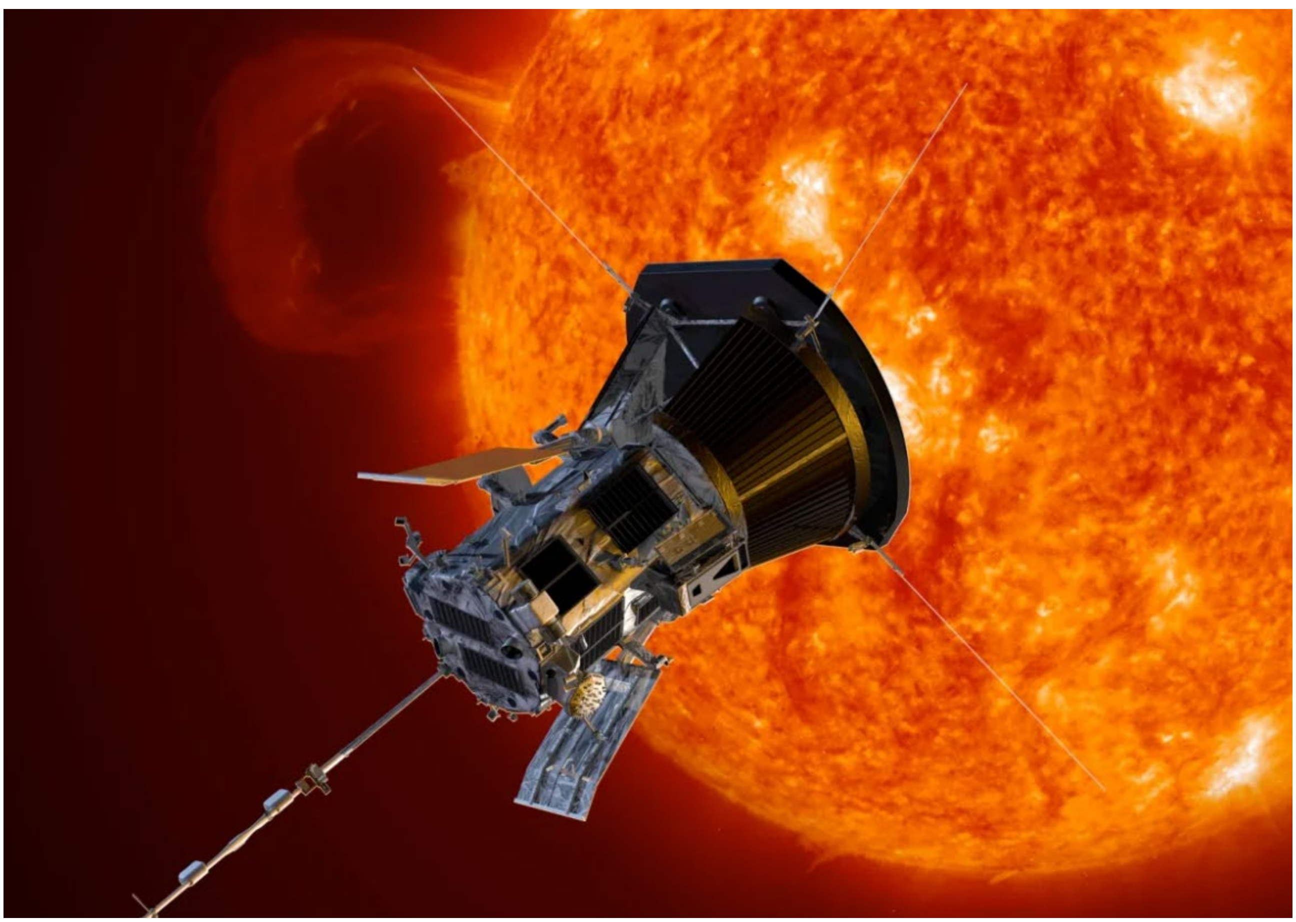
National Aeronautics and Space Administration

Math Problems Related to

Technology - III



Introduction

This book provides an assortment of 65 mathematics problems that cover aspects of technology such as rocket launches, telescopes, unit conversions, solar power, ion rockets, orbit decay, solar energy and other space technology topics.

Page	Level	Math Skill	Topic
1	Advanced	Algebra II	Exploring the Third Belt with the Van Allen Probes.
2	Advanced	Algebra II	An Improved Model for Van Allen Belt Radiation Dose.
3	Advanced	Graphing, Geometry	Navigating in a Magnetic World!
4	Advanced	Vectors, Conversions	A Practical Application of Vector Dot and Cross Products.
5	Advanced	Logarithms	Cassini Sees Earth from Space - How Bright is it?
6	Advanced	Ratios, Geometry	The Basic Mathematics of Rocket Engines - I
7	Advanced	Scientific Notation	Scientific Notation - An Astronomical Perspective.
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16	Advanced	Scale, Geometry	Hubble: Seeing a Dwarf Planet Clearly
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18	Advanced	Geometry, Formulas	Solid Rocket Boosters
19	Advanced	Geometry, Formulas	Solid Rocket Boosters and Thrust
20	Advanced	Algebra II	The Ares-V Cargo Rocket
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25	Advanced	Data Analysis	Exploring the Ares 1-X Launch: Downrange Distance
26	Advanced	Algebra II	Exploring the Ares 1-X Launch: Parametrics
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29	Advanced	Conversions	Unit Conversions; Energy; Power and Flux
30	Advanced	Geometry	The International Space Station and a Sunspot: Exploring angular scales.
31	Advanced	Geometry	The Hexagonal Tiles in the Webb Space Telescope Mirror
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34	Advanced	Functions	The Parker Solar Probe: Having a hot time near the sun!
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45	Advanced	Geometry, Formulas	Exploring the Interior of Mars with Spheres and Shells
46	Advanced	Equations, Geometry	The Distance to the Martian Horizon
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63	Advanced	Calculus	Fluid Level in a Spherical Tank
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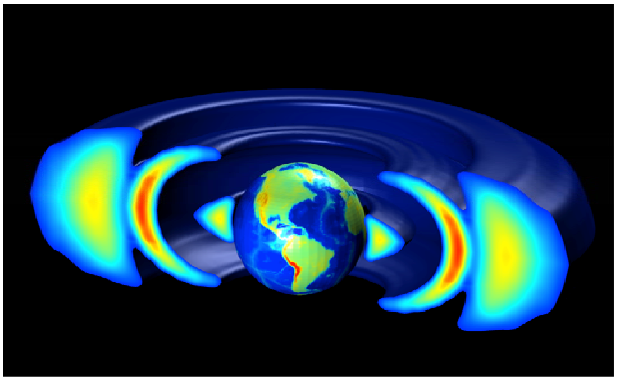


(Image credit: Yuri Shprits/UCLA)

The Van Allen Probes spacecraft travel in an elliptical orbit through the Van Allen belts. Soon after launch, they detected a third radiation belt shown by the middle crescent in the figure to the left.

In this problem, we predict when the spacecraft will encounter this third belt along their orbit, so that scientists can schedule observations of this new region.

The equation for the orbit of the spacecraft is given in Standard Form by

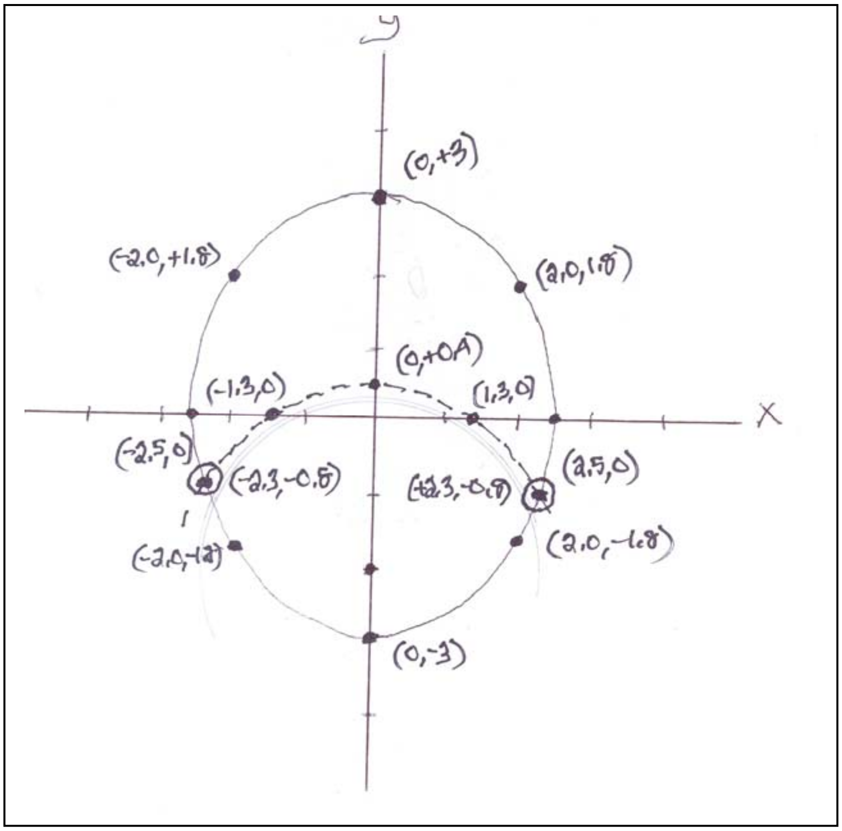
$$1 = \frac{x^2}{6.25} + \frac{y^2}{9.0}$$

The equation for the location of the third Van Allen belt projected into the plane of the elliptical orbit and concentric with the orbit focus centered on Earth (0,-2) is given by

$$5.8 = x^2 + (y+2)^2$$

Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth.

Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth.



Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth. Answer: See plot above (-2.3, -0.8), (+2.3, -0.8).

Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth. Answer: Expand out all terms and simplify

Equation of circle: $5.8 = x^2 + y^2 + 4y + 4.0$ so
$1.8 = x^2 + y^2 + 4y$ Equation of ellipse: $9.0 = 1.54x^2 + y^2$

Eliminate x^2 in equation of circle by using equation of ellipse solved for x^2.

$1.8 = \mathbf{(9.0 - y^2)/1.54} + y^2 + 4y$ $1.8 - 5.84 = -0.65y^2 + y^2 + 4y$ then $4.0 = 0.35y^2 - 4y$

Solve the quadratic equation in y for its two roots: $0 = 0.35y^2 - 4y - 4.0$ A = +0.35, b = -4.0 c = -4.0 then

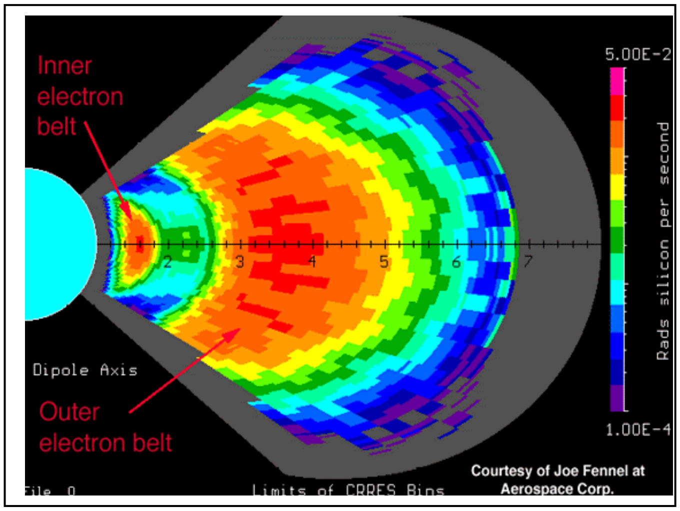
$Y = [4.0 +/- (16+5.6)^{1/2}]/0.70$ so Y1 = +12.3 Y2 = -0.86

Then from the equation for the ellipse we get the x coordinates:

Y1 = +12.3 : $X1 = +/- ((9.0 - (12.3)^2)/1.54)^{1/2}$ x1 = <u>imaginary solution</u>! (+/- 9.6 i)

Y2 = -0.86 : $X2 = +/- ((9.0 - (0.86)^2)/1.54)^{1/2}$ = +/- 2.30

Rounded to the nearest tenth we have +/- 2.8 so the real coordinates are **(+2.3, -0.9) and (-2.3, -0.9)**



Spacecraft engineers design spacecraft by considering the radiation environment, planned duration of the research program, and how much shielding is needed for the spacecraft to survive to the end of its mission.

In this problem we will work with a more realistic model for the orbit path and radiation belt dose rates, and perform a simple graphical integration by estimating areas under curves.

Problem 1 - The distance from the center of Earth to the spacecraft is given by the function

$$R(\theta) = 5.7 - \left[\frac{210}{100 - 55\cos\theta}\right]$$ in Earth radii units (where 1.0 = 6378 km)

$$T(\theta) = \frac{9}{2\pi}\left(\theta - 0.55\sin\theta\right)$$ hours

A better model of radiation dose rates into an unshielded silicon material G(R) in Grays/hour is given by the sixth-order function

G(R) = $0.136R^6 - 2.194R^5 + 13.89\,R^4 - 43.73\,R^3 + 71.78\,R^2 - 57.95R + 18.15$ Gys/hour

where R is the distance in multiples of Earth's radius. What is the function G(T)?

Problem 2 - For most realistic situations, it is almost impossible to find an analytic solution to the required integral to exactly determine the area under the curve, which in this case is the total integrated dose to the satellite in one orbit. It is common to use numerical integration techniques to construct an approximate solution, or to estimate the area under the curve by 'counting squares' graphically.

A) Graph the function G(T) over one complete orbit over the domain T: [0 , 9 hours].

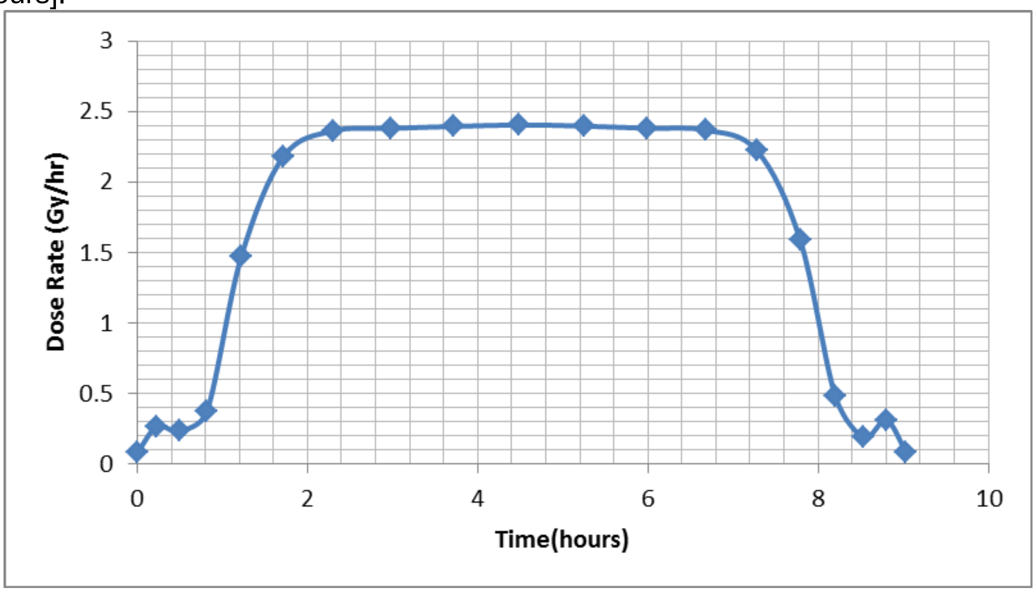
B) Estimate the geometric area under the curve using any method. What are the units of your estimated area rounded to 2 significant figure accuracy?

Problem 3 - A spacecraft will be at the end of its life after it has accumulated about 1000 Grays of radiation dose. A 1cm thickness of aluminum will reduce the radiation by 15 times. How many cm of shielding will be needed to reduce the total dose to 1000 Grays over 5 years?

Problem 1 - Answer. A students first approach is to replace 'R' in the function G(R) with the function R(θ), and then replace θ by inverting the equation for T(θ) so that we have G(R(θ(T))). For realistic functions, this is a depressing and frustrating approach. The student should realize that a perfectly reasonable alternative is to define a function table where the columns are Θ, T(θ), R(θ), and G(R). A sample of such a table is shown here:

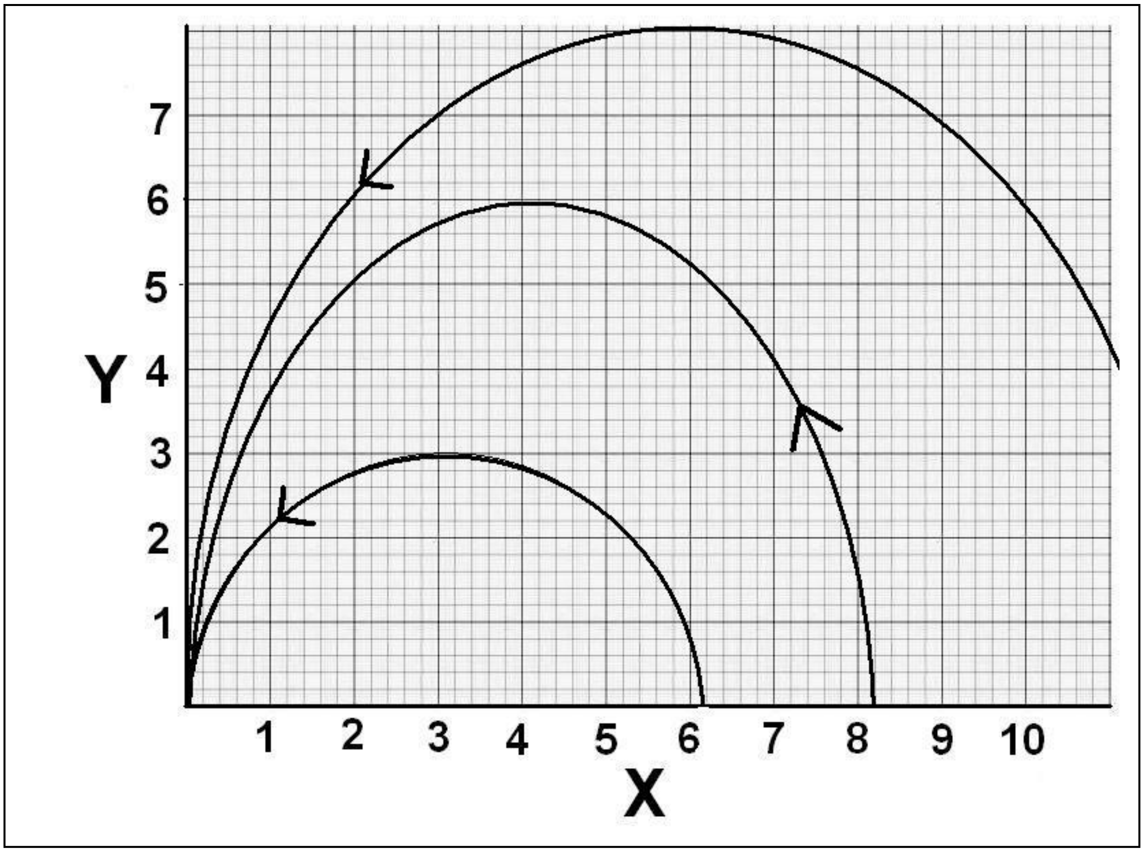
Θ in degrees	T: Time (hrs)	R in Re	G in Grays/hr
0	0.0	1.0	0.08
40	0.5	2.1	0.23
80	1.2	3.4	1.5
120	2.3	4.0	2.4
160	3.7	4.3	2.4
200	5.3	4.3	2.4
240	6.7	4.1	2.4
280	7.8	3.4	1.6
320	8.5	2.1	0.19

Problem 2 - A) Graph the dose rate function G(T) over one complete orbit over the domain T: [0 , 9 hours].



B) Answer: Graph: Total squares = 2x[½(2x4) + 4x4 + ½(4x20)+24x5.5)] = 384 squares
Table: 1 hr x (0.08+0.23+1.5+2.4+2.4+2.4+2.4+1.6+0.19) = **13.2** Grays/orbit
Area for 1 square = (0.4 hours) x (0.1 Grays/hr) = 0.04 Grays.
More accurate total dose is 0.04 x 384 squares= **15.4** Grays every 9 hour orbit.

Problem 3 - A spacecraft will be at the end of its life after it has accumulated about 1000 Grays of radiation dose. A 1cm thickness of aluminum will reduce the radiation by 15 times. How many cm of shielding will be needed to reduce the total dose to 1000 Grays over 5 years?
Answer: 15.4 Grays/10 hours x (8760 hours/1 year) x 5 years = 67500 Grays. Will need 67500/1000 = a factor of 68 reduction. 1 cm x (68/15) = **4.5 cm of shielding**



The figure above shows three magnetic field lines in space. The three arrows show the direction that a compass needle will point in the magnetic north direction. The X-axis lies along the west-east direction with east towards the right. The Y-axis lies along the geographic north-south direction with north at the top.

Problem 1 – Describe what happens to the compass needle as a spacecraft moves from point (+6.2, 0) to (+6.2, +5.0) to (+6.2, +8.0).

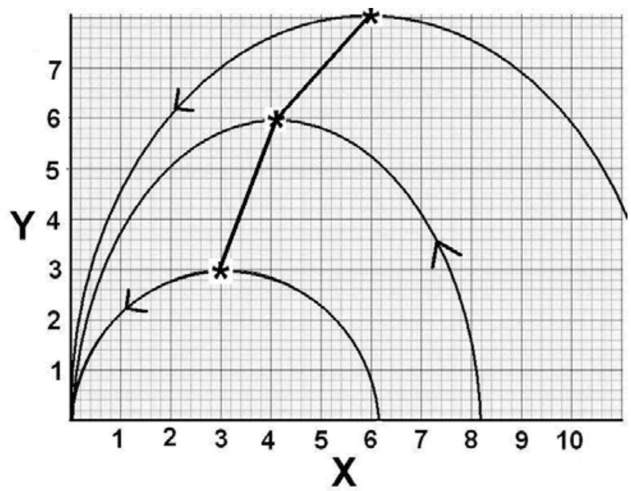
Problem 2 - Draw a possible spacecraft path so that the compass needle always points to geographic west in the figure.

Problem 3 – A satellite is launched from point (+1.0, +2.0) and travels horizontally to point (+10.0, +2.0). Plot a graph that shows how its instruments will record the direction changes of the magnetic field as it travels. At what location is the magnetic field pointed due-west? (Note: You may approximate angle measurements by interpolation as needed.)

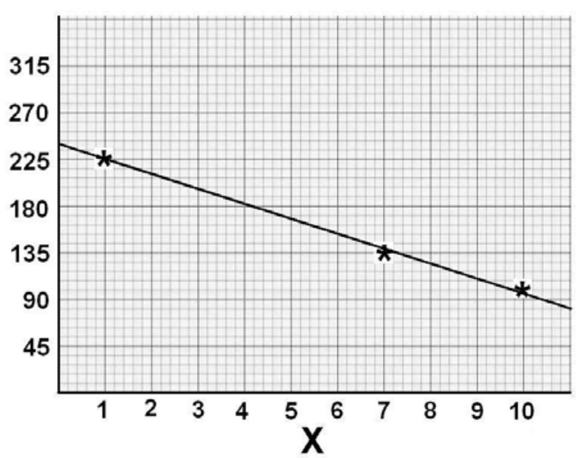
Problem 1 – Describe what happens to the compass needle as a spacecraft moves from point (+6.2, 0) to (+6.2, +5.0) to (+6.2, +8.0).

Answer: First the compass needle points vertically due-North, then it points 45degrees west of north, then it points due-west!

Problem 2 - Draw a possible spacecraft path so that the compass needle always points to geographic west in the figure.



Problem 3 – A satellite is launched from point (+1.0, +4.0) and travels horizontally to point (+10.0, +4.0). Plot a graph that shows how its instruments will record the direction changes of the magnetic field as it travels. At what location is the magnetic field pointed due-west? (Note: You may approximate angle measurements by interpolation as needed.)



To point due-west, the angle must be 180 degrees. From the graph this happens when the satellite is close to x=+4.0, so its coordinate on its path is (+4.0, +4.0)

Solar panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A simple application of vector dot and cross products lets us predict the amount of electrical power the panels can produce.

A surveyor on the sidewalk uses his instruments to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture shown above, suppose the points are labeled counter clockwise from the roof corner nearest the camera in units of meters: P1(6, 8, 4); P2(21, 8, 4); P3(21, 16, 10) and P4(6, 16, 10)

Problem 1 – What are the components to the two edge vectors defined by **A** = **P**2-**P**1 and **B** = **P**4-**P**1. Write the vector in standard notation with **x**, **y** and **z** being the coordinate unit vectors.

Problem 2 – What are the magnitudes of the vectors **A** and **B,** and in what units?

Problem 3 – What are the components to the vector, **N**, perpendicular to **A** and **B** and the surface of the roof?

Problem 4 – What is the magnitude of **N** and its units?

Problem 5 – The sun is located along the unit vector **S** = ½ **x** – 6/7 **y** + 1/7**z**. If the flow of solar energy is given by the vector **F** = 910 **S** in units of watts/meter^2, what is the dot product of **F** with **N**, and the units for this quantity?

Problem 6 – What is the angle between **N** and **S**? What is the elevation angle of the sun above the plane of the roof?

Problem 1 – What are the components to the two edge vectors defined by **A** = P2-P1 and **B** = P4-P1. Write the vector in standard notation with **x**, **y** and **z** being the coordinate unit vectors.

Answer: **A** = (21-6)**x** +(8-8)**y** + (4-4)**z** so **A =** 15**x**
B = (6-6)**x** + (16-8)**y** + (10-4)**z** so **B =** 8**y +** 6**z**

Problem 2 – What are the magnitudes of the vectors **A** and **B**, and in what units?

Answer: ||**A**|| = **15 meters** ||**B**|| = $(8^2 + 6^2)^{1/2}$ = **10 meters**

Problem 3 – What are the components to the vector, **N**, perpendicular to **A** and **B** and the surface of the roof?

Answer: Use the vector cross product: **N** = **A** x **B** so **N** = 0**x** – 90**y** +120**z**

Problem 4 – What is the magnitude of **N** and its units?
Answer: ||**N**|| = $((-90)^2 + (120)^2)^{1/2}$ so ||**N**|| = 150 meters^2 which is the area of the roof

Problem 5 – The sun is located along the unit vector **S** = ½ **x** – 6/7 **y** + 1/7**z.** If the flow of solar energy is given by the vector **F** = 910 **S** in units of watts/meter^2, what is the dot product of **F** with **N**, and the units for this quantity?

Answer: **F** = 910 (1/2**x** -6/7**y** +1/7**z**) = 455**x** – 780**y** + 130**z**.

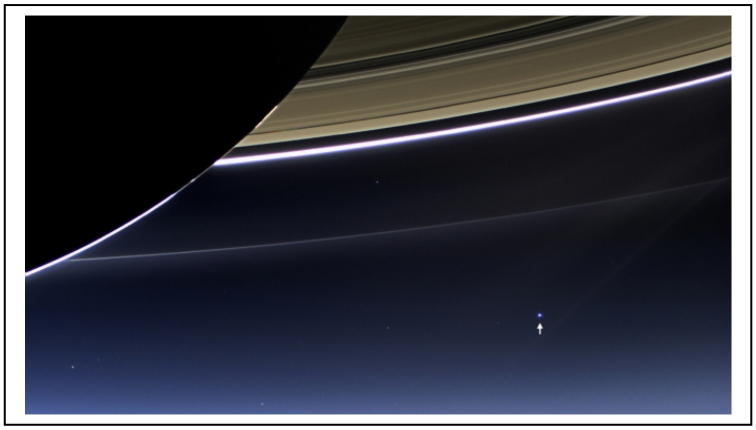
The dot product is just **F**dot**N** = 455*(0) -780*(-90) + 130*120 = **85,800 watts**.

Problem 6 – What is the angle between **N** and **S**? What is the elevation angle of the sun above the plane of the roof?

Answer: From the definition of dot product**:** **F** dot **N** = ||**F**|| ||**N**|| sin θ

Then since ||**F**|| = 910 and ||**N**|| = 150 and **F**dot**N** = 85,800 we have

Sinθ = (85800/(910x150)) = 0.629 and so θ **= 39^o**. This is the angle between the normal to the surface and the incident solar rays. The compliment of this is the elevation of the sun above the plane of the roof or 90-39 = 51^o.



The Cassini spacecraft captured this image of Earth seen through Saturn's rings (arrow).

At the distance of Saturn, our planet looks like many other bright stars in the sky, but with a noticeable blue color due to its oceans.

At the time of the photo, Earth was located 898 million miles (1.4 billion km) from Saturn.

Problem 1 –If Mars is about 250 million km from Earth and Saturn is about 1440 million km from Earth, using the Inverse-Square Law, how much fainter will Earth be from Saturn?

Astronomers use a logarithmic brightness scale called apparent magnitude, m. The formula that relates brightness to magnitude is given by

$$B = 10^{-.4m}$$

For example, a brightness change of 100-times means a magnitude difference of exactly 5.0 because $1/100 = 10^{-0.4(m)}$ so $10^{-2} = 10^{-0.4(m)}$ and solving for m, you get m = +5.0.

Problem 2 - At the distance of Mars, Earth appears as a star of magnitude -2.5, making it about as bright as Venus seen from Earth. How bright will Earth be from Saturn?

NASA Releases Images of Earth Taken by Distant Spacecraft
http://www.nasa.gov/mission_pages/cassini/whycassini/cassini20130722.html
July 22, 2013

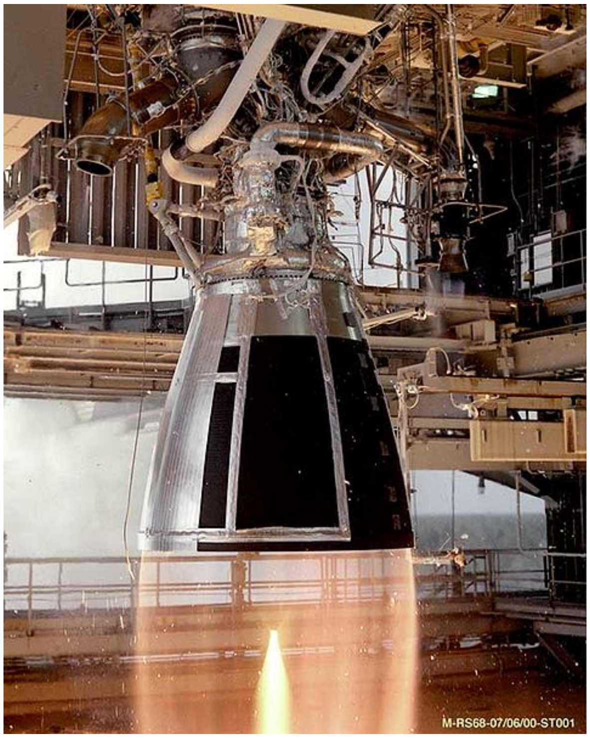
Problem 1 –If Mars is about 250 million km from Earth and Saturn is about 1440 million km from Earth, using the Inverse-Square Law, how much fainter will Earth be from Saturn?

Answer: Brightness = $(250/1440)^2$ = 0.030, so Earth will be about **33 times fainter** from Saturn than from Mars.

Problem 2 - At the distance of Mars, Earth appears as a star of magnitude -2.5, making it about as bright as Venus seen from Earth. How bright will Earth be from Saturn?

Answer: It will be 33 times fainter than from Mars so 33 = $10^{-.4m}$ so m = +3.8 m fainter than seen at Mars, and so -2.5 + 3.8 = **+1.3m at Saturn**.

Note: This assumes that Earth is seen under the same illumination from both planets. In fact from Mars the disk of Earth is about 90% illuminated, while from Saturn, Earth is only 50% illuminated. This makes Earth appear 50%/90% = 0.55 = $10^{-.4m}$, m = +0.6 fainter or +1.9 m from Saturn.



Rocket engines work by throwing matter out the back end of a rocket as fast as they can. Usually this matter is in the form of gas. This causes the rocket to move forward in the opposite direction from the ejected matter, thanks to Newton's Third Law of Motion which says that '*For every Action, there is an equal and opposite Reaction*'.

To lift tons of cargo into space, rocket engines have to be very powerful. Engineers usually compare different engines by their thrust and by their specific impulse. This makes it easy to decide which kind of engine to use to put different payloads into space in the most efficient and low-cost way possible.

Problem 1 – Specific Impulse Isp = V/g is the ratio of the exhaust speed of the engine (V) divided by the acceleration of gravity at Earth's surface (g), where g is 9.8 meters/sec^2 and V is measured in meters/sec. The J-2X rocket engine has an Isp=421 seconds. How fast are the exhaust gases leaving the bottom of the engine in A) kilometers/sec? B) feet per second? C) miles per hour? (1 kilometer = 3280 feet; 1 mile = 5280 feet)

Problem 2 – Thrust is a measure of the force that the rocket can produce to move an object against the pull of gravity. It is measured in Newtons and is defined by Thrust = F x V where F is the rate of flow of fuel in kilograms/sec and V is the exhaust speed of the combusted gases. The new J-2X rocket engines being designed and tested by NASA have an Isp=421 seconds and a thrust of T = 1,310,000 Newtons. At what rate is mass leaving the rocket engine in A) kilograms/second? B) pounds/second? (1 kilogram = 2.2 pounds).

Problem 3 – Ion rocket engines: Instead of chemical rockets using liquid fuels, the Dawn spacecraft has an ion rocket with Isp=3,100 seconds. If the same ion technology was used to design a replacement for the liquid-fueled J-2X engine, what must be the fuel flow rate, F, in order to produce the same thrust as the J-2X engine?

Problem 4 – Rocket Design: What would the Isp and thrust be for an engine that had V = 10,000 meters/sec and F = 500 kg/sec?

http://www.qrg.northwestern.edu/projects/vss/docs/propulsion/3-how-you-calculate-specific-impulse.html

Problem 1 – Specific Impulse Isp = V/g is the ratio of the exhaust speed of the engine (V) divided by the acceleration of gravity at Earth's surface (g), where g is 9.8 meters/sec^2 and V is measured in meters/sec. The J-2X rocket engine has an Isp=421 seconds. How fast are the exhaust gases leaving the bottom of the engine in A) kilometers/sec? B) feet per second? C) miles per hour? (1 meter = 3.280 feet; 1 mile = 5280 feet)

Answer: A) Isp=421 seconds and g = 9.8 m/sec^2 so V = Isp x g and
V = **4,126 meters/second**.
B) 1 meter = 3.28 feet and so V = 4126 m/s (3.28 feet/1 meter) so
V = **13,533 feet/sec**.
C) 1 mile = 5280 feet so V = 13,533/5280 = 2.56 miles/sec and for 3600 seconds in 1 hour, we have **V = 9,216 miles/hour**.

Problem 2 – Thrust is a measure of the force that the rocket can produce to move an object against the pull of gravity. It is measured in Newtons and is defined by Thrust = F x V where F is the rate of flow of fuel in kilograms/sec and V is the exhaust speed of the combusted gases. The new J-2X rocket engines being designed and tested by NASA have an Isp=421 seconds and a thrust of T = 1,310,000 Newtons. At what rate is mass leaving the rocket engine in A) kilograms/second? B) pounds/second? (1 kilogram = 2.2 pounds).

Answer: A) Isp = V/g so V = 421 x 9.8 m/sec2 = 4,126 meters/sec, then T = F x V and F = T/V so F = 1,310,000/4126 = **317 kg/second**. B) 317.5 kg/sec x (2.2 pounds/1kg) = **698 pounds/second**.

Problem 3 – Ion rocket engines: Instead of chemical rockets using liquid fuels, the Dawn spacecraft has an ion rocket with Isp=3,100 seconds. If the same ion technology was used to design a replacement for the liquid-fueled J-2X engine, what must be the fuel flow rate, F, in order to produce the same thrust as the J-2X engine?

Answer: Isp = 3100 sec so V = 3100 sec/9.8 m/sec2 = 316 m/sec. T = F x V and T = 1,310,000 Newtons and so F = 1,310,000/316 = **4,145 kg/sec**.

Problem 4 – Rocket Design: What would the Isp and thrust be for an engine that had V = 10,000 meters/sec and F = 5000 kg/sec?

Answer: Isp = 10,000/9.8 = **1020 sec**, and T = 5000 x 1020 = **5,100,000 Newtons**

Scientific notation is an important way to represent very big, and very small, numbers. Here is a sample of astronomical problems that will test your skill in using this number representation.

Problem 1: The sun produces 3.9×10^{33} ergs per second of radiant energy. How much energy does it produce in one year (3.2×10^{7} seconds)?

Problem 2: One gram of matter converted into energy yields 3.0×10^{20} ergs of energy. How many tons of matter in the sun is annihilated every second to produce its luminosity of 3.9×10^{33} ergs per second? (One metric ton = 10^{6} grams)

Problem 3: The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the sun?

