speeds of the two clouds in kilometers/sec.

Answer: Cloud A W(rest) = 1.30130 mm W(observed) = 1.30132 mm so S(A) = 300000 (1.30132-1.30130)/1.30130 = **+4.6 km/sec**. Cloud B W(observed) = 1.30111 mm, so S(B) = 300000 (1.30111 - 1.30130)/1.30130 = **-43.8 km/sec**

Problem 5 - Draw a possible sketch of the locations of these clouds and how they are moving. Can you create an interpretation of what the gas clouds are doing in this star-forming region?

Answer: The two clouds were detected in the same direction in the sky so there are two possible orderings of the clouds relative to Earth.

Earth	Cloud A -43.8 k/s Towards	Cloud B +4.6 k/s Away
Earth	Cloud B +4.6 k/s Away	Cloud A -43.8 k/s Towards

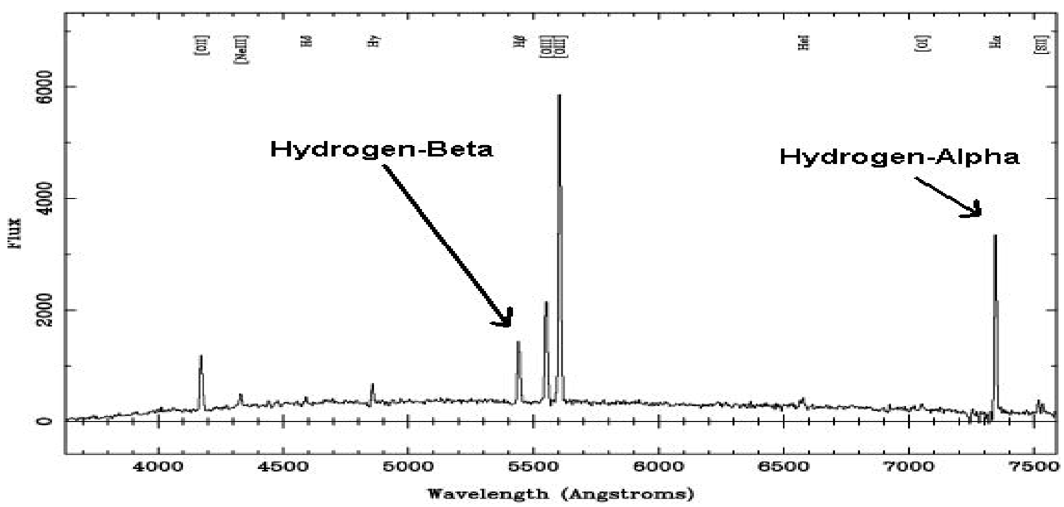
In the first case, the clouds could be expanding away from a point in between the two clouds. In the second case, the clouds could be falling into a region between them, which has an average speed of (-43.8 + 4.6)/2 = -19.6 km/s. When stars form, gas often falls into a denser region rather than expanding away from the region, so an astronomer might consider the second model more consistent with the existence of a star-forming region with infalling gas.

The Speed of Galaxy Q2125-431

The Doppler Shift is an important physical phenomenon that astronomers use to measure the speeds of distant stars and galaxies. When an ambulance approaches you, its siren seems to be pitched higher than normal, and as it passes and travels away from you, the pitch becomes lower. A careful measurement of the pitch change can let you determine the speed of the ambulance once you know the speed of the sound wave. A very similar method can be used when analyzing light waves from distant stars and galaxies. The basic formula for slow-speed motion (that is, speeds much slower than the speed of light) is:

$$\text{Speed} = 299{,}792 \; \frac{W_O - W_R}{W_R}$$

The speed of the object in km/s can be found by measuring the wavelength of the signal that you observe (Wo), and knowing what the rest wavelength of the signal is (Wr), with wavelength measured in units of Angstroms (1.0×10^{-10} meters).



This above plot is a small part of the spectrum of the Seyfert galaxy Q2125-431 in the constellation Microscopium. An astronomer has identified the spectral lines for Hydrogen Alpha, which is known to have a rest wavelength of Wr = 6563 Angstroms, and Hydrogen-Beta for which Wr = 5007 Angstroms.

Problem 1: From the graph of the spectrum above, use you millimeter ruler to determine the scale of the figure in angstroms per millimeter. What are the observed wavelengths, Wo, of the Hydrogen-Alpha and Hydrogen-Beta lines?

