National Aeronautics and Space Administration

Math Problems Related to

Technology - II

Introduction

This book provides an assortment of 69 mathematics problems that cover aspects of technology such as rocket launches, telescopes, unit conversions, solar power, orbit decay and using scales to analyze images among other topics.

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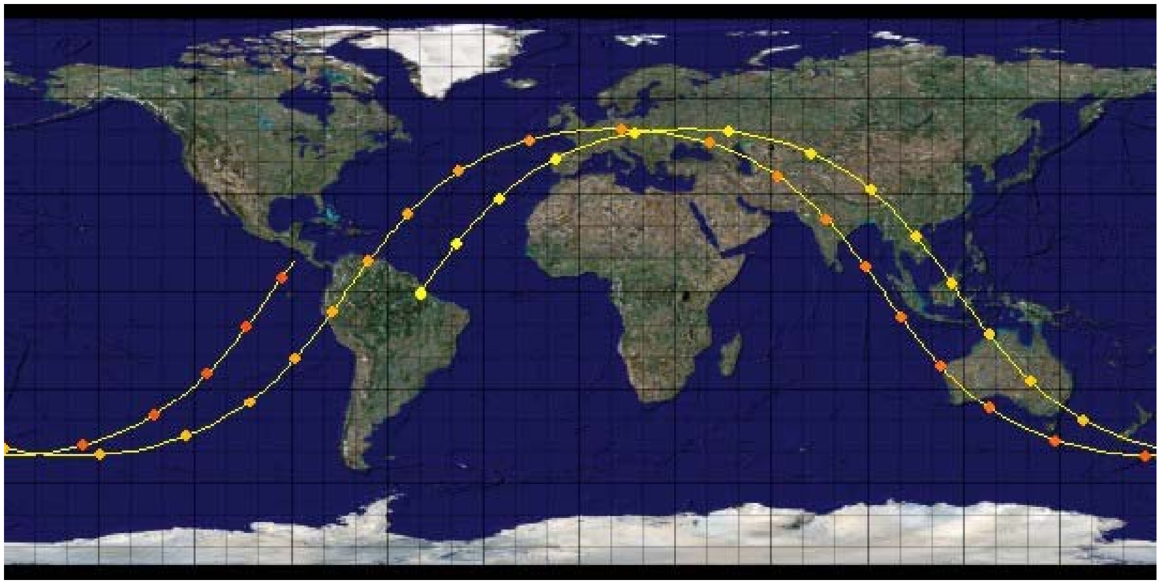
The Ground Track of the International Space Station



As the ISS orbits Earth every 90 minutes, Earth rotates 'underneath' the orbit of the ISS once every 24 hours. This means that each ISS orbit advances in longitude by a fixed amount in longitude. The figure above shows the ground track of the ISS for two orbits. Each square is 10 degrees on a side, with the Equator running horizontally across the middle of the diagram. The squares are shown at intervals of 5 minutes.

Problem 1 - If Earth rotates 360 degrees in 24 hours, by how many degrees does it shift during the orbit of the ISS?

Problem 2 – How many sunsets and sunrises will the SAGE-III instrument on the ISS observe each 24-hour day?

Problem 3 – How many orbits will it take before the ISS passes over the same spot on the Equator?

Problem 4 - During each orbit, the ISS will cross the Equator traveling north to south, then after ½ orbit (45 minutes) it will cross the Equator traveling south to north. How long will you wait before you see the ISS from the ground traveling north-to-south across the sky directly overhead?

Problem 1 - If Earth rotates 360 degrees in 24 hours, by how many degrees does it shift during the orbit if the ISS?

Answer: 360/24 = X/1.5 so **X = 22.5 degrees**.

Problem 2 – How many sunsets and sunrises will the SAGE-III instrument on the ISS observe each 24-hour day?

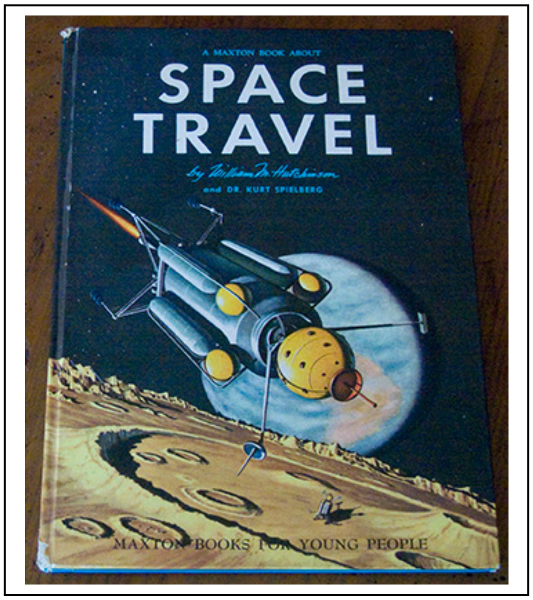
Answer: 24 h /1.5 h = **16 sunrises, and an equal number of sunsets** for a total of 32 events. A sunrise is followed by a sunset every 90/2 = 45 minutes!

Problem 3 – How many orbits will it take before the ISS passes over the same spot on the Equator?

Answer: The orbit advances 22.5 degrees in longitude every orbit, so it will take 360/22.5 = 16 orbits or one full day to return to the same longitude. However, because the ISS crosses the equator twice every orbit, it actually takes only **8 orbits or 12 hours**.

Problem 4 - During each orbit, the ISS will cross the Equator traveling north to south, then after ½ orbit (45 minutes) it will cross the Equator traveling south to north. How long will you wait before you see the ISS from the ground traveling north-to-south across the sky directly overhead?

Answer: Every **16 orbits** as seen from the ground, the ISS is traveling in the same direction, so you will have to wait **24 hours**.



Most science fiction stories often have spaceships with powerful, or exotic, rockets that can let space travelers visit the distant planets in less than a day's journey. The sad thing is that we are not quite there in the Real World. This is because our solar system is so vast, and our rockets can't produce quite enough speed to make journeys short.

NASA has been working on this problem for over 50 years and has come up with many possible solutions. Each one is more expensive than just using ordinary fuels and engines like the ones used on most rockets!

Problem 1 – The entire International Space Station orbits Earth at a speed of 28,000 kilometers per hour (17,000 mph). At this speed, how many days would it take to travel to the sun from Earth, located at a distance of 149 million kilometers?

Problem 2 – The planet Neptune is located 4.5 billion kilometers from Earth. How many years would it take a rocket traveling at the speed of the International Space Station to make this journey?

Problem 3 – The fastest unmanned spacecraft, Helios-2, traveled at a speed of 253,000 km/hr. In the table below, use proportional math to fill in the travel times from the sun to each planet traveling at the speed of Helios-2. Give your answers to the nearest tenth in appropriate units of days or years.

Planet	Distance in millions of kilometers	Time
Mercury	57	
Venus	108	
Earth	149	
Mars	228	
Jupiter	780	
Saturn	1437	
Uranus	2871	
Neptune	4530	

Problem 1 – The entire International Space Station orbits Earth at a speed of 28,000 kilometers per hour (17,000 mph). At this speed, how many days would it take to travel to the sun from Earth, located at a distance of 149 million kilometers?

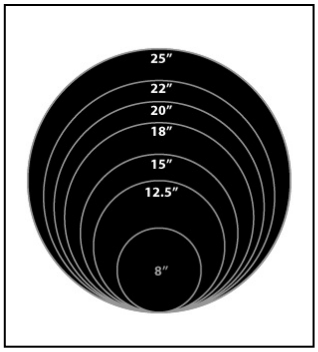
Answer: Time = Distance/speed so
Time = 149,000,000 km/ 28,000
= 5321 hours or **222 days**.

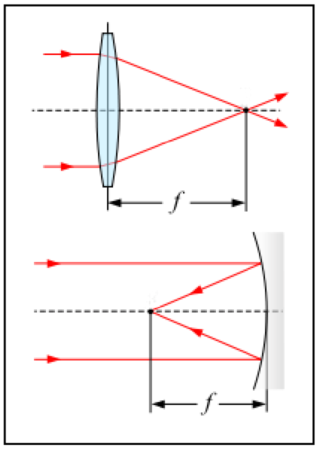
Problem 2 – The planet Neptune is located 4.5 billion kilometers from Earth. How many years would it take a rocket traveling at the speed of the International Space Station to make this journey?

Answer: Time = 4,500,000,000 km / 28,000 km/h
= 160714 hours or 6696 days or **18.3 years**.

Problem 3 – The fastest unmanned spacecraft, Helios-2, traveled at a speed of 253,000 km/hr. In the table below, use proportional math to fill in the travel times from the sun to each planet traveling at the speed of Helios-2. Give your answers to the nearest tenth in appropriate units of days or years.

Planet	Distance in millions of kilometers	Time
Mercury	57	9.4 days
Venus	108	17.8 days
Earth	149	24.5 days
Mars	228	37.5 days
Jupiter	780	128.5 days
Saturn	1437	236.7 days
Uranus	2871	1.3 years
Neptune	4530	2.0 years





Any optical system, such as a telescope, camera or microscope, can be described by a just few basic numbers.

Aperture is the main lens or mirror that gathers the light to a focus. Aperture diameter, D, is commonly measured in inches, millimeters, centimeters or even meters. The larger the aperture, the more light the system gathers and the finer details it can see. The top figure shows various aperture diameters for telescopes that can be bought.

Focal length is the distance between the center of the aperture and the point in space where distant light rays come to a focus. In the figure, both a lens and a properly-curved mirror can have focal points. The symbol, f, represents the focal length.

F/ number is a measure of the speed and clarity of the optical system. It is the ratio of the focal distance to the aperture size. Fast systems have small F/numbers such as F/1, F/2 or F/3. Slow systems have large F/ numbers such as F/8, F/15 or even F/20. In photography these are also called F-stops.

$$F/ = f / D$$

Problem 1 – An astronomer wants to design a telescope that takes up the least amount of space in a research satellite. The aperture has to be 254 millimeters in order to gather the most light possible and provide the clearest images. The light path between the mirror center and the focus can be folded 3 times between mirrors separated by 500 millimeters. What is the focal length of this system and the F/ number? Is this a fast or slow system?

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

System 1 :	F/2.0		f = 100 mm
System 2 :		D= 10-inches,	f = 1270 mm
System 3 :	F/15.0	D = 50 mm	

Fill-in the missing quantities and describe the pros and cons of each system.

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Answer: The focal length is 3 x 1500 millimeters = 1500 millimeters, and F/ = 1500/254 = 5.9. It is a slow system because F/ > 3.0.

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

System 1 :	F/2.0	D=200 mm	f = 100 mm
System 2 :	F/5.0	D= 254 mm	f = 1270 mm
System 3 :	F/15.0	D = 50 mm	f = 750 mm

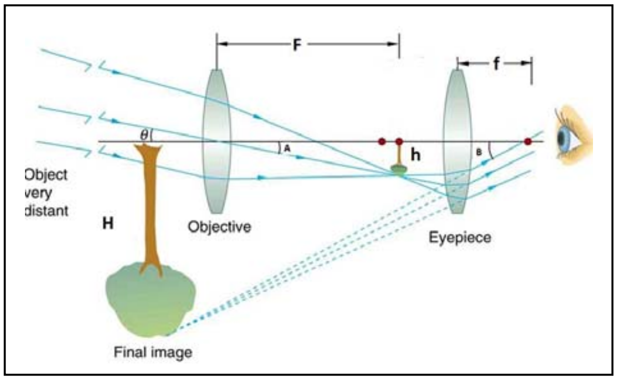
Fill-in the missing quantities and describe the pros and cons of each system.

Answer: System 1: D = 100 mm x 2.0 = 200 mm. System 2: D = 10 inches x 25.4 mm/inch = 254 millimeters so F = 1270/254 = 5.0; System 3: f = 50 mm x 15.0 = 750 mm.

System 3 is the slowest optical system in the group, and has the smallest aperture, which means that it gathers the least amount of light and so images will appear fainter and show less detail.

System 1 and 2 are very similar in aperture so they gather about the same amount of light, however, System 1 is nearly 3 times faster and so will provide the clearest images. System 1 is also shorter that System 2 (100 mm vs 1270 mm) so it would be easier and lighter to operate.

Calculating the Magnification of a Telescope



A telescope consists of an objective mirror or lens and an eyepiece. The role of the eyepiece is to change the angle, A, of the rays from the objective as they enter the eye. As the figure shows, when B > A, it appears as though the image of the tree is bigger than its actual image at the focus of the telescope objective. A simple proportion relates the image sizes to the focal lengths of the lenses:

$$\frac{H}{h} = \frac{F}{f}$$

For example, if the telescope objective has a focal length of 2000 millimeters and the eyepiece has a focal length of 4 millimeters, H/h = 2000/4 = 500, so the image h has been magnified by 500 times. The quantity F/f is the magnification.

Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information. Focal lengths and aperture dimensions are given in millimeters.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203		1198	10	
Obsession-20	Reflector	508	5.0		8	
1-meter	Reflector	1000	17.0			850
David Dunlap	Reflector	1880	17.3		100	
Hubble	Reflector	2400		57600		2880
Mt Palomar	Reflector		3.3	16830	28	
Yale	Refractor	1020	19.0		4	
Subaru	Reflector	8200		15000		7500
Keck	Reflector		1.75	17500	1	

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

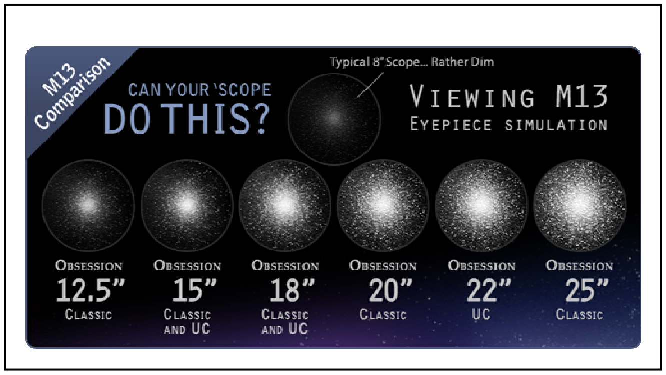
Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203	**5.9**	1198	10	**120**
Obsession-20	Reflector	508	5.0	**2540**	8	**317**
1-meter	Reflector	1000	17.0	**17000**	**20**	850
David Dunlap	Reflector	1880	17.3	32,524	**100**	**325**
Hubble	Reflector	2400	**24**	57600	**20**	2880
Mt Palomar	Reflector	**5100**	3.3	16830	28	**601**
Yale	Refractor	1020	19.0	**19400**	4	**4850**
Subaru	Reflector	8200	**1.83**	15000	**2**	7500
Keck	Reflector	**10000**	1.75	17500	1	**17500**

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

Answer: The focal length of the Obsession-20 mirror is 2540 mm, and the eye's focal length is 250 mm, so the magnification is only about **10 times**. This means that if you look at the moon in this way, it will appear 10 times bigger 'in the sky' than without the telescope.

Note: A rule-of-thumb is that you do not use a higher magnification than 2 times the aperture size in millimeters. At higher magnifications the image remains blurry and you do not see additional details. In the table in Problem 1, the only eyepiece that violates this rule is the one selected for the Yale Telescope, and so an eyepiece with a longer focal length and lower magnification is the best to use.



This figure shows a comparison of the same faint star cluster seen with telescopes of increasing aperture size and LGA. Notice the big change between an 8-inch and am 18-inch!

The human eye is a small lens that lets-in only a small amount of light. This is useful when you are looking at a bright daytime scene, but when you are studying faint stars this becomes a problem.

A telescope has a much larger aperture than the eye and allows more light to be brought to a focus to study. This means that even stars too faint to be detected by the eye can easily be 'brightened' by the telescope so that they are easy to detect and study.

Light Gathering Ability is the property of an optical system that tells you how much brighter things will appear than what the human eye can see. It is the ratio of the area of the objective to the area of the human eye lens.

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Answer Key

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

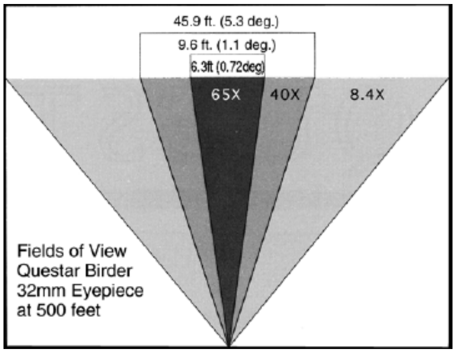
Answer: LGA = $(50/7)^2$ = 51 times more light.

Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Answer: The magnitude difference is m = 5.0, so the brightness difference is a factor of $(2.512)^5$ = 100 times.

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Answer: The magnitude difference is +16.0 - +1.0 = +10.0, which is a brightness factor of 100x100 = 10,000. You need a telescope that provides a LGA of 10000 so
10000 = $(D/7mm)^2$ and so D = 700 millimeters (27-inches in diameter).



Although magnifying an object makes it appear larger, the view also gets smaller as the diagram to the left shows. Every combination of telescope and eyepiece produces its own Field of View (FOV), usually stated in angular terms. For example, a combination that gives an angular FOV of 10° in diameter will easily let you see the entire full moon, which is only 0.5° in diameter. But if you use a lens with 40 times more magnification, the FOV is now only 1/4° so you only see ½ of the full disk of the moon in the eyepiece.

