Separation-Space Study of Wide Binaries

WINGS SIT Workshop 2021

Chris Kervick

Nov 15, 2021

Carnegie Mellon University

Background

Motivation

- Wide Binaries provide probe for DM potential in dSphs[1]

[1] Peñarrubia *et al.*, 2016

Motivation

- Wide Binaries provide probe for DM potential in dSphs[1]
- Easily disrupted by host tidal field

[1] Peñarrubia *et al.*, 2016

Motivation

- Wide Binaries provide probe for DM potential in dSphs[1]
- Easily disrupted by host tidal field
    - DM-dominated in ultra-faint dSphs

[1] Peñarrubia *et al.*, 2016

Motivation

- Wide Binaries provide probe for DM potential in dSphs[1]
- Easily disrupted by host tidal field
    - DM-dominated in ultra-faint dSphs
- Distribution of wide binaries sensitive to slope of DM halo profile at $R \sim R_h$, half-light radius

[1] Peñarrubia *et al.*, 2016

Motivation

- Wide Binaries provide probe for DM potential in dSphs[1]
- Easily disrupted by host tidal field
    - DM-dominated in ultra-faint dSphs
- Distribution of wide binaries sensitive to slope of DM halo profile at $R \sim R_h$, half-light radius

[1] Peñarrubia *et al.*, 2016

Motivation

- Wide Binaries provide probe for DM potential in dSphs[1]
- Easily disrupted by host tidal field
    - DM-dominated in ultra-faint dSphs
- Distribution of wide binaries sensitive to slope of DM halo profile at $R \sim R_h$, half-light radius

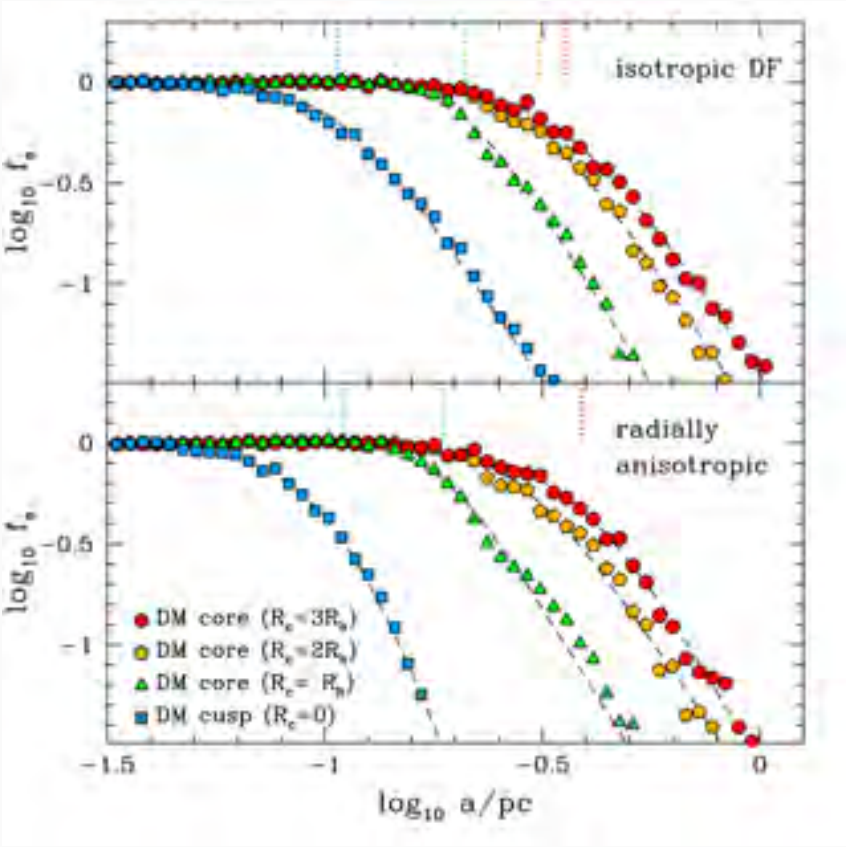

Figure 1: Fraction of surviving binaries vs semi-major axis for several DM halo profiles from N-body simulations[1]

[1] Peñarrubia *et al.*, 2016

Separation Distribution of Binary Stars

- In solar neighbourhood, wide binary separation function well described by a broken power law (BPL) [2]:

$$\phi_b(s) = \phi_0 \left(\frac{s}{s_b}\right)^{\gamma_1} \left[\frac{1}{2}\left(1 + \left(\frac{s}{s_b}\right)^{\frac{1}{\Lambda}}\right)\right]^{(\gamma_2-\gamma_1)\Lambda} \tag{1}$$

  where s_b is the break separation, γ_1 and γ_2 are the two power law indices, and Λ is a smoothness parameter.

[2] Tian *et al.*, 2019

Separation Distribution of Ultrawide dSph Binaries

Three models for binary separations within dSphs[3]:

- Broken Power Law: Smooth transition from $\gamma_1 = 0.5$ to $\gamma_2 = -1$

All three can be modelled by BPL equation via different choice of parameters.

[3] Peñarrubia, 2021

Separation Distribution of Ultrawide dSph Binaries

Three models for binary separations within dSphs[3]:

- Broken Power Law: Smooth transition from $\gamma_1 = 0.5$ to $\gamma_2 = -1$
- Truncation: Sharp truncation from $\gamma_1 = 0.5$ at break radius

All three can be modelled by BPL equation via different choice of parameters.

[3] Peñarrubia, 2021

Separation Distribution of Ultrawide dSph Binaries

Three models for binary separations within dSphs[3]:

- Broken Power Law: Smooth transition from $\gamma_1 = 0.5$ to $\gamma_2 = -1$
- Truncation: Sharp truncation from $\gamma_1 = 0.5$ at break radius
- Öpik's law: No transition, $\gamma_1 = \gamma_2 = -1$

All three can be modelled by BPL equation via different choice of parameters.

[3] Peñarrubia, 2021

Analysing Separation Distribution

Interested in determining parameters of the binary separation function.

- Usual Method: two-point correlation functions

Analysing Separation Distribution

Interested in determining parameters of the binary separation function.