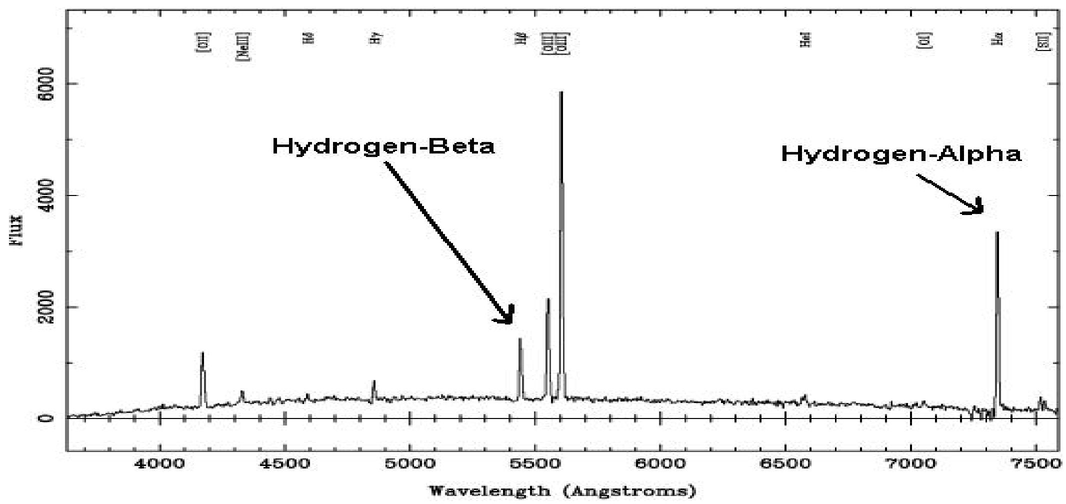
Problem 2: What are the rest wavelengths, Wr, of the Hydrogen-Alpha and Hydrogen-Beta lines?

Problem 3: What two velocities do you calculate for each line from the formula above?

Problem 4: From your answer to question 3, what is the average of the two velocities?

Problem 5: Is the galaxy moving towards or away from the Milky Way? Explain.

Answer Key



This figure was obtained from the paper “Naked Active Galactic Nuclei” by M. Hawkins, University of Edinburg, Scotland. (Astronomy and Astrophysics, 2004, vol. 424, p. 519)

Problem 1: From the graph of the spectrum, use you millimeter ruler to determine the scale of the figure in angstroms per millimeter. Answer: On the Student’s page, the wavelength scale from 4000 to 7000 Angstroms measures 100 millimeters, so the scale is (7000-4000)/100 = 30 Angstroms/mm. The observed wavelengths of the Hydrogen-Alpha and Hydrogen-Beta lines are then Alpha: 7000 A + 30 x (11.5mm) so **W0= 7345 A**. Beta: 5000 A + 30 x (14.5 mm) so **W0= 5435 A.**

Problem 2: What are the rest wavelengths, Wr, of the Hydrogen-Alpha and Hydrogen-Beta lines? **Answer:** Alpha = 6563 Angstroms; Beta = 5007 Angstroms.

Problem 3: What velocity do you calculate for each line from the formula above? **Answer:** For the Alpha line; Wr=6563 A, Wo = 7345 A. so from the formula Speed = 299792 x (7345-6563)/6563 = **35721 km/**s. For the Beta line: Wr = 5007 A, Wo = 5435 A. so Speed = 299792 x (5435-5007)/5007 = **25626 km/s**.

Problem 4: From your answer to Problem 3, what is the average of the two velocities? **Answer:** (35721 + 25626)/2 = **30,700 km/s**. **Note to Teacher:** Because it is hard to measure the wavelengths of these lines to less than 1 mm accuracy, the line wavelengths will be uncertain to about 30 Angstroms. This works out to a speed uncertainty of 299792 x (30/5007) = 1,800 km/sec. The actual difference in the two speeds is 35721-25626 = 10,095 km/sec which is much higher than the measurement uncertainty, and may mean that the regions of gas producing the H-Alpha and H-Beta emission are not moving at the same speeds within the galaxy.

Problem 5: Is the galaxy moving towards or away from the Milky Way? Explain. **Answer:** Because the observed wavelength of each line is LONGER then the wavelength for the gas at rest, the source is **moving away** from the observer (just as the pitch of an ambulance siren is lower as it moves away from you).

