National Aeronautics and Space Administration

Math Problems Related to

The Universe

Introduction

This book provides an assortment of mathematics problems that deal with our universe, the Big Bang, distant galaxies and unusual objects discovered by NASA telescopes. Our sun and solar system are part of a vast system of matter called the universe, which has been in existence for 13.8 billion years. These math problems introduce the scale of this system and how math is used to investigate various phenomena in physics, which are responsible for the shape and structure of galaxies.

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Galaxies to Scale

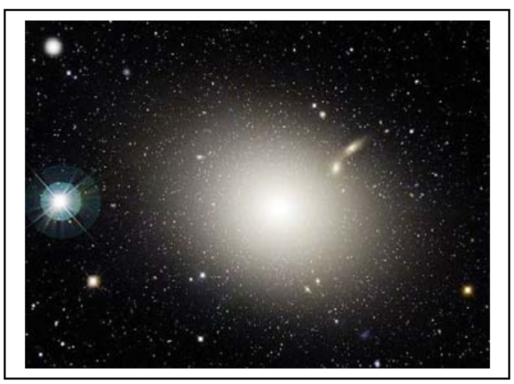
The Milky Way is a spiral galaxy. There are many other kinds of galaxies, some much larger then the Milky Way, and some much smaller. This exercise lets you create a scale model of the various kinds, and learn a little about working with fractions too!

Problem 1 - The irregular galaxy IC-1613 is twice as large as the elliptical galaxy M-32, but 10 times smaller than the spiral galaxy NGC-4565. How much larger is NGC-4565 than M-32?

Problem 2 - The spiral galaxy Andromeda is three times as large as the elliptical galaxy NGC-5128, and NGC-5128 is 4 times as large as the Large Magellanic Cloud, which is an irregular galaxy. How much larger is the Andromeda galaxy than the Large Magellanic Cloud?

Problem 3 - The Milky Way spiral galaxy is 13 times larger than the irregular galaxy IC-1613. How much larger than NGC-4565 is the Milky Way?

Problem 4 - The elliptical galaxy Leo-1 is 1/4 as large as the elliptical galaxy Messier-32, and the spiral galaxy Messier-33 is 9 times larger than Messier-32. How large is Leo-1 compared to Messier-33?



Problem 5 - The elliptical galaxy NGC-205 is 2/3 as large as the Large Magellanic Cloud. How large is NGC-205 compared to the Andromeda galaxy?

Problem 6 - The irregular galaxy NGC-6822 is 8/5 the diameter of Messier-32, and Messier-32 is 20 times smaller than NGC-4565. How large is NGC-6822 compared to IC-1613?

Problem 7 - Draw a scale model of these galaxies showing their relative sizes and their shapes.

Images: Top: The spiral galaxy Messier 74 taken by the Gemini Observatory; Bottom: The elliptical galaxy Messier-87 obtained at the Canada-France-Hawaii Telescope (copyright@cfht.hawaii.edu);

Answer Key

The galaxies used in this exercise, with the diameter given in light years, and relative to Messier-32:

Name	Type	Diameter	M-32
Large Magellanic Cloud	Irregular	15,000 LY	3
NGC-5128	Elliptical	65,000	13
NGC-4565	Spiral	100,000	20
IC-1613	Irregular	10,000	2
Andromeda	Spiral	200,000	40
NGC-205	Elliptical	10,000	2
Messier-32	Elliptical	5,000	1
Milky Way	Spiral	130,000	26
Messier-33	Spiral	45,000	9
Leo-1	Elliptical	1,000	1/4
NGC-6822	Irregular	8,000	8/5

Problem 1 - IC-1613/M-32 = 2.0 , NGC-4565/IC-1613 = 10 so NGC-4565/M-32 = 10 x 2 = **20 times**

Problem 2 - Andromeda/NGC-5128 = 3 and NGC-5128/LMC = 4 so Andromeda/LMC = 3 x 4 =**12 times**

Problem 3 - MW/IC-1613 = 13 and also NGC-4565/IC-1613 = 10, so Milky Way / NGC-4565 = 13 x 1/10 = **1.3 times.**

Problem 4 - Leo-1 / M-32 = 1/4 and M-33 / M-32 = 9, so Leo-1 / M-33 = 1/4 x 1/9 = **1/36 times smaller**.

Problem 5 - NGC-205 / LMC = 2/3 and Andromeda/LMC = 12 so NGC-205 / Andromeda = 2/3 x 1/12 = **2/36 or 1/18 as large.**

Problem 6 - NGC-6822 / M-32 = 8/5 and M-32 / NGC-4565 = 1/20 and NGC-4565 / IC-1613 = 10 so NGC-6822 / IC-1613 = 8/5 x 1/20 x 10 = 8/5 x 1/2 = **8/10 or 4/5 as large.**

Problem 7 - Students can use the ratios in the problems, together with the ones they derived, to create a table that gives the relative sizes for each galaxy. The table at the top gives the 'official' numbers, and the relative sizes in the last column.

Galaxy Distances and Mixed Fractions

Our Milky Way galaxy is not alone in the universe, but has many neighbors.

The distances between galaxies in the universe are so large that astronomers use the unit 'megaparsec' (mpc) to describe distances.

One *mpc* is about 3 1/4 million light years.

Hubble picture of a Ring Galaxy (AM 0644 741) at a distance of 92 mpc.

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way?

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy?

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy?

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy?

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy?

Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55?

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?

Answer Key

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way? Answer: 12 mpc - 3/4 mpc = **11 1/4 mpc**

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy? Answer: 12 mpc - 3 4/5 mpc = **8 1/5 mpc**

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy? Answer: 19 mpc - 3 4/5 mpc = **15 1/5 mpc.**

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 3 1/5 mpc - 3/4 mpc = 16/5 mpc - 3/4 mpc = 64/20 mpc - 15/20 mpc = 49/20 mpc = **2 9/20 mpc.**

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 4 2/5 mpc - 3/4 mpc = 88/5 mpc - 15/20 mpc = 73/20 mpc = **3 13/20 mpc**

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy? Answer: 4 1/5 mpc - 3 4/5 mpc = 21/5 mpc - 19/5 mpc = **2/5 mpc.**

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy? Answer: 2 1/3 mpc - 3/4 mpc = 7/3 mpc - 3/4 mpc = 28/12 mpc - 9/12 mpc = 19/12 mpc = **1 7/12 mpc**.

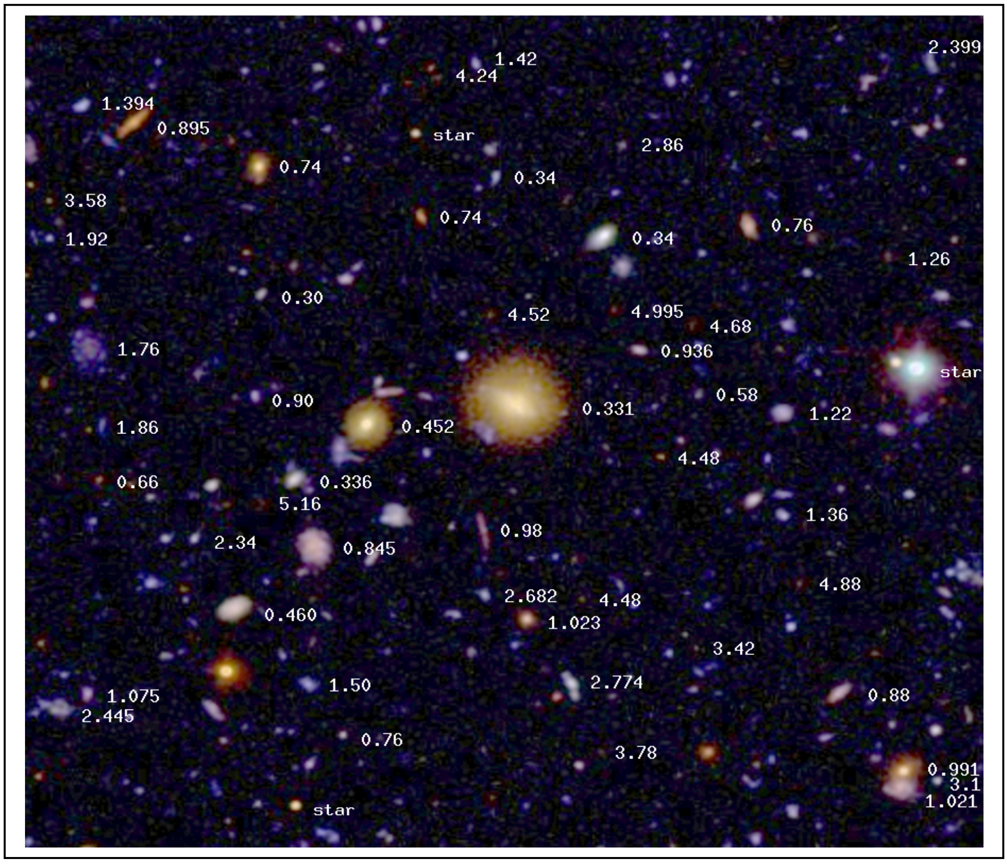
Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55? Answer; NGC-55 is located 2 1/3 mpc from the Milky Way, so the mystery galaxy is located 2 1/3 mpc + 2 1/15 mpc = 7/3 mpc + 31/15 mpc = 35/15 mpc + 31/15 mpc = 66/15 mpc = 4 6/15 mpc or **4 2/5 mpc**. This is the distance to the Centaurus-A galaxy.

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?
Answer: The distance is 19 megaparsecs, but 1 parsec equals 3 1/4 light years, so the distance to the Virgo Cluster is

19 million parsecs x (3 1/4 lightyears/parsec) = 19 x 3 1/4 = 19 x 12/4 = 228/4 = **57 million light years**

Note: Galaxies are actually located in 3-dimensional space, but to make this problem work we have assumed that the galaxies are all located along a straight like with the Milky Way at the center.

In 2004, astronomer Immo Appenzeller and his colleagues from Germany and the United States used the FORS camera at the ESA-VLT observatory in Chile to create a Deep Field image of a small piece of the sky. The goal of this research was to find the most distant, and therefore youngest, galaxies possible so that they could study how the earliest generations of stars in the universe were formed. They obtained the photograph below, and the redshifts are noted for some of the galaxies they were later able to identify and obtain spectra. (http://www.lsw.uni-heidelberg.de/users/jheidt/fdf/pics/pics.html).



Question 1: The numbers indicate the redshifts of the galaxies identified in the field. What does the histogram of the redshifts look like if you bin the number of galaxies at intervals of 0.5 in Z?

Question 2: What is the largest redshift seen for any galaxy in this field? What is the smallest? What is the average redshift? What are the mode and median redshifts?

Question 3: Use the redshift calculator at http://sa1.star.uclan.ac.uk/~cph/redshift.html to determine the time into the past (look-back time) that each galaxy image represents. For instance, the look-back time for our sun is 8.5 minutes, and the nearest star is 4.3 years. Use Omega-M = 0.3 and Lambda =0.7 with H0 = 71 km/sec/mpc which are the parameters that define our universe. Calculate from your answers to Question 2 the corresponding look-back times. Note that at 13.7 billion years, you are looking back to the formation of the universe in the Big Bang.

Question 4: Our best model for the universe indicates an age of 13.7 billion years. What is the longest look-back time you found for a galaxy in Question 3? How long after the Big Bang did this galaxy form?

Question 1: The numbers indicate the redshifts of the galaxies identified in the field. What does the histogram of the redshifts look like if you bin the number of galaxies at intervals of 0.5 in Z?
Answer: Count the number of galaxies in each interval of Z, and plot the histogram (bar graph).

Z	Number of galaxies
0.0 < Z < 0.5	6
0.5 < Z < 1.0	15
1.0 < Z < 1.5	8
1.5 < Z < 2.0	3
2.0 < Z < 2.5	3
2.5 < Z < 3.0	3
3.0 < Z < 3.5	2
3.5 < Z < 4.0	2
4.0 < Z < 4.5	3
4.5 < Z < 5.0	4
5.0 < Z < 5.5	1

Question 2: What is the largest redshift seen for any galaxy in this field? What is the smallest? What is the average redshift? What are the mode and median redshifts? **Answer: Largest redshift z= 5.16. Smallest redshift z= 0.30.**

Question 3: Use the redshift calculator at http://sa1.star.uclan.ac.uk/~cph/redshift.html to determine the time into the past (look-back time) that each galaxy image represents. For instance, the look-back time for our sun is 8.5 minutes, and the nearest star is 4.3 years. Use Omega-M = 0.3 and Lambda-=0.7 with H0 = 71 km/sec/mpc to represent our universe. Calculate from your answers to Question 2 the corresponding look-back times. Note that at 13.7 billion years, you are looking back to the formation of the universe in the Big Bang.
Answer: Largest redshift of z=5.16 yields a look-back time of 12.2 billion years. Smallest redshift z=0.30 yields a look-back time of 3.4 billion years.

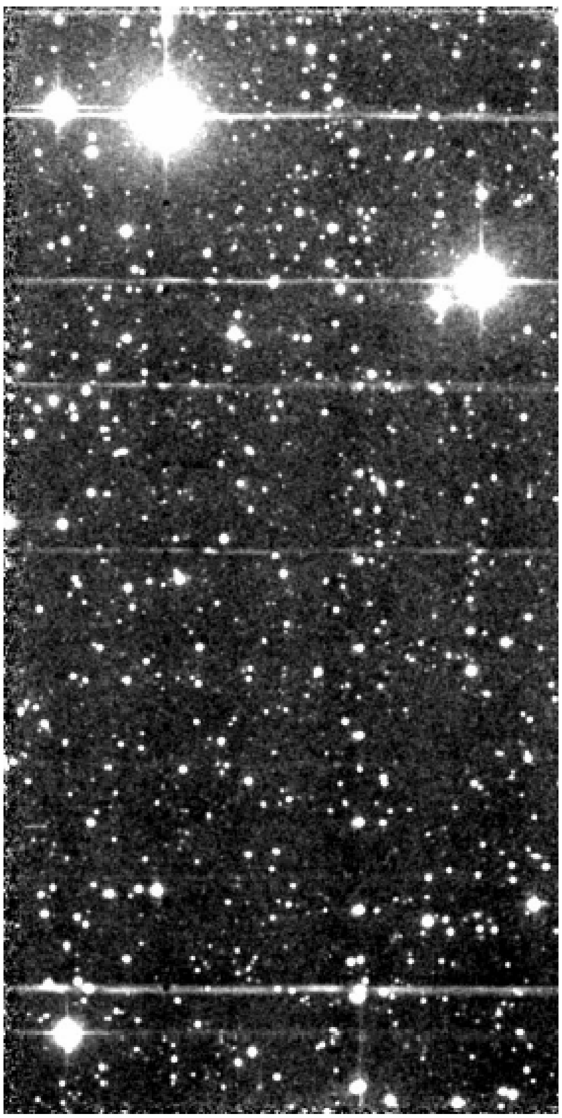
Question 4: Our best model for the universe indicates an age of 13.7 billion years. What is the longest look-back time you found for a galaxy in Question 3? How long after the Big Bang did this galaxy form? **Answer: The longest look-back time was 12.2 billion years. This galaxy would have formed about 13.7-12.2 = 1.5 billion years after the Big Bang.**

Note to Teacher. The age of our Milky Way galaxy is between 12.0 and 12.8 billion years, so this distant galaxy formed about the same time as the Milky Way did. The age of our Earth, 4.5 billion years, and the light we are seeing from galaxies with a redshift of Z = 0.45 is about this old! Astronomers use the redshift and light travel age interchangeably.

Teachers: Have your students play with the online redshift calculator by trying different universes than our own (values for Lambda, Omega and Hubble constant) and see how a galaxy's estimated distance and look-back time depends on the kind of universe model you select. This is why, according to general relativity, you cannot determine distances INDEPENDENTLY of first assuming a geometry for the universe.

On a clear night in the city, you can see hundreds of stars across the sky. From a location in the distant countryside, you can see thousands of much fainter stars. With a telescope you can see millions of stars. In this activity, you will use a recent photograph of a small part of the sky to estimate how many stars could be seen by astronomers using a modern telescope to photograph the entire sky.

In 1999, the 2-Micron Astronomical Sky Survey (2MASS) photographed a small section of sky in the constellation Hercules. Follow the step-by-step procedure to estimate from this photograph about how many stars there are in the sky that are one million times fainter than what the human eye can detect.



1 – This image is 0.15 degrees wide and 0.29 degrees long. How many square degrees in size is this picture?

2 – The sky has an area of 41,260 square degrees. How many of these picture 'tiles' would be needed to cover the entire sky?

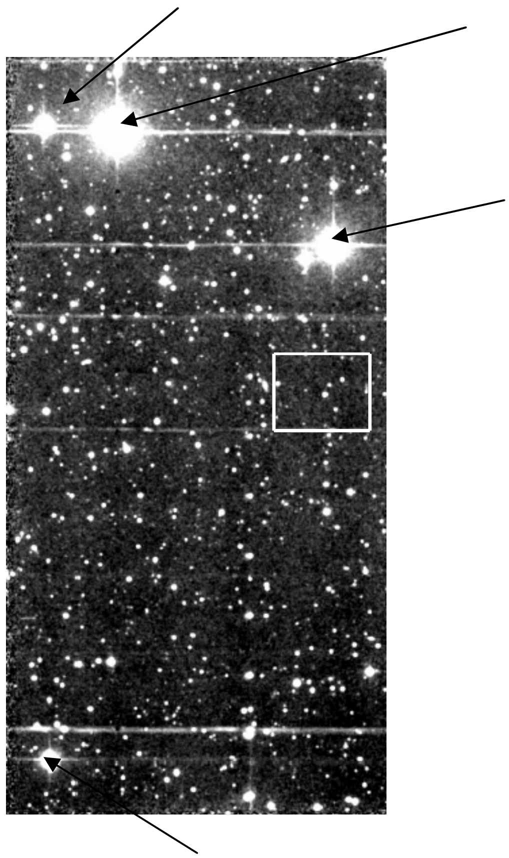
3 – How many bright stars do you see in the picture?

4 – How many faint stars do you see in the picture?

5 – If this picture were typical of the tiles in Question 2, about how many bright stars would you expect to find across the entire sky?

6 – How many of the faint stars you counted in Question 4 would you expect to see across the entire sky?

7 – The human eye should be able to see about 8,000 stars across the entire sky. Based on the numbers you estimated in questions 5 and 6, are any of the stars in the picture to the left likely to be visible to the eye?



1 – This image is 0.15 degrees wide and 0.29 degrees long. How many square degrees in size is this picture? **Answer:** 0.15 x 0.29 = 0.044 square degrees.

2 – The sky has an area of 41,260 square degrees. How many of these picture 'tiles' would be needed to cover the entire sky? **Answer:** 41,260/0.0435 = 948,500 tiles.

3 – How many bright stars do you see in the picture? **Answer:** Estimates may vary. The arrows indicate the four reasonable choices.

4 – How many faint stars do you see in the picture? **Answer:** Students will have to count by hand all of the 'spots' in this photograph. Answers may vary, but should be in the range from 500 to 600. Break the photograph into 4 x 8 equal-sized patches and count the number of stars in one of these, then multiply by the number of patches (32) to get an estimate. For example, in the patch I highlighted in the picture above, I counted 17 stars. There are 32 of these patches covering the picture, so I estimate that there are 32 x 17 = 544 stars in this picture.

5 – If this picture were typical of the tiles you estimated in Question 2, about how many bright stars would you expect to find across the entire sky? **Answer:** It the picture is typical, with 4 bright stars in its field of view, the number of similar stars would be 4 x 948,500 = 3.8 million.

6 – How many of the faint stars you counted in Question 4 would you expect to see across the entire sky? **Answer:** If we chose an average of 550 stars in this picture, then for 948,500 fields there would be about 550 x 948,500 = 521 million stars covering the entire sky.

7 – The human eye should be able to see about 8,000 stars across the entire sky. Based on the numbers you estimated in questions 5 and 6, are any of the stars in the picture to the left likely to be visible to the eye? **Answer:** No, because even the brightest stars seen in the photograph, if present across the sky, are far more common (there are 3.8 million of them!) than the stars you can see with the eye (about 8,000!), so they must be much fainter.

Our Neighborhood in the Milky Way

The Milky Way galaxy is a flat disk about 100,000 light years in diameter and 1000 light years thick. All of the bright stars, clusters and nebula we see are actually very close-by. Let's have a look at a few of these familiar landmarks!

The table below gives the distances and angles to a few familiar nebulae and star clusters within 7,000 light years of the Sun. Plot them on a paper with a scale of 1 centimeter = 500 light years, and with the Sun at the origin.

Object	Type	Distance		Angle
Pleiades	star cluster	410	ly	60
Orion Nebula	nebula	1500		80
Betelgeuse	star	650		90
Deneb	star	1600		310
Antares	star	420		245
Cygnus Loop	supernova remnant	2000		315
Ring Nebula	nebula	2300		280
Owl Nebula	nebula	1900		170
Crab Nebula	supernova remnant	6300		80
Praesepe	star cluster	520		130
Rosette Nebula	nebula	3,600		100
Eta Carina	nebula	7,000		160
Lagoon Nebula	nebula	4,000		270
Jewel Box	star cluster	6,500		190

Problem 1 - If you only wanted to visit the three bright stars, how many light years would you have to travel for a round-trip tour?

Problem 2 - If you only wanted to visit all of the nebulas how long would your round-trip journey be?

Problem 1 - Sun - Betelgeuse - Deneb - Antares - sun would measure 10 centimeters, and from the scale of 1 cm=500 ly, it would be a **5,000 light year journey.**

Problem 2 - One possibility would be Sun - Cygnus Loop - Lagoon Nebula - Ring Nebula - Owl Nebula - Eta Carina Nebula - Rosette Nebula - Crab Nebula - Orion Nebula - Sun. This would be a journey of 45 centimeters or 45 x 500 = **22,500 light years.**

Name ______________________________

The galaxy NGC-1232 was photographed by the ESA, Very Large Telescope in May, 2003. From resources at your library or the Internet, fill-in the following information:

1) Type of galaxy ---------- Number of arms ----------
2) Name of galaxy cluster it is a member of --------
3) Distance in parsecs and in light-years ----------
4) Right Ascension ------- Declination------
5) Constellation ---------
6) Diameter in light years ------
7) Diameter in arcminutes -----
8) Apparent visual magnitude ----------

From the photograph below, and an assumed diameter of 200,000 light years, answer these questions:

9) Diameter of nuclear region in light years ---------------
10) Average width of arms in light years ----------
11) Average spacing between arms in light years--------
12) Diameter of brightest star clusters (bright knots) in light years ------------
13) Describe in 500 words why this galaxy is so interesting.

1) Type of galaxy ---------- Spiral 'Sc' Number of arms ---------- About 5
2) Name of galaxy cluster it is a member of -------- Eridanus Galaxy Group
3) Distance in light-years ---------- 70 to 100 million Light-years
This question introduces students to the many different values that can be cited, especially on the internet. Earlier estimates (70 million) sometimes are given even though more recent estimates (100 million) are available. For research purposes, astronomers will always use the most current estimate and will state why they do so.
4) Right Ascension ------- 3h 9m 45s Declination------ -20d 34'
5) Constellation ----- Eridanus
6) Diameter in light years ------ 100,000 light years (or 200,000 lightyears)
The diameter is based on the angular sized (which is fixed) and the distance (which depends on whether you use the 70 or 100 million light year estimate). At a distance of 100 million light years, the diameter is about 200,000 light years. At 70 million light years, the diameter is (70/100)*200,000 = 140,000 light years. Students may find articles where authors cite either 100,000 or 200,000 light years. This might be a good time to ask students which resources preferred one estimate over another, and why there is such uncertainty. How do astronomers measure the distances to galaxies?
7) Diameter in arcminutes ----- 7 arc minutes (moon is 30 arcminutes!)
This is the diameter of the galaxy as you would see it in the sky from Earth. The moon is 30 arcminutes in diameter, so NGC-1232 is about ¼ the diameter of the full moon. Students may need to be reminded that there are 360 degrees in a full circle, and each degree consists of 60 minutes of arc.
8) Apparent visual magnitude ---------- +10.6
This is a measure of how bright the galaxy appears in the sky. The faintest stars you can see in a rural dark sky are about +6, while in an urban setting this limit is about +3. Because each magnitude corresponds to a brightness factor of 2.5, the galaxy is about 8 magnitudes fainter than the brightest star you can see in a city, or a factor of 2.5 x 2.5 x 2.5 x 2.5 x 2.5 x 2.5 x 2.5 x 2.5 = 1500 fainter.

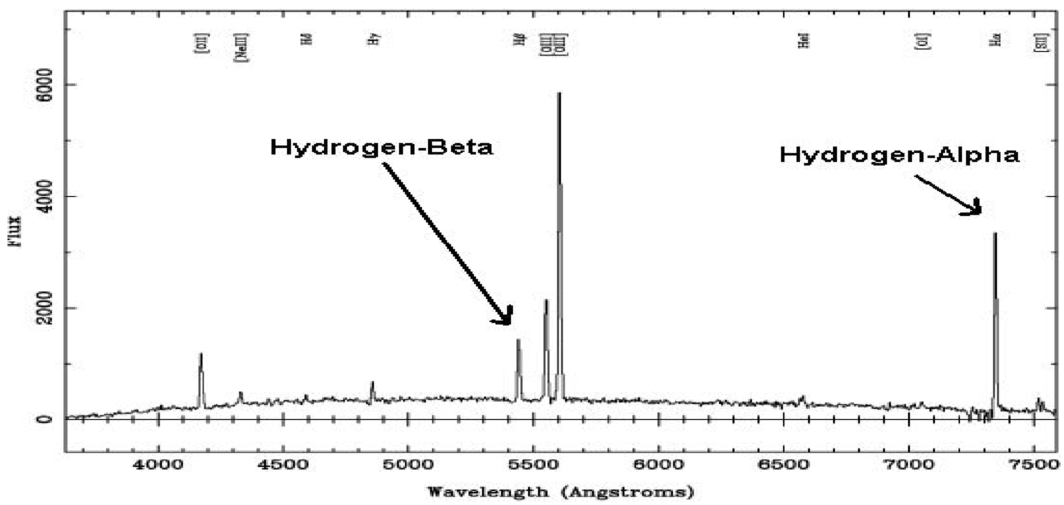
From the photograph, and an assumed diameter of 200,000 light years, answer these questions:
The diameter of the galaxy photograph is about 100 millimeters, so the scale of the photograph is 200,000 light years / 100 mm = 2,000 light years per millimeter.

9) Diameter of nuclear region. About 10 millimeters or 20,000 light years.
10) Average width of arms. Students will find that the arm widths near the nucleus are narrower than in the more distant regions. They vary from about 4mm (8,000 light years) to 10 mm (20,000 light years).
11) Average spacing between arms. As for the arm widths, this can vary from 4mm (8,000 light years) to 25 mm (50,000 light years).
12) Diameter of brightest star clusters (bright knots). About 2 mm (4,000 light years).
13) Describe in 500 words why this galaxy is so interesting. Students may cite many features, including its similarity to the Milky Way, the complexity of its arms, the many star clusters, the complexity and shape of the nuclear region.

The Doppler Shift is an important physical phenomenon that astronomers use to measure the speeds of distant stars and galaxies. When an ambulance approaches you, its siren seems to be pitched higher than normal, and as it passes and travels away from you, the pitch becomes lower. A careful measurement of the pitch change can let you determine the speed of the ambulance once you know the speed of the sound wave. A very similar method can be used when analyzing light waves from distant stars and galaxies. The basic formula for slow-speed motion (that is, speeds much slower than the speed of light) is:

$$\textbf{Speed} = 299{,}792 \; \frac{W_O - W_R}{W_R}$$

The speed of the object in km/s can be found by measuring the wavelength of the signal that you observe (Wo), and knowing what the rest wavelength of the signal is (Wr), with wavelength measured in units of Angstroms (1.0×10^{-10} meters).