- Usual Method: two-point correlation functions
- Basic Estimator

$$\xi(s) = \frac{DD(s)}{DR(s)} - 1$$

(or Landy-Szalay, etc)

Analysing Separation Distribution

Interested in determining parameters of the binary separation function.

- Usual Method: two-point correlation functions
- Basic Estimator

$$\xi(s) = \frac{DD(s)}{DR(s)} - 1$$

(or Landy-Szalay, etc)

  - $DD =$ number of binned pairs at separation s in data

Analysing Separation Distribution

Interested in determining parameters of the binary separation function.

- Usual Method: two-point correlation functions
- Basic Estimator

$$\xi(s) = \frac{DD(s)}{DR(s)} - 1$$

(or Landy-Szalay, etc)

- $DD =$ number of binned pairs at separation s in data
- $DR =$ number of binned pairs between stars in data set and a random, uniform mock data set

Analysing Separation Distribution

However, this method has several drawbacks:

- Requires averaging over many mock data sets for statistically significant results

Analysing Separation Distribution

However, this method has several drawbacks:

- Requires averaging over many mock data sets for statistically significant results
- Computationally expensive for large number of stars

Analysing Separation Distribution

However, this method has several drawbacks:

- Requires averaging over many mock data sets for statistically significant results
- Computationally expensive for large number of stars
- Sensitive to choice of bin size

Analysing Separation Distribution

However, this method has several drawbacks:

- Requires averaging over many mock data sets for statistically significant results
- Computationally expensive for large number of stars
- Sensitive to choice of bin size
- Introduces potential bias (over/under estimation at various scales)

Analysing Separation Distribution

- New Method: Fit separation distribution of stars directly, $\phi(s)$.

Analysing Separation Distribution

- New Method: Fit separation distribution of stars directly, $\phi(s)$.
- Total separation distibution becomes sum of, e.g. Plummer-Plummer separations + binary separations.

$$\phi'(s) \approx (1 + f_{\rm b})^2 \, N^2 \, \phi(s) + 2 \, f_{\rm b} N \, \phi_{\rm b}(s). \tag{2}$$

Analysing Separation Distribution

- New Method: Fit separation distribution of stars directly, $\phi(s)$.
- Total separation distibution becomes sum of, e.g. Plummer-Plummer separations + binary separations.

$$\phi'(s) \approx (1 + f_{\rm b})^2 \, N^2 \, \phi(s) + 2 \, f_{\rm b} N \, \phi_{\rm b}(s). \qquad (2)$$

- Ideally want an analytical expression for this distribution for use in fitting algorithms (to avoid Numerical Integration)

Direct Separation Functions

Direct Separation Functions

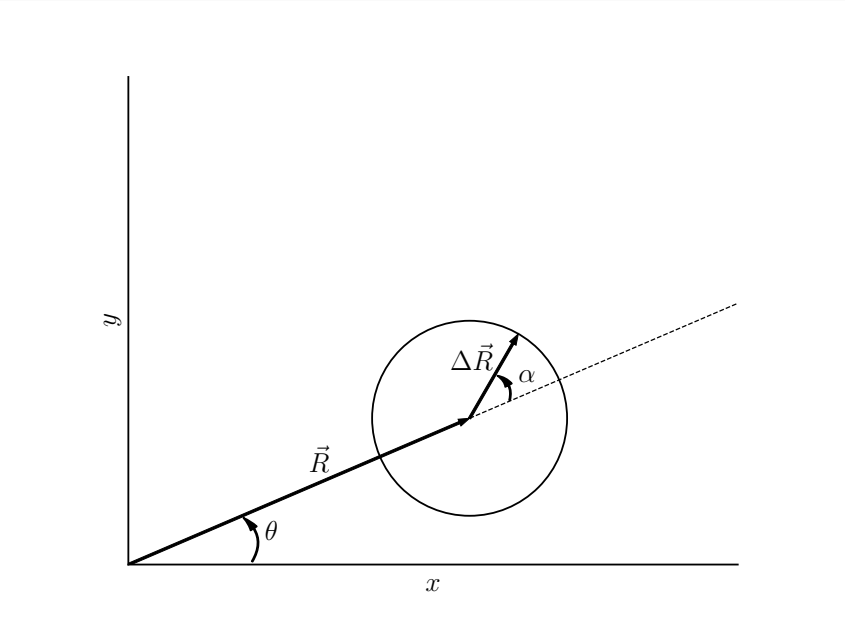

Figure 2: Geometry for the integral

- Perform integral of joint probability function to find PDF for pair separation

Direct Separation Functions

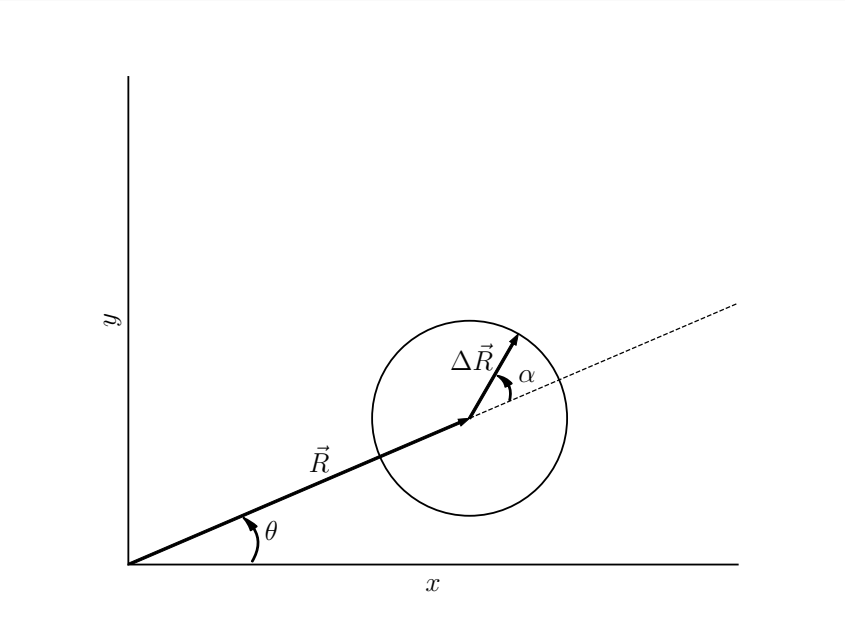

Figure 2: Geometry for the integral

- Perform integral of joint probability function to find PDF for pair separation
- Yields analytical results for Uniform distribution and Plummer distributions, along with cross terms.