By itself, an eyepiece allows incoming light to be brought to a focus for the human eye or camera. The incoming light rays can come from many different directions within a cone whose vertex is the focus point for the lens. The angle of the cone's vertex defines the FOV for the eyepiece. The table below shows the FOVs for various eyepieces that are used with telescopes:

Vendor	Model	Focal Length	Apparent FOV (°)	Actual FOV (°)	Magnification	Price
Orion	Optilux 2"	40	60	1.18	51	$140
Televue	Panoptic 2"	35	68	1.17	58	$370
Orion	FMC Plössl 2"	50	45	1.11	41	$120
Orion	DeepView 2"	42	52	1.07	48	$70
Edmund Optics	RKE Erfle 2"	32	68	1.07	64	$225
Meade	SWA 2"	32	67	1.06	64	$240
Orion	Optilux 2"	32	60	0.94	64	$140
Teleview	Panoptic 2"	27	68	0.90	75	$330
Teleview	Plössl 1.25"	40	43	0.85	51	$110
Celestron	Ultima 1.25"	35	49	0.84	58	$108

The apparent FOV for each eyepiece ranges from 45° to 68° and is a result of how the eyepiece is designed. When used in this example with an 8-inch telescope with a focal length of 2032 millimeters, the magnifications range from 41x to 75x. The resulting telescope FOV is then just FOV = Eyepiece FOV/magnification. For the Optilix 2" eyepiece, the FOV is then 60°/51 = 1.18°.

Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60°. What magnification will give him the desired FOV?

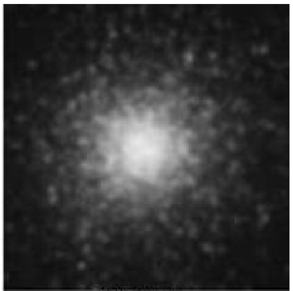
Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50°. His old telescope provided a 2.0° FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

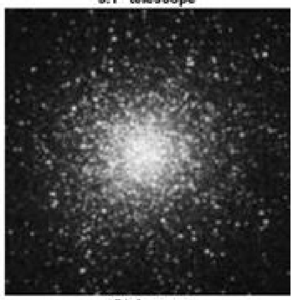
Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60^o. What magnification will give him the desired FOV?

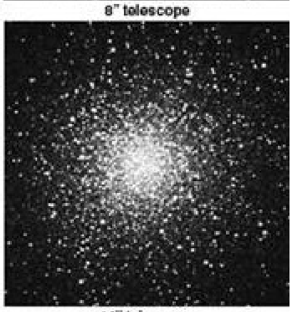
Answer: $0.5^o = 60^o / M$ so the magnification is M = **120x**.

Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50^o. His old telescope provided a 2.0^o FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

Answer: Because FOV = eyepiece FOV/magnification, if the new telescope provides magnifications that are twice the older system, then the FOV will be half as large for this eyepiece or 1.0^o.


3.1" telescope


8" telescope


14" telescope

The images to the left show what the star cluster Messier-13 would look like to three different telescopes with apertures of 3.1, 8.0 and 14.0 inches. Notice that as the aperture increases, the fuzzy smudges seen by the smallest telescope become increasingly more clear to see as the aperture increases. This is an example of Optical Resolution, which is sometimes called the <u>Resolving Power</u> of a telescope.

To make the clearest photographs of stars, planets, or even people, it helps to use the largest lens or aperture to make crisp, clear images. Astronomers also want the highest resolutions possible so that they can study the smallest details on a planet surface, or in a distant galaxy.

Telescope resolution at optical wavelengths can be calculated using the simple formula:

$$R = \frac{134}{D}$$

where D is the diameter of the objective in millimeters, and R is the resolution in seconds of arc. (There are 3600 seconds of arc in 1 angular degree).

For example, a pair of binoculars with D = 50 mm, provides a resolution limit of R = 2.8 arcseconds. A small 8-inch telescope for which D = 200 mm, provides R = 0.67 arcseconds.

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a single telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

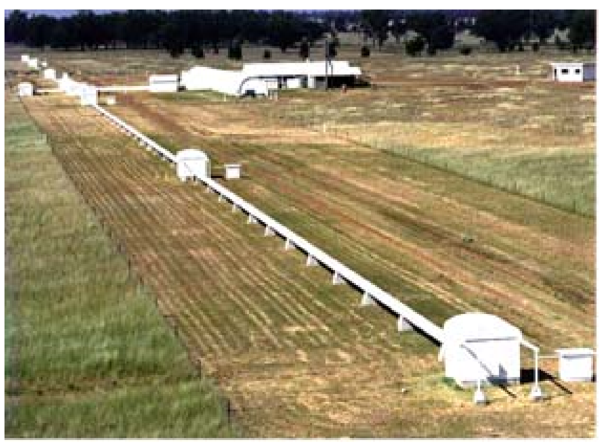
Answer: 0.1 = 134/D so D = **1340 millimeters (53 inches)**.

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Answer: R = 134/2400 = **0.06 arcseconds** or 60 milliarcseconds

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Answer: R = 134/640000 = **0.0002 arcseconds or 0.2 milliarcseconds**.



A photo of the Sydney University Stellar Interferometer (SUSI) is a long-baseline optical interferometer located approximately 20km west of the town of Narrabri in northern New South Wales, Australia. The equivalent diameter of the optical aperture for this instrument is 640 meters.

Telescope diameter = D
Telescope focal length = F
Eyepiece focal length = f
Eyepiece field of view = FOVe
Magnification = M
Resolution = R

D, F, f are in millimeters
FOVe is in degrees
R is in arcseconds

$$M = \frac{F}{f}$$

$$F/\text{number} = \frac{F}{D}$$

$$R = \frac{134}{D}$$

Astronomers don't just go out and buy a telescope and then use it. Whether it is for use on a satellite orbiting Saturn, or in an observatory, telescopes are designed 'from the ground up' by starting from a set of goals that the research needs to accomplish. The telescope is mathematically designed to meet these research goals.

The table to the left gives the basic quantities and formulae for designing a simple telescope system. Let's see how two different research goals can lead to very different telescope systems!

System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossl with a FOV of 68^{o}, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

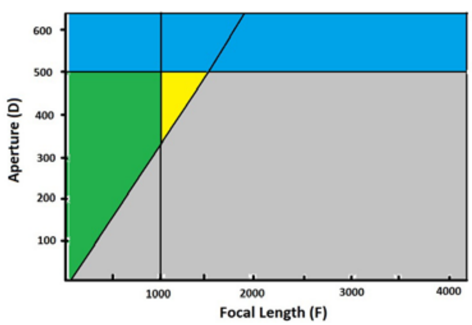
To design these systems, create a graph with the aperture diameter (vertical) and the focal length (horizontal).

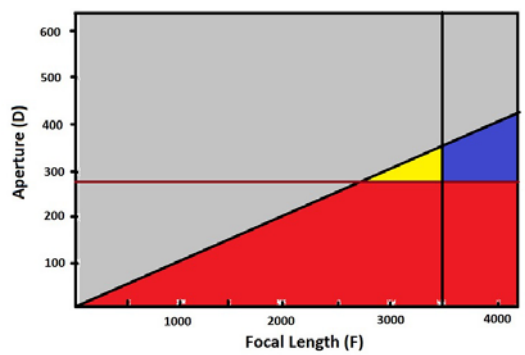
System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossl with a FOV of 68°, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

Answer: D < 20 inches (500 mm); F/number < 3.0; f = 20 mm, FOVe=68°, M> 50x

On the D vs F graph, draw a line representing F/D = 3.0 or F = 3.0D and shade the excluded region below this line (grey), which represents F/n > 3.0. Now draw a horizontal line for D = 500mm and shade (blue) the region above this line which represents D > 500mm. Magnification = F/f so we have F/f > 50 and F > 50f. For f = 20mm, the constraint is F > 1000 mm. Draw a vertical line at F = 1000mm and shade (green) all points to the left as the excluded region. The permitted regions is the one shown in yellow. The final plot (below left) should look like the one below.

An optimal system is near the middle of the yellow permitted region for which D = 450 mm and F = 1250mm. We then have F/number = 2.8, a magnification of 62, a telescope FOV of 68/62 = 1 degree, and a resolution of 134/450 = 0.3 arcseconds.

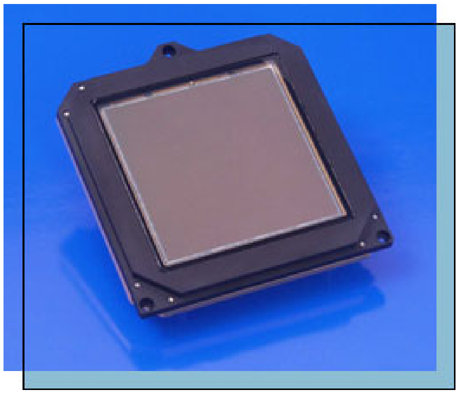




System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

Answer: F/number > 10; R = 1/3 to ½ arcseconds; M < 700x.
Draw a line representing F/number = 10 and shade (grey) the excluded area above this line which indicates F/number <10. For R = ½ arcseconds, draw a horizontal line at D = 270mm and shade the region (red) below this line that represents R > ½. The magnification M = F/5mm so for M=700 we have F < 3500mm. Draw a vertical line at F = 3500 and shade (blue) the excluded area to the right. The allowed region is in yellow in the diagram on the upper right.

The best system lies in the center of the triangle with D = 300mm and F = 3250 mm. This gives F/10.8; Resolution = 134/300 = 0.4 arcseconds; Lens magnifications of 650, 325 and 162.



This is a 9200x9200 image sensor for a digital camera. The grey area contains the individual 'pixel' elements. Each square pixel is about 8 micrometers (8 microns) wide, and is sensitive to light. The pixels accumulate electrons as light falls on them, and computers read out each pixel and create the picture.

Digital cameras are everywhere! They are in your cell phones, computers, iPads and countless other applications that you may not even be aware of.

In astronomy, digital cameras were first developed in the 1970s to replace and extend photographic film techniques for detecting faint objects. Digital cameras are not only easy to operate and require no chemicals to make the images, but the data is already in digital form so that computers can quickly process the images.

Commercially, digital cameras are referred to by the total number of pixels they contain. A '1 megapixel camera' can have a square-shaped sensor with 1024x1024 pixels. This says nothing about the sensitivity of the camera, only how big an image it can create from the camera lenses. Although the largest commercial digital camera has 80 megapixels in a 10328x7760 format, the largest astronomical camera developed for the Large Synoptic Survey Telescope uses 3200 megapixels (3.2 gigapixels)!

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

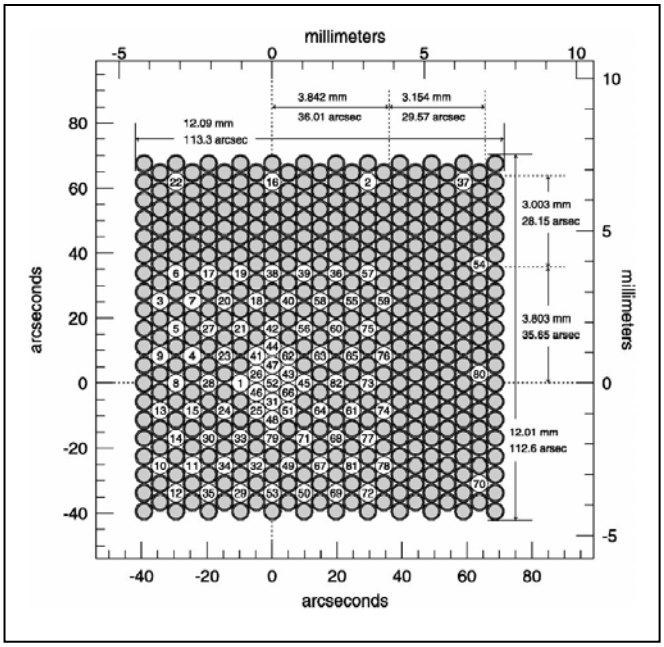
Answer: This is a square array, so s2 = 6100000 pixels and so s = 2469 pixels. The format is **2469 x 2469 pixels**. Since the width of a side is 20 mm, each pixel is about 20 mm/2469 = 0.0000081 meters or **8.1 microns** on a side.

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Answer: 1800 arcseconds/2469 pixels = **0.7 arcseconds/pixel**.

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Answer: 1 degree = 3600 arcseconds, so 3.5 degrees = 12600 arcseconds. Then 12600 arcseconds/10328 pixels = **1.2 arcseconds/pixel.**



The image of an astronomical object forms at the focus of a telescope. It can be measured either in terms of its angular size in degrees or arcseconds, or in terms of the number of millimeters. For astronomy, you are interested in its 'angular size' but if you want to photograph it, you are interested in how big it will be compared to the size of your film or digital sensor array (called the CCD).

A very simple formula defines how to convert from angular size to millimeters at the focus of a telescope:

$$\text{Scale} = \frac{206265}{F}$$

where F is the focal length in millimeters and 'Scale' is the image scale in arcseconds per millimeter.

Problem 1 – Two telescopes are used for astrophotography. The first has a mirror diameter of 20-inches and a focal length of 5000 mm, the second one is more portable and has a diameter of 10-inches and a focal length of 2000 mm. What are the image scales of each telescope?

Problem 2 – An astronomer wants to design a camera so that each pixel views an angle of only 0.5 arcseconds. If the width of each pixel is 8 micrometers (0.008 millimeters), what is the image scale he needs for the telescope, and what will be its focal length?

Problem 3 - If a digital camera array measures 20 millimeters across and consists of 2048 pixels, what will the image scale have to be so that the array can be used to photograph a star cluster with a diameter of ¼ degree? What will the telescope focal length have to be.

Answer Key

Problem 1 – Two telescopes are used for astrophotography. The first has a mirror diameter of 20-inches and a focal length of 5000 mm, the second one is more portable and has a diameter of 10-inches and a focal length of 2000 mm. What are the image scales of each telescope?

Answer: The first one is Scale = 206265/5000 = **41 arcseconds/mm**. The second one has 206265/2000mm = **103 arcseconds/mm**.

Problem 2 – An astronomer wants to design a camera so that each pixel views an angle of only 0.5 arcseconds. If the width of each pixel is 8 micrometers (0.008 millimeters), what is the image scale he needs for the telescope, and what will be its focal length?

Answer: He needs a scale of 0.5 arcseconds/0.008 mm) = 62.5 arcseconds/mm. The focal length will be 62.5 = 206265/F so **F = 3300 millimeters**.

Problem 3 - If a digital camera array measures 20 millimeters across and consists of 2048 pixels, what will the image scale have to be so that the array can be used to photograph a star cluster with a diameter of ¼ degree? What will the telescope focal length have to be.

Answer: ¼ degree/20 millimeters = 3600 arcsec (1/4)/20 mm = 45 arcseconds/mm. The Focal length is 45 = 206265/F so **F = 4583 millimeters**.

Buying a Telescope!

OK…So those wonderful pictures of planets, star clusters and galaxies have got your curiosity on fire. You want to have your own telescope so that you can see the universe for yourself! All you have to do is spend a few minutes on the Internet and you will see a bewildering number of choices for telescopes you can buy. Some are pretty inexpensive and cost less than \$70.00, but others can cost \$500.00 or more. How do you decide which one is right for you?

Remember, the bigger the objective lens or mirror, the fainter you can see stars in the sky. The longer the focal length, the higher will be the magnification. The only limit to either of these is that you should not use magnifications higher than 50x the diameter of the objective, or 2 times its diameter in millimeters. Higher magnifications only make images look worse!

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost
Reflector	7.6	300		\$64.95
Refractor	6.0	700		\$54.95
Refractor	8.9	910		\$300.00
Reflector	11.4	900		\$129.95
Reflector	15.2	610		\$319.95
Refractor	10.0	900		\$749.95
Reflector	20.3	1000		\$699.95
Refractor	15.2	1219		\$1,199.00
Reflector	50.8	2032		\$4,400.00

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost	Cost per area
Reflector	7.6	300	152x	$64.95	1.4
Refractor	6.0	700	120x	$54.95	1.9
Refractor	8.9	910	178x	$300.00	4.7
Reflector	11.4	900	228x	$129.95	1.3
Reflector	15.2	610	304x	$319.95	1.8
Refractor	10.0	900	200x	$749.95	9.6
Reflector	20.3	1000	406x	$699.95	2.2
Refractor	15.2	1219	304x	$1,199.00	6.6
Reflector	50.8	2032	1016x	$4,400.00	2.2

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

Answer: The limit would be set by the 2mm eyepiece. For the telescope focal lengths in the table, this eyepiece could not be used with the telescopes shaded in yellow in the table, Telescopes 2, 3, 4, 6, 7 and 8. The 4mm would not be used on telescopes 2, 3, 4, and 6.