Useful Constants

Speed of light (c) 300,000 km/sec

1 electron Volt (eV) 1.6×10^{-19} Joules

Average solar power at top of Earth's atmosphere
1,300 watts/meter^2

Prefixes by x1000:

	Power	
Yotta	+24	1,000,000,000,000,000,000,000,000
Zetta	+21	1,000,000,000,000,000,000,000
Exa	+18	1,000,000,000,000,000,000
Peta	+15	1,000,000,000,000,000
Tera	+12	1,000,000,000,000
Giga	+ 9	1,000,000,000
Mega	+ 6	1,000,000
Kilo	+ 3	1,000
Unit	+ 0	1
Milli	- 3	0.001
Micro	- 6	0.000001
Nano	- 9	0.000000001
Pico	-12	0.000000000001
Femto	-15	0.000000000000001
Atto	-18	0.000000000000000001
Zepto	-21	0.000000000000000000001
Yocto	-24	0.000000000000000000000001

Additional Resources

The Electromagnetic Spectrum - The Landsat program created this resource to introduce students to the many ways that we learn about Earth by looking at it at different wavelengths. A DVD, poster and explanatory guide are also available for this resource.

http://landsat.gsfc.nasa.gov/education/resources.html

The Mulitwavelength Milky Way - see images of the Milky Way galaxy from radio to gamma-ray wavelengths. Downloadable posters, slides and education resources to introduce students to how astronomers study astronomical objects to learn more about them.

http://mwmw.gsfc.nasa.gov/

The Electromagnetic Spectrum -Provides a short introduction to basic concepts of wavelength and the various spectral regions.

http://imagine.gsfc.nasa.gov/docs/science/know_l1/emspectrum.html

The Electromagnetic Spectrum - An introduction to the various regions of the spectrum plus images from satellites and observatories.

http://science.hq.nasa.gov/kids/imagers/ems/index.html

Chandra Observatory - Modeling the Electromagnetic Spectrum -

This NASA resource provides information and hands-on demonstrations for middle and high school students on the EM spectrum,

http://chandra.harvard.edu/edu/formal/ems/

Land of the Magic Windows at Space Place - For younger students, this is an interactive guide to the EM spectrum using the idea that we use different windows to see the universe.

http://spaceplace.nasa.gov/en/kids/chandra.shtml

A Note from the Author

Dear Teacher and Student,

Electromagnetic radiation….even the sound of this term is a bit mysterious, and a common stock term used in low-budget science fiction movies. In fact, scientists have learned a lot about it in the 150 years they have been investigating it, but just like gravity and magnetism, the general public remains uncomfortable thinking about these ideas too closely. We still have a gut feeling that these are utterly alien concepts, despite the fact that they are the most commonplace ingredients to our lives. Let's look at two of the genuine 'mysteries' of electromagnetic radiation.

A basic property of electromagnetic radiation is that the fastest speed at which it travels is exactly 299,792.4 kilometers/sec. It cannot travel 350,000 km/sec, or even 299,795 kilometers/sec. In fact there is no physical phenomenon in our universe that can travel 'faster than light'. Other than saying that this speed limit is 'just' the way that the universe was put together at the Big Bang, there is no scientific explanation for why this speed limit is this exact value.

Because of the quantum nature of matter and energy, electromagnetic radiation is fundamentally a quantum phenomenon in which energy is in the form of particles called photons. Each photon carries a specific amount of energy. But amazingly when you calculate what this energy is, you must use terms such as wavelength or frequency, which assume that light is not based upon photons. You cannot simultaneously describe light in terms of a wave-like phenomenon and also as a quantum 'photon' phenomenon.

In this book, I have tried to show some of the simple kinds of mathematics we use to understand light, and how we can use light in its different forms to study the properties of distant objects. This is an exciting subject when you take ideas out of the laboratory and start to apply them to learning about your environment. When mathematics is applied to deciphering your world, you can learn much more about how it is put together than by just looking at 'pretty pictures' !!