Problem 4: The approximate volume of the visible universe (A sphere with a radius of about 14 billion light years) is 1.1×10^{31} cubic light-years. If a light-year equals 9.5×10^{17} centimeters, how many cubic centimeters does the visible universe occupy?

Problem 5: A coronal mass ejection from the sun travels 1.5×10^{13} centimeters in 17 hours. What is its speed in kilometers per second?

Problem 6: The NASA data archive at the Goddard Space Flight Center contains 25 terabytes of data from over 1000 science missions and investigations. (1 terabyte = 10^{12} bytes). How many CD-roms does this equal if the capacity of a CD-rom is about 6×10^{8} bytes? How long would it take, in years, to transfer this data by a dial-up modem operating at 56,000 bits/second? (Note: one byte = 8 bits).

Problem 7: Pluto is located at a distance of 5.9×10^{14} centimeters from Earth. At the speed of light (2.99×10^{10} cm/sec) how long does it take a light signal (or radio message) to travel to Pluto and return?

Problem 8: The planet HD209458b, now known as Osiris, was discovered by astronomers in 1999 and is at a distance of 150 light-years (1 light-year = 9.5×10^{12} kilometers). If an interstellar probe were sent to investigate this world up close, traveling at a maximum speed of 700 km/sec (about 10 times faster than our fastest spacecraft: Helios-1), how long would it take to reach Osiris?

Problem 1: The sun produces 3.9×10^{33} ergs per second of radiant energy. How much energy does it produce in one year (3.2×10^{7} seconds)? **Answer:** $3.9 \times 10^{33} \times 3.2 \times 10^{7} = 1.2 \times 10^{41}$ ergs.

Problem 2: One gram of matter converted into energy yields 3.0×10^{20} ergs of energy. How many tons of matter in the sun is annihilated every second to produce its luminosity of 3.9×10^{33} ergs per second? (One metric ton = 10^{6} grams). **Answer:** $3.9 \times 10^{33}/3.0 \times 10^{20} = 1.3 \times 10^{13}$ grams per second, or $1.3 \times 10^{13}/10^{6} = 1.3 \times 10^{7}$ metric tons of mass.

Problem 3: The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the sun? **Answer:** $1.98 \times 10^{33}/1.6 \times 10^{-24} = 1.2 \times 10^{57}$ protons.

Problem 4: The approximate volume of the visible universe (A sphere with a radius of about 14 billion light years) is 1.1×10^{31} cubic light-years. If a light-year equals 9.5×10^{17} centimeters, how many cubic centimeters does the visible universe occupy? **Answer:** 1 cubic light year = $(9.5 \times 10^{17})^{3} = 8.6 \times 10^{53}$ cubic centimeters, so the universe contains $8.6 \times 10^{53} \times 1.1 \times 10^{31} = 9.5 \times 10^{84}$ cubic centimeters.

Problem 5: A coronal mass ejection from the sun travels 1.5×10^{13} centimeters in 17 hours. What is its speed in kilometers per second? **Answer:** $1.5 \times 10^{13} / (17 \times 3.6 \times 10^{3}) = 2.4 \times 10^{8}$ cm/sec = 2,400 km/sec.

Problem 6: The NASA data archive at the Goddard Space Flight Center contains 25 terabytes of data from over 1000 science missions and investigations. (1 terabyte = 10^{12} bytes). How many CD-roms does this equal if the capacity of a CD-rom is about 6×10^{8} bytes? How long would it take, in years, to transfer this data by a dial-up modem operating at 56,000 bits/second? (Note: one byte = 8 bits). **Answer:** $2.5 \times 10^{13} / 6 \times 10^{8} = 4.2 \times 10^{4}$ Cdroms. It would take $2.5 \times 10^{13}/7{,}000 = 3.6 \times 10^{9}$ seconds or about 110 years.

Problem 7: Pluto is located at a distance of 5.9×10^{14} centimeters from Earth. At the speed of light (2.99×10^{10} cm/sec) how long does it take a light signal (or radio message) to travel to Pluto and return? **Answer:** $2 \times 5.9 \times 10^{14}/2.99 \times 10^{10} = 3.9 \times 10^{4}$ seconds or 11 hours.

Problem 8: The planet HD209458b, now known as Osiris, was discovered by astronomers in 1999 and is at a distance of 150 light-years (1 light-year = 9.5×10^{12} kilometers). If an interstellar probe were sent to investigate this world up close, traveling at a maximum speed of 700 km/sec (about 10 times faster than our fastest spacecraft: Helios-1), how long would it take to reach Osiris? **Answer:** $150 \times 9.5 \times 10^{12}/700 = 2.0 \times 10^{12}$ seconds or about 63,000 years!

Compound Interest

How it works: Suppose this year I put $100.00 in the bank. The bank invests this money and at the end of the year gives me $4.00 back in addition to what I gave them. I now have $104.00. My initial $100.00 increased in value by 100% x ($104.00 - $100.00)/ $100.00) = 4%. Suppose I gave all of this back to the bank and they reinvested in again. At the end of the second year they have me another 4% increase. How much money do I now have? I get back an additional 4%, but this time it is 4% of $104.00 which is $104.00 x 0.04 = $4.16. Another way to write this after the second year is:

$$\$100.00 \times (1.04) \times (1.04) = \$108.16.$$

After 6 years, at a gain of 4% each year, my original $100.00 is now worth:

$$\$100.00 \times (1.04) \times (1.04) \times (1.04) \times (1.04) \times (1.04) \times (1.04) = \$126.53$$

Do you see the pattern? The basic formula that lets you calculate this 'compound interest' easily is:

$$F = B \times (1 + P/100)^{T}$$

where :
B = the starting amount, P= the annual percentage increase, T = number of investment years.

Question: In the formula, why did we divide the interest percentage by 100 and then add it to 1?

Problem 1: The US Space program invested $26 billion to build the Apollo Program to send 7 missions to land on the Moon.

A) What was the average cost for each Apollo mission?

B) You have probably heard your parents complain that 'prices have sure gone up this year!'. This is because, each year, the price for food, gasoline, and other things you buy as a family have been increasing each year by about 3% .This is called Inflation. It means that this year you have to pay $1.03 for something you bought for $1.00 last year. Since the last moon landing in 1972, inflation has averaged about 4% each year. From you answer to A), how much would it cost to do the same Apollo moon landing in 2007?

Problem 2: A NASA satellite program was originally supposed to cost $250 million when it started in 2000. Because of delays in approvals by Congress and NASA, the program didn't get started until 2005. If the inflation rate was 5% per year, A) how much more did the mission cost in 2005 because of the delays? B) Was it a good idea to delay the mission to save money in 2000?

Problem 3: A scientist began his career with a salary of $40,000 in 1980, and by 2000 this had grown to $100,000. A) What was his annual salary gain each year? B) If the annual inflation rate was 3%, why do you think that his salary gain was faster than inflation during this time?

Do you see the pattern? Each year you invest the money, you multiply what you started with the year before by 1.04.

$$F = B \times (1 + P/100)^T$$

Question: In the formula, why did we divide the interest percentage by 100 and then add it to 1? Because if each year you are increasing what you started with by 4%, you will have 4% more at the end of the year, so you have to write this as 1 + 4/100 = 1.04 to multiply it by the amount you started with.

Problem 1: The US Space program invested $26 billion to build the Apollo Program to send 7 missions to land on the Moon. A) What was the average cost for each Apollo mission?

Answer : $26 billion/7 = $3.7 billion.

B) Answer: The number of years is 2007-1972 = 35 years. Using the formula, and a calculator:

$$F = \$3.7 \text{ billion} \times (1 + 4/100)^{35} = \$3.7 \text{ billion} \times (1.04)^{35} = \$14.6 \text{ billion.}$$

Problem 2: A) Answer: The delay was 5 years, so

$$F = \$250 \text{ million} \times (1 + 5/100)^{5} = \$250 \text{ million} \times (1.28) = \$319 \text{ million}$$

The mission cost $69 million more because of the 5-year delay.

B) No, because you can't save money starting an expensive mission at a later time. Because of inflation, missions always cost more when they take longer to start, or when they take longer to finish.

Problem 3: A scientist began his career with a salary of $40,000 in 1980, and by 2000 this had grown to $100,000. A) What was his annual salary gain each year? Answer A) The salary grew for 20 years, so using the formula and a calculator, solve for X the annual growth:

$$\$100{,}000 = \$40{,}000 \times (X)^{20} \qquad X = (100{,}000/40{,}000)^{1/20} \qquad X = 1.047$$

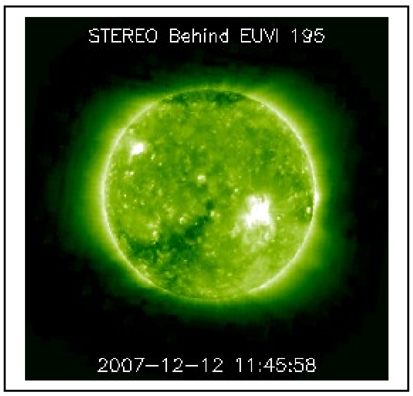
So his salary grew by about 4.7% each year, which is a bit faster than inflation.

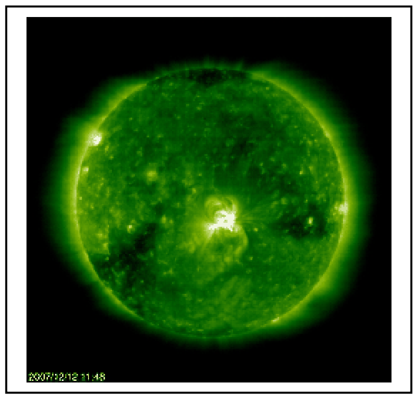
B) If the inflation rate was 3%, why do you think that his salary gain was faster than inflation during this time? Answer: His salary grew faster than inflation because his employers valued his scientific research and gave him average raises of 1.5% over inflation each year!

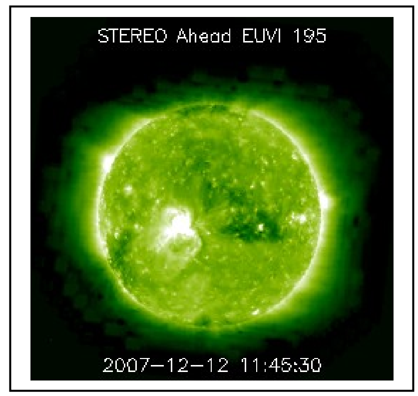
An Application of the Parallax Effect

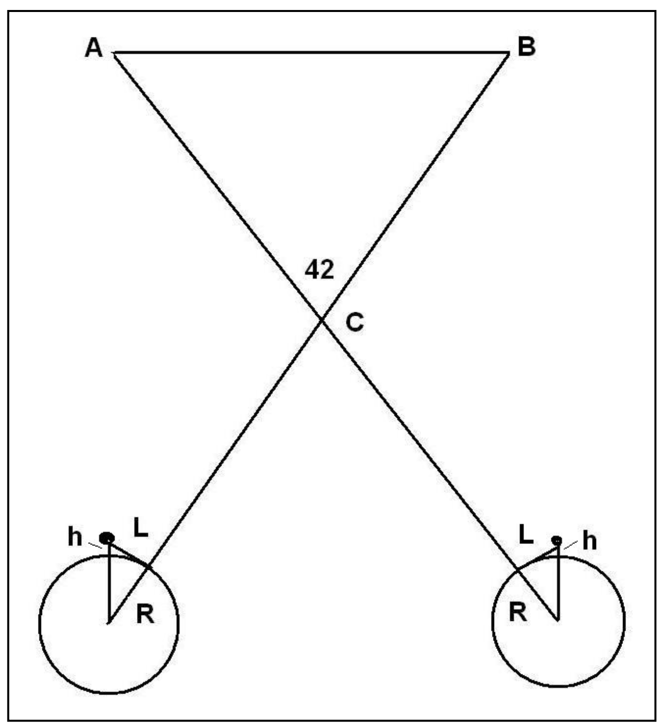
Two NASA, STEREO satellites take images of the sun and its surroundings from two separate vantage points along Earth's orbit. From these two locations, one located ahead of the Earth, and the other located behind the Earth along its orbit, they can create stereo images of the 3-dimensional locations of coronal mass ejections (CMEs) and storms on or near the solar surface.

The three images below, taken on December 12, 2007, combine the data from the two STEREO satellites (left and right) taken from these two locations, with the single image taken by the SOHO satellite located half-way between the two STEREO satellites (middle). Notice that there is a large storm event, called Active Region 978, located on the sun. The changing location of AR978 with respect to the SOHO image shows the perspective change seen from the STEREO satellites. You can experience the same *Parallax Effect* by holding your thumb at arms length, and looking at it, first with the left eye, then with the right eye. The location of your thumb will shift in relation to background objects in the room.









The diagram to the right shows the relevant parallax geometry for the two satellites A and B, separated by an angle of 42 degrees as seen from the sun. The diagram lengths are not drawn to scale. The radius of the sun is 696,000 km.

Problem 1: With a millimeter ruler, determine the scale of each image in km/mm. How many kilometers did AR978 shift from the center position (SOHO location for AR) between the two STEREO images? This is the average measure of 'L' in the diagram.

Problem 2: Using the Pythagorean Theorem, determine the equation for the height, h, in terms of R and L. Assume the relevant triangle is a right triangle.

Problem 3: How high (h) above the sun's surface, called the photosphere, was the AR978 viewed by STEREO and SOHO on December 12, 2007?

Answer Key

Problem 1: With a millimeter ruler, determine the scale of each image in km/mm. How many kilometers did AR978 shift from the center position (SOHO location for AR) between the two STEREO images? This is the measure of 'L' in the diagram.

Answer:

STEREO-Left image, sun diameter = 28 mm, actual = 1,392,000 km, so the scale is
1392000 km / 28mm = **49,700 km/mm**
SOHO-center sun diameter = 36 mm, so the scale is
1392000 km/36mm = **38,700 km/mm**
STEREO-right sun diameter = 29 mm, so the scale is
1392000 km / 29 mm = **48,000 km/mm**

Taking the location of the SOHO image for AR978 as the reference, the left-hand image shows that AR978 is about 5 mm to the right of the SOHO location which equals 5 mm x 49,700 km/mm = 248,000 km. From the right-hand STEREO image, we see that AR978 is about 5 mm to the left of the SOHO position or 5 mm x 48,000 km/mm = 240,000 km.
The average is 244,000 kilometers.

Problem 2: Using the Pythagorean Theorem, determine the equation for the height, h, in terms of R and L.

Answer: $(R + h)^2 = R^2 + L^2$

$$h = (R^2 + L^2)^{1/2} - R$$

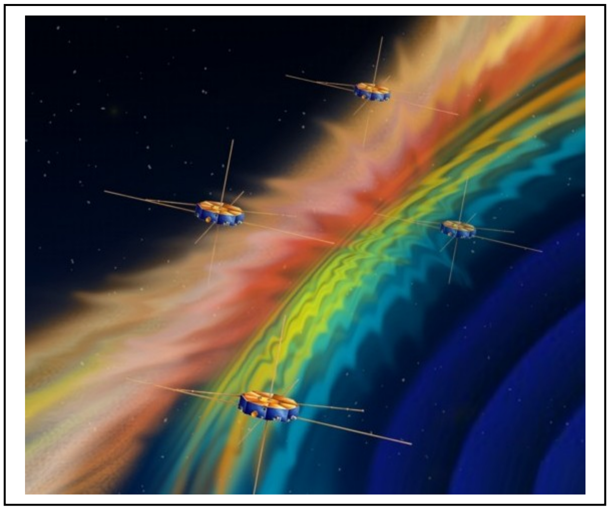
Problem 3: How high (h) above the sun's surface, called the photosphere, was the AR978 viewed by STEREO and SOHO on December 12, 2007?

Answer: $h = ((244{,}000)^2 + (696{,}000)^2)^{1/2} - 696{,}000$

$$h = 737{,}500 - 696{,}000$$

$$h = 41{,}500 \text{ kilometers}$$

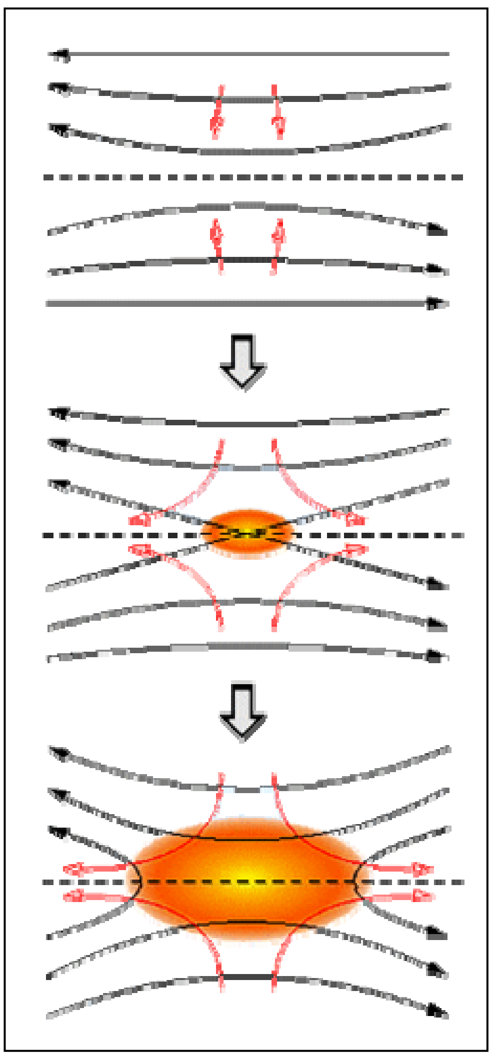
NASA Juggles Four Satellites at Once:



In 2013, NASA will launch the Magnetosphere Multi-Scale (MMS) satellite constellation; a set of four identical satellites that will study the 3-dimensional properties of Earth's magnetosphere. The spacecraft will travel in a tetrahedral formation with inter-spacecraft distances that will vary from ten kilometers to tens of thousands of kilometers.

At the center of the investigations is magnetic reconnection. This process, as shown in the sequence of figures below, breaks and reconnects magnetic lines of force. This converts stored magnetic energy into kinetic energy that causes trapped particles (the expanding spot in the figure) to also gain energy. MMS scientists hope that they will witness a few of these events inside the tetrahedral volume.

MMS will measure the physical properties of the reconnection process including the factors that control it, such as its distribution in space, and its minute-by-minute changes in time. This will help scientists test various theories of how this important process is triggered, evolves in time, and how magnetic energy is transferred into particle motion.



Problem 1: The volume of a regular tetrahedron with an edge length of A is given by the formula:

$$V = \frac{1}{12} A^3 (2)^{1/2}$$

What will be the volume of the MMS constellation: A) at its minimum size of A = 10 km? B) at its maximum size of A = 10,000 km?

Problem 2: The speed of the reconnection event is predicted to be about 500 kilometers/sec. About how long will it take disturbances to travel across the region of space being studied by MMS at its largest and its smallest satellite configuration?

Problem 3: Magnetic reconnection process (see the figure to left) releases stored magnetic energy (shaded region) which can then be transformed into kinetic energy in the motion of trapped, charged particles. The amount of stored magnetic energy is given by the formula:

$$E = \frac{1}{8\pi} B^2 \times V \quad \text{ergs}$$

Where B is the magnetic field strength in Gauss, and V is the volume of the magnetic field in cubic centimeters. If B = 0.0002 Gauss, how much energy could be stored within the tetrahedral volumes of the largest and smallest configurations of MMS?

Answer Key:

Problem 1: The volume of a regular tetrahedron with an edge length of A is given by the formula:

$$V = \frac{1}{12} A^{3} (2)^{1/2}$$

What will be the volume of the MMS constellation:

A) at its minimum size of A = 10 km? Answer = 118 cubic kilometers
B) at its maximum size of A = 10,000 km? Answer = 118 billion cubic kilometers

Problem 2: The speed of the reconnection event is predicted to be about 500 kilometers/sec. About how long will it take disturbances to travel across the region of space being studied by MMS at its largest and its smallest satellite configuration?

Answer: Largest: 10,000 km / (500 km/sec) = **20 seconds.**
Smallest 10 km / (500 km/sec) = **0.02 seconds.**

Problem 3: Magnetic reconnection releases stored magnetic energy which can then be transformed into kinetic energy in the motion of trapped, charged particles. The amount of stored magnetic energy is given by the formula:

$$E = \frac{1}{8\pi} B^{2} \times V \quad \text{ergs}$$

Where B is the magnetic field strength in Gauss, and V is the volume of the magnetic field in cubic centimeters. If B = 0.0002 Gauss, how much energy could be stored within the tetrahedral volumes of the largest and smallest configurations of MMS?

Largest volume:

$$V = 118 \text{ billion km}^{3} \times (1.0 \times 10^{15} \text{ cm}^{3}/\text{km}^{3}) = 1.2 \times 10^{26} \text{ cm}^{3}$$

$$E = 0.039 \times (0.0002)^{2} \times 1.2 \times 10^{26} \text{ cm}^{3} = \mathbf{1.9 \times 10^{17} \text{ ergs}}$$

Smallest volume:

$$V = 118 \text{ km}^{3} \times (1.0 \times 10^{15} \text{ cm}^{3}/\text{km}^{3}) = 1.2 \times 10^{17} \text{ cm}^{3}$$

$$E = 0.039 \times (0.0002)^{2} \times 1.2 \times 10^{17} \text{ cm}^{3} = \mathbf{1.9 \times 10^{8} \text{ ergs}}$$

The Limiting Behavior of Selected Functions

In astrophysics, many different kinds of formulae are derived for the interaction of radiation with matter. Often, astrophysicists want to know the 'limiting behavior' of the equations for extreme conditions. Here are a few examples.

$$\sigma = \frac{3}{4} s \left(\frac{1+x}{x^2} \left[\frac{2(1+x)}{1+2x} - \frac{1}{x} \ln(1+2x) \right] + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right)$$

Problem 1 : The equation above is the Klien and Nishina formula for the interaction of a high-energy photon with an electron. X is the ratio of the energy carried by the photon ($E = h\nu$) compared to the rest mass energy of the electron ($E = mc^2$). What is the form of this equation in the limit for large X?

Problem 1 – The formula can be simplified by noticing that as X becomes very large compared to 1, (1 + x) becomes x, and (1 + 2x) becomes 2x, (1 + 3x) becomes 3x, and the formula simplifies to

$$\sigma = 3/4\ s\ (1/x)[\ 2x/2x – (1/x)\ln(2x)] + (1/2x)\ln(2x) – 3x/(2x)^2).$$

This becomes $\sigma = 3/4\ s\ (1/x – (1/x^2)\ln(2x) + (1/2x)\ln(2x) – 3/4x)$

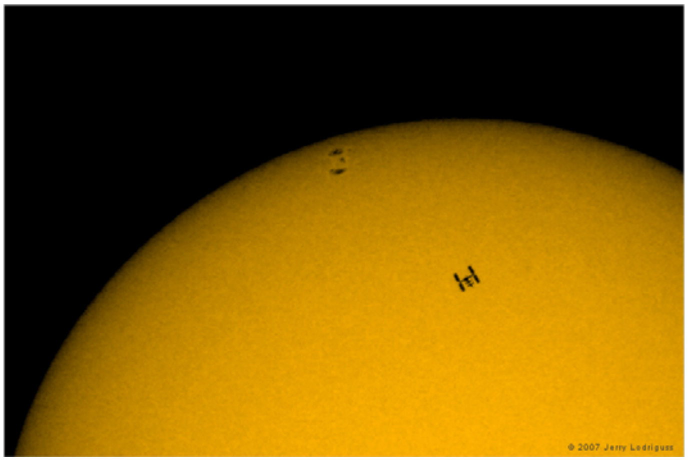
Because terms involving $1/x^2$ diminish faster than terms with 1/x, this leaves us with

$$\sigma = 3/4\ s\ [(1/2x)\ln(2x) + 1/4x]$$

which further simplifies to

$$\boldsymbol{\sigma = (3s/8x)[\ \ln(2x) + 1/2\ ]}$$

Angular Size and Velocity



(Photo courtesy Jerry Lodriguss (Copyright 2007, http://www.astropix.com/HTML/SHOW_DIG/055.HTM)

The relationship between the distance to an object, **R**, the objects size, **L**, and the angle that it subtends at that distance, θ, is given by:

$$\theta = 57.29 \frac{L}{R} \text{ degrees}$$

$$\theta = 3{,}438 \frac{L}{R} \text{ arcminutes}$$

$$\theta = 206{,}265 \frac{L}{R} \text{ arcseconds}$$

To use these formulae, the units for length, L, and distance, R, must be identical.

Problem 1 - You spot your friend (L = 2 meters) at a distance of 100 meters. What is her angular size in arcminutes?

Problem 2 - The Sun is located 150 million kilometers from Earth and has a radius of 696.000 kilometers, what is its angular diameter in arcminutes?

Problem 3 - How far away, in meters, would a dime (1 centimeter) have to be so that its angular size is exactly one arcminute?

Problem 4 - The spectacular photo above shows the International Space Station streaking across the disk of the Sun. If the ISS was located 379 kilometers from the camera, and the ISS measured 73 meters across, what was its angular size in arcseconds?

Problem 5 - The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) what was the angle, in arcminutes, that it moved through in one second as seen from the location of the camera? B) What was its angular speed in arcminutes/second?

Problem 6 - Given the diameter of the Sun in arcminutes (Problem 2), and the ISS angular speed (Problem 5) how long, in seconds, did it take the ISS to travel across the face of the sun?

Answer Key

Problem 1 - Answer: Angle = 3,438 x (2 meters/100 meters) = **69 arcminutes**.

Problem 2 - Answer: 3,438 x (696,000/150 million) = 15.9 arcminutes in radius, so the diameter is 2 x 15.9 = **32 arcminutes**.

Problem 3 - Answer: From the second formula R = 3438 * L/A = 3438 * 1 cm/1 arcminute so R = 3,438 centimeters or a distance of **34.4 meters**.

Problem 4 - Answer: From the third formula, Angle = 206,265 * (73 meters/379,000 meters) = **40 arcseconds**.

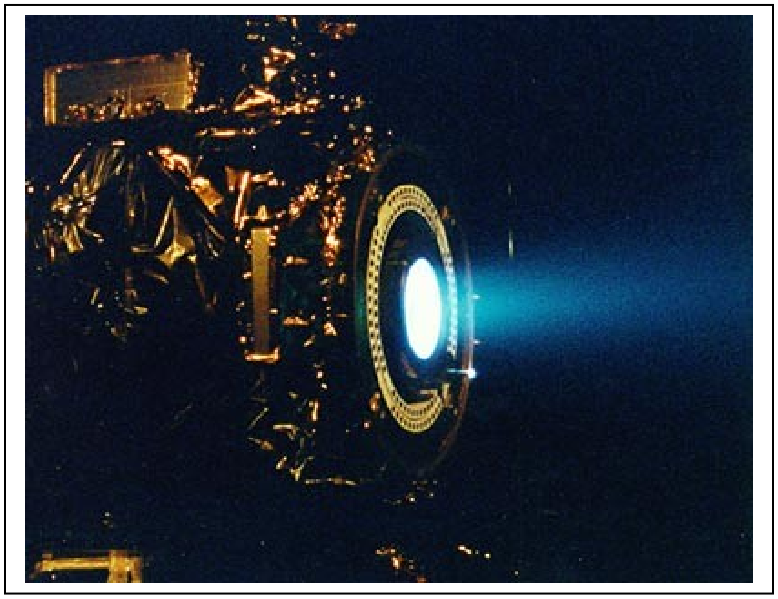
Problem 5 - Answer: The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) The ISS traveled L = 7.4 kilometers so from the second formula Angle = 3,438 * (7.4 km/379 km) = **67 arcminutes**. B) The angular speed is just **67 arcminutes per second**.

Problem 6 - Answer: The time required is T = 31.8 arcminutes/ (67 arcminutes/sec) = **0.47 seconds.**

The spectacular photo by Jerry Lodriguss had to be taken with careful planning beforehand. He had to know, to the second, when the Sun and ISS would be in the right configuration in the sky as viewed from his exact geographic location. Here's an example of the photography considerations in his own words:

" I considered trying to monitor the transit visually with a remote release in my hand and just firing (the camera) when I saw the ISS in my guidescope. Then I worked out the numbers. I know that my reaction time is 0.19 seconds. This is actually quite good, but I make my living shooting sports where this is critical, so I better be good at it. I also know that the Canon 1D Mark IIn has a shutter lag of 55 milliseconds. Adding these together, plus a little bit of a fudge factor, the best I could hope for was about 1/4 of a second from when I saw it to when the shutter opened. Since the entire duration of the transit was only 1/2 of a second, in theory, I could capture the ISS at about the center of the disk if I fired as soon as I saw it start to cross. This was not much of a margin for error. I could easily blink and miss the whole thing... Out of 49 frames that the Mark IIn recorded, the ISS is visible in exactly one frame."

The Mathematics of Ion Rocket Engines



Believe it or not, NASA has been using ion engines for decades, and most commercial satellites use them too!

This image of a xenon ion engine, being tested at NASA's Jet Propulsion Laboratory, shows the faint blue glow of charged atoms being emitted from the engine. It was used in both the Deep Space 1 and Dawn satellites in their historic journeys to explore asteroids. The operating principle is simple.

Heavy atoms such as cesium, xenon are ionized, accelerated through a high-voltage grid, and ejected out the back of the thruster. The momentum of the ejected heavy atoms, when multiplied by the trillions of atoms in the beam, produces a steady, constant thrust that can be maintained for years at a time. Because the speed of the atoms is thousands of times higher than the exhaust from chemical rocket engines, very little mass is needed in the beam of atoms to generate a large thrust over time. For the Dawn spacecraft, launched in 2007, the 'fuel' mass is only 425 kilograms, but ejected steadily for 8 years, the 1,200 kilogram satellite will reach a speed of over 36,000 km/hour (22,300 miles/hour). This is equal to 315 million kilometers/year or the distance to the sun and back from Earth! Here is some of the mathematics, in a highly simplified form, that will take you through the basic ideas behind these exciting rocket technologies!

Problem 1 - Charged particles gain speed in an electric field - The kinetic energy of a particle is given by K.E. = $1/2\ mv^2$. The energy a charged particle gains from falling through a potential difference of V volts is given by E = qV. The NSTAR ion engine developed for the Deep Space 1 satellite uses xenon atoms with a mass of 2.2×10^{-25} kg, and a charge of q = 1.6×10^{-19} coulombs. What will be the speed of the atom, in kilometers/hour, if the voltage grid of the ion engine is 1,300 volts?

Problem 2 - The smaller the grid separation, the higher the acceleration - The NSTAR engine has a grid separation of 0.7 mm. From your answer to Problem 1, A) what is the average acceleration of the ions as the leave the grid? B) What is the force they experience, in Newtons?

Problem 3 - The thrust depends on particle flow rate - How many particles have to be ejected in the time it takes to cross the grid, to create a thrust of 0.90 Newtons? (Express the answer in particles per second).

Problem 4 - Charged particle flows produce electrical currents - If each particle carries exactly one unit of charge, and 1 Ampere = 6.25 x 1018 particles/sec, what is the current needed in the beam to give the thrust in Problem 3?

Problem 5 - Currents require power to maintain them - What is the beam power, in watts, defined by Power = Voltage x Amperage?

Answer Key

Problem 1 - Equate K.E = E and solve for v to get:

$$v^2 = 2\,qV/m = 2\,(1.6 \times 10^{-19})(1{,}300) / (2.2 \times 10^{-25}) = 2.0 \times 10^9$$

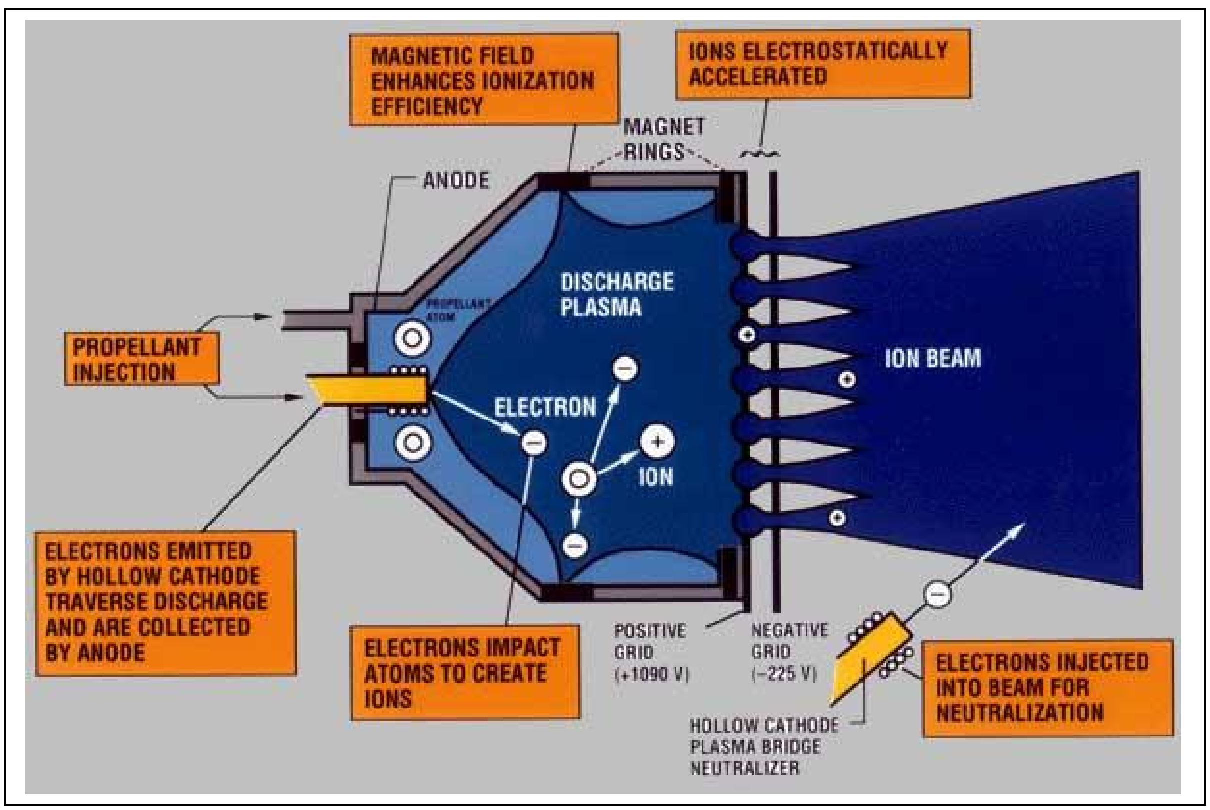
so v = 44.2 km/sec or 159,000 km/hr

Problem 2 - Answer: A) At an average speed of 44.2 km/sec, the atoms take about T = 0.7 mm/ (22.1 km/sec) = 3.1×10^{-8} seconds to travel the distance. This is an average acceleration of 22,100 m/sec / 3.1×10^{-8} sec = 7.1×10^{11} meters/sec^2. B) The force F = (2.2×10^{-25}) $(7.1 \times 10^{11}$ meters/sec$^2)$ = 1.6×10^{-13} Newtons/particle.

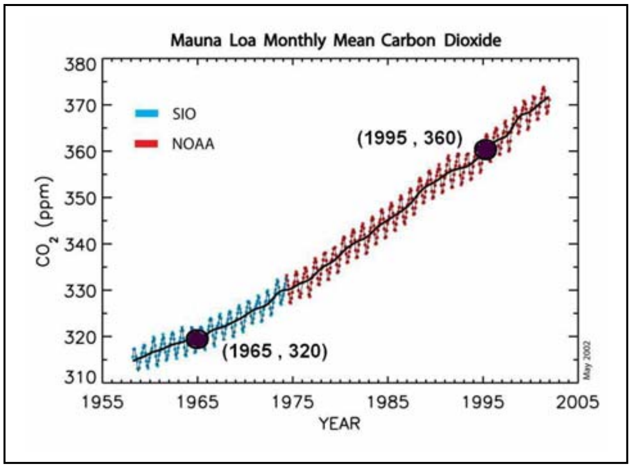
Problem 3 - 0.090 Nt = N x 1.6×10^{-13} Newtons/particle so N = 5.6×10^{11} particles. The time to cross the grid is 3.1×10^{-8} seconds so the particle flow has to be 5.6×10^{11} particles / 3.1×10^{-8} seconds = 1.8×10^{19} particles/second.

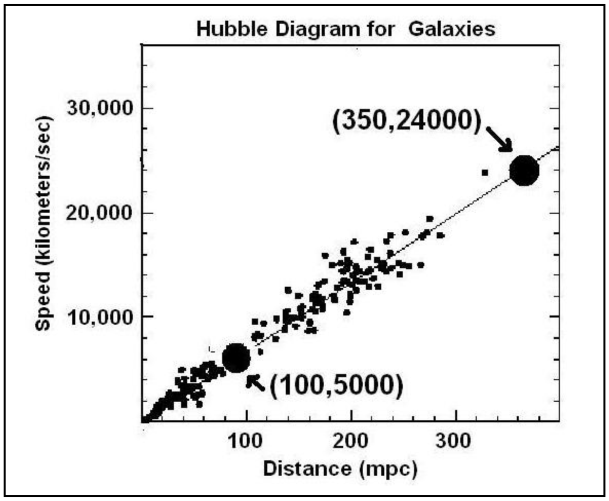
Problem 4 - 1.8×10^{19} particles/second / $(6.25 \times 10^{18}$ particles/sec//Ampere) = 2.9 Amperes!