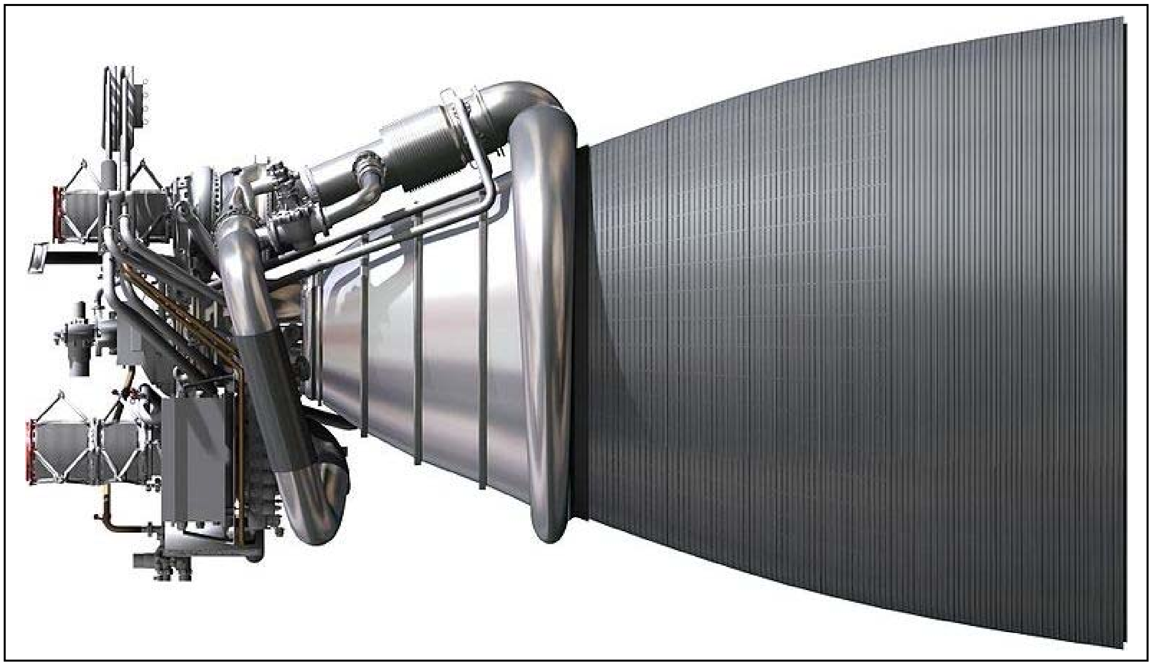
Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

Answer: You can get a reflector with a larger area than you can a refractor. Refractors are more expensive per unit area. From the table, reflectors cost 1.3 to 2.2 dollars per square centimeter, while refractors cost 1.9 to 9.6 dollars per square centimeter.

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Answer: A 15.2 cm refractor costs about 6.6 dollars per square centimeter, so a 50.8-cm refractor would cost $\pi\,(50.8/2)^2 \times 6.6 =$ **$13,374.**

Calculating the Volume of the J-2x Rocket Engine Bell



Problem 1 – The volume of a cone is given by $V = 1/3\pi R^2 h$. If the base of the rocket nozzle has a radius of r = 1.53 meters (5-feet) and a height of h = 4.37 meters (13-feet), what is the approximate total volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Problem 2 – A better approximation to the shape of the curved nozzle is to break the volume up into a conical top section (h=2.18 meters) and a cylindrical bottom section (h=2.18 meters). If the base of the cone in the top section has a radius of 1.00 meter, and the average radius of the cylinder in the bottom section is R=(1.00 + 1.53)/2 = 1.27 meters, what is the total volume of the total nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Problem 3 – The exact volume of the volume for this curved nozzle is given by the function

$$V(x) = 0.0094\, x^4 - 0.141\, x^3 + 1.10\, x^2 + 0.14\, x$$

where V is in cubic meters and x is the distance from the vertex of the cone to its base. What is the exact volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters ?

Problem 1 – The volume of a cone is given by $V = 1/3\pi R^2 h$. If the base of the rocket nozzle has a radius of r = 1.53 meters and a height of h = 4.37 meters, what is the approximate total volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Answer: $V = 1/3\ (3.14)\ (1.53)^2\ (4.37) =$ **10.7 meters^3**

Problem 2 – A better approximation to the shape of the curved nozzle is to break the volume up into a conical top section (l=2.18 meters) and a cylindrical bottom section (L=2.18 meters). If the base of the cone in the top section has a radius of 1.00 meter, and the average radius of the cylinder in the bottom section is (1.00 + 1.53)/2 = 1.27 meters, what is the total volume of the total nozzle to the nearest 0.1 cubic meters? (Use $\pi = 3.14$)

Cone = $1/3\ (3.14)(1.0)^2(2.18) =$ 2.28 meters^3
Cylinder = $3.14\ (1.27)^2\ (2.18) = 11.04$ meters^3
Approximate total volume = **13.3 meters^3**.

Problem 3 – The exact volume of the volume for thus curved nozzle is given by the function $V(x) = 0.0094x^4 - 0.141x^3 + 1.10x^2 + 0.14x$ where V is in cubic meters and x is the distance in meters from the vertex of the cone to its base. What is the exact volume of the J-2X rocket nozzle to the nearest 0.1 cubic meters?

Answer: $V(4.37\text{meters}) = 0.0094(4.37)^4 - 0.141(4.37)^3 + 1.10\ (4.37)^2 + 0.14(4.37)$
= 3.43 - 11.77 + 21.00 + 0.61
= 13.27
= **13.3 meters^3**.

A Simple Gauge in a Tank - II

For the convenience of storing some types of solids (grains, sand) or liquids, some tanks have a conical shape. The volume of a cone is given by

$$V = \frac{1}{3}\pi R^2 h$$

In this picture, the diameter of the top of the tank is 6 meters and its height is 7.5 meters. What is the total volume of this tank in gallons if 1 gallon is 3.85 liters and 1meter^3 = 1000 liters.

Answer: $V = 0.33\ (3.14)(3\ )^2\ (7.5)$
$= 70$ meters^3
= **18,182 gallons**

Problem 1 – What is the linear equation that gives the radius of the tank, R, at a height h in meters above the ground?

Problem 2 - An engineer wants to store an expensive solvent in this tank and needs to know when there is only 200 gallons remaining so that he can re-order. He will install a gauge at a height, Z, in the tank that will be triggered when the solvent level is just under the gauge.

Problem 3 – How high up on the slanted side of the tank from its vertex will the gauge be located?

Problem 1 – What is the linear equation that gives the radius of the tank, R, at a height h in meters above the ground?

Answer: At h = 7.5 meters, R = 3 meters, so the slope of the linear equation for h is just m = 3/7.5 = 0.4 and so **R(h) = 0.4h**.

Problem 2 - An engineer wants to store an expensive solvent in this tank and needs to know when there is only 200 gallons remaining so that he can re-order. He will install a gauge at a height, Z, in the tank that will be triggered when the solvent level is just under the gauge.

Answer: $V = 1/3\ (3.14)\ (0.4h)^2\ (h)$ so $V = 0.167\ h^3$
200 gallons equals 770 liters or 0.77 $meters^3$, then
$0.77 = 0.167\ z^3$ and solving for z we get **z = 1.66 meters**.

Problem 3 – How high up on the slanted side of the tank from its vertex will the gauge be located?

Answer: The slope of the side of the tank is just 1/0.4 = 2.5 so the 'hypotenuse' of the tank side makes an angle of $Tan(\theta) = 2.5$ or 68 degrees. Since Hsin(68) = 1.66 meters, we have **H = 1.8 meters**.

Conical storage tanks come in many different sizes, from grain storage silos like the one top-left, to chemical storage and separating funnels like the one shown top-right. The nice thing about cones is that they have a wide base area that is easy to poor things into, and a valve at the conical tip lets you remove carefully-measured amounts of whatever is being stored. Recall that the volume of a cone is given by $V = 1/3\ \pi\ R^2\ h$ where R is the base radius and H is the vertical height (not the slant height along the side of the cone!).

Problem 1 – Suppose that the function that relates the radius of the cone to the vertical height of the cone is given by R(h) = 0.5h, where h and R are in meters. The maximum height of the storage vessel is 3.0 meters. What is the radius of the upside-down conical tank at its maximum height?

Problem 2 – To the nearest tenth of a cubic meter, what is the maximum volume of this conical tank?

Problem 3 – At what height, H, should the astronaut place a mark on the outside of the tank to indicate a level of ½ the volume of the conical tank?

Answer Key

Problem 1 – Suppose that the function that relates the radius of the cone to the vertical height of the cone is given by R(h) = 0.5h, where h and R are in meters. The maximum height of the storage vessil is 3.0 meters. What is the radius of the upside-down conical tank at its maximum height?

Answer: R(2.5) = 0.5 x 3.0 = **1.5 meters**.

Problem 2 – To the nearest tenth of a cubic meter, what is the maximum volume of this conical tank?

Answer: H = 3.0 meters, R = 1.5 meters

so $V = 1/3\ (3.141)\ (1.5)^2\ (3.0)$

$= \mathbf{7.1\ meters^3}$.

Problem 3 – At what height should the astronaut place a mark on the outside of the tank to indicate a level of ½ the volume of the conical tank?

Answer: We want $V = ½ \times 7.1 m^3 = 3.55\ m^3$

But R = 0.5H

So $V = 1/3\ \pi\ (0.5H)^2\ H = 0.333(3.141)(0.25)\ H^3$ and so $V = 0.26H^3$

Then $3.55\ m^3 = 0.26\ H^3$ and so solving for H we get **H = 2.4 meters**.

Satellites use electricity to run their various systems and experiments. Since the dawn of the Space Age, engineers have used solar cells to generate this energy from sunlight.

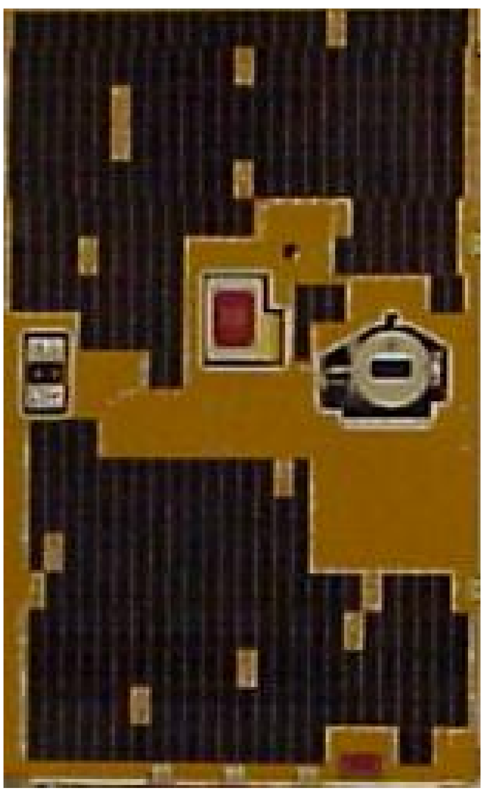
In this exercise, you will calculate how much power the IMAGE satellite can generate from one of its 8 hexagonal faces, allowing for the areas lost by instrument windows and other blank areas on the satellite. **Note: The solar cells used by the IMAGE satellite can generate 0.03 watts per square centimeter of area**.

Question 1: What is the usable area of the satellite's face shown below?

Question 2: What electrical power can be generated by the panel?

Question 3: If there are 8 similar panels on the satellite, what is the approximate total power that can be generated if all faces are fully illuminated, and have about the same number of solar cells?

IMAGE satellite Face 1

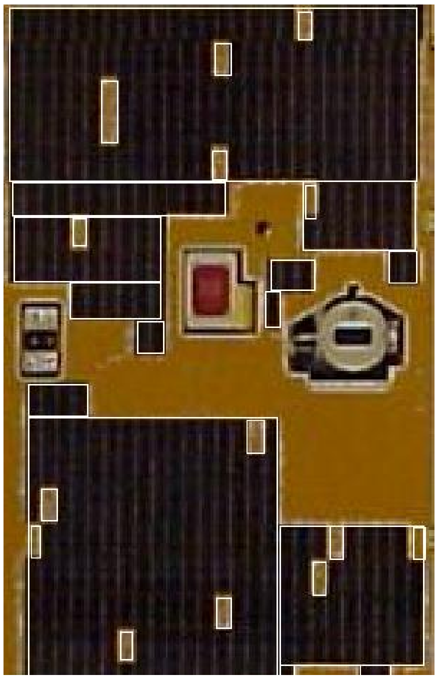


The panel above is 136 centimeters long and 90 centimeters wide. The solar cells that generate the electricity are shown in black. The brown-colored areas do not generate electricity.

Come up with a plan to determine the sizes of the black areas from the given information and image, then answer the three questions above after performing the required calculations.

Goal: Students will calculate the area of a satellite solar panel and estimate the total electrical power that can be generated. Students will use the images and dimensions provided to create a scaled drawing of each satellite face, and from this determine the scaled dimensions of the dark solar cell areas.

Note: If you want to make a full-sized model of the satellite visit the *IMAGE Satellite Scaled Model* page at
http://image.gsfc.nasa.gov/poetry/workbook/page14.html



A

B,C
D,E,F
G,H
I
J

K,L

As a benchmark: The maximum possible area of the panel is 136 cm x 90 cm = 12,240 sq cm. The maximum power is therefore (0.03 watts/sq cm) x 12,240 sq cm = 367 watts if the panel is fully illuminated.

The scale factor of the students image is 137 cm (actual) / 10.2 cm (picture) or **13.4**

Suggested Method: Determine the black area by breaking the panel into rectangles as indicated by the letters from left to right. Subtract from each rectangle the area of the non-black regions. There are 15 small rectangles within the boxed black regions. Each have the same size = 0.3cmx0.5cm (image). Note, perform all area calculations in 'image' units, then convert final area answer to actual units by multiplying by (13.4)^2.

ID	W	L	A	ID	W	L	A	ID	W	L	A
A	6.0	3.5	21.0	**E**	0.7	0.5	0.4	**I**	0.4	0.5	0.2
B	0.5	3.2	1.6	**F**	0.5	0.5	0.3	**J**	0.9	0.5	0.5
C	1.0	1.7	1.7	**G**	1.4	0.5	0.7	**K**	3.7	4.0	14.8
D	1.0	2.3	2.3	**H**	0.2	0.5	0.1	**L**	1.9	2.0	3.8

W, L = image width and height in cm

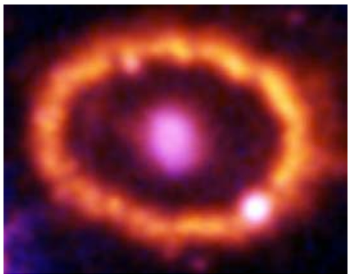
A = image area in sq.cm.

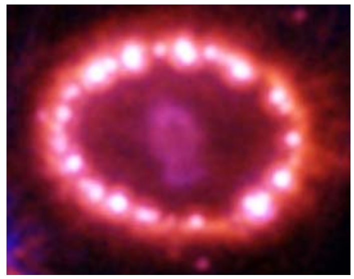
Question 1: What is the usable area of the satellite's face? **Answer**: Add A-L areas to get 47.4 sq cm, then subtract the areas of the 15 non-celled rectangles (15 x 0.15 = 2.3) and get 47.4 – 2.3 = 45.1 square cm in image units. Convert to actual area by multiplying by 13.4 x 13.4 = 179.6. The total area of the solar cells is then 45.1 x 179.6 = 8100 sq. cm. Note, the maximum panel area is 12,240 sq. cm, so (8100/12240)x100% = 66% of the panel is covered by solar cells.

Question 2: What electrical power can be generated by the panel? **Answer**: 0.03 W/sq.cm x 8100 sq. cm = 243 watts.

Question 3: If there are 8 similar panels on the satellite, what is the approximate total power that can be generated if 4 faces are fully illuminated at a given time, and have about the same number of solar cells? **Answer:** 4 x (243 W) = 972 Watts.

Objects in space move. To figure out how fast they move, astronomers use many different techniques depending on what they are investigating. In this activity, you will measure the speed of astronomical phenomena using the scaling clues and the time intervals between photographs of three phenomena: A supernova explosion, a coronal mass ejection, and a solar flare shock wave.



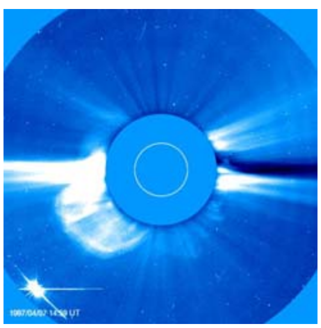


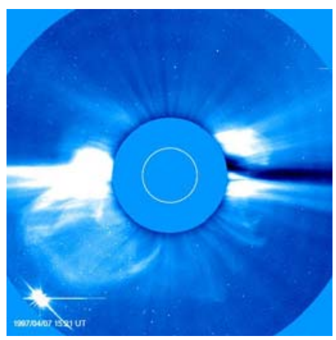
Problem 1: Supernova 1987A was photographed 7.7 years apart to study its expanding shell of gas. What is the speed of this material shown in the photographs in:

A) light-years per year?
B) kilometers per second?

Measure the change in longest dimension of the inner blob of light. The outer ring is about one light-year in diameter. The left-hand image was taken by the Hubble Space Telescope in March, 1995. The right-hand image was taken in November, 2003. Note 1 light-year = 9.2 trillion km.