Plummer and Uniform Separation Functions

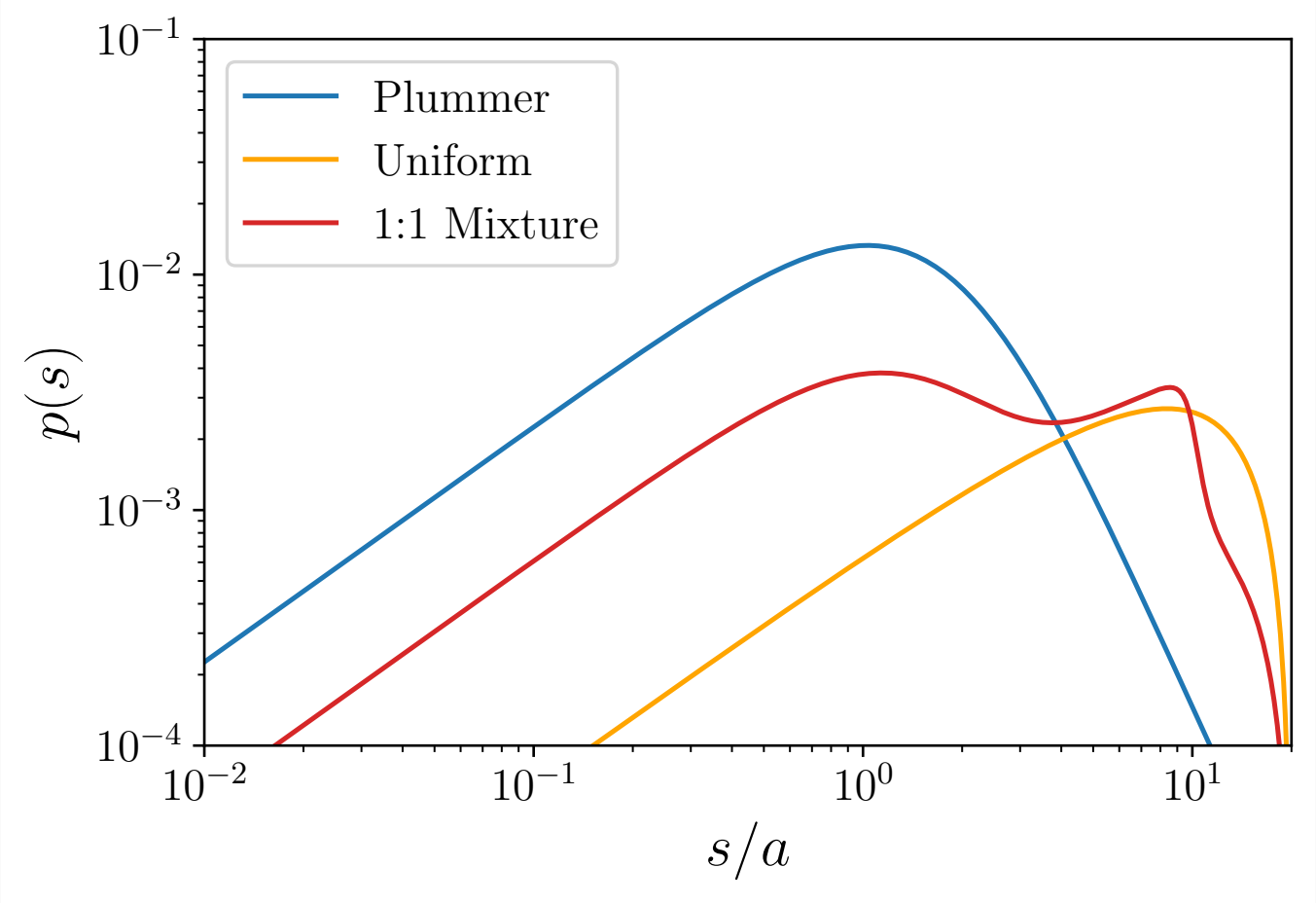

Figure 3: Separation function for Plummer (blue), Uniform (orange) and 1:1 mixture (red).

Analysing the Binary Separation Distribution

Analysing the Binary Separation Distribution

Can extract more information than just number of binaries:

- Using Markov-Chain Monte Carlo (MCMC) methods, maximize a suitable log-likelihood function to fit for parameters of Plummer function and broken power law

[4] Buchner *et al.*, 2014

Analysing the Binary Separation Distribution

Can extract more information than just number of binaries:

- Using Markov-Chain Monte Carlo (MCMC) methods, maximize a suitable log-likelihood function to fit for parameters of Plummer function and broken power law
- We use MultiNest[4] to perform the fitting

[4] Buchner *et al.*, 2014

Analysing the Binary Separation Distribution

Can extract more information than just number of binaries:

- Using Markov-Chain Monte Carlo (MCMC) methods, maximize a suitable log-likelihood function to fit for parameters of Plummer function and broken power law
- We use MultiNest[4] to perform the fitting
- In practise, use joint position-separation function for fitting (population probability is position-dependent, accounts for covariance.)

[4] Buchner *et al.*, 2014

Analysing the Binary Separation Distribution

We apply our method to a set of 40 dwarf spheroidal galaxies[5]

- Populate a mock data set for each galaxy using isochrone files[6]

[5] Moskowitz and Walker, 2020

[6] Bressan *et al.*, 2012

Analysing the Binary Separation Distribution

We apply our method to a set of 40 dwarf spheroidal galaxies[5]

- Populate a mock data set for each galaxy using isochrone files[6]
- Stellar positions: 50% according to a Plummer law, 50% uniform foreground.