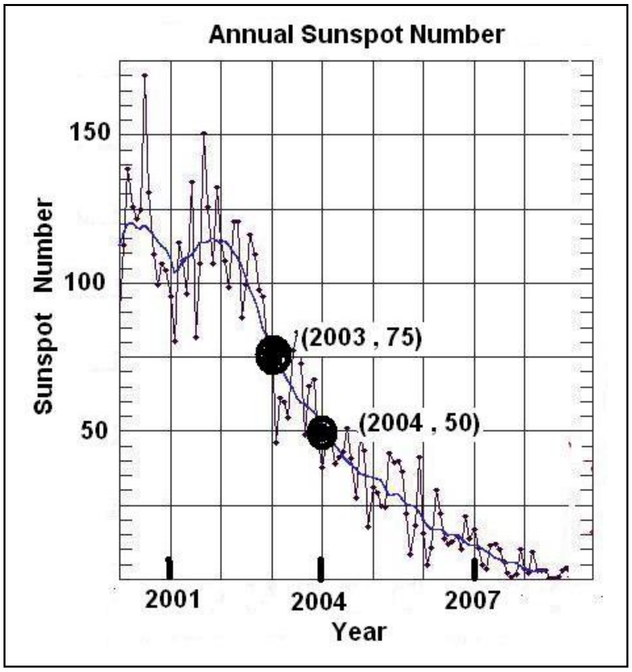
Problem 5 - P = 1,300 Volts x 2.9 Amperes = 3,800 watts.



Rates and Slopes: An Astronomical Perspective







A 'rate' is defined as the ratio of two quantities which have different units of measurement.

For example, if you travel in a car 200 kilometers in 2 hours, the rate is R = 200 kilometers/2 hours or R = 100 kilometers/hour. You recognize this particular rate as just the speed of the car! Scientists work with other kinds of rates as well.

Graphically, a rate is a measure of the difference between two values along the Y-axis, divided by the difference between two corresponding values along the X-axis. It also represents the slope of a curve plotted on a graph.

For example, let's look at the top graph to the left. It shows how the amount of carbon dioxide in the atmosphere is increasing between 1955 and 2005. The two points along the data curve can be used to find the rate of change of the carbon dioxide in time, which is the slope of the line connecting these two points.

The change along the X-axis is just the difference '1995-1965' or +30 years. The difference along the Y-axis corresponding to these same years is just '360 ppm -320 ppm' or +40 ppm. The rate is then R = +40 ppm/+30 years or +1.3 ppm/year.

Note that we have kept careful track of the signs and units in the calculations. This is because rates can represent both increases (positive) or decreases (negative) changes.

Problem 1 - Calculate the Rate corresponding to the speed of the galaxies in the Hubble Diagram. (Called the Hubble Constant, it is a measure of how fast the universe is expanding).

Problem 2 - Calculate the rate of sunspot number change between the indicated years.

Answer Key

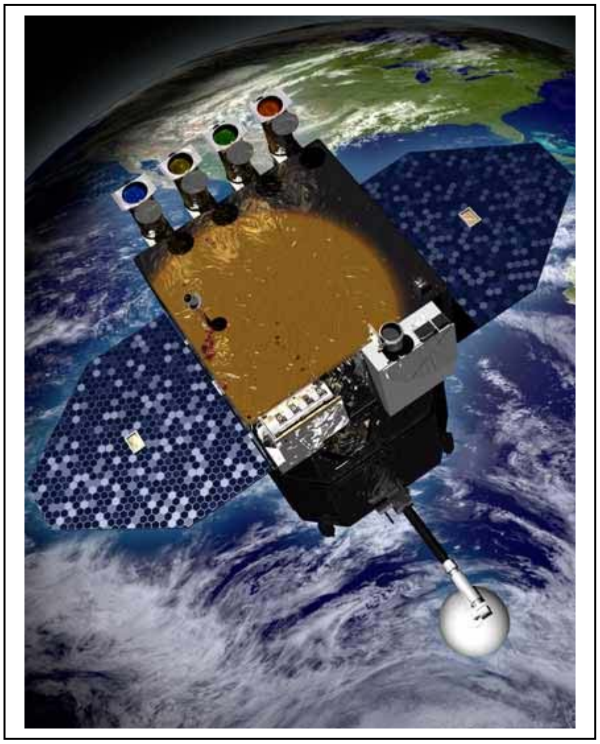
Problem 1 - Calculate the rate corresponding to the speed of the galaxies in the Hubble Diagram. (Called the Hubble Constant, it is a measure of how fast the universe is expanding).

Answer: The two points have coordinates (x1,y1) = (100 mpc, 5000 km/s) and (x2,y2) = (350 mpc, 24000 km/s). The difference in the y coordinates is y2-y1 = 24000-5000 = 19000km/s. The difference in the x coordinates is x2-x1 = 350 mpc - 100 mpc = 250 mpc. The Rate is then R = (19000km/sec) / (250 mpc) = **76 km/sec/mpc**. This is read as '76 kilometers per second per megaparsec'. Another way to write this complicated mixed-unit ratio is

$$H = 76 \frac{\text{km}}{\text{sec mpc}}$$

Problem 2 - Calculate the rate of sunspot number change between the indicated years.
Answer: The two points have coordinates (x1,y1) = (2003, 75 spots) and (x2,y2) = (2004, 50 spots). The difference in the y coordinates is y2-y1 = 50 spots - 75 spots = -25 spots. The difference in the x coordinates is x2-x1 = 2004 - 2003= 1 year. The Rate is then R = -25 spots / (1 year) = **-25 spots/year.**

This rate is negative because the number of spots is decreasing as time goes forward. This is reflected in the slope of the data being negative in the graph.



The 15 instruments on NASA's latest solar observatory will usher in a new era of solar observation by providing scientists with HD-quality viewing of the solar surface in nearly a dozen different wavelength bands.

One of the biggest challenges is how to handle all the data that the satellite will return to Earth 24/7/365! It is no wonder that the design and construction of this data handling network has taken nearly 10 years to put together! To make sense of the rest of this story, here are some units and prefixes you need to recall (1 byte = 8 bits):

Kilo = 1 thousand
Mega = 1 million
Giga = 1 billion
Tera = 1 trillion
Peta = 1,000 trillion
Exa = 1 million trillion

Problem 1 - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models?

Problem 2 - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute, uncompressed, MPEG-4 song occupies 8 megbytes, about A) How many uncompressed songs can be stored on the HD? B) How many hours of music can be stored on the HD? (Note: music is actually stored in a compressed format so typically several thousand songs can be stored on a large HD)

Problem 3 - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?

Problem 4 - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 5 gigabytes of information, how many DVDs-worth of data will be generated by the SDO: A) Each day? B) Each year?

Problem 5 - How many petabytes of data will SDO generate during its planned 5-year mission?

Problem 6 - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal?

Problem 1 - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models? Answer: Take the ratio of the modern HD to the one in 1982 to get 2.5 billion / 25 million = 2,500,000,000/25,000,000 = 2,500/25 = **100 times.**

Problem 2 - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute MPEG-4 song occupies 8 megbytes, about A) how many songs can be stored on the HD? B) How many hours of music can be stored on the HD?
Answer: A) Number of songs = 2.5 gigabytes/8 megabytes = 2,500 megabytes/8 megabytes = 312 B) Time = 312 songs x 4 minutes/song = 1,248 minutes = **20.8 hours.**

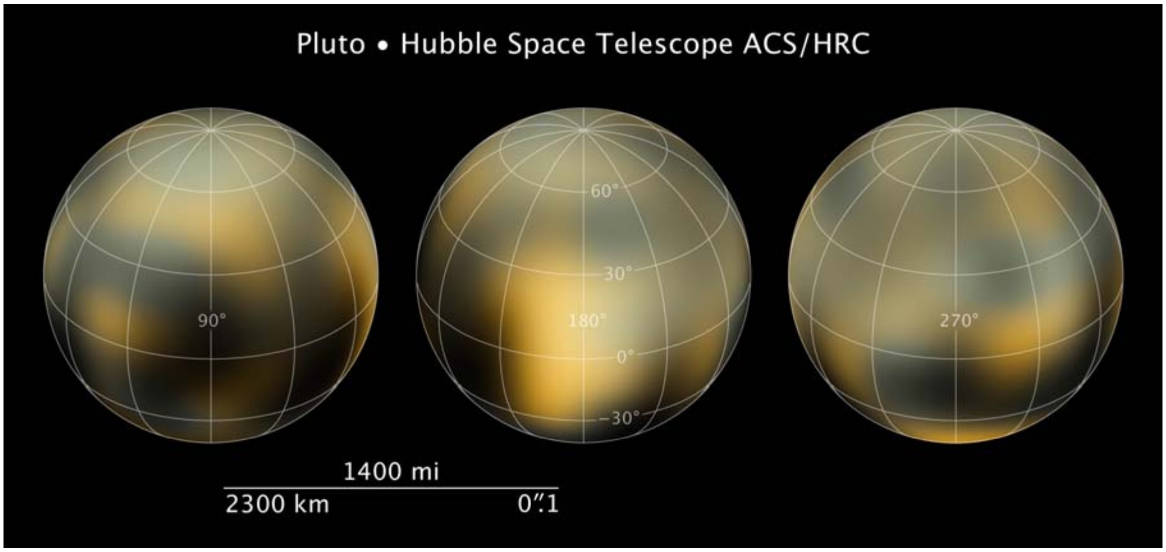
Problem 3 - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?
Answer; A) 2 gigabytes x 8 bits/1 byte = 16 gibabits. Then 16,000,000,000 bits x (1 second/56,000 bits) = 285,714 seconds or **79.4 hours**. B) 16,000,000,000 bits x (1 second/16,000,000 bits) = 1,000 seconds or about **17 minutes.**

Problem 4 - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 5 gigabytes of information, how many DVDs-worth of data will be generated by the SDO each A) Day? B) Year? Answer: Adding up the data rates for the three instruments plus the satellite housekeeping we get 67+63+20 = 150 megabits/sec. A) In one day this is 150 megabits/sec x (86,400 sec/1day) = 12.96 terabites or since 1 byte = 8 bits we have 1.6 terabytes. This equals 1,600 gigabytes x (1 DVD/8 gigabytes) = **200 DVDs each day**. B) In one year this equals 365 days x 1.6 terabytes/day = 584 terabytes per year or 365 days/1 year x 200 DVDs/1 year = **73,000 DVDs/year.**

Problem 5 - How many petabytes of data will SDO generate during its planned 5-year mission? Answer: In 5 years it will generate 5 years x 584 terabytes/year = 2,920 terabytes. Since 1 petabyte = 1,000 terabytes, this becomes **2.9 petabytes**.

Problem 6 - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal? Answer: 1 exabyte = 1,000 petabytes = 1,000,000 terabytes = 1,000,000,000 gigabytes. So 50 exabytes equals 50 billion gigabytes. One DVD stores 5 gigabytes, so the total human information 'stream' would occupy 50 billion / 5 billon = **10 billion DVDs.**

Seeing a Dwarf Planet Clearly: Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. The images, created at the very limits of Hubble's resolving power, show enigmatic light and dark regions that are probably organic compounds (dark areas) and methane or water-ice deposits (light areas). Since these photos are all that we are likely to get until NASA's New Horizons spacecraft arrives in 2015, let's see what we can learn from the image!

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter?

Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?

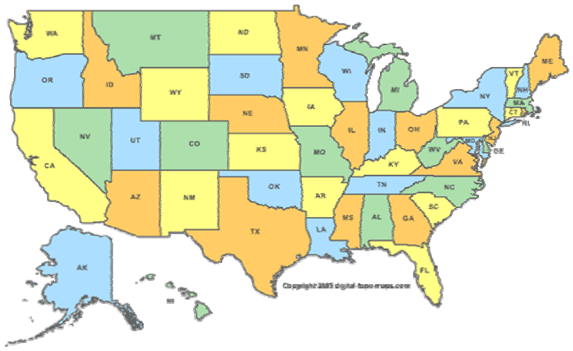
Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)?

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Answer Key

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter? Answer: The Legend bar indicates 2,300 km and is 43 millimeters long so the scale is 2300/43 = **53 km/mm**.

Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?
Answer; Student's selection will vary, but on the first image to the lower right a feature measures about 8 mm in diameter which is 8 mm x (53 km/1mm) = **424 kilometers wide**. This is about the same size as the **state of Utah**!



Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)? Answer: From the image, Pluto is a sphere with a diameter of 2,300 km, so its volume will be $V = 4/3\ \pi\ (1{,}250{,}000)^3 = 8.2 \times 10^{18}$ meters^3. Then its density is just D = M/V = (1.3 x 10^{22} kilograms)/ (8.2 x 10^{18} meters^3) so **D = 1,600 kg/m^3**. This would be about the density of **a mixture of rock and water-ice**.

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Answer: We want to match the density of Pluto (1,600 km/m^3) by using ice (917 kg/m^3) and rock (2300 kg/m^3). Suppose we made Pluto out of half-rock and half-ice **by mass**. The volume this would occupy would be V = (0.5* 1.3 x 10^{22} kilograms/917 kg/m^3) = 7.1x10^{18} meters^3 for the ice, and V = (0.5* 1.3 x 10^{22} kilograms/3000 kg/m^3) = 2.2 x10^{18} meters^3 for the rock, for a total volume of 9.3 x 10^{18} meters^3 for both. This is a bit larger then the actual volume of Pluto (8.2 x 10^{18} meters^3) so we have to increase the mass occupied by ice, and lower the 50% by mass occupied by the rock component. The result, from student trials and errors should yield after a few iterations **about 40% ice and 60% rock.** This can be done very quickly using an Excel spreadsheet. For advanced students, it can also be solved exactly using a bit of algebra.

ISS - Orbit Altitude Changes

The International Space Station is a 400-ton, $160 billion platform that supports an international team of 3-5 astronauts for tours of duty lasting up to 6 months at a time. Like all satellites that orbit close to Earth, the atmosphere causes the ISS orbit to decay steadily every day, so the ISS has to be 're-boosted' every few months to prevent it from burning up in the atmosphere.

Problem 1 - Based on the following information, what is the altitude of the ISS by April 2009?

"In January, the altitude was 340 kilometers. By March it has lost 8 kilometers before the Progress-59 supply ship raised its altitude by 5 kilometers. In May, the ISS lost 4 1/2 kilometers and was re-boosted by the Progess-60 supply ship by 5 1/2 kilometers. Again the ISS continued to lose altitude by 5 1/2 kilometers by July when the Progress-61 supply ship raised its orbit by 9 1/2 kilometers. The ISS altitude then fell by 3 kilometers by October when the Soyuz TMA-11 mission re-boosted the station by 5 kilometers. The ISS continued to lose altitude until late December, 2007 when it had lost a total of 8 1/2 kilometers since its last re-boost by Soyuz. Since December 2007, the total of all the declines and re-boosts added up to a net change of + 11 1/2 kilometers by April 2009. "

Answer Key

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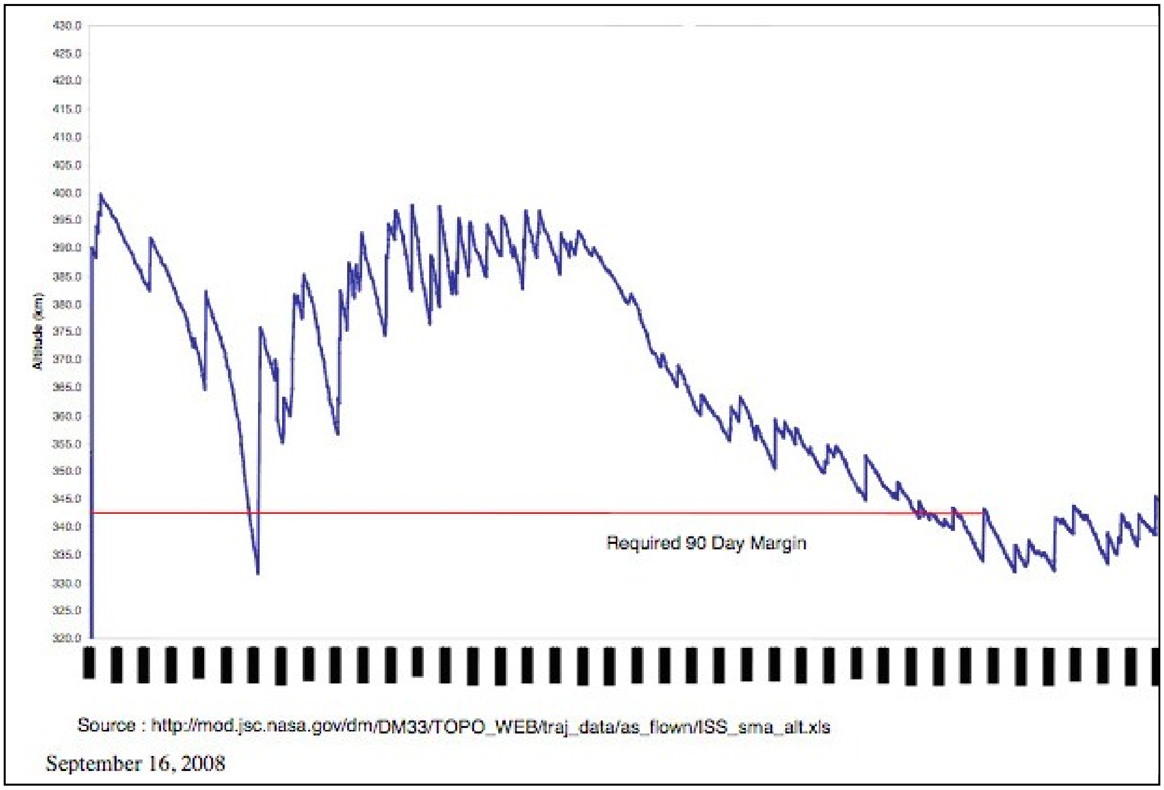
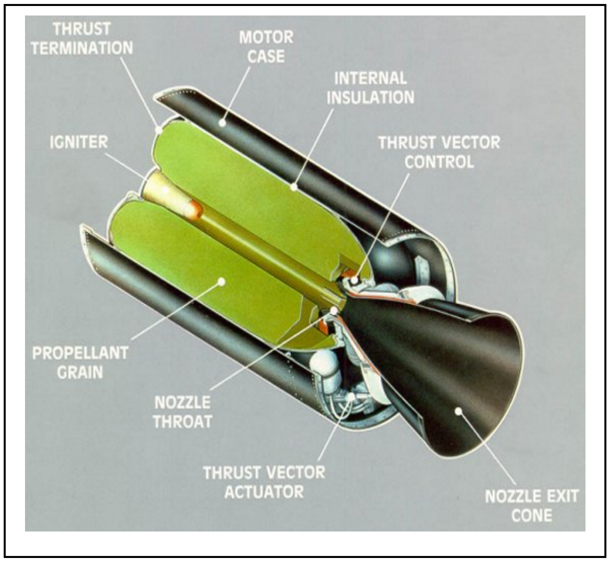
Answer: 340 kilometers - 8 kilometers + 5 kilometers - 4 1/2 kilometers + 5 1/2 kilometers - 5 1/2 kilometers + 9 1/2 kilometers - 3 kilometers + 5 kilometers - 8 1/2 kilometers + 11 1/2 kilometers = 347 kilometers.

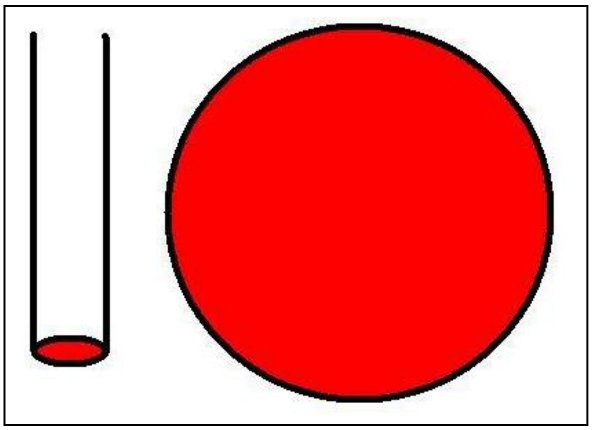
Note to Teacher: The figure below shows the altitude changes between November 1998 and July 2008.



Solid Rocket Boosters



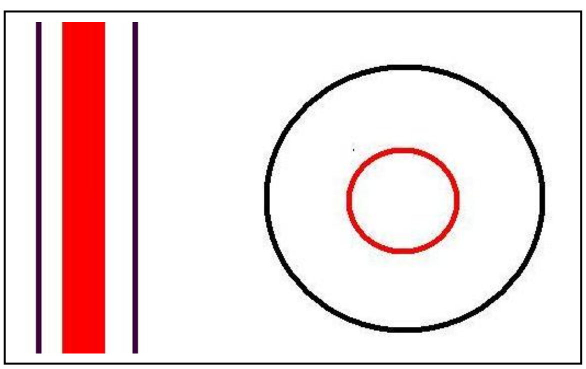
The two solid rocket boosters (SRBs) for the Ares-V launch vehicle will each generate a total thrust of 3.8 million pounds (16.9 megaNewtons). The SRBs from tip to ground are 193 feet long and have a diameter of 12.2 feet. The actual fuel occupies a cylindrical volume about 180 feet (60 meters) long and 11.5 feet (3.7 meters) in diameter. They will 'burn' for a total of 126 seconds. You might think that the fuel burns from the bottom of the cylinder to the top, the way that a candle consumes its wax. The fuel in an SRB is actually designed to burn from the central axis of the cylinder to the outside casing! Let's see why this is a much better way to launch a rocket!



Problem 1 - Thrust is created by burning the exposed surface area of the fuel in the SRB. To launch the 3.4 million pound Ares-V rocket, they have a combined burn rate of 8,740 kg of fuel each second. The density of the fuel is 1770 kg/m^3. If the exposed fuel area is just the circular cross section of the cylinder (see red area in the figure to left), and the burn depth is 0.025 meters each second;

A) What is the total burn rate?

B) Is this enough to launch Ares-V?



Problem 2 - Suppose, instead, that a cylindrical core (red circle) with a diameter of 0.6 meters is cut out along the axis of the booster from top to bottom. The figure to the left shows the red areas where the fuel is burning.

A) What is the surface area of the exposed fuel in the core region?

B) If the burn depth is 0.025 meters each second, what is the mass rate in kg/sec?

C) Is this enough to launch Ares-V?

Answer Key

Problem 1 - Thrust is created by burning the exposed surface area of the fuel in the SRB. To launch the 3.4 million pound Ares-V rocket, they have to have a combined burn rate of 8,740 kg of fuel each second. The density of the fuel is 1770 kg/m^3. If the exposed fuel area is just the circular cross section of the cylinder (see red area in the figure to left), and the burn depth is 0.025 meters each second,

A) What is the total burn rate? Answer: A) Area = 3.14 x (3.7m/2)^2 = 10.7 m^2. Volume change of disk = 10.7 m^2 x 0.025 m/sec = 0.27 m^3/sec. Mass = 1770 x 0.27 = 480 kg/sec to two significant figures. For both SRBs this yields a total of **960 kg/sec**.

B) Is this enough to launch Ares-V? Answer: **This is not a fast enough** fuel burn rate to ensure the proper thrust, so an SRB cannot provide enough thrust by burning only the fuel exposed by the cylindrical cross-section at the tail-end of the booster.

Problem 2 - Suppose, instead, that a cylindrical core with a diameter of 0.6 meters is cut out along the axis of the booster from top to bottom.

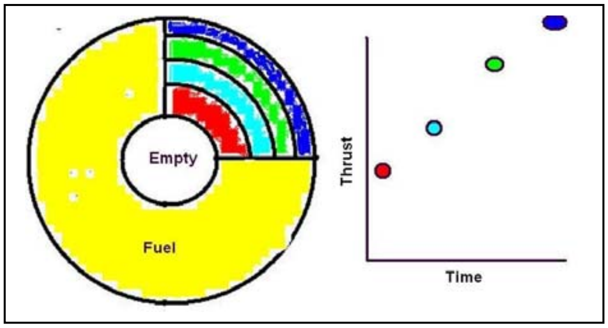
A) What is the surface area of the exposed fuel in the core region? Answer: Burn Area = 2πRh = 2 x 3.14 x (0.6 meters/2) x 60 meters = **113 m^2**.

B) If the burn rate is 0.025 meters each second, what is the mass rate in kg/sec? Answer: 113 m^2 x 0.025 m/sec x 1770 kg/m^3 = 5,000 kg/sec. For both SRBs this equals **10,000 kg/sec.**

C) Ares-V needs 8,740 kg/sec to launch so the 10,000 kg/sec produced by burning along the cylindrical core surface **provides more than enough thrust!**

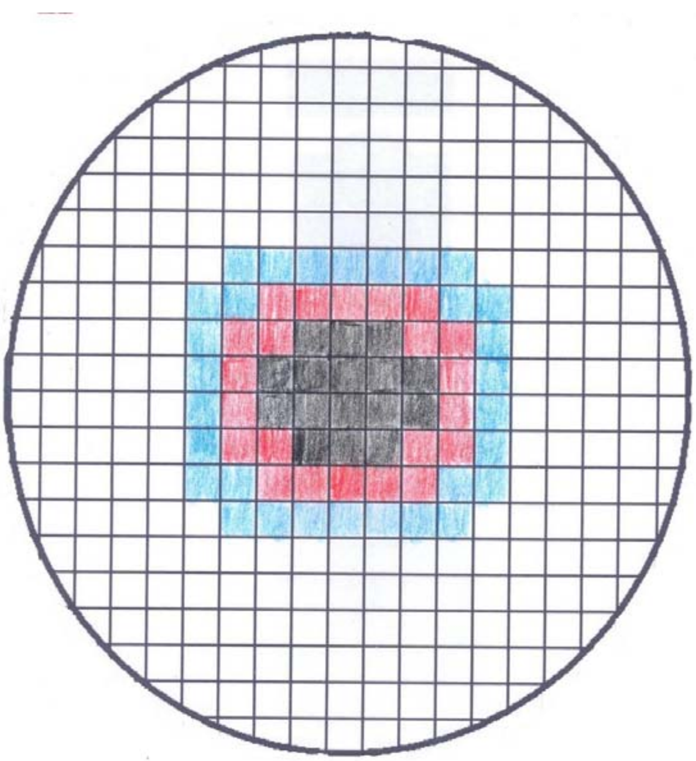
Note to Teacher: Problem 2 used the 'tubular' method. Sketch this cross section on the board and ask your students to predict what the thrust profile will look like as the fuel burns outward from the core to the outer layer of the cylinder. Repeat this exercise for other kinds of fuel shapes and cross-sections.

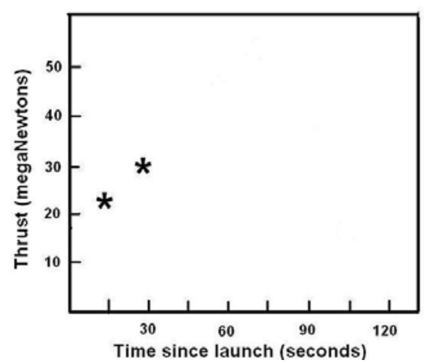
Solid Rocket Boosters - II



The example above shows the thrust curve for fuel burned in shells concentric with a cylindrical empty cavity along the axis of the booster from the inner (red) zone to the outer (blue) zone.

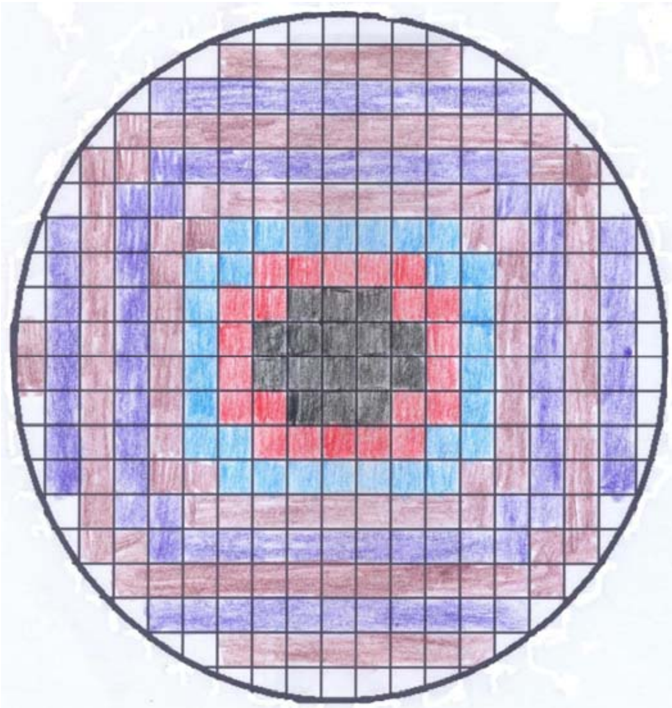
As the fuel in a solid rocket booster burns, it produces gas that exits the nozzle at very high pressure. This produces the thrust needed to launch a rocket. The area under combustion is a hollow core along the long axis of the booster from top to bottom. Depending on the shape of this empty tube, different volumes of gas will be produced from second to second, leading to different patterns of thrust for the rocket during its flight. The curve that describes a rocket engine's 'thrust versus time' is called the **thrust curve**. The more volume of fuel that is burned, the more thrust is produced.

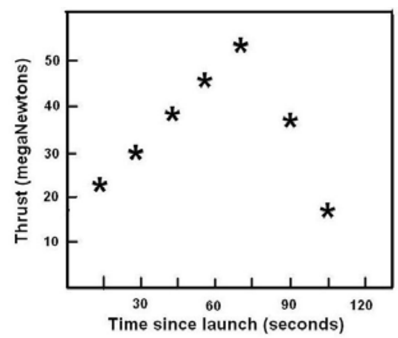




Problem 1 - The grid shows the cross section of a proposed SRB that is a cylinder 80-meters tall. The black squares represent the empty core region. As the fuel burns its way from the core to the outside circle, complete the shading of the rings of combustion at each 15-second time step. Count the number of shaded squares in each ring. The first two rings in red and blue are shown as an example. The graph represents the thrust curve and gives the number of squares shaded (red=22 and blue = 30) at each time step.

Problem 2 - How many seconds after launch does the SRB produce its maximum thrust?





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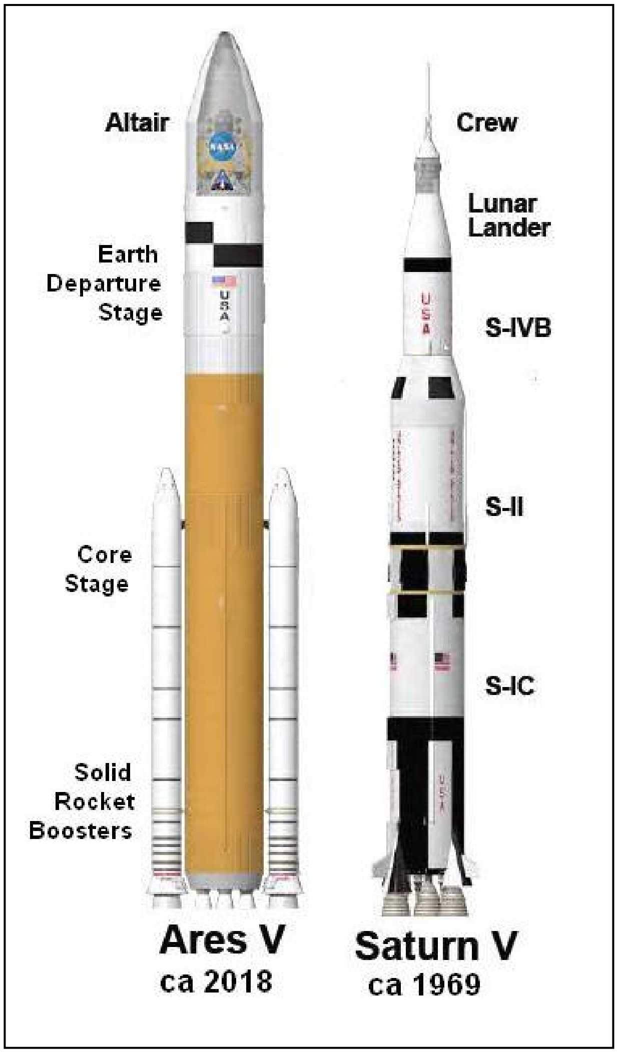
Answer: see above shaded rings. Note, the rings are only 1-box wide. Students shading may vary. Students may also estimate the areas of the partial boxes near the outer ring as an additional exercise in completing the graph.

Problem 2 - How many seconds after launch does the SRB produce its maximum thrust?

Answer: The thrust curve shows that the booster reaches a maximum thrust of about 54 megaNewtons about 75 seconds after launch.

Note to Teacher: Although the Ares-V rocket boosters are based on a 'star' shaped empty core pattern, it will reach its maximum thrust about 80 seconds after launch. Students may also experiment with other shapes for the empty core region (square, hexagon, triangle) and see what other thrust curves they can produce by the square-counting exercise.

The Ares-V Cargo Rocket



The Ares-V rocket, now being developed by NASA, will weigh 3,700 tons at lift-off, and be able to ferry 75 tons of supplies, equipment and up to 4 astronauts to the moon. As a multi-purpose launch vehicle, it will also be able to launch complex, and very heavy, scientific payloads to Mars and beyond. To do this, the rockets on the Core Stage and Solid Rocket Boosters (SRBs) deliver a combined thrust of 47 million Newtons (11 million pounds). For the rocket, let's define:

T(t) = thrust at time-t
m(t) = mass at time-t
a(t) = acceleration at time-t

so that:

$$a(t) = \frac{T(t)}{m(t)}$$

The launch takes 200 seconds. Suppose that over the time interval [0,200], T(t) and m(t) are approximately given as follows:

$$T(x) = 8x^3 - 16x^2 - x^4 + 47$$

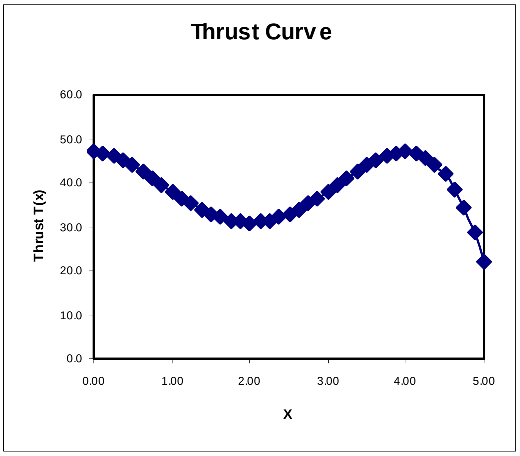
$$m(x) = 35 - x^2$$

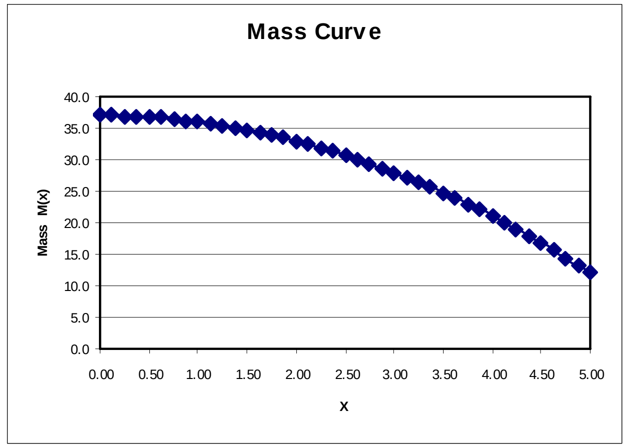
where t = 40 x

Problem 1 - Graph the thrust curve T(x), and the mass curve m(x) and find all minima, maxima inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 2 - Graph the acceleration curve a(x) and find all maxima, minima, inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? How many seconds will this be after launch? You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

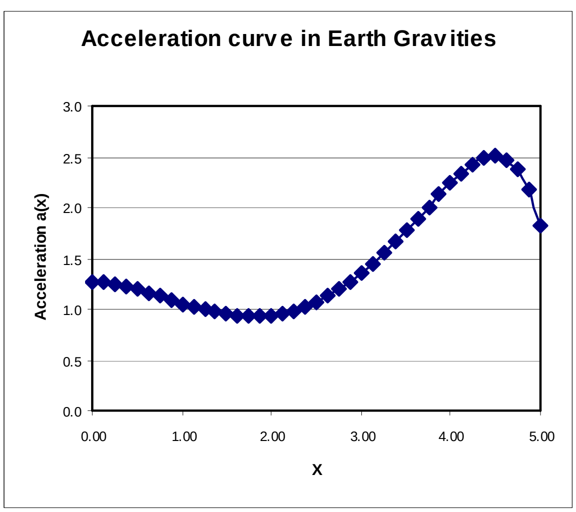




Problem 1 - The above graphs show T(x) and m(x) graphed with Excel. Similar graphs will be rendered using a graphing calculator. For the thrust curve, T(x), the relative maxima are at (0, 47) and (4,47). The relative minimum is at (2,31).