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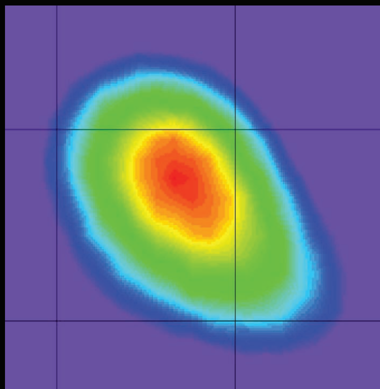
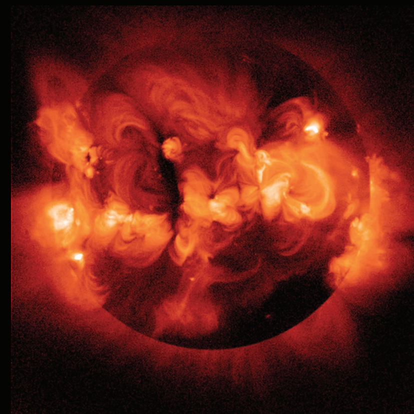
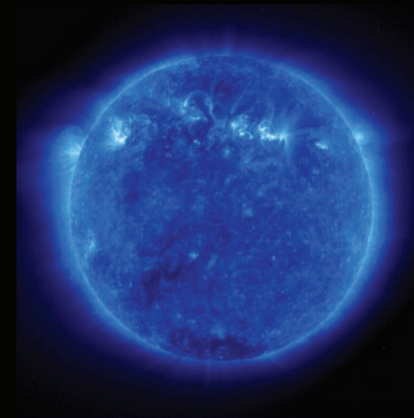
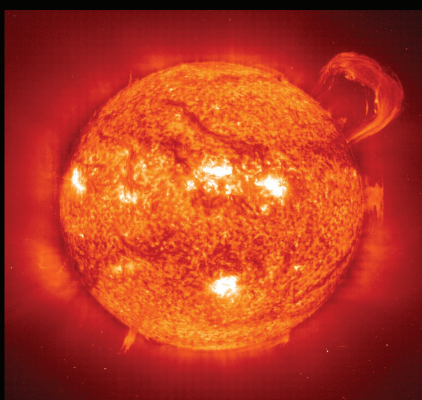
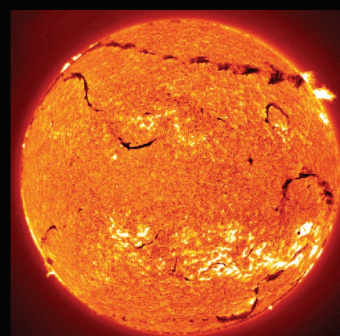
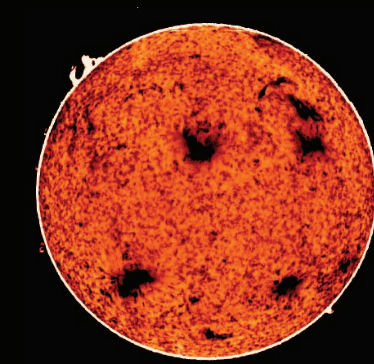
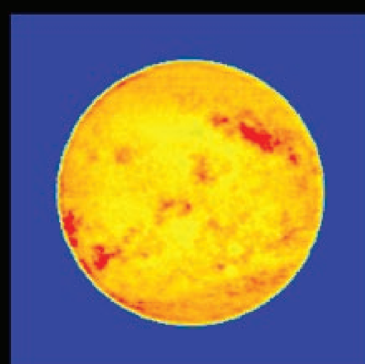
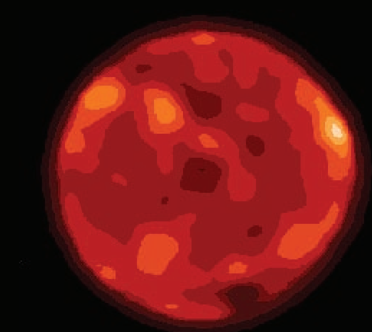
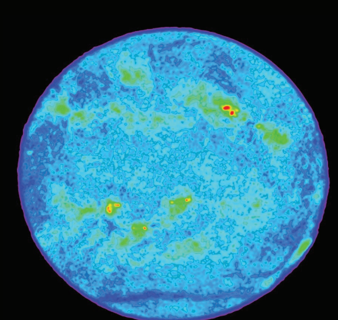
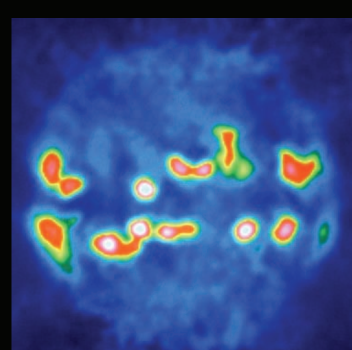
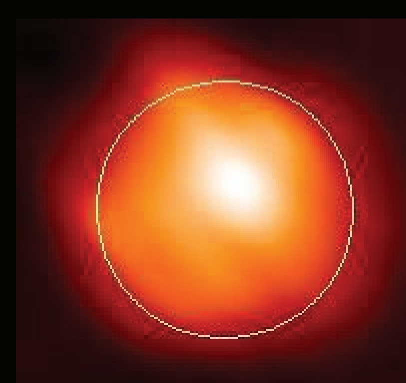
Sincerely,

Dr. Sten Odenwald
Space Math @ NASA

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