[5] Moskowitz and Walker, 2020

[6] Bressan *et al.*, 2012

Analysing the Binary Separation Distribution

We apply our method to a set of 40 dwarf spheroidal galaxies[5]

- Populate a mock data set for each galaxy using isochrone files[6]
- Stellar positions: 50% according to a Plummer law, 50% uniform foreground.
- To both populations add binaries with positions drawn according the the relevant broken power law. Binary fraction $f_b = 0.1$ for both populations.

[5] Moskowitz and Walker, 2020

[6] Bressan *et al.*, 2012

Analysing the Binary Separation Distribution

We apply our method to a set of 40 dwarf spheroidal galaxies[5]

- Populate a mock data set for each galaxy using isochrone files[6]
- Stellar positions: 50% according to a Plummer law, 50% uniform foreground.
- To both populations add binaries with positions drawn according the the relevant broken power law. Binary fraction $f_b = 0.1$ for both populations.
- Resolution cut at 0.05 arcsec, magnitude cut at 27th magnitude in V-band (550nm)

[5] Moskowitz and Walker, 2020

[6] Bressan *et al.*, 2012

Results

Binary Fit Posteriors

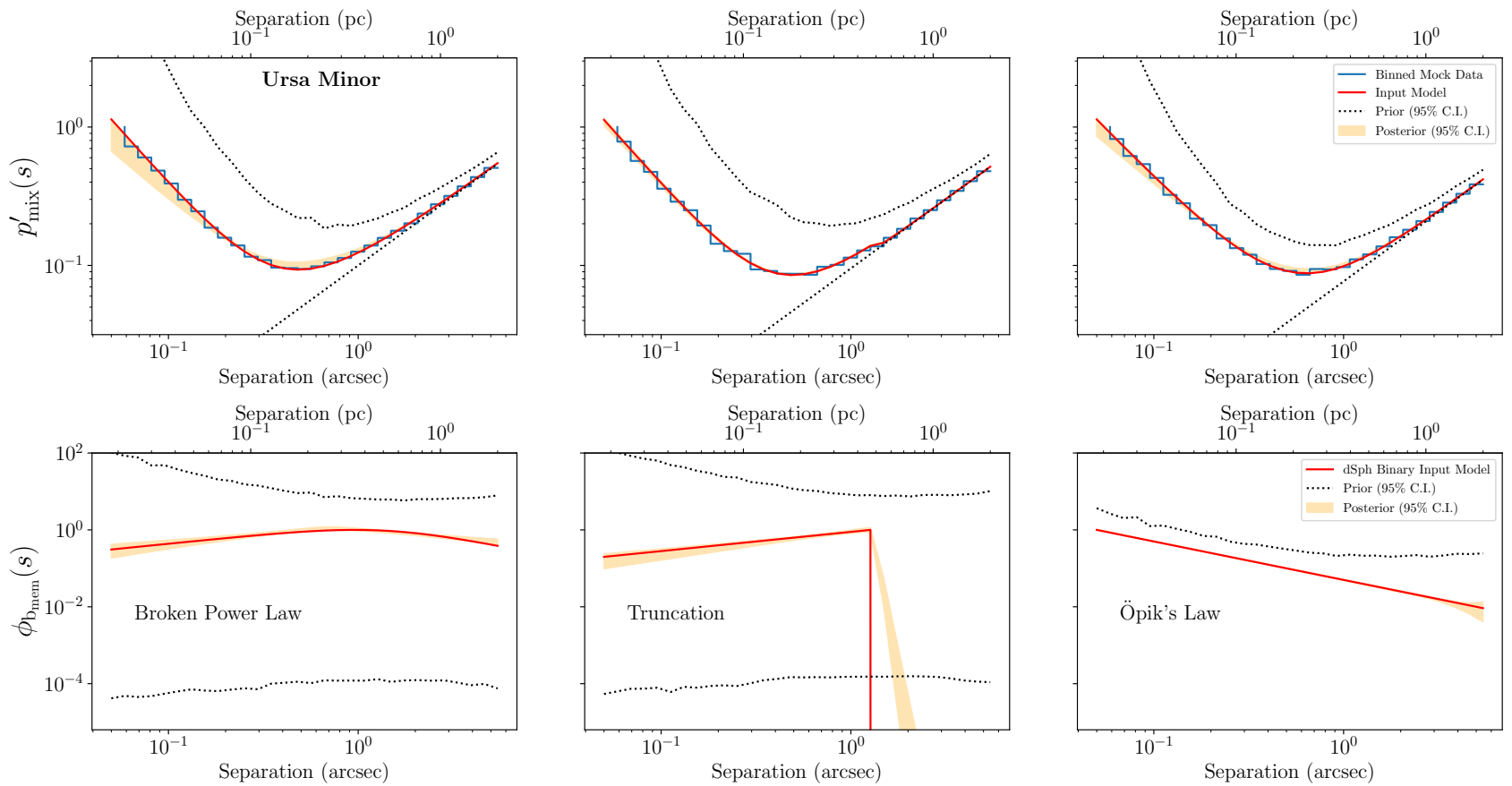

Figure 4: Histogrammed Separations and Posterior Fits for Ursa Minor (3000 detectable binaries)