Problem 2 - For the mass curve, M(x), the absolute maximum is at (0,37).

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? You may use a graphing calculator, or Excel spreadsheet, or differential calculus. How many seconds will this be after launch?

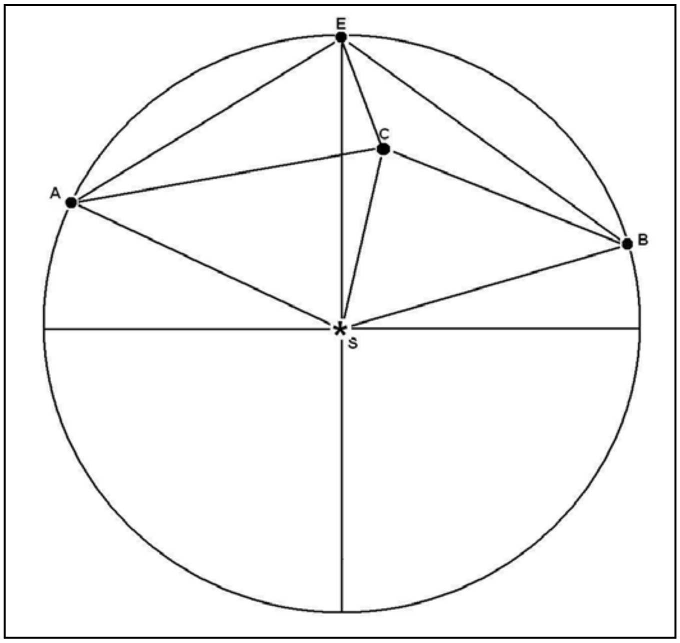


Answer: The curve reaches its maximum acceleration near (4.5,2.5). Because t = 40 X, this occurs about 40 x 4.5 = 180 seconds after launch. ***Note to teacher:*** *The units for acceleration are in Earth Gravities (1 G = 9.8 meters/sec^2) so astronauts will feel approximately 2.5 times their normal weight at this point in the curve.* U sing the Quotient Rule in differential calculus, and setting da(x)/dx = 0 we get:

$$0 = \frac{2x^5 - 8x^4 - 140x^3 + 840x^2 - 1026x}{(35 - x^2)^2}$$

Although after factoring out 'x' from the numerator we see that X=0 is a trivial solution, the locations of the remaining two extrema near x=1.5-2.0 and x=4.0-5.0 have to be found by solving a 4th-order equation. A graphing calculator can be used to find these two points at x ~ 1.8 and x ~ 4.56 where da/dt ~ 0. Since t = 40x, we see that the <u>absolute maximum</u> acceleration occurs near t = 4.56 x 40 = 182 seconds after launch.

Seeing Solar Storms in STEREO - I



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical unit (150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. The CME is ejected from the sun at the angle mESC=14 degrees.

Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:

SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Answer Key

Givens from satellite orbits:

SB = SA = SE = 150 million km   AE = 136 million km   BE = 122 million km

mASE = 54 degrees   mBSE = 48 degrees

mEAS = 63 degrees   mEBS = 66 degrees   mAEB = 129 degrees

use units of megakilometers i.e. 150 million km = 150 Mkm.

Method 1: Students construct a scaled model of the diagram based on the angles and measures, then use a protractor to measure the missing angles, and from the scale of the figure (in millions of kilometers per millimeter) they can measure the required segments. Segment EC is about 49 Mkm at an angle, mSEB of 28 degrees.

Method 2 use the Law of Cosines and the Law of Sines to solve for the angles and segment lengths exactly.

mEAC = mEAS - mSAC = 63 - 45 = 18 degrees

mASC = mASE + mESC = 54 + 14 = 68 degrees

mASB = mASE + mBSE = 54 + 48 = 102 degrees

mCSB = mASB - mASE - mESC = 102 - 54 - 14 = 34 degrees

SC from Law of Sines: Sin(45)/SC = Sin(67)/150Mkm so SC = 115 Mkm.

CB from Law of Cosines: $CB^2 = 115^2 + 150^2 - 2(115)(150)\cos(34)$ so CB = 84 Mkm

mEBC = mEBS - mSBC = 66 - 50 = 16 degrees

EC from law of Cosines: $EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(16)$

EC = 47 Mkm.

mCEB from Law of Cosines: $84^2 = 122^2 + 47^2 - 2(122)(47)\cos(mCEB)$

mCEB = 29 degrees

And since mAES = 180 - mASE - mEAS = 180 - 54 - 63 = 63 degrees

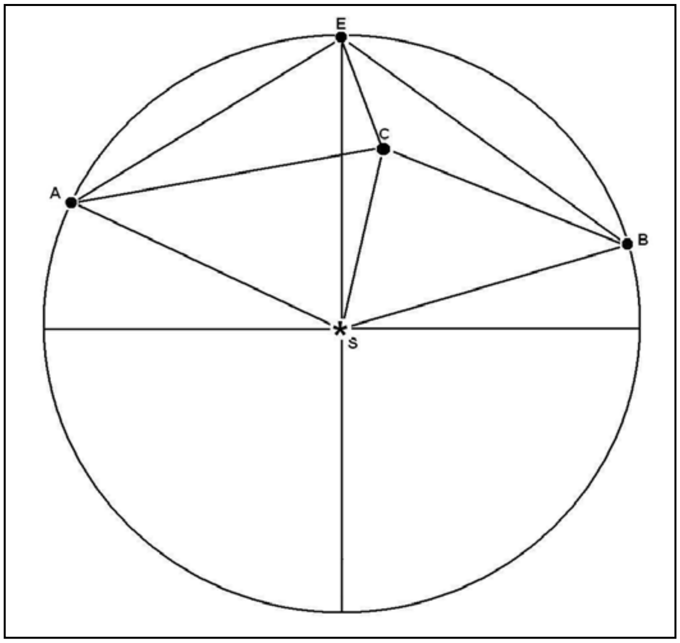
so mSEC = mAEB - mAES - mCEB = 129 - 63 - 29 = 37 degrees

So, the two satellites are able to determine that the CME is 49 million kilometers from Earth and approaching at an angle of 37 degrees from the sun.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Answer: 115 million kilometers / 2 million km/hr = 58 hours or 2.4 days.

Seeing Solar Storms in STEREO - II



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

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Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

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Method 2 use the Law of Cosines and the Law of Sines to solve for the angles and segment lengths exactly.

mASB = mASE+mBSE = 102 degrees
mASC = theta
mACS = 360 - mCAS - mASC = 315 - theta
mBSC = mASB - theta = 102 - theta
mBCS = 360 - mCBS - mBSC = 208 + theta

Use the Law of Sines to get

sin(mCAS)/L = sin(mACS)/150 Mkm   and   sin(mBCS)/L = sin(mBCS)/150 Mkm.

Eliminate L :   150sin(45)/sin(315-Theta) = 150 sin(50)/sin(208+theta)

Re-write using angle-addition and angle-subtraction:

sin50 [sin(315)cos(theta)-cos(315)sin(theta)] = sin(45) [sin(208)cos(theta) + cos(208)sin(theta)]

Compute numerical factors by taking indicated sines and cosines:

-0.541cos(theta) - 0.541sin(theta) = -0.332cos(theta) - 0.624sin(theta)

Simplify: cos(theta) = 0.397sin(theta)

Use definition of sine: $\cos(\theta)^{\mathbf{2}}$ = 0.158 (1 - $\cos(\theta)^{\mathbf{2}}$)

Solve for cosine: cos(theta) = $(0.158/1.158)^{1/2}$ so theta = 68 degrees. And so mASC=68

Now compute segment CS = 150 sin(45)/sin(315-68) = 115 Mkm.

BC = 115 sin(102-68)/sin(50) = 84 Mkm.

Then $EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(mEBS-mCBS)$

$EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(66-50)$ So EC = 47 Mkm.

mCEB from Law of Cosines: $84^2 = 122^2 + 47^2 - 2(122)(47)\cos(mCEB)$ so mCEB = 29 degrees

And since   mAES = 180 - mASE - mEAS = 180 - 54 - 63 = 63 degrees

so mSEC = mAEB - mAES - mCEB = 129 - 63 - 29 = 37 degrees

So, the two satellites are able to determine that the CME is 49 million kilometers from Earth and approaching at an angle of 37 degrees from the sun.

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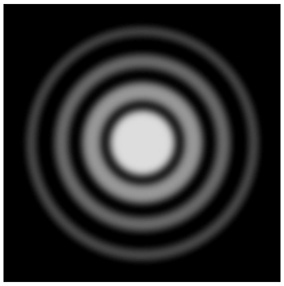
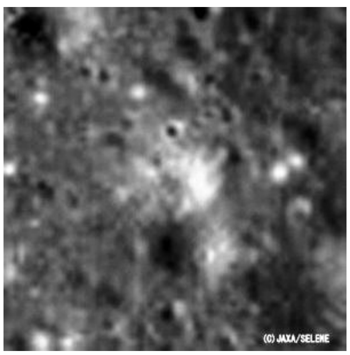
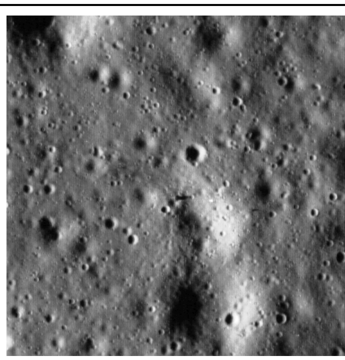
Answer: 115 million kilometers / 2 million km/hr = 58 hours or 2.4 days.

$$R = 1.22\frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (0.5 meters/pixel at a 50km orbit elevation: aperture =). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer Key

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = 0.14 degrees. And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = 8.4 arcminutes.

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is 0.069 microradians. B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = 14 milliarcseconds.

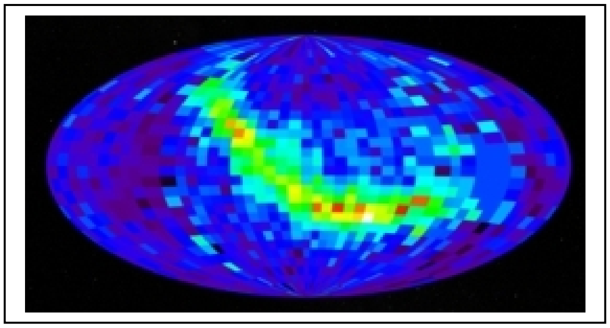
Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/(0.0000048 radians) = 5.1 meters.



$$K.E. = \frac{1}{2} mV^2$$

NASA's IBEX satellite has detected fast-moving atoms streaking into the solar system from interstellar space. The energetic neutral atoms (called ENAs) are created in an area of our solar system known as the interstellar boundary region. This region is where charged particles from the sun, called the solar wind, flow outward far beyond the orbits of the planets and collide with material between stars.

The NASA Press release says that the ENAs "…travel inward toward the sun from interstellar space at speeds from 100,000 mph to more than 2.4 million mph."

Question: How do scientists know the speeds of these particles?

Answer: It's all about Kinetic Energy!

The IBEX satellite detects energetic neutral atoms with a kinetic energy (K.E.) of 1,000 electron volts (1 keV), where 1 electron volt (eV) equals 1.6 x 10^{-19} Joules of energy. In the formula above, with KE expressed in Joules and the particle mass, m, expressed in kilograms, the speed of the particle, V, will be in meters/sec.

Problem 1 - What is the formula for the particle speed, V, in terms of the particle's mass and kinetic energy?

Problem 2 - Show that, if the particle is a proton with a mass of 1.7 x 10^{-27} kg and it has a speed of 450 kilometers/sec, to two significant figures, its energy is A) 1.7 x 10^{-16} Joules, or B) 1,100 eV.

Problem 3 - The IBEX satellite measures ENAs with an energy of 1 keV in order to make the image shown above. The most common element in the universe is hydrogen (1 proton). If the detected ENAs are all protons, what is the speed of the protons, in kilometers/sec, detected by IBEX to two significant figures?

Problem 1 - What is the formula for the particle speed, V, in terms of the particle's mass and kinetic energy? Answer:

$$E = \frac{1}{2}mV^2$$

$$so$$

$$V = \sqrt{\frac{2E}{m}}$$

Problem 2 - Show that, if the particle is a proton with a mass of 1.7 x 10^{-27} kg and it has a speed of 450 kilometers/sec, to two significant figures its energy is A) 1.7 x 10^{-16} Joules, or B) 1,076 eV.

Answer A) In order to use the formula for K.E., we have to convert km/s to meters/sec, so 450 km/s x (1000 m/km) = 450,000 m/s, then K.E = 1/2 (1.7 x 10^{-27} kg) x (4.5 x 10^{5} m/s)^2 = 1.7 x 10^{-16} Joules. B) Since 1 eV = 1.6 x $10^{\mathbf{-19}}$ Joules we have
1.7 x 10^{-16} Joules x (1 eV/1.6 x $10^{\mathbf{-19}}$ Joules) = 1,063 eV
which to two significant figures is 1,100 eV.

Problem 3 - The IBEX satellite measures ENAs with an energy of 1 keV in order to make the image shows above. The most common element in the universe is hydrogen (1 proton). If the detected ENAs are all protons, what is the speed of the protons in kilometers/sec, detected by IBEX to two significant figures?

Answer: 1 keV = 1,000 eV. The equivalent energy in Joules is 1,000 eV x (1.6 x $10^{\mathbf{-19}}$ Joules/eV) = 1.6 x $10^{\mathbf{-16}}$ Joules. The mass of a proton is 1.7 x 10^{-27} kg, so from the formula derived in Problem 1,

$$V = \sqrt{\frac{2\text{x}(1.6\text{x}10^{-16})}{1.7\text{x}10^{-27}}}$$

$$V = \sqrt{1.88x10^{11}}$$

So V = 433,861 m/s by the calculator display, or to 2 significant figures we get 430,000 m/s or 430 kilometers/sec.

Note: 430 km/sec is equivalent to 960,000 miles/hour or nearly 1 million miles per hour. Also, IBEX measures higher-energy ENAs with energies up to 6 keV, which corresponds to protons with speeds of 2,400,000 million miles/hour as stated in most press releases.

IBEX Press Release: http://www.nasa.gov/mission_pages/ibex/index.html

At 11:30 AM EST, NASA successfully launched the Ares 1-X rocket from Cape Canaveral. The top image is an artists illustration of the launch and the bottom photo shows the actual launch from Pad 39B. The flight reached the target sub-orbital altitude of 150,000 feet. The next launch will be in March 2014 of Ares 1-Y.
(Images courtesy NASA)

The first half of the flight just before the rocket reached its peak altitude was powered by the first stage. Once the first stage was jettisoned 150 seconds after launch, the capsule traveled under its own inertia under the influence of Earth's gravity. The capsule reached a maximum altitude of 28 miles (45 km) at a point 40 miles (64 km) downrange from the launch pad, traveling at a horizontal speed of 4000 mph (6400 km/hr). We can approximate the capsule's path by a portion of a parabolic arc.

The time that it takes a body to fall to the ground is given by

$$H = \frac{1}{2} g T^2$$

where H is the altitude in meters, T is the time in seconds and g is the acceleration of gravity given by g = 9.8 meters/sec^2.

Problem 1 - To two significant figures, how many seconds did it take the Ares 1-X capsule to fall from an altitude of 45 kilometers?

Problem 2 - At the horizontal speed that the capsule was traveling, and to two significant figures, how far from the launch pad did it come to earth if the capsule reached its maximum altitude 64 kilometers downrange from the launch pad?

Problem 1 - To two significant figures, how many seconds did it take the Ares 1-X capsule to fall from an altitude of 45 kilometers?

Answer: H = 45 kilometers or 45,000 meters
g = 9.8 meters/$\sec^2$

And $$H = \frac{1}{2} g T^2$$

So solving for T we get

$$T = \sqrt{\frac{2H}{g}}$$

$$T = \sqrt{\frac{2x45000}{9.8}}$$

$$T = \sqrt{9184}$$

T = 95.833
T = 96 seconds to 2 significant figures.

Problem 2 - At the horizontal speed that the capsule was traveling, and to two significant figures, how far from the launch pad did it come to earth if the capsule reached its maximum altitude 64 kilometers downrange from the launch pad?

Answer: The time taken was 96 seconds which is 96 sec x (1 hour/3600 sec) = 0.027 hours. Then V = 6,400 km/hour and the initial distance was 40 kilometers, so in 96 seconds, the capsule lands

X = x0 + VT

X = 64 km + (6400 km/hr) x (0.027 hours)

X = 237 kilometers (or 147 miles) downrange

To 2 significant figures, this becomes 240 kilometers or 150 miles downrange from the launch pad.

The neat thing about ballistic problems (flying baseballs or rockets) is that their motion in the vertical dimension is independent of their motion in the horizontal dimension. This means we can write one equation that describes the movement in time along the x axis, and a second equation that describes the movement in time along the y axis. In function notation, we write these as y(t) and y(t) where t is the independent variable representing time.

To draw the curve representing the trajectory, we have a choice to make. We can either create a table for X and Y at various instants in time, or we can simply eliminate the independent variable, t, and plot the curve y(x).

Problem 1 - The Ares 1X underwent powered flight while its first stage rocket engines were operating, but after it reached the highest point in its trajectory (apogee) the Ares 1X capsule coasted back to Earth for a water landing. The parametric equations that defined its horizontal downrange location (x) and its altitude (y) in meters are given by

$$x(t) = 64,000 + 1800t$$

$$y(t) = 45,000 - 4.9t^2$$

Using the method of substitution, create the new function y(x) by eliminating the variable t.

Problem 2 - Determine how far downrange from launch pad 39A at Cape Canaveral the capsule landed, (y(x)=0), giving your answer in both meters and kilometers to two significant figures.

Problem 3 - Why is it sometimes easier not to work with the parametric form of the motion of a rocket or projectile?

Problem 1 - Answer: The parametric equations that defined its horizontal downrange location (x) and its altitude (y) in meters are given by

$$x(t) = 64,000 + 1800t$$
$$y(t) = 45,000 - 4.9t^2$$

We want to eliminate the variable, t, from y(t) and we do this by solving the equation x(t) for t and substituting this into the equation for y(t) to get y(t(x)) or just y(x).

$$t = \frac{x - 64,000}{1800}$$

$$y = 45,000 - 4.9\left[\frac{x - 64,000}{1800}\right]^2$$

$$y = 38,800 + 0.19x - 0.0000015x^2$$

Problem 2 - Determine how far downrange from the launch pad the capsule landed, (y(x)=0), giving your answer in both meters and kilometers to two significant figures. Answer: Using the Quadratic Formula, find the two roots of the equation y(x)=0, and select the root with the largest positive value.

$$x_{1,2} = \frac{-0.19 \pm \sqrt{0.036 - 4(-0.0000015)38800}}{2(-0.0000015)} \text{ meters}$$

so x1 = -109,000 meters or -109 kilometers
x2 = +237,000 meters or 237 kilometers.

The second root, x2, is the answer that is physically consistent with the given information. Students may wonder why the mathematical model gives a second answer of -109 kilometers. This is because the parabolic model was only designed to accurately represent the physical circumstances of the coasting phase of the capsule's descent from its apogee at a distance of 64 kilometers from the launch pad. Any extrapolations to times and positions earlier than the moment of apogee are 'unphysical'.

Problem 3 - Why is it sometimes easier not to work with the parametric form of the motion of a rocket or projectile? Answer: In order to determine the trajectory in space, you need to make twice as many calculations for the parametric form than for the functional form y(x) since each point is defined by (x(t), y(t)) vs (x, y(x)).

Potential energy is the energy that a body possesses due to its **location** in space, while kinetic energy is the energy that it has depending on its **speed** through space. For locations within a few hundred kilometers of Earth's surface, and for speeds that are small compared to that of light, we have the two energy formulae:

$$P.E = mgh \qquad K.E = \frac{1}{2}mV^2$$

where g is the acceleration of gravity near Earth's surface and has a value of 9.8 meters/sec^2. If we use units of mass, **m**, in kilograms, height above the ground, **h**, in meters, and the body's speed, **V**, in meters/sec, the units of energy (P.E and K.E.) are Joules.

As a baseball, a coasting rocket, or a stone dropped from a bridge moves along its trajectory back to the ground, it is constantly exchanging, joule by joule, potential energy for kinetic energy. Before it falls, its energy is 100% P.E, while in the instant just before it lands, its energy is 100% K.E.

Problem 1 - A baseball with m = 0.145 kilograms falls from the top of its arc to the ground; a distance of 100 meters. A) What was its K.E., in joules, at the top of its arc? B) What was the baseball's P.E. in joules at the top of the arc?

Problem 2 - The Ares 1-X capsule had a mass of 5,000 kilograms. If the capsule fell 45 kilometers from the top of its trajectory 'arc', how much kinetic energy did it have at the moment of impact with the ground?

Problem 3 - Suppose that the baseball in Problem 1 was dropped from the same height as the Ares 1-X capsule. What would its K.E. be at the moment of impact?

Problem 4 - From the formula for K.E. and your answers to Problems 2 and 3, in meters/sec to two significant figures; A) What was the speed of the baseball when it hit the ground? B) What was the speed of the Ares 1-X capsule when it landed? C) Discuss how your answers do not seem to make 'common sense'.

Problem 5 - Use the formulae for P.E. and K.E. to explain your answers to Problem 4. (Hint: Don't worry about air resistance!!)

Problem 1 - A baseball with m = 0.145 kilograms falls from the top of its arc to the ground; a distance of 100 meters. A) What was its K.E., in joules, at the top of its arc? B) What was the baseball's P.E. in joules at the top of the arc?
Answer: A) K.E = 0 B) P.E. = mgh = (0.145) x (9.8) x (100) = 142 joules.

Problem 2 - The Ares 1-X capsule had a mass of 5,000 kilograms. If the capsule fell 45 kilometers from the top of its trajectory 'arc', how much kinetic energy did it have at the moment of impact with the ground?
Answer: At the ground, the capsule has exchanged all of its potential energy for 100% kinetic energy so K.E. = P.E. = mgh. Then K.E. = (5,000 kg) x (9.8) x (45,000 meters) = 2.2 billion joules for the Ares 1-X capsule.

Problem 3 - Suppose that the baseball in Problem 1 was dropped from the same height as the Ares 1-X capsule. What would its K.E. be at the moment of impact?
Answer: its K.E. would equal 100% of its original P.E. so K.E = mgh = (0.145 kg) x (9.8) x (45,000 meters) = 64,000 joules for the baseball.

Problem 4 - From the formula for K.E. and your answers to Problems 2 and 3, in meters/sec to two significant figures; A) What was the speed of the baseball when it hit the ground? B) What was the speed of the Ares 1-X capsule when it landed? C) Discuss how your answers do not seem to make 'common sense'.

Answer; A) Baseball: $E = 1/2\ m\ V^{2}$

$V = (2E/m)^{1/2}$

$= (2(64000)/0.145))^{1/2}$

= 940 meters/sec.

B) Capsule: $V = (2\ (2{,}200{,}000{,}000)/5{,}000)^{1/2}$

= 940 meters/sec

C) Our intuition suggests that the much heavier Ares 1-X capsule should have struck the ground at a far-faster speed!

Problem 5 - Use the formulae for P.E. and K.E. to explain your answers to Problem 4.
Answer: The P.E. that it starts with will be equal to the K.E when it lands, so P.E. = K.E. Comparing the formulae, we see that

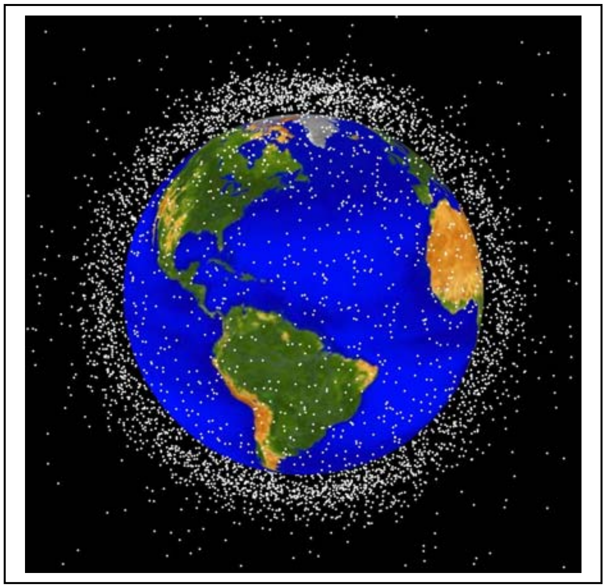
$$mgh = \frac{1}{2} mV^2$$

And since, by algebra, the variables representing mass, m, cancel from both sides, we get

$$gh = \frac{1}{2} V^2$$

so the speed upon impact, V, depends only upon the constant g, and the height from which the body fell, h. Q.E.D.

Note: The above discussion is modified slightly when air resistance is considered.



We live at the bottom of a deep 'gravity well' that takes an enormous amount of energy to climb out of. The figure shows the locations of 12,000 currently in Low Earth Orbit (LEO). To reach any altitude above sea level, we have to work against gravity. The higher we want to climb, the more energy we need to use to reach that altitude.

We can measure this energy in terms of the number of joules per kilogram (J/kg) that is needed to reach an elevation of h meters. The formula is given very simply by

$$E = 9.8\ h \quad \text{J/kg}$$

Problem 1 - A mountain climber hikes from sea level to the top of Mt Everest, which has an altitude of 8,848 meters. A) How many Joules/kg will he need for the trip? B) If his total mass is 75 kg, how many Joules of energy will he have to expend to reach the summit?

Problem 2 - The Ares 1-X rocket was launched on Wednesday, October 28 at Cape Canaveral. If the payload reached a maximum altitude of 45 kilometers, A) how many megaJoules/kg were needed for the payload to reach this altitude? B) If the payload mass was 5,000 kg, how many megaJoules were required?

Problem 3 - At what altitudes will the mountain climber and the Ares 1-X need to expend the same number of Joules/kg?

Problem 4 - How many megaJoules/kg will the Ares 1-X need to expend to reach Low Earth Orbit at an altitude of 200 miles? (1 mile = 1.62 kilometers).

Problem 5 - In terms of megaJoules/kg, by what factor is the Ares 1-X energy requirement higher to get into LEO than to reach an altitude of 45 kilometers?

Problem 1 - A mountain climber hikes from sea level to the top of Mt Everest, which has an altitude of 8,848 meters. A) How many Joules/kg will he need for the trip? B) If his total mass is 75 kg, how many Joules of energy will he have to expend to reach the summit?

Answer; A) E = 9.8 x 8848 = 87,000 J/kg.

B) E = 87,000 J/kg x (75 kg) = 6,500,000 Joules.

Problem 2 - The Ares 1-X rocket was launched on Wednesday, October 28 at Cape Canaveral. If the payload reached a maximum altitude of 45 kilometers, A) how many megaJoules/kg were needed for the payload to reach this altitude? B) If the payload mass was 5,000 kg, how many megaJoules were required?

Answer: A) E = 9.8 x 45 km x (1000 m/km) = 440,000 J/kg.
B) E = 440,000 J/kg x (5,000 kg)
= 2,200,000,000 Joules = 2,200 megaJoules.

Problem 3 - At what altitudes will the mountain climber and the Ares 1-X need to expend the same number of Joules/kg?

Answer: Because E = 9.8 x h and does not depend on the mass of the body, the mountain climber and the Ares 1-X payload will need the SAME number of Joules/kg to reach all elevations that they have in common up to 8,848 meters.

Problem 4 - How many megaJoules/kg will the Ares 1-X need to expend to reach Low Earth Orbit at an altitude of 200 miles? (1 mile = 1.62 kilometers).

Answer: First we have to convert 200 miles to meters:

200 miles x (1.61 km/mile) x (1000 meters/1 km) = 322,000 meters.

Then E = 9.8 x (322,000) = 3,200,000 Joules/kg or 3.2 megaJoules/kg.

Problem 5 - In terms of megaJoules/kg, by what factor is the energy requirement for Ares 1-X higher to get into LEO than to reach an altitude of 45 kilometers?

Answer: To reach 45 km requires 0.44 megaJoules/kg while for LEO it requires 3.2 megaJoules/kg so the energy factor per kilogram of payload mass is 3.2/0.44 = 7.3 times higher.

Teacher Note: The unit Joules/kg is also called the 'energy density'. Different fuels have different energy densities. A peanut butter and jelly sandwich (500 calories and 0.1 kg) has E = 21,000 Joules/kg, while solid rocket fuel has E = 5 megaJoules/kg!

Energy is measured in a number of ways depending on what property is being represented.

Total Energy - Joules and ergs - The total amount of energy in various forms (kinetic, potential, magnetic, thermal, gravitational)

Power - Watts, Joules/second or ergs/second – the rate at which energy is produced or consumed in time. Power = Energy/Time

Flux - Watts/meter^2, Joules/sec/meter^2 or ergs/sec/meter^2 – the rate with which energy flows through a given area in given amount of time: Flux=Power/Area

1 Joule = 10 million ergs
1 Watt = 1 Joule/1 second
1 hour = 3600 seconds
1 kilowatt = 1,000 watts
1 megaJoule = 1,000,000 Joules
3 feet = 1.0 meters

Example: A 5-watt flashlight is left on for 1 hour: Convert its energy consumption of 5 watt-hours to Joules.

$$5 \text{ Watt-hours} \times \frac{1 \text{ Joule}}{1 \text{ sec } 1 \text{ watt}} \times \frac{3{,}600 \text{ sec}}{1 \text{ hour}} = 18{,}000 \text{ Joules}$$

Notice how the compound unit 'watt' is handled so that the appropriate colored units cancel. The canceling units are color-coded for convenience

Problem 1 – The flux of sunlight at Earth's surface is 1300 Watts/meter^2. Convert this flux to ergs/sec/cm^2.

Problem 2 – A 100-watt bulb shines light over a wall with a surface area of 25 meters^2. What is the flux of light energy in Joules/sec/meter^2?

Problem 3 – The common energy unit for electricity is the watt-hour (Wh), which can be written as 1 watt x 1 hour. How many megajoules equal 1 kilowatt-hour (1 kWh)?

Problem 4 – How many ergs of energy are collected from a solar panel on a roof, if the sunlight provides a flux of 300 Joules/sec/meter^2, the solar panels have an area of 27 square feet, and are operating for 8 hours during the day?

Problem 1 –

Answer: $$1300 \ \frac{\text{Watts}}{\text{meter}^2} \times \frac{\text{10 million ergs}}{\text{sec Watt}} \times \frac{\text{1 meter}}{\text{100 cm}} \times \frac{\text{1 meter}}{\text{100 cm}} = \mathbf{1.3 \times 10^6} \ \frac{\textbf{ergs}}{\textbf{sec cm}^{\mathbf{2}}}$$

Problem 2 –

Answer: $$\text{100 watts} \times \left(\frac{\text{1 Joule}}{\text{1 sec watt}}\right) \times \frac{1}{\text{25 meters}^2} = \mathbf{4.0} \ \frac{\textbf{Joules}}{\textbf{sec meters}^{\mathbf{2}}}$$

Problem 3 –

Answer: $$\text{1 kilowatt-hour} \times \frac{\text{1 Joule}}{\text{1 sec 1 Watt}} = 1{,}000 \ \frac{\text{Joules}}{\text{sec}} \times \text{1 hour}$$

$$1{,}000 \ \frac{\text{Joules}}{\text{sec}} \times \text{1 hour} \times \left(\frac{\text{3,600 sec}}{\text{1 hour}}\right)$$

$$1{,}000 \ \frac{\text{Joules}}{\text{sec}} \times \text{3,600 seconds}$$

$$\text{3,600,000 Joules} \times \frac{\text{1 megaJoule}}{\text{1,000,000 Joules}} = \textbf{3.6 megaJoules}$$

Problem 4 -

Answer: Area of roof = $27 \text{ feet}^2 \times (\text{1 meter/ 3 feet}) \times (\text{1 meter/ 3 feet}) = 3 \text{ meter}^2$

Flux = Power/Area so Power = Flux x Area:

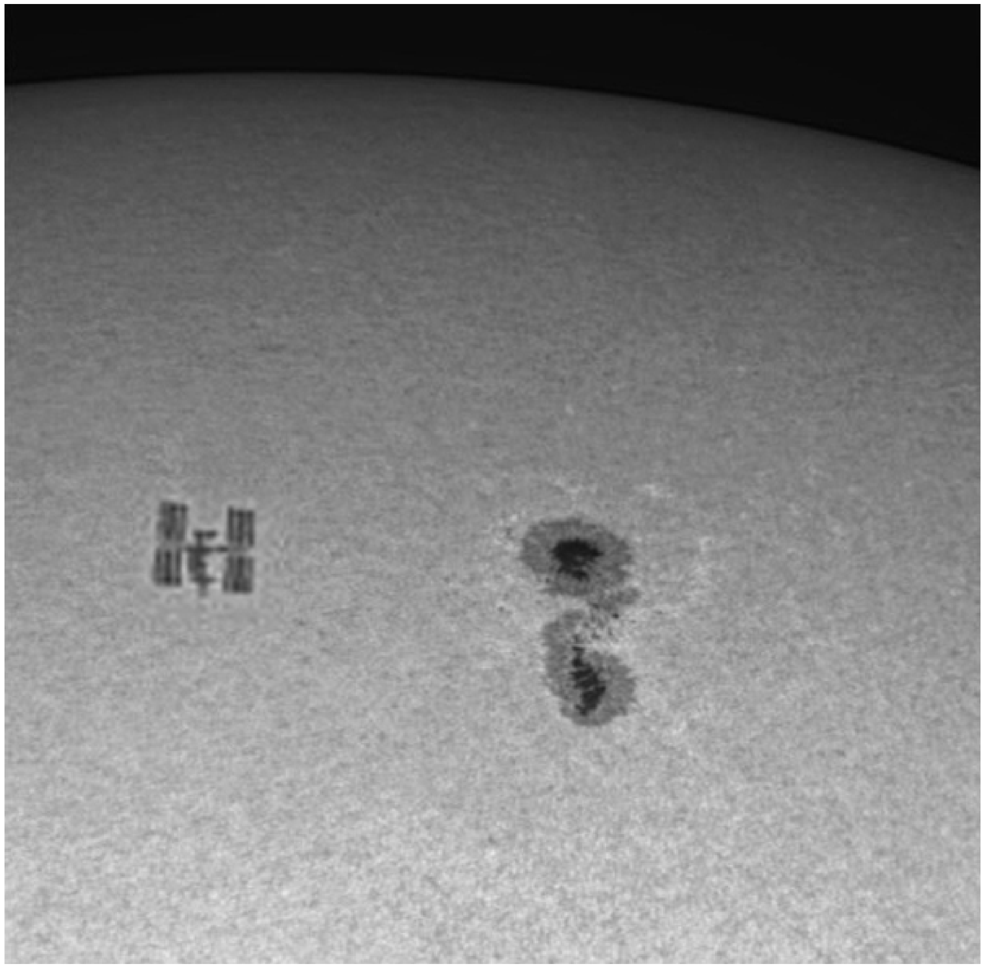
$$\text{Power} = 300 \ \frac{\text{Joules}}{\text{sec meter}^2} \times 3 \text{ meters}^2 = 900 \ \frac{\text{Joules}}{\text{sec}}$$

Energy = Power x Time

$$= 900 \ \frac{\text{Joules}}{\text{sec}} \times \left(\frac{\text{3,600 sec}}{\text{1 hour}}\right) \times \text{8 hours} = \text{25,920,000 Joules}$$

$$= \text{25,920,000 Joules} \times \left(\frac{\text{10 million ergs}}{\text{1 Joule}}\right) = \textbf{258.2 trillion ergs}$$

ISS and a Sunspot - Angular Scales



Photographer John Stetson took this photo on March 3, 2010 by carefully tracking his telescope at the right moment as the International Space Station passed across the disk of the sun.

The angular size, θ, in arcseconds of an object with a length of L meters at a distance of D meters is given by

$$\theta = 206265 \frac{L}{D}$$

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Answer: As viewed from the ground, the ISS subtends an angle of
Angle = 206265 x (108 meters/350,000 meters) so
Angle = 63 arcseconds.

At the distance of the sun, which is 150 million kilometers, the angular size of the ISS corresponds to a physical length of
L = 150 million kilometers x (63/206265) so
L = 46,000 kilometers.

The sunspot is comparable in width to that of the ISS and has a length about twice that of he ISS so its size is about **46,000 km x 92,000 km**.