For more information, visit: http://hubblesite.org/newscenter/newsdesk/archive/releases/2004/



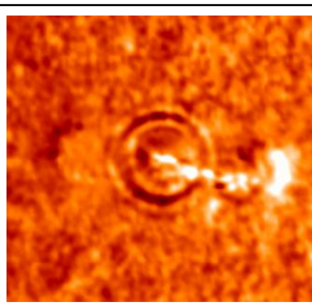


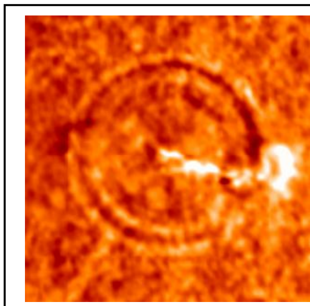
Problem 2: Closer to Earth, solar storms provide another example of violent motion. How fast did this coronal mass ejection travel in:

A) kilometers per second?
B) kilometers per day?

The white circle is the diameter of the sun (1.4 million km). Images taken at 14:59 UT (left) and 15:21 UT (right).

CME observed by the SOHO satellite on April 7, 1997.





Problem 3 – A solar flare on July 9, 1996 caused a phenomenon called a Morton Wave to travel across the sun's surface. What was its speed in:

A) kilometers per hour?
B) kilometers per second?

Each picture is 150 million meters on a side. The difference in time between the images is one hour.

More information: http://www.solarviews.com/eng/sohopr3.htm

Problem 1: Supernova 1987A was photographed 7.7 years apart to study its expanding shell of gas. What is the speed of this material shown in the photographs in: A) light-years (LY) per year? B) kilometers per second?

Answer: Using a millimeter ruler and the stated size of the image, the scale is about 30mm = 1 LY. The central 'blob' which is the supernova shell has an initial diameter of 5mm and a final largest diameter of 10mm, so its **radius** has increased by 2.5mm which equals (2.5mm/30mm)x 1LY = 0.085 LY. The difference in time between the images is 7.7 years so A) 0.085 LY/7.7 yrs = 0.010 Light-years/year, and for 3.1 x 10^7 seconds in a year, B) $9.2 \times 10^{12} \times 0.010/3.1 \times 10^7 =$ 3,100 km/sec.

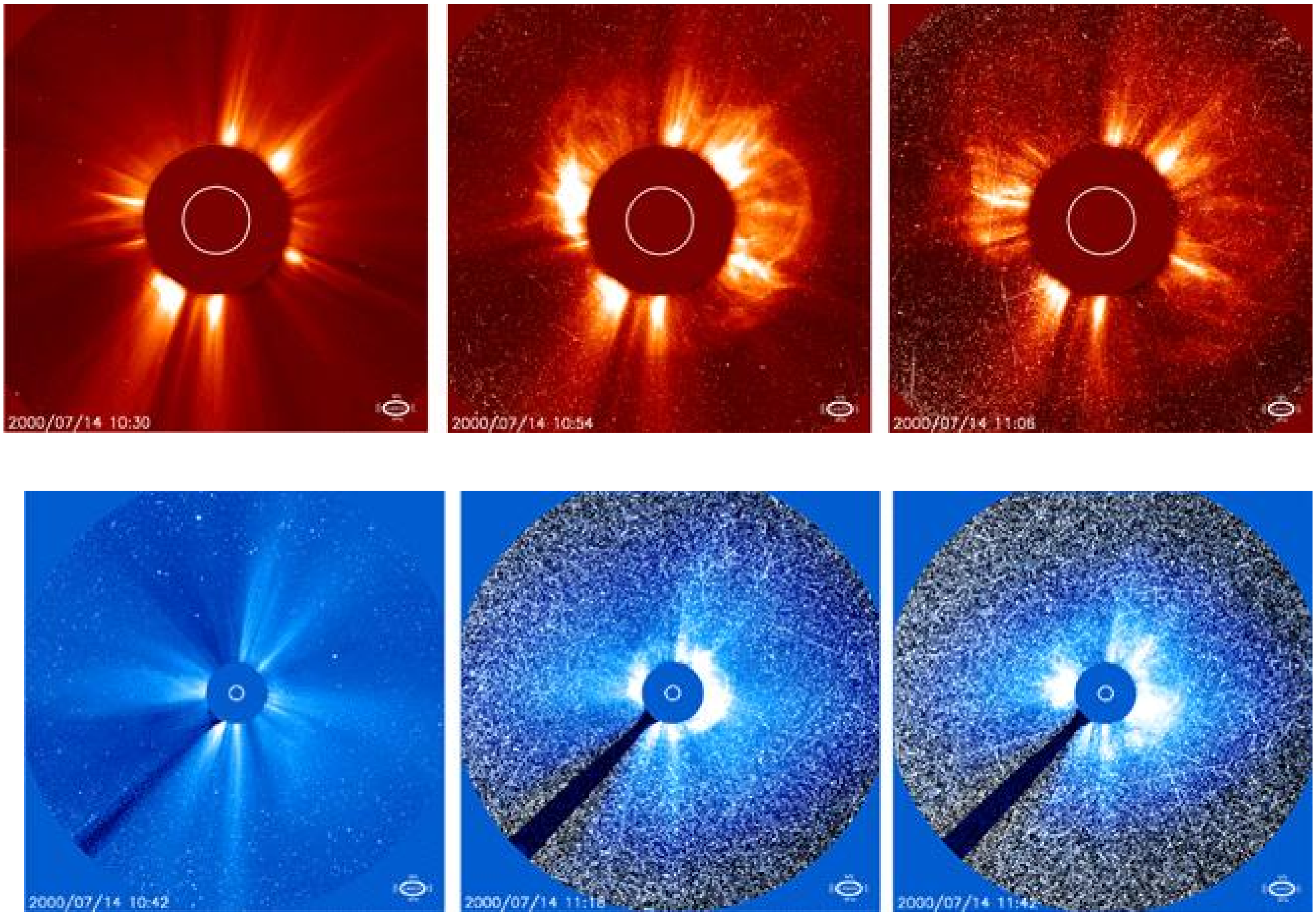
Problem 2: Closer to Earth, solar storms provide another example of violent motion. How fast did this coronal mass ejection travel in: A) kilometers per second? B) kilometers per day?

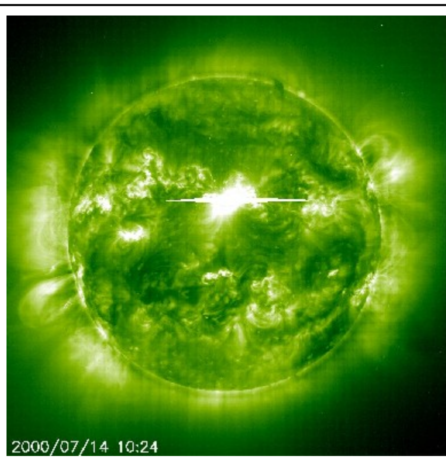
Answer: The scale of the prints is about 7 mm = 1.4 million km. If you measure the distance from the center of the sun circle to the outer edge of the CME in the lower left corner of each picture, you get about 13.5 mm and 19 mm respectively. This equals a **change** in distance of (5.5mm/7mm) x 1.4 million km = 1.1 million km. The difference in time between the two images is 22 minutes or 1320 seconds, so A) the speed is about 1.1 million km/1320 sec = 833 km/sec. There are 24x60x60 = 86400 seconds in a day, so B) is about 833 x 86400 = 72 million km/day.

Problem 3 – A solar flare on July 9, 1996 caused a phenomenon called a Morton Wave to travel across the sun's surface. What was its speed in: A) kilometers per second? B) kilometers per hour?

Answer: The image scale is 37 mm = 150 million meters. The circles represent the shock wave, and the outer ring radius has increased from 7mm to 12mm in one hour. This is a distance change of (5mm/37mm)x 150 million meters = 20 million meters or 20,000 kilometers. A) 20,000 kilometers/hour. B) There are 3600 seconds in an hour so 20,000/3600 = 5.6 km/sec.

Data Corruption by High-Energy Particles





Solar flares can severely affect sensitive instruments in space and corrupt the data that they produce. On July 14, 2000 the sun produced a powerful X-class flare, which was captured by instruments onboard the Solar and Heliospheric Observatory (SOHO). The EIT imager operating at a wavelength of 195 Angstroms, showed a brilliant flash of light (left image). When these particles arrived at the SOHO satellite some time later, they caused the imaging equipment to develop 'snow' as the individual particles streaked through the sensitive electronic equipment. The above images taken by the SOHO LASCO c2 and c3 imagers show what happened to that instrument when this shower of particles arrived. The date and time information (hr : min) is given in the lower left corner of each image, and give the approximate times of the events.

Problem 1: At about what time did the solar flare first erupt on the sun?

Problem 2: At about what time did the LASCO imagers begin to show significant signs of the particles having arrived?

Problem 3: If the SOHO satellite was located 147 million kilometers from the sun, about what was the speed of the arriving particles?

Problem 4: If the speed of light is 300,000 km/sec, what percentage of light-speed were the particles traveling?

Answer Key:

Problem 1: At about what time did the solar flare first erupt on the sun?

Answer: The EIT image time says 10:24 or 10 hours and 24 minutes Universal Time
The reason this is not an exact time is because the images were taken at set times, and not at the exact times of the start or end of the events. To within the 24-minute interval between successive EIT images, we will assume that 10:24 UT is the closest time.

Problem 2: At about what time did the LASCO imagers begin to show significant signs of the particles having arrived?

Answer: The top sequence shows that the 'snow began to fall' at 10:54 UT. The second sequence suggests a later time near 11:18. However, the 11:18 time is later than the 10:54 time. The time interval between exposures is 24 minutes, but the top series started at 10:30 and ended at 10:54 UT, while the lower series started at 10:42 and ended at 11:18. That means, comparing the exposures between the two series, the snow arrived between 10:42 and 10:54 UT. We can split the difference and assume that the snow began around 10:48 UT.

Problem 3: If the SOHO satellite was located 147 million kilometers from the sun, about what was the speed of the arriving particles?

Answer: The elapsed time between the sighting of the flare by EIT (10:24 UT) and the beginning of the snow seen by LASCO (10:48 UT) is 10:48 UT – 10:24 UT = 24 minutes. The speed of the particles was about 147 million km/24 minutes or 6.1 million km/minute.

Problem 4: If the speed of light is 300,000 km/sec, what percentage of light-speed were the particles traveling?

Answer: Converting 6.1 million km/minute into km/sec we get 6,100,000 km/sec x (1 min /60 sec) or 102,000 kilometers/sec. Comparing this to the speed of light we see that the particles traveled at (102,000/300,000)x100% = 34% the speed of light!

Note: Because these damaging high-speed particles can arrive only a half-hour after the x-ray flash is first seen on the sun, it can be very difficult to protect sensitive equipment from these storms of particles if you wait for the first sighting of the solar flare flash. In some cases, science research satellites have actually been permanently damaged by these particle storms.

Are U Nuts?

Converting from one set of units to another is something that scientists do every day. We have made this easier by adopting metric units wherever possible, and re-defining our standard units of measure so that they are compatible with the new metric units wherever possible.

In the western world, certain older units have been replaced by the modern ones, which are now adopted the world over. (see Wikipedia under 'English Units' for more examples). In this exercise, you will convert from…

Conversion Table:

4 Gallons = 1 Bucket	142.065 cubic cm = 1 Noggin
9 Gallons = 1 Firkin	1.296 grams = 1 Scruple
126 Gallons = 1 Butt	201.168 meters = 1 Furlong
34.07 Liters = 1 Firkin	14 days = 1 Fortnight
0.0685 Slugs = 1 Kilogram	0.2 grams = 1 Carat

1) A typical aquarium holds 25 gallons of water. Convert this to A) Firkins; B) Liters, and C) Buckets.

2) John weighs 7.2 Slugs, and Mary weighs 53 kilograms. Who weighs the most kilograms?

3) The passenger volume of a car is about 5.4 cubic meters. How many Noggins can fit inside the car?

4) Sven weighs 105 kilograms and finished a diet of pickled herring, losing 3.8 kilograms. A) How many Scruples did he lose? B) How many Scruples did he start out with?

5) The density of water is 1.0 grams/cm^3. How many Scruples per Noggin is this?

6) Evelyn finished the Diamond Man Marathon by walking 400 kilometers in 18 days. What was her average speed in Furlongs per Fortnight?

7) A swimming pool holds 50,000 gallons of water. How many Butts were in the pool?

8) If a Fathom is 72 inches, and there are 2.5 centimeters per inch, how many kilometers are there in 3.6 Leagues if 1 League = 2640 Fathoms?

9) The original Cullinan Diamond was discovered in 1904 and weighs 3,106.75 Carats. A) How many grams is this? B) The polished Cullinan Diamond I (Great Star of Africa) weighs 530.2 Carats and is worth $386 million. What is the approximate worth of the original Cullinan Diamond? C) What is the going rate for diamonds in terms of dollars per Carat? D) Dollars per gram?

Answer Key:

Conversion Table:

4 Gallons = 1 Bucket	142.065 cubic centimeters = 1 Noggin
9 Gallons = 1 Firkin	1.296 grams = 1 Scruple
126 Gallons = 1 Butt	201.168 meters = 1 Furlong
34.07 Liters = 1 Firkin	14 days = 1 Fortnight
0.0685 Slugs = 1 Kilogram	200 milligrams = 1 Carat

1) A typical aquarium holds 25 gallons of water. Convert this to
A) Firkins; 25 Gallons x (1 Firkin/9 Gallons) = **2.8 Firkins**
B) Liters, and 2.8 Firkins x (34.07 Liters/ 1 Firkin) = **95.4 Liters**
C) Buckets. 25 Gallons x (1 Bucket/4 gallons) = **6.3 Buckets.**

2) John weighs 7.2 Slugs, and Mary weighs 53 kilograms. Who weighs the most kilograms?
John = = 7.2 Slugs x (1 kg/0.0685 Slugs) = **105 kg so John weighs the most kilograms.**

3) The passenger volume of a car is about 5.4 cubic meters. How many Noggins can fit inside the car?
5.4 cubic meters x (1,000,000 cubic cm/1 cubic meter) x(1 Noggin/142.065 cubic cm) = **38,028 Noggins!**

4) Sven weighs 105 kilograms and finished a diet of pickled herring, losing 3.8 kilograms.
A) How many Scruples did he lose? 3.8 kg x (1,000 gm/ 1kg) x (1 Scruple/1.296 grams) = **2,932 Scruples.**
B) How many Scruples did he start out with? 105 kg x(1,000 gm/1 kg) x 1 Scruple/1.296 grams) = **81,018 Scruples**

5) The density of water is 1.0 grams per cubic centimeter. How many Scruples per Noggin is this?
1 gram x (1 Scruple/1.296 grams) = 0.771 Scruples.
1 cubic centimeter x (1 Noggin/142.065 cubic cm) = 0.007 Noggins.
Dividing the two you get 0.771 Scruples/0.007 Noggins = **110 Scruples/Noggin.**

6) Evelyn finished the Diamond Man Marathon by walking 400 kilometers in 18 days. What was her average speed in Furlongs per Fortnight?
400 kilometers x (1,000 meters/ 1 km) x (1 Furlong/201 meters) = 1,990 Furlongs.
18 days x (1 Fortnight/14 days) = 1.28 Fortnights.
Dividing the two you get 1,990 Furlongs/1.28 Fortnights = **1,555 Furlongs per fortnihght.**

7) A swimming pool holds 50,000 gallons of water. How many Butts were in the pool?

50,000 gallons x (1 Butt / 126 gallons) = **397 Butts.**

8) If a Fathom is 72 inches, and there are 2.5 centimeters per inch, how many kilometers are there in 3.6 Leagues if 1 League = 2640 Fathoms?
3.6 Leagues x (2640 Fathoms/ 1 League) x (72 Inches / 1 Fathom) = 684,288 inches.

684,288 inches x (2.5 cm/1 inch) x (1 meter/100 cm) x (1 kilometer/1000 meters) = **17.1 kilometers.**

9) The original Cullinan Diamond was discovered in 1904 and weighs 3,106.75 Carats.