This is a small part of the spectrum of the Seyfert galaxy Q2125-431 in the constellation Microscopium. An astronomer has identified the spectral lines for Hydrogen Alpha (Wr = 6563 Angstroms), and Beta (Wr = 5007 Angstroms).

Question 1: From the graph of the spectrum above, use you millimeter ruler to determine the scale of the figure in angstroms per millimeter.

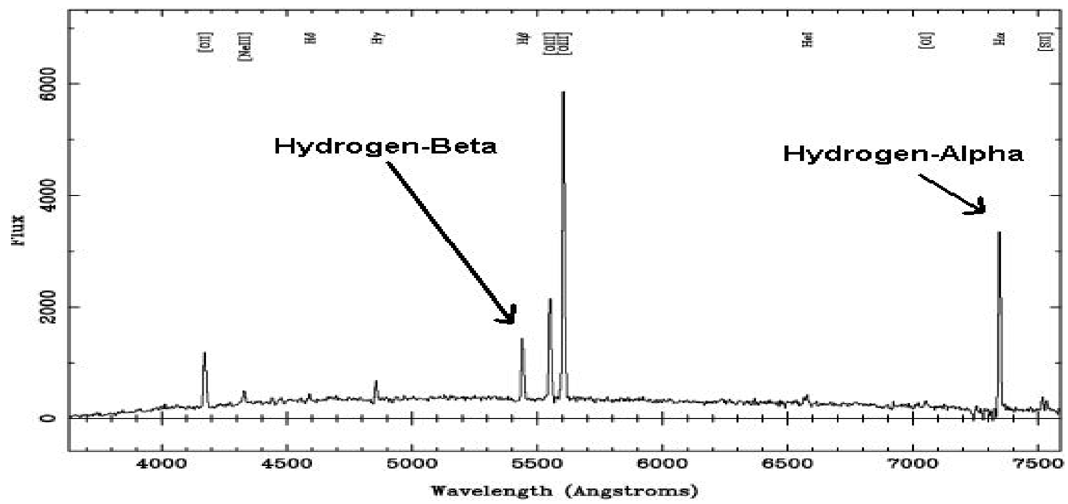
Question 2: What are the observed wavelengths of the Hydrogen-Alpha and Hydrogen-Beta lines?

Question 3: What are the rest wavelengths of the Hydrogen-Alpha and Hydrogen-Beta lines?

Question 4: If Wr is defined by your answers to question 3, and Wo is defined from your answers to question 2, what velocity do you calculate for each line from the formula above?

Question 5: From your answer to question 4, what is the average of the two velocities?

Question 6: Is the galaxy moving towards or away from the Milky Way? Explain.



This figure was obtained from the paper "Naked Active Galactic Nuclei" by M. Hawkins, University of Edinburg, Scotland. (Astronomy and Astrophysics, 2004, vol. 424, p. 519)

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Question 1: From the graph of the spectrum, use you millimeter ruler to determine the scale of the figure in angstroms per millimeter. Answer: On the Student's page, the wavelength scale from 4000 to 7000 Angstroms measures 100 millimeters, so the scale is (7000-4000)/100 = 30 Angstroms/mm.

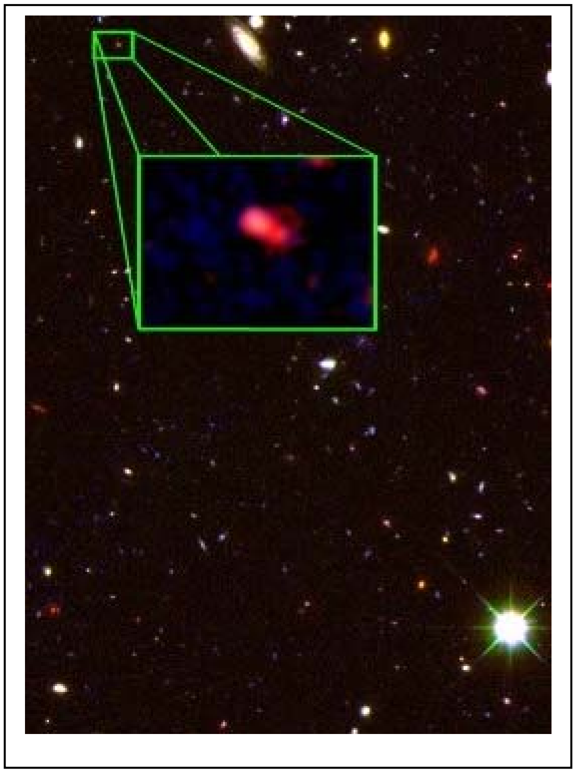
Question 2: What are the observed wavelengths of the Hydrogen-Alpha and Hydrogen-Beta lines? **Answer:** Alpha: 7000 A + 30 x (11.5mm) = 7345 A. Beta: 5000 A + 30 x (14.5 mm) = 5435 A.

Question 3: What are the rest wavelengths of the Hydrogen-Alpha and Hydrogen-Beta lines? **Answer:** Alpha = 6563 Angstroms; Beta = 5007 Angstroms.

Question 4: If Wr is defined by your answers to Question 3, and Wo is defined from your answers to question 2, what velocity do you calculate for each line from the formula above? **Answer:** For the Alpha line; Wr=6563 A, Wo = 7345 A. so from the formula Speed = 299792 x (7345-6563)/6563 = 35721 km/s. For the Beta line: Wr = 5007 A, Wo = 5435 A. so Speed = 299792 x (5435-5007)/5007 = 25626 km/s.

Question 5: From your answer to Question 4, what is the average of the two velocities? **Answer:** (35721 + 25626)/2 = 30,700 km/s. **Note to Teacher:** Because it is hard to measure the wavelengths of these lines to less than 1 mm accuracy, the line wavelengths will be uncertain to about 30 Angstroms. This works out to a speed uncertainty of 299792 x (30/5007) = 1,800 km/sec. The actual difference in the two speeds is 35721-25626 = 10,095 km/sec which is <u>much higher</u> than the measurement uncertainty, and may mean that the regions of gas producing the H-Alpha and H-Beta emission are not moving at the same speeds within the galaxy.

Question 6: Is the galaxy moving towards or away from the Milky Way? Explain. **Answer:** Because the observed wavelength of each line is LONGER then the wavelength for the gas at rest, the source is moving away from the observer (just as the pitch of an ambulance siren is lower as it moves away from you).



Astronomers have recently discovered one of the most distant galaxies in our universe using a list of candidates from the Hubble Space Telescope's Cosmic Assembly Near-infrared Deep Extragalactic Legacy Survey (CANDELS). This survey found 42 objects that seemed to be good choices for a follow-up study to determine their exact distances.

Astronomers used the MOSFIRE infrared camera on the Keck Telescope in Hawaii to study each of these candidates spectroscopically. One of these objects called z8_GND_5296 was then discovered to be the most distant galaxy known after studying the Lyman-alpha spectral line of hydrogen. This line should have appeared at an ultraviolet wavelength of 121 nanometers. Instead, thanks to the expansion of the universe, this line was detected at a wavelength of 1029 nanometers in the infrared part of the light spectrum.

Problem 1 – Astronomers define a quantity called the redshift by the formula

$$Z = (\lambda_m - \lambda_r)/\lambda_r$$

where λ_m is the observed wavelength of a spectral line and λ_r is its wavelength when measured under laboratory conditions. What was the redshift of the new galaxy based on the lyman-alpha spectral line?

Problem 2 – According to the Big Bang theory, the redshift of a galaxy is related to the time since its light left the object, called the look-back time, T, by the approximate formula T(Z) = 12.65 + 0.06Z. What is the look-back time for the galaxy z8_GND_5296?

Problem 3 – The Big Bang occurred 13.8 billion years ago. How soon after the Big Bang, in millions of years, did the light from z8_GND_5296 begin its journey?

http://hubblesite.org/newscenter/archive/releases/2013/39/image/a/
Galaxy Found in Hubble Survey Has Farthest Confirmed Distance
October 23, 2013

Problem 1 – Astronomers define a quantity called the redshift by the formula $Z = (\lambda_m - \lambda_r)/\lambda_r$ where λ_m is the observed wavelength of a spectral line and λ_r is its wavelength when measured under laboratory conditions. What was the redshift of the new galaxy based on the lyman-alpha spectral line?

Answer: λ_r = 1029 nanometers λ_r = 121 nanometers, so z = (1029-121)/121 = **7.5**

Problem 2 – According to the Big Bang theory, the redshift of a galaxy is related to the time since its light left the object, called the look-back time, T, by the approximate formula T(Z) = 12.65 + 0.06Z. What is the look-back time for the galaxy z8_GND_5296?

Answer: T = 12.65 + 0.06(7.5) = **13.1 billion years**.

Problem 3 – The Big Bang occurred 13.8 billion years ago. How soon after the Big Bang, in millions of years, did the light from z8_GND_5296 begin its journey?

Answer: 13.8 – 13.1 = 0.7 billion years or **700 million years**.

The Origin of Our Universe Number Puzzle

About 14 billion years ago, our entire universe came into existence in an event called the 1)____________. Because this state of matter, energy, light, space and time were so unimaginably extreme, astronomers prefer to call it the 2)____________. Before this event, mathematical models predict that there was no space, time, matter or energy at all. Just a pure 3)____________. No sooner did the event begin, but space and time began to 4)__________, causing the matter and energy to cool at a fantastic rate. After the first second following the Big Bang, matter and energy were still 1000 times hotter than the center of our sun. No matter where you looked in the universe, all you would see was this intensely hot matter and radiation far brighter than our own 5)_______. It took over three minutes for the universe to cool to the temperature of our own sun's interior. Through 6)___________ reactions, only hydrogen and 7)_______ nuclei were formed, but the intense 8)________ of light still existed everywhere in space. It would take another million years before the universe had cooled to a few thousand degrees. This allowed then free 9)__________ to combine with the hydrogen and helium nuclei to form normal atoms. After another 100 million years had passed, the intense fireball light had finally dimmed to 10)__________ and the universe entered the 11)____________. During this time, the universe was completely dark with no visible light anywhere. Within this darkness, clouds of hydrogen and helium gas began to form and collapse under their own gravity forming the first generations of stars. These stars exploded as 12)___________ and littered the universe with carbon, 13)________ and other elements. After 200 million years the first 14)_________ formed, and after another 9 billion years our own sun and 15)___________ formed.

Word Bank

+6 Earth	+2 nuclear	-3 expand
-2 Dark Ages	+9 fireball	+8 helium
-1 sun	+1 galaxies	+15 supernova
+10 Singularity	+4 electrons	+27 invisibility
+5 Big Bang	+7 nothingness	+19 oxygen

Solve these questions and use the integer to find the word from the Word Bank

1) The distance between two galaxies increases from 100 to 500. What is the dilation factor?
2) Distance between the points (+3,+5) and (-3, +13)
3) Solve for x: $3x10^3 \times 4x10^x = 1.2x10^{11}$
4) y=3x+5 and y=2x+2 intersect at the point (x,-4) what is x?
5) Use 2-point distance formula to solve for x: (x,+3) and (+5,+11) where distance = 10.
6) A figure is dilated by 8 and contracted by 4, what is its final dilation factor?
7) Solve for x: $6x10^{15}/3x10^x = 2x10^7$
8) $T=10^{10}/t^{1/2}$. What is our universe's temperature T after t = 100 seconds: $1x10^x$.
9) Slope of the line perpendicular to y = -0.25x+3
10) After universe expanded three times, what was the volume increase?
11) Largest factor (a or b) of x^2+5x+6 : (x+a)(x+b)
12) Distance from (4,2) to (1,-2) after a dilation of 3x
13) T = 2.7 (1+z). At what redshift, z, will the cosmic temperature equal 54 degrees?
14) Solve for intersection of x^2+3x-1 and y=2x+1 (x,+3)
15) Distance between (3,6) and (0,3) after a dilation of $2^{1/2}$

Answer Key

About 14 billion years ago, our entire universe came into existence in an event called the 1)__**Big Bang**__. Because this state of matter, energy, light, space and time were so unimaginably extreme, astronomers prefer to call it the 2)_**Singularity**__. Before this event, mathematical models predict that there was no space, time ,matter or energy at all. Just a pure 3)_**Nothingness**__. No sooner did the event begin, but space and time began to 4)_**expand**, causing the matter and energy to cool at a fantastic rate. After the first second following the Big Bang, matter and energy were still 1000 times hotter than the center of our sun. No matter where you looked in the universe, all you would see was this intensely hot matter and radiation far brighter than our own 5)__**sun**__. It took over three minutes for the universe to cool to the temperature of our own sun's interior. Through 6)__**nuclear**__ reactions, only hydrogen and 7)_ **helium**__ nuclei were formed, but the intense 8)_**fireball**__ of light still existed everywhere in space. It would take another million years before the universe had cooled to a few thousand degrees. This allowed the free 9)_**electrons**__ to combine with the hydrogen and helium nuclei to form normal atoms. After another 100 million years had passed, the intense fireball light had finally dimmed to 10)__**invisibility**__ and the universe entered the 11)_**Dark Ages**__. During this time, the universe was completely dark with no visible light anywhere. Within this darkness, clouds of hydrogen and helium gas began to form and collapse under their own gravity forming the first generations of stars. These stars exploded as 12)_**supernova**__ and littered the universe with carbon, 13)__**oxygen**__ and other elements. After 200 million years the first 14)__**galaxies**__ formed, and after another 9 billion years our own sun and 15)__**Earth**__ formed.

1) The distance between two galaxies increases from 100 to 500. What is the dilation factor? Answer: 500/100 = +5, word = **Big Bang**

2) Distance between the points (+3,+5) and (-3, +13): $d^2 = (-3-3)^2 + (13-5)^2 = 100$ so d = +10 word = **Singularity**

3) Solve for x: $3x10^3 \times 4x10^x = 1.2x10^{11}$ so 3+x +1 = 11, so x = +7 word = **Nothingness**

4) y=3x+5 and y=2x+2 intersect at the point (x,-4). 3x+5 = 2x+2, so x=-3 and word = **expand**

5) Use 2-point distance formula to solve for x: (x,+3) and (+5,+11) where distance = 10. $100 = (5-x)^2 + (11-3)^2$ so $100 - 64 = (5-x)^2$; $36 = (5-x)^2$ so x = -1 and word = **sun**

6) A figure is dilated by 8 and contracted by 4, what is its final dilation factor? 8/4 = 2 word = **nuclear**.

7) Solve for x: $6x10^{15}/3x10^x = 2x10^7$ answer: 15-x = 7 so x = +8 and word = **helium**

8) $T=10^{10}/t^{1/2}$. What is the universe temperature T after t = 100 seconds: 1x10x. $T = 10^{10}/(100)^{1/2} = 10^9$ so x = +9 and word = **fireball**

9) Slope of the line perpendicular to y = -0.25x+3: answer: m = 4.0 word **= electrons**

10) After universe expanded three times, what was the volume increase? $3^3 = 27$ so word = **invisibility**

11) Largest factor (a or b) of x^2+5x+6 : (x+a)(x+b) a=+3, b=+2 so x=-3, x=-2 and –2 is largest so word = **Dark Ages**.

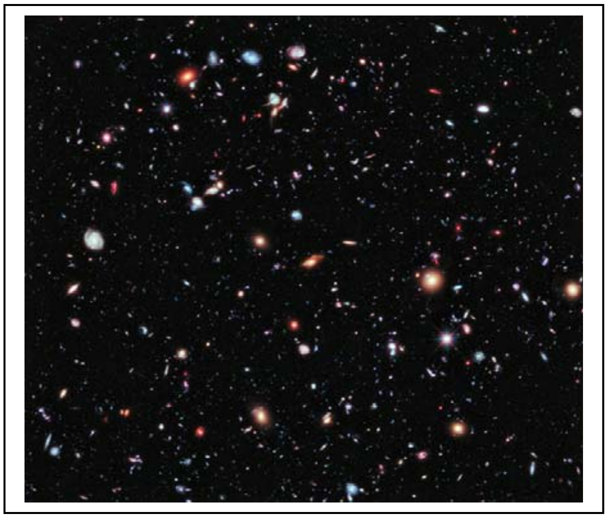
12) Distance from (4,2) to (1,-2) after a dilation of 3x. Answer: $d^2 = (1-4)^2 + (-2-2)^2 = 25$, d = 5 and 5x3 = 15. Word = **supernova.**

13) T = 2.7 (1+z). At what redshift, z, will the cosmic temperature equal 54 degrees? Answer: 54=2.7(1+z) so z = +19. The word = **oxygen**

14) Solve for intersection of x^2+3x-1 and y=2x+1 (x,+3) : Answer: $x^2+3x-1 = 2x+1$ so x^2+x-2 factors (x+2)(x-1) x=-2, x=+1 The two intersection points are (-2, -3) and (+1, +3) so x = +1 and word = **galaxies**.

15) Distance between (3,6) and (0,3) after a dilation of $2^{1/2}$. Answer: $d^2 = (0-3)^2 + (3-6)^2 = 18$ $d = (18)^{1/2}$ then after dilation $d = (18)^{1/2} \times (2)^{1/2} = (36)^{1/2} = +6$ or -6, but since no word exists for x=-6, we have word = **Earth**.

The Hubble eXtreme Deep Field – Counting Galaxies



The eXtreme Deep Field (XDF) image was created by astronomers by combining thousands of images of the same spot in the sky. When the data from 2,000 images were carefully combined, astronomers could identify 5,500 galaxies in the combined image.

Some of these galaxies, which appear as the large round spots in the photograph, are nearby galaxies. But in between these are the faint spots of light from the more distant galaxies. Some of these were formed only 500 million years after the Big Bang!

A simple study of this image can tell us a lot about our universe!

Problem 1 - Astronomers use angular measure when referring to locations and areas in the sky. In ordinary angle measure 1 degree can be subdivided into 60 arcminutes. 1 arc minute can be subdivided into 60 arcseconds. How many arcseconds are there in one degree of angular measure?

Problem 2 – The dimensions of the XDF patch are approximately 2.3 arcminutes wide and 2.0 arcminutes tall. What are the dimensions of this patch of the sky in degrees?

Problem 3 - What is the area of this patch of sky in square degrees?

Problem 4 – The full surface area of the entire sky can be found using the formula for the surface area of a sphere, where the 'radius' is equal to 1.0 radians. A radian is an angular measure equal to $180/\pi = 57.296$ degrees. What is the surface area of the entire sky in square degrees?

Problem 5 – How many of the XDF sky patches would cover the entire sky?

Problem 6 - If astronomers counted 5,500 galaxies in the XDF, about how many galaxies would you estimate across the entire sky?

Problem 7 - Astronomers studying the galaxies in the XDF have found about 10% are seen as they were less than 5 billion years ago, 30% are seen as they were between 5 billion and 9 billion years ago, and 60% are being seen as they were more than 9 billion years ago. Across the entire sky, how many galaxies might there be that are more than 9 billion years?

Problem 1 - 1 degree x (60 arcmin/1 deg) x (60 arcses/1 arcmin) = **3600 arcseconds.**

Problem 2 – 2.3 arcmin x (1 deg/60 arcmin) = **0.038 degrees wide** and
2.0 arcmin x (1 deg /60 arcmin) = **0.033 degrees tall**.

Problem 3 - 0.038 degrees x 0.033 degrees = **0.00125 square degrees**.

Problem 4 – $4 \pi (57.296)^2$ = **41,253 square degrees.**

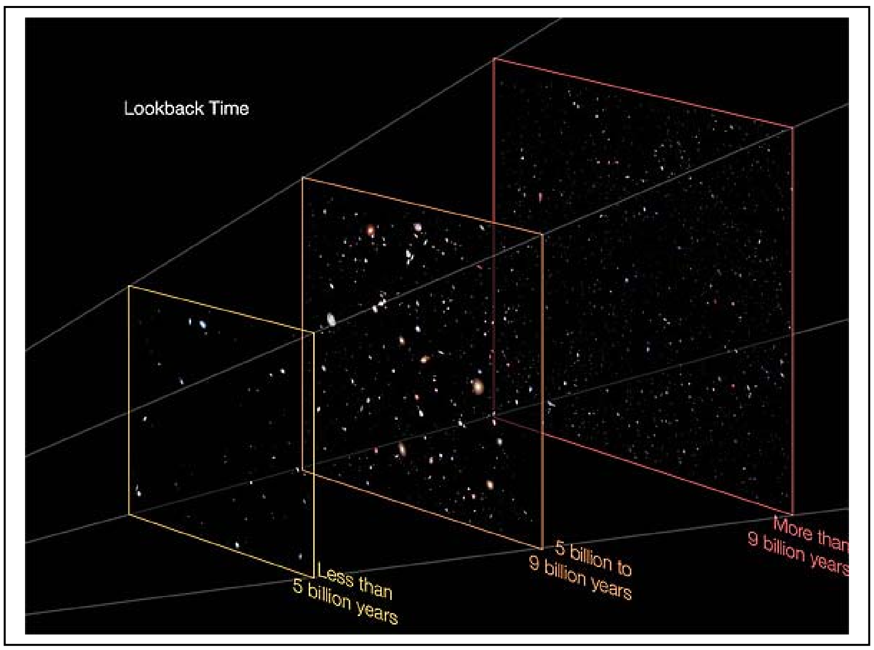
Problem 5 – How many of the XDF sky patches would cover the entire sky?
41,253 / 0.00125 = **33,000,000 patches**. (for 2 significant figure accuracy)

Problem 6 - If astronomers counted 5,500 galaxies in the XDF, about how many galaxies would you estimate across the entire sky? N = 5,500 galaxies/patch x (33000000 patches) = **182 billion galaxies**! With 2 significant figure accuracy this is 180 billion galaxies.

Problem 7 - Astronomers studying the galaxies in the XDF have found about 10% are seen as they were less than 5 billion years ago, 30% are seen as they were between 5 billion and 9 billion years ago, and 60% are being seen as they were more than 9 billion years ago. Across the entire sky, how many galaxies might there be that are more than 9 billion years?

60% of the total would be galaxies seen as they were more than 9 billion years ago, so 0.60 x 182 billion = **109 billion galaxies**. With 2 significant figure accuracy this is 110 billion.

Note: Since the Big Bang occurred 13.7 billion years ago, these galaxies are 13.7 - 9 or less than 4.7 billion years old!



The Most Distant Objects in the Visible Universe

Discovery Year	Redshift Z	Age (millions of yrs)	Object Name or Catalog Number	Type
1930	Infinity	13,700	Big Bang	Creation
2011	10	13,200	UDFj-39546284	Galaxy fragment
2010	8.6	13,100	UDFy-38135539	Galaxy fragment
2009	8.2	13,070	GRB090423	Gamma ray burst
2008	7.6	13,000	A1689-zD1	Starburst Galaxy
2006	6.9	12,880	IOK-1	Galaxy
2007	6.4	12,700	CFHQS J2329-0301	Black Hole/Quasar
2011	5.3	12,600	COSMOS-AzTEC3	Galaxy Cluster

The universe was born 13.7 billion years ago in the 'Big Bang' and since then has been expanding to its present, enormous size. By using sensitive telescopes such as NASA's Hubble Space Telescope, astronomers can study the images of ancient galaxies, whose light has been traveling towards Earth for billions of years. For the most distant objects, their light has been traveling towards Earth for over 10 billion years, allowing astronomers to see what these distant galaxies looked like 10 billion years ago. The 'race' has been on to detect the most ancient images of galaxies so that astronomers can learn about the events that created the first galaxies and stars in the universe. The table above shows the light travel times for various known objects that have been identified by 2011. For the problems below, assume that these objects are common examples of their types at their estimated ages.

Problem 1 – Create a timeline that shows the age of each object since the Big Bang.

Problem 2 – How many million years after the Big Bang did the first galaxy fragments begin to form?

Problem 3 – About how many million years after the formation of the first galaxy fragments did the formation of the first galaxies begin?

Problem 4 – The quasar CFHQS J2329-0301 contains a black hole with a mass estimated to be 500 million times the mass of our sun. If this 'supermassive' black hole began to form when the first galaxy fragments appeared, about how many years did it take to form this black hole?

Problem 5 – About how many years elapsed between the appearance of the first galaxy fragments, and the first cluster of galaxies in the universe?

Problem 6 – In your timeline, what is the largest gap in time for which we, as yet, have not observed any candidate objects?

For more information online:
Cosmic Background Radiation http://wmap.gsfc.nasa.gov/
UDFj-39546284 **http://www.nasa.gov/mission_pages/hubble/science/farthest-galaxy.html**
UDFy-38135539 http://www.hubblephotoprints.com/blog/hubble-telescope-reveals-the-most-distant-galaxy-ever-found/
GRB090423 **http://science.nasa.gov/science-news/science-at-nasa/2009/28apr_grbsmash/**
A1689-zD1 **http://www.spacetelescope.org/news/heic0805/**
IOK-1 **http://www.physorg.com/news176737523.html**
CFHQS J2329-0301 http://www.gemini.edu/index.php?q=node/254
COSMOS-AzTEC3 http://www.nasa.gov/mission_pages/spitzer/news/spitzercluster20110112.html

Problem 1 – Create a timeline that shows the age of each object since the Big Bang. Answer:

Age (millions of yrs)	Years since the Big Bang	Object Type
13,700	0	Creation of time, space and matter
13,200	500 million	Galaxy fragments begin to form
13,100	600 million	Galaxy fragments still present
13,070	630 million	Gamma ray burst appears - supernovae
13,000	700 million	Starburst Galaxy and intense star formation
12,880	820 million	Galaxies begin to form
12,700	1 billion	Supermassive black holes appear
12,600	1.1 billion	Galaxy Clusters begin to form

Note: The Object Type can be used to indicate what kinds of events are taking place. Shown above is an example.

Problem 2 – How many million years after the Big Bang did the first galaxy fragments begin to form? Answer: that would be the time between the Big Bang (13.7 billion years ago) and UDFj-39546284 (13.2 billion years ago) or **500 million years.**

Problem 3 – About how many million years after the formation of the first galaxy fragments did the formation of the first galaxies begin? Answer: That would be the time between 13.2 billion years ago when we are seeing one of the first galaxies, UDFj-39546284, and the appearance of IOK-1 at 12.880 billion years ago or about **820 million years** after we first start seeing fragments of galaxies.

Problem 4 – The quasar CFHQS J2329-0301 contains a black hole with a mass estimated to be 500 million times the mass of our sun. If this 'supermassive' black hole began to form when the first galaxy fragments appeared, about how many years did it take to form this black hole? Answer: Galaxy fragments first appeared about 13.2 billion years ago (UDFj-39546284) and the quasar image is 12.7 billion years old, so the difference in time is about **1 billion years**.

Problem 5 – About how many millions of years elapsed between the appearance of the first galaxy fragments, and the first cluster of galaxies in the universe? Answer: Fragments formed about 13.2 billion years ago; clusters appeared about 12.6 billion years ago, so it took **about 600 million years**.

Problem 6 – In your timeline, what is the largest gap in time for which we, as yet, have not observed any candidate objects? Answer: **Between the Big Bang (13. 7 billion years ago) and the appearance of the first galaxy fragments (13.2 billion years ago) or 500 million years.**

A 'Galactic City' in the Far Reaches of our Universe

Astronomers have recently discovered a massive cluster of young galaxies that formed when the universe was only about 1 billion years old. The light from these galaxies has taken over 12 billion years to reach Earth. The growing galactic metropolis, called COSMOS-AzTEC3 is the most distant known massive "proto-cluster" of galaxies known today. The circled red smudges in the above image are the individual galaxies that are a part of the cluster. In the time since the light started on its journey, these galaxies have probably fallen together under the influence of gravity to form a large galaxy about the size of our Milky Way. At the distance of the galaxy cluster, the width of this photograph, which was taken by the Japanese Subaru Telescope on Mauna Kea, is about 25 million light years across. (Image Credit: Subaru/ NASA / JPL-Caltech)

Problem 1 - Assuming all of the galaxies lie in the same plane, and at the same distance, to two significant figures: A) What is the greatest distance between the galaxies in this photograph? B) What is the smallest distance between galaxies in this photograph?

Problem 2 - The Milky Way galaxy and the Andromeda galaxy are separated by about 2,200,000 light years in diameter. How many millimeters apart would they appear if they were viewed at the same distance as this cluster?