As a comparison, Earth has a diameter of 13,000 km so the sunspot is about **3 times the diameter of Earth in width, and 6 times the diameter of Earth in length.**

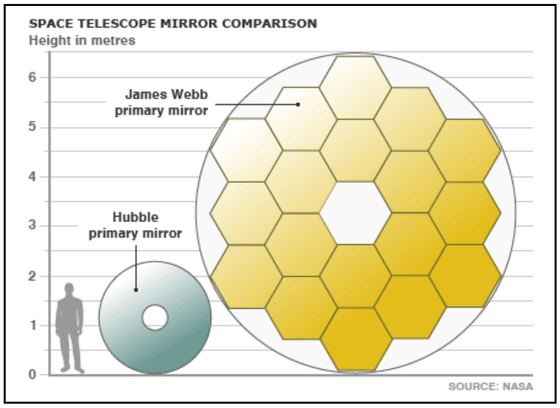
Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Answer: From the ground, convert the speed of the ISS in km/sec to an angular speed in arcseconds/sec.

In one second, the ISS travels 10 km along its orbit. From the ground this corresponds to an angular distance of
Angle = 206265 x (10 km/350 km)
= 5900 arcseconds.

The speed is then 5900 arcseconds/sec. The diameter of the sun is 0.5 degrees which is 30 arcminutes or 1800 arcseconds. To cover this angular distance, the ISS will take
T = 1800 arcseconds / (5900 arcseconds/s) so
T = 0.3 seconds!

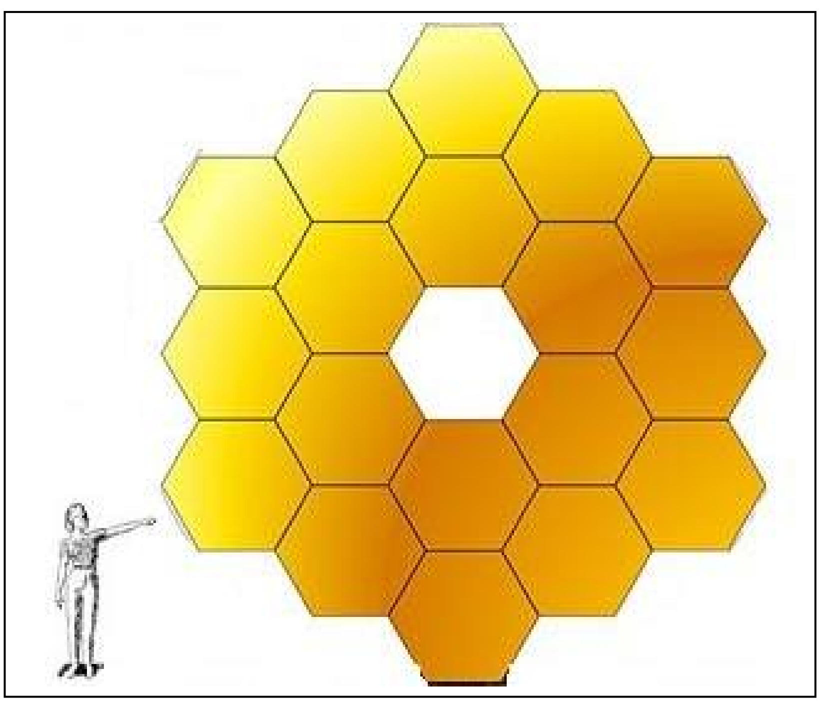
The Hexagonal Tiles in the Webb Telescope Mirror



The James Webb Space Telescope, to be launched by NASA in 2014, is a telescope designed to explore galaxies and stars that formed soon after the Big Bang.

Its unique design features a large mirror that consists of 18 hexagonal tiles; each tile is its own mirror. The 18 mirror tiles work together to form a single large mirror to collect faint star light.

An important feature of a telescope mirror is its surface area. The more surface area a mirror has, the more light it can collect. To make faint stars and galaxies appear bright enough to study in detail, mirrors with large collecting areas are needed.

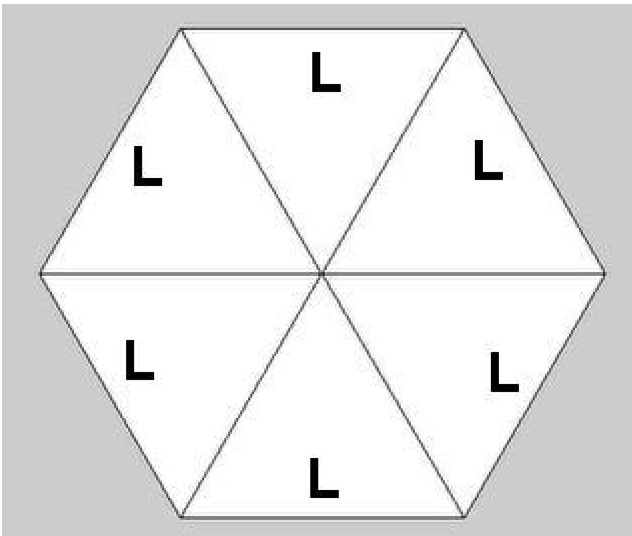


Problem 1 – For a regular hexagon with a side length of L, how many equilateral triangles with a side length of L can you fit into one hexagon?

Problem 2 – From the drawing of the Webb Space Telescope Mirror on the left, how many equilateral triangles are formed by the 18 hexagonal tiles?

Problem 1 – For a regular hexagon with a side length of L, how many equilateral triangles with a side length of L can you fit into one hexagon?

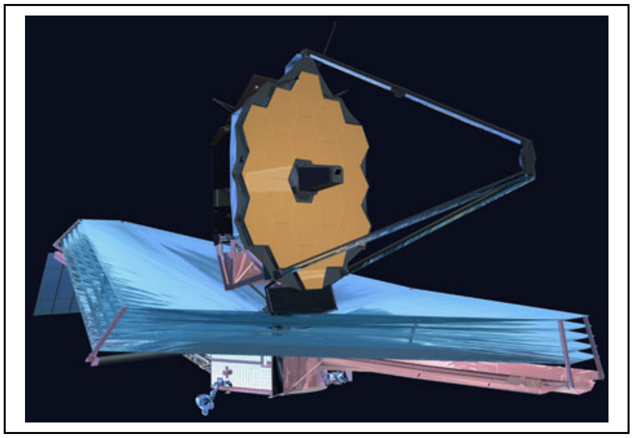
Answer: A total of six as shown below.



Problem 2 – From the drawing of the Webb Space Telescope Mirror on the left, how many equilateral triangles are formed by the 18 hexagonal tiles?

Answer. Students may draw the triangles inside each of the 18 hexagons and then count them, or can recognize that (6 triangles in each hexagon) x 18 hexagons = **108 equalateral triangles.**

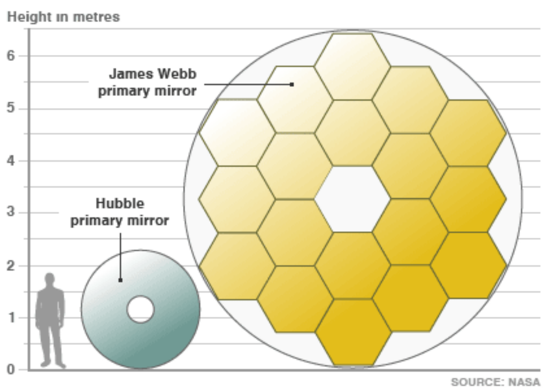
Scaling the Webb Space Telescope Mirror -III



The James Webb Space Telescope, to be launched by NASA in 2014, is a telescope designed to explore galaxies and stars that formed soon after the Big Bang. Its unique design features a large mirror that consists of 18 hexagonal tiles; each tile is its own mirror.

Many of the largest telescope mirrors now being built for ground-based observatories use the hexagonal 'segmented' design. A single 1-meter wide hexagon, replicated dozens of times, is a lot easier to make than a single large mirror!

Suppose that in the problems below, the length of a side of the hexagon is L = 0.76 meters. New mirror designs are created from the Webb Space Telescope design by adding enough mirror tiles to complete a new outer ring. For example, the Webb Space Telescope mirror consists of two complete rings of hexagonal tiles.



Problem 1 - Using the sketch to the left as a guide, how many tiles will be in the assembled mirror if 1, 2 or 3 additional rings of hexagonal tiles are added?

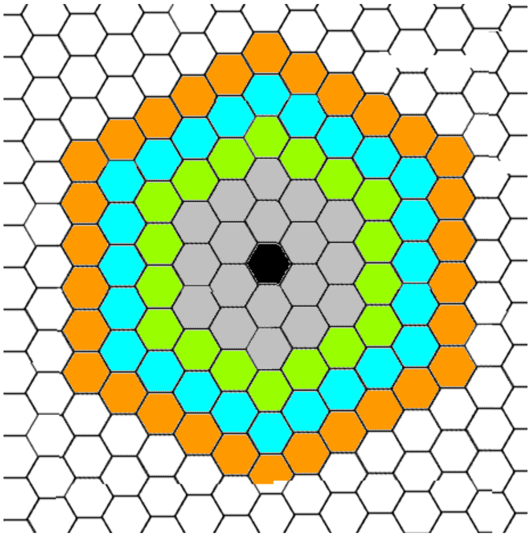
Problem 2 - What is the total area of each mirror design if the area of a single hexagon is

$$A = \frac{3}{2}\sqrt{3}L^2$$

Compared to the Webb Space Telescope design of 18 tiles, by what factor do the three new mirror designs exceed the Webb Space Telescope collecting area?

Problem 1 - Using the sketch to the left as a guide, how many tiles will be in the assembled mirror if 1, 2 or 3 additional rings of hexagonal tiles are added?

Answer: From the shaded rings indicated below: One additional ring (green) = 18 + 18 = **36 tiles**. Two rings = 36 + 24 = **60 tiles**. Three rings = 60 + 30 = **90 tiles**.



Problem 2 - What is the total area of each mirror design if the area of a single hexagon is

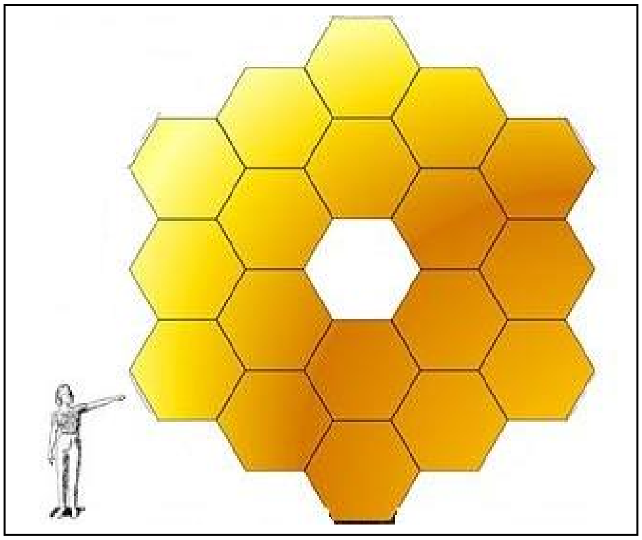
$$A = \frac{3}{2}\sqrt{3}L^2$$

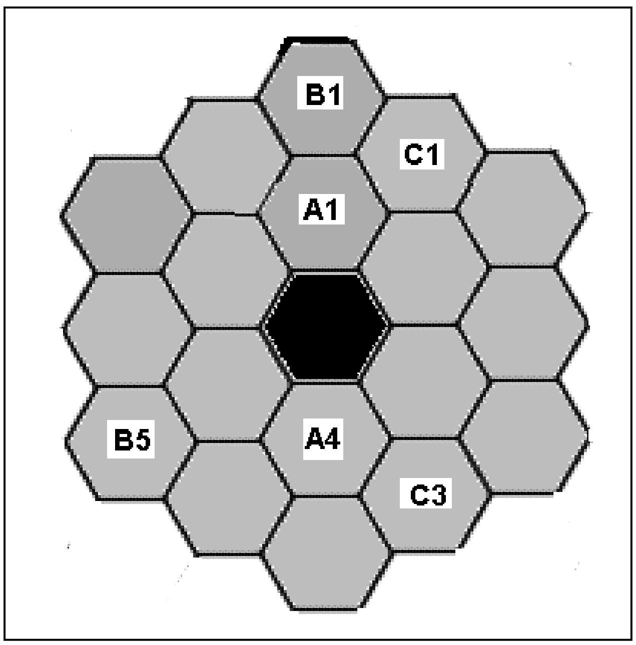
Compared to the Webb Space Telescope design of 18 tiles, by what factor do the three new mirror designs exceed the Webb Space Telescope collecting area?

Answer: The hexagon area is A = 1.5 (1.732) $(0.76)^2$ = 1.5 meters^2.
Webb Space Telescope Mirror: 18 tiles, Area = 18 x 1.5 = 27 meters^2.

One additional ring: Area = 36 x 1.5 = **54 meters^2.**
Two additional rings: Area= 60 x 1.5 = **90 meters^2.**
Three additional rings: Area = 90 x 1.5 = **135 meters^2**.

The areas increase by factors of **2.0, 3.3 and 5.0 times** the area of the Webb Space Telescope design.





The Webb Space Telescope segmented mirror consists of 18 hexagonal mirror tiles assembled to make a larger mirror just over 6 meters in diameter. The placement of these tiles is not random, however.

Tiles located at a specific distance from the center of the mirror are manufactured with exactly the same optical properties. For example, in the drawing to the left, each of the inner ring of 6 tiles has an identical 'twin' to each of the other tiles in this ring.

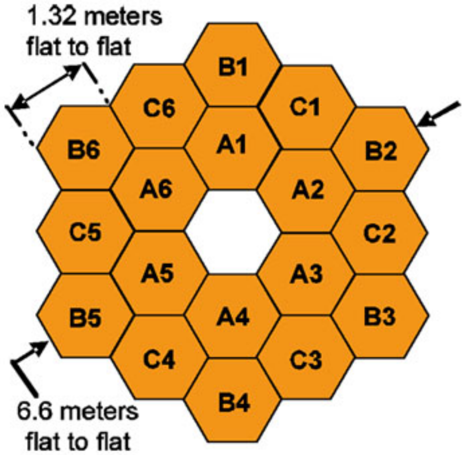
Another property of regular hexagons is that, when you rotate them by 60 degrees, the pattern looks identical to the one you started with. Suppose you labeled one of the six sides, Side A, and placed it at the top of the pattern. If you rotate the hexagon in steps of 60 degrees, it will take exactly 6 shifts to bring Side A back to the top of the pattern. The assembled Webb Space telescope mirror shows this same pattern among the tiles.

Problem 1 - Using the tile labeling shown above, find all other tiles that follow 6-fold symmetry and label them using the same scheme. How many classes can you identify, and how many tiles per class?

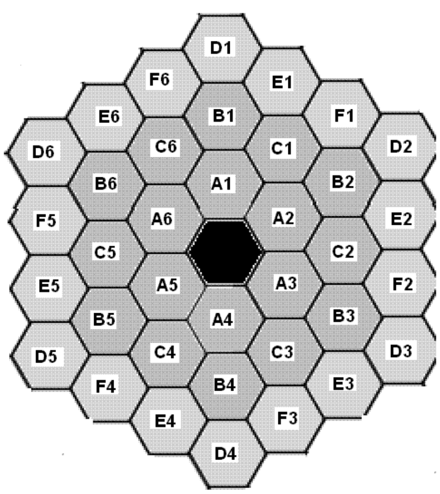
Problem 2 - If you were to add one more ring of hexagonal tiles to the outer edge of the Webb Space Telescope mirror, how many different kinds of mirror tiles would there be, and in each class, how many identical mirror tiles would be present with the same optical properties?

Problem 1 - Using the tile labeling shown above, find all other tiles that follow 6-fold symmetry and label them using the same scheme. How many classes can you identify, and how many tiles per class?

Answer: See figure below. **There are exactly three different tile classes, A, B and C, and 6 tiles per class.**



Problem 2 - If you were to add one more ring of hexagonal tiles to the outer edge of the Webb Space Telescope mirror, how many different kinds of mirror tiles would there be, and in each class, how many identical mirror tiles would be present with the same optical properties?



There are 6 unique mirror classes (A, B, C, D, E, F) and six identical mirror per class. Each mirror class is at a unique distance from the center of the mirror and has the same optical properties. This means that A1 can be interchanged with A5, but that A1 cannot be swapped for B1, C1 etc.

The Solar Probe Plus mission will be launched in 2018 for a rendezvous with the sun in 2024. To lose enough energy to reach the sun, the spacecraft will make seven fly-bys of Venus.

As the spacecraft approaches the sun, its heat shield must withstand temperatures exceeding 2500 F and blasts of intense radiation.

The 480 kg spacecraft, costing $740 million, will approach the sun to a distance of 6 million km. Protected by the heat shield will be five instruments that will peak over the edge of the heat shield and measure the particles and radiation fields in the outer solar corona.

A simple formula that predicts the temperature, in Kelvins, of a surface exposed to solar radiation is given by

$$T = 396\frac{(1-A)^{\frac{1}{4}}}{\sqrt{R}}$$

where R is the distance to the solar surface in Astronomical Units, and A is the fraction of the incoming radiation that is reflected by the surface. Because only the amount of absorbed sunlight determines how hot a body becomes, the final temperature depends on the quantity (1 – A) rather than A alone. (Note 1 Astronomical Unit is the distance of Earth from the center of the sun; 147 million kilometers).

Problem 1 – An astronaut's white spacesuit reflects 80% of the incoming radiation at Earth's orbit (1 Astronomical Unit). From the formula, about what is the temperature of the surface of the spacesuit?

Problem 2 - The Solar Probe Plus spacecraft will use a heat shield facing the sun with a reflectivity of about A = 0.60. What will be the temperature of the heat shield, called the Thermal Protection System or TPS, at the distance of 5.9 million km (0.040 AU), A) In Kelvins? B) In degrees Centigrade? C) In degrees Fahrenheit?

Problem 3 – Suppose that the TPS consisted of a highly-reflective mirrored coating with a reflectivity of 99%. What would be the temperature in Kelvins, of the back of the heat shield when the Solar Probe is closest to the sun at a distance of 0.04 AU?

Problem 1 – An astronaut's white spacesuit reflects 80% of the incoming radiation at Earth's orbit (1 Astronomical Unit). From the formula, about what is the temperature of the surface of the spacesuit?

Answer: $T = 396\frac{(1-0.8)^{\frac{1}{4}}}{\sqrt{1AU}}$ so **T = 265 Kelvin**.

Problem 2 - The Solar Probe Plus spacecraft will use a heat shield facing the sun with a reflectivity of about A = 0.60. What will be the temperature of the heat shield, called the Thermal Protection System or TPS, at the distance of 5.9 million km (0.040 AU), A) In Kelvins? B) In degrees Centigrade? C) In degrees Fahrenheit?

Answer: A) **T = 1600 K**.
B) Tc = Tk – 273 so **Tc = 1,300 C**
C) Tf = 9/5 (Tc) + 32 so **Tf = 2,400 F**.

Problem 3 – Suppose that the TPS consisted of a highly-reflective mirrored coating with a reflectivity of 99%. What would be the temperature in Kelvins, of the back of the heat shield when the Solar Probe is closest to the sun at a distance of 0.04 AU?

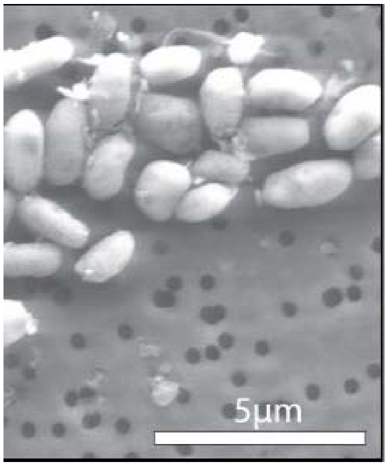
Answer: A = 0.99 then

$T = 396\frac{(1-0.99)^{\frac{1}{4}}}{\sqrt{0.04AU}}$ so Tk = **630 Kelvins**.

For more mission details, visit:

http://solarprobe.jhuapl.edu/

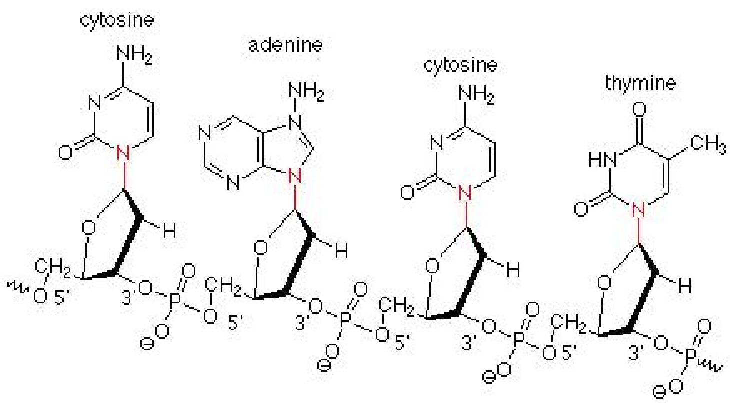
http://www.nasa.gov/topics/solarsystem/sunearthsystem/main/solarprobeplus.html



Microphotograph of the new bacterium GFAJ-1 that subsists on the toxic element arsenic.

NASA researchers exploring extremophile bacteria in Mono Lake, California claimed to have discovered a new strain of bacterium GFAJ-1 in the Gammaproteobacteria group, which not only feeds on the poisonous element arsenic, but incorporates this element in its DNA as a replacement for normal phosphorus. All other known life forms on Earth use 'standard' DNA chemistry based upon the common elements carbon, oxygen, nitrogen and phosphorus.

In the search for life on other worlds, knowing that 'life' can exist that is fundamentally different than Earth life now broadens the possible places to search for the chemistry of life in the universe.



This diagram shows the elements that make up a small section of normal DNA containing the four bases represented from top to bottom by the sequence 'CACT'. They are held together by a 'phosphate backbone' consisting of a phosphor atom, P, bonded to four oxygen atoms, O. Each phosphor group (called a phosphodiester) links together two sugar molecules (dioxyribose), which in turn bond to each of the bases by a nitrogen atom, N.

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4 ? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4?

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4 ?

Answer: A) PO_4 = 1 Phosphorus + 4 Oxygen
= 1 x 31 AMU + 4 x 16 AMU
= **95 AMU**

B) AsO_4 = 1 Arsenic + 4 Oxygen
= 1 x 75 AMU + 4 x 16 AMU
= **139 AMU**

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Answer: The arsenic-substituted ester has a mass of 139 AMU compared to the phosphorus-based ester with 95 AMU, so the new molecule AsO_4 is 100% x (95/139) = 68% more massive than PO_4.

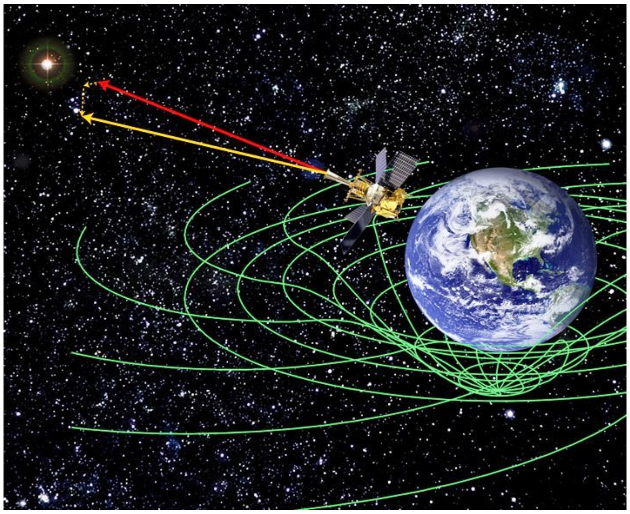
Since in the normal DNA the PO_4 contributes 30% of the total DNA mass, the non-PO_4 molecules contribute 70% of the normal mass.

This is added to the new arsenic-based molecule mass for AsO_4 of 30% x1.68 = 50% to get a new mass that is 70% + 50% = **120% heavier** than the original, 'normal' DNA based on PO_4.

So we would predict that the DNA of the twin arsenic-based organism is only 20% more massive than the DNA of the original phosphate-based organism.

Note: Students may have a better sense of the calculation if they start with a concrete amount of 100 grams of normal DNA. Then 70 grams are in the non-PO_4 molecules and 30 grams is in the PO_4 molecules. Because AsO_4 is 68% more massive than PO_4, its contribution would be 30 grams x 1.68 = 50 grams. Then adding this to the 70 grams you get 120 grams with is 20 grams more massive than normal DNA for a gain of 120%.

New research published in 2012 now disputes the claim that the organism is truly an arsenic-based life form.



Copyright James Overduin, Pancho Eekers and Bob Kahn.

Physicists at Stanford University have recently completed their analysis of data from the Gravity Probe-B (GP-B) satellite, launched in 2004, and have confirmed two predictions of Albert Einstein's relativistic theory of gravity called General Relativity.

The pointing direction of a high-precision gyroscope was measured for over 50 weeks as it orbited Earth. If Newton's theory of gravity were correct, the pointing direction should stay absolutely the same. If Einstein's theory was correct, it should point in a slightly different direction.

The effect is called 'frame dragging' and was first predicted in 1918 by Austrian physicists Josef Lense (1890-1985) and Hans Thirring (1888-1976) using Einstein's mathematical theory of gravity published in 1915. The rate at which the pointing angle will change is given by the formula for Ω, in degrees/sec, shown below:

$$\Omega = \frac{GJ}{2c^2 a^3 \left(1-e^2\right)^{3/2}} \left(\frac{360}{2\pi}\right)$$

c is the speed of light: $c = 300{,}000{,}000$ m/s
J is the angular momentum of Earth: $J = 5.861 \times 10^{33}\ \text{m}^2\ \text{kg sec}^{-1}$
G is the Newtonian Gravitational constant $G = 6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$
a is the semi-major axis of the satellite orbit
e is the eccentricity of the satellite orbit

Problem 1- The GP-B satellite orbits at a distance from Earth's center of a = 7020 km, in a circular orbit for which e=0. To two significant figures, what is the value for Omega in A) degrees per second? B) arcseconds per year? (Note 1 degree = 3600 arcseconds and 1 year = 3.1×10^7 seconds)

Problem 2 - The GP-B spacecraft took observations for 50 weeks. About what would be the accumulated angular shift by the end of this time to two significant figures?

Problem 1 - The GP-B satellite orbits at a distance from Earth's center of a = 7020 km, in a circular orbit for which e=0. To two significant figures, what is the value for Omega in A) degrees per second? B) arcseconds per year? (Note 1 degree = 3600 arcseconds and 1 year = 3.1×10^7 seconds)

$$\Omega = \frac{GJ}{2c^2a^3\left(1-e^2\right)^{3/2}}\left(\frac{360}{2\pi}\right)$$ in degrees/sec

$$\Omega = \frac{(6.67x10^{-11})(5.861x10^{33})(360)}{4(3.141)\left(3.0x10^8\right)^2\left(7.02x10^6\right)^3\left(1-0^2\right)^{3/2}}$$ **3.66×10^{-13}** degrees/sec

Answer
A) $\Omega =$ **3.6×10^{-13} degrees/sec**
B) $\Omega =$ **3.6×10^{-13}** degrees/sec x (3600 arcsec/1 degree) x (3.1×10^7 sec/1 year)
= **0.04 arcseconds/year**

Problem 2 - The GP-B spacecraft took observations for 50 weeks. About what would be the accumulated angular shift by the end of this time to two significant figures?

Answer: 50 weeks is 50/52 = 0.96 years, so the total shift is just
$\Theta = \Omega$ x 0.96
= 0.04 arcseconds/year x (0.96 years)
= 0.038 arcseconds.

Note: This calculation is an approximation to the actual models used to represent the complex spacecraft motion and Earth's gravitational field. Because a more detailed model for Earth and the satellite's motion was used by the GP-B science team, the actual shift detected by the Gravity Probe-B satellite was 0.041 arcseconds, in agreement to within 1% with refined calculations from Einstein's theory.

The equation used in this problem, which predicts the rate of advance of the right ascension of the ascending node of the spacecraft's orbit due to the Lens-Thirring Effect, was obtained from the article:

"*Gravitation, Relativity and Precise Experimentation*" by C.W. Everitt, Proceedings of the First Marcel Grossmann Meeting on General Relativity, pp. 545-615, North Holland, 1977 (p. 567, Equation 22). See the archive of scientific papers at the GP-B website **http://einstein.stanford.edu/content/sci_papers/index.html**

The Upper Atmosphere Research Satellite (UARS) was launched in 1991. After 15 years of research, in 2005 its orbit was lowered from 550 km to 360 km as part of the orbital disposal of this retired satellite. Through atmospheric drag, the satellite began a slow continued descent from 352 km in 2008 to 320 km in 2011. By July, the pace quickened as the satellite steadily encountered a denser atmosphere. At the extrapolated rate of descent, it was predicted that the satellite would finally burn-up by September 23/24 - an event that intrigued and concerned millions of people who worried that some part of the satellite might strike them or do significant property damage.

The table below gives the perigee altitude in kilometers of the satellite at various dates in 2011 during its descent from orbit.

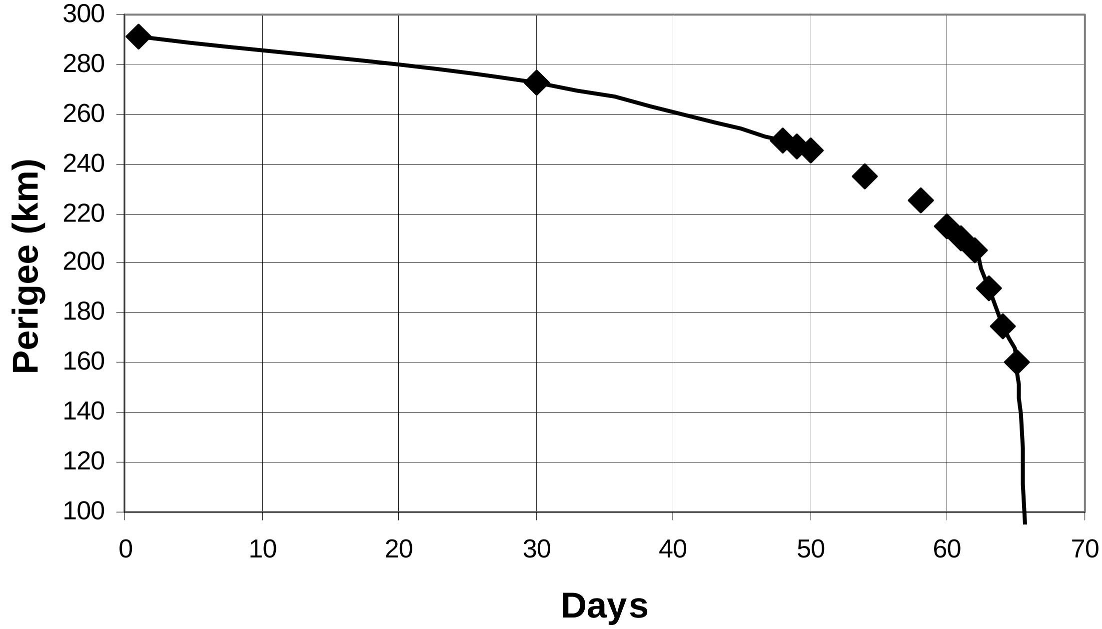
Date	Day	Altitude	Date	Day	Altitude
July 20	1	291	Sept. 17	59	220
August 19	30	273	Sept. 18	60	215
Sept. 6	48	249	Sept. 19	61	210
Sept. 7	49	247	Sept. 20	62	205
Sept. 8	50	245	Sept. 21	63	190
Sept. 12	54	235	Sept. 22	64	175
Sept. 15	57	230	Sept. 23	65	160
Sept. 16	58	225			

Problem 1 - Graph these points and connect the points with a smooth curve.

Problem 2 - What is the rate of altitude loss in meters per hour between A) August 19 and September 6? B) September 17 and 18? C) September 22 and 23?

Problem 3 - When do you predict the satellite reaches zero altitude?

Problem 1 - Graph these points and connect the points with a line.



Problem 2 - What is the rate of altitude loss in meters per hour between A) August 19 and September 6? B) September 17 and 18? C) September 22 and 23?

Answer: A) R = (249 - 273)/(48 - 30)
= - 1.33 km/day x (1000 m/1 km)x(1 day/24h)
= **-56 meters/hour**.

B) R = **-208 meters/hour.**

C) R = **- 625 meters/hour.**

Problem 3 - When do you predict the satellite reaches zero altitude?
Answer: Students can use their hand-drawn curve to estimate that the 'zero point' is reached on Day 66, which is September 24. Students can also program the data into an Excel spreadsheet and use various 'Trendlines' to extrapolate the data. Students should quickly realize that because the rate of descent changes constantly, a linear equation using a constant 'slope' will not work.

Exploring the Night-time Launch of STEREO

This spectacular night-time image taken by photographer Dominic Agostini shows the launch of the STEREO mission from Pad 17B at the Kennedy Space Center. The duration of the time-lapse image was 2.5 minutes. The distance to the horizon was 10 km, and the width of the image was 40 degrees. Assume that the trajectory is at the distance of the horizon, and in the plane of the photograph (the camera was exactly perpendicular to the plane of the launch trajectory).

Problem 1 – By using trigonometry, what is the horizontal scale of this image in meters/mm at the distance of the horizon?

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launch pad?

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this portion of the rocket's trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements)

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

More of Dominic's excellent night launch images can be found at www.dominicphoto.com

Problem 1 – By using trigonometry, what is the horizontal scale of this image in meters/mm at the distance of the horizon?
Answer: Solve the right-triangle to determine the hypotenuse h as h = 10 km/cos(20) = 11 km, then the width of the field of view at the horizon is just 2(11sin(20)) = 2(3.8 km) = 7.6 km. This is the width of the photograph along the distant horizon. With a millimeter ruler, the width of the mage is 125 mm, so the scale of this image at the horizon is about 7600m/125mm = **61 meters/mm**.

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launchpad? Answer: Draw a vertical line through the center of the image. Measure the length of this line between the distant horizon and the point on the trajectory. Typical values should be about 40 mm. Then from the scale of the image of 61 meters/mm, we get an altitude of H = 40 mm x 61 m/mm = **2440 meters or 2.4 kilometers**.

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements with the launch pad as the origin of the coordinates)

Answer: Because the launch pad is the origin of the coordinates, one of the two x-intercepts must be at (0,0) so the value for c = 0. We already know from Problem 2 that a second point is (2.4,2.4) so that $2.4 = a(2.4)^2 + b(2.4)$ and so the first equation for the solution is just
1 = 2.4a + b. We only need one additional point to solve for a and b. If we select a point at (1.2,1.5) we have a second equation $1.5 = a(1.2)^2 + 1.2b$ so that 1.5 = 1.4a + 1.2b. Our two equations to solve are now just

2.4 a + 1.0 b = 1.0
1.4 a + 1.2 b = 1.5

which we can solve by elimination of b to get a = -0.2 and then b = 1.48
so $\mathbf{H(x) = -0.2\,x^2 + 1.48\,x}$

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

Answer: Students can use a ruler to determine this as about **2.7 kilometers**. Alternatively they can use the formula they derived in Problem 3 to get the vertex coordinates at
x = -b/2a
= +3.7 km and so h = **2.7 km**.

This sequence of stills was obtained from a YouTube.com video of the launch of MSL by United Space Alliance available at

http://www.youtube.com/watch?v=0cxsvVBemHY

This sequence shows the launch of the MSL mission from the Kennedy Space Center Launch Complex 49 on November 27, 2011 at 10:02 EST. The four images were taken, from bottom to top, at times 10:02:48 EST, 10:02:50 EST, 10:02:51 EST and 10:02:52 EST. At the distance of the launch pad, the width of each image is 400 meters.

Problem 1 - With the help of a millimeter ruler, what is the scale of each image in meters/mm?

Problem 2 - For each image, what is the distance between the bottom of the image and the base of the rocket nozzle for the Atlas V rocket in each scene?

Problem 3 – What is the estimated distance from the base of the launch pad to the rocket nozzle in each image?

Problem 4 – From the time information, what is the average speed of the rocket between A) Image 1 and 2? B) Image 2 and 3? C) Image 3 and 4?

Problem 5 – From the speed information in Problem 4, what is the average acceleration between A) Image 1 and Image 3? B) Image 2 and Image 4?

Problem 6 – Graph the height of the rocket versus the time in seconds since launch.

Problem 7 – Graph the speed of the rocket versus time in seconds after launch. For the time, use the midpoint time for each speed interval.

Problem 8 – Graph the acceleration of the rocket versus time in seconds after launch. For the time, use the midpoint time for each acceleration interval.

Problem 1 - Answer: Width = 69 mm, so scale = 400 m/69 mm = **5.8 meters/mm**

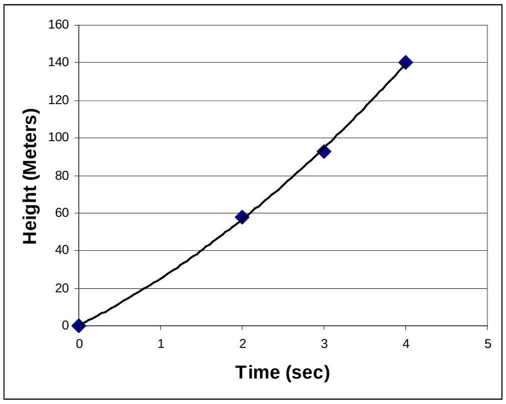
Problem 2 - Answer: 8mm, 18mm, 24mm and 32mm so using the scale of the image, the actual distances are **46m, 104m, 139m and 186 meters**.