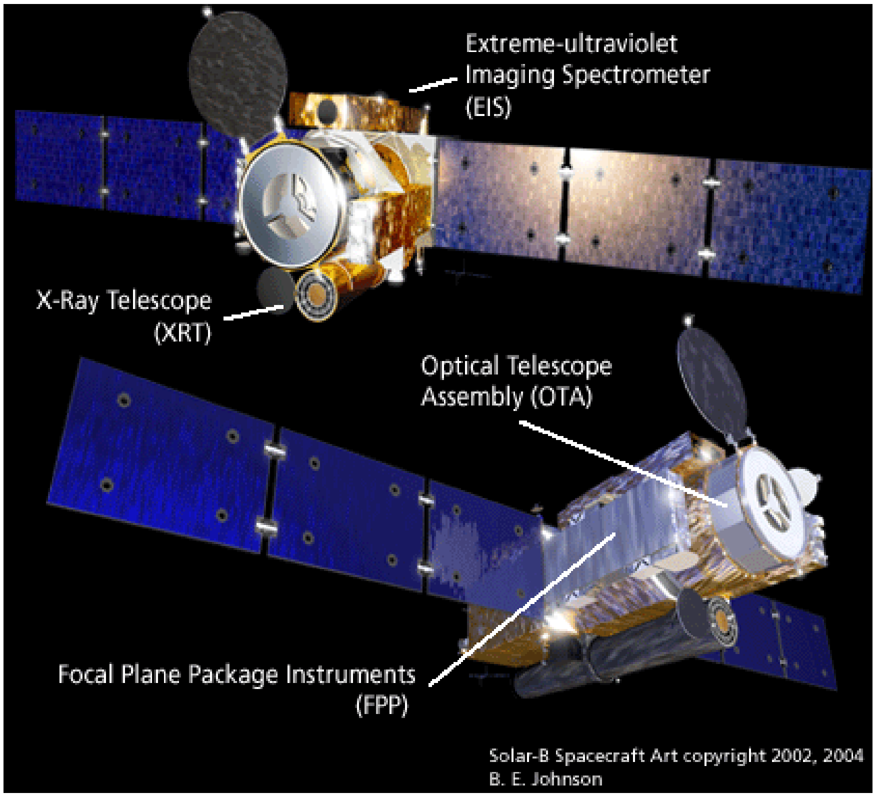
A) How many grams is this?
3,106.75 carats x (0.2 grams / 1 carat) = **621.2 grams.**

B) The polished Cullinan Diamond I (Great Star of Africa) weighs 530.2 Carats and is worth $386 million. What is the approximate worth of the original Cullinan Diamond?
386 million dollars x (3106.75 carats / 530.2 carats) = **$2.26 billion.**

C) What is the going rate for diamonds in terms of dollars per Carat?
386 million dollars / 530.2 carats = **$728,027 per carat.**

D) Dollars per gram?
728,027 dollars / carat x (1 carat / 0.2 grams) = **$ 3.6 million / gram.**

Hinode Satellite Power



The Hinode satellite weighs approximately 700 kg (dry) and carries 170 kg of gas for its steering thrusters, which help to maintain the satellite in a polar, sun-synchronous orbit for up to two years. The satellite has two solar panels (blue) that produce all of the spacecraft's power. The panels are 4 meters long and 1 meter wide, and are covered on both sides by solar cells.

Problem 1 - What is the total area of the solar panels covered by solar cells in square centimeters?

Problem 2 - If a solar cell produces 0.03 watts of power for each square centimeter of area, what is the total power produced by the solar panels when facing the sun? Can the satellite supply enough power to operate the experiments which require 1,150 watts?

Problem 3 - Suppose engineers decided to cover the surface of the cylindrical satellite body with solar cells instead. If the satellite is 4 meters long and a diameter of 1 meter, how much power could it produce if only half of the area was in sunlight at a time? Can the satellite supply enough power to keep the experiments running, which require 1,150 watts?

Answer Key:

Problem 1 - What is the total area of the solar panels covered by solar cells in square centimeters?

Answer: The surface area of a single panel is 4 meters x 1 meter = 4 square meter per side. There are two sides, so the total area of one panel is 8 square meters. There are two solar panels, so the total surface area covered by solar cells is 16 square meters. Converting this to square centimeters:

$$16 \text{ square meters} \times (10{,}000\ cm^2/m^2) = 160{,}000\ cm^2$$

Problem 2 - If a solar cell produces 0.03 watts of power for each square centimeter of area, what is the total power produced by the solar panels when facing the sun? Can the satellite supply enough power to operate the experiments which require 1,150 watts?

Answer: Only half of the solar cells can be fully illuminated at a time, so the total exposed area is 80,000 cm^2. The power produced is then:

$$\text{Power} = 80{,}000\ cm^2 \times 0.03\ \text{watts}/cm^2 = 2{,}400 \text{ watts.}$$

Yes, the satellite solar panels can keep the experiments running, with 2400-1150 = 1,250 watts to spare!

Problem 3 - Suppose engineers decided to cover the surface of the cylindrical satellite body with solar cells instead. If the satellite is 4 meters long and a diameter of 1 meter, how much power could it produce if only half of the area was in sunlight at a time? Can the satellite supply enough power to keep the experiments running, which require 1,150 watts?

Answer - Surface area of a cylinder = Area of 2 circular end caps + area of side of cylinder

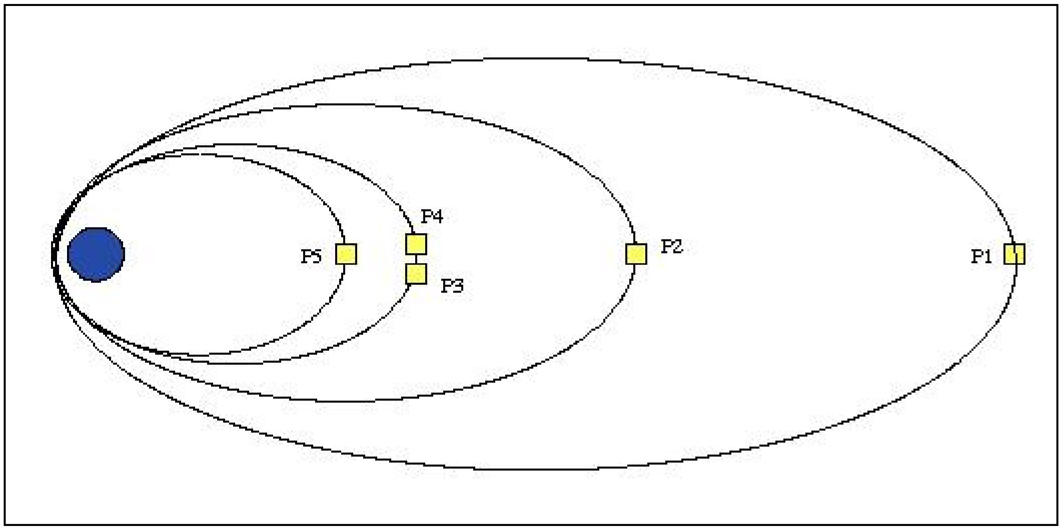
$$= 2 \pi R^2 + 2 \pi R h$$

$S = 2 \times (3.14)\ (0.5 \text{ meters})^2 + 2 \times (3.14)\ (0.5 \text{ meters})\ (4 \text{ meters})$
= 1.57 square meters + 12.56 square meters
= 14.13 square meters.

Only half of the solar cells can be illuminated, so the usable area is 7.06 square meters or 70,600 square centimeters. The power produced is 70600 x 0.03 = 2,100 watts.

Yes..the satellite can keep the experiments running with this solar cell configuration.

A Problem in Satellite Synchrony



The THEMIS program has launched five satellites that will orbit earth at different distances to study Earth's magnetic field. The goal is to have them line up a few times each year so that they can study how magnetic storms are triggered and evolve in time.

The following problems will be easier to solve if you use a spreadsheet, but they CAN be worked out by hand by finding the Least Common Multiples. Can you answer them both ways?

Problem 1 - Suppose you had two satellites orbiting Earth in the same plane. The closer-in satellite had a period of 4 hours, and the second satellite had a period of 8 hours. Looking down at Earth from the North Pole, suppose you started the satellites together at the same longitude, say 100 degrees West. How many hours later would they both be together again?

Problem 2 - Suppose the faster satellite had a period of 120 minutes, and the slower satellite had a period of 17 hours. After how many hours would they come together again in longitude?

Problem 3 - Suppose you had three satellites with periods of 2 hours, 4 hours and 6 hours. Using the Least Common Multiple, find how long would it take for them to return to their original line-up at the same longitude.

Problem 4 - The THEMIS mission uses five satellites with periods of about 19 hours, 24 hours, 24 hours, 48 hours, and 96 hours, how many hours would it take for them all to get together again at the same longitude? Give your answer in days.

Answer Key:

Problem 1 - Suppose you had two satellites orbiting Earth. The closer-in satellite had a period of 4 hours, and the second satellite had a period of 8 hours. Looking down at Earth from the North Pole, suppose you started the satellites together at the same longitude, say 100 degrees West. How many hours later would they both be together again?

Answer: Create a time line and mark of the times, starting from 00:00, when the satellites will return to West 100.
Satellite A 00:00………04:00…………08:00………..12:00……16:00………..20:00……..24:00…etc
Satellite B 00:00………………………..08:00…………………….16:00……………………..24:00…etc

Another method: If you know the periods of the two satellites, look for the Lowest Common Multiple between them, which in this case is 8 hours.

Problem 2 - Suppose the faster satellite had a period of 2 hours, and the slower satellite had a period of 17 hours. After how many hours would the come together again in longitude?

In hours:
Sat A: 0, 2, 4, 6, 8, 10 ,12, 14, 16, 18, 20, …., 30, 32, 34, 36, 38, ………50, 52, 54, …..66, 68, 70, ….
Sat B: 0, 17, 34, 51, ………… 68, ….

Answer: Students can use the spreadsheet approach, or sketch out a timeline. The satellite periods, in hours, are 2 hours and 17 hours. The LCM, in this case, is 34 hours. So every 34 hours, the satellites will return to the same longitude they started at in synchrony.

Teachers; another, more algebraic method is as follows:
The angular speed of the two satellites is A: 360 degrees/2 hours = 180 degrees/hour and B: 360 degrees/17 hours = 21.176 degrees/hour. As time goes on, the speed with which the faster one pulls ahead of the slower one is given by V = 180 degrees/hr - 21.176 degrees/hour = 158.824 degrees/hour. How much time does it take for the angular distance between the satellites to equal exactly 360 degrees or an exact integer multiple (360 ,720, 1080, etc) ?

32hr x 21.176 d/hr = 677.63d ,
34hr x 21.176 d/hr = 719.98 d.

So, after 34 hours, the angular distance between the two satellites is 720 degrees, with is 2 x 360, so the satellites meet up at the same longitude they started from.

Problem 3: Suppose you had three satellites with periods of 2 hours, 4 hours and 6 hours. How long would it take for them to return to their original line-up at the same longitude?

Answer: The prime factors are 2, 2x2 and 2x3 so the LCM is 2x2x3 = 12, so after 12 hours the satellites will come together again.

Problem 4 - The THEMIS mission uses five satellites with periods of about 19 hours, 24 hours, 24 hours, 48 hours, and 96 hours, how many hours would it take for them all to get together again at the same longitude? Give your answer in days.

Answer: Find the prime factors 19: 19 , 24: 2x2x2x3 , 48: 2x2x2x2x3 , 96: 2x2x2x2x2x3
42: then 19 x 2x2x2x2x2 x3 = 1824 hours. Then to check: 1824/19 = 96, 1824/24 = 76, 1824/48 = 38, 1824/96 = 19. So, it will take 1,824 hours or 76 days!

Scientific Notation I

Astronomers rely on scientific notation in order to work with 'big' things in the universe. The rules for using this notation are pretty straight-forward, and are commonly taught in most 7th-grade math classes as part of the National Education Standards for Mathematics.

The following problems involve the conversion of decimal numbers into SN form, and are taken from common astronomical applications and quantities.

1) Length of a year. 31,560,000.0 seconds

2) Speed of light: 299,792.4 kilometers/sec

3) Mass of the sun: 1,989,000,000,000,000,000,000,000,000,000,000 grams

4) Mass of Earth: 5,974,000,000,000,000,000,000,000 kilograms

5) One light-year : 9,460,500,000,000 kilometers

6) Power output of sun : 382,700,000,000,000,000,000,000,000 watts

7) Mass of an electron: 0.00000000000000000000000000000091096 kilograms

8) Energy equivalent of one electron-Volt : 0.00000000000000000016022 joules

9) Ratio of proton to electron mass: 1,836.2

10) Planck's Constant: 0.000000000000000000000000006626068 ergs seconds

11) Radius of hydrogen atom : 0.00000000529177 centimeters

12) Radius of Earth's orbit: 14,959,789,200,000 centimeters

13) Smallest unit of physical distance: 0.0000000000000000000000000000000016 centimeters

14) Diameter of Visible Universe: 26,000,000,000,000,000,000,000,000,000 centimeters

Answer Key:

1) Length of a year. 31,560,000.0 seconds

Answer: 3.156×10^{7} seconds

2) Speed of light: 299,792.4 kilometers/sec

Answer: 2.997924×10^{5} km/sec

3) Mass of the sun: 1,989,000,000,000,000,000,000,000,000,000,000 grams

Answer: 1.989×10^{33} grams

4) Mass of Earth: 5,974,000,000,000,000,000,000,000 kilograms

Answer: 5.974×10^{24} kg

5) One light-year : 9,460,500,000,000 kilometers

Answer: 9.4605×10^{12} km

6) Power output of sun : 382,700,000,000,000,000,000,000,000 watts

Answer: 3.827×10^{26} watts

7) Mass of an electron: 0.00000000000000000000000000000091096 kilograms

Answer: 9.1096×10^{-31} kg

8) Energy equivalent of one electron-Volt : 0.00000000000000000016022 joules

Answer: 1.6022×10^{-19} Joules

9) Ratio of proton to electron mass: 1,836.2

Answer; 1.8362×10^{3}

10) Planck's Constant: 0.000000000000000000000000006626068 ergs seconds

Answer: 6.626068×10^{-27} ergs seconds

11) Radius of hydrogen atom : 0.00000000529177 centimeters

Answer: 5.29177×10^{-9} cm

12) Radius of Earth's orbit: 14,959,789,200,000 centimeters

Answer: $1.49597892 \times 10^{13}$ cm

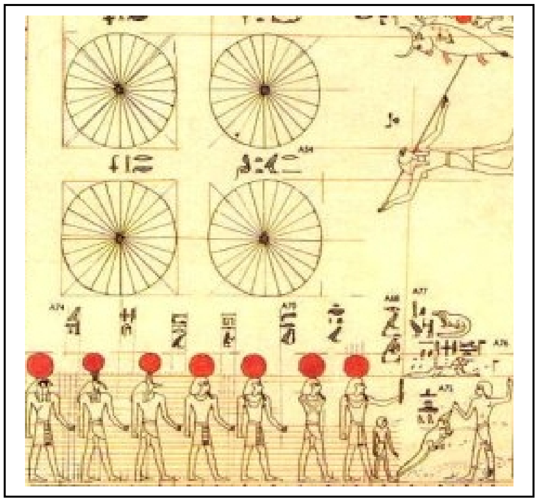
13) Smallest unit of physical distance: 0.0000000000000000000000000000000016 centimeters

Answer: 1.6×10^{-33} cm

14) Diameter of Visible Universe: 26,000,000,000,000,000,000,000,000,000 centimeters

Answer; 2.6×10^{28} cm

Scientific Notation II



Astronomers rely on scientific notation in order to work with 'big' things in the universe. The rules for using this notation are pretty straight-forward, and are commonly taught in most 7th-grade math classes as part of the National Education Standards for Mathematics.

The following problems involve the addition and subtraction of numbers expressed in Scientific Notation. For example:

$$\begin{aligned} 1.34 \times 10^{8} + 4.5 \times 10^{6} &= 134.0 \times 10^{6} + 4.5 \times 10^{6} \\ &= (134.0 + 4.5) \times 10^{6} \\ &= 138.5 \times 10^{6} \\ &= 1.385 \times 10^{8} \end{aligned}$$

1) $1.34 \times 10^{14} + 1.3 \times 10^{12} =$

2) $9.7821 \times 10^{-17} + 3.14 \times 10^{-18} =$

3) $4.29754 \times 10^{3} + 1.34 \times 10^{2} =$

4) $7.523 \times 10^{25} - 6.32 \times 10^{22} + 1.34 \times 10^{24} =$

5) $6.5 \times 10^{-67} - 3.1 \times 10^{-65} =$

6) $3.872 \times 10^{11} - 2.874 \times 10^{13} =$

7) $8.713 \times 10^{-15} + 8.713 \times 10^{-17} =$

8) $1.245 \times 10^{2} - 5.1 \times 10^{-1} =$

9) $3.64567 \times 10^{137} - 4.305 \times 10^{135} + 1.856 \times 10^{136} =$

10) $1.765 \times 10^{4} - 3.492 \times 10^{2} + 3.159 \times 10^{-1} =$

Answer Key:

1) $1.34 \times 10^{14} + 1.3 \times 10^{12} = (134 + 1.3) \times 10^{12} =$ **1.353×10^{14}**

2) $9.7821 \times 10^{-17} + 3.14 \times 10^{-18} = (97.821 + 3.14) \times 10^{-18} =$ **1.00961×10^{-16}**

3) $4.29754 \times 10^{3} + 1.34 \times 10^{2} = (42.9754 + 1.34) \times 10^{2} =$ **4.43154×10^{3}**

4) $7.523 \times 10^{25} - 6.32 \times 10^{22} + 1.34 \times 10^{24} = (7523 - 6.32 + 134) \times 10^{22} =$ **7.65068×10^{25}**

5) $6.5 \times 10^{-67} - 3.1 \times 10^{-65} = (6.5 - 310) \times 10^{-67} =$ **-3.035×10^{-65}**

6) $3.872 \times 10^{11} - 2.874 \times 10^{13} = (3.872 - 287.4) \times 10^{11} =$ **2.83528×10^{13}**

7) $8.713 \times 10^{-15} + 8.713 \times 10^{-17} = (871.3 + 8.713) \times 10^{-17} =$ **8.80013×10^{-15}**

8) $1.245 \times 10^{2} - 5.1 \times 10^{-1} = (1245.0 - 5.1) \times 10^{-1} =$ **1.2399×10^{2}**