Problem 3 - If the average speed of a galaxy in this cluster is about 1000 light years in 1 million years, how many years will it take for all the galaxies to fall to the center of the cluster?

Problem 1 - Assuming all of the galaxies lie in the same plane, and at the same distance. To two significant figures: A) What is the greatest distance between the galaxies in this photograph? B) What is the smallest distance between galaxies in this photograph?

Answer: With a millimeter ruler, the width of the image is about 153 millimeters, which corresponds to an actual distance of 25 million light years. The scale of the image is then 25 million/153 mm = 160,000 light years /mm, then:

A) The farthest distance between the 11 identified galaxies is about 96 mm, for a distance of 96 mm x (160,000 ly/1 mm) = **15,000,000 light years**. B) The closest pair of galaxies are about 2 mm apart, or 2 x 160,000 = **320,000 light years**.

Problem 2 - The Milky Way galaxy and the Andromeda galaxy are separated by about 2,200,000 light years in diameter. How many millimeters apart would they appear if they were viewed at the same distance as this cluster?

Answer: 2,200,000 light years x (1 mm/160,000 light years) = **14 millimeters**.

Problem 3 - If the average speed of a galaxy in this cluster is about 1000 light years in 1 million years, how many years will it take for all the galaxies to fall to the center of the cluster?

Answer: For the two most distant galaxies to fall to the center, a point half way between them in the photograph, they must fall a distance of 7,500,000 light years. Since the speed is stated as 1000 light years / 1 million years, the time it would take is about

T = 7,500,000 light years x (1 million years / 1000 light years)

= 7,500 million years or **7.5 billion years**.

Note that galaxies closer together will have fallen together much sooner, so the most distant pair of galaxies defines the approximate free-fall age of this cluster. Since the universe is 13.7 billion years, and the free-fall age of this cluster is only 7.5 billion years, it must have had enough time to collapse into a smaller size long ago...perhaps even before our own Earth was formed!

The Big Bang - Hubble's Law

Galaxy	Distance (mpc)	Speed (km/s)
NGC-5357	0.45	200
NGC-3627	0.9	650
NGC-5236	0.9	500
NGC-4151	1.7	960
NGC-4472	2.0	850
NGC-4486	2.0	800
NGC-4649	2.0	1090

In 1921, Astronomer Edwin Hubble was measuring the speeds of nearby galaxies when he noticed a puzzling thing. When he plotted the speed of the galaxy against its distance, the points from each of the galaxies in his sample seemed to follow an increasing 'straight' line.

This turned out to be the first important clue that the universe was expanding. Each galaxy was moving away from its neighbor. The farther away the galaxy was from the Milky Way, the faster it was moving away from us.

The table shows the distance and speed of 7 galaxies. The distances are given in megaparsecs (mpc). One megaparsec equals 3.26 million light years. The speed is given in kilometers per second. Note, the speed of light is 300,000 kilometers/sec.

Problem 1 - Create a graph that presents the distance to each galaxy in mpc on the horizontal axis, and the speed in kilometers/sec on the vertical axis.

Problem 2 - What is the range of distances to the galaxies in this sample in light years?

Problem 3 - Does the data show that the distances and speeds of the galaxies are correlated, anti-correlated or uncorrelated (random)?

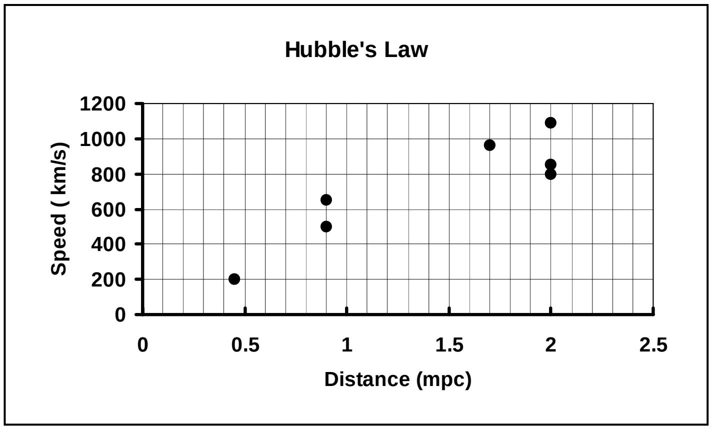
Problem 4 - By using a calculator, or using an Excel Spreadsheet, plot the data and use the 'Tools' to determine a best-fit linear regression. Alternatively, you may use the graph you created in Problem 1 to draw a best-fit line through the data points.

Problem 5 - The slope of the line in this plot is called Hubble's Constant. What is your estimate for Hubble's Constant from the data you used?

Problem 6 - An astronomer measures the speed of a galaxy as 2500 kilometers/sec. What would its distance be using your linear regression (now called Hubble's Law)?

Answer Key:

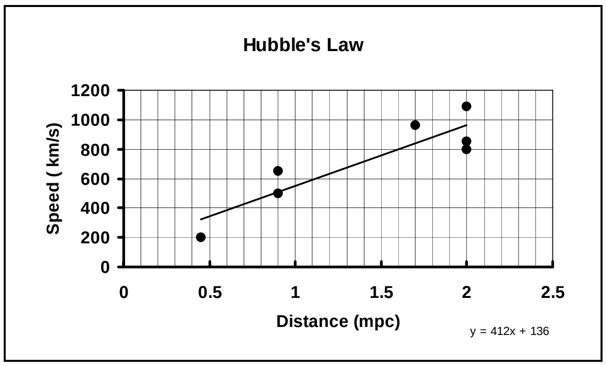
Problem 1 - Plot looks like this:



Problem 2 - The range is from 0.45 to 2 megaparsecs. Since 1 megaparses = 3.26 light years, the range is 450,000 parsecs x 3.26 light years/parsec = 1,467,000 light years to 2,000,000 parsecs x 3.26 light years/parsec = 6,520,000 light years.

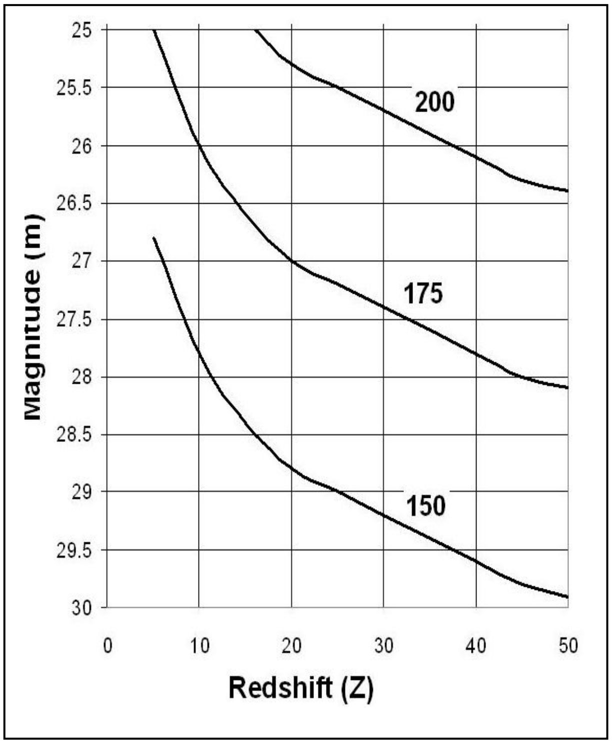
Problem 3 - Because the data points show an increasing speed with increasing distance, the data indicate a correlated relationship between these two quantities.

Problem 4 - The result of using Excel 'Trendlines' is below:



Problem 5 - The regression line has a slope of '412' and the units will be kilometers/sec per megaparsec.

Problem 6 - The regression equation is Speed (km/s) = 412 x distance(mpc) + 136 so solving for distance you get Distance = (Speed - 136)/412 and so for Speed = 2500 km/s the distance is (2500-136)/412 = 5.74 megaparsecs (or 18,700,000 light years).



When the Webb Space Telescope begins operation in ca 2015, one of its first scientific goals will be to detect, if they exist, the most distant supernova in the universe.

Astronomers predict that soon after the Big Bang, stars more massive than our own sun were commonly formed - but exploded as supernova within a few million years after birth. This is a crucial phase in the evolution of our universe. Without them, the elements that now form planets and living systems would not exist.

These supernovae, surrounded by dense clouds of dust, will be very faint, and can only be observed at infrared wavelengths, and with very large telescopes like the Webb Space Telescope.

The graph to the left shows the predicted brightness of these massive 'Population III' supernovae for stars with different masses: 200 (top), 175 (middle) and 150 (bottom) times the mass of our sun.

The vertical scale is in terms of the stellar magnitude scale, which indicates the logarithmic brightness of a star. On this scale, our sun has a brightness of about -26.0. The faintest star visible with the human eye is about +6.0. Each magnitude step represents a factor of 2.512 in brightness change so that 5 magnitudes of change is exactly a factor of 100.0.

The horizontal scale gives the distance to the supernova in terms of its cosmological redshift. On this scale, the distance to the nearest galaxy, Andromeda, is about z = 0.001 and the most distant known quasar is at about z = 3.0. It is expected that infant stars will be detected by their light at a distance of z between 20 - 50, which corresponds to a time about 100 million years after the Big Bang. At a distance of z > 1000 the universe was opaque to starlight when the universe was less than 300,000 years old.

Problem 1 - For a 10,000-second exposure, the NIR Camera on the Webb Space Telescope can detect objects brighter than a magnitude of +29.0. Out to what redshift will the NIR camera be able to see supernova from stars about 150 times the mass of our sun?

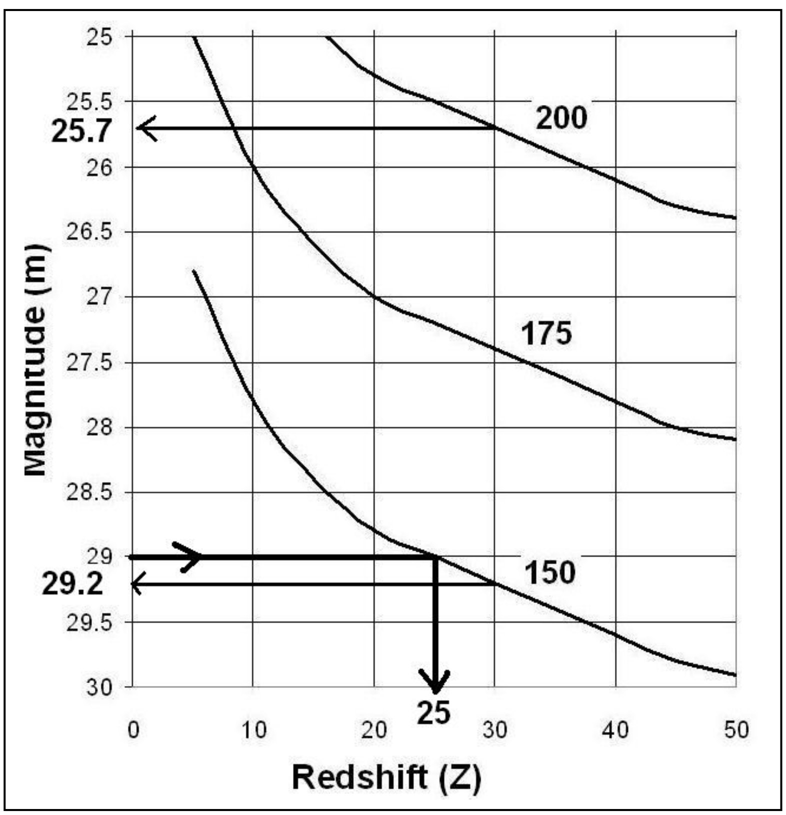
Problem 2 - At a redshift of z = 30, by what factor is the supernova of a star with a mass of 200 times the sun, brighter than the supernova of a star with a mass of 150 times the sun?

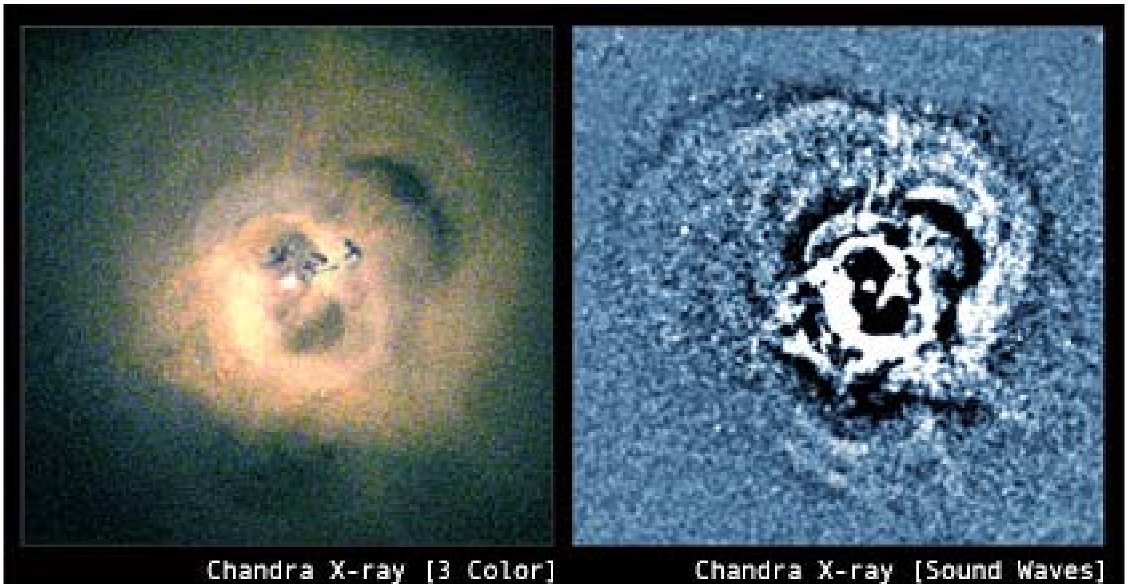
Problem 1 - For a 10,000-second exposure, the NIR Camera on the Webb Space Telescope can detect objects brighter than a magnitude of +29.0. Out to what redshift will it be able to see supernova from stars about 150 times the mass of our sun?
Answer: See graph below: **z = 25** is where the limit curve of m = +29.0 intersects the predicted supernova magnitude curve at this mass.

Problem 2 - At a redshift of z = 30, by what factor is the supernova of a star with a mass of 200 times the sun, brighter than the supernova of a star with a mass of 150 times the sun?

Answer: According to the graph below, at a redshift of Z = 30, the curve for a 150 solar mass star has a value of m = +29.2, and for a 200 solar mass star at the same Z we have m = +25.7. The difference in brightness is 29.2 - 25.7 = + 3.5. The brightness factor is between
(2.512)(2.512)(2.512) = 16 times and
(2.512)(2.512)(2.512)(2.512) = 40 times. (The actual factor, is 25 times)

At this redshift, the supernova from a 200 solar mass star is between 16 and 40 times brighter than the supernova from a 150 solar mass star.





In September 2003, the Chandra Observatory took an x-ray image of a massive black hole in the Perseus Galaxy Cluster located 250 million light years from Earth. Although it could not see the black hole, it did detect the x-ray light from the million-degree gas in the core of the cluster. Instead of a featureless blob, the scientists detected a series of partial concentric rings which they interpreted as sound waves rushing out from the vicinity of the black hole as it swallowed gas in a series of explosions. The image above left shows the x-ray image, and to the right, an enhanced version that reveals the details more clearly.

Problem 1 - The image has a physical width of 350,000 light years. Using a millimeter ruler, what is the scale of the image in light years/millimeter?

Problem 2 - Examine the image on the right very carefully and estimate how far apart the consecutive crests of the sound wave are in millimeters. What is the wave length of the sound wave in light years?

Problem 3 - The wavelength of middle-C on a piano is 1.3 meters. If 1 light year = 9.5×10^{15} meters, and if 1 octave represents a change by a factor of 1/2 change in wavelength, how many octaves below middle-C is the sound wave detected by Chandra?

Problem 1 - The image has a physical width of 350,000 light years. Using a millimeter ruler, what is the scale of the image in light years/millimeter?

Answer: The width is approximately 70 millimeters wide, so the scale is 350,000 light years / 70 millimeters = **5,000 light years/millimeter**.

Problem 2 - Examine the image on the right very carefully and estimate how far apart the consecutive crests of the sound wave are in millimeters. What is the wave length of the sound wave in light years?

Answer: Depending on where the student makes the measurement, such as the set of two bright parallel features on the lower part (7-o'clock position) of the image, the separations will be about 6 millimeters, so the wavelength is 6 mm x (5,000 light years/ 1 mm) = **30,000 light years!**

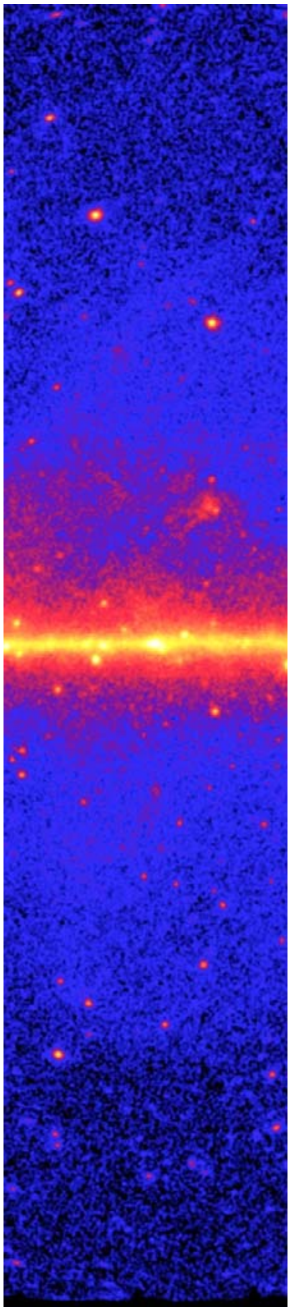
Problem 3 - The wavelength of middle-C on a piano is 1.3 meters. If 1 light year = 9.5×10^{15} meters, and if 1 octave represents a change by a factor of 1/2 change in wavelength ,how many octaves below middle-C is the sound wave detected by Chandra?

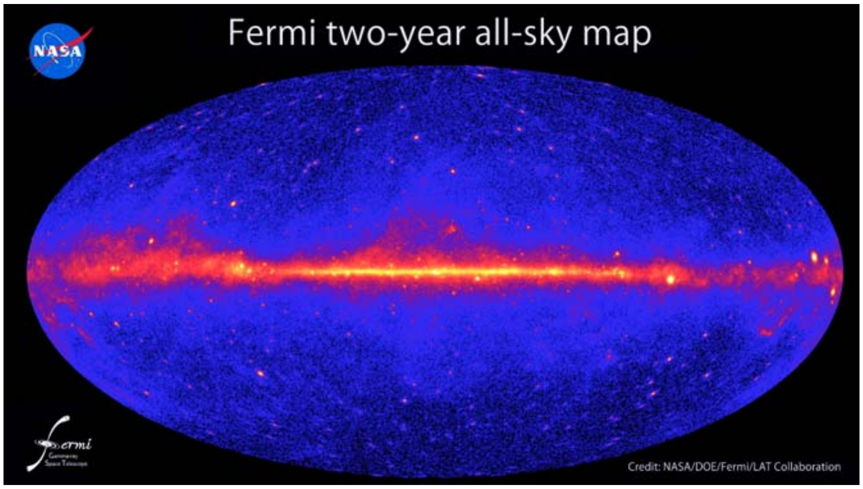
Answer: The sound wave has a wavelength of 30,000 light years x (9.5×10^{15} meters / 1 light year) = 2.9×10^{20} meters. Octaves are determined in terms of powers of two changes in sound waves so that a wavelength change from 32 meters to 16 meters = 1/2, from 32 meters to 8 meters = 1/2 x 1/2 = 1/4 or 2^{-2}, and so on. Middle-C has a wavelength of 1.3 meters, so the Perseus Cluster sound wave differs from middle-C by a factor of 2.9×10^{20} meters /1.3 meters = 2.2×10^{20} times. We have to find N such that $2^N = 2.2 \times 10^{20}$. Using a calculator and repetitive multiplications (or visit the power-of-two table at http://web.njit.edu/~walsh/powers/) we get the table below:

N	factor
10	1,024
30	1.07×10^{9}
40	1.1×10^{12}
57	1.4×10^{17}
68	2.9×10^{20}

So the answer is approximately **68 octaves below middle-C**.

Note to Teacher: A tremendous amount of energy is needed to generate the cavities, as much as the combined energy from 100 million supernovae. For more information, visit the Chandra web page at http://chandra.harvard.edu/press/03_releases/press_090903.html





This all-sky image, constructed from two years of observations by NASA's Fermi Gamma-ray Space Telescope, shows how the sky appears at light energies greater than 1 billion electron volts (1 GeV). As a comparison, the x-rays used by your dentist to search for cavities have energies of only about 5,000 electron volts (5 KeV).

In the false-color diagram above, brighter colors like red orange and yellow, indicate brighter gamma-ray sources. A diffuse glow fills the sky and is brightest along the plane of our galaxy (middle). The point-like gamma-ray 'spots' are believed to be pulsars and supernova remnants within our galaxy, as well as distant galaxies powered by supermassive black holes. (Credit: NASA/DOE/Fermi LAT Collaboration)

Earlier this year, the Fermi team released its second catalog of sources detected by the satellite's Large Area Telescope (LAT), producing an inventory of 1,873 gamma-ray point-sources found in their survey. The resulting classifications of the sources are shown in the lower-left table.

A much smaller study of 11 of these 572 unidentified Fermi/LAT sources by the Japanese Suzaku X-ray Observatory revealed that 6 could be identified as pulsars, 3 were unknown, 1 was a normal flaring star and 1 was a blazar-type galaxy.

Type of Object	Number
Blazar galaxy	1069
Pulsars	115
Supernovae	77
Active Galaxies	20
Normal galaxies and stars	20
Unknown objects	572

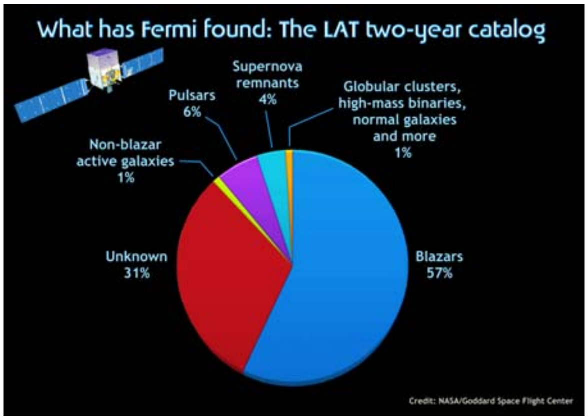
Problem 1 – Create a pie graph showing the percentage of gamma-ray sources in each of the six different categories listed in the original Fermi/LAT survey.

Problem 2 – Based upon the results from the second Suzaku Survey of the 'unknown' objects, and assuming that the Suzaku objects were randomly selected from the Fermi/LAT 'unknowns' , what are the new percentages for the 6 categories?

Problem 3 – What is the probability that the remaining unknown Fermi/LAT survey sources are very faint blazer galaxies and pulsars?

Problem 1 –Answer: See the percentages listed in the table below and the actual pie graph provided by the Fermi/LAT research report.

Type of Object	Number	Percentage
Blazar galaxy	1069	**57%**
Pulsars	115	**6%**
Supernovae	77	**4%**
Active Galaxies	20	**1%**
Normal galaxies and stars	20	**1%**
Unknown objects	572	**31%**



This pie graph is from
http://www.nasa.gov/mission_pages/GLAST/news/gamma-ray-census.html

Problem 2 – Answer: The 11 Suzaku survey says that the 572 Fermi/LAT unknowns should be distributed as follows among the Fermi/LAT source types:

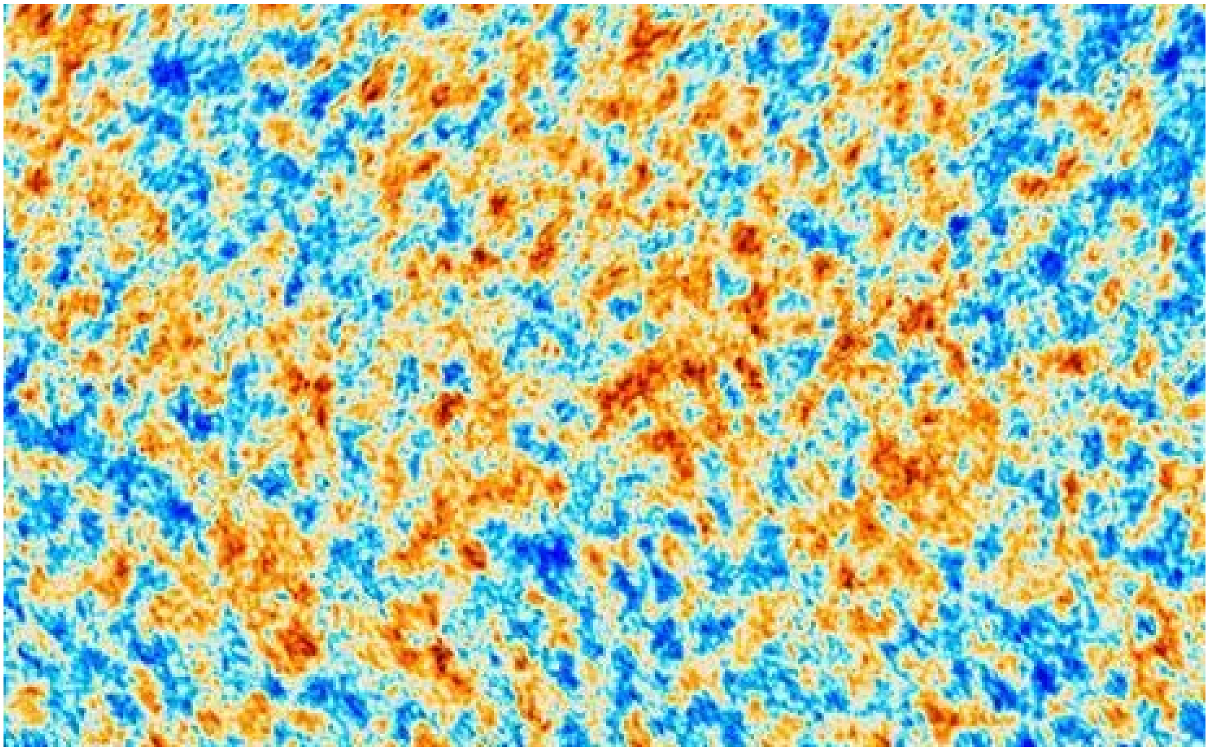
Pulsars = 6/11 x 572 = 312 sources
Unknown = 3/11 x 572 = 156 sources
Normal star = 1/11 x 572 = 52 sources
Blazar = 1/11 x 572 = 52 sources.

Adding these to the already identified Fermi/LAT source types we get the new distribution:

Type of Object	Number (N=1873)	Percentage
Blazar galaxy	1069 +52 = 1121	**60%**
Pulsars	115 + 312=427	**23%**
Supernovae	77=77	**4%**
Active Galaxies	20=20	**1%**
Normal galaxies and stars	20 + 52=72	**4%**
Unknown objects	156	**8%**

Problem 3 – What is the probability that the remaining unknown Fermi/LAT survey sources are very faint blazer galaxies and pulsars? Answer: 60% + 23% = **83%**

Planck Mission Sees Ancient Universe Clearly



The Planck Mission, like the COBE and WMAP missions before it, has created this even-clearer image of the cosmic fireball radiation (cosmic background radiation). This image shows the minute temperature differences in the cosmic fireball radiation at a time about 370,000 years after the Big Bang. The differences are caused by the lumpiness of matter that was present by this time." At this time, the average temperature of matter was about 3,000$^{\mathbf{o}}$ Celsius. The image spans a portion of the sky 45^{o} wide.

Problem 1 – How many degrees wide are the smallest red flecks in the image?

Problem 2 – The full moon is 0.5 degrees wide. How large are the red flecks that you measured in Problem 1 in terms of the full moon diameter?