arXiv: 2107.07554

Binary Fit Posteriors

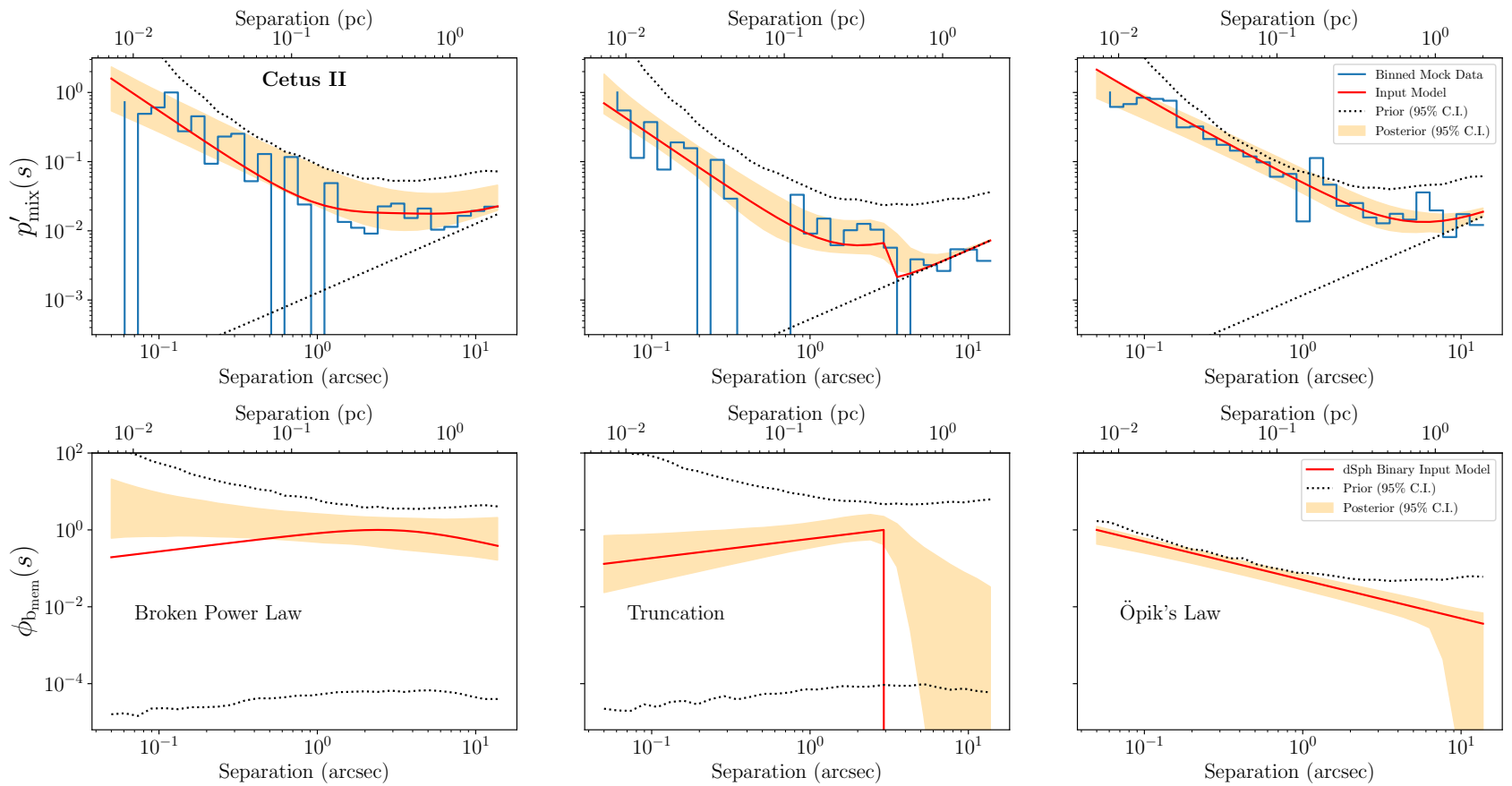

Figure 5: Histogrammed Separations and Posterior Fits for Cetus II ($10 - 100$ detectable binaries)