9) $3.64567 \times 10^{137} - 4.305 \times 10^{135} + 1.856 \times 10^{136} = (364.567 - 4.305 + 18.56) \times 10^{135} =$
3.78822×10^{137}

10) $1.765 \times 10^{4} - 3.492 \times 10^{2} + 3.159 \times 10^{-1} = (17650.0 - 3492 + 3.159) \times 10^{-1} =$
1.4161159×10^{4}

Scientific Notation III

The following problems involve the multiplication and division of numbers expressed in Scientific Notation. Report all answers to two significant figures. For example:

$1.34 \times 10^8 \times 4.5 \times 10^6 = (1.34 \times 4.5) \times 10^{(8+6)}$
$= 6.03 \times 10^{14}$

To 2 significant figures this becomes… 6.0×10^{14}

3.45×10^{-5} / $2.1 \times 10^6 = (3.45/2.1) \times 10^{(-5 - (6))}$
$= 1.643 \times 10^{-11}$

To 2 significant figures this becomes… 1.6×10^{14}

1) Number of nuclear particles in the sun: 2.0×10^{33} grams / 1.7×10^{-24} grams/particle

2) Number of stars in the visible universe: 2.0×10^{11} stars/galaxy x 8.0×10^{10} galaxies

3) Age of universe in seconds: 1.4×10^{10} years x 3.156×10^7 seconds/year

4) Number of electron orbits in one year: (3.1×10^7 seconds/year) / (2.4×10^{-24} seconds/orbit)

5) Energy carried by visible light: (6.6×10^{-27} ergs/cycle) x 5×10^{14} cycles

6) Lengthening of Earth day in 1 billion years: (1.0×10^9 years) x 1.5×10^{-5} sec/year

7) Tons of TNT needed to make crater 100 km across: $4.0 \times 10^{13} \times (1.0 \times 10^{15})/(4.2 \times 10^{16})$

8) Average density of the Sun: 1.9×10^{33} grams / 1.4×10^{33} cm^3

9) Number of sun-like stars within 300 light years: (2.0×10^{-3} stars) x 4.0×10^6 cubic light-yrs

10) Density of the Orion Nebula: ($3.0 \times 10^2 \times 2.0 \times 10^{33}$ grams) / (5.4×10^{56} cm^3)

Answer Key:

1) Number of nuclear particles in the sun: 2.0×10^{33} grams / 1.7×10^{-24} grams/particle

1.2×10^{57} particles (protons and neutrons)

2) Number of stars in the visible universe: 2.0×10^{11} stars/galaxy x 8.0×10^{10} galaxies

1.6×10^{22} stars

3) Age of universe in seconds: 1.4×10^{10} years x 3.156×10^{7} seconds/year

4.4×10^{17} seconds

4) Number of electron orbits in one year: (3.1×10^{7} seconds/year) / (2.4×10^{-24} seconds/orbit)

1.3×10^{31} orbits of the electron around the nucleus

5) Energy carried by visible light: (6.6×10^{-27} ergs/cycle) x 5×10^{14} cycles

3.3×10^{-12} ergs

6) Lengthening of Earth day in 1 billion years: (1.0×10^{9} years) x 1.5×10^{-5} sec/year

1.5×10^{4} seconds or 4.2 hours longer

7) Tons of TNT needed to make crater 100 km across: $4.0 \times 10^{13} \times (1.0 \times 10^{15})/(4.2 \times 10^{16})$

9.5×10^{11} tons of TNT (equals 950,000 hydrogen bombs!)

8) Average density of the Sun: 1.9×10^{33} grams / 1.4×10^{33} cm^3

1.4 grams/cm^3

9) Number of sun-like stars within 300 light years: (2.0×10^{-3} stars) x 4.0×10^{6} cubic light-yrs

8.0×10^{3} stars like the sun.

10) Density of the Orion Nebula: ($3.0 \times 10^{2} \times 2.0 \times 10^{33}$ grams) / (5.4×10^{56} cm^3)

1.1×10^{-21} grams/cm^3

A Trillion Here...A Trillion There

A light-year is nearly 9.5 trillion kilometers. This is an unbelievably large number that is so huge, no one can possibly comprehend just how large it is compared to a walk to the local store, or a drive to Grandma's! Even by astronomical standards, astronomers rarely work with numbers this large. The total number of stars in the Milky Way is 'only' measured in the billions. The distance to the farthest reaches of the visible universe is 'only' 14 billion light years. Let's bring this huge number 'one trillion' down to Earth by looking at the 2006-2007 finances of the United States and the world.

US Statistics for 134 million taxpayers:

1…Federal Income Tax revenue	$ 2.2	trillion.
2...Federal Government expenses	$ 2.6	trillion.
3…The Gross Domestic Product (National wealth)	$12.9	trillion
4…Consumer debt (credit cards, loans etc)	$ 2.5	trillion
5…Total Mortgage debt (commercial and residential)	$ 6.5	trillion
6…Total federal debt (including unfunded programs)	$59.1	trillion
7…External debt owed to other countries	$16.3	trillion
8…Total retirement assets (401k, private pensions, etc)	$16.1	trillion
9…Annual income for US taxpayers	$ 7.4	trillion

World Statistics for 6 billion people:

10…Gross World Product	$65.8	trillion
11…Debt to World banking	$40.0	trillion
12…Value of all the shares on all the world stock markets	$66.6	trillion
13…Yearly exports	$ 6.6	trillion
14… Insurance premiums (life and non-life)	$ 3.7	trillion
15…Assets of 9.5 million wealthiest individuals.	$37.2	trillion

Problem 1 - What was the average Gross Domestic Product wealth for each US taxpayer?

Problem 2 - What was the average consumer debt for each US Taxpayer?

Problem 3 - About how much mortgage debt does each US taxpayer have?

Problem 4 - What is the total federal debt for each taxpayer including unfunded programs (like Social Security and Medicaid)?

Problem 5 - What percentage of the taxpayer annual income is paid in the federal Income Tax each year?

Problem 6 - What percentage of the world's wealth (Gross World Product) is the US wealth?

Problem 7 - About how much money does each taxpayer have in retirement assets in the US?

Problem 8 - Comparing the US taxes to the US government expenses, how much debt does the US generate each year?

Problem 9 - What is the average wealth per person in the world?

Problem 10 - What percentage of the world population includes the richest individuals?

Problem 11 - What is the average wealth of the non-richest individuals?

Answer Key:

US Statistics for 134 million taxpayers:

1…Federal Income Tax revenue	\$ 2.2	trillion.
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14… Insurance premiums (life and non-life)	\$ 3.7	trillion
15…Assets of 9.5 million wealthiest individuals.	\$37.2	trillion

Problem 1 - What was the average Gross Domestic Product wealth for each US taxpayer?

Answer: \$12.9 trillion/134 million = \$12,900,000 million/134 million = \$96,300.

Problem 2 - What was the average consumer dept for each US Taxpayer?

Answer: \$2.5 trillion/134 million = \$2,500,000 million/134 million = \$18,600

Problem 3 - About how much mortgage debt does each US taxpayer have?

Answer: \$6.5 trillion/134 million = \$6,500,000 million/134 million = \$48,500

Problem 4 - What is the total federal debt for each taxpayer including unfunded programs (like Social Security and Medicaid)?

Answer: \$59.1 trillion/134 million = \$59,100,000 million/134 million = \$441,000

Problem 5 - What percentage of the taxpayer annual income is paid in the federal Income Tax each year?

Answer: (\$2.2 trillion / \$7.4 trillion) x 100% = 29.7%

Problem 6 - What percentage of the world's wealth (Gross World Product) is the US wealth?

Answer: (\$12.9 trillion / \$65.8 trillion) x 100% = 19.6 %

Problem 7 - About how much money does each taxpayer have in retirement assets in the US?

Answer: \$16.1 trillion / 134 million = \$16,100,000 million/134 million = \$120,000

Problem 8 - Comparing the US taxes to the US government expenses, how much debt does the US generate each year?

Answer: \$2.2 trillion - \$2.6 trillion = \$0.4 trillion or \$400 billion

Problem 9 - What is the average wealth per person in the world?

Answer: \$65.8 trillion / 6 billion = \$65,800 billion/6 billion = \$11,000 per person

Problem 10 - What percentage of the world population includes the richest individuals?

Answer: (9.5 million / 6 billion) x 100% = 0.16 %

Problem 11 - About what is the average wealth of the non-richest individuals?

Answer: (\$65.8 trillion - \$37.2 trillion) / 6 billion people = \$4,700 per person

This is a picture taken by International Space Station astronauts of Washington, DC, and can be found among many other pictures at http://eol.jsc.nasa.gov/Coll/EarthObservatory/PostedSort.htm. The bridge at the bottom-center of the image is the George Mason Bridge (1) and it is 0.75 kilometers from end to end across the main part of the Potomac River (2).

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. It is the most important number to determine because without it, you don't know how big the objects in the image are!

Step 1: Measure the length of the George Mason Bridge with a metric ruler. How many millimeters long is the image of the bridge?

Step 2: The information in the introduction says that the bridge is actually 0.75 kilometers long. Convert this number into meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: About what is the distance between the US Capitol Building (3) and the Washington Monument (4)?
Question 2: About how wide are the major boulevards and roadways?
Question 3: About how wide is the Potomac River?
Question 4: How big is the smallest feature you can measure, and what do you think it is?
Question 5: How big is the area covered by this image in kilometers rounded to the nearest tenth?
Question 6: What other features can you recognize in this image?

You can use GOOGLE-Earth to help find other interesting landmarks in the image!

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. It is the most important number to determine because without it, you don't know how big the objects in the image are!

It is highly recommended that students use GOOGLE-Earth and dial-in 'Washington DC' to zoom-in on this area in higher resolution. They can use the various tools to bring up the labels for roads, buildings and geographic features.

Step 1: Measure the length of the Mason Bridge with a metric ruler. How many millimeters long is the image of the bridge? Answer: 15 millimeters

Step 2: The information in the introduction says that the bridge is actually 0.75 kilometers long. Convert this number into meters. Answer: 750 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: The image scale is 50 meters/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: About what is the distance between the US Capitol Building and the Washington Monument?
Answer: 72 millimeters on the image x 50 meters/mm = 3,600 meters or 3.6 kilometers.

Question 2: About how wide are the major boulevards and roadways?
Answer: The thick black lines are about 1.0 millimeter wide or 1.0mm x 50 meters/mm = 50 meters.

Question 3: About how wide is the Potomac River?
Answer: The river banks are about 12 millimeters apart along most of the river, so their true width is 12 mm x 50 meters/mm = 600 meters or 0.6 kilometers.

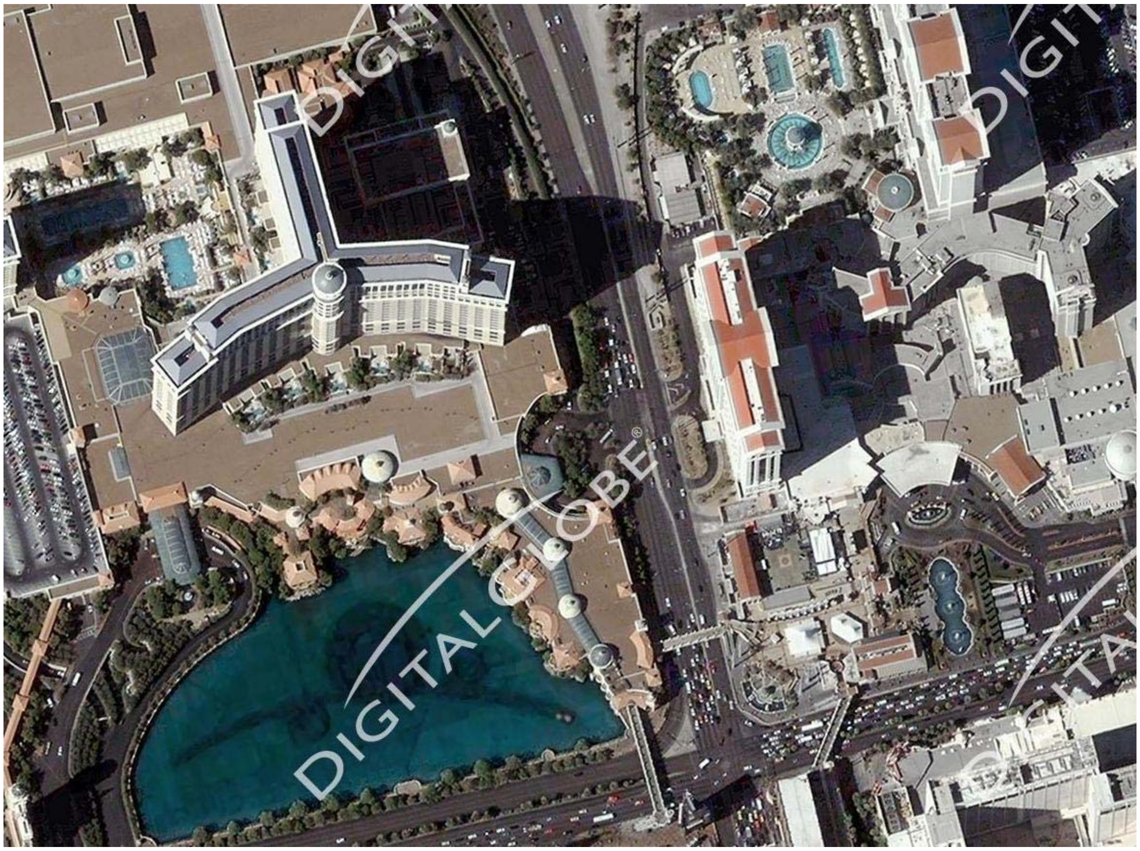
Question 4: How big is the smallest feature you can measure…and what do you think it is?
Answer: Students should be able to find many buildings that look like white spots with barely a square shape. These would be about 1 millimeter wide or 50 meters in true physical size.

Question 5: How big is the area covered by this image in kilometers rounded to the nearest tenth?
Answer; The field is 169 millimeters by 97 millimeters which is 8.5 kilometers x 4.9 kilometers in true size.

Question 6: What other features can you recognize in this image?
Answer: Students should be able to figure out the following features without using GOOGLE:

1. Rivers and waterways
2. Large and small buildings
3. Major boulevards
4. Minor streets
5. Bridges
6. Areas with trees and plant life

Exploring a City from Space



This QuickBird Satellite image was taken of downtown Las Vegas Nevada from an altitude of 450 kilometers. Private companies such as Digital Globe (http://www.digitalglobe.com) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellaggio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters. The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?
Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.
Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

Problem 1: How long is each of the three wings of the Bellaggio Hotel in meters?

Problem 2: What is the length of a car on the street in meters?

Problem 3: How wide are the streets entering the main intersection?

Problem 4: What is the smallest feature you can see, in meters?

Problem 5: What kinds of familiar objects can you identify in this image?

Answer Key:

This QuickBird Satellite image was taken of downtown Las Vegas Nevada on October 14, 2005 from an altitude of 450 kilometers. Private companies such as Digital Globe (http://www.digitalglobe.com) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellaggio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?
Answer: 150 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.
Answer: The information in the introduction says that the image is 700 meters long.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter.
Answer: 700 meters / 150 millimeters = 4.7 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

Problem 1: How long is each of the three wings of the Bellaggio Hotel in meters?
Answer: About 25 millimeters on the image or 25 mm x (4.7 meters/mm) = 117.5 meters.

Problem 2: What is the length of a car on the street in meters?
Answer: About 1 millimeter on the image or 1 mm x 4.7 meters/mm = 4.7 meters.

Problem 3: How wide are the streets entering the main intersection?
Answer: About 8 millimeters on the image or 8 mm x 4.7 meters/mm = 37 meters.

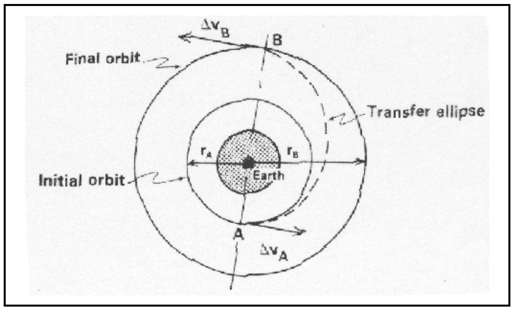
Problem 4: What is the smallest feature you can see, in meters?
Answer: Some of the small dots on the roof tops are about 0.2 millimeters across which equals 1 meter.

Problem 5: What kinds of familiar objects can you identify in this image?
Answer: Will vary depending on student.

1. Cars, busses
2. swimming pools and reflecting ponds
3. Trees
4. lane dividers
5. Shadows of people walking across the plaza to the Hotel.

Note: Ask the students to use image clues to determine the time of day (morning, afternoon, noon); Whether it is rush-hour or not; Time of year, etc.

Fly me to the Moon!



If spacecraft had rockets that could make them travel at any speed, we could fly to the Moon from Earth in a straight line, and make the trip in a few minutes. In the Real World, we can't do that even with the most powerful rockets we have. Instead, we have to obey Newton's Laws of Motion and take more leisurely, round-about routes!

To see how this works, you need a compass, metric ruler, a large piece of paper, a string, a thumbtack, and a pencil.