Problem 3 - At the distance that this 'surface' is located from Earth, the scale is 62 parsecs/arcsecond. If 1 degree = 3600 arcseconds, how many parsecs wide is the smallest clump?

Problem 4 – The Milky Way has a diameter of about 35,000 parsecs. How large are the features in the Planck image compared to the Milky Way?

Problem 1 – How many degrees wide are the smallest red flecks in the image?

Answer: ½ mm = about 0.14 degrees

Problem 2 – The full moon is 0.5 degrees wide. How large are the red flecks that you measured in Problem 1 in terms of the full moon diameter?

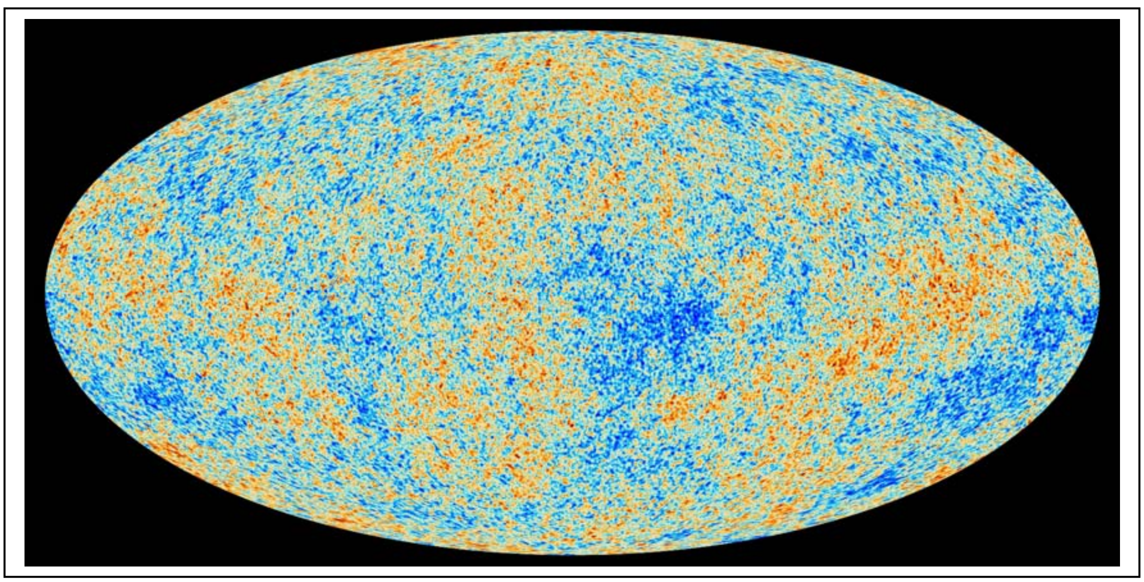
Answer: 0.5/0.14 = about 1/3 the diameter of the full moon.

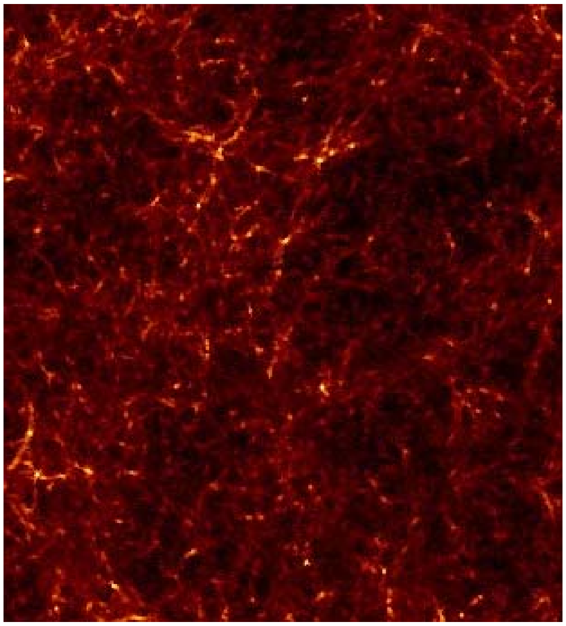
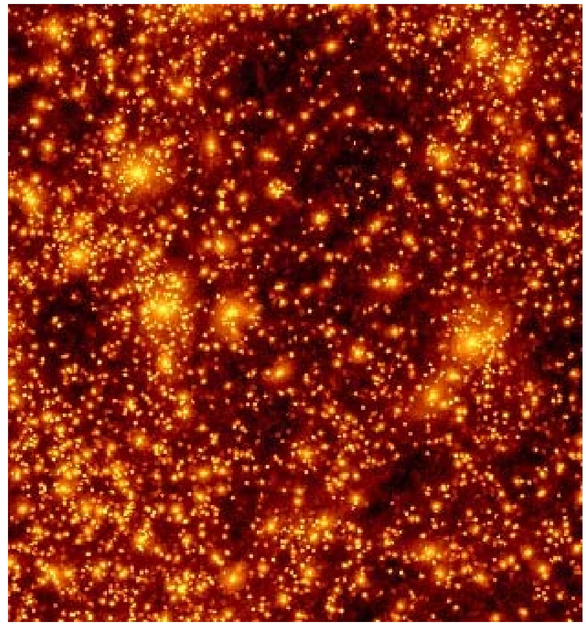
Problem 3 - At the distance that this 'surface' is located from Earth, the scale is 62 parsecs/arcsecond. If 1 degree = 3600 arcseconds, how many parsecs wide is the smallest clump?

Answer: 0.14 degrees x 3600 x 62 parsecs = 31,200 parsecs.

Problem 4 – The Milky Way has a diameter of about 35,000 parsecs. How large are the features in the Planck image compared to the Milky Way?

Answer: The Milky Way is about the same size as one of the smaller spots in the image!



Supercomputers can do trillions of calculations each second, and follow the movement of billions of particles over time. In 2008, scientists at the University of Chicago used supercomputer simulations to investigate how dark matter. Dark matter is an invisible material of unknown composition that is estimated to be 5 times more abundant in our universe than ordinary matter. Astrophysicists believe that dark matter may have herded luminous matter in the universe from its initial smooth state into the cosmic web of galaxies and galaxy clusters that populate the universe today. These two images from the simulation of the evolution of the universe show a cubic volume of the universe measuring approximately 200 million light years wide. The images show how dark matter caused the distribution of the luminous matter to change from 470 million years after the big bang (left) to today, some 13.7 billion years after the big bang (right). (*Courtesy: University of Chicago: Andrey Kravtsov, Charlie Conroy and Risa Weschlser*).

Astronomers can use their catalogs of millions of distant galaxies to check these supercomputer calculations. The goal is to mathematically model the earliest epoch of galaxy formation and to use telescopes like the Webb Space Telescope to confirm the details of these models to within 20 million years after the big bang. Since no luminous stars existed then, this period from 1 million years to about 20 million years after the big bang is called the Dark Ages.

Problem 1 - Using a millimeter ruler; A) what is the scale of these two images in light years per millimeter? B) How large would the Milky Way be on the scale of these supercomputer images? C) The Local Group of galaxies, which is 10 million light years in diameter?

Problem 2 - The smallest mathematical feature that the supercomputer can follow in these calculations is about 100 light years across. If the entire volume modeled was 280 million light years on a side, how many cubic cells 100 light years wide are in this entire volume?

Problem 3 - If the time step between calculations was 1000 years, how many time steps did it take to complete a full 13.7 billion year calculation?

Problem 1 - Using a millimeter ruler; A) what is the scale of these two images in light years per millimeter? B) How large would the Milky Way be on the scale of these supercomputer images? C) The Local Group of galaxies, which is 10 million light years in diameter?

Answer: A) The width of each image is about 108 millimeters, so the scale is 200 million/108mm = **1.9 million light years/millimeter**.

B) The Milky Way would be about 100,000 light years x (1 mm/1.9 million) = **0.05 millimeters across!**

C) The Local Group = 10 million light years x (1 millimeter/1.9 million light years)

= **5.3 mm**.

Problem 2 - The smallest mathematical feature that the supercomputer can follow in these calculations is about 100 light years across. If the entire volume modeled was 280 million light years on a side, how many cubic cells 100 light years wide are in this entire volume?

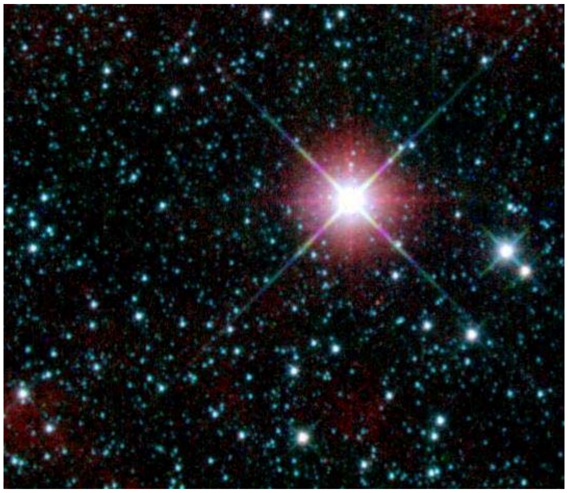
Answer: The large volume has a width of 280,000,000 / 100 = 2,800,000 cells ,so the total number of cells in the volume is just N = $(2,800,000)^3$ = **2.2×10^{19} cells**

Problem 3 - If the time step between calculations was 1000 years, how many time steps did it take to complete a full 13.7 billion year calculation?

Answer: The total number of time steps is 13,700,000,000 / 1000 = **13.7 million**.

Note: The actual calculation used what is called a variable cell mesh so that for volumes of space where not much was happening, a large cell size many times larger than 100 light years was used. Only in regions where things were changing rapidly was a smaller cell size used, so the actual number of cells followed in the entire simulation was much less than 2.2×10^{19}. A similar variable mesh process was used for the time steps.

The Local Group of galaxies consists of the Milky Way, the Andromeda galaxy, and about 30 other smaller galaxies located within 5 million light years of the Milky Way.



There are many situations in astrophysics when two distinct functions are multiplied together to form a new function.

If there are N light bulbs, each with a brightness of W watts, then the total brightness, T of all these bulbs is just N x W. For N=3 bulbs and W = 100 watts we have T = 300 watts.

Suppose N(m) tells us the number of stars in an area of the sky with a brightness of m. Let a second function, S(m), represent the number of watts per square meter at the Earth that a star with a brightness of m produces. Then N(m)S(m) will be the total number of watts/meter^2 produced by the stars in the sample that have a brightness of m.

NASA's, Wide-Field Infrared Survey Explorer (WISE) satellite is surveying the sky to catalog stars visible at a wavelength of 3.5 microns in the infrared spectrum. If the differential star count function $N(m)=0.000005\ m^{+7.0}$ stars, and the star brightness function is defined by $S(m)= 350\ 10^{-0.4m}$ Janskys. Use this information to answer the following problems:

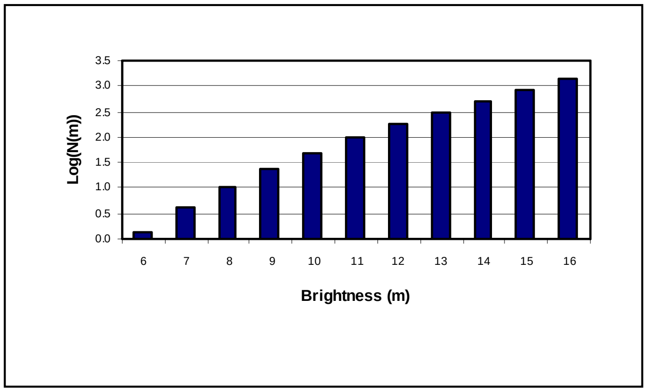
Problem 1 - Graph the functions Log(N(m)) and Log(S(m)) as individual histograms over the domain m:[+6, +16] for integer values of m.

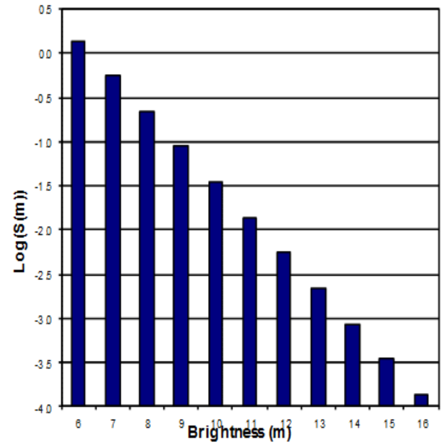
Problem 2 - Graph the product of these functions N(m)S(m) over the domain m:[+6, +16] for integer values of m.

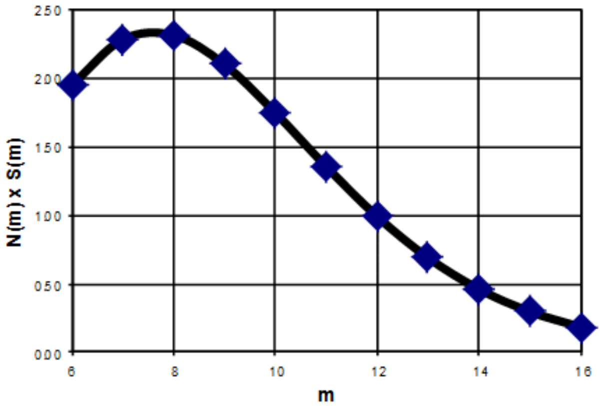
Problem 3 - What is the sum, T, of N(m)S(m) for each integer value of m in the domain m:[+6, +16], and how does this sum relate to the area under the curve for N(m)S(m)?

Problem 4 - What is the integral of N(m)S(m) from m=+6 to m= +16? You do not need to evaluate it!

Problem 1 and 2 - Answer: **See below.**







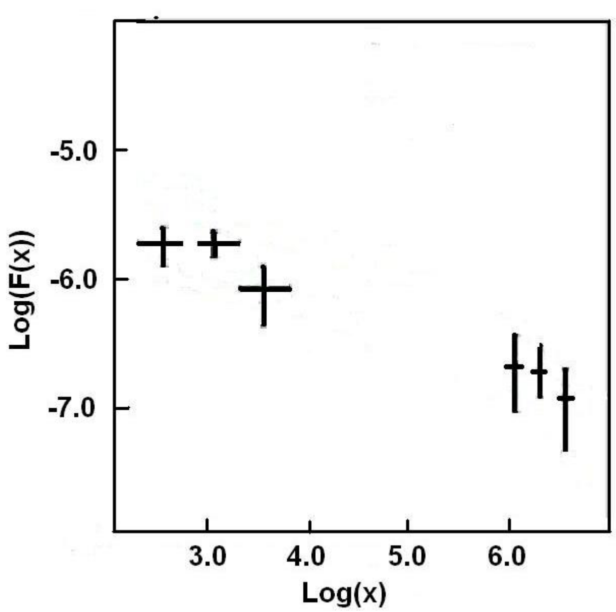
Problem 3 - Answer: The sum is T = N(6)S(6) + N(7)S(7) + … N(16)S(16)
T = 1.95 + 2.28 + 2.32 + 2.10 + 1.75 + 1.36 + 0.99 + 0.69 + 0.46 + 0.30 + 0.19
T = 14.39 Janskys. **This sum represents the approximate area under the curve N(m)S(m) vs m using vertical rectangles with a width of m= +1.0 and a height of N(m)S(m).**

Problem 4 - Answer: $T = \int_{6}^{16} (5x10^{-6})m^{+7}(350)10^{-0.4m}\,dm$ so $T = 0.00175\int_{6}^{16} m^{7}e^{-(2.3)0.4m}\,dm$

Changing variables to y=0.92m so dy = 0.92 dm we have $T = 0.0034\int_{5.5}^{14.7} y^{7}e^{-y}\,dy$

This integral can be evaluated by approximation, as we did in Problem 3 using large rectangles with a base size of 1.0. Improved approximations can be created with base sizes of 1/2m, 1/4m, 1/8m…etc until a limit is reached for a desired degree of accuracy. Note: This integral can actually 'looked up' by advanced students, and its solution will be found to involve a recursive integral of $x^{m}e^{ax}$ where m=7 and a = -1. With computers, it is actually faster to evaluate it by successive approximation!

M82, or the Cigar Galaxy, is a starburst galaxy about 12 million light-years away from Earth. In the galaxy's center, stars are being born 10 times faster than they are inside the entire Milky Way galaxy. The high stellar birth and death rate made M82 a good test case for the theory that cosmic rays are generated in supernovae explosions. In this false-color image, X-ray data recorded by the Chandra X-ray observatory is blue; infrared light recorded by the Spitzer infrared telescope is red; Hubble space telescope observations of hydrogen line emission is orange, and the bluest visible light is yellow-green. (Credit: NASA/JPL-Caltech, STScI, CXC, Uof A, ESA, AURA, JHU)



The Fermi Gamma-Ray Space Telescope has recently confirmed that the nearby galaxy, Messier 82, is the major source of high-energy gamma-rays seen at Earth: over 12 million light years away!

This galaxy is a mini-quasar with an active core in which a super-massive black hole is absorbing matter and turning it into energy at a ferocious rate.

The gamma-rays arrive at Earth, at a rate of about one or two per hour, and span a range of energies (x given in MeV) that are shown in the Log-Log plot to the left. F(x) is related to the number of gamma-rays detected per second over an area of 1 square centimeter.

Power-Laws

A surprising number of physical phenomena can be mathematically represented, at least over a part of their range, in terms of a power-law function **$F(x) = ax^n$**. We are going to explore some interesting, and convenient, features of power-law functions in analyzing data.

Problem 1 - Show that the graph of Log(F(x)) vs Log(x) is a straight line.

Problem 2 - The graph to the left shows the gamma-ray energy spectrum measured by Fermi. The data points are presented as crosses (called error bars), with the measured value being at the center of the cross. The size of each error bar shows the acceptable range of the measurement. Using a ruler, what linear equation passes through the crosses for the entire collection of data?

Problem 3 - From your answer to Problem 2, what is the 'best fit' power-law, F(x), defined by the linear equation you derived from the data?

Problem 4 - Evaluate F(4.0) and F(5.0) to determine the number of gamma rays/sec/cm^2 at energies of 10,000 and 100,000 MeV.

Problem 1 - Show that the graph of Log(F(x)) vs Log(x) is a straight line.
Answer: $F(x) = ax^n$ so Log(F) = Log(a) + n Log(x) . This is of the form y(x) = b + mx which is the equation for a line.

Problem 2 - Using a ruler, what linear equation passes through the crosses for the entire collection of data? Answer: Students may 'fit' several different lines to the data. One possibility is shown below and has the form Y = mX + b. Using the 'Point-Slope Form' of a linear equation where Y = Log(F(x)) and X = Log(x):

$$Y - Y1 = \frac{Y2 - Y1}{X2 - X1}(X - X1)$$ where X1 = 2.0, Y1 = -5.6, X2 = 7.0 and Y2 = -7.0 we have

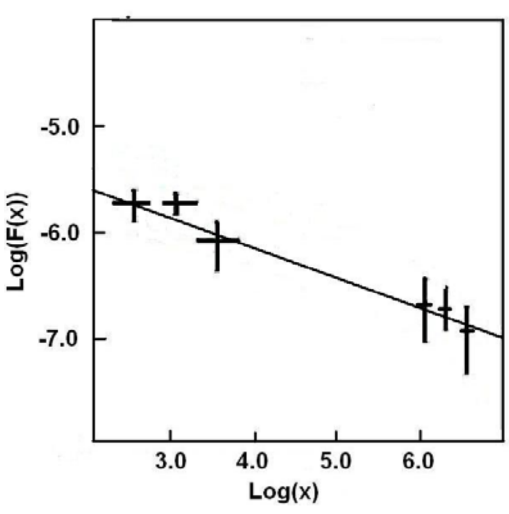
Y = -0.28 X - 5.0

Problem 3 - From your answer to Problem 2, what is the 'best fit' power-law, F(x), defined by the linear equation you derived from the data? Answer: From the above linear example, Y = Log(F(x)) and X = Log(x), so Log(F(x)) = -0.28Log(x) - 5.0 and

$10^{Log(F(X))} = 10^{-0.28Log(x)-5.0}$ becomes $F(x)=10^{-5}\, x^{-0.28}$ or $\mathbf{F(x)=0.00001x^{-0.28}}$

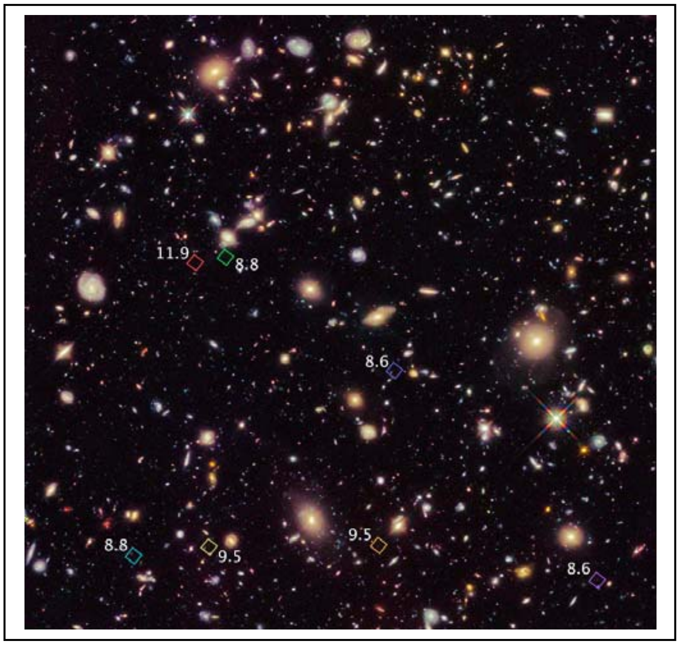
Problem 4 - Evaluate F(4.0) and F(5.0) to determine the number of gamma rays/sec/cm^{2} at energies of 10,000 and 100,000 MeV. Answer:
$F(10{,}000) = 0.00001(10{,}000)^{-0.28} = 7.58\times10^{-7}$ gamma rays/sec/cm^{2} (Log(F) = -6.1)
$F(100{,}000) = 0.00001(100{,}000)^{-0.28} = 3.98\times10^{-7}$ gamma rays/sec/cm^{2} (Log(F) = -6.4)

Note: that these two points, plotted on the Log-Log graph are (+4.0, -6.1) and (+5.0, -6.4) and 'fill-in' the gap between the data points. This is an example of interpolation.



Graph adapted from Abdo, A. A. et al, 2009, 'Detection of Gamma-Ray Emission from the Starburst Galaxies M-82 and NGC-253 with the Large Area Telescope on Fermi', (http://arxiv.org/PS_cache/arxiv/pdf/0911/0911.5327v1.pdf)

Note x is the gamma-ray energy in millions of electron volts (MeV), so '3.0' is 1,000 MeV, and F(x) is in units of MeV/cm^{2}/sec.



Using the Ultra Deep Field Image created by NASA's Hubble Space Telescope, astronomers have uncovered a previously unseen population of seven primitive galaxies that formed more than 13 billion years ago.

The age of the universe is 13.7 billion years, so we are seeing these galaxies as they were when the universe was only 4 percent of its present age. This is like an 80 year old human seeing a picture of themself when they were only 3 years old!

Among the predictions of Big Bang cosmology is a mathematical relationship T(Z) between the redshift of a galaxy denoted by the variable Z, and the time since the light was emitted by that galaxy, T, so that it can arrive at Earth today, some 13.7 billion years after the Big Bang occurred. During this time, the space within the universe has expanded, and Z indicates the amount of stretching of space that has occurred between the time when the light was emitted and the current era.

Using the exact solution for T(Z) from Big Bang theory, an astronomer wants to create a faster way to compute T(Z) that he can use on his hand calculator. He derives an approximation to the function T(Z) for Z between 2 and 15 given by

$$\mathbf{T(Z) = 0.000154z^5 - 0.0072Z^4 + 0.1301Z^3 - 1.143Z^2 + 5.0141Z + 3.7677}$$

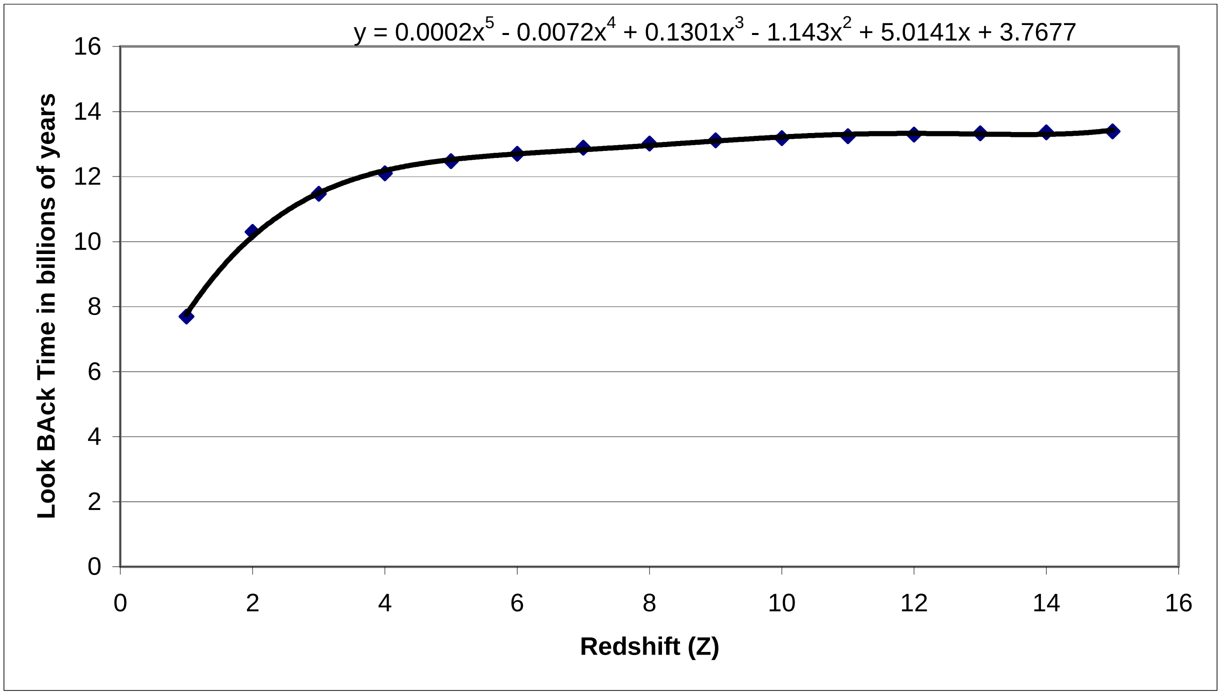
Problem 1 - Graph this function over the redshift range Z:[2.0, 15.0].

Problem 2 - Most of the galaxy redshift measurements the astronomer will make will be made over the much smaller range from 9.0 to 12.0. What is the slope of T(Z) over the range Z:[9.0, 12.0]?

Problem 3 - What is the linear equation that matches the redshift formula over Z:[9.0, 12.0]?

Problem 4 - If the astronomer uses the 'linear approximation' to T(Z) over Z:[9.0, 12.0] by what percentage will his estimates for the look-back time T differ from the original equation over the range Z:[5.0 ,15.0]?

Problem 1 - Graph this function over the redshift range Z:[2.0, 15.0].



Problem 2 - Most of the galaxy redshift measurements will be made over the much smaller range from 9.0 to 12.0. What is the slope of T(Z) over the range Z:[9.0, 12.0]?

Answer: change in Z = 12-9 = 3.0, T(12) = 13.18 billion years T(9)= 13.00 billion years, Slope M = 13.18-13.00/3.0 = 0.06.

Problem 3 - What is the linear equation that matches the redshift formula over Z:[9.0, 12.0]?

Answer: T = mZ + b, T(9) = 13.00 and T(12) = 13.18. Use the two-point formula: y-y1 - (x-x1) (y2-y1)/(x2-x1) then T - 13.00 = (z-9) (13.18-13.00)/(12-9) so T = 13.00 +(0.06(z-9) and so **T(Z) = 12.46 + 0.06Z**.

Problem 4 - If the astronomer uses the new 'linear approximation' to T(Z) over Z:[9.0, 12.0] by what percentage will his estimates for the look-back time T differ from the original equation over the range Z:[5.0 ,15.0]?