Problem 3 – Answer: Take the differences in the measurements relative to the first image at the moment of launch to get h1 = 46m-46m = **0m**, h2=104m-46m = **58m**, h3=139m-46m = **93 m** and h4 =186m-46m = **140 m**.

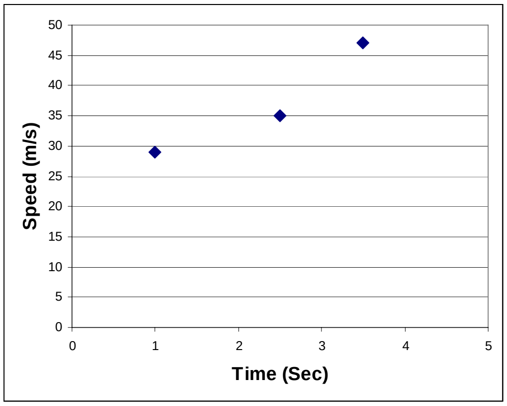
Problem 4 – Answer: A) v= distance/time, v1 = (58m-0m)/2sec = **29m/sec** B) v2 =(93m-58m)/1sec = **35 m/sec**, C) v3=(140m-93m)/1sec = **47 m/sec**.

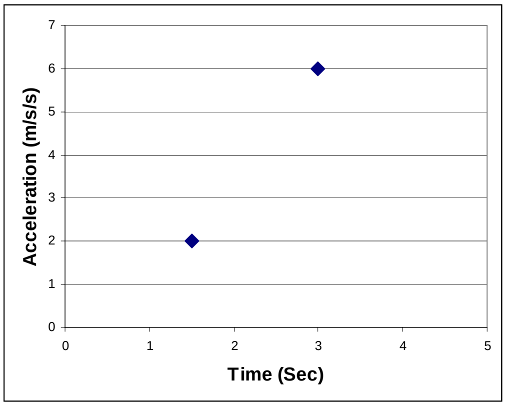
Problem 5 – Answer: A) a1 = (v2-v1)/3sec = (35-29)/3 = 6/3 = **2 m/sec^2**. B) a2 = (V3-v2)/2sec = (47-35)/2sec = **6 m/sec^2**.

Problem 6 – Graph the height of the rocket versus the time in seconds since launch.

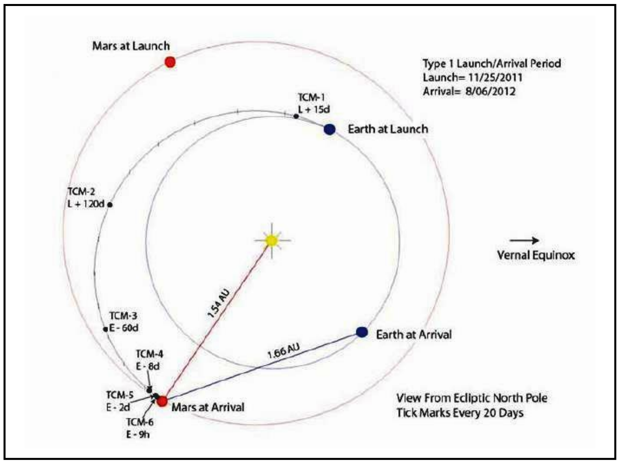


Problem 7 – Graph the speed of the rocket versus time in seconds after launch. For the time, value, use the midpoint time for each speed interval. Answer: Left Above. For the first speed, the two height measurements are made at T=0 and T=2, so the speed V1 will be plotted at the midpoint time: T=(2-0)/2 = 1 sec





Problem 8 – Graph the acceleration of the rocket versus time in seconds after launch. For the time value, use the midpoint time for each acceleration interval. Answer: Right Above.



The Mars Science Laboratory was launched from Cape Canaveral on November 26, 2011 for a 251-day journey to Mars along an orbital path 567 million kilometers in length. The path is along a portion of an elliptical orbit called a Hohmann Transfer Orbit.

Hohmann Transfer Orbits require the least amount of fuel to transfer a spacecraft from Earth's orbit around the sun to the orbit of Mars.

A Hohmann Transfer Orbit to Mars has a perihelion distance of 1.0 Astronomical Units, and an aphelion distance equal to the distance of Mars from the sun at the time of interception, which for August 6, 2012 is 1.5 Astronomical Units. One Astronomical Units (1 AU) equals 149 million km. One focus of the transfer orbit is located on the sun, as are the elliptical orbits of all the other planets. The relationship between the aphelion and perihelion distances, A and P, and the semi-major axis, A, and eccentricity, e, of the corresponding ellipse is given by

$P = a - c \qquad A = a + c$

where a is the semi-major axis and also the semi-minor axis is $b = (a^2 - c^2)^{1/2}$

Problem 1 – For the desired transfer orbit, what is the equation of the required elliptical orbit in standard form?

Problem 2 – From the diagram above, where would Earth be in its orbit if the spacecraft could complete its original transfer orbit and attempt to return to Earth?

Problem 3 - To the proper number of significant figures, what is the average speed of the Mars Science Laboratory spacecraft in its journey to Mars A) in kilometers/hr? B) miles/hour?

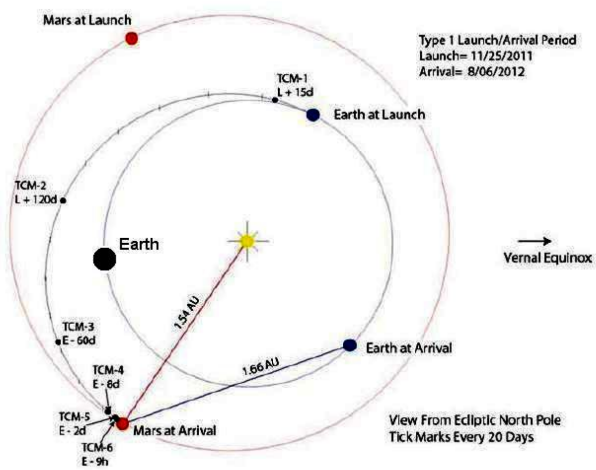
Problem 1 – For the desired transfer orbit, what is the equation of the required elliptical orbit in standard form?

Answer: The major axis has a total length of 1.0 + 1.0 + 0.5 = 2.5 AU, so a = 2.5/2 = 1.25 AU. Then from P = a – c and A = a + c we have 1.0 = 1.25 –c and 1.5 = 1.25 + c so that c = 0.25 AU. Then b = $(1.25^2 - 0.25^2)^{1/2}$ = 1.2 AU. Then from the standard form for an ellipse we get:

$$1 = \frac{x^2}{(1.25)^2} + \frac{y^2}{(1.2)^2}$$

Problem 2 – From the diagram above, where would Earth be in its orbit if the spacecraft could complete its original transfer orbit and attempt to return to Earth?

Answer: By symmetry, the dot on the orbit for Earth on August 6, 2012 would have to move an equal distance to its journey since November 25, 2011, which would place it near the indicated point in the diagram below:



Problem 3 - To the proper number of significant figures, what is the average speed of the Mars Science Laboratory spacecraft in its journey to Mars A) in kilometers/hr? B) miles/hour?

Answer: The 567 million km journey will take 251 days, so the average speed will be A) 567 million km/251 days = 2.26 million km/day or **94,200 km/hr** B) **58,400 miles/hr**. (since 1 km = 0.62 miles)

The SpaceX Falcon 9 rocket soared into space from Space Launch Complex-40 on Cape Canaveral Air Force Station in Florida, carrying the Dragon capsule (left) to orbit on May 22, 2012.

During the flight, there were a series of check-out procedures to test and prove Dragon's systems, including rendezvous and berthing with the International Space Station. If the capsule performed as planned, the cargo and experiments it was carrying would be transferred to the station.

Problem 1 – The Dragon Capsule has the shape shown in the photo above (Courtesy NASA). The diameter of the base just above the curved head shield at its bottom is 3.2 meters. The diameter of the top is 2.2 meters. The capsule is 2.3 meters tall, and it would be 5.2 meters tall to its apex at the top if it were the shape of an upside-down ice cream cone.

The volume of an ice cream cone with a base radius of R and a height of h is given by the formula

$$V = \frac{1}{3}\pi R^2 h$$

From the information provided, what is the volume of the Dragon Capsule in cubic meters to the nearest tenth?

Problem 2 - Suppose you had a room in your house with an 8-foot (2.7 meter) ceiling. If the floor area were a perfect square, what would be the dimensions of the floor so that the volume of this room were the same as the volume of the Dragon Capsule A) in meters? B) in feet?

Problem 1 – The Dragon Capsule has the shape shown in the photo above (Courtesy NASA). The diameter of the base just above the curved head shield at its bottom is 3.2 meters. The diameter of the top is 2.2 meters. The capsule is 2.3 meters tall, and it would be 5.2 meters tall to its apex at the top if it were the shape of an upside-down ice cream cone. From the information provided, what is the volume of the Dragon Capsule in cubic meters to the nearest tenth?

Answer: Students should first compute the volume of the full 'ice cream cone' with a base radius of R = (3.2/2)=1.6 meters and a height h = 5.2 meters, then subtract the cone with a height of h = (5.2-2.3) = 2.9 meters and a base radius of R = (2.2/2)=1.1 meters. The difference is the volume of the capsule.

$$V = 1/3\ \pi\ (1.6)^2\ (5.2) - 1/3\ \pi\ (1.1)^2\ (2.9)$$

$$= 13.91 - 3.67$$

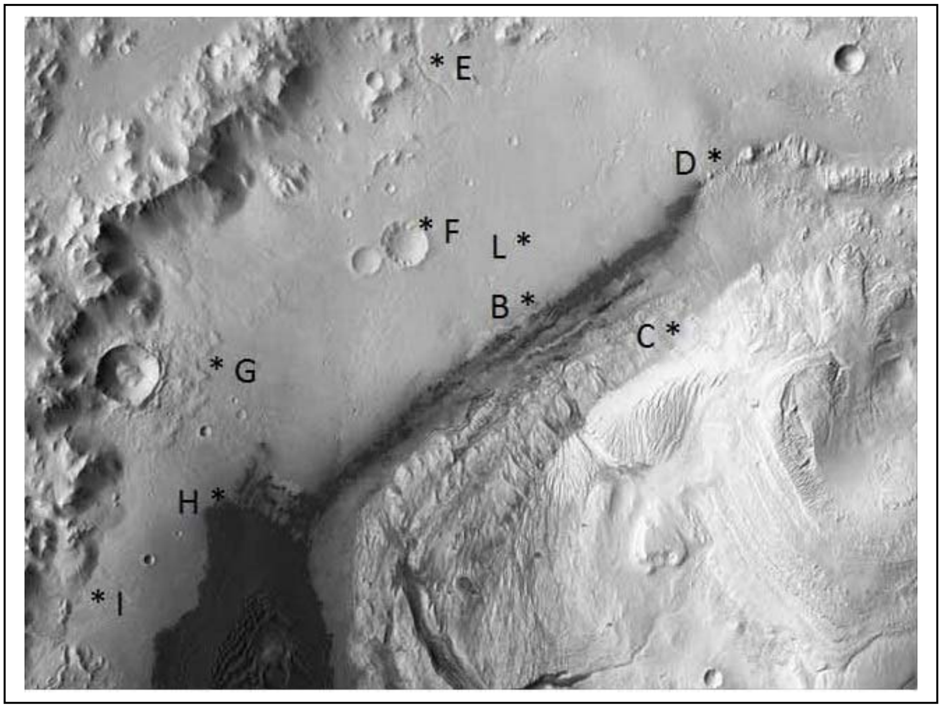
= **10.2 cubic meters**

Problem 2 - Suppose you had a room in your house with an 8-foot (2.7 meter) ceiling. If the floor area were a perfect square, what would be the dimensions of the floor so that the volume of this room were the same as the volume of the Dragon Capsule A) in meters? B) in feet?

Answer: A) The volume of the room would be V = 2.7 x floor area, and since Area = L^2 for a square floor, the length would be given by 10.8 = 2.7 x L^2, so **L = 2 meters**.

B) 1 meter = 3 feet, so the length would be **L = 6 feet**. The dimensions of the room would be 2m x 2m x 2.7m or about 6 feet x 6 feet x 8 feet.

Note: Have the students try to imagine 5 astronauts lying on couches in this volume with computer equipment and spacesuits too!



The table below gives the coordinates for the locations visited by the Curiosity Rover shown in the figure above. The X and Y coordinate units are in kilometers. Although Curiosity is free to travel between most points in the map, Point C is at a much higher elevation than the other points, and a steep cliff wall exists between Point B and C and runs diagonally to the lower left.

Label	Name	(X,Y)	Label	Name	(X,Y)
L	Landing Area	(45,40)	F	Crater Wall	(38,43)
B	Layered Wall	(50,35)	G	Mudslide	(17,30)
C	Alluvial Fan	(60,32)	H	Dark Sands	(17,19)
D	Summit Access	(65,50)	I	Mystery Valley	(5,10)
E	River Bed	(37,58)			

Problem 1 – Curiosity can travel a top speed of 300 meters/hr. When it landed, it was instructed to move as quickly as possible to Point B in case the mission malfunctioned. What is the distance between Point L and Point B, and how long did it take Curiosity to get there?

Problem 2 – To the nearest kilometer, what is the distance directly from Point D to Point I as, and how long would it take Curiosity to make this trip without stopping to do any scientific research?

Problem 3 – One possible path Curiosity might take connects all of the points in the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days would this journey take?

Problem 4 – Can you think of a different trip, and include a 3-day stay at each point along the way?

Answer Key

Problem 1 – Curiosity can travel a top speed of 300 meters/hr. When it landed, it was instructed to move as quickly as possible to Point B in case the mission malfunctioned. What is the distance between Point L and Point B, and how long did it take Curiosity to get there?

Answer: L(45,40) and B(50,35). Using the Pythagorean distance formula, $D = ((50-45)^2 + (35-40)^2)^{1/2}$ = **7 kilometers**. The time taken is just T = 7000 meters/300 m/h = **23 hours.**

Problem 2 – To the nearest kilometer, what is the distance directly from Point D to Point I as, and how long would it take Curiosity to make this trip without stopping to do any scientific research?

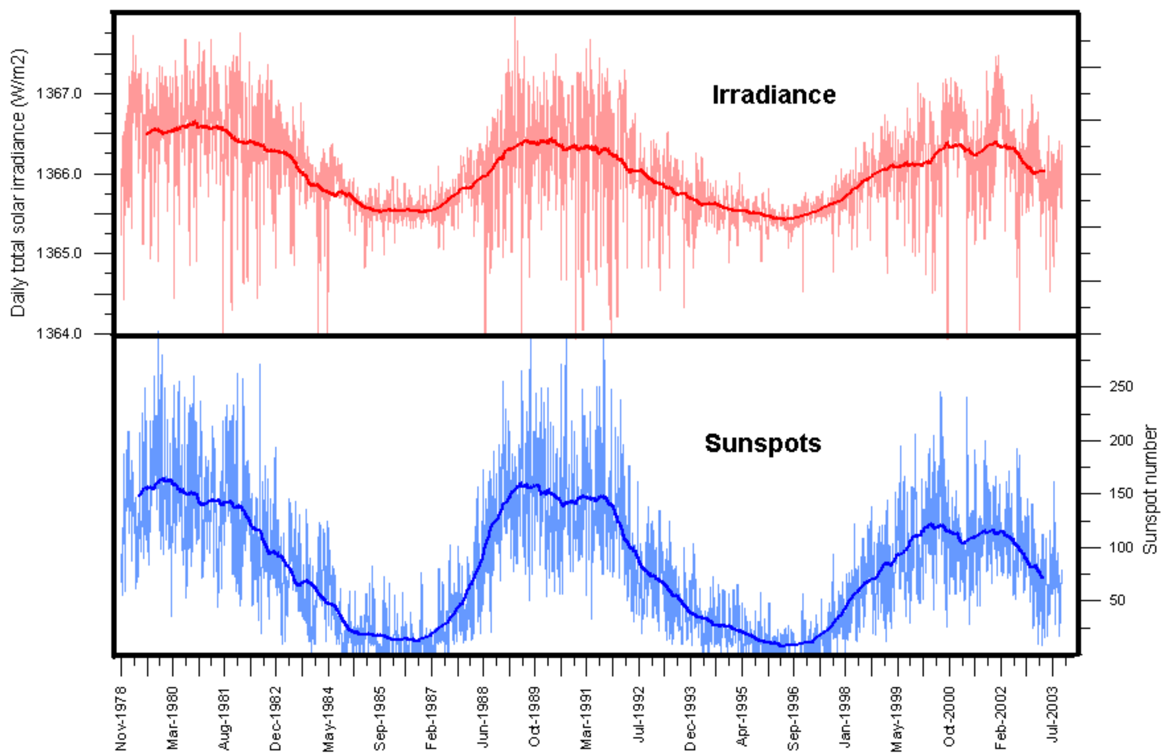
Answer: Point D(65,50) and Point I(5,10) than d = ((5-65)2 + (10-50)2)1/2 = **72 kilometers**. This takes T = 72000 meters/300 m/h = 240 hours or **10 days**.

Problem 3 – One possible path Curiosity might take connects all of the points in the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days would this journey take?

Answer: Using the distance formula between consecutive point pairs we get:
D(LB) = 7 km
D(BD) = 21 km
D(DC) = 19 km
D(CD) = 19 km
D(DE)= 29 km
D(EF) = 15 km
D(FB) = 14 km
D(BG)= 33 km
D(GH)=11 km
D(HI) = 15 km Total distance = 183 km, time = 183,000/300 = 610 hrs = **25.4 days**.

Problem 4 – Can you think of a different trip, and include a 3-day stay at each point along the way?

Answer: Students may take the points in any interesting order. One strategy is to do the most interesting points first to make sure that the mission science goals are achieved. Be cafreful of the big cliff between Points B and C!



Irradiance (also called insolation) is a measure of the amount of sunlight power that falls upon one square meter of exposed surface, usually measured at the 'top' of Earth's atmosphere. This energy increases and decreases with the season and with your latitude on Earth, being lower in the winter and higher in the summer, and also lower at the poles and higher at the equator. But the sun's energy output also changes during the sunspot cycle!

The figure above shows the solar irradiance and sunspot number since January 1979 according to NOAA's National Geophysical Data Center (NGDC). The thin lines indicate the daily irradiance (red) and sunspot number (blue), while the thick lines indicate the running annual average for these two parameters. The total variation in solar irradiance is about 1.3 watts per square meter during one sunspot cycle. This is a small change compared to the 100s of watts we experience during seasonal and latitude differences, but it may have an impact on our climate. The solar irradiance data obtained by the ACRIM satellite, measures the total number of watts of sunlight that strike Earth's upper atmosphere before being absorbed by the atmosphere and ground.

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance?

Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

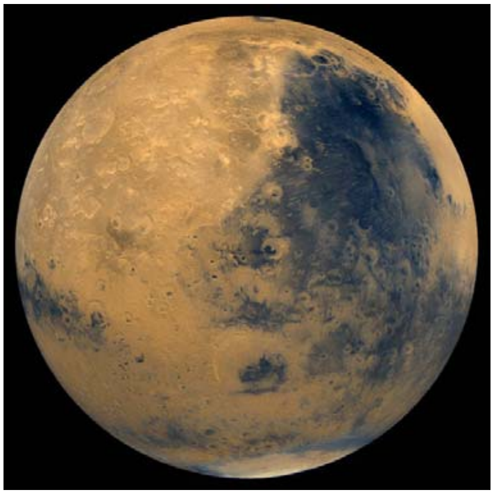
Answer Key

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003? Answer: Draw a horizontal line across the upper graph that is mid-way between the highest and lowest points on the curve. An approximate answer would be 1366.3 watts per square meter.

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance? Answer: When there are a lot of sunspots on the sun (called sunspot maximum) the amount of solar radiation is higher than when there are fewer sunspots. The solar irradiance changes follow the 11-year sunspot cycle.

Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

Answer: In 1985, the amount of insolation was about 1365.5 watts per square meter when the photovoltaic system was built. Because of changes in the sunspot cycle, in 1989 the insolation increased to about 1366.5 watts per square meter. This insolation change was a factor of 1366.5/1365.5 = 1.0007. That means that by scaling, if the system was generating 3,000 kilowatt-hours of electricity in 1985, it will have generated 1.0007 x 3,000 kWh = 2 kWh more in 1989 during sunspot maximum! That is equal to running one 60-watt bulb for about 1 day (actually 33 hours).



Astronomers can determine the mass of a planet by using Kepler's Third Law, which is written in algebraic form as follows:

$$p^2 = \frac{4\pi^2}{G\,M}\,a^3$$

where G is Newton's constant of gravity equal to 6.67×10^{-11} $m^3/kg\ sec^2$, M is the mass of the planet in kilograms, a is the average orbit radius in meters and T is the orbit period in seconds.

Here are some problems that let you work with this very important formula in astronomy.

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of 9380 km and a period of 0.32 days, what is the mass of Mars?

Problem 2 – The martian moon Deimos has an orbit period of 1.26 days. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of 35.5 hours and an average radius of 25,889 km. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of 3700 km. What is the final orbit period of MRO in the new circular orbit?

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of 9380 km and a period of 0.32 days, what is the mass of Mars?

Answer: First solve the algebraic equation for M to get M = $(4\pi^2/G)(a^3/p^2)$. Then use a = 9380 km x 1000m/km = 9.38x106 m, and p = 0.32days x 24h/d x 3600s/hr = 27648 seconds, to get

M = $[4 \times (3.141)2/6.67\times10^{-11}] (9.38\times10^{6})^{3}/(27648)^{2}$ = $\mathbf{6.39\times10^{23}}$ **kg**.

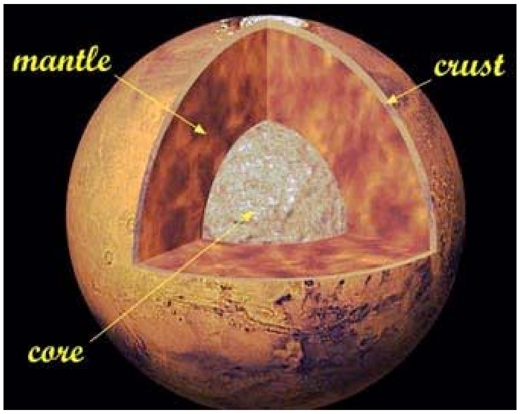
Problem 2 – The martian moon Diemos has an orbit period of 1.26 days. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Answer: In this case we know M and p and need to solve for a:

$a^3 = GMp^2/4\pi^2$ so p = 1.26d x 24h/d x 3600 s/h = 108864 seconds, and so $a^3 = (6.67\times10^{-11})(6.39\times10^{23})(108864)^2/(4 \times 3.141^2) = 1.27\times10^{22}$ so a = 23352 kilometers and so a = **23,400 kilometers**.

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of 35.5 hours and an average radius of 25,889 km. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of 3700 km. What is the final orbit period in minutes of MRO in the new circular orbit?

Answer: Solving for p we get $p^2 = (4\pi^2/GM)\, a^3$, and so for a = 3,700,000 meters and M = 6.39×10^{23} kg we get $p^2 = 4.69\times10^7$ and so p = 6848 seconds or **114 minutes**.



Once astronomers have measured the diameter and mass of a planet, they can determine the average density of the planet by dividing its mass by its volume. This is a valuable 'first look' into the interior of a planet because if the average density is close to 1000 kg/m^3, then most of the planet consists of light materials and gas or even water and ice like Saturn and Uranus. If the value is large and near 4000 kg/m^3, then the planet may consist mostly of rocky materials like Mercury and Earth.

Problem 1 - The mass of Mars is known to be 6.39×10^{23} kilograms, and the outer radius of the planet is 3400 kilometers. What is the average density of Mars in kilograms/meter^3? What would you estimate as the composition of the martian interior if ice has a density of 917 kg/m^3, granite has a density of 2700 kg/m^3 and iron ore has a density of 7000 kg/m^3?

Problem 2 – The interior of mars can be represented by three main geologic regions: The core is a spherical region with a radius of about 1800 km; the mantle is a spherical shell with an outer radius of 3300 km, and the crust is a 100 km spherical shell located above the mantle. The crust of Mars has been sampled by several NASA landers including Viking, Spirit, Opportunity, Phoenix and Curiosity. The density of the surface rocks appears to be about 2000 kg/m^3. If models of the core of Mars suggest a density of 6400 kg/m^3, what is the average density of the rocks in the martian mantle zone to two significant figures?

Problem 1 - The mass of Mars is known to be 6.39×10^{23} kilograms, and the outer radius of the planet is 3400 kilometers. What is the average density of Mars in kilograms/meter^3? What would you estimate as the composition of the martian interior if ice has a density of 917 kg/m^3, granite has a density of 2700 kg/m^3 and iron ore has a density of 7000 kg/m^3?

Answer: The volume of mars as a sphere is given by $V = 4/3 \pi R^3$ so $V = 1.333 \times 3.141 \times (3400000)^3 = 1.65 \times 10^{20}$ m^3, then the density is just $D = 6.39 \times 10^{23}$ kg/1.65×10^{20} m^3 = **3872 kg/m^3**. This is between the density of granite and iron, but closer to granite, so on average there is probably very little iron in the interior of Mars.

Problem 2 – The interior of mars can be represented by three main geologic regions: The core is a spherical region with a radius of about 1800 km; the mantle is a spherical shell with an outer radius of 3300 km, and the crust is a 100 km spherical shell located above the mantle. The crust of Mars has been sampled by several NASA landers including Viking, Spirit, Opportunity, Phoenix and Curiosity. The density of the surface rocks appears to be about 2000 kg/m^3. If models of the core of Mars suggest a density of 6400 kg/m^3, what is the average density of the rocks in the martian mantle zone to two significant figures?

Answer: We know:

The total mass of mars is 6.39×10^{23} kg.
The radius of the core is 1800 km.
The inner and outer radius of the mantle shell as 1800 km and 3300 km.
The inner and outer radius of the crust shell as 3300 km and 3400 km.
The density of the core as 6400 kg/m^3
The density of the crust as 2000 kg/m^3.

So we subtract from the mass of Mars the mass of the core and the crust to get the mass of the mantle. From the mantle shell volume we can then determine it density:

Mcore = $6400 \times 4/3 \pi (1800000)^3 = 1.56 \times 10^{23}$ kg.
Mcrust = $2000 \times 4/3\pi (3400000^3 - 3300000^3) = 2.82 \times 10^{22}$ kg
Mmantle = 6.39×10^{23} kg – 1.56×10^{23} kg – 2.82×10^{22} kg = 4.55×10^{23} kg.
Volume(mantle) = $4/3\pi (3300000^3 - 1800000^3) = 1.26 \times 10^{20}$ m^3.

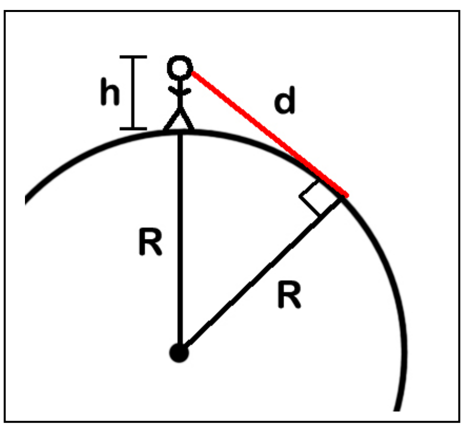
So Density = 4.55×10^{23} kg/1.26×10^{20}m^3 = **3600 kg/m^3**.

Imagine you and your friend standing on the surface of a perfectly flat planet. Your friend starts walking away from you, and you see her size get smaller and smaller until at a distance of 100 kilometers you can't even see her at all.

Now imagine the same experiment on a spherical planet. As many sea-farers discovered 1000 years ago, because Earth is curved, you will see the ships hull disappear from the bottom upwards, then the last thing that vanishes is the top of the main mast.

The image above comes from Johannes de Sacrobosco's Tractatus de Sphaera (*On the Sphere of the World*) written in 1230 AD. It showcases the knowledge that the appearance of ships on the horizon testified to a curved earth. A bit of simple geometry, and some help from the Pythagorean Theorem, will let you calculate the distance to the horizon on Mars as viewed from the InSight Lander!

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.



Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.

Answer: $d^2 = (R+h)^2 - R^2$

So $d^2 = 2Rh + h^2$

And so $d = (2Rh + h^2)^{1/2}$

Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Answer: For the formula to work, R and h must be in the same units of meters (or kilometers!). When h is much less then R, the quantity h^2 is always much, much smaller than 2Rh, so the formula simplifies to $d = (2Rh)^{1/2}$.

For Mars, h = 1 meter and R = 3378000 meters and so **d = 2599 meters or about 2.6 kilometers.**

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Answer: h = 100 meters or 0.1 km, so the horizon distance for one cell tower has a radius of $d = (2 \times 3378 \text{ km} \times 0.1 \text{ km})^{1/2}$ = 26 kilometers. The area of this reception circle is $A = \pi R^2 = 2123 \text{ km}^2$. The total surface area of Mars is $A = 4 \times \pi \times (3378)^2 = 1.43\times10^8 \text{ km}^2$.

Then dividing the surface area of Mars by the cell tower reception area we get $N = 1.43\times10^8/2123$ = **67,530 cell towers**.

As of 2013, there are over 200,000 cell towers in the Unites States alone!

Estimating the Mass of a Martian Dust Devil!

This image shows a Martian dust devil tearing across the surface of Mars in the region called Amazonis Planitia. The image was obtained by the Mars Reconnaissance Orbiter on February 16, 2012. The dust devil rises up more than a half mile high (1000 meters) and is about 100 feet (30 meters) wide. Although rare in the equatorial region where NASA's InSight lander will be located, the movement of so much mass near the sensitive seismometer will disturb its measurements. So, how much mass is in a dust devil like the one above?

Problem 1 – The average martian dust devil observed by the Mars Pathfinder Rover has a diameter of about 200 meters. If its height is 1000 meters and is cylindrically shaped, what is its total volume in cubic meters? (Use $\pi = 3.14$)

Problem 2 – If the average dust density is about 0.0000035 kg/m^3, how many kilograms of dust would be present in an average dust devil to the nearest kilogram? How much is this in pounds to the nearest pound? (1 kg = 2.2 pounds)

Sources:

http://aoss-research.engin.umich.edu/planetaryenvironmentresearchlaboratory/docs/Ferri.et.al.JGR03.pdf

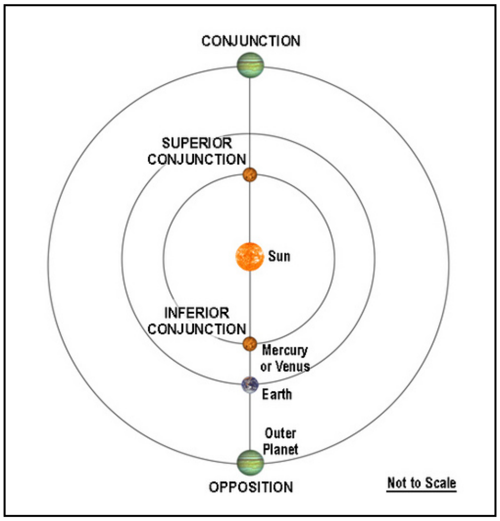
Problem 1 – The average martian dust devil observed by the Mars Pathfinder Rover has a diameter of about 200 meters. If its height is 1000 meters and is cylindrically shaped, what is its total volume in cubic meters?

Answer: $V = \pi R^2 h$ so $V = (3.14)\ (100m)^2\ (1000m) =$ **$3.1x10^7$ meters^3**.

Problem 2 – If the average dust density is about 0.0000035 kg/m^3, how many kilograms of dust would be present in an average dust devil, to the nearest kilogram? How much is this in pounds, to the nearest pound? (1 kg = 2.2 pounds)

Answer: Mass = density x volume, so $M = 3.5x10^{-6}\ x\ 3.1\ x\ 10^7 =$ **109 kilograms**.

In pounds this is M = 2.2 x 109 = **240 pounds**.



The InSight Lander needs to be in constant communication with Earth every day the data it gathers can be sent back to Earth. As viewed from the surface of Mars, Earth never gets very far from the sun. Over the course of about 780 days, Earth travels from its farthest westward position in the sky (morning star) of 46^{o} W, to its farthest eastward position (evening star) of 41^{o} E and back in about 780 days.

When the Earth-Sun angle is near zero as viewed from Mars, Earth can either be between Mars and the sun (called Inferior Conjunction) or Earth can be on the opposite side of the sun as viewed from Mars (called Superior Conjunction).

When Mars and Earth are in inferior conjunction, Earth can receive signals from the Lander, but the Lander will have to broadcast its data almost directly at the sun, which is a hazard for the transmitter. When Earth and Mars are in superior conjunction, Neither InSight nor the Earth radar can transmit or receive data with Earth behind the sun.

Problem 1 – The following table gives the Earth-Sun angle viewed from Mars during the time InSight is operating on the martian surface. During this time, inferior conjunction occurred on May 22, 2016 and July 26, 2018, with superior conjunction on July 27, 2017. When will the next inferior conjunction occur after July 26, 2018?

Month	Angle	Month	Angle	Month	Angle	Month	Angle
9/16	+46	3/17	+24	9/17	-10	3/18	-40
10/16	+45	4/17	+18	10/17	-17	4/18	-41
11/16	+42	5/17	+13	11/17	-23	5/18	-37
12/16	+38	6/17	+7	12/17	-28	6/18	-27
1/17	+34	7/17	+1	1/18	-33	7/18	-7
2/17	+29	8/17	-5	2/18	-38		

Angular data obtained from *Eyes on the Solar System* (February 28, 2013).

Problem 2 – InSight will end its operations after one full martian year (687 days). If it lands on September 20, 2016, during which months of operation will Earth be within 10 degrees of the sun at conjunction, and unable to communicate with Earth?

Problem 3 - Graph the data in the table. During which months will the Earth-Mars angle A) be changing the most rapidly? B) the slowest?

Problem 1 – When will the next superior conjunction occur after July 26, 2018?

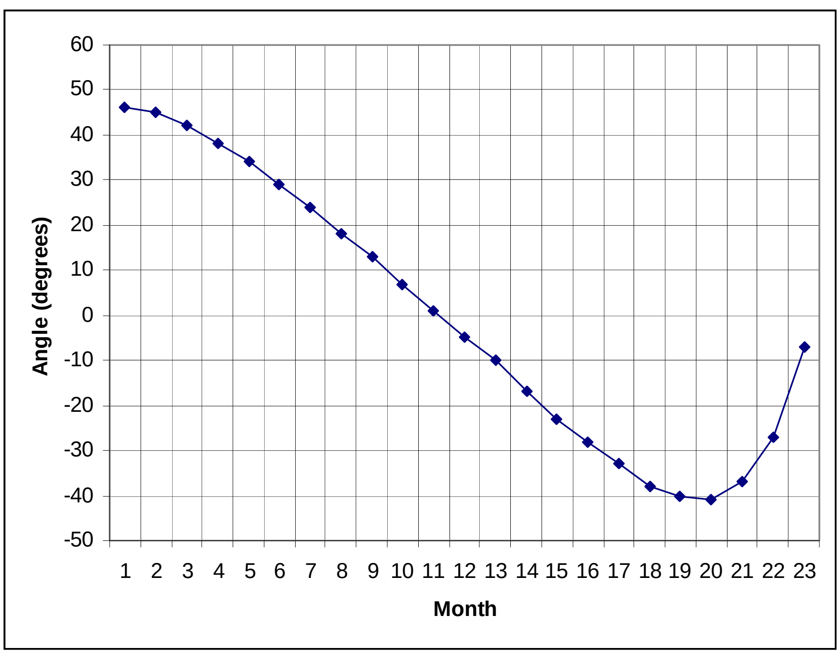
Answer: Each pair of superior and inferior conjunctions happen in cycles. The time for one cycle can be found by the time between the dates of the two listed inferior conjunctions on 5/22/2016 and 7/26/2018. Students can find the number of days between these dates manually (hard), or they can use an online calculator (fun!) like http://www.timeanddate.com/date/duration.html to get 796 days. To find the next superior conjunction, add 796 days to July 27, 2017 (eg. http://www.timeanddate.com/date/dateadd.html) to get **May 4, 2019**.