Step 1 - With your compass, draw a circle 1/2-centimeter in radius. Label the inside of this 'Earth'. **Step 2** - Draw a second circle centered on Earth with a radius of 1 centimeter. Label this 'Earth Orbit'. **Step 3** - Using the string and thumbtack, draw a second circle with a radius of 30 centimeters. Label this 'Orbit of Moon'. **Step 4** - Draw a line connecting the center of earth and a point on the lunar orbit. Label the lunar orbit Point B. **Step 5** - Extend the line so that it intersects a point on the Earth orbit circle in the opposite direction from Earth's center. There should be two intersection points. The first will be between Earth and the lunar orbit. Label this Point C. The second will be behind Earth. Label this Point A. **Step 6** - As carefully as you can, draw a free-hand ellipse with one focus centered on Earth that arcs between Point A and Point B. This is called the major axis of the ellipse. See the above figure for comparison.

What you have drawn is a simple rocket trajectory, called a Hohmann Transfer orbit, that connects a spacecraft orbiting Earth, with a point on the lunar orbit path. If you had unlimited rocket energy, you could travel the path from Point C to Point B in a few hours or less. If you had less energy, you would need to take a path that looks more like the one from Point A to Point B and is slower, so it takes more time. Even less energy would involve a spiral path that connects Point A and Point B but may loop one or more times around Earth as it makes its way to lunar orbit. Can you draw such a path?

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Astrodynamicists are the experts that calculate orbits for spacecraft. One of the most important factors is the total speed change, called the delta-V, to get from one orbit to another. For a rocket to get into Earth orbit requires a delta-V of 8600 m/sec. To go from Earth orbit to the Moon takes an additional delta-V of 4100 meters/sec.

Problem 2 - To enter a Lunar Transfer Orbit, a spacecraft has enough fuel to make a total speed change of 3500 m/sec. If it needs to make a speed change of 2000 m/s in the horizontal direction, and 3000 m/sec in the vertical direction to enter the correct orbit, is there enough fuel to reach the Moon in this way? [Hint, use the Pythagorean Theorem]

Answer Key

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Answer - Distance = speed x time, so 380,000 km = 10 km/s x time. Solving for Time you get 38,000 seconds. Since there are 60 minute/hour x 60 seconds/minute = 3600 seconds/hour, 38,000 / 3600 = 10.6 hours.

Note: In reality, it takes several days for spacecraft to make the trip under the influence of gravity and allowing for conservation of energy and momentum.

Problem 2 - To enter a Lunar Transfer Orbit, a spacecraft has enough fuel to make a total speed change of 3500 m/sec. If it needs to make a speed change of 2000 m/sec in the horizontal direction, and 3000 m/sec in the vertical direction to enter the correct orbit, is there enough fuel to reach the moon in this way? [Hint, use the Pythagorean Theorem, or solve graphically]

Answer:

<u>Method 1</u>: From the lengths of the horizontal and vertical speeds, we want to find the length of the hypotenuse of a right triangle. Using the Pythagorean Theorem

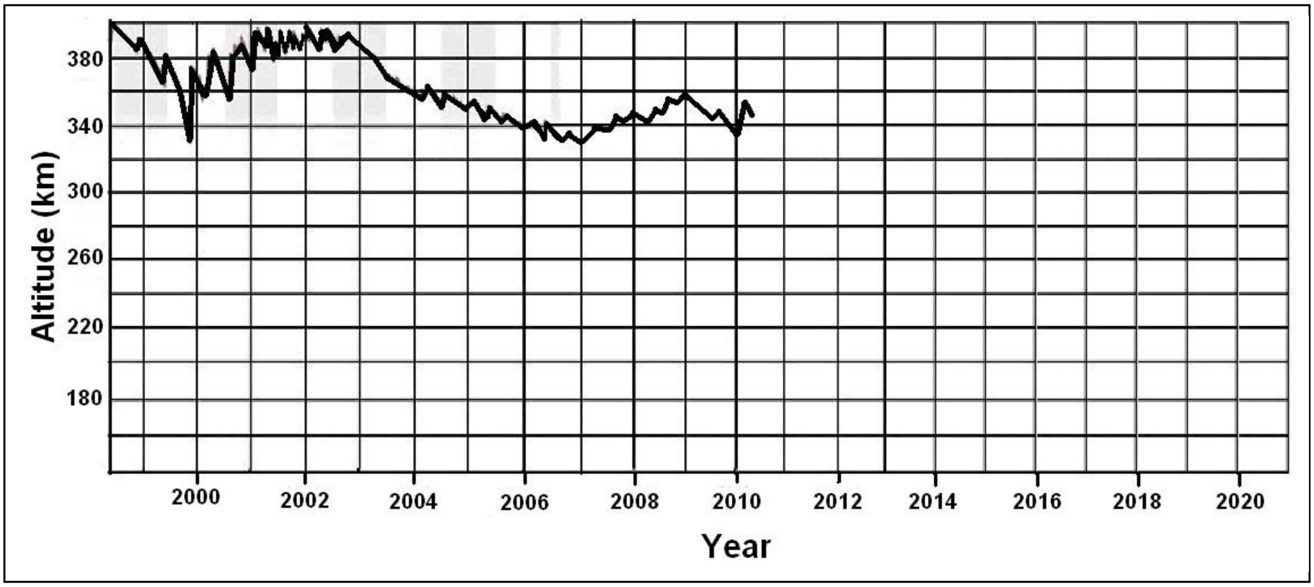
$$\text{total speed} = (2000^2 + 3000^2)^{1/2} = 3605 \text{ m/sec}$$

This is the total change of speed that is required, but there is only enough fuel for 3500 m/sec so the spacecraft cannot enter the Transfer Orbit.

<u>Method 2</u>: Graphically, draw a right triangle to the proper scale, for example, 1 centimeter = 1000 m/sec. Then the two sides of the triangle have lengths of 2 cm and 3 cm. Measure the length of the hypotenuse to get 3.6 cm, then convert this to a speed by multiplying by 1000 m/sec to get the answer.

The International Space Station - Follow that graph!

At the present time, the International Space Station is losing about 300 feet (90 meters) of altitude every day. Its current altitude is about 345 km after a 7.0-km re-boost by the Automated Transfer Vehicle, Jules Vern spacecraft on June 20, 2008. The graph below shows the ISS altitude since 1999.



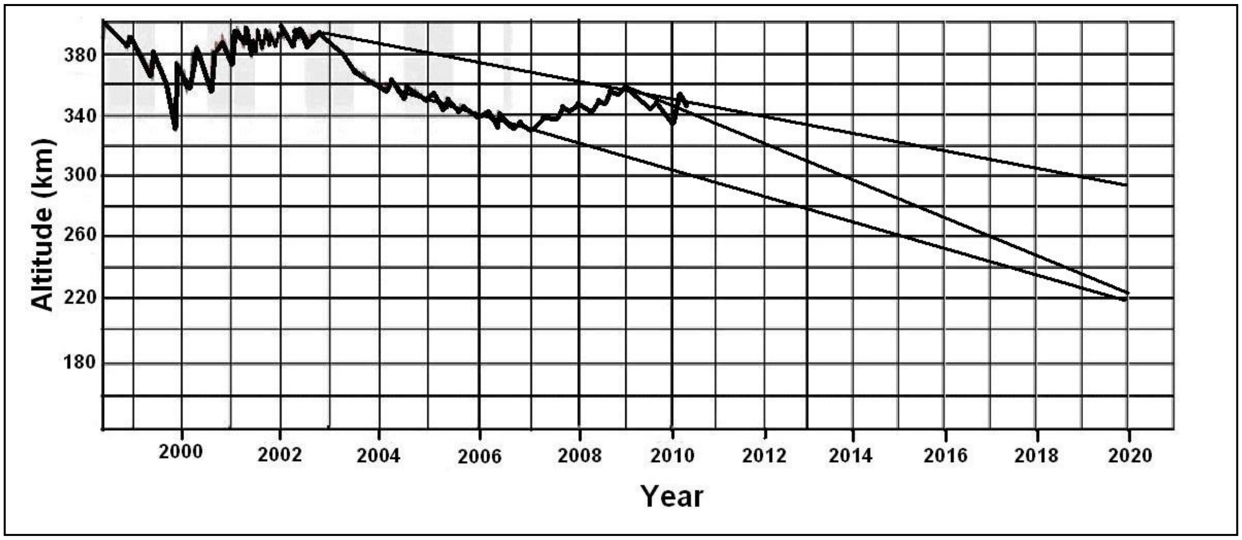
The drag of Earth's atmosphere causes the ISS altitude to decrease each day, and this is accelerated during sunspot maximum (between 2000-2001) when the dense atmosphere extends to a much higher altitude. At altitudes below about 200 km, spacecraft orbits decay and burn up within a week.

Problem 1 - From the present trends, what do you expect the altitude of the ISS to be between 2010 until its retirement year around 2020?

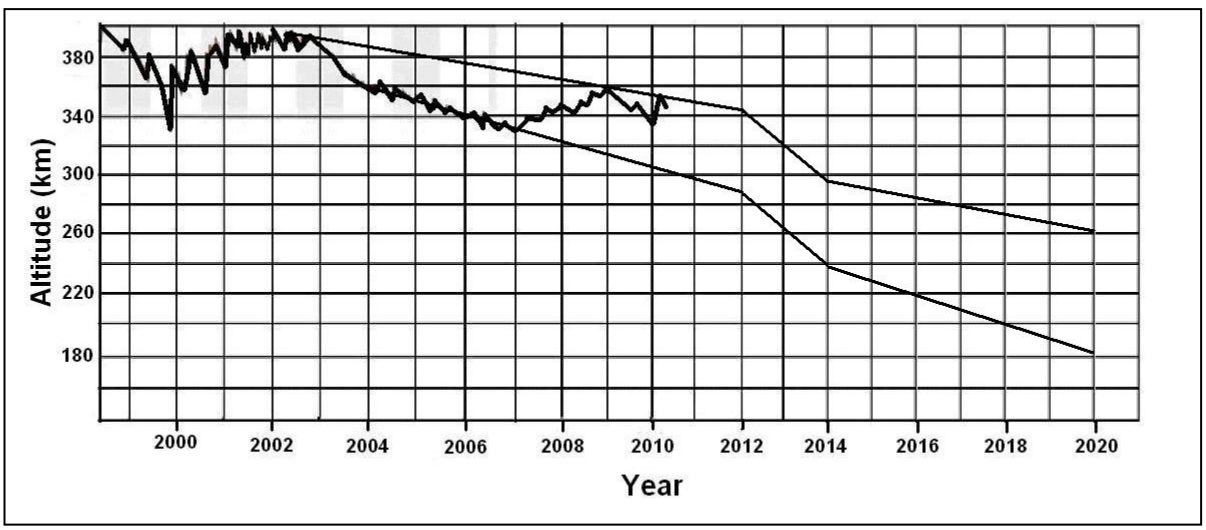
Problem 2 - Sunspot maximum will occur between 2012-2014, and we might expect a 50-km decline in altitude during this period if the solar activity weaker than the peak in 2000, which is currently forecasted. Including this effect, what might be the altitude of the ISS in 2020? Is the ISS in danger of atmospheric burn-up?

Problem 3 - What are the uncertainties in predicting ISS re-entry, and what strategy would you use if you were the Program Manager for the ISS?

Problem 1 – **Answer: The graph below shows several plausible linear trends depending on which features you use as a model for the slope. The predicted altitude would be between 220 and 300 km** .



Problem 2 - The graph below shows, for example, two forecasts that follow the extremes of the general decline trend, but then assume all of the altitude loss occurred between 2012-2014 at 50 km. Note that the range of altitudes in the graph in either case is 180-260 km. This places the ISS in danger of burn-up before its retirement year in 2014.



Problem 3 - The largest uncertainty is the strength of the next solar activity (sunspot) cycle. If it is stronger than the previous maximum between 2000-2001, the ISS altitude losses will be even larger in 2012-2014, and the ISS will be in extreme danger of re-entry before 2020.

The period after 2010, when the US losses access to space travel and has to rely on Russian low-capacity shuttles, will be a critical time for the ISS, and an intensely worrisome one for ISS managers.

Are U Still Nuts?

That's right… It's time for more unit conversion exercises!

Problem 1: The Solar Constant is the amount of energy that the sun delivers to the surface of Earth each second. If it is measured to be 1350000 ergs/cm^2 each second, how many watts per square meter is this? (1 watt = 10,000,000 ergs each second).

Problem 2: A supermassive black hole in the center of the quasar 3C273 swallows one star a year, and the heated gases emit 1.3 x 10^{53} ergs of energy. How much energy does 3C273 emit, in watts? (1 year = 31,000,000 seconds).

Problem 3: An astronaut is preparing a meal that includes 50 grams of cocoa mixed with 8 ounces of milk. What is the concentration of the chocolate in kilograms per liter? (128 oz = 1 gallon; and 9 gallons = 34 liters)

Answer Key

Problem 1: The solar constant is an important number if you are trying to build a solar, hot water heater or generate electricity using solar panels. Although astronomers use ergs and centimeter units, solar energy system design uses watts and square-meters for greater convenience.

In 1 second:

$$\frac{1350000 \text{ ergs}}{\text{cm}^2} \times \frac{1 \text{ watt}}{10{,}000{,}000 \text{ ergs}} \times \frac{100 \text{ cm}}{1 \text{ meter}} \times \frac{100 \text{ cm}}{1 \text{ meter}} = \mathbf{1350 \text{ watts/m}^2}$$

Notice how the units cancel, leaving behind the units of watt / (meter x meter) which can be re-written as watts/meter^{2}. This problem is a bit tricky because 1 watt is equal to 10,000,000 ergs per second, and not just 'ergs'. To avoid confusing students with compound unit conversions which come up all the time in science, the above conversion is for 1 second of time.

Problem 2 :

$$\frac{1.3 \times 10^{53} \text{ ergs}}{1 \text{ year}} \times \frac{1 \text{ year}}{31{,}000{,}000 \text{ sec}} \times \frac{1 \text{ watt}}{10{,}000{,}000 \text{ ergs/s}}$$

The first two rungs of the ladder convert the energy emitted into units of power in terms of ergs/sec. Note that the unit 'year' cancels in the numerator and denominator. The third rung converts the power units from ergs/s to watts. The answer is 4.2×10^{38} watts. Note that the sun produces 3.8×10^{26} watts, so 3C273 is about 1 trillion times as powerful as a single star.

Problem 3:

$$\frac{50 \text{ grams}}{8 \text{ ounces}} \times \frac{128 \text{ ounces}}{1 \text{ gallon}} \times \frac{9 \text{ gallon}}{34 \text{ liters}} \times \frac{1 \text{ kilogram}}{1000 \text{ grams}}$$

Note that all but the units kilogram/liter cancel out, leaving the answer **0.21 kg/liter.**

'Can you hear me now?': getting the message across

Have you ever wondered why some radio stations come in clearly in your radio, while others can barely be heard no matter how you crank up the volume? The reception of a radio signal depends on two very important quantities. The first is how much power the radio station is broadcasting. The second is how far that station is from you.

Some radio stations broadcast only at 100 watts, while others transmit over 50,000 watts of radio power. Imagine a 5-watt light bulb and a 100-watt light bulb. Which one do you think will be easier to see from across the room, or at 100-meters?

The brightness of a lamp, or a radio station, is measured by the amount of power that is delivered to a square-meter of area. We call this physical unit 'intensity' and measure its quantity in watts per square-meter (W/m^2). Because most kinds of lights and radio stations broadcast their power over a spherical volume, the intensity of a source is easily computed by dividing its power, P, by the surface area of a sphere whose radius, D, equals your distance from the source. The formula is just $P / 4\pi D^2$.

Transmitter	Distance (km)	Power (watts)	Intensity (watts/m^2)
AM Station	1000	50,000	**3.9×10^{-9}**
TV Station	100	50,000	
Cell Phone	1	0.3	
THEMIS P1	160,000	3	
STEREO A	15 million	10	
ACE	1.5 million	5	
MESSENGER	50 million	15	
Mars Orbiter	220 million	100	
Cassini	1.4 billion	20	
Ulysses	800 million	5	
Voyager 2	13 billion	40	

Problem 1 - Fill out the last column in the table to find the intensity of the radio signal at Earth. (Use Scientific Notation to an accuracy of one decimal place.)

Problem 2 - The signal from the Voyager-2 spacecraft, located beyond the orbit of Pluto, is just detectable by sensitive receivers of the Deep Space network on Earth. How far away would the AM radio station be to just be detectable by the DSN? Express the answer in kilometers, Astronomical Units (1 AU = 150 million kilometers), and light years. (Note 1 light year = 9.5 trillion km).