Z	Actual	Predicted	Difference	Percent
5	12.47	12.76	- 0.29	2.3%
15	13.39	13.36	+0.03	0.2%

So using the new, faster to compute, linear approximation only gives answers that differ by no more than 3% from the original, slower to compute, fifth-order polynomial approximation for T(Z).

$$\frac{dR}{dt} = \sqrt{\frac{8\pi G\rho}{3R} + \frac{\Lambda}{3}R^2}$$

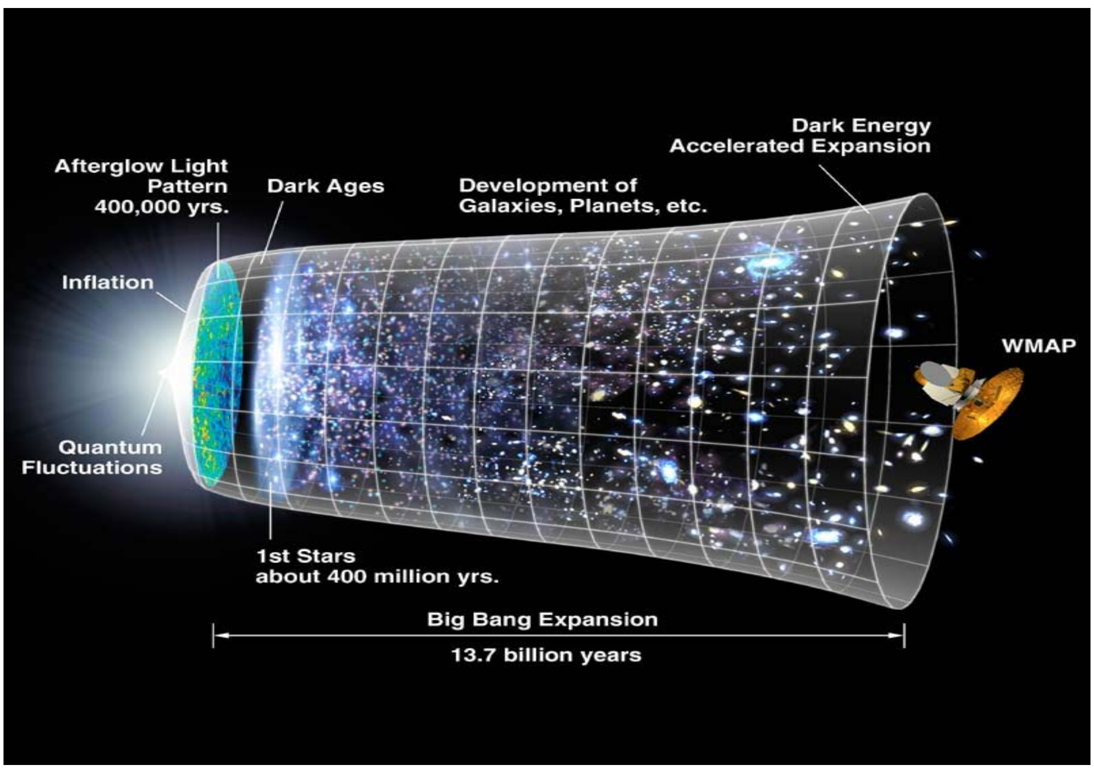
According to Big Bang theory, the scale of the universe increases with time at a rate that depends on the density of matter, ρ, and the size of the cosmological constant, Λ. This is defined by the fundamental equation to the left.

Problem 1 - Determine the general form of the integral that relates the time, t, to the value of the scale factor, R; Solve the integral for the time, t, but do not solve the integral for R.

Problem 2 - Transform the integral for R to a new variable, U, such that U = $(A/C)^{1/3}$ R where A = Λ /3 and C = $8\pi G\rho/3$.

Problem 3 - Solve the integral for two special cases A) The Inflationary Universe case where U >> 1 and B) the matter-dominated universe case where U << 1.

Problem 4 - Hubbel's Constant is a measure of the rate of expansion of the universe. It is defined as H = 1/R (dR/dt). Find the formula for Hubbel's Constant for the two cosmological cases described in Problem 3.



Answer Key

Problem 1 - The integral equation is then

$$\int dt = \int \frac{dR}{\sqrt{\frac{8\pi G\rho}{3R} + \frac{\Lambda}{3}R^2}}$$

Problem 2 First clean up the rather cumbersome radical expression so that it only involves R to positive powers and the constants A and C, by factoring out $(1/R)^{1/2}$ to get $(1/R)^{1/2}(C + A R^3)^{1/2}$. Factor out the constant C from the square-root so that the denominator of the integrand becomes $(1/R)^{1/2} C^{1/2} (1 + A/C\, R^3)^{1/2}$ and replace with $U = (A/C)^{1/3} R$ to get

$$C^{1/2} (A/C)^{1/6}\, U^{-1/2} (1 + U^3)^{1/2}$$

Note that we have also transformed the $(1/R)^{1/2}$ factor by replacing it with $(A/C)^{1/6}\, U^{-1/2}$. Since $dU = (A/C)^{1/3} dR$, we can now re-write the complete integrand as

$$(1/C)^{1/2} (C/A)^{1/6} (C/A)^{1/3}\; U^{1/2}\, dU / (1 + U^3)^{1/2}$$

After combining the constants A and C and replacing them with their definitions the integrand simplifies to $(3/\Lambda)^{1/2}\; U^{1/2}\, dU / (1 + U^3)^{1/2}$ and the integral becomes

$$t = \sqrt{\frac{3}{\Lambda}} \int \frac{U^{1/2}\, dU}{\sqrt{U^3 + 1}}$$

Problem 3 A) If U > 1, then the term under the square-root is essentially U^3, so we get $U^{1/2} / U^{3/2} = 1/U$. This leads to an integrand of $(3/\Lambda)^{1/2}\; 1/U\, dU$ which is a fundamental integral whose solution is $t = (3/\Lambda)^{1/2} \operatorname{Ln} U + C$. This can be re-written as $U(t) = e\,[(\Lambda/3)^{1/2}\, t\,]$. From the definition for U we get

$$R(t) = \left(\frac{8\pi G\rho}{\Lambda}\right)^{\frac{1}{3}} e^{\left(\frac{\Lambda}{3}\right)^{1/2} t}$$

This represents a universe that expands at an exponential rate because of the positive pressure provided by the cosmological constant - a property of the energy of empty space. This solution is thought to describe our universe during its' inflationary' era shortly after the Big Bang.

Problem 3 B) In this case, U << 1 so the term under the square-root is essentially 1, and the integrand becomes $(3/\Lambda)^{1/2}\; U^{1/2}\, dU$. This is easily integrated to get $t = (3/\Lambda)^{1/2}\; U^{3/2}$. After substituting for the definition of U we get $t = (3/\Lambda)^{1/2}\; (\Lambda/8\pi G\rho)^{1/2} R^{3/2}$ so that $t = (3/8\pi G\rho)^{1/2} R^{3/2}$. This can be easily inverted to get

$$\mathbf{R(t) = (8\pi G\tilde{\rho} / 3)^{1/3}\; t^{2/3}}$$

This solution is the 'matter-dominated' cosmology represented by Big Bang cosmology, and applies to the modern expansion of the universe.

Problem 4 A) $\mathbf{H = (\Lambda/3)^{1/2}}$ and B) $\mathbf{H = 2/3\,(1/t)}$. In the inflationary case, the rate of expansion is constant in time, but in the matter-dominated case, the expansion rate decreases in proportion to the age of the universe !

Colliding Galaxies – The future of our Milky Way

Looking towards the constellation of Triangulum (The Triangle), in the northern sky we see two very similar galaxies, named PGC 9074 and PGC 9071 located about 490 million light years from Earth. Both are spiral galaxies, and are presented to our eyes face-on, so we are able to appreciate their distinctive shapes. After the collision, these two galaxies will eventually merge together into a new, larger galaxy.

The distance between the centers of these two galaxies is about 110,000 light years. The galaxy diameters are about 75,000 light years.

Problem 1 – Astronomers measure the speeds of stars and galaxies in units of kilometers/sec, but because of the scale of the universe, sometimes it is helpful to use the equivalent speed unit if 'parsecs per million years' to make certain calculations easier and faster. One parsec equals 3.26 light years and one light year equals 9.46×10^{12} kilometers. If there are 3.1×10^{7} seconds in one year, what is the equivalent speed unit for 1 kilometer/sec in pc/myr?

Problem 2 – The two galaxies in the above photo are currently about 110,000 light years apart, and are approaching each other at a speed of 500 km/sec. In about how many million years will the two galaxies have completely collided if they continued at the same speed?

Problem 3 – Like a ball falling to the ground, the gravity of each galaxy will cause the galaxies to move faster and faster as they get closer together. The following equation describes this accelerated motion:

$$D = d_0 - V_0T - 1/2aT^2$$

where T is the time in millions of years (myr) from the present, V_0 is the current speed in pc/myr, and a is the acceleration of gravity in pc/myr^2. If d_0 = 34,000 parsecs, V_0 = 500 pc/myr, and a = 0.4 pc/myr^2, in how millions of years will the galaxies have collided?

http://www.nasa.gov/content/inseparable-galactic-twins/
July2, 2013
Inseparable Galactic Twins

Problem 1 – Astronomers measure the speeds of stars and galaxies in units of kilometers/sec, but because of the scale of the universe, sometimes it is helpful to use the equivalent speed unit if 'parsecs per million years' to make certain calculations easier and faster. One parsec equals 3.26 light years and one light year equals 9.46×10^{12} kilometers. If there are 3.1×10^{7} seconds in one year, what is the equivalent speed unit for 1 kilometer/sec in pc/myr?

Answer: 1 km/sec x (1 ly/9.46×10^{12} km) x (1 pc/3.26 ly) x (3.1×10^{7} sec/1 year) x (10^{6} yr/1 myr) = **1.0 pc/myr**.

Problem 2 – The two galaxies in the above photo are currently about 110,000 light years apart, and are approaching each other at a speed of 500 km/sec. In about how many million years will the two galaxies have completely collided if they continued at the same speed?

Answer: The galaxies are 110,000 ly apart, which is 110,000 ly x 1pc/3.26 ly = 34,000 parsecs. They are traveling at 500 km/sec which is 500 pc/myr, so the time required is T = distance/speed = 34,000 pc/500 = **68 million years**.

Problem 3 – Like a ball falling to the ground, the gravity of each galaxy will cause the galaxies to move faster and faster as they get closer together. The following equation describes this accelerated motion:

$$D = d_0 - V_0 T - 1/2 a T^2$$

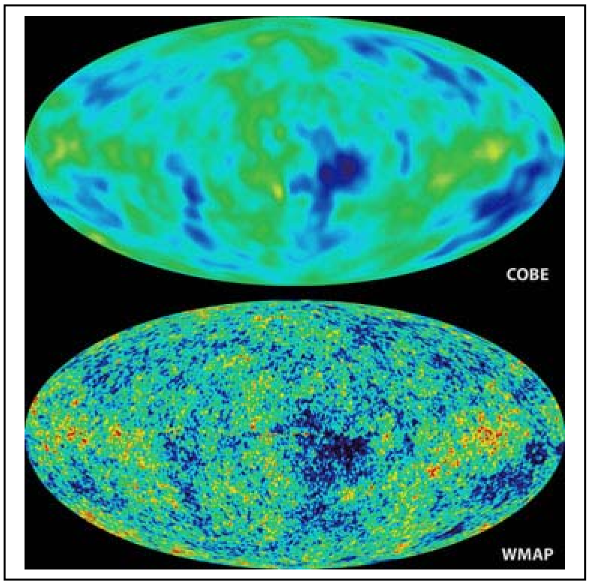
where T is the time in millions of years (myr) from the present, V_0 is the current speed in pc/myr, and a is the acceleration of gravity in pc/myr^2. If d_0 = 34,000 parsecs, V_0 = 500 pc/myr, and a = 0.4 pc/myr^2, in how millions of years will the galaxies have collided?

Answer: The formula will look like $D = 34000 - 500\,T - 0.2T^2$. We want the condition that D = 0 for the collision. The value for T can be found by graphing this equation and finding the point (T,0), or by using the quadratic roots formula.

Method 2: a = -0.2 b = -500 and c = 34000, then

$X = [-b +/- (b^2 - 4ac)^{1/2}] / 2a$

X = [500 +/- 526] / -0.4 so X1 = -2565 and X2 = +65
Only the solution '+65 million years' is a real solution about a future (positive) time so the required root is **65 million years**. This is only slightly smaller than the answer in Problem 2 which assumed constant collision speeds, so the gravitational acceleration between these two galaxies is not a significant factor.



The Webb Space Telescope uses a very large mirror to enable astronomers to see distant objects more clearly. This is a very important goal of this telescope since astronomers know so little about how objects look far from Earth. In the universe, viewing objects at a great distance also means that you are seeing them as they were long ago.

The scientific mission of the Webb Space Telescope is to study how galaxies like the Milky Way formed when the universe was only a few million years old compared to its current age of over 13 billion years.

The image above shows the sky imaged by the COBE satellite with a 7-degree resolution compared to the WMAP satellite with a 1/2-degree resolution. Note the increased detail with WMAP.

The sky is measured in angular units (degrees, minutes, seconds) but we would actually like to know how many kilometers a given angular measurements corresponds to. The simple formula below relates size to distance and angular width:

$$L = \frac{\theta}{206265} d$$

where θ is the angular diameter in arcseconds, d is the distance to the object in light years and L is the actual diameter of the object in light years. Note that for closer objects, L and d will both be in units kilometers.

The smallest feature that the human eye can see on the moon has an angular width of about 2 arcminutes or 120 arcseconds. The smallest feature that the Webb Space Telescope Near Infrared Camera (NIRcam) can see clearly has a width of about 0.032 arcseconds per pixel.

Problem 1 - What is the width of the smallest feature that the human eye can see on the moon at the distance of Earth, d = 384,000 kilometers?

Problem 2 - How far, d, would a planet the size of Earth (L = 12,800 km) have to be in order for the Webb Space Telescope to just see it (θ = 0.032 arcseconds)?

Problem 3 - Suppose that the most distant object that can be detected by the Webb Space Telescope is located 13 billion light years from Earth. What would be the minimum diameter of this object, L, at the maximum resolution of the telescope?

Problem 1 - What is the width of the smallest feature that the human eye can see on the moon at the distance of Earth, d = 384,000 kilometers?

Answer: $L = \frac{120}{206265} x384,000km$ so L = **220 kilometers**

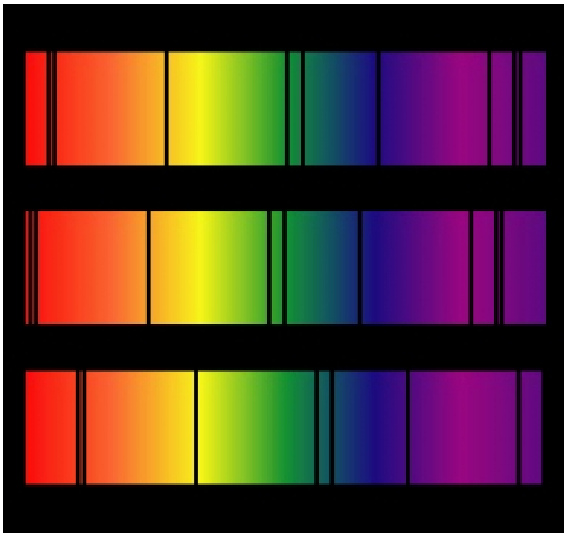
Problem 2 - How far, d, would a planet the size of Earth (L = 12,800 km) have to be in order for the Webb Space Telescope to just see it (θ = 0.032 arcseconds)?

Answer: $d = \frac{206265}{0.032} x12,800km$ so **d = 83 billion km**.

Problem 3 - Suppose that the most distant object that can be detected by the Webb Space Telescope is located 13 billion light years from Earth. What would be the minimum diameter of this object ,L, at the maximum resolution of the telescope?

$L = \frac{0.032}{206265} x13 \text{ billion light years}$ so **L = 2,000 light years**

Note: In Problem 3, no correction has been made for the fact that over these great distances, the curvature of space causes the relationship between angular size and distance to be different than the formula used, which is only valid for the geometry of flat 'Euclidean' space.



Top: Normal 'dark' spectral line positions at rest.
Middle: Source moving away from observer.
Bottom: Source moving towards observer.

We have all heard the sound of an ambulance siren as it passes-by. The high-pitch as it approaches is replaced by a low-pitch as it passes-by. This is an example of the Doppler Shift, which is a phenomenon found in many astronomical settings as well, except that instead of the frequency of sound waves, it's the frequency of light waves that is affected.

The figure to the left shows how the wavelength of various atomic spectral lines, normally found in the top locations, are shifted to the red (long wavelength; lower pitch) for a receding source, and to the blue (short wavelength; higher pitch) for an approaching source of light.

For very distant galaxies, the effects of curved space causes the wavelengths of light to be increasingly red-shifted as the distance to Earth increases. This is a Doppler-like effect, but it has nothing to do with the speed of the galaxy or star, but on the c hanging geometry of space over cosmological distances.

Just as in the Doppler Effect, where we measure the size of the Doppler 'red'-shift in terms of the speed of the object emitting the sound waves, for distant galaxies we measure their redshifts in terms of the cosmological factor, z. Close-by galaxies have z-values much less than 1.0, but very distant galaxies can have z=6 or higher.

Observing distant galaxies is a challenge because the wavelengths where most of the light from the galaxy are emitted, are shifted from visible wavelengths near 500 nanometers (0.5 microns) to much longer wavelengths. This actually makes distant galaxies very dim in the visible spectrum, but very bright at longer infrared wavelengths. For example, for redshifts of z = 3, the maximum light from a normal galaxy is shifted to a wavelength of L = 0.5 microns x (1+z) = 2.0 microns!

Problem 1 - The Webb Space Telescope, Mid-Infrared Instrument (MIR) can detect galaxies between wavelengths of 5.0 and 25.0 microns. Over what redshift interval can it detect normal galaxies like our Milky Way?

Problem 2 - An astronomer wants to study an event called Reionization, which occurred between 5.0 < z < 7.0. What wavelength range does this correspond to in normal galaxies?

Problem 3 - The NIRcam is sensitive to radiation between 0.6-5.0 microns, the MIR instrument range is 5.0 to 25.0 microns, and the Fine Guidance Sensor-Tunable Filter Camera detects light between 1 to 5 microns. Which instruments can study the Reionization event in normal galaxies?

Problem 1 - The Webb Space Telescope, Mid-Infrared Instrument (MIR) can detect galaxies between wavelengths of 5.0 and 25.0 microns. Over what redshift interval can it detect normal galaxies like our Milky Way?

Answer: L = 5.0, so 5.0 = 0.5 x (1+z) and so z = 9.0
L = 25, so 25 = 0.5 x (1+z), and so z = 49.0
The redshift interval is then z = [9,49]

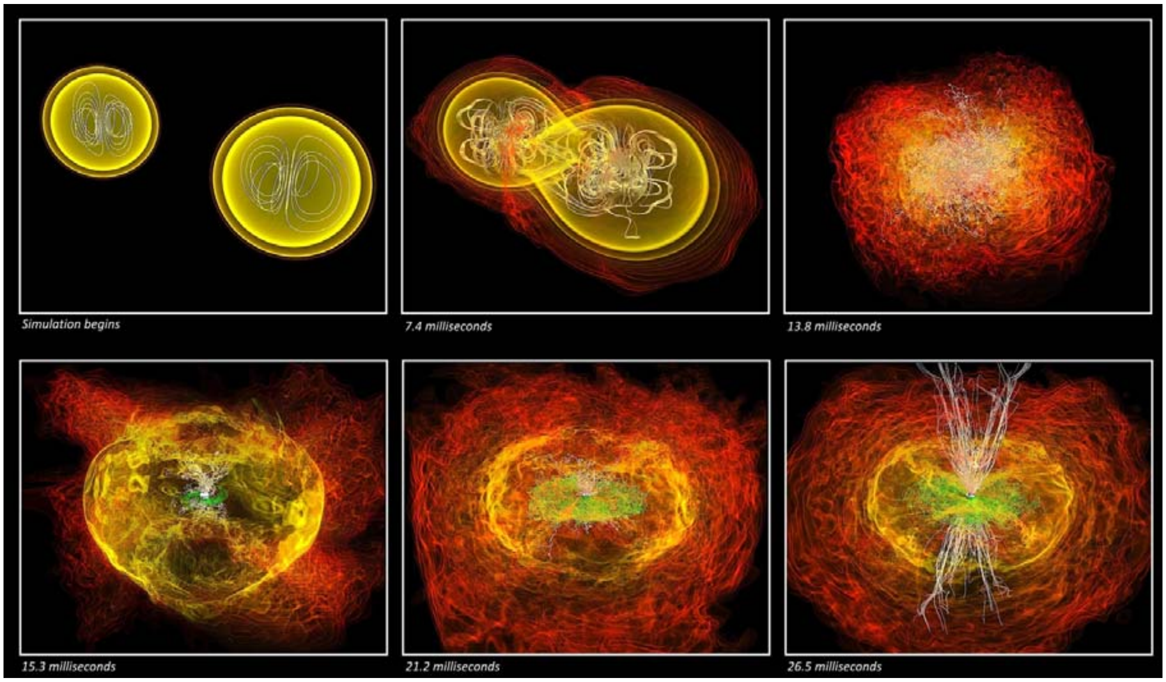
Problem 2 - An astronomer wants to study an event called Reionization, which occurred between 5.0 < z < 7.0. What wavelength range does this correspond to in normal galaxies?

Answer: Z=5.0 corresponds to L = 0.5 x (1 + 5) so L = **3.0 microns**
Z = 7.0 L = 0.5 x (1 + 7) = **4.0 microns**.

Problem 3 - The NIRcam is sensitive to radiation between 0.6-5.0 microns, the MIR instrument range is 5.0 to 25.0 microns, and the Fine Guidance Sensor-Tunable Filter Camera detects light between 1 to 5 microns. Which instruments can study the Reionization event in normal galaxies?

Answer: The Reionization event can be detected between 3.0 and 4.0 microns ,which is within the observing ranges of both the NIRcam and the FGS-TFC instruments, but not the MIR camera.

Note: The Reionization event is believed to be the result of an intense period of supernova activity in millions of galaxies across the universe, which caused intense amounts of ultraviolet radiation sufficient to ionize hydrogen gas clouds. The first time the universe was ionized to this degree ended about 400,000 years after the Big Bang when the universe cooled down during its expansion. The term 're' ionization indicates that the expanding cooled hydrogen gas clouds once again became ionized about 100 million years after the Big Bang in a second ionization event.



Once astrophysicists understand the physics involved, they can use supercomputers to model events that can never be observed first-hand. The panel of images above was created by Dr's Koppits and Rezolla, and shows what happens to two neutron stars when they collide and merge. They wanted to find out whether the magnetic fields re-arrange themselves to allow jets of matter to be 'beamed out' of the collision area. Each image, from the upper left to the lower right, represents snapshots of the event calculated by the supercomputer at 0.0, 0.007, 0.014, 0.015, 0.021 and 0.026 seconds. The width of each image is about 48 kilometers. Each neutron star, the dense core of a star after a supernova, has a mass of about 1.5 times our own sun.

Problem 1 - How fast were the two neutron stars approaching each other between 0.007 and 0.014 seconds after the star of the calculation?

Problem 2 - The radius of a black hole with a mass of M solar masses is given by the formula R = 3.0 M, where R is the radius of the event horizon in kilometers. If the last calculated image at 0.026 seconds represents the final size of the neutron star merger, will it become a black hole?

Problem 3 - The double cone formed by the neutron star's magnetic field is shown in the last computed image at 0.026 seconds after the start of the collision. Astrophysicists predict that high-speed particles and radiation will flow out of the neutron stars within this 'beaming channel' formed by these concentrated magnetic fields. From the measured angle, and by using any method, what will be the width of this beam by the time it leaves its galaxy and reaches our Milky Way located 5 billion light years away in A) light years? B) Milky Way diameters if 1 MW = 100,000 light years?

Problem 1 - How fast were the two neutron stars approaching each other between 0.007 and 0.014 seconds after the star of the calculation?

Answer: The width of each image is 48 kilometers, and using a millimeter ruler, the width is about 48 millimeters, so the scale is 1 kilometer/mm. The center to center separation between the neutron stars was 18 millimeters or 18 kilometers at 0.007 seconds, and 0 millimeters at 0.014 seconds, so the distance traveled was 18 kilometers in (0.014 – 0.007) = 0.007 seconds. The speed is then S = 18 km/0.007 sec or **2600 kilometers/sec**.

Problem 2 - The radius of a black hole with a mass of M solar masses is given by the formula R = 3.0 M, where R is the radius of the event horizon in kilometers. If the last calculated image at 0.026 seconds represents the final size of the neutron star merger, will it become a black hole?

Answer; If no mass is lost, the final mass is 3.0 solar masses, so the radius of the black hole will be R = 9 kilometers. For a black hole to form, the mass must be located inside this radius. The last image shows that the matter in the collision has an extent of about 20 km, so a black home will **probably not form**.

Problem 3 - The double cone formed by the neutron star's magnetic field is shown in the last computed image at 0.026 seconds. Astrophysicists predict that high-speed particles and radiation will flow out of the neutron stars within this 'beaming channel' formed by these concentrated magnetic fields. From the measured angle, and by using any method, what will be the width of this beam by the time it leaves its galaxy and reaches our Milky Way located 5 billion light years away in: A) light years? B) Milky Way diameters if 1 MW = 100,000 light years?

Answer: Students may use a protractor to measure this angle. Answers near 30 degrees are acceptable.

A) Method 1: Students may draw a scaled drawing of this event.

Method 2 : The full circumference of the circle is 360 degrees, so a 30-degree segment represents about (30/360) = 1/12 of the full circumference. The circumference is 2 p d = 6.24 x 5 billion = 31 billion light years, so 1/12 of this is **about 2.6 billion light years**.

Method 3: The tangent of one-half the angle is equal to the ½ of the width of the beam divided by the distance to the Milky Way, so w = 2 d x Tan(θ/2). For θ = 30 and d = 5 billion light years, w = **2.7 billion light years**.

B) Width = 2.7 billion x (1 MW/100,000 ly) = **27,000 Milky Way diameters!**

Note: The outflowing energy tends to be even more tightly beamed than this rather large angle. Estimates of about 0.6 degrees are not uncommon when actual beaming events are observed and analyzed. The width of this beam at a distance of 5 billion light years is more like (0.6/30)x2.7 billion light years or about 50 million light years. This is about 500 times the width of the Milky Way!

Rotation Velocity of a Galaxy

Spiral galaxy M-101 showing its bright nucleus and spiral arms. The radius of M-101 is about 90,000 light years, which corresponds to x=9 in the formula for V(x). (Hubble image)

Stars orbit the center of a galaxy with speeds that decrease as their orbital distances increase. A simple function, V(x) can model the orbital speeds of stars as a function of their distance, x, from the nucleus of the galaxy:

$$V(x) = \frac{350x}{\left(1+x^2\right)^{\frac{3}{4}}}$$

For example: At a distance of 10,000 light years from the center, x = 1.0 and the rotation speed is V(1.0) = 208 kilometers/sec.

Problem 1 – For small x (i.e. x < 1), what is the limiting form of V(x)?

Problem 2 – For large x, (i.e. x > 1) what is the limiting form of V(x)?

Problem 3 - The radius of M-101 is 90,000 light years. How fast are stars orbiting the center of M-101 according to V(x)? (Hint: At a radius of 90,000 light years, x=9.0. If the units of V(x) are kilometers/sec, what is V(x) at x = 9.0?)

Problem 4 – For what value of x is V(x) maximum?

Problem 5 – For x=1 the physical distance is 10,000 light years. How many years does it take a star to complete one circular orbit at x=1.0 if 1 light year equals 9.5 x 10^{12} km, and there are 3.1 x 10^{7} seconds in a year?