arXiv: 2107.07554

Binary Fit Posteriors

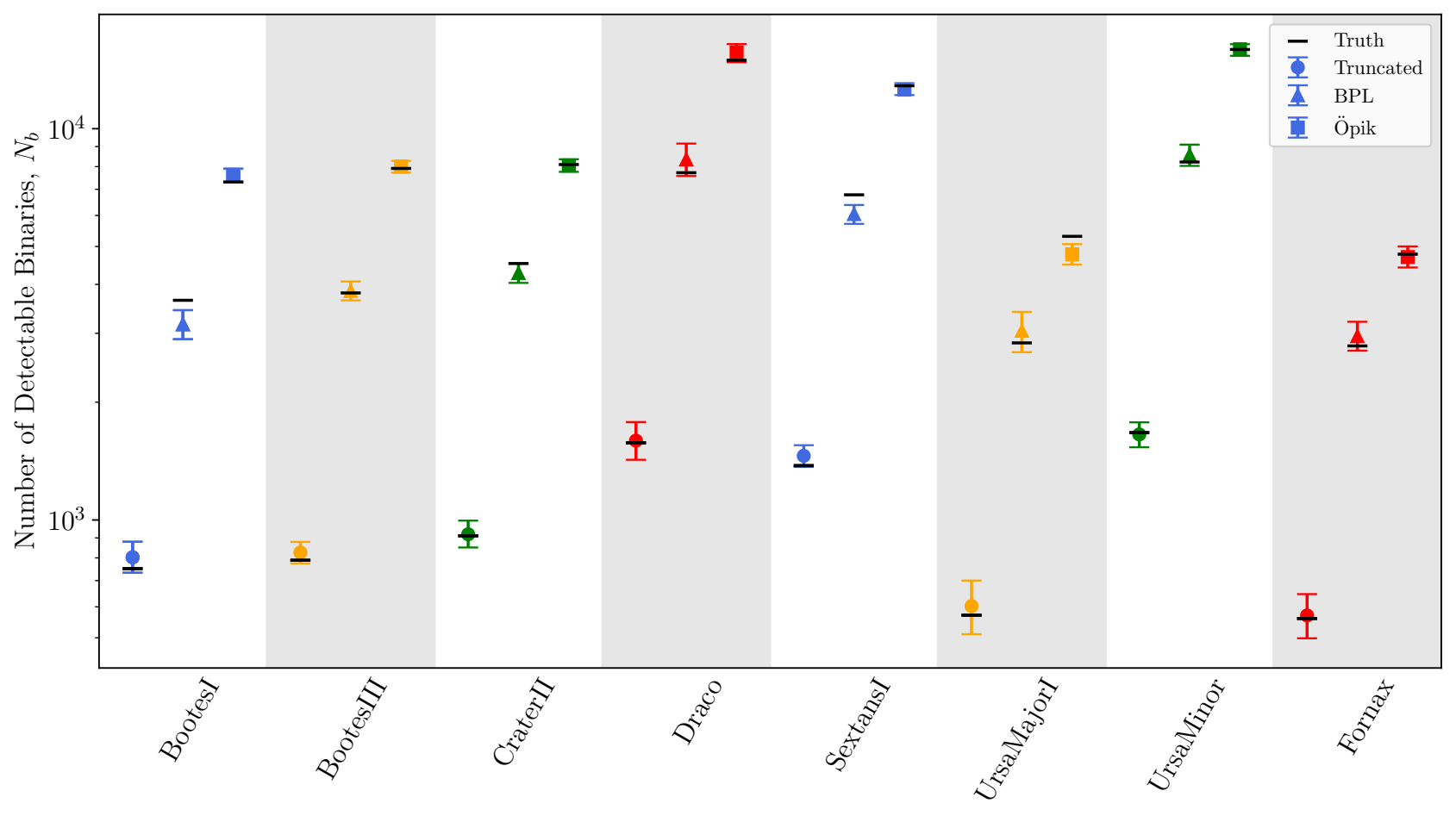

Figure 6: MCMC fit: Number of Binary Pairs ($N_b > 500$)
arXiv: 2107.07554

Binary Fit Posteriors

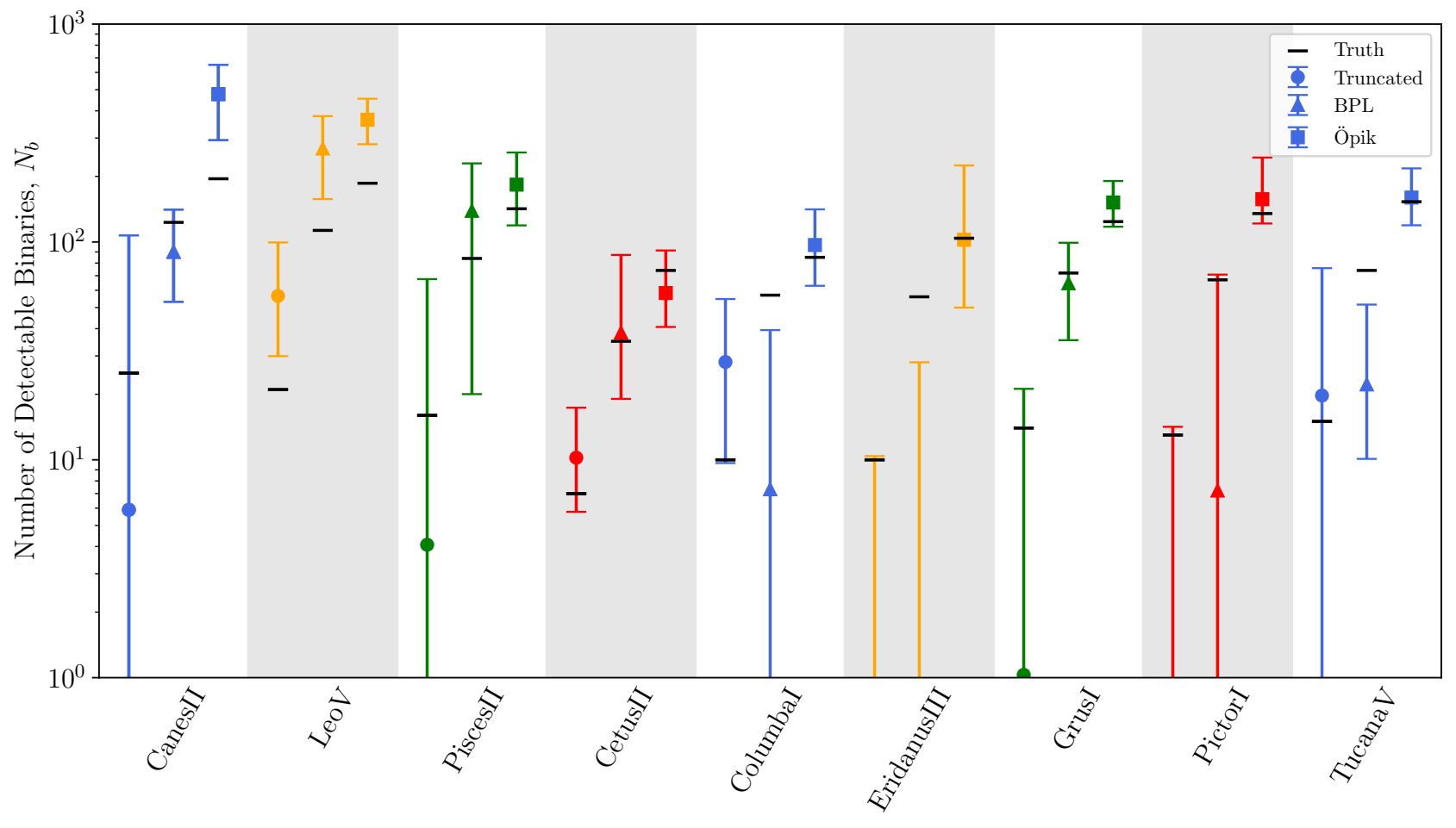

Figure 7: MCMC fit: Number of Binary Pairs ($N_b < 200$)
arXiv: 2107.07554

Binary Fit Posteriors

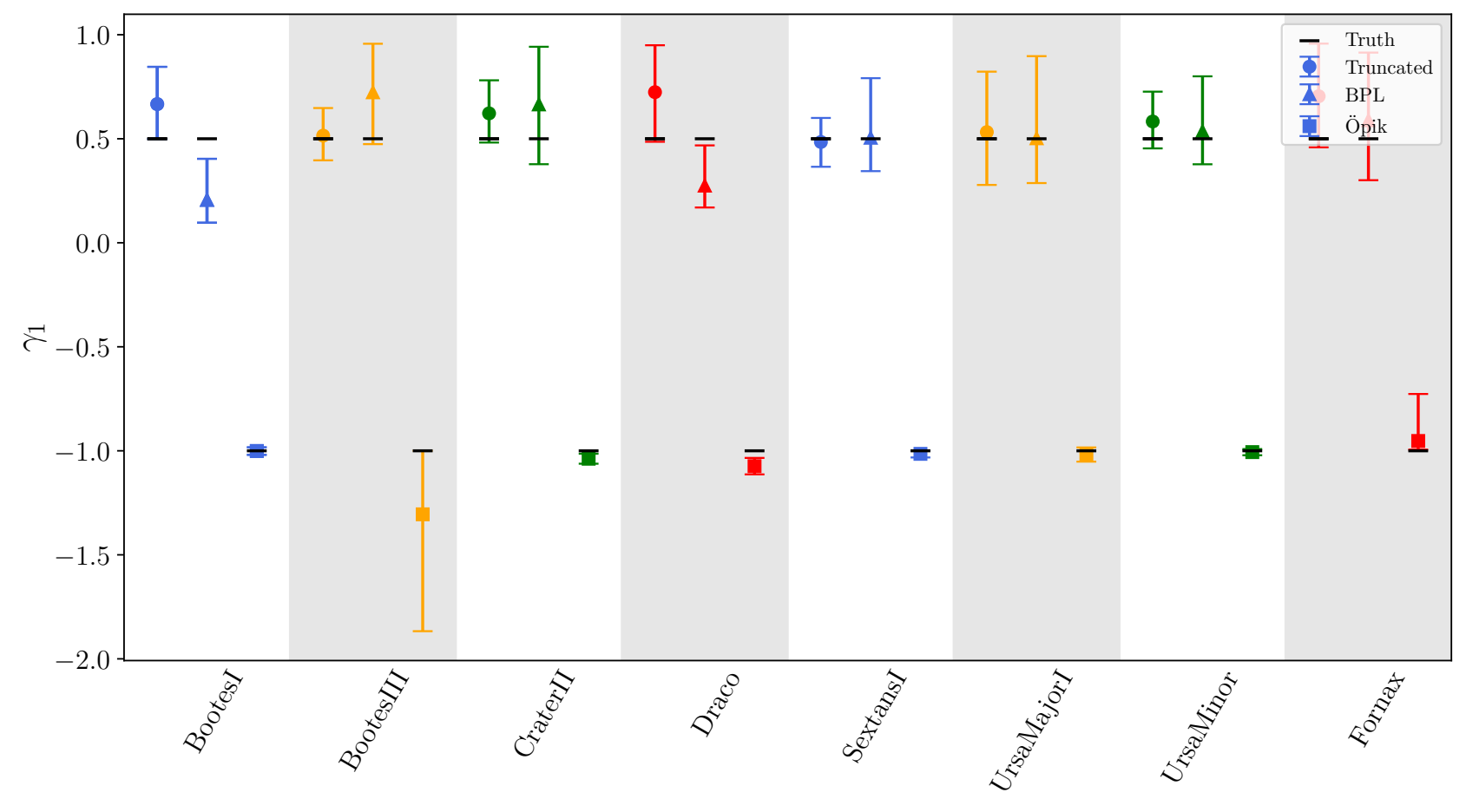

Figure 8: MCMC fit: Power Law Index γ_1
arXiv: 2107.07554

Binary Fit Posteriors

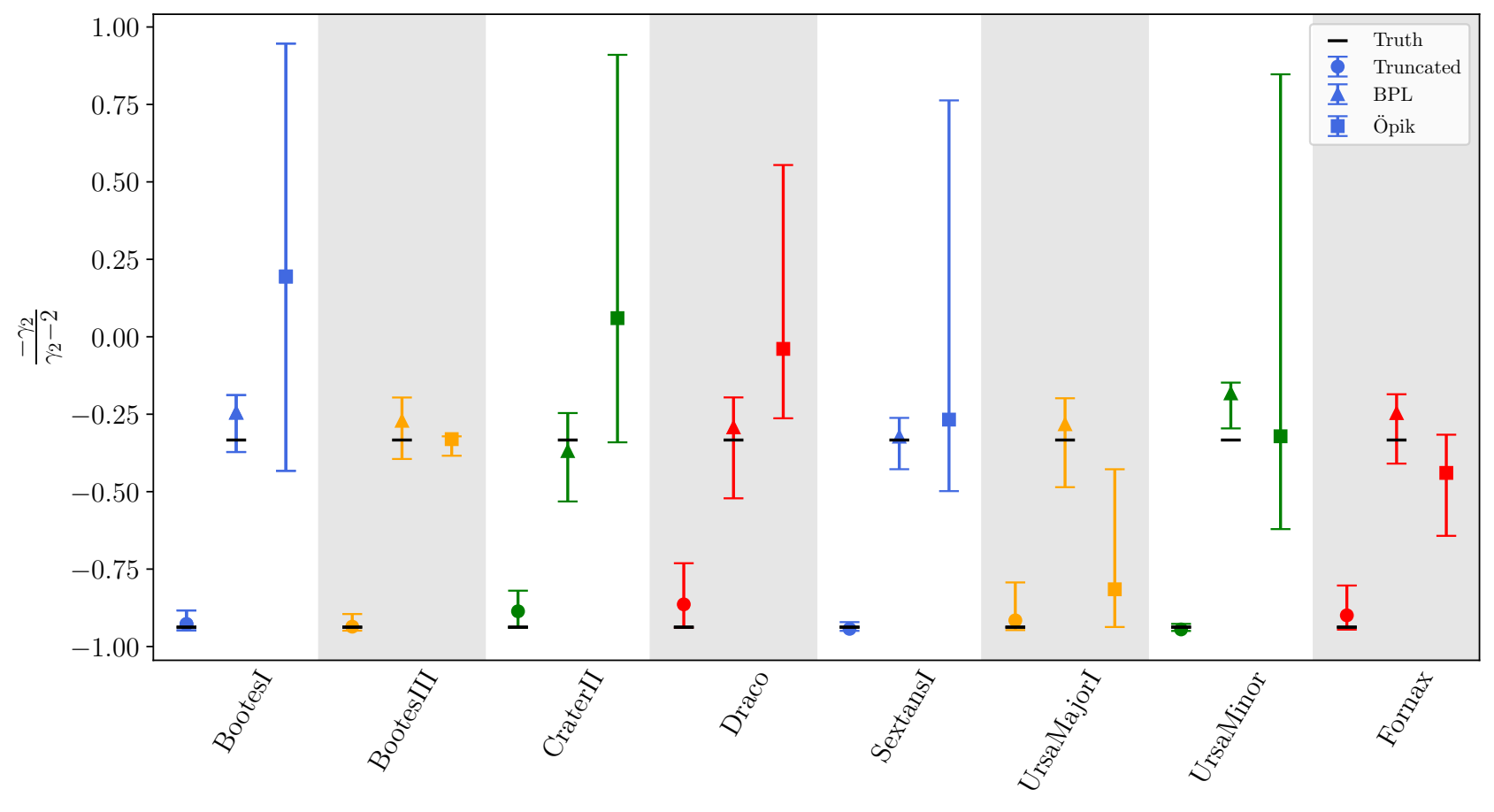

Figure 9: MCMC fit: Power Law Index γ_2
arXiv: 2107.07554

Binary Fit Posteriors

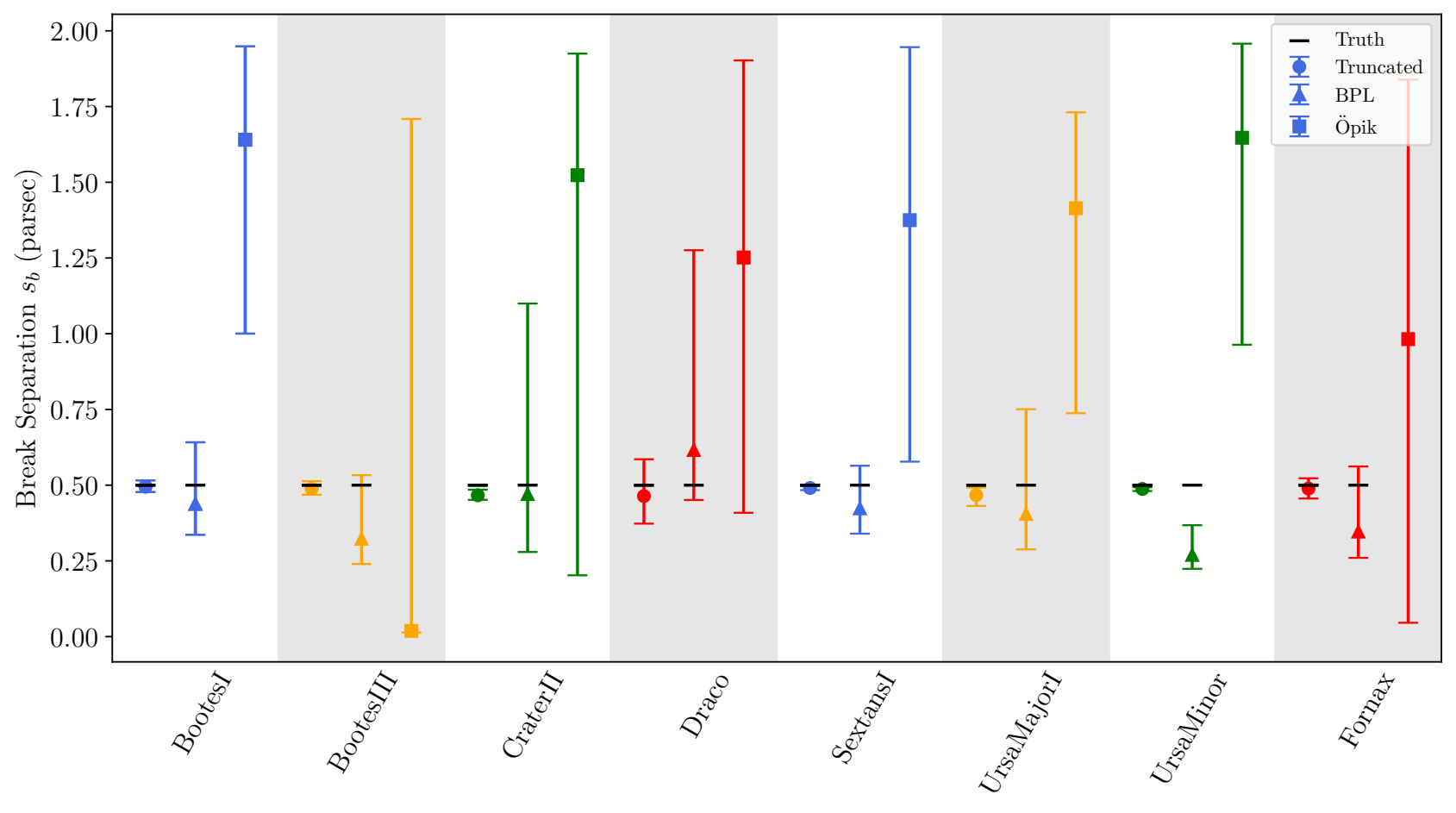

Figure 10: MCMC fit: Break Separation s_b
arXiv: 2107.07554

Conclusions

Conclusions

- Using MCMC, we can recover properties of binary separation PDF for mock data sets w/ ROMAN-like mag cuts + res limits