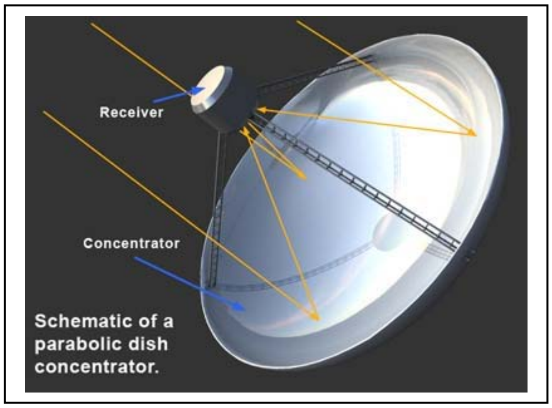
Problem 2 – InSight will end its operations after one full martian year (687 days). If it lands on September 20, 2016, during which months of operation will Earth be within 10 degrees of the sun at conjunction, and unable to communicate with Earth?

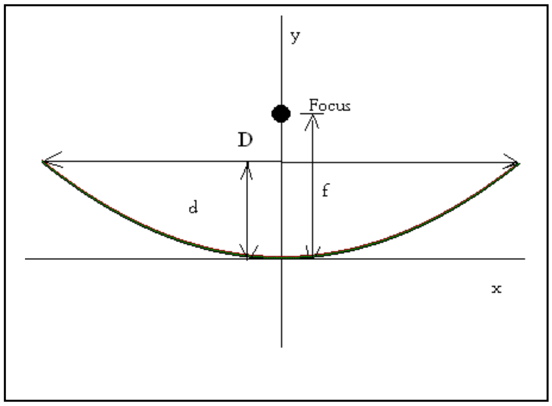
Answer: Conjunction occurs on July 27, 2017. From the table, Earth is within 10 degrees of the sun during the months of **June 2017 to August 2017** so it will be difficult to directly transmit or receive data during this time.

Problem 3 - Graph the data in the table. During which months will the Earth-Mars angle A) be changing the most rapidly? B) the slowest?

Answer: Students may use Microsoft Excel. X-axis may be the month number.
A) June-July, 2018.
B) During September-November, 2016 and February-April, 2018. Slow changes correspond to a nearly flat 'slope' while fast changes correspond to the steepest slope.







A very beautiful property of parabolas is that at a point called the FOCUS, all of the lines entering the parabola parallel to its axis are 'reflected' from the parabolic curve and intersect the focus. This property is used by astronomers to design telescopes, and by radio engineers to design satellite dishes.

The top figure to the left shows a satellite dish with a radio receiver located at the focus of the parabola. The radio rays are reflected from the parabolic surface and concentrated at the focus. This focusing and amplification property of parabolic reflectors is also used for solar heating and generating solar electricity.

The bottom figure defines the distance, f, of the focus from the bottom of the dish, and the diameter, D, of the dish.

Suppose you wanted to design a parabolic dish with a depth, d, of 1 meter and a radius of 5 meters. Where would the focus be located? If the basic equation of a parabola is $y = ax^2$. The location of the focus will be at f = 1/(4a). Since we know that the point (5.0,1.0) is on the curve of the parabola, that means that we can solve for a for this particular dish. We get $a (5.0)^2 = 1.0$ so a = 1/25. Then the focus will be at f = 1/(4/25) so f = 25/4 = 6 ¼ meters above the bottom of the dish. Let's design a few parabolic reflectors that we can use to reflect and concentrate sound waves, or sunlight!

Problem 1 – A bird watcher wants to record bird songs from a distance. He goes to the cooking store and finds a parabola-shaped bowl that is light-weight. It has a diameter of 12 inches and a depth of 5 inches. How far below the edge of the bowel does he have to mount the microphone to use this as a sound amplifier?

Problem 2 – A hobbiest has several hundred small flat mirrors and wants to build a solar cooker by mounting the mirrors on the inside surface of a parabolic shaped form. The diameter of the parabola is 3 meters and the depth of the form is 1/2 meter. How far above the center of the form will the sunlight be the most concentrated?

Problem 1 – A bird watcher wants to record bird songs from a distance. He goes to the cooking store and finds a parabola-shaped bowl that is light-weight. It has a diameter of 12 inches and a depth of 5 inches. How far below the edge of the bowel does he have to mount the microphone to use this as a sound amplifier?

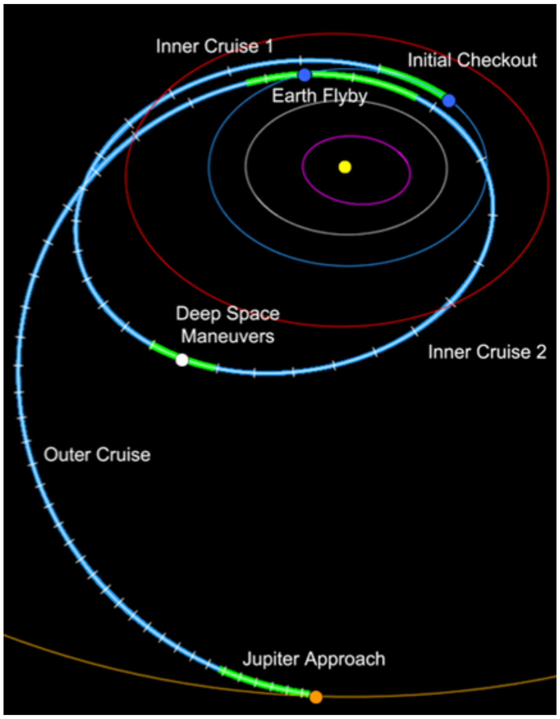
Answer: The radius of the bowl is x=6.0 inches, and at this location y = 5.0 inches. The equation is $y = a x^2$, and we know that the point (6.0, 5.0) is on this curve, so $5.0 = a (6.0)^2$ so a = 0.139. The focus distance is then f = 1/(4x0.139) = 1.8 inches from the bottom of the bowl, or 5.0 – 1.8 = **3.2 inches** from the top edge of the bowl.

Problem 2 – A hobbiest has several hundred small flat mirrors and wants to build a solar cooker by mounting the mirrors on the inside surface of a parabolic shaped form. The diameter of the parabola is 3.0 meters and the depth of the form is 1/2 meters. How far above the center of the form will the sunlight be the most concentrated?

Answer: The diameter is 3.0 meters so the radius is 1.5 meters. The point (1.5, 0.50) is on the parabolic curve, so $0.50 = a (1.5)^2$ and so a = 0.22. Then the focus is at f = 1/(4x0.22) = **1.1 meters from the bottom of the form**.

Note: The light from the sun falls on the parabola over an area of 2 square meters, and is concentrated into an area at the focus of 1/100 square meters, so the sunlight is amplified by 2.0/0.01 = 200 times, making this a very good solar cooker!

Investigating Juno's Elliptical Transfer Orbit



The Juno spacecraft was initially placed in an elliptical orbit near Earth soon after its launch on August 5, 2011. The orbit was elliptical and designed so that a Deep Space Maneuver in August 2012 would send the spacecraft into a flyby of Earth in 2013. This encounter with Earth would boost the spacecraft's speed and place it into an elliptical transfer orbit that would intersect Jupiter's orbit in 2016.

This added speed would not require extra fuel by the spacecraft making it a free resource that keeps the cost of the mission small. These kinds of 'billiard shot' gravitational assists are commonly used by NASA to place spacecraft in trajectories to the outer solar system.

An approximate equation for the transfer orbit is given by the formula:
$5.15x^2 + 9.61y^2 = 49.49$. The units for x and y are given in terms of Astronomical Units where 1 AU = 150 million kilometers, which is the average orbit distance of Earth from the Sun.

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?

Problem 2 – What is the semimajor axis length in AU?

Problem 3 – What is the semiminor axis length in AU?

Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?

Problem 5 - What is the eccentricity, e, of the orbit?

Problem 6 – What are the spacecraft's aphelion and perihelion distances?

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years.If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?
Answer:
$5.15x^2 + 9.61y^2 = 49.49$ Divide both sides by 49.49 to get

$$\frac{x^2}{9.61}+\frac{y^2}{5.15}=1$$

Problem 2 – What is the semimajor axis length in AU?
Answer: For an ellipse written in standard form: $x^2/a^2 + y^2/b^2 = 1$
Comparing with the equation from Problem 1 we get that the longest axis of the ellipse is along the x axis so the semimajor axis is $a^2 = 9.61$ so **a = 3.1 AU**

Problem 3 – What is the semiminor axis length in AU?
Answer: The semiminor axis is along the y axis so $b^2 = 85.15$ and **b = 2.3 AU**

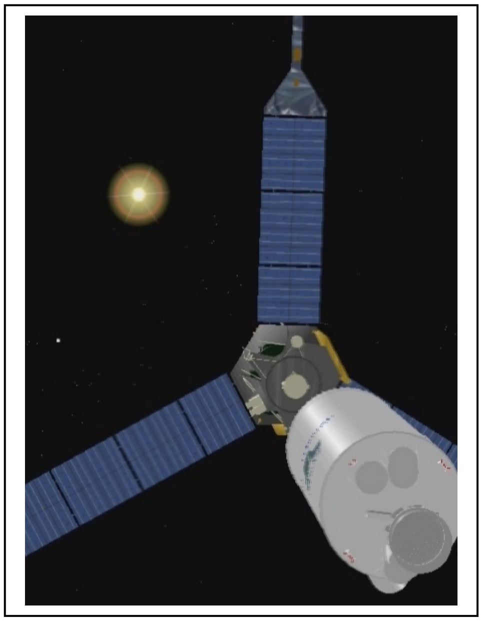
Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?
Answer: $c = (a^2 - b^2)^{1/2}$. With a= 3.1 and b=2.3 we have **c = 2.1**.

Problem 5 - What is the eccentricity, e, of the orbit?
Answer: The eccentricity e = c/a so e = 2.1/3.1 and so **e = 0.68**

Problem 6 – What are the spacecraft's aphelion and perihelion distances?
Answer: The closest distance to the focus along the orbit is given by a – c so the perihelion distance is 3.1 – 2.1 = **1.00 AU**. The farthest distance is a + c = 3.1 + 2.1 = **5.2 AU**. *Note the perihelion distance is at Earth's orbit and the aphelion distance is at Jupiter's orbit.*

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years. If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Answer: Since a = 3.1 we have $P = 3.1^{3/2} = 5.5$ years for a full orbit. For Juno it spands 5.5/2 = 2.8 years in the elliptical transfer orbit, so it arrives at Jupiter in October 2013 + 2.8 years = October, 2013 + 2 years 8 months = **June, 2016**.



As the Juno spacecraft travels to Jupiter, it gets farther from the sun every day. Because the spacecraft generates its electrical power using solar cells, as the sun gets farther away, the amount of power constantly diminishes. At Earth, the solar panels generate about 12,700 watts. Because the spacecraft's trajectory is a portion of an ellipse, the formula for its distance, r, from the sun, located at one of the ellipse's foci, is given by the formula:

$$r(\theta) = \frac{a(1-e^2)}{1-e\cos\theta}$$

where r is in Astronomical Units, a is the semi-major axis length and e is the orbit eccentricity.

The specific equation for the Juno spacecraft can be approximately represented by a = 3.0 AU and e = 2/3, where all distances are given in units of the Astronomical Unit. The Astronomical Unit is a measure of the distance between Earth and the sun; a physical distance of 150 million km.

Problem 1 – What is the simplified form for R given the initial parameters, a and e, for the Juno spacecraft?

Problem 2 – If the amount of solar energy falling on the Juno solar panels is determined by the inverse-square law, and the amount of solar energy generated by the solar panels at r = 1.0 AU is exactly 12,690 watts, what is the formula for the solar panel power at any distance defined by the function P(r)?

Problem 3 – For what angle, θ, will the spacecraft be able to generate only ¼ of the electrical power it did when it left Earth orbit?

Problem 4 – What will the spacecraft power be when it reaches Jupiter at its maximum distance from the sun?

Problem 1 – What is the simplified form for R given the initial parameters, a and e, for the Juno spacecraft?

$$r(\theta) = \frac{5}{3 - 2\cos\theta}$$

Problem 2 – If the amount of solar energy falling on the Juno solar panels is determined by the inverse-square Law, and the amount of solar energy generated by the solar panels at r = 1.0 AU is 12,000 watts, what is the formula for the solar panel power at any distance defined by the function P(r)?

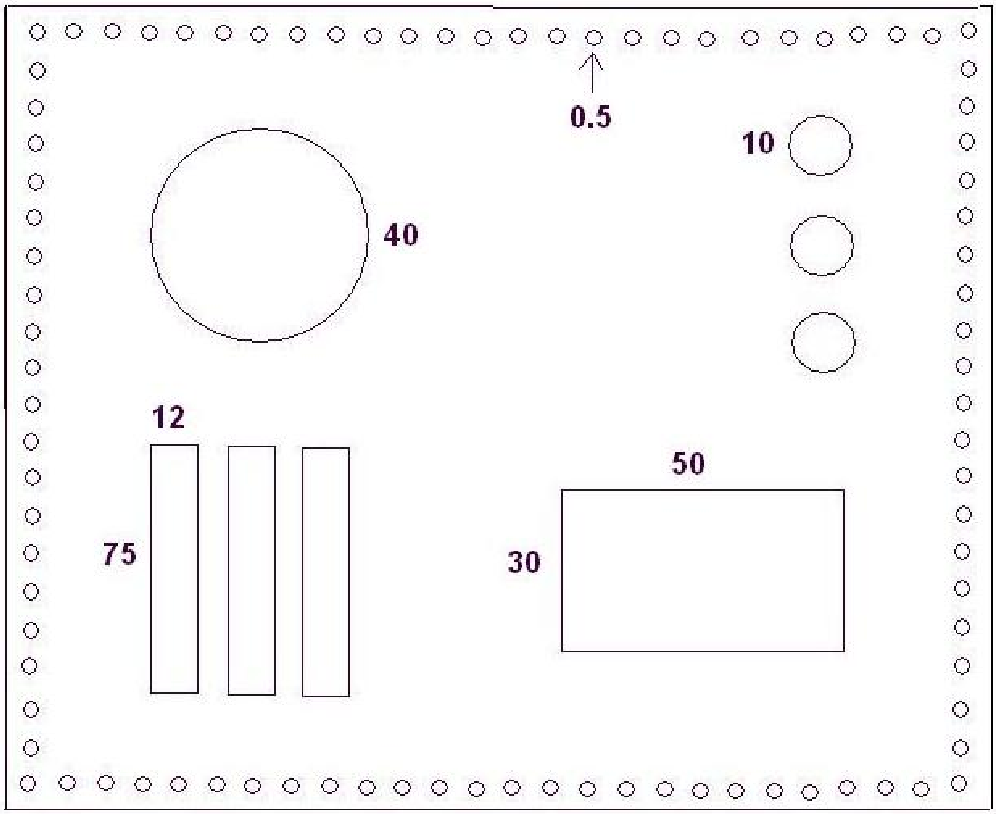
$$P(r) = 480(3 - 2\cos\theta)^2$$

Problem 3 – For what angle, θ in degrees, will the spacecraft be able to generate only ¼ of the electrical power it did when it left Earth orbit?

P = 12000/4 = 3000 watts
$3000 = 480\ (3-2\cos\theta)^2$ so $2.5 = 3 - 2\cos\theta$ and $\cos\theta = 0.25$ so **θ= 76 degrees**

Problem 4 – What will the spacecraft power be when it reaches Jupiter at its maximum distance from the sun?

Answer: The maximum value for r occurs at $\theta = 0$, so **P = 480 watts**.



This is a diagram of a panel from a spacecraft showing all of the openings. All units are in centimeters. The panel is 190 centimeters wide and 150 centimeters tall. The diameters of each circular opening is also given. The small holes around the circumference are for the screws that fasten the panels together.

Problem 1 - To the nearest square-centimeter, what is the total area of the panel in square-centimeters before the openings were made?

Problem 2 - To the nearest square-centimeter, what is the total area of all of the openings in the panel? (Use $\pi = 3.1415$)

Problem 3 - To the nearest square-centimeter, what is the area of the panel after the openings were made?

Problem 4 - The panel is 1.5 centimeters thick. To the nearest cubic-centimeter, what is the volume of the finished panel?

Problem 5 - To the nearest cubic-centimeter, what is the volume of the material that was removed to make the holes?

Problem 6 - If the density of the aluminum in the panel is 2.7 grams/cm^3, to the nearest tenth of a kilogram, what is the mass of the finished panel?

Problem 7 - To the nearest tenth of a kilogram, how many grams of aluminum were removed to make all of the openings?

Problem 1 - To the nearest square-centimeter, what is the total area of the panel in square-centimeters before the openings were made?

Answer: A = w x h so A = 190 x 150 = **28,500 cm^2**

Problem 2 - To the nearest square-centimeter, what is the total area of all of the openings in the panel?

Answer:

A = (30 x 50) + 3 (12 x 75) + 1 $(3.1415)(40/2)^2$ + 3 x $(3.1415)(10/2)^2$ + 90x$(3.1415)(0.5/2)^2$
A = 1500 + 2700 + 1256.6000 + 235.6125 + 17.6709
A = 5709.8834
A = 5710 cm^2.

Problem 3 - To the nearest square-centimeter, what is the area of the panel after the openings were made?

Answer: 28,500 - 5710 = **22,790 cm^2**

Problem 4 - The panel is 1.5 centimeters thick. To the nearest cubic-centimeter, what is the volume of the finished panel?
Answer: Volume = Area x Thickness
= 22,790 cm^2 x 1.5 cm
= **34,185 cm^3**

Problem 5 - To the nearest cubic-centimeter, what is the volume of the material that was removed to make the holes?

Answer: Volume = 5710 cm^2 x 1.5 cm = **8,565 cm^3**

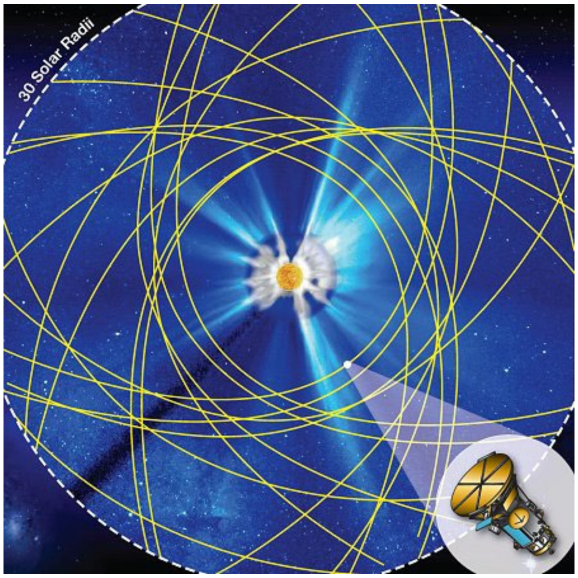
Problem 6 - If the density of the aluminum in the panel is 2.7 grams/cm^3, to the nearest tenth of a kilogram, what is the mass of the finished panel?

Answer: Mass = Density x volume
= 2.7 grams/cm^3 x 34,185 cm^3
= 92,299.5 grams
= **92.3 kilograms**

Problem 7 - To the nearest tenth of a kilogram, how many grams of aluminum were removed to make all of the openings?

Answer: Mass = 2.7 grams/cm^3 x 8,565 cm^3
= 23,125.5 grams
= **23.1 kilograms**

Exploring Interplanetary Dust with the ***Parker Solar Probe***



Spacecraft orbits near the sun

The Parker Solar Probe spacecraft will go closer to the Sun than any manmade object has gone before, which has required the development of new thermal and micrometeoroid protection technologies.

During the 24 solar orbits of the mission, the spacecraft will encounter a thermal environment that is 50 times more severe than any previous spacecraft. It will also travel through a dust environment previously unexplored, and be subject to particle hypervelocity impacts at speeds much faster than anything previously encountered by NASA spacecraft.

The FIELDS instrument on the Parker Solar Probe will make direct measurements of electric and magnetic fields, radio emissions, and shock waves which course through the sun's atmospheric plasma. FIELDS also turns the spacecraft into a giant dust detector, registering voltage signatures when specks of space dust hit the spacecraft's antenna.

Problem 1 – Astronomical models of interplanetary dust predict that near perihelion, the flux of dust grains with a diameter of 1 micron will be about 0.005 dust grains/meter^2 per second. Near aphelion at the orbit of Venus the flux will drop to about 0.0008 dust grains/meter^2 per second. If the Parker Solar Probe spends 10 days near the perihelion distance, and 20 days near aphelion each orbit, about how long between dust grain impacts will it be if the instrument collecting area is 140 cm^2?

Problem 2 – Each impact will produce a crater about 10 microns in diameter. What percentage of the surface area of the detector will be cratered after the mission completes 24 orbits?

Answer Key

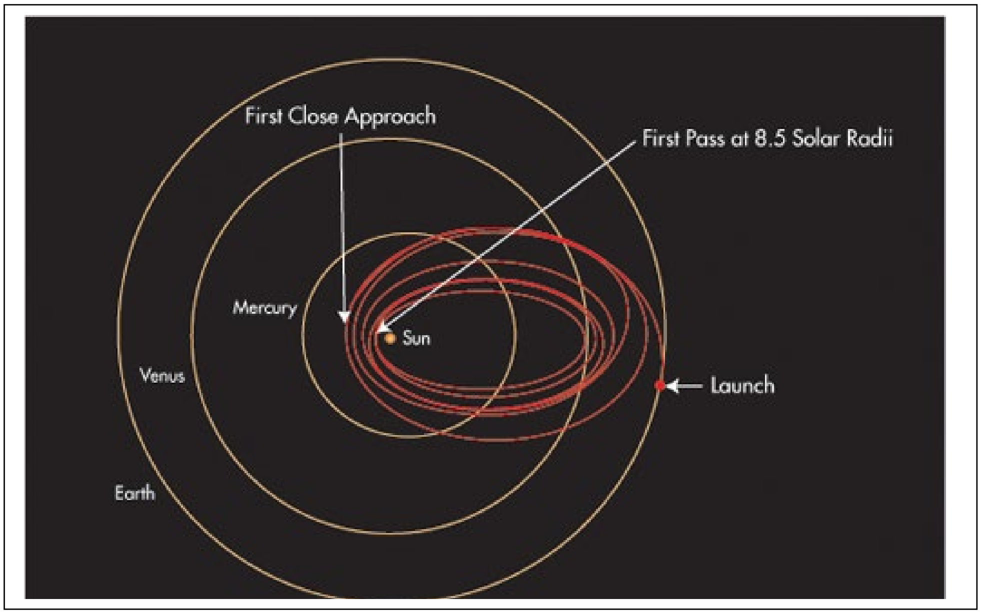
Problem 1 – Astronomical models of interplanetary dust predict that near perihelion, the flux of dust grains with a diameter of 1 micron will be about 0.005 dust grains/meter^2 per second. Near aphelion at the orbit of Venus the flux will drop to about 0.0008 dust grains/meter^2 per second. If the Parker Solar Probe spends 10 days near the perihelion distance, and 20 days near aphelion, about how long between dust grain impacts will it be if the instrument collecting area is 140 cm^2?

Answer: The collecting area is 140 cm^2 x (1 meter^2/10000cm^2) = 0.014 meters^2, so the dust grain fluxes result in 0.005 x 0.014 = 0.00007 impacts per second at perihelion, and 0.00008 x 0.014 = 0.0000011 impacts per second at aphelion. The time between impacts is just the reciprocal of these rates or about 1 impact every 4 hours at perihelion, and one impact every 252 hours at aphelion. During the times spent at each distance, at perihelion there will be about 10days x24hrs /4hrs = **60 impacts**. For the 20 days spent at aphelion, 20 days x 24 hrs/252 hrs = **2 impacts.**

Problem 2 – Each impact will produce a crater about 10 microns in diameter. What percentage of the surface area of the detector will be cratered after the mission completes 24 orbits?

Answer: The area of the detector is 0.014 meters^2. Each circular crater has an area of π (10x10^{-6} meters/2)^2 = 7.9x10^{-11} meters^2. At perihelion there are 60 impacts per orbit so there will be a total of 60x24 = 1440 impacts, each with an area of 7.9x10^{-11} meters^2, for a total area of 1.1x10^{-7} meters^2. The percentage of area cratered is then 100% (1.1x10^{-11}/0.014) = **0.0008 %**.

Exploring Orbital Speeds with the ***Parker Solar Probe***!



The Parker Solar Probe spacecraft will be placed in a series of elliptical orbits around the sun. As the spacecraft travels around the elliptical path, its speed changes from a very fast pace nearest the sun (called the perihelion), to a very slow speed at its farthest point (called the aphelion). The speed at any distance from the sun along the orbit can be computed from the following equation:

$$V = 1.15x10^{10}\sqrt{\frac{2}{r} - \frac{1}{a}}$$

r and a are distances in meters, and v is in meter/sec

Problem 1 - The elliptical orbit of the Parker Solar Probe begins at Earth at aphelion, at a distance of $1.49x10^{11}$ meters, and a perihelion distance of $2.23x10^{10}$ meters. If the elliptical orbit has a semi-major axis of a = $8.57x10^{10}$ meters, what are the spacecraft speeds at perihelion and aphelion in km/sec rounded to 2 significant figures?

Answer Key

r and a are distances in meters, and v is in meter/sec

Problem 1 - The elliptical orbit of the Parker Solar Probe begins at Earth at aphelion, at a distance of 1.49×10^{11} meters, and a perihelion distance of 2.23×10^{10} meters. If the elliptical orbit has a semi-major axis of a = 8.57×10^{10} meters, what are the spacecraft speeds at perihelion and aphelion in km/sec rounded to 2 significant figures?

Answer: Perihelion

$$V = 1.15x10^{10}\sqrt{\frac{2}{2.23x10^{10}} - \frac{1}{8.57x10^{10}}}$$

= $1.15 \times 10^{10} (7.8 \times 10^{-11})^{1/2}$

= 1.0×10^{5} meters/sec

= **100 km/sec.**

Aphelion:

$$V = 1.15x10^{10}\sqrt{\frac{2}{1.49x10^{11}} - \frac{1}{8.57x10^{10}}}$$

= $1.15 \times 10^{10} (1.7 \times 10^{-12})^{1/2}$

= 14,994 m/sec

= **15.0 km/sec.**

Exploring Equations with the **Parker Solar Probe**

Table of mass definitions

System	Formula
Instruments	x
Power	y
Heat Shield	z
Hardware	$\frac{1}{7}x$
Telemetry	$\frac{1}{4}y$
Guidance	$\frac{2}{3}x$
Thermal Control	$\frac{1}{3}x$
Avionics	$\frac{1}{10}y$
Propulsion	$\frac{1}{4}z$
Structure	$\frac{1}{2}y$
Harness	$\frac{1}{4}z$
Fuel	$\frac{5}{8}z$

The Solar Probe-Plus spacecraft consists of 11 separate systems and the fuel for the thrusters. All of these systems have to work together to create a functioning spacecraft that can conduct scientific research.

The table to the left shows each system along with the approximate mass of the system represented by a formula. For example, the Heat Shield has a mass of 'z', and the Propulsion System has a mass of '1/4z'.

The masses of the Instrument, Power and Heat Shield systems have been selected as the reference masses for the other systems in the spacecraft.

Problem 1 - What is the simplified equation for the total mass of the entire payload?

Problem 2 - What algebraic fraction represents the sum of the Instrument, Power and Heath Shield Systems to the total mass of the spacecraft?

Problem 3 - If the mass of the Instrument System is 52 kg, the Power System is 135 kg and the Thermal System is 80 kg, what is the total mass of the spacecraft in kilograms?

Problem 4 - What percentage of the spacecraft is represented by the sum of the Instrument, Power and heat Shield Systems?

Answer Key

Problem 1 - What is the simplified equation for the total mass of the entire payload?

Answer: Add up the equations defining the spacecraft systems by grouping like terms together and simplifying the fractions.

Total mass = $\left(1+\frac{1}{7}+\frac{2}{3}+\frac{1}{3}\right)x+\left(1+\frac{1}{4}+\frac{1}{10}+\frac{1}{2}\right)y+\left(1+\frac{5}{8}+\frac{1}{4}+\frac{1}{4}\right)z$

= $\frac{15}{7}x+\frac{37}{20}y+\frac{17}{8}z$

Problem 2 - What algebraic fraction represents the sum of the Instrument, Power and Heath Shield Systems to the total mass of the spacecraft?

Fraction = $\dfrac{x+y+z}{\frac{15}{7}x+\frac{37}{20}y+\frac{17}{8}z}$

Problem 3 - If the mass of the Instrument System is 52 kg, the Power System is 135 kg and the Thermal System is 80 kg, what is the total mass of the spacecraft in kilograms?

Answer:

Total mass = $\frac{15}{7}(52kg)+\frac{37}{20}(135kg)+\frac{17}{8}(80kg)$ = 531 kilograms

Problem 4 - What percentage of the spacecraft is represented by the sum of the Instrument, Power and heat Shield Systems?

Answer: The sum of the three systems is x+y+z = (52+135+80) = 267 kilograms.

The percentage of the total mass is then P = 100% (267 kg / 531 kg) = 50%

Extra Problem: If y = 3x and z=8/5x, write the total mass of the spacecraft in terms of only the Instrument System mass, x. Express the coefficient of x as a fraction, and as a decimal rounded to the nearest hundredth.

Answer: $\frac{15}{7}x+\frac{37}{20}(3x)+\frac{17}{8}\left(\frac{8}{5}x\right)$ = $\left(\frac{15}{7}+\frac{111}{20}+\frac{17}{8}\right)x$ = $\frac{10996}{1120}x$ = 9.82x

Exploring Solar Energy with the ***Parker Solar Probe***

NASA's mission to the sun will be powered by two different solar panel systems. The first system, shown deployed in the figure, will operate when the spacecraft is between the orbit of Earth and Mercury. These panels will be retracted behind the heat shield to avoid damage. A second solar panel system will then take over and carefully poke out from behind the heat shield to generate just enough electricity to run the 480-watt spacecraft as the spacecraft travels to 9.5 million kilometers from the sun. Why do we need two systems?

Problem 1 – At Earth, located 149 million kilometers from the sun, sunlight delivers 1350 watts of energy to every square meter of surface area, tilted so that the area is exactly perpendicular to the direction of the sunlight. The Parker Solar Probe spacecraft uses two solar panels, each with an area of 1.5 meters^2, that can convert 28% of this sunlight energy into electricity. How much electrical energy will the spacecraft generate in this way?

Problem 2 – The intensity of sunlight increases with distance according to the inverse-square law. At half the distance, the intensity is four times as great. When the spacecraft reaches the orbit of Mercury located 50 million km from the sun, what will be the electrical power produced by the solar panels in Problem 1?

Problem 3 – Suppose the same solar panel system was used when the spacecraft reached its closest distance to the sun of 9.5 million kilometers. How much more intense will the sunlight be at this distance compared to near Earth, and how much power will the panels produce?

Problem 4 – During its journey to the orbit of Mercury, the solar panel system cannot generate more than 1200 watts without damaging the electrical system. How would you design the solar panels so that as the distance to the sun decreased, the panels never generated more than 1200 watts of energy?

Answer Key

Problem 1 – At Earth, located 149 million kilometers from the sun, sunlight delivers 1350 watts of energy to every square meter of surface area, tilted so that the area is exactly perpendicular to the direction of the sunlight. The Parker Solar Probe spacecraft uses two solar panels, each with an area of 1.5 meters^2, that can convert 28% of this sunlight energy into electricity. How much electrical energy will the spacecraft generate in this way?

Answer: 1350 watts/m^2 x 2 panels x 1.5 m^2/panel x 0.28 = **1100 watts**.

Problem 2 – The intensity of sunlight increases with distance according to the inverse-square law. At half the distance, the intensity is four times as great. When the spacecraft reaches the orbit of Mercury located 50 million km from the sun, what will be the electrical power produced by the solar panels in Problem 1?

Answer: The intensity of sunlight will be (149 million/50 million)^2 = **8.9 times as great**, so the electrical power produced will be 8.9 x 1100 watts = **10,000 watts**.

Problem 3 – Suppose the same solar panel system was used when the spacecraft reached its closest distance to the sun of 9.5 million kilometers. How much more intense will the sunlight be at this distance compared to near Earth, and how much power will the panels produce?

Answer: (149 million km/9.5 million km)^2 = **250 times more intense**, and the solar panel power will be 250 x 1100 watts = **280,000 watts**.

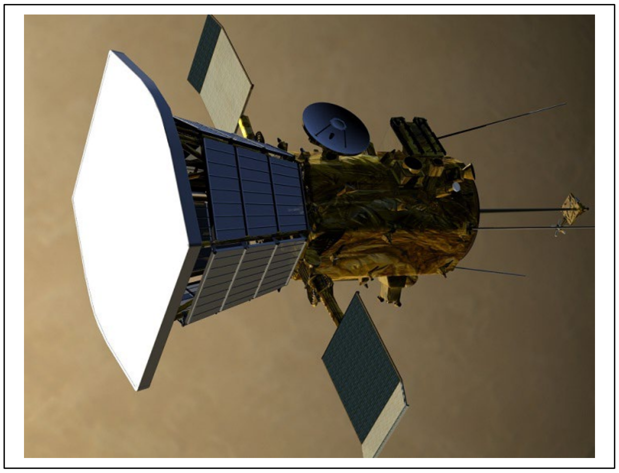
Problem 4 – During its journey to the orbit of Mercury, the solar panel system cannot generate more than 1200 watts without damaging the electrical system. How would you design the solar panels so that as the distance to the sun decreased, the panels never generated more than 1200 watts of energy?

Answer: From Problem 2 we have to reduce the area of the arrays by a factor of 10,000/1200 = 8.3 times. Because we can't change the efficiency of the solar cells (28%), we have to adjust the area of the solar panels that the sun sees. We can do this by 1) using a shade that blocks out some of the surface of the panels, or 2) rotate the panels so that they are no longer perpendicular to the sun.

The first system could involve a window shade that rolls up across the panels changing their area from 1.5 m^2 at Earth to 0.18 m^2 near Mercury.

The second system would rotate the panel from 90 degrees at Earth to a smaller angle near Mercury. From basic geometry, **I** = 10000sin(theta) so to get **I** = 1200 watts, we need theta = 7 degrees. This is almost 'edge on' to the direction of the sunlight. The area of the array that the sun sees at this angle is only 1.5 meter^2 x sin(7) = 0.18 m^2.

Exploring Solar Heating with the ***Parker Solar Probe***



Any surface in space near the sun will heat up as it absorbs solar energy. It will absorb sunlight at all wavelengths, but it can only cool off by emitting infrared radiation, which we experience as 'heat' radiation making the body warm to the touch. The mathematical relationship between the amount of energy it absorbs and the temperature it reaches is given by the Stefan-Boltzman Law:

$$F = 5.67\times10^{-8}\ (1-A)\ T^4$$

Where F is the power per unit area (watts/meter^2), T is its temperature in kelvin units, and A is the reflectivity of the surface.

Problem 1 – At the distance of Earth, F = 1378 watts/meter^2. If the sun-facing heat shield on the spacecraft has a reflectivity of exactly 60%, how hot will the heat shield get in space?

Problem 2 – The previous calculation assumed that the heat shield absorbed radiation from the sun, and only emitted its heat energy from the same face. In fact, both the front and shaded back side of the heat shield are emitting the heat energy, so the correct formula to use is

$$F = 2 \times (5.67\times10^{-8}\ (1-A)T^4)$$

What is the more accurate answer for the heat shield temperature in Problem 1?

Problem 3 – At its closest distance to the sun, the intensity of sunlight will be 250 times what it is near Earth. What will be the heat shield temperature at that point?

Problem 4 – The spacecraft Thermal Control System uses a set of cooling vanes and ablation of material from the heat shield to dump most of the heat energy. If the heat shield has an area of 7.8 meter^2 and the final temperature at the back of the heat shield is 400 kelvins, how many watts of heat energy have to be eliminated by the TCS at perihelion?

Answer Key

Problem 1 – At the distance of Earth, F = 1378 watts/meter^2. If the sun-facing heat shield on the spacecraft has a reflectivity of exactly 60%, how hot will the heat shield get in space?

Answer: $1378 = 5.67 \times 10^{-8} (1-0.60) T^4$

So $T^4 = 1378/(0.4 \times 5.67 \times 10^{-8}) = 6.1 \times 10^{10}$ and so **T = 500 kelvins**.

Problem 2 – The previous calculation assumed that the heat shield absorbed radiation from the sun, and only emitted its heat energy from the <u>same face</u>. In fact, both the front and shaded back side of the heat shield are emitting the heat energy, so the correct formula to use is

$$F = 2 \times (5.67 \times 10^{-8} (1-A)T^4)$$

What is the more accurate answer for the heat shield temperature in Problem 1?

Answer: $1378 = 2 \times 5.67 \times 10^{-8} (1-0.60) T^4$

So $T^4 = 3.1 \times 10^{10}$ and so

T = 420 kelvins.

Problem 3 – At its closest distance to the sun, the intensity of sunlight will be 250 times what it is near Earth. What will be the heat shield temperature at that point?

Answer: $1378 \times 250 = 2 \times 5.67 \times 10^{-8} (1-0.60)T^4$

$T^4 = 7.6 \times 10^{12}$ so

T = 1700 kelvins.