Note: This example of V(x) is for galaxies in which most of the mass is concentrated within their central regions (x < 1), however, astronomers know that this model is not completely accurate. Beyond x = 1, the rotation speeds for some galaxies, including the Milky Way, do not decrease rapidly as suggested by V(x), but actually remain constant. This implies that some galaxies contain substantial amounts of 'Dark Matter' that is not in the form of stars or other known forms of matter.

Problem 1 – For small x, what is the limiting form of V(x)?
Answer: The denominator approaches 1 and so **V(x) = 350x**

Problem 2 – For large x, what is the limiting form of V(x)?
Answer: In the denominator, x^2 dominates over 1 so the denominator approaches $x^{3/2}$ and so $V(x) = 350x/x^{3/2}$ becomes:

$$V(x) = \frac{350}{\sqrt{x}}$$

Problem 3 - The radius of M-101 is 90,000 light years. How fast are stars orbiting the center of M-101 according to V(x)? (Hint: At a radius of 90,000 light years, x=9.0. If the units of V(x) are kilometers/sec, what is V(x) at x = 9.0?)
Answer:

$$V(9) = \frac{350(9)}{(1+9^2)^{\frac{3}{4}}}$$ = **26 kilometers/sec**

Note: X is a pure number. It represents the ratio X = (d /10,000 light years) where d is a physical distance in units of light years. Example: at a physical distance of 40,000 light years from the center of the galaxy, x = 40,000 LY/10,000 LY so x = 4.0. The rotation speed of stars at this distance is just $V(4) = 350(4)/(1+4^2)^{\mathbf{3/4}}$ = 167 kilometers/sec.

Problem 4 – For what value of x is V(x) maximum?
Answer: Students can graph this function on a calculator. The maximum should occur near **x = 1.4** with a value V(x) = 217 km/sec.

Advanced students can use differential calculus and solve for x in the equation dV(x)/dx = 0.

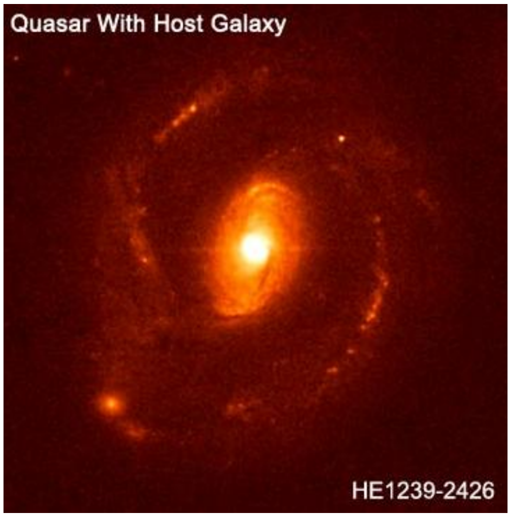
$$\frac{dV(x)}{dx} = \frac{-(350x)(3/4)(1+x^2)^{\frac{-1}{4}}(2x) + 350(1+x^2)^{\frac{3}{4}}}{(1+x^2)^{\frac{3}{2}}}$$

so after some algebra:

$$0 = 1 - \frac{3x^2}{2(1+x^2)}$$ so $2 + 2x^2 = 3x^2$ and $x = (2)^{1/2}$ = **1.414**

Problem 5 – For x=1 the physical distance is 10,000 light years. How many years does it take a star to complete one circular orbit at x=1.0 if 1 light year equals 9.5×10^{12} km, and there are 3.1×10^7 seconds in a year? Answer: For x=1 the physical distance is 10,000 light years or 9×10^{16} kilometers. The circumference of the orbit is $2 \pi R = 2\ (3.141)\ (9.5 \times 10^{16}$ km) = 6.0×10^{17} kilometers. The speed is V(1) = 208 km/sec, so the time in seconds is $T = 6 \times 10^{17}$ kilometers / (208 km/sec) = 2.9×10^{15} seconds. Since there are 3.1×10^7 seconds/year, it will take **93 million years for a star to orbit once-around the center of M-101**.

How Many Quasars Are There?



This Hubble Space Telescope image shows the brilliant 'quasar' core of a distant galaxy located 1.5 billion light years from Earth in the direction of the constellation Corvus the Crow.

Quasars are among the most luminous galaxies in the universe, and because of this, astronomers can detect them to great distances exceeding nearly 10 billion light years from Earth. Originally discovered as peculiar faint 'blue stars' in 1964, astronomers have counted up their numbers as a function of their brightness, expressed according to the logarithmic stellar apparent magnitude scale, m. The function that provides a good match to the quasar surface density, N(m), defined over the domain [+13.0, +23.0], is given by:

$$\frac{dN(m)}{dm} = 10^{f(m)}$$

where f(m) is a piecewise function defined as follows:

f(m) = +0.7m -12.0 for $+13.0 \le m \le +18.0$
f(m) = +0.5m -9.0 for $+18.0 < m \le +23.0$

Problem 1 - The integral of dN(m)/dm is the total number of quasars present in one square degree of the sky. As a comparison, the full moon subtends an area of about 0.19 square degree. How many quasars does the function predict that have magnitudes between +13.0 and +18.0 inclusively, over an area of the sky similar to the area of the full moon?

Problem 2 - How many quasars does the function predict that have magnitudes between +18.0 and +23.0 inclusively, over an area of the sky similar to the area of the full moon?

Problem 3 - An astronomer wants to study quasars in the brightness interval +13.0 < m < +18.0. What is the minimum sky area, in square degrees, that she needs to photograph in order to have one quasar present in the photograph?

Problem 4 - Combining the quasar sky densities from Problem 1 and 2, and the fact that the sky has a total angular area of 41,253 deg^2 how many quasars are there with magnitudes in the range from +13 to +23, inclusively?

Problem 1 - The integral of dN(m)/dm is the total number of quasars present in one square degree of the sky. As a comparison, the full moon subtends an area of 0.19 square degree. How many quasars does the function predict that have magnitudes between +13.0 and +18.0 inclusively, over an area of the sky similar to the area of the full moon?

Answer: $N = \int_{13}^{18} \frac{dN(m)}{dm} dm$ so from the definition of f(m) in the appropriate interval,

we get $N = \int_{13}^{18} 10^{0.7m-12} dm$ With u = (0.7m-12) , du = 0.7 dm, and using $10^x = e^{2.3u}$ the

equivalent integral becomes: $N = \frac{1}{0.7} \int_{-2.9}^{0.6} e^{2.3u} du$ or with x = 2.3u $N = \frac{1}{0.7(2.3)} \int_{-6.67}^{1.38} e^{x} dx$

so N = 0.62 [$e^{1.38} - e^{-6.67}$] = 0.62[3.97 - 0.001] = 2.46 quasars per deg^2. The full moon has an area of 0.19 square degrees, so there will be (0.19 deg^2) x (2.46 quasars/deg^2) = **1.92 quasars in this sky area. (Note: because you can't have a fraction of a quasar, this rounds to 2 quasars!)**

Problem 2 - How many quasars does the function predict that have magnitudes between +18.0 and +23.0 inclusively, over an area of the sky similar to f the area of the full moon?

Answer; $N = \int_{18}^{23} 10^{0.5m-9} dm$ With u = (0.5m-9) , du = 0.5 dm, and using $10^x = e^{2.3u}$

the equivalent integral becomes: $N = \frac{1}{0.5(2.3)} \int_{0}^{5.75} e^{x} dx$

so N = 0.87 [$e^{5.75} - e^{0}$] = 0.87 [314 - 1] = 313 quasars/deg^2. For an area equal to the full moon, there are (0.19 deg^2) x (313 quasars/deg^2) = **59 quasars in this sky area.**

Problem 3 - An astronomer wants to study quasars in the brightness interval +13.0 < m < +18.0. What is the minimum sky area, in square degrees, that she needs to photograph in order to have at least one quasar present in the photograph? Answer; From Problem 1, the integral over this magnitude range gives an surface area of 1.92 quasars per square degree. To find one quasar, you need to photograph an area of 1/1.92 square degrees or **0.5 square degrees**.

Problem 4 - Answer: The combined quasar density is 1.92 + 313 = 311 quasars/deg^2 , so over the entire sky, there are about 311 x 41,253 = **12.8 million quasars** in this magnitude range.

$$B = B_0 10^{-0.4m}$$

Since the time of the Greek astronomer Hipparchus (190BC), astronomers have used a 'magnitude' scale to indicate the brightness of stars. A star of the First Magnitude (+1.0) is brighter than a star of the Second Magnitude (+2.0) and so on, which means that, the more positive a star's magnitude is, the fainter is the star! Although modern astronomers would have preferred a more conventional scale, we are basically stuck with this ancient convention.

On the stellar magnitude scale, a difference of 5 magnitudes is exactly a brightness difference of 100 times. This magnitude scale, **m**, is related to a physical brightness scale, **B**, using the formula shown to the left.

The star field image is from the Hubble Space Telescope Exoplanet Survey and shows a range of stellar brightnesses spanning a magnitude range from about +13 to +17

Problem 1 - In this formula, to what magnitude does a brightness of B_0 correspond?

Problem 2 - The sun has a magnitude of -26.5, and the faintest star detected by the Hubble Space Telescope has a magnitude of +28.5 A) What is the magnitude difference between these two objects? B) By what factor to they differ in brightness?

Problem 3 - An astronomer has a digital camera that can accommodate a brightness level change of about 10 million from the brightest to the faintest object that can be imaged without 'saturating' the camera. What magnitude difference does this range correspond to?

Problem 4 - NASA's WISE infrared sky survey satellite will detect stars at a wavelength of 4.6 microns (called M-band) to a brightness limit of 160 microJanskys. If B0 is 180 Janskys at this wavelength, and for the visual magnitude scale (called V-band) B0 = 3,781 Janskys. What will be the equivalent magnitude limit of the WISE survey at 4.6 microns, and in the visual band?

Problem 1 - In this formula, to what magnitude does a brightness of B_0 correspond?

Answer: It corresponds to m=0 .This means that B_0 establishes the 'zero-point' for this magnitude scale.

Problem 2 - The sun has a magnitude of -26.5, and the faintest star detected by the Hubble Space Telescope has a magnitude of +28.5 A) What is the magnitude difference between these two objects? B) By what factor do they differ in brightness?

Answer: A) 28.5 - (-26.5) = **55 magnitudes**! B) From the formula, the brightness ratio will be $10^{0.4(55)}$ = **10^{22} times** (or alternately 10^{-22} times).

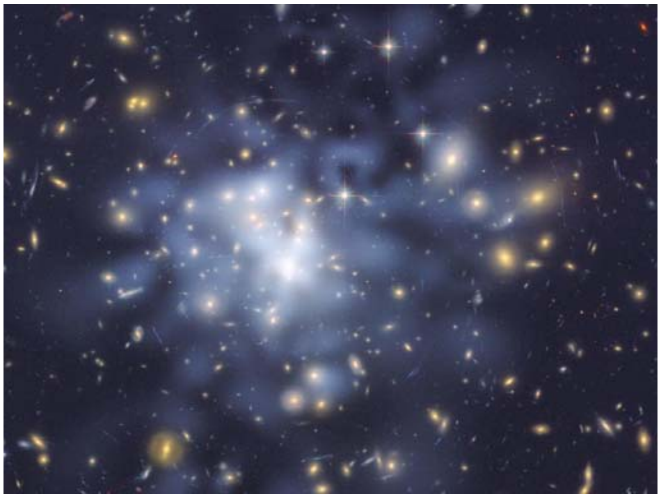
Problem 3 - An astronomer has a digital camera that can accommodates a brightness level change of about 10 million from the brightest to the faintest object that can be imaged without 'saturating' the camera. What magnitude difference does this range correspond?

Answer $10^7 = 10^{0.4(m)}$ so m = **17.5 magnitudes**.

Problem 4 - NASA's WISE infrared sky survey satellite will detect stars at a wavelength of 4.6 microns (called M-band) to a brightness limit of 160 microJanskys. If B0 is 180 Janskys at this wavelength, and for the visual magnitude scale (called V-band) B0 = 3,781 Janskys. What will be the equivalent magnitude limit of the WISE survey at 4.6 microns, and in the visual band?

Answer: For M-band, 0.000160 Jy = 180 Jy $10^{-0.4m}$ so m(M-band) = **+15.1**
For V-band, 0.000160 Jy = 3,781 Jy $10^{-0.4m}$ so m(V-band) = **+18.4**. So, the same brightness level in V-band corresponds to a magnitude of +18.4 which is +3.3 magnitudes fainter than in M-band.

Hubble Detects More Dark Matter



This NASA Hubble Space Telescope image shows the distribution of dark matter in the center of the giant galaxy cluster Abell 1689. This cluster contains about 1,000 galaxies and trillions of stars, and is located 2.2 billion light-years from Earth.

Dark matter is an invisible form of matter that accounts for most of the universe's mass. Hubble cannot see the dark matter directly, but used the distorted images of background galaxies to infer its location.

Astronomers used the observed positions of 135 distorted images of 42 background galaxies to calculate the location and amount of dark matter in the cluster. They superimposed a map of these inferred dark matter concentrations, tinted blue, on an image of the cluster taken by Hubble's Advanced Camera for Surveys. If the cluster's gravity came only from the visible galaxies, the galaxy image distortions would be much weaker, and the corresponding area in the image above would be black. The map reveals that the densest concentration of dark matter is in the cluster's core.

Problem 1 - The escape speed of a body located R meters from a second body with a mass of M kilograms can be given by the basic 'escape velocity' equation:

$$V = \sqrt{\frac{2GM}{R}}$$

where the constant of gravity given by G = 6.66 x 10^{-11}. We can re-write this more conveniently for a cluster of N galaxies each with an average mass of 10 billion stars, and an average star mass of 2 x 10^{30} kg, and a cluster radius of R light years (1 light year = 9.4 x 10^{15} meters)

$$V = 17,000\sqrt{\frac{N}{R}} \quad \text{km/sec}$$

Show that the second equation is related to the first one as indicated.

Problem 2 - An astronomer counts the galaxies in two clusters and measures the average radius of each galaxy cluster. The astronomer also measures the average speeds of the galaxies in each cluster and finds that for Cluster A: R= 10 million, N= 1000, V= 300 km/s, and for Cluster B: R= 5 million light years, N= 350, V= 140 km/s. For which cluster does the measured average speed not match the expected speed based on the visible, countable, masses found in the galaxies?

Problem 3 - How many extra 'invisible' galaxies would the discrepant cluster have to have in order for the mass of the cluster to be consistent with the observed speeds of its visible galaxies? (This is the 'dark matter' problem, originally called the 'missing mass' problem, and indicates that extra gravitating mass is present that is not in the form of ordinary stars and gas within the visible galaxies.)

Problem 1 - Answer: The escape speed of a body located R meters from a second body with a mass of M kilograms can be given by the basic 'escape velocity' equation:

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where the constant of gravity given by G = 6.66 x 10^{-11}. We can re-write this more conveniently for a cluster of N galaxies each with an average mass of 10 billion stars, and an average star mass of 2 x 10^{30} kg, and a cluster radius of R light years (1 light year = 9.4 x 10^{15} meters)

$$V = \sqrt{\frac{2(6.66x10^{-11})(N)(10billion)(2x10^{30})}{9.4x10^{15}(R)}} \quad \text{so} \quad V = 17,000,000\sqrt{\frac{N}{R}} \text{ meters/sec}$$

And so in kilometers/sec we have:

$$V = 17,000\sqrt{\frac{N}{R}} \text{ km/sec}$$

Problem 2 - An astronomer counts the galaxies in two clusters and measures the average radius of each galaxy cluster. The astronomer also measures the average speeds of the galaxies in each cluster and finds that for Cluster A: R= 10 million, N= 1000, V= 300 km/s, and for Cluster B: R= 5 million light years, N= 350, V= 140 km/s. For which cluster does the measured average speed not match the expected speed based on the visible, countable, masses found in the galaxies?

Answer: Cluster A: V(predicted) = 17,000 $(1000/10 \text{ million})^{1/2}$ = 170 km/s

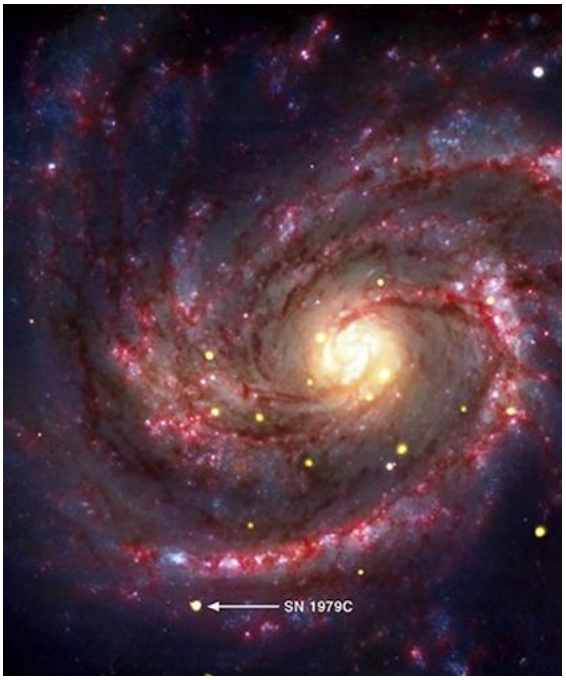
Cluster B: V(predicted) = 17,000 $(350/5 \text{ million})^{1/2}$ = 140 km/s

Although the average measured speed of the galaxies in Cluster B are identical to the predicted speed given the size and mass of this cluster, for Cluster A, its measured galaxy speeds are much higher than the speed deduced from the clusters visible galaxies and size, so **Cluster A is the discrepant one**.

Problem 3 - How many extra 'invisible' galaxies would the discrepant cluster have to have in order for the mass of the cluster to be consistent with the observed speeds of its visible galaxies? (This is the 'dark matter' problem, originally called the 'missing mass' problem, and indicates that extra gravitating mass is present that is not in the form of ordinary stars and gas within the visible galaxies.)

Answer: In order to make the average speeds of the galaxies in Cluster A (300 km/s) match the expected speed for a cluster with this many visible galaxies (V=170 km/s) we have to add more mass in the form of Dark Matter.

$$300 = 17,000\sqrt{\frac{n}{10million}}$$ so n = 3114 galaxies-worth of equivalent mass. Since only N=1000 is accounted for, we need to add a **Dark Matter equivalent to about 2114 extra galaxies,** or about m(Dark) = 2114 galaxies x 10 billion stars/galaxy = 2.1 x 10^{13} stellar masses-worth of Dark Matter.



The Chandra X-Ray Observatory recently found evidence for an infant black hole in the nearby galaxy Messiar-100. The black hole is thought to have been produced when a star with a mass of about 20 times that of the sun exploded and left behind a black hole with a mass about 8 times the sun's mass.

The satellite observatory has detected x-rays from the gasses in the orbiting accretion disk that are falling into this young black hole. Infalling gas can be heated to over 100,000,000 K as atoms collide at higher and higher speed during the infall process. The temperature of this x-ray emitting gas is related to its distance from the black hole.

At a distance of R kilometers from a black hole with a mass of M times the sun, suppose that the two equations below relate the temperature of the gas, T, and the wavelength, L, at which the in-flowing gas emits most of its light:

Equation 1 - $T = 100,000,000\left(\frac{M}{R^3}\right)^{1/4}$ Kelvin

Equation 2 - $L = \frac{3,600,000}{T}$ nanometers

where M is in solar mass units, and R is in kilometers.

Problem 1 - Combining these equations using the method of substitution, what is the new formula L(R,M), for the wavelength, L, emitted by the gas as a function of its distance from the black hole center, R, and the mass of the black hole, M?

Problem 2 – X-rays are detected from the vicinity of the SN 1979C black hole at a wavelength of 0.53 nanometers (2,300 electronVolts). If the mass of the black hole is 8 times the sun, at what distance from the center of the black hole is the gas being detected?

Problem 3 – The Event Horizon of a black hole that is not rotating (called a Schwarzschild black hole) is located at a distance of Rs = 3.0 M from the center of the black hole, where M is the mass of the black hole in units of our sun, and Rs is in units of kilometers. What is Rs for the SN 1979C black hole, and where is the x-ray emitting gas in relation to the Event Horizon?

NASA Press release 'Youngest Nearby Black Hole' November 15, 2010

"Data from Chandra, as well as NASA's Swift, the European Space Agency's XMM-Newton and the German ROSAT observatory revealed a bright source of X-rays that has remained steady for the 12 years from 1995 to 2007 over which it has been observed. This behavior and the X-ray spectrum, or distribution of X-rays with energy, support the idea that the object in SN 1979C is a black hole being fed either by material falling back into the black hole after the supernova, or from a binary companion.

The scientists think that SN 1979C formed when a star about 20 times more massive than the Sun collapsed. It was a particular type of supernova where the exploded star had ejected some, but not all of its outer, hydrogen-rich envelope before the explosion, so it is unlikely to have been associated with a gamma-ray burst (GRB). Supernovas have sometimes been associated with GRBs, but only where the exploded star had completely lost its hydrogen envelope. Since most black holes should form when the core of a star collapses and a gamma-ray burst is not produced, this may be the first time that the common way of making a black hole has been observed.

The very young age of about 30 years for the black hole is the observed value, that is the age of the remnant as it appears in the image. Astronomers quote ages in this way because of the observational nature of their field, where their knowledge of the Universe is based almost entirely on the electromagnetic radiation received by telescopes."
(**http://www.nasa.gov/mission_pages/chandra/multimedia/photoH-10-299.html)**

Problem 1 - Answer: Substitute Equation 1 into Equation 2 to eliminate T,

$$L(R,M) = \frac{3,600,000}{100,000,000}\left(\frac{R^3}{M}\right)^{1/4}$$

so $L(R,M) = 0.036\left(\frac{R^3}{M}\right)^{1/4}$ nanometers.

Problem 2 – Answer: $0.53 = 0.036\ (8^{-1/4})\ R^{3/4}$ so solve for R to get

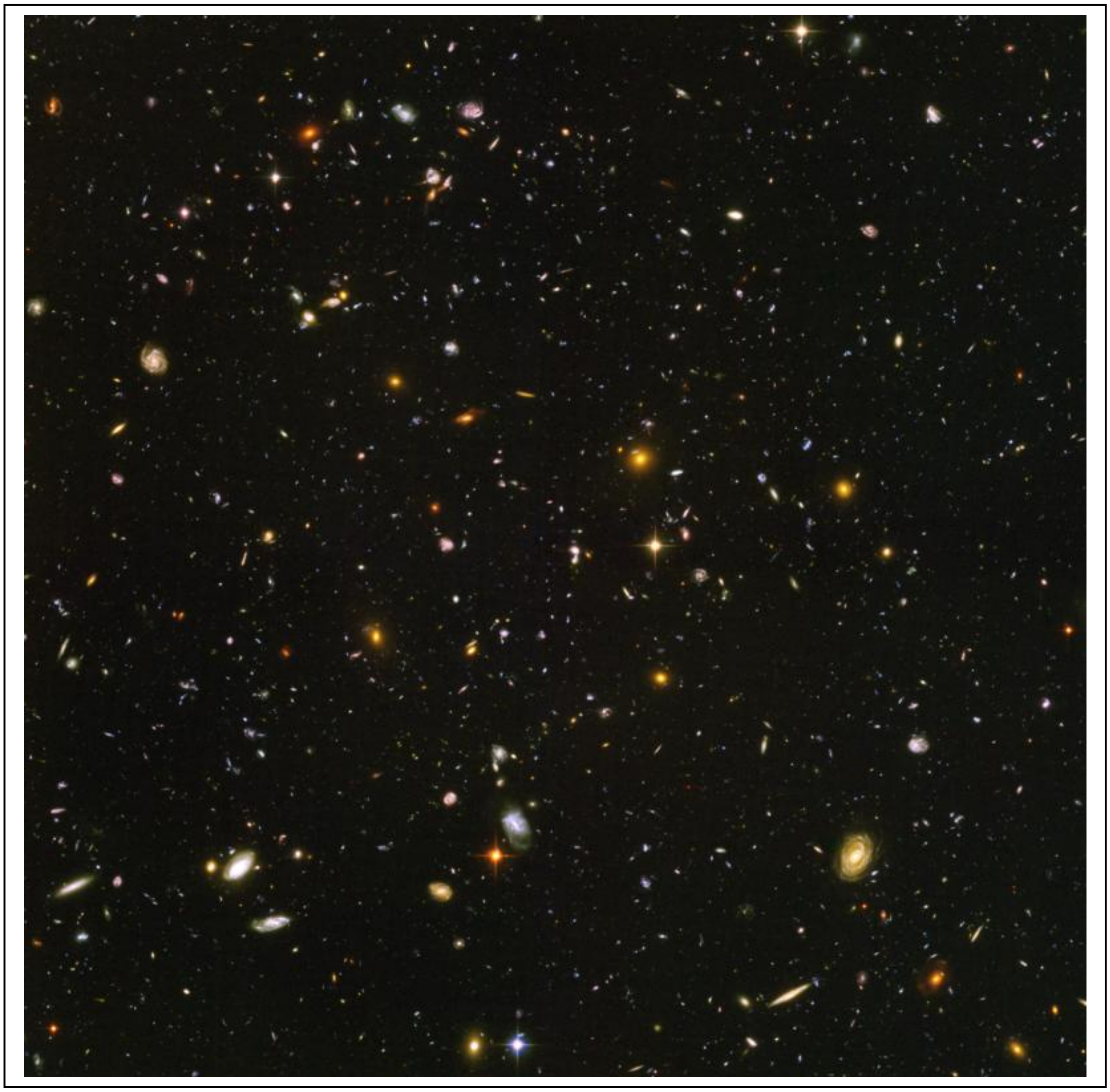
$R = (24.8)^{4/3}$

and so R = **72 km**.

Problem 3 – Answer: The Event Horizon is at Rs = 3.0 x 8 = 24 kilometers. **The x-ray emitting gas is located at R = 72 km, just outside the Event Horizon at a distance of about 48 kilometers.**

Counting Galaxies with the Hubble Space Telescope

In 2004, the Hubble Space Telescope took a million-second exposure of a small part of the sky to detect as many galaxies as possible. Here's what they saw!



Problem 1 – Divide the field into 16 equal areas. Label the grids alphabetically from A to P starting from the top left cell. Count the number of 'smudges' in four randomly selected cells. What is the average number of galaxies in of one of the cells in the picture? What uncertainties can you identify in counting these galaxies?

Problem 2 – One square degree equals 3,600 square arcminutes. If Hubble Ultra Deep Field picture is 3 arcminutes on a side, what is the area of one of your cells in square degrees?

Problem 3 – There are 41,250 square degrees in the sky, about how many galaxies are in the full sky as faint as the faintest galaxy that Hubble detected in the Deep Field image?

Answer Key

Problem 1 – Divide the field into 16 equal areas. Count the number of 'smudges' in four randomly selected cells. What is the average number of galaxies in an area of the sky equal to the size of one of the cells in the picture? What uncertainties can you identify in counting these galaxies?

Answer: Students counting will vary depending on how many 'spots' they can easily see. There should be vigorous discussions about which smudges are galaxies and which ones are photocopying artifacts. Note that, depending on the quality of the laser printer used, the number of galaxies will vary considerably.

The full image is about 145 millimeters wide, so the cells should measure about 145/4 = 36 millimeters wide. For a typical photocopy quality, here are some typical counts in 4 cells:
D=160, F=170; K=112; M=164. The average is (160+170+112+164)/4 = 151 galaxies/cell. Students estimates may vary depending on the number of galaxies they could discern in the photocopy of the Hubble Ultra-Deep Field image. A reasonable range is from 50 – 200 galaxies per cell on average.

Problem 2 – One square degree equals 3,600 square arcminutes. If the Hubble Ultra Deep Field picture is 3 arcminutes on a side, what is the area of one of your cells in square degrees?

Answer: The field is 3 arcminutes wide, so one cell is 3 / 4 = 0.75 arcminutes on a side. Since there are 60 arcminutes in one degree, this equals 0.75 x 1/60 = 0.0125 degrees. The area of the cell is 0.0125 x 0.0125 = 0.000156 square degrees.