Conclusions

- Using MCMC, we can recover properties of binary separation PDF for mock data sets w/ ROMAN-like mag cuts + res limits
- Analysing these results helps us identify good observation candidates for investigating properties of wide binaries in dSphs

Conclusions

- Using MCMC, we can recover properties of binary separation PDF for mock data sets w/ ROMAN-like mag cuts + res limits
- Analysing these results helps us identify good observation candidates for investigating properties of wide binaries in dSphs
    - No false-positives when $N_b = 0$

Conclusions

- Using MCMC, we can recover properties of binary separation PDF for mock data sets w/ ROMAN-like mag cuts + res limits
- Analysing these results helps us identify good observation candidates for investigating properties of wide binaries in dSphs
  - No false-positives when $N_b = 0$
- WIP: Elliptical dSphs, finite field size, arbitrary field shape (all require numerical integration as separation function no longer analytically integratable).

Conclusions

- Using MCMC, we can recover properties of binary separation PDF for mock data sets w/ ROMAN-like mag cuts + res limits
- Analysing these results helps us identify good observation candidates for investigating properties of wide binaries in dSphs
  - No false-positives when $N_b = 0$
- WIP: Elliptical dSphs, finite field size, arbitrary field shape (all require numerical integration as separation function no longer analytically integratable).
- WIP: Introduce magnitude/contrast- dependent selection function.