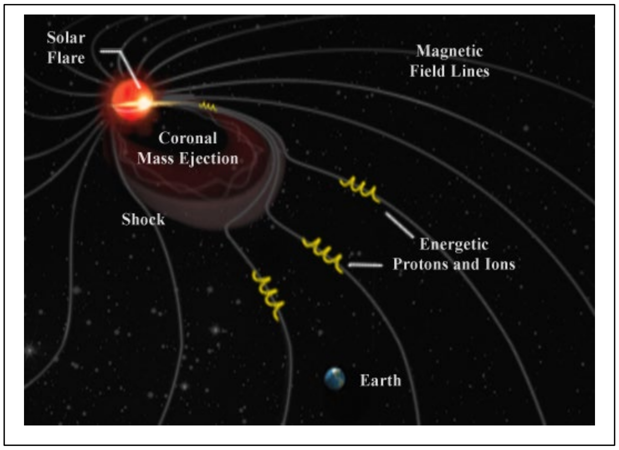
Problem 4 – The spacecraft Thermal Control System uses a set of cooling vanes and ablation of material from the heat shield to dump most of the heat energy. If the heat shield has an area of 7.8 meter^2 and the final temperature at the back of the heat shield is 400 kelvins, how many watts of heat energy have to be eliminated by the TCS at perihelion?

Answer: At perihelion, the solar energy is 250 x 1378 watts/meter^2 = 340000 watts/m^2. The power received by the heat shield is then P = 340000 watts/m^2 x 7.8 meters^2 = 2,700,000 watts.

The back of the heat shield radiates $5.67 \times 10^{-8} (1-0.6)(400)^4 = 580$ watts/meter^2, so for 7.8 meters^2 area this is 4500 watts.

So the TCS has to eliminate 2,700,000 – 4500 = **2,700,000 watts of heat energy!**

Exploring Proton Storms with the ***Parker Solar Probe***



Solar Proton Events (SPEs) are intense bursts of high-energy protons created as dense clouds of plasma are ejected from the sun. The Parker Solar Probe will have a ring-side seat to observe how these events are caused. This will help scientists create better models to forecast when they will occur, and how damaging they will be to expensive satellites when they arrive at Earth.

SPEs have been observed at Earth for decades and carefully measured. The following table shows the seven events that occurred during 2011. The intensity is indicated by the number of particles per second that pass through a given square-centimeter of area at Earth orbit. Each event lasts about 2 hours. There were no events between 2007-2010

Date	Intensity	Date	Intensity
March 8, 2011	50	August 9, 2011	26
March 21, 2011	14	September 23, 2011	35
June 7, 2011	72	November 26, 2011	80
August 4, 2011	96		

Problem 1 – The intensity of SPEs is known to approximately follow an inverse-square law. If Earth is 149 million kilometers from the sun, and the Parker Solar Probe spacecraft will be as close as 9.5 million kilometers, how much more intense will the tabulated SPEs be for the Parker Solar Probe?

Problem 2 – If the Parker Solar Probe will spend about 8 months of its mission closest to the sun in its orbit, about how many events will it encounter more intense than 15,000 units?

Problem 3 – SPEs are known to damage solar panels on satellites in orbit around Earth. If the Parker Solar Probe loses 2% of its power for events stronger than 15,000, how much power will the spacecraft lose in 7 years?

Answer Key

Problem 1 – The intensity of SPEs is known to approximately follow an inverse-square law. If Earth is 149 million kilometers from the sun, and the Parker Solar Probe spacecraft will be as close as 9.5 million kilometers, how much more intense will the tabulated SPEs be for the Parker Solar Probe?

Answer: $(149/9.5)^2$ = 246 times greater, so the events will span a range from 14x246 = 3,444 pFUs to 96x246= 23,616 pFUs.

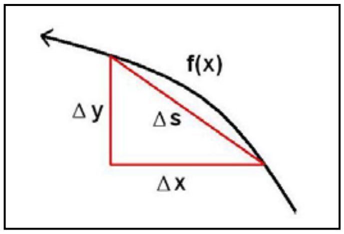
Problem 2 – If the Parker Solar Probe will spend about 6 months of its mission closest to the sun in its orbit, about how many events will it encounter more intense than 15,000 units?

Answer : The table shows 3 events brighter than (58x256=15000) in 12 months, so you have about 1 event every 4 months, and so in 8 months you will get about 2 events.

Problem 3 – SPEs are known to damage solar panels on satellites in orbit around Earth. If the Parker Solar Probe loses 2% of its power for events stronger than 15,000, how much power will the spacecraft lose in 7 years?

Answer: At about 1 event every 4 months you have 21 events in 7 years, so the loss is 42%.

Calculating Arc Lengths of Simple Functions- I



Spirals are found in many different places in astronomy, from the shape of the arms in a 'spiral' galaxy, to the trajectory of a spacecraft traveling outward from Earth's orbit at constant velocity. Figuring out spiral lengths requires a bit of calculus. Here's how it's done:

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Step 1: Study the figure above, and use the Pythagorean Theorem to determine the hypotenuse length in terms of the other two sides. It should look like the equation to the left.

$$\Delta s = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$$

Step 2: Factor out the **Δx** to get a new formula.

$$S = \int_A^B \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

Step 3: Following the basic techniques of calculus, 'take the limit' and allow the deltas to become differentials, then use the integral calculus to sum-up all of the differentials along the curve defined by y = F(x), and between points A and B, to get the fundamental arc-length formula.

$$S = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$$

The arc length formula can be re-written in polar coordinates too. In this case, the function, y = F(x) has been replaced by the polar function **r(θ).**

Problem 1) Find the arclength for the line y = mx + b from x=3 to x=10.

Problem 2) Find the arclength for the parabolic arc defined by $y = x^2$ from x=1 to x=5.

Problem 3) Find the arclength for the logarithmic spiral $R(\theta) = e^{b\theta}$ from $\theta = 0$ to $\theta = 4\pi$ if b = 1/2.

Problem 4) The spiral track on a CDROM is defined by the simple formula $R = k\theta/2\pi$, where k represents the width of each track of data. If k = 1.5 microns, how long is the spiral track, in meters, for a standard r =6.0-cm disk if only the radius between 2.5cm and 5.8 cm is used? (use π = 3.14159)

Answer Key

Problem 1) $dy/dx = m$, so the integrand becomes $(1 + m^{\mathbf{2}})^{\mathbf{1/2}}\,dx$. Because m is a constant independent of x, the integral is just $(1 + m^{\mathbf{2}})^{\mathbf{1/2}}$ $(10 - 3)$ = $\mathbf{7\,(1+ m^{2})^{1/2}}$**.**

Problem 2) $dy/dx = 2x$ and the integrand becomes $(1 + 4x^{\mathbf{2}})^{\mathbf{1/2}}\,dx$. This can be integrated by using the substitution $2x = \sinh(u)$, and $dx = (1/2)\cosh(u)\,du$, so that the integrand becomes $1/2\,\cosh^{\mathbf{2}}(u)\,du$. This is a fundamental integral with the solution **1/2 [sinh(2u)/4 + u/2 + C]**.

Limits: The limits go from x=1 to x=5, but since $x = 1/2\,\sinh(u)$, the limits re-expressed in terms of u become $u_{\mathbf{1}} = \sinh^{\mathbf{-1}}(2) = 1.44$ and $u_{\mathbf{2}} = \sinh^{\mathbf{-1}}(10) = 3.00$ so evaluating the definite integral leads to 1/2 (sinh(6.00)/4 + 3.00/2) - 1/2 (sinh(2.88)/4 + 1.44/2) = 25.96 - 1.47 = 24.49. Because the limits are provided to three significant figures, the answer to the same number of significant figures will be 24.5

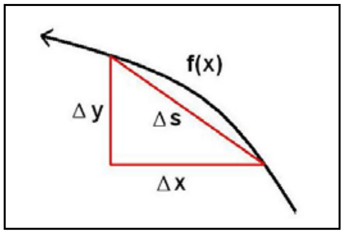
Problem 3) We use the polar form of the arclength formula. First perform the differentiation of $r(\theta)$ to get $dr/d\theta = b\,e^{\mathbf{b\theta}}$. Then after substitution, the integrand becomes $(e^{\mathbf{2b\theta}} + b^{\mathbf{2}}e^{\mathbf{2b\theta}})^{\mathbf{1/2}}\,d\theta$ which after simplification then becomes $e^{\mathbf{b\theta}}\,(1 + b^{\mathbf{2}})^{\mathbf{1/2}}\,d\theta$. This is easily integrated to get $(1/b)\,(1 + b^{\mathbf{2}})^{\mathbf{1/2}}\,e^{\mathbf{b\theta}}$ + **C**. Since b = 1/2, we get the simpler form $2.24\,e^{\mathbf{\theta/2}} + C$. This can be evaluated between the two limits for θ to get 2.24 (534.86 - 1) = 1,195.85. Because the limits are provided to three significant figures, the answer to the same number of significant figures will be 1,200.

Problem 4) The radial function is designed so that every 2π radians, the radius advances by k units in length. Because $r = k\theta/2\pi$, the integrand becomes $((k/2\pi)^2\theta^2 + (k/2\pi)^2)^{\mathbf{1/2}}\,d\theta$ or $(k/2\pi)\,(1 + \theta^2)^{\mathbf{1/2}}\,d\theta$. This can be simplified using the hyperbolic trig identity $1 + \sinh^{\mathbf{2}}(x) = \cosh^{\mathbf{2}}(x)$ where we have used the substitution $\theta = \sinh(x)$. This also means that $d\theta = \cosh(x)\,dx$. Then the integrand becomes $\cosh^{\mathbf{2}}(x)\,dx$. The integral is then a fundamental integral with the solution (**k/2π) [sinh(2x)/4 + x/2 + C] which as for all indefinite integrals includes the constant of integration C**.

Limits: How many radians does the spiral take up? Lower integral limit: $\theta = 2\pi \times 2.5$ cm/1.5 microns = $33{,}333\,\pi$. Upper integral limit: $\theta = 2\pi \times 5.8$ cm/1.5 microns = $77{,}333\,\pi$. But $\theta = \sinh(x)$ so the limits become x(lower) = $\sinh^{\mathbf{-1}}(33{,}333\,\pi) = 12.252$ to x(upper) = $\sinh^{\mathbf{-1}}(77{,}333\,\pi) = 13.094$.

The definite integral is then (1.5 microns/2π) x [(sinh(26.188)/4 +13.094/2 + C) - (sinh(24.504)/4 + 12.252/2 + C) = (0.0000015 meters/2π) (2.95 - 0.548) x 10^{10} = 5734 meters or 5.734 kilometers. Because the problem only gives 1.5 and 6.0 to two significant figures, this becomes the maximum accuracy of the available numbers, so the answer to the same number of significant figures will be **5.7 kilometers!**

The Ant and the Turntable - Frames of Reference



$$s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Figuring out spiral lengths requires a bit of calculus. In the previous problem, we saw that the arc length integral can be written in polar coordinates where the function, y = F(x) is replaced by the polar function **r(θ).**

Because this formula is completely general, the variable, θ, can refer to angle, time or any other independent variable which leads to an arc-like geometry in the dependent variable defined by r(θ). Here is an interesting application.

An ant takes a journey from the center of a CDrom (r=0) to a point at its edge (r=5cm) at a leisurely pace of 5 cm/minute. Meanwhile, the turntable is spinning at 1 rotation per minute.

Problem 1 - Draw a sketch, to scale, of the turntable that shows the ant's motion from its own, stationary, perspective.

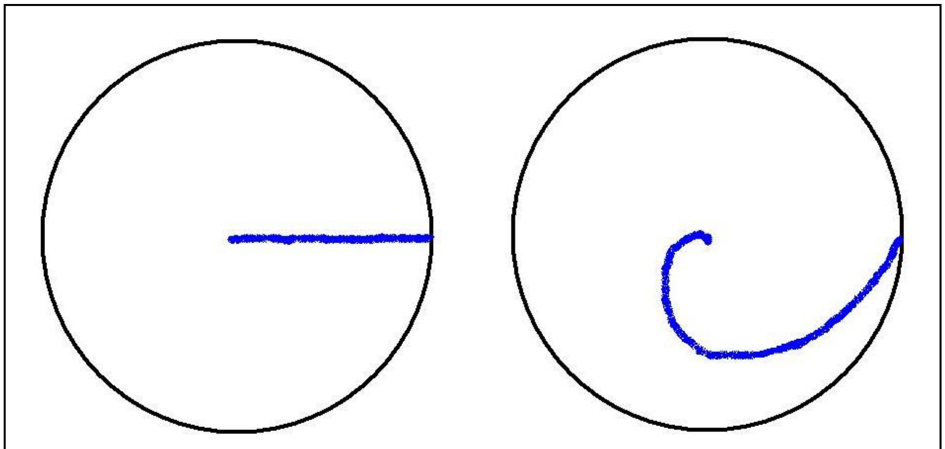
Problem 2 - Draw a free-hand sketch, to scale, of the turntable that shows the ant's motion from the perspective of an outside, stationary observer.

Problem 3 - What is the equation that describes the radial motion of the ant with respect to time?

Problem 4 - How far did the ant travel in the radial direction in A) 15 seconds?, B) 60 seconds?

Problem 5 - Evaluate the arc length integral formula for S(t) to determine the length of the ant's arc between r=0 and r = 5 centimeters.

Inquiry: Explain A) how it can be that there are two path lengths and travel times in the problem, but there is only one ant? and B) which of the two answers is correct?



Problem 3) **R(T) = (5 cm/minute) T** where T is given in minutes of time.

Problem 4) A) 0.25 minutes x 5 cm/minute = 1.25 centimeters,
B) 1 minute x 5 cm/minute **= 5 centimeters.**

Problem 5) dr/dT= 5 cm/m so the integrand becomes by substitution, $dS = (r^2 + (dr/dt)^2)^{1/2}$ or $dS = ((5 T)^2 + 5^2)^{1/2} dT$. This simplifies to $dS = 5 (1 + T^2)^{1/2} dT$. The integral then becomes,

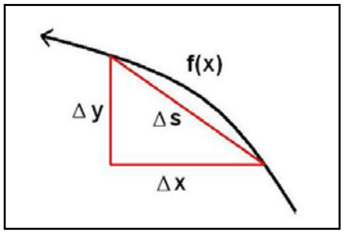
$$S(t) = 5\int_0^t \sqrt{1 + T^2}\, dT$$

This can be integrated by using the substitution T = sinh(x) where dT = cosh(x) dx so that $(1 + T^2)^{1/2}$ dT becomes cosh(x) cosh(x) dx, or $\cosh^2(x)dx$. The integral is a fundamental form whose solution is sinh(2x)/4 + x/2 + C. Evaluating from T =0 to T=t one gets the general arc length formula for the ant of S(t) = 5 cm/m x (sinh(2t)/4 + t/2) where t is the elapsed time expressed in minutes.

To travel from r=0 to r=5cm, the ant takes 1 minute, so S(1) = 5 cm/minute x (3.63/4 + 1/2) = 5 cm/m (1.4 minutes) = 7.0 centimeters.

Inquiry: There are two reference frames, and the movement of the ant will appear different to the two observers. Only in the 'proper' frame of reference locked to the ant will the movement of the ant appear to be 'simple' as it travels in a straight line from the center to the edge of the CDROM according to R(T) = 5 T. In all other 'improper' reference frames, the motion will appear more complicated, and trace out more complex paths because of the action of fictitious forces. For example, if you are sitting in a moving car on the freeway and you drop a ball to the floor, from your perspective, it travels 1/2 meter straight down. For someone standing by the side of the road, the ball travels in a long 100-meter arc.

B) They are both correct, but they apply to two different frames of reference who are seeing the movement differently because of their relative state of motion.

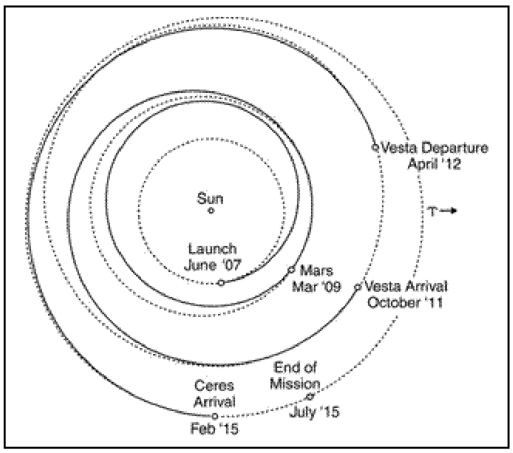


Ion rocket motors provide a small but steady thrust, which causes a spacecraft to accelerate. The shape of the orbit for the spacecraft as it undergoes constant acceleration is a spiral path. The length of this path can be computed using calculus.

$$s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

The arc length integral can be written in polar coordinates where the function, y = F(x) is replaced by the polar function **r(θ).**

Because the integrand is generally a messy one for most realistic cases, in the following problems, we will explore some simpler approximations.



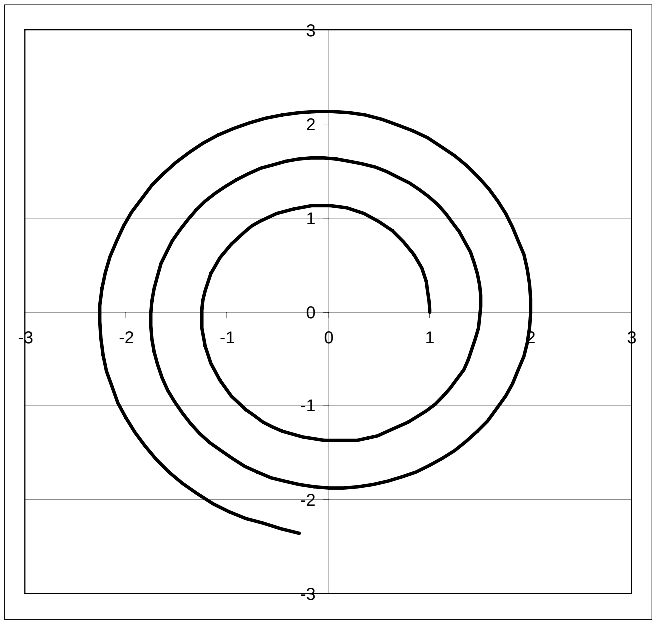
The Dawn spacecraft was launched on September 27, 2007, and will take a spiral journey to visit the asteroid Vesta in February 2015. Earth is located at a distance of 1.0 Astronomical Units from the Sun (1 AU = 150 million kilometers) and Vesta is located 2.36 AU from the Sun. The journey will take about 66,000 hours and make about 3 loops around Earth's orbit in its outward spiral as shown in the figure to the left.

Problem 1 - Suppose that the Dawn spacecraft travels at a constant outward speed from Earth's orbit. If we approximate the motion of the spacecraft by $X = R \cos\theta$, $Y=R\sin\theta$ and $R = 1 + 0.08\,\theta$, where the angular measure is in radians, show that the path taken by Dawn is a simple spiral.

Problem 2 - From the equation for R(θ), compute the total path length of the spiral from R=1.0 to R = 2.36 AU, and give the answer in kilometers. About what is the spacecraft's average speed during the journey in kilometers/hour? [Note: Feel free to use a Table of Integrals!]

Problem 3 - The previous two problems were purely 'kinematic' which means that the spiral path was determined, not by the action of physical forces, but by employing a mathematical approximation. The equation for R(θ) is based on constant-speed motion, and not upon actual accelerations caused by gravity or the action of ion engine itself. Let's improve this kinematic model by approximating the radial motion by a uniform acceleration given by $R(\theta) = 1/2\, A\, \theta^2$ where we will approximate the net acceleration of the spacecraft in its journey as A = 0.009. What is the total distance traveled by Dawn in kilometers, and its average speed in kilometers/hour?

Problem 1) Answer computed using Excel spreadsheet.

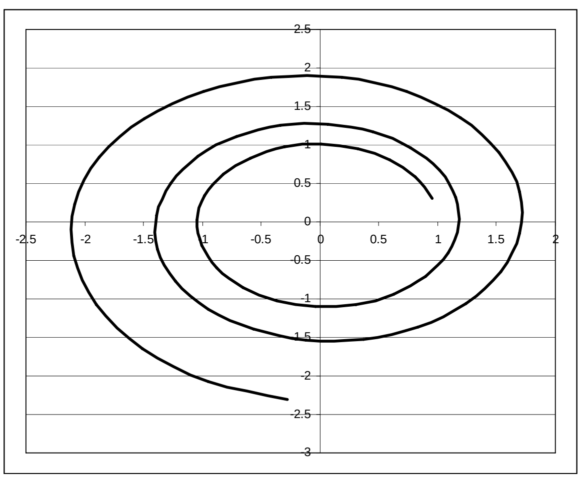


Problem 2: $R = 1.0 + 0.08\,\theta$ and so $dR/d\theta = 0.08$ and $d\theta/dR = 12.5$. The integrand becomes $(1+156R^2)^{1/2}\,dR$.

If we use the substitution $U = 12.5R$ $dU = 12.5\,dR$ and the integrand becomes $0.08\,(1 + U^2)^{1/2}\,dU$. A table of integrals yields the answer

$1/2\,[\,U\,(1 + U^2)^{1/2} + \ln(U + (1+U^2)^{1/2}\,)$.

The limits to the integral are Ui = 12.5 x 1.0 = 12.5 and Uf = 12.5 x 2.36 = 29.5, and when the integral is evaluated we get 1/25 [29.5 (29.5) + ln (29.5 + (29.5)) - 12.5 (12.5) - ln(12.5 + (12.5))] = 1/25 (870 + 4.1 - 156 - 3.2) = 28.6 Astronomical Units or 28.6 x 150 million km = 4.3 billion kilometers! The averages speed would be about 4.3 billion/66000 hrs = **65,100 kilometers/hour.**



Problem 3 - $dR/d\theta = A\,\theta$ so that

$d\theta/dR = 1/(A\,\theta)$.

From $R(\theta)$, we can re-write $d\theta/dR$ solely in terms of R as $d\theta/dR = (1/(2Ar))^{1/2}$ so that the integrand becomes $(1 + R/(2A))^{1/2}\,dR$.

Unlike the integral in Problem 1 ,this integral can be easily performed by noting that if we substitute

$U = 1 + R/(2A)$, and $dU = dR/2A$,

we get the integrand $2A\,U^{1/2}\,dU$ and so
$S = (4A/3)\,U^{3/2} + C$.

The limits to this integral are Ui = 1 + 1.0/2A = 56. and Uf = 1 + 2.36/2A = 132.

Then the definite integral becomes
$S = (4 \times 0.009/3)\,[\,132^{3/2} - 56^{3/2}] = 0.012\,[1516 - 419] = 13.2$ AU .Since 1 AU = 150 million km, the spiral path has a length of **2.0 billion kilometers.** The averages speed would be about 2.0 billion km/66000 hours = **30,300 km/hour**. The trip takes less time because the 'kinematic' motion is speeded up towards the end of the journey.

Space Shuttle Launch Trajectory - I

The trajectory of the Space Shuttle during the first 5 minutes of the launch of STS-30 can be represented by an equation for its altitude

$$h(T) = 2008 - 0.047\, T^3 + 18.3\, T^2 - 345T$$

and an equation for its down-range distance due-east

$$R(T) = 4680\, e^{0.029T}$$

where the distances are provided in units of feet commonly used by NASA engineers for describing trajectories near Earth. The problems below will use these 'parametric equations of motion' to determine the time of the highest acceleration.

Image: Shuttle launch as seen from a NASA aircraft.

Problem 1 - Use the parametric equations for h(T) and R(T) to determine the equation for the speed, S, of the Shuttle along its trajectory where $dS/dt = ((dh/dt)^2 + (dR/dt)^2)^{1/2}$

Problem 2 - Determine the formula for the magnitude of the acceleration of the Shuttle using the second time derivatives of the parametric equations.

Problem 3 - From your answer to Problem 2, A) find the time at which the acceleration is an extremum, and specifically, a maximum along the modeled trajectory. B) What is the acceleration in feet/sec^2 at this time? C) If the acceleration of gravity at the earth's surface is 32 feet/sec^2, how many 'Gs' did the astronauts pull at this time?

Problem 1 - Use the parametric equations for h(T) and R(T) to determine the equation for the speed, S, of the Shuttle along its trajectory where $dS/dt = ((dh/dt)^2 + (dR/dt)^2)^{1/2}$

$dh/dt = - 0.142 T^2 + 36.6 T - 345$

$dR/dt = 135.7 e^{0.029T}$

Then

$$\mathbf{dS/dt = ((- 0.142 T^2 + 36.6 T - 345)^2 + (135.7 e^{0.029T})^2)^{1/2}}$$

Problem 2 - The components of the acceleration vector, a, are given by
$a_h = d^2h/dT^2$ and $a_R = d^2R/dT^2$

so $D^2h/dt^2 = 36.6 - 0.284T$ $d^2R/dt^2 = 3.93 e^{0.029T}$

Then the magnitude of the acceleration vector is just
$|a| = (a_h{}^2 + a_R{}^2)^{1/2} = \mathbf{(0.08 T^2 - 20.8 T + 1339 + 15.4 e^{0.058 T})^{1/2}}$

Problem 3 - A) The minimum or maximum acceleration is found by solving for T when d |a|/dT = 0

$0 = \mathbf{1/2 (0.08 T^2 - 20.8 T + 1339 + 15.4 e^{0.058 T})^{-1/2} (0.16T - 20.8 + 0.89e^{0.058T})}$

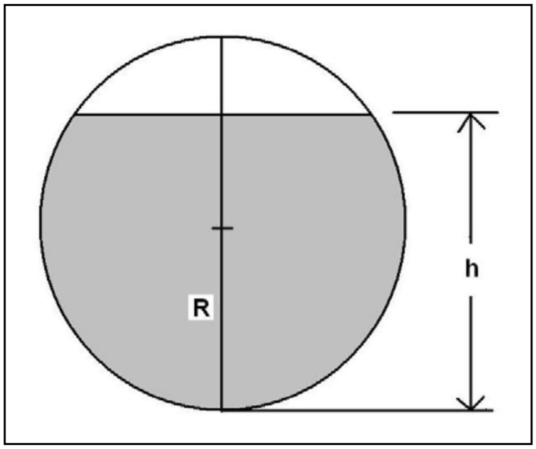
Then $\mathbf{0 = 0.16T - 20.8 + 0.89e^{0.058T}}$ or $\mathbf{20.8 = 0.16 T + 0.89e^{0.058T}}$

A calculator can be used to plot this curve and determine that for **T = 46.7 seconds** the condition d |a|/dT = 0 is obtained. This is near the time usually recorded by engineers as the Maximum Dynamic Pressure point 'Max-Q'.

B) Evaluating the answer to Problem 2 at T=46.7 seconds we get $\mathbf{|a| = (773)^{1/2} = 28}$ **feet/sec^2.**

C) The number of Gs = 28/32 = **0.9 Gs**.

Fuel Level in a Spherical Tank



Spherical tanks are found in many different situations, from the storage of cryogenic liquids, to fuel tanks. Under the influence of gravity, or acceleration, the liquid will settle in a way such that it fills the interior of the tank up to a height, h. We would like to know how full the tank is by measuring h and relating it to the remaining volume of the liquid. A sensor can then be designed to measure where the surface of the liquid is, and from this derive h.

Problem 1 - Slice the fluid into a series of vertically stacked disks with a radius r(h) and a thickness dh. What is the general formula for the radius of each disk?

Problem 2 - Set up the integral for the volume of the fluid and solve the integral.

Problem 3 - Assume that fluid is being withdrawn from the tank at a fixed rate $dV/dt = -F$. What is the equation for the change in the height of the fluid volume with respect to time? A) Solve for the limits $h \ll R$ and $h \gg R$. B) Solve graphically for R=1 meter, F=100 cm^3/min. (Hint: select values for h and solve for t).

Answer Key

Problem 1 - $r(h)^2 = R^2 - (R-h)^2$ so $r(h)^2 =$ $\mathbf{2Rh - h^2}$

Problem 2 - The integrand will be $\pi (2Rh - h^2)\, dh$ and the solution is $\mathbf{\pi R h^2 - 1/3 \pi h^3}$

Problem 3 -

$$\frac{dV}{dt} = 2 \pi R h \frac{dh}{dt} - \pi h^2 \frac{dh}{dt} \quad \text{so} \quad \frac{dV}{dt} = (2 \pi R h - \pi h^2) \frac{dh}{dt} = -F$$

Then

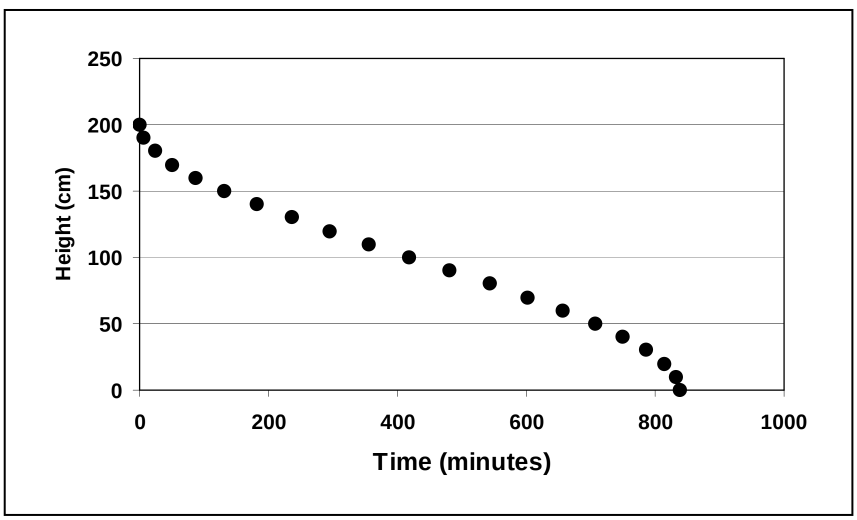
$$\frac{dh}{dt} = \frac{-F}{(2 \pi R h - \pi h^2)}$$

The integrands become: $(2 \pi R h - \pi h^2)\, dh = -F\, dt$. This can be integrated from t=0 to t = T to obtain $\pi R h^2 - 1/3 \pi h^3 = -F T$ and simplified to get

$$\mathbf{h^3 - 3 R h^2 - (3 F T)/\pi = 0}$$

We would normally like to invert this equation to get h(T), but cubic equations of the form $x^3 - \alpha x^2 + \beta = 0$ cannot be solved analytically. We can solve it for two limiting cases. Case 1 for a tank nearly empty where h << R. This yields $\mathbf{h(T) = (F T/R)^{1/2}}$ Case 2 is for a tank nearly full so that h >> R, and we get $h^3 = 3 F T/\pi$ and $\mathbf{h(T) = (3 F T/\pi)^{1/3}}$. The full solution for h(T) can be solved graphically. Since R is a constant, we can select a new variable U = h/R and re-write the equation in terms of the magnitude of h relative to the radius of the tank.

$U^3 - 3 U^2 = (3 F T)/\pi R^3$ and plot this for selected combinations of (U,T) where time, T, is the dependent variable. The solution below is for F = 100 cm^3/minute, R = 1 meter, with the intervals in h spaced 10 cm. The plot was generated using an *Excel* spreadsheet.



Optimization

Satellites are designed to optimize the number of experiments they can carry, while at the same time keeping the mass and power requirements at a minimum. The picture shows the IMAGE satellite with the dark solar cells attached to its surface.

Here is one example of a simple problem that can be encountered by a satellite designer.

A satellite is designed to fit inside the nose-cone (shroud) of a Delta II rocket. There is only enough room for a single satellite, and it cannot have deployable solar panels to generate electricity using solar cells. Instead, the solar cells have to be mounted on the exterior surface of the satellite. At the same time, the satellite configuration is that of a hexagonal prism. The total volume of the satellite is 10 cubic meters. The solar cells will be mounted on the hexagonal top, bottom, and the rectangular side panels of the satellite.

Problem 1 - If the width of a panel is W, and the height of the satellite is H, what are the dimensions of the satellite that minimize the surface area and hence the available power that can be generated by the solar cells?

Problem 2 - If only 1/2 of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the power that this satellite can provide to the experiments and operating systems?

Problem 3 - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite?

Problem 4 – If the launch cost is $10,000 per pound, how much will it cost to place this satellite into orbit? (Note, 1 pound = 0.453 kilograms)

Problem 1 - If the width of a panel is W, and the height of the satellite is H, what are the dimensions of the satellite that minimize the surface area and hence the available power that can be generated by the solar cells?

Answer:
The volume of a hexagonal prism is given by

$$V = \frac{3\ (3)^{1/2}\ H\ W^2}{2}$$

The surface area is given by

$$A = 6WH + 3\ (3)^{1/2}\ W^2$$

Since V = 10 cubic meters, we can solve for H to get

$$H = \frac{20}{3\ (3)^{1/2}\ W^2}.$$

Eliminate H from the equation for surface area to get

$$A = \frac{40}{(3)^{1/2}\ W} + 3\ (3)^{1/2}\ W^2$$

Differentiate A with respect to W, set this equal to zero.

$$\frac{40}{(3)^{1/2}\ W^2} = 6\ (3)^{1/2}\ W$$

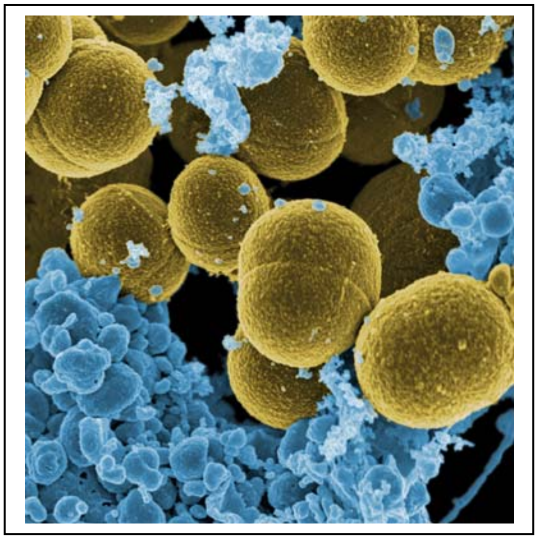
A bit of algebra gives us $W^3 = 40/(6*3)$ so **W = 1.3** meters and from the definition for H we have **H = 2.3 meters**.

Problem 2 - If only 1/2 of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the maximum power that this satellite can provide to the experiments and operating systems? Answer: The total surface area is $A = 6\ (1.3)\ (2.3) + 3\ (3)^{1/2}\ (1.3)^2 = 17.9 + 8.8 = 26.7$ square meters. Only ½ are available for sunlight, and so the total power will be about 26.7 x 0.5 x 100 = **1,335 watts**.

Problem 3 - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite? Answer: 3 x 26.7 = **80.1 kilograms.**

Problem 4 – If the launch cost is $10,000 per pound, how much will it cost to place this satellite into orbit? (Note, 1 pound = 0.453 kilograms). Answer: The cost to launch is 80 kilograms x 10,000 dollars/pound x (1 pound/ 0.453 kg) = $800,000/0.453 = **$1.8 million dollars.**

Estimating Maximum Cell Sizes



A simple living cell generates wastes from the volume of cytoplasm inside its cell wall, and passes the wastes out side its wall by passive diffusion.

If a cell cannot remove the waste fast enough, toxins will build up that eventually disrupt cellular functioning. The balance between waste generation and diffusion, therefore, determines how much volume a cell may have and therefore its typical size.

Photo: *S. aureus* bacteria escaping destruction by human white blood cells. (Credit: NIAID / RML)

Suppose the cell has a spherical volume, and that it generates waste at a rate of **a** molecules per cubic micron per second. Suppose it removes the waste through its surface by passive diffusion at a rate of b molecules per square micron per second, where 1 micron is 0.000001 meters.

Problem 1 - What is the equation that defines the rate, R, at which the organism changes the amount of its net waste products?

Problem 2 - For what value for the cell's radius will the net change be zero ,which means the cell is in equilibrium?

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Problem 4 - Two organisms are discovered that have a size of 1 micron and 10 microns. A) How does the ratio of their diffusion rates compare? B) If the surface waste diffusion rates are the same, how do their metabolic rates of waste production compare?

Answer Key

Problem 1 - What is the equation that defines the rate, R, at which the organism changes its net waste products?

Answer:

$$R = \frac{4}{3}\pi r^3 a - 4\pi r^2 b$$

Problem 2 - At what value for the cell's radius will the net change be zero?
Answer: Set R = 0, and then solve for r in terms of *a* and *b* to get:

$$\frac{4}{3}\pi r^3 a = 4\pi r^2 b$$

$$\frac{ra}{3} = b$$

$$r = \frac{3b}{a}$$

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Answer: r = 3 (2000/800) = 7.5 microns in radius.

Problem 4 - Two organisms are discovered that have a size of 1 micron and 10 microns.

A) How does the ratio of their diffusion rates compare? Answer: The smaller organism has 1/10 the ratio of the diffusion rates, b / a, as the larger organism.

B) If the surface diffusion rates are the same, how do their metabolic rates of waste production compare? Answer: If the surface waste diffusion rate, b, is the same, and the smaller organism has the lower ratio, then the smaller organism must have 10 times the waste production rate, a, if only passive diffusion is involved.