Problem 3 – There are 41,250 square degrees in the sky, about how many galaxies are in the full sky as faint as the faintest galaxy that Hubble detected in the Deep Field image?

Answer: The number of these cells in the full sky is 41,250 square degrees/0.000156 square degrees = 264 million. Since there are on average 151 galaxies per cell, the total number of galaxies in the sky is 151 galaxies/cell x 264 million cells/full sky = 39.4 billion galaxies.

Note, astronomers using the original image data counted an average of 625 galaxies in each cell, for an estimated total of 165 billion galaxies in the full sky.

The photo on the left shows what the universe may have looked like a few million years after the Big Bang: A clumpy soup of dimly glowing matter. The image on the right shows how one of those clumps may have evolved into a recognizable galaxy today. In Big Bang cosmology, the universe expands, and space stretches. An important consequence of this is that the density of matter in space is also decreasing!

Problem 1 - The volume of the Milky Way can be approximated by a disk with a thickness of 1000 light years and a radius of 50,000 light years. Compute the volume of the Milky Way in cubic centimeters. (1 light year = 9.5×10^{17} centimeters.)

Problem 2 - The mass of the Milky Way is approximately equal to 300 billion stars, each with the mass of the Sun: 2×10^{33} grams. Compute the total mass of the Milky Way.

Problem 3 - If you were to take all of the stars and gas in the Milky Way and spread them out throughout the entire volume of the Milky Way, about what would be the density of the Milky Way in: A) grams/cm^3 B) kilograms/m^3

Problem 4 - If the average density of the matter in the universe was at one time equal to that of the Milky Way (Problem 3), by what factor would the volume of the universe have to increase in order for it to be 4.6×10^{-31} grams/cm^3 today?

Problem 5 - By what factor would the size of the universe have had to expand by today, and how far apart would the Milky Way and the Andromeda galaxy have been at that time if their current separation is 2.2 million light years?

Answer Key

Problem 1 - The volume of the Milky Way can be approximated by a disk with a thickness of 1000 light years and a radius of 50,000 light years. Compute the volume of the Milky Way in cubic centimeters.

Answer: $V = \pi R^2 h = \pi (50,000)^2 (1000) = 7.9 \times 10^{12}$ cubic lightyears.

$= 7.9 \times 10^{12} \times (5.9 \times 10^{17}\ \text{cm/ly})^3 =$ $\mathbf{1.6 \times 10^{66}\ cm^3}$

Problem 2 - The mass of the Milky Way is approximately equal to 300 billion stars, each with the mass of our sun: 2×10^{33} grams. Compute the total mass of the Milky Way.

Answer: $M = 3 \times 10^9 \times 2 \times 10^{33}$ grams = $\mathbf{2 \times 10^{42}}$ **grams**

Problem 3 - If you were to take all of the stars and gas in the Milky Way and spread them out throughout the entire volume of the Milky Way, about what would be the density of the Milky Way in: A) grams/cm^3 B) kilograms/m^3

Answer A) Density = M/V = 2×10^{42} grams / $\mathbf{1.6 \times 10^{66}\ cm^3}$ = $\mathbf{1.3 \times 10^{-24}}$ **grams/$\mathbf{cm^3}$** . B) 1.3×10^{-24} grams/cm^3 x (1 kg/1000 gm) x (1000cm/1 meter)3 = $\mathbf{1.3 \times 10^{-21}}$ **kilograms/$\mathbf{meter^3}$**

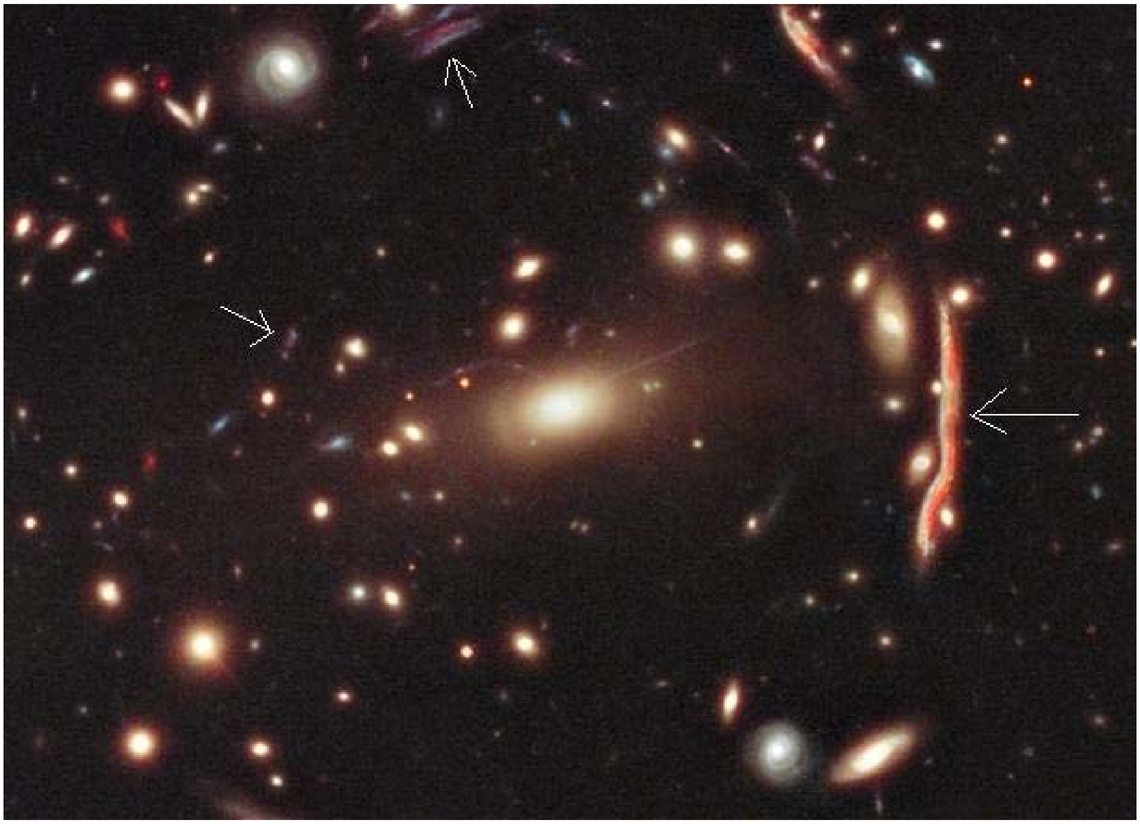
Problem 4 - If the average density of the matter in the universe was at one time equal to that of the Milky Way (Problem 3), A) by what factor would the volume of the universe have to increase in order for it to be 4.6×10^{-31} grams/cm^3 today?

Answer: A) Density then / density now = 1.3×10^{-24} grams/cm^3 / 4.6×10^{-31} grams/cm^3 = **2.8 million times.**

Problem 5 - By what factor would the size of the universe have had to expand by today, and how far apart would the Milky Way and the Andromeda galaxy have been at that time if their current separation is 2.2 million light years?

Answer: Because volume is proportional to the cube of a length, the factor by which the universe would have to increase in size is S = (2.8 million times)1/3 = **140 times.** That means that the Milky Way and the Andromeda galaxy would have been at a distance of 2.2 million light years / 140 = **16,000 light years!**

Mapping Dark Matter in a Distant Cluster



Decades after its discovery, astronomers still do not know the nature of dark matter; a form of 'matter' unlike anything familiar in every-day life to humans. It is commonly found mixed with ordinary matter inside great clusters of galaxies, however, there is often ten times more of it than luminous matter in stars. A detailed study of galaxy clusters such as MACS J1206.2 shown in the Hubble Telescope image above now shows how much dark matter can exist, and how it is spread through out such clusters. This cluster is located 4.5 billion light years from our Milky Way. It contains over 100 individual galaxies.

When light passes through a strong gravitational field it is bent. This causes the image of the source to become distorted. Astronomers have counted in the cluster image a total of 47 'ghost' images from 12 background galaxies behind this massive cluster. A few of these images is shown by the arrows above. A careful study of the number and shapes of these images lets astronomers estimate the total mass of the cluster inside spheres of different radii centered on the massive galaxy at the center of the cluster. They found that inside a radius of 515,000 light years the total mass is 1.3×10^{14} Msun; inside 312,000 light years the total mass is about 8.0×10^{13} Msun. Only about 10% of the total mass is in the form of stars and galaxies. (Note: The Milky Way has a mass of about 5.0×10^{11} Msun)

Problem 1 – From the mass data within the two radii, and comparing the density of matter found within the two concentric volumes, is the dark matter uniformly mixed within each cubic light year of the cluster's volume?

http://www.nasa.gov/mission_pages/hubble/science/dark-matter-survey.html
Ambitious Hubble Survey Obtaining New Dark Matter Census
Oct 13, 2011

Problem 1 – From the mass data within the two radii, is the dark matter uniformly mixed within each cubic light year of the cluster's volume?

Answer: To determine whether dark matter is uniformly distributed in space, we can compare the average density of the total mass inside the first volume, with the total mass found within the shell of the second sphere.

Inner sphere:

Volume = $4/3\ \pi\ (312{,}000\ \text{ly})^3 = 1.3\text{x}10^{17}\ \text{ly}^3$.

Mass = 1.3×10^{14} Msun

Density = Mass/Volume
= $(8.0 \times 10^{13}\ \text{Msun}) / (1.3\text{x}10^{17}\ \text{ly}^3)$
= **0.00062 Msun/ $\mathbf{ly^3}$**

Outer shell between 515,000 and 312,000 light years

$V = 4/3\ \pi\ (515{,}000)^3 - 4/3\ \pi\ (312{,}000)^3 = 4.4 \times 10^{17}\ \text{ly}^3$

The difference in mass residing in this shell volume is
$M = 13 \times 10^{13}\ \text{Msun} - 8.0 \times 10^{13}\ \text{Msun} = 5 \times 10^{13}\ \text{Msun}$.

So the density of mass in this outer shell is just
$D = 5.0 \times 10^{13}\ \text{Msun} / 4.4 \times 10^{17}\ \text{Ly}^3$
= **0.00011 Msun/ $\mathbf{Ly^3}$**.

From this we see that there is about 6 times more mass in the core volume than in the shell volume, so dark matter is not evenly spread through out every volume of space within the cluster.

The universe is a BIG place…but it also has some very small ingredients! Astronomers and physicists often find linear plotting scales very cumbersome to use because the quantities you would most like to graph differ by powers of 10 in size, temperature or mass. Log-Log graphs are commonly used to see the 'big picture'. Instead of a linear scale '1 kilometer, 2 kilometers 3 kilometers etc' a Logarithmic scale is used where '1' represents 10^1, '2' represents 10^2 …'20' represents 10^{20} etc. A calculator easily lets you determine the Log of any decimal number. Just enter the number, n, and hit the 'log' key to get m = log(n). Then just plot a point with 'm' as the coordinate number!

Below we will work with a Log(m) log(r) graph where m is the mass of an object in kilograms, and r is its size in meters.

Problem 1 - Plot some or all of the objects listed in the table below on a LogLog graph with the 'x' axis being Log(M) and 'y' being Log(R).

Problem 2 - Draw a line that represents all objects that have a density of A) nuclear matter (4×10^{17} kg/m^3), and B) water (1000 kg/m^3).

Problem 3 - Black holes are defined by the simple formula R = 3.0 M, where r is the radius in kilometers, and M is the mass in multiples of the sun's mass (1 M = 2.0×10^{30} kilograms). Shade-in the region of the LogLog plot that represents the condition that no object of a given mass may have a radius smaller than that of a black hole.

Problem 4 - The lowest density achievable in our universe is set by the density of the cosmic fireball radiation field of 4×10^{-31} kg/m^3. Draw a line that identifies the locus of objects with this density, and shade the region that excludes densities lower than this.

	Object	R (meters)	M (kg)
1	You	2.0	60
2	Mosquito	2×10^{-3}	2×10^{-6}
3	Proton	2×10^{-15}	2×10^{-27}
4	Electron	4×10^{-18}	1×10^{-30}
5	Z boson	1×10^{-18}	2×10^{-25}
6	Earth	6×10^{6}	6×10^{24}
7	Sun	1×10^{9}	2×10^{30}
8	Jupiter	4×10^{8}	2×10^{27}
9	Betelgeuse	8×10^{11}	6×10^{31}
10	Milky Way galaxy	1×10^{21}	5×10^{41}
11	Uranium atom	2×10^{-14}	4×10^{-25}
12	Solar system	1×10^{13}	2×10^{30}
13	Ameba	6×10^{-5}	1×10^{-12}
14	100-watt bulb	5×10^{-2}	5×10^{-2}
15	Sirius B white dwarf.	6×10^{6}	2×10^{30}
16	Orion nebula	3×10^{18}	2×10^{34}
17	Neutron star	4×10^{4}	4×10^{30}
18	Binary star system	1×10^{13}	4×10^{30}
19	Globular cluster M13	1×10^{18}	2×10^{35}
20	Cluster of galaxies	5×10^{23}	5×10^{44}
21	Entire visible universe	2×10^{26}	2×10^{54}

The figure below shows the various items plotted, and excluded regions cross-hatched. Students may color or shade-in the permitted region. This wedge represents all of the known objects and systems in our universe; a domain that spans a range of 85 orders of magnitude (10^{85}) in mass and 47 orders of magnitude (10^{47}) in size!

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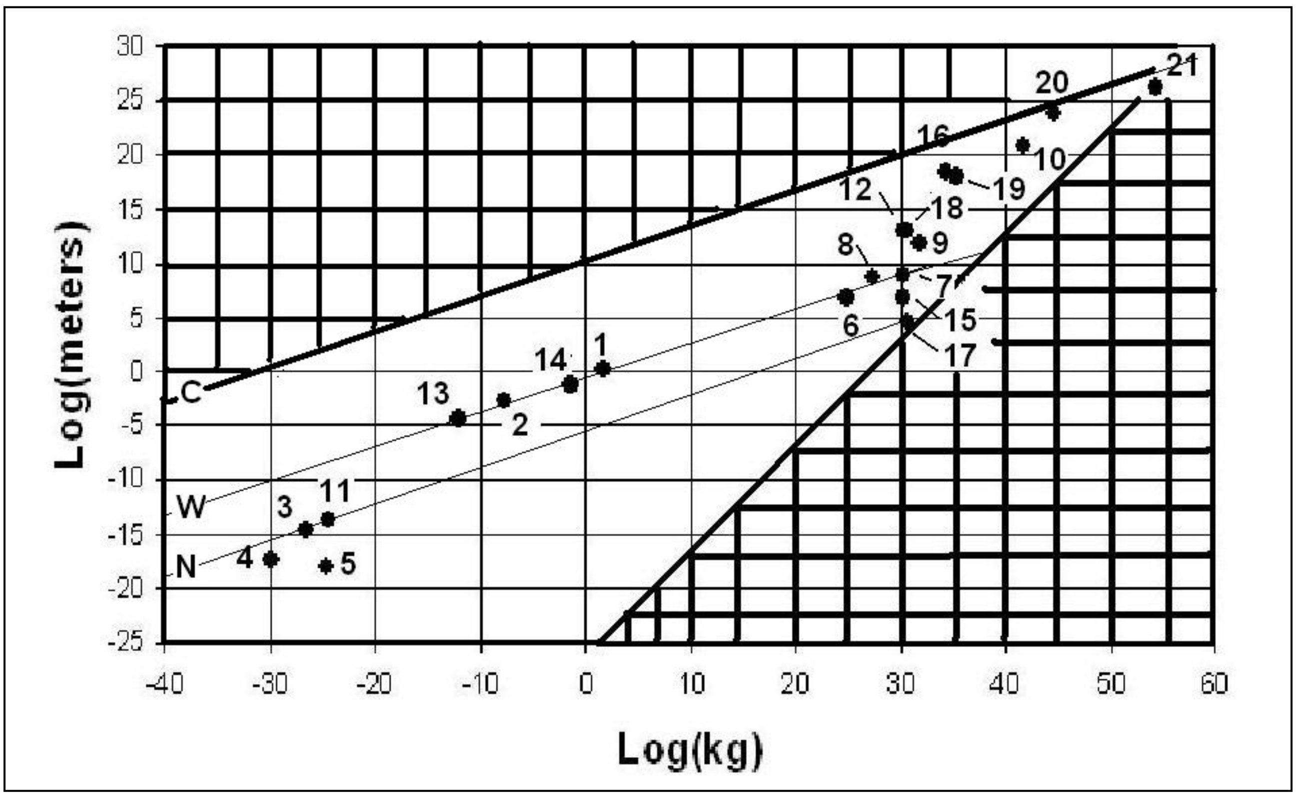
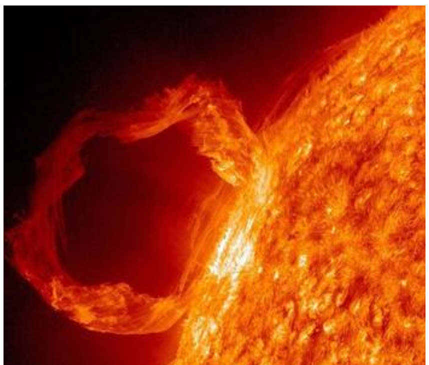
Inquiry: Can you or your students come up with more examples of objects or system that occupy some of the seemingly 'barren' regions of the permitted area?



Mass = $9.5\text{x}10^{31}$ kg
Volume = $5.0\text{x}10^{47}$ m^3

Mass = $5.1\text{x}10^{18}$ kg
Volume = $4.2\text{x}10^{18}$ m^3



Mass = $1.2\text{x}10^{25}$ kg
Volume = $6.0\text{x}10^{28}$ m^3

Mass = $2.0\text{x}10^{39}$ kg
Volume = $1.6\text{x}10^{60}$ m^3

In addition to mass and volume, density is the next most important feature of matter that we can easily determine. Density is just the mass of an object divided by its volume. Although density is usually given in terms of the unit kilograms/meter^3, in astronomy we prefer to use the number of particles per cubic meter. It is easy to imagine how a cubic meter might contain 1000 atoms, but the equivalent density of $1.6\text{x}10^{-24}$ kg/m^3 seems mysterious and doesn't provide much of a clue for how to think about it physically!

Problem 1 – Complete the table below by calculating the density of each astronomical object in terms of atoms per cubic meter.

Name	Volume (m^3)	Mass (kg)	Density (atoms/m^3)
Atmosphere of Earth	$4.2\text{x}10^{18}$	$5.1\text{x}10^{18}$	
Red supergiant star	$2.3\text{x}10^{33}$	$4.0\text{x}10^{31}$	
Surface of our Sun	$6.0\text{x}10^{28}$	$1.2\text{x}10^{25}$	
Atmosphere of Moon	$1.9\text{x}10^{15}$	$1.0\text{x}10^{4}$	
Solar Corona	$9.0\text{x}10^{26}$	$8.9\text{x}10^{13}$	
Interstellar Cloud	$5.0\text{x}10^{47}$	$9.5\text{x}10^{31}$	
Orion Nebula	$6.2\text{x}10^{51}$	$1.5\text{x}10^{33}$	
Solar Wind	$1.4\text{x}10^{34}$	$4.5\text{x}10^{14}$	
Milky Way galaxy	$1.6\text{x}10^{60}$	$2.0\text{x}10^{39}$	
Van Allen radiation belts	$1.3\text{x}10^{23}$	$1.1\text{x}10^{-2}$	

Problem 1 – Complete the table below by calculating the density of each astronomical object in terms of atoms per cubic meter.

Answer: example Solar Corona:

$D = M/V = 8.9\times10^{13}$ kg$/9.0\times10^{26}$ m^3 $= 9.9\times10^{-14}$ kg/m^3

$N = 9.9\times10^{-14} / 1.6\times10^{-27} = 6.2\times10^{13}$ atoms/m^3

Name	Volume (m^3)	Mass (kg)	Density (atoms/m^3)
Atmosphere of Earth	4.2×10^{18}	5.1×10^{18}	7.5×10^{26}
Red supergiant star	2.3×10^{33}	4.0×10^{31}	1.1×10^{25}
Surface of our Sun	6.0×10^{28}	1.2×10^{25}	1.3×10^{23}
Atmosphere of Moon	1.9×10^{15}	1.0×10^{4}	3.3×10^{15}
Solar Corona	9.0×10^{26}	8.9×10^{13}	6.2×10^{13}
Interstellar Cloud	5.0×10^{47}	9.5×10^{31}	1.2×10^{11}
Orion Nebula	6.2×10^{51}	1.5×10^{33}	1.5×10^{8}
Solar Wind	1.4×10^{34}	4.5×10^{14}	2.0×10^{7}
Milky Way galaxy	1.6×10^{60}	2.0×10^{39}	8.1×10^{4}
Van Allen radiation belts	1.3×10^{23}	1.1×10^{-2}	53

Students can also be asked to compare how many times more dense is Object A than Object B? Example: Atmosphere of Moon / Milky Way galaxy = $3.3\times10^{15}/8.1\times10^{4}$ = 41 billion times!

The Sombrero Galaxy (a.k.a Messier-104 and NGC-4594) is a spiral galaxy located 28 million light years from the Milky Way in the direction of the constellation Virgo, and contains over 800 billion stars. It is the most massive galaxy in the Virgo Cluster, which itself contains over 2,000 galaxies spanning a volume of space 10 million light years in diameter. M-104 is about the same age as the Milky Way (~12 billion years), but instead of only having about 130 globular clusters, M-104 has over 2,000; each cluster contains about 100,000 stars. Its diameter is about 50,000 light years. The infrared and optical photographs below were taken with the Spitzer Infrared Telescope (main) and the Hubble Space Telescope (inset). The optical image shows mainly the locations of stars (whitish haze) and absorption of starlight by the dust disk. The infrared image shows mainly the dust (colored red) heated by starlight (colored blue), and can penetrate deeper into the interior of a galaxy that visible light.

Question 1 – What is the scale of the image in light years per millimeter?
Question 2 – What is the radius of the stellar (blue) component to the galaxy in light years?
Question 3 – What is the inner and outer radius of the ring of dust in light years?
Question 4 - What is the thickness of the dust disk in light years?
Question 5 – What is the radius of the faint inner disk in light years?
Question 6 – What is the diameter of the bright nuclear core containing the black hole?
Question 7 – How many globular star clusters (the star-like spots) can you count in this image?
Question 8 - How big, in light years, is the smallest dust cloud you can see in the outer disk?
Question 9 – Compare the infrared and optical photographs, and describe their similarities and differences. Why would an astronomer want to study the infrared photograph?

You can obtain much higher resolution (and gorgeous!!!) images of M-104 from the Spitzer website

http://www.spitzer.caltech.edu/Media/releases/ssc2005-11/release.shtml

and from Hubble website

http://hubblesite.org/newscenter/newsdesk/archive/releases/2003/28/image/a

The following answers are relevant to the large 'Spitzer' image. Note, the smallest detail you can reliably measure with a millimeter ruler will be about 0.5 mm or 330x0.5 = 160 light years across. The above, enlarged photos will let you see features less than 50 light years (about 2 image pixels) across.

Question 1 – What is the scale of the image in light years per millimeter? **Answer:** Galaxy diameter 50,000 lys = 150 mm so the scale is about 330 light years per millimeter.

Question 2 – What is the radius of the stellar (blue) component to the galaxy in light years?
Answer: Depending on the quality of your laser printer, the distance from center to the outer edge of the blue haze (stars!!) will be about 40 to 60 mm, for a radius of 40x330 = 13,000 ly to 60x330 = 19,800 light years. The distance from our sun to the center of the Milky Way is about 25,000 light years.

Question 3 – What is the inner and outer radius of the ring of dust in light years?
Answer: The inner radius is about 45mm or 15,000 light years. The outer radius is about 70mm or 23,000 light years.

Question 4 - What is the thickness of the dust disk in light years?
Answer: The thickness of the dust disk can be estimated from the smallest width of the ring which is near the center of the picture. This width is about 3 mm or 1,000 light years. This is similar to the width of the disk of the Milky Way.

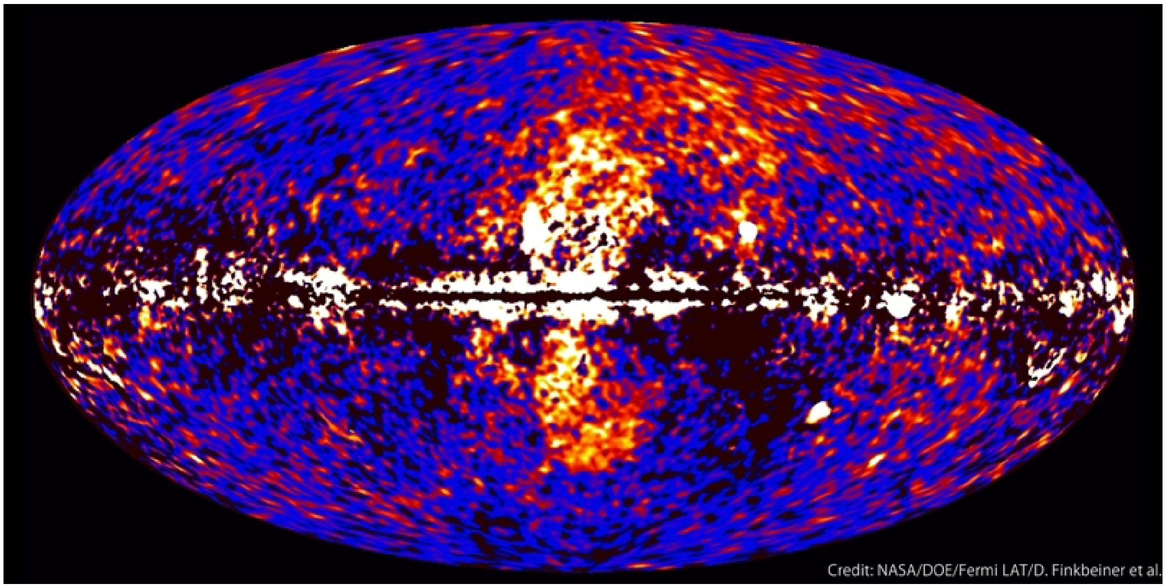
Question 5 – What is the radius of the faint inner disk in light years?
Answer: It may be difficult to see this inner disk depending on the quality of the photocopy. Its radius is about 25 mm or 8,000 light years.

Question 6 – What is the diameter of the bright nuclear core containing the black hole?
Answer: This is the star-like bright spot at the center of the Spitzer image. It is about 3 mm in diameter or 1,000 light years across. This is similar to the inner core region of our Milky Way which also contains a massive black hole. In both cases, the black hole size is 0.0001 light years and cannot be seen in photographs like this.

Question 7 – How many globular star clusters can you count in this image?
Answer: This may depend on the quality of the photocopy. Students should count all the star-like spots in the photo that are 'faint'. There are a few bright stars in the field, like the one in the lower right corner. Answers may range from 100 to 300. More can be seen in higher quality data.

Question 8 - How big, in light years, is the smallest dust cloud you can see in the outer disk?
Answer: Look in the dust ring and find the smallest 'blob'. You should be able to see interstellar clouds only 0.5 millimeters across or 160 light years. As a comparison, the Orion Nebula is about 40 light years across.

Question 9 – Compare the infrared and optical photographs, and describe their similarities and differences. Why would an astronomer want to study the infrared photograph? **Answer:** Both show stars, but the infrared image shows the dust ring more clearly, including a clearer view of the core region and inner disk. The optical photo shows the globular star clusters more clearly, and also the details inside the dust ring, which appear black. Astronomers would want the infrared photo because it lets them see more details in the dust clouds, especially when looking deep inside a galaxy where the stars would usually 'white out' the entire field and hide these details. Optical instruments can resolve finer details, so these images give better views of star clusters and the outsides of dust clouds.



NASA's Fermi Gamma-ray Space Telescope has unveiled a previously unseen structure centered in the Milky Way. The feature spans 50,000 light-years and may be the remnant of an eruption from a supersized black hole at the center of our galaxy, or the gas ejected by a burst of star formation in the center of the galaxy several million years ago. The bubbles extend 25,000 light years above the plane of the Milky Way, and appear to be symmetric in shape.

Problem 1 - Approximate the volume of these two bubbles as two spheres with diameters of 25,000 light years. What is the total volume of the two bubbles in cubic light years?

Problem 2 - The average density of interstellar gas is about 1 hydrogen atom per cubic centimeter. If 1 light year = 9.5×10^{17} centimeters, and one hydrogen atom has a mass of 1.6×10^{-27} kg, how much mass may have been displaced by the formation of these bubbles A) in kilograms? B) in solar mass units if 1 SMU = 2.0×10^{30} kg?

Problem 3 - Suppose that the density of the gas ejected by the activity at the center of our Milky Way had a density of about 10% of the normal interstellar gas. If the process took 1,000,000 years to form the bubbles, at what rate would gas from the core of our Milky Way have to be produced in Earth mass units per year?

Problem 1 - Approximate the volume of these two bubbles as two spheres with diameters of 25,000 light years. What is the total volume of the two bubbles in cubic light years?

Answer: For one bubble: $V = 4/3\, \pi\, R^3$ so

$V = 1.33\ (3.14)\ (25{,}000/2)^3$ cubic light years

$V = 8.2 \times 10^{12}$ cubic light years.

For two bubbles $2V =$ **1.6×10^{13} cubic light years.**

Problem 2 - The average density of interstellar gas is about 1 hydrogen atom per cubic centimeter. If 1 light year = 9.5×10^{17} centimeters, and one hydrogen atom has a mass of 1.6×10^{-27} kg, how much mass may have been displaced by the formation of these bubbles A) in kilograms? B) in solar mass units if 1 SMU = 2.0×10^{30} kg?

Answer: A) Mass in one cubic light year = 1 atoms/cm^3 x $1.6\text{x}10^{-27}$ kg/atom x (9.5e^{17} cm/light year)^3 = 1.4×10^{27} kg. The volume of the bubbles is 1.6×10^{13} ly^3, so the mass is $M=1.4\text{x}10^{27}$ x ($1.6\text{x}10^{13}$) and so M= **2.2×10^{40} kg**.

B) In solar mass units M= 1.4×10^{27} kg x (1 SMU/$2.0\text{x}\ 10^{30}$ kg)

= **1.1×10^{10} solar masses.**

Problem 3 - Suppose that the density of the gas ejected by the activity at the center of our Milky Way had a density of about 10% of the normal interstellar gas. If the process took 1,000,000 years to form the bubbles, at what rate would gas from the core of our Milky Way have to be produced in Earth mass units per year?

Answer: The density is 0.10 x the normal interstellar gas, so from Problem 2(B) the amount of material in the bubble volume would be 0.1 x 1.1×10^{10} solar masses = $1.1\text{x}10^9$ solar masses. It took 1,000,000 years to inject this gas into the bubbles, so the rate is just

R =$1.1\text{x}10^9$ solar masses/1,000,000 years

= **1100 solar masses/year**.

$$Fg = \frac{G\,M\,m}{R^2}$$

$$Fc = \frac{m\,V^2}{R}$$

$$V = \frac{2\,\pi\,R}{T}$$

One of the neatest things in astronomy is being able to figure out the mass of a distant object, without having to 'go there'. Astronomers do this by employing a very simple technique. It depends only on measuring the separation and period of a pair of bodies orbiting each other. In fact, Sir Issac Newton showed us how to do this over 300 years ago!

Imagine a massive body such as a star, and around it there is a small planet in orbit. We know that the force of gravity, **Fg**, of the star will be pulling the planet inwards, but there will also be a centrifugal force, **Fc**, pushing the planet outwards.

This is because the planet is traveling at a particular speed, **V**, in its orbit. When the force of gravity and the centrifugal force on the planet are exactly equal so that **Fg = Fc**, the planet will travel in a circular path around the star with the star exactly at the center of the orbit.

Problem 1) Use the three equations above to derive the mass of the primary body, M, given the period, T, and radius, R, of the companion's circular orbit.

Problem 2) Use the formula $\mathbf{M = 4\,\pi^2 R^3 / (G\,T^2)}$ where G = 6.6726×10^{-11} N-m^2/kg^2 and M is the mass of the primary in kilograms, R is the orbit radius in meters and T is the orbit period in seconds, to find the masses of the primary bodies in the table below. (Note: Make sure all units are in meters and seconds first! 1 light years = 9.5 trillion kilometers)

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	
Earth	Moon	27.3 days	385,000 km	
Jupiter	Callisto	16.7 days	1.9 million km	
Pluto	Charon	6.38 days	17,530 km	
Mars	Phobos	7.6 hrs	9,400 km	
Sun	Earth	365 days	149 million km	
Sun	Neptune	163.7 yrs	4.5 million km	
Sirius A	Sirius B	50.1 yrs	20 AU	
Polaris A	Polaris B	30.5 yrs	290 million miles	
Milky Way	Sun	225 million yrs	26,000 light years	

Answer Key

Problem 1: Answer

$$\frac{G\,M\,m}{R^2} = \frac{m\,V^2}{R}$$

Cancil the companion mass, m, on both sides, and isolate the primary mass, M, on the left side:

$$M = \frac{R\,V^2}{G}$$

Now use the definition of V to eliminate it from the equation,

$$M = \frac{R}{G}\left(\frac{2\pi R}{T}\right)^2$$

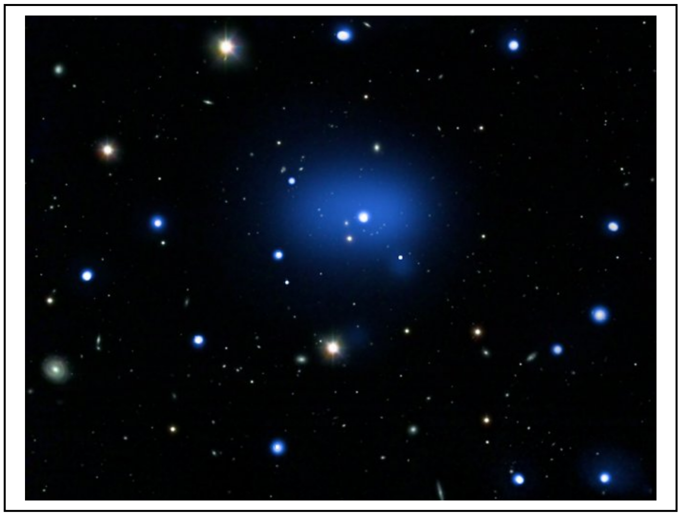
and simplify

$$M = \frac{4\pi^2 R^3}{G\,T^2}$$

Problem 2:

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	**6.1×10^{24} kg**
Earth	Moon	27.3 days	385,000 km	**6.1×10^{24} kg**
Jupiter	Callisto	16.7 days	1.9 million km	**1.9×10^{27} kg**
Pluto	Charon	6.38 days	17,530 km	**1.3×10^{22} kg**
Mars	Phobos	7.6 hrs	9,400 km	**6.4×10^{23} kg**
Sun	Earth	365 days	149 million km	**1.9×10^{30} kg**
Sun	Neptune	163.7 yrs	4.5 million km	**2.1×10^{30} kg**
Sirius A	Sirius B	50.1 yrs	298 million km	**6.6×10^{30} kg**
Polaris A	Polaris B	30.5 yrs	453 million km	**6.2×10^{28} kg**
Milky Way	Sun	225 million yrs	26,000 light years	**1.7×10^{41} kg**

Note: The masses for Sirius A and Polaris A are estimates because the companion star has a mass nearly equal to the primary so that our mass formula becomes less reliable.



The most distant galaxy cluster yet has been discovered by combining data from NASA's Chandra X-ray Observatory and optical and infrared telescopes. The cluster is located about 10.2 billion light years away, and is observed as it was when the Universe was only about a quarter of its present age. The galaxy cluster, known as JKCS041, beats the previous record holder by about a billion light years. Galaxy clusters are the largest gravitationally bound objects in the Universe.

JKCS041 was originally detected in 2006 in a survey from the United Kingdom Infrared Telescope (UKIRT). The Chandra data were the final - but crucial - piece of evidence that showed JKCS041 was, indeed, a genuine galaxy cluster. Clusters of galaxies have such strong gravitational fields that they can serve as a bottle for very high temperature gas. These gases often emit x-ray light that can be detected by observatories such as Chandra. The discovery of such a high-temperature gas between the galaxies in JKCS041 supports the original idea that the galaxies seen in that direction are, in fact, members of a cluster. From the X-ray information, astronomers can also measure the total mass of the entire cluster that is responsible for creating the gravitational field holding the gas in place.

Problem 1 - The Chandra satellite detected x-rays coming from the region of the sky containing the galaxy cluster JKS041. The electrons in the gas are emitting the X-rays, and colliding at high speed with the protons in the gas. The energy of the x-rays at the time they were emitted by the hot gas was 21,400 electron Volts (eV). This energy is shared equally between the electrons and protons. The speed of a proton is related to its kinetic energy by $E = 1/2mV^2$ where E is the energy in Joules, V is the proton speed in meters/sec, and m is the mass of a proton ($m = 1.7 \times 10^{-27}$ kg). About how fast are the protons moving? (Note: 1 eV = 1.6×10^{-19} Joules)

Problem 2 -The escape velocity (in km/s) from a body is given by $V = 0.17\ (M/R)^{1/2}$ where M is the mass in multiples of the mass of our sun, and R is the average distance, in light years, between the body and the gas particle. Example, for the Milky Way, R = 50,000 light years and M = 300 billion so V = 420 km/sec. Compared to the sun, about how much mass do you need to confine the gas cloud observed by Chandra, if the cluster of galaxies has a radius of about 1 million light years A) in units of the sun's mass? B) In terms of the number of Milky Way galaxies where 1 Milky Way is about 2×10^{12} solar masses?

Problem 1 - The Chandra satellite detected x-rays coming from the region of the sky containing the galaxy cluster JKS041. The electrons in the gas are emitting the X-rays, and colliding at high speed with the protons in the gas. The energy of the x-rays at the time they were emitted by the hot gas was 21,400 electron Volts (eV). This energy is shared equally between the electrons and protons. The speed of a proton is related to its kinetic energy by $E = 1/2mV^2$ where E is the energy in Joules, V is the proton speed in meters/sec, and m is the mass of a proton ($m = 1.7 \times 10^{-27}$ kg). About how fast are the protons moving that are producing the X-ray light seen by Chandra? (Note: 1 eV = 1.6×10^{-19} Joules)

Answer: The information given in the problem is that:

The x-ray energy is 21,400 eV
1 eV = 1.6×10^{-19} Joules of energy
The electrons carry ½ of the x-ray energy
The protons carry ½ of the x-ray energy
The mass of a proton is 1.7×10^{-27} kilograms

The formula requires the energy, E, in units of Joules, so we first have to convert 21,400 eV to Joules. E = 21,400 eV x (1.6×10^{-19} Joules /1 eV)
= 3.4×10^{-15} Joules.
This is the total energy, so protons only carry half of this so that E = **1.7×10^{-15} Joules**

Next, we use the formula $E = 1/2mV^2$ and solve for V to get $V = (2E/m)^{1/2}$ and substitute the known values for m and E to get $V = (2 \times 1.7 \times 10^{-15} \text{ Joules}/ 1.7 \times 10^{-27} \text{ kg})^{1/2}$ = **1,400,000 m/sec or 1,400 km/s.**

Problem 2 -The escape velocity (in km/s) from a body is given by $V = 0.17 (M/R)^{1/2}$ where M is the mass in multiples of the mass of our sun, and R is the average distance, in light years, between the body and the gas particle. Example, for the Milky Way, R = 50,000 light years and M = 300 billion so V = 420 km/sec. Compared to the sun, about how much mass do you need to confine the gas cloud observed by Chandra, if the cluster of galaxies has a radius of about 1 million light years, A) in units of the sun's mass? B) In terms of the number of Milky Way galaxies where 1 Milky Way is about 2×10^{12} solar masses?

Answer: Solve the stated equation for M to get $M = R (V/0.17)^2$. For R = 1 million light years and V = 1,400 km/sec, then A) **M = 70 trillion suns** and B) N = 70 trillion suns x (1 Milky Way/2 trillion suns) so this is about **35 Milky Ways**. Note, some of this mass is actually in the 16 large galaxies that make up the cluster. Some of it is in the hot cloud of x-ray emitting gas, and some of it may be in Dark Matter.

Note: For more information about this discovery, read the Chandra Press release at: http://www.nasa.gov/mission_pages/chandra/news/09-086.html

According to the research paper by S. Andreon, B. Maughan, G. Trinchieri, and J. Kirk "JKCS041: a color-detected galaxy cluster at z=1.9 with deep potential well as confirmed by x-ray data" published in the journal Astronomy and Astrophysics, October 2009, the estimated mass based on careful modeling of the data indicated a range between 30 trillion and 670 trillion suns.

$$T = \frac{10^{10}}{\sqrt{t}} \quad {}^{o}\text{Kelvin}$$

$$E = \frac{860{,}000}{\sqrt{t}} \quad \text{eVolts}$$

T is the temperature in degrees Kelvin that was reached **t** seconds after the Big Bang.

E is the energy in electron Volts that was reached **t** seconds after the Big Bang.

Moments after the Big Bang, the universe was brilliantly hot, but steadily cooled with every passing second. All of the matter we see around us today was once fragmented into its individual parts, and we can re-create many of the original Big Bang conditions in our laboratories today. All we have to do is heat up matter so that the parts collide with the same energy as they would have during the Big Bang.

Physicists do this by using 'atom smashers' that accelerate individual particles such as electrons and protons, and then collide them. Thanks to Relativity, and Einstein's famous equation $E = mc^2$, all of the energy of the collision is then available for creating new kinds of particles…if they exist.

Astronomers can also use the Big Bang model to calculate at exactly what time after the Big Bang particles of matter typically had the energies being explored under laboratory conditions. There are two different formula, depending on whether you want to predict the temperature of the gas, or the average energy of the particles in the gas, as shown on the left.

Problem 1 - At 100 seconds after the Big Bang, what was the temperature of the universe, and what was the average collision energy, in kilovolts, of the particles at that time?

Problem 2 - Collisions between particles with a combined energy of 2 billion Volts (2 GeV) can produce a pair of particles, one proton and one anti-proton, out of pure energy. How many seconds after the Big Bang were particles colliding with these energies?

Problem 3 - How hot did the Big Bang have to be in order for it to create particles as massive as a pair of Top Quarks (E = 175 GeV for one top quark), and how long after the Big Bang was this temperature achieved?

Problem 4 - The Large Hadron Collider at CERN in Switzerland was recently 'powered-up' and achieved collision energies of 1.2 TeV, with an ultimate goal of about 15 TeV when it is fully operational in 2010. If 1 TeV = 1 trillion electron Volts (1 Terra EV), A) how many seconds after the Big Bang will the LHC be able to explore the state of matter at the lower and upper energy limits achieved in 2009 and 2010? B) What will be the temperature of matter at these two times?

Problem 1 - At 100 seconds after the Big Bang, what was the temperature of the universe, and what was the average collision energy, in kilovolts, of the particles at that time?
Answer;
T = 10 billion / $(100)^{1/2}$ = 10 billion/10 = **1 billion K**.
E = 860,000 eV / $(100)^{1/2}$ = 860,000/10 = 86,000 eV or **86 kiloVolts**. (abbrev. 86 keV)

Problem 2 - Collisions between particles with a combined energy of 2 billion Volts (2 GeV) can produce a pair of particles, one proton and one anti-proton, out of pure energy. How many seconds after the Big Bang were particles colliding with these energies?

Answer: From the equation for E, we can solve it for t to get t = $(860{,}000/E)^2$ .Since E = 2,000,000,000 eV, we get t = $(860{,}000/2{,}000{,}000{,}000)^2$ = 0.00000018 seconds or **0.18 microseconds after the Big Bang**.

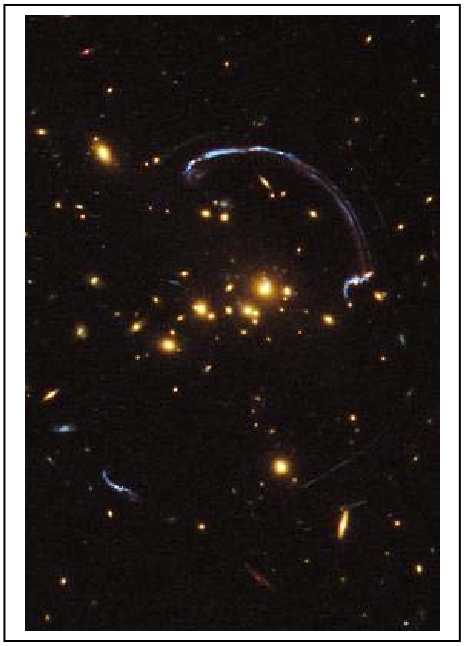
Problem 3 - How hot did the Big Bang have to be in order for it to create particles as massive as a pair of Top Quarks (E = 175 GeV for one top quark), and how long after the Big Bang was this temperature achieved?

Answer: E = 2 x 175 GeV = 350,000,000,000 eV. Then since t = $(860{,}000/E)^2$ we have t = $(860{,}000 / 350{,}000{,}000{,}000)^2$ = 0.000000000006 seconds or **6.0 trillionths of a second** after the Big Bang (also written as 6 picoseconds). At this time, the temperature was T = 10^{10} K/$(6 \times 10^{-12}\ \text{sec})^{1/2}$ = **4.1×10^{15} Kelvin or 4,100 trillion degrees K**.

Problem 4 - The Large Hadron Collider at CERN in Switzerland was recently 'powered-up' and achieved collision energies of 1.2 TeV, with an ultimate goal of about 15 TeV when it is fully operational in 2010. If 1 TeV = 1 trillion electron Volts (1 Terra EV), A) how many seconds after the Big Bang will the LHC be able to explore the state of matter at the lower and upper energy limits achieved in 2009 and 2010? B) What will be the temperature of matter at these two times?

Answer: A) E = 1.2 TeV so t = $(860{,}000/1{,}200{,}000{,}000{,}000)^2$ = **5.1×10^{-13} seconds**
And for E = 15 TeV; $(860{,}000/15{,}000{,}000{,}000{,}000)^2$ = **3.3×10^{-15} seconds**
The temperature will be T = $10^{10}/(5.1 \times 10^{-13})^{1/2}$ = **1.4×10^{16} Kelvin**
And T = $10^{10}/(3.3 \times 10^{-15})^{1/2}$ = **1.7×10^{17} Kelvin**

Using a Gravity Lens to 'Weigh' a Cluster of Galaxies



The Hubble Space Telescope recently photographed a distant galaxy being 'lensed' by the gravity field of a foreground cluster of galaxies called RCS2-032727-132623 located 5.4 billion light years away in the southern constellation Cetus the Whale. The cluster lies directly between the more distant galaxy and Earth. Its gravity causes the image of the distant galaxy to be distorted into several arcs (blue).

Thanks to the magnification provided by this 'gravity lens' astronomers can study the blue galaxy located 9.7 billion light years from Earth – details that would have been impossible to see otherwise.

Astronomers can also use the geometry of the arc and its radius to determine the total mass of the cluster of galaxies, including any dark matter that it might contain!

The diameter of a gravity lens ring is related to the mass of the object producing the gravitational field by the formula:

$$M = \frac{\theta^2 c^2 (dD)}{4G(d-D)}$$

where θ is the angular diameter in radians of the ring, D is the distance in meters from Earth to the cluster of galaxies, d is the distance in meters between Earth and the galaxy whose image is being lensed, c is the speed of light (3×10^8 m/s) and G is the constant of gravity (6.67×10^{-11} m^3 kg^{-1} sec^{-2})

Problem 1 - For convenience, astronomers often re-write equations in the common units of astronomy where angles, θ, are measured in arcseconds (1 radian = 206265 arcseconds), distances (d and D) are measured in light years and masses (M) are given in multiples of the sun's mass). If 1 light year = 9.5×10^{15} meters and Msun = 2.0×10^{30} kg, what is the value of the constant, C, in the new equation

$$M = C\left[\frac{\theta^2 (dD)}{d-D}\right]$$

Problem 2 - The diameter of the gravitational lens ring in RCS2-032727-132623 is 38 arcseconds. What is the mass of the galaxy cluster?

Hubble Zooms in on a Magnified Galaxy
http://www.nasa.gov/mission_pages/hubble/science/magnified-galaxy.html
February 2, 2012

Problem 1 - For convenience, astronomers often re-write equations in the common units of astronomy where distances (d and D) are measured in light years and masses (M) are given in multiples of the sun's mass). If 1 light year = 9.5×10^{15} meters and Msun = 2.0×10^{30} kg and 1 radian = 2.0×10^{5} arcseconds, constant, C, in the new equation.

Answer: First insert the indicated units into the formula to get the mass, M, in kilograms. Note 1 arcsecond = $1/2.0\times10^5 = 5\times10^{-6}$ radians so

$$M = \frac{(5.0x10^{-6})^2(3.0x10^8)^2(9.5x10^{15})^2}{4(6.67x10^{-11})(9.5x10^{15})} \quad = \quad 8.0 \times 10^{31}$$

Then convert M to solar mass units by dividing by 2.0×10^{30} to get C = 40

Problem 2 - The diameter of the gravitational lens ring in RCS2-032727-132623 is 38 arcseconds. What is the mass of the galaxy cluster?

Answer; From the text, d = 9.7 billion light years, D = 5.4 billion light years and θ = 38 arcseconds, so

M = $(37.7)\,(38)^2$ (9.7 billion)(5.4 billion) / (9.7-5.4) billion

M = **6.5×10^{14} solar masses**!

Note: Another independent way to measure the mass of the cluster is by using the formula for the speed of an object in a circular orbit:

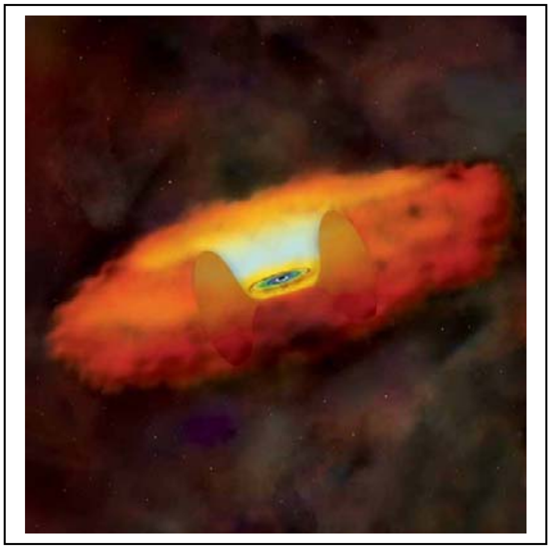
$$V^2 = \frac{GM}{R}$$

The average speed of the galaxies in the cluster is V = 988 km/sec and the average radius, R, of the cluster is 9.5 million light years

So M = $(9.5 \times 10^{15})(9.5 \times 10^{6})(988{,}000)^2 / (6.67\times10^{-11})$
M = 1.3×10^{45} kg

Or M = 6.6×10^{14} solar masses

Black Hole Power…X



Black holes are sometimes surrounded by a disk of orbiting matter. This disk is very hot. As matter finally falls into the black hole from the inner edge of that disk, it releases about 7% of its rest-mass energy in the form of light. Some of this energy was already lost as the matter passed through, and heated up, the gases in the surrounding disk. But the over-all energy from the infalling matter is about 7% of its rest-mass in all forms (heat+ light).

The power produced by a black hole is phenomenal, with far more energy per gram being created than by ordinary nuclear fusion, which powers the sun.

Illustration of black hole accretion disk cut-away, showing the central black hole. Courtesy NASA/Chandra/ M.Weiss (CXC)

Problem 1 - The Event Horizon of a black hole has a radius of 2.93 M kilometers, where M is the mass of the black hole in multiples of the sun's mass. Assume the Event Horizon is a spherical surface, so its surface area is $S = 4 \pi R^2$. What is the surface area of A) a stellar black hole with a mass of 10 solar masses? B) a supermassive black hole with a mass of 100 million suns?

Problem 2 - What is the volume of a spherical shell with the surface area of the black holes in Problem 1, with a thickness of one centimeter?

Problem 3 - If the density of gas near the horizon is 10^{10} atoms/cc of hydrogen, how much matter is in each black hole shell, if the mass of a hydrogen atom is 1.6×10^{-24} grams?

Problem 4 - If $E = m c^2$ is the rest mass energy, E, in ergs, for a particle with a mass of m in grams, what is the rest mass energy equal to the masses in Problem 3 if $c = 3 \times 10^{10}$ cm/sec is the speed of light and only 7% of the mass produced energy?

Problem 5 - Suppose the material was traveling at 1/2 the speed of light as it crossed the horizon, how much time does it take to travel one centimeter if $c = 3 \times 10^{10}$ cm/sec is the speed of light?

Problem 6 - The power produced is equal to the energy in Problem 4, divided by the time in Problem 5. What is the percentage of power produced by each black hole compared to the sun's power of 3.8×10^{33} ergs/sec?

Answer Key:

Problem 1 - The Event Horizon of a black hole has a radius of 2.93 M kilometers, where M is the mass of the black hole in multiples of the sun's mass. Assume the Event Horizon is a spherical surface, so its surface area is $S = 4 \pi R^2$. What is the surface area of A0 a stellar black hole with a mass of 10 solar masses? B) a supermassive black hole with a mass of 100 million suns?

Answer; A) The radius is 2.93 x 10 = 29.3 kilometers. The surface area $S = 4 \times 3.14 \times (2.93 \times 10^6)^2 = 1.1 \times 10^{14}$ cm^2, B) 2.93×10^{13} cm $S = 4 \times 3.14 \times (2.93 \times 10^{13})^2 = 1.1 \times 10^{28}$ cm^2,

Problem 2 - What is the volume of a spherical shell with the surface area of the black holes in Problem 1, with a thickness of one centimeter?
Answer: Stellar black hole, V = Surface area x 1 cm = 1.1×10^{14} cm^3; Supermassive bklack hole, V = 1.1×10^{28} cm^3.

Problem 3 - If the density of gas near the horizon is 10^{10} atoms/cc of hydrogen, how much matter is in each black hole shell, if the mass of a hydrogen atom is 1.6 x 10-24 grams?

Answer - Stellar: M = (1.1×10^{14} cm^3) x (1.0×10^{10} atoms/cm^3) x (1.6×10^{-24} grams/atom = 1.76 grams. Supermassive: M = (1.1×10^{28} cm^3) x (1.0×10^{10} atoms/cm^3) x (1.6×10^{-24} grams/atom = 1.76×10^{14} grams.

Problem 4 - If $E = m c^2$ is the rest mass energy, E, in ergs, for a particle with a mass of m in grams, what is the rest mass energy equal to the masses in Problem 3 if $c = 3 \times 10^{10}$ cm/sec is the speed of light and only 7% of the mass produced energy?
Answer: Stellar: E = 0.07 x (1.76 grams x ($3 \times 10^{10})^2$) = 1.1×10^{20} ergs.;
Supermassive E = 0.07 x (1.76×10^{14} grams x ($3 \times 10^{10})^2$) = 1.1×10^{34} ergs

Problem 5 - Suppose the material was traveling at 1/2 the speed of light as it crossed the horizon, how much time does it take to travel one centimeter if $c = 3 \times 10^{10}$ cm/sec is the speed of light?
Answer; 1 cm / 3×10^{10} cm/sec = 3.3×10^{-11} seconds.

Problem 6 - The power produced is equal to the energy in Problem 4, divided by the time in Problem 5. What is the percentage of power produced by each black hole compared to the sun's power of 3.8×10^{33} ergs/sec?
Answer Stellar: 1.1×10^{20} ergs / 3.3×10^{-11} seconds = 3.3×10^{30} ergs/sec
Percent = 100% x (3.3×10^{30} / 3.8×10^{33}) = 0.08 %

Supermassive: 1.1×10^{34} ergs / 3.3×10^{-11} seconds = 3.3×10^{44} ergs/sec
= (3.3×10^{44} / 3.8×10^{33}) = 86 billion times the sun's power!