Answer Key

Problem 1 – Answer in last column

Transmitter	Distance (km)	Power (watts)	Intensity (watts/m^2)
AM Station	1000	50,000	**3.9×10^{-9}**
TV Station	100	50,000	**3.9×10^{-7}**
Cell Phone	1	0.3	**3.0×10^{-7}**
THEMIS P1	160,000	3	**9.2×10^{-18}**
STEREO A	15 million	10	**3.5×10^{-22}**
ACE	1.5 million	5	**1.8×10^{-19}**
MESSENGER	50 million	15	**4.7×10^{-22}**
Mars Orbiter	220 million	100	**1.6×10^{-22}**
Cassini	1.4 billion	20	**8.1×10^{-25}**
Ulysses	800 million	5	**6.2×10^{-25}**
Voyager 2	13 billion	40	**1.9×10^{-26}**

Problem 2 - If the radio station were at the distance of the Voyager 2 spacecraft, I = 2.3×10^{-23} watts/m^2. The desired Intensity = 1.9×10^{-26} watts/m^2, is 1230 times weaker. Since the signal intensity is proportional to the inverse-square of the distance, the distance would be increased by $(1230)^{1/2}$ = 35 times so that the radio station signal is now only 1.9×10^{-26} watts/m^2. The distance would then be 35 x 13 billion km or **456 billion km**, which equals **3040 AU**, or **0.001 light years**.

The STEREO Mission: getting the message across

In 2007, NASA launched two satellites, STEREO-A and STEREO-B that were to slowly drift away from Earth in opposite directions, taking 'stereo' images of the Sun as they went. Getting the information and images back to Earth posed a challenge because the farther away they drifted, the weaker the signal got. The table below gives the distance of STEREO-A from Earth. The satellite used a 10-watt transmitter operating at a frequency of 2,300 Megahertz .This, by the way, is about 10-times the frequency of a normal UHF TV station.

Date	Distance (Million km)	Intensity ($\times 10^{-24}$ W/m^2)	Received Power ($\times 10^{-20}$ Watts)
June-2007	24.6	1316	**126**
August-2007	40.3	490	
October-2007	51.0	306	
December-2007	55.2	261	
February-2008	58.4	234	
April-2008	67.0	177	
June-2008	81.9	119	
August-2008	97.6	84	
October-2008	107.3	69	
December-2008	111.2	64	

Problem 1 - How much weaker was the radio signal from STEREO-A in December 2008 than in June 2007?

Problem 2 - The Deep Space network radio dish has a diameter of 70-meters. In Column 4 calculate how many watts the dish collected from the signals sent during each month.

Problem 3 - If the satellite sends one bit of data ('1' or '0') every 5 seconds, how much energy is detected (in Joules) per bit sent in A)June 2007? and B) December 2008? (1 watt = 1 Joule per second).

Problem 4 - Suppose the receiver cannot detect less radio energy less than 2 x 10^{-25} Joules each bit. What is the largest number of bits that can be detected each second in A) June 2007 and B) December 2008?

Problem 5 - As a spacecraft gets further and further away from Earth, what kinds of strategies do engineers and scientists have to use to get their data back to Earth?

Answer Key

Problem 1 - How much weaker was the radio signal from STEREO-A in December 2008 than in June 2007? **Answer: The signal was 126/6 = 21 times weaker in December than in June.**

Problem 2 - Answer:

Date	Distance (Million km)	Intensity ($x10^{-24}$ W/m^2)	Receiver Power ($x10^{-20}$ Watts)
June-2007	24.6	1316	126
August-2007	40.3	490	**47**
October-2007	51.0	306	**29**
December-2007	55.2	261	**25**
February-2008	58.4	234	**22**
April-2008	67.0	177	**17**
June-2008	81.9	119	**11**
August-2008	97.6	84	**8**
October-2008	107.3	69	**7**
December-2008	111.2	64	**6**

Problem 3 - If the satellite sends one bit of data ('1' or '0') every 5 seconds, how much energy is detected (in Joules) per bit sent in A)June 2007? and B)December 2008? (1 watt = 1 Joule per second). Answer: A) 1.26 x 10^{-18} Joules/sec x 5 sec = **6.3 x 10^{-18} Joules**. B) 6 x 10^{-20} Joules/sec x 5 sec = **3.0 x 10^{-19} Joules.**

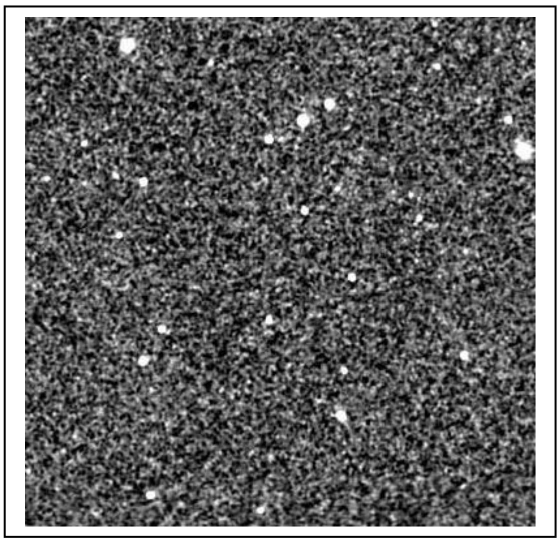
Problem 4 - Suppose the receiver cannot detect less radio energy less than 2 x 10^{-25} Joules each bit. What is the largest number of bits that can be detected each second in A) June 2007 and B) December 2008? Answer: A) 1.26 x 10^{-18} Joules/sec / (2.0 x 10^{-25} Joules/bit) = **6.3 million bits/sec**. B) 6 x 10^{-20} Joules/sec / (2.0 x 10^{-25} Joules/bit) = **300,000 bits/sec**.

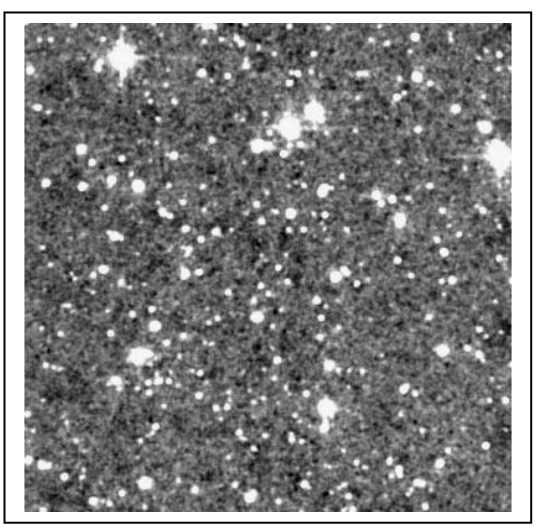
Problem 5 - Answer: **Here are just a few possibilities: 1) They can use larger radio dishes to increase the signal strength at the receiver. 2) They can slow down the data rates so that the energy per bit is larger making the data more easily detectable. 3) They can design the spacecraft with a more powerful transmitter. 4) They can do all of the above.**

How to make faint things stand out in a bright world!

You are, by now, probably familiar with the mathematical procedure of averaging numbers together. When we combine images together, we can use data averaging to make faint things stand out more clearly. To see why this happens, let's imagine a picture that consists of a string of just 5 pixels (out of the 3 million pixels that might exist in a typical digital camera image!). Let's take a snapshot of exactly the same scene 9 times without moving the camera, and note the values of the intensity numbers in each pixel. Here's what you might get:

Pixel	1	2	3	4	5	6	7	8	9	Average
1	120	122	120	123	110	114	112	110	110	**116**
2	125	115	110	130	115	110	113	110	109	
3	130	133	131	128	130	130	131	129	130	
4	122	125	123	120	110	105	115	120	110	
5	122	125	123	109	110	114	120	105	115	





Problem 1 - Calculate the average value of the 9 images for each pixel by completing the table. The first Pixel has been done already.

Problem 2 - Scientists discriminate between background 'noise' and 'source' whenever they look at an image. Background noise has the property that it averages to a relatively constant intensity that is nearly the same everywhere in the picture. A Source, however, tends to stand out in only a few pixels, and with an intensity brighter than the background. From the five pixel image, which pixels do you think have mostly Noise, and which have mostly Source?

Problem 3 - How easy would it have been if you only had Pictures 1 and 4 to work with in trying to study the faint source in the field?

Problem 4 - If you are trying to detect a faint source against a bright background, what is a good Rule of Thumb to use?

Problem 5 - The two images are from the 2MASS infrared sky survey. The bottom image is an average of over 5000 images like the one at the top. Can you find 5 stars that are present in the 'coadded' image below but not seen in the single image?

Answer Key

Problem 1 - Answer:

Pixel	1	2	3	4	5	6	7	8	9	Average
1	120	122	120	123	110	114	112	110	110	116
2	125	115	110	130	115	110	113	110	109	**115**
3	130	133	131	128	130	130	131	129	130	**130**
4	122	125	123	120	110	105	115	120	110	**117**
5	122	125	123	109	110	114	120	105	115	**116**

Problem 2 - Answer: From your averages in Column 11, you can see that Pixels 1, 2, 4 and 5 have very similar averages, while Pixel 3 has a very different average value. This means that Pixels 1,2, 4 and 5 behave like Noise in a roughly constant intensity background, while Pixel 3 looks a lot like a Source.

Problem 3 - Answer: It would have been very difficult because the Background Noise level that was detected in Pixel 2 in Picture 1 and Picture 4 sometimes got as intense as the faint source itself in Pixel 3.

Problem 4 - Answer: A good Rule of Thumb would be to take as many pictures of it as you can, and then average the pictures together to make the faint source stand out more clearly.

Problem 5 - Answer: Students will have an easy time of finding many such candidates. The top image was taken by a 1.5-meter telescope that can detect light at an infrared wavelength of 2.2 microns. (Sunlight has a wavelength of 0.6 microns). The exposure was only 7.8 seconds. The bottom image represents an exposure of 7.8 x 5000 = 39,000 seconds and is able to see stars nearly 10,000 times fainter than the shorter exposure image.

Working with Rates

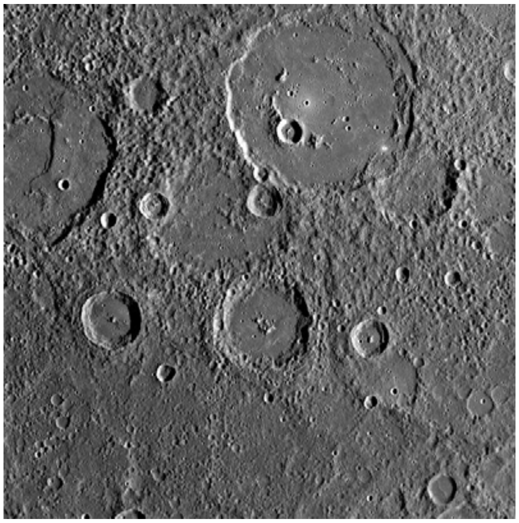


Image of craters on Mercury taken by the MESSENGER spacecraft.

Because things change in the universe, astronomers often have to work with mathematical quantities that describe complex rates.

Definition: A rate is the ratio of two quantities with different units.

In the problems below, convert the indicated quantities into a rate.

Example: 15 solar storms in 2 weeks becomes the rate:

$$R = \frac{\text{15 solar storms}}{\text{2 weeks}} = \frac{15}{2}$$

R = 7 solar storms/week.
or 7 solar storms per week.

Problem 1 - 15 meteor impacts in 3 months.
Problem 2 - 2,555 days in 7 years
Problem 3 - 1,000 atomic collisions in 10 seconds
Problem 4 - 36 galaxies in 2 two clusters
Problem 5 - 1600 novas in 800 years
Problem 6 - 416 gamma-ray bursts spotted in 52 weeks
Problem 7 - 3000 kilometers traveled in 200 hours.
Problem 8 - 320 planets orbiting 160 stars.
Problem 9 - 30 Joules of energy consumed in 2 seconds

Compound Units:

Problem 10 - 240 craters covering 8 square miles of area
Problem 11 - 16,000 watts of energy collected over 16 square meters.
Problem 12 - 380 kilograms in a volume of 20 cubic meters
Problem 13 - 6 million years for 30 magnetic reversals
Problem 14 - 1,820 Joules over 20 square meters of area
Problem 15 - A speed change of 50 kilometers/sec in 10 seconds.

Scientific Notation:

Problem 16 - 3×10^{13} kilometers traveled in 3×10^{7} seconds.
Problem 17 - 70,000 tons of gas accumulated over 20 million square kilometers
Problem 18 - 360,000 Newtons of force over an area of 1.2×10^{6} square meters
Problem 19 - 1.5×10^{8} kilometers traveled in 50 hours
Problem 20 - 4.5×10^{5} stars in a cluster with a volume of 1.5×10^{3} cubic lightyears

Answer Key

Problem 1 - 15 meteor impacts in 3 months. = **5 meteor impacts/month.**
Problem 2 - 2,555 days in 7 years = 2,555 days / 7 years = **365 days/year**
Problem 3 - 1,000 atomic collisions in 10 seconds = **100 atomic collisions/second**
Problem 4 - 36 galaxies in 2 two clusters = **18 galaxies/cluster**
Problem 5 - 1600 novas in 800 years = **2 novas/year**
Problem 6 - 416 gamma-ray bursts spotted in 52 weeks = **8 gamma-ray bursts/week**
Problem 7 - 3000 kilometers traveled in 200 hours. = **15 kilometers/hour**
Problem 8 - 320 planets orbiting 160 stars. = **2 planets/star**
Problem 9 - 30 Joules of energy consumed in 2 seconds = **15 Joules/second**

Compound Units:

Problem 10 - 240 craters covering 8 square miles of area = **30 craters/km^2**
Problem 11 - 16,000 watts of energy collected over 16 square meters. = **1000 watts/km^2**
Problem 12 - 380 kilograms in a volume of 30 cubic meters = **19 kilograms/m^3**
Problem 13 - 6 million years for 30 magnetic reversals = **200,000 years/reversal**
Problem 14 - 1,820 Joules over 20 square meters of area = **91 Joules/m^2**
Problem 15 - A speed change of 50 kilometers/sec in 10 seconds. = **5 km/sec^2**

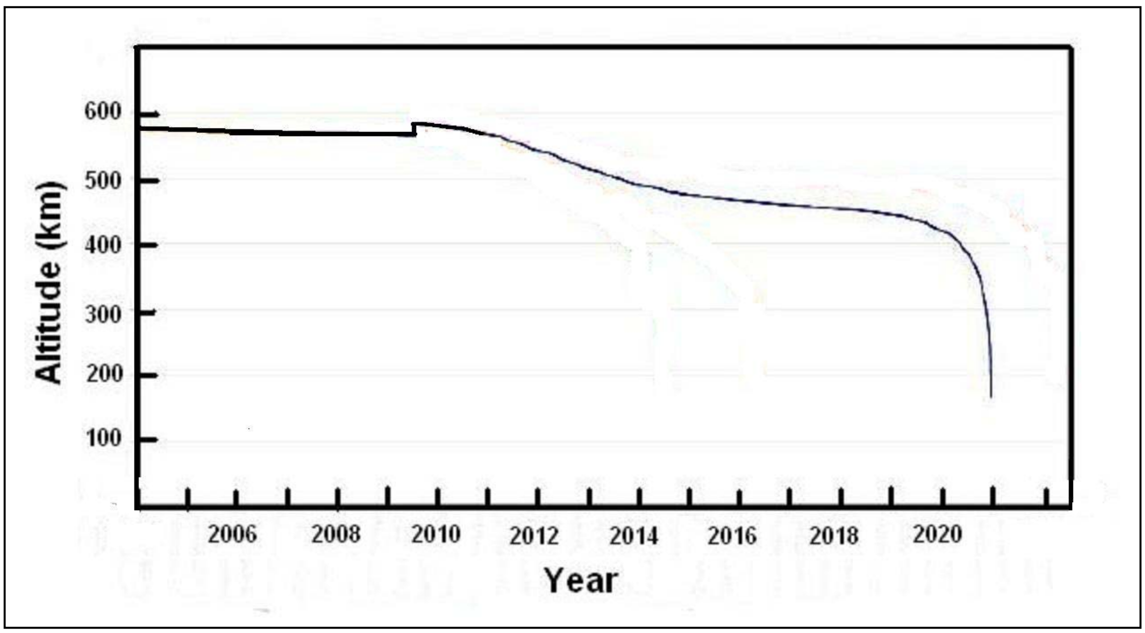
Scientific Notation:

Problem 16 - 3×10^{13} kilometers traveled in 3×10^{7} seconds.
= **$1.0\text{x}10^{6}$ kilometers/sec**
Problem 17 - 70,000 tons of gas accumulated over 20 million square kilometers
= 70,000 tons/20 million km^2 = **0.0035 tons/km^2**
Problem 18 - 360,000 Newtons of force over an area of 1.2×10^{6} square meters
= 392,000 Newtons/1,200,000 m^2 = **0.3 Newtons/m^2**
Problem 19 - 1.5×10^{8} kilometers traveled in 50 hours
= 1.5×10^{8} km/50 hrs = **3 million km/hr**
Problem 20 - 4.5×10^{5} stars in a cluster with a volume of 1.5×10^{3} cubic lightyears
= **300 stars/cubic lightyear**

Satellite Drag and the Hubble Space Telescope

The Hubble Space Telescope was never designed to operate forever. What to do with the observatory remains a challenge for NASA once its scientific mission is completed in 2012. Originally, a Space Shuttle was proposed to safely return it to Earth, where it would be given to the National Air and Space Museum in Washington DC. Unfortunately, after the last Servicing Mission, STS-125, in May, 2009, no further Shuttle visits are planned. As solar activity increases, the upper atmosphere heats up and expands, causing greater friction for low-orbiting satellites like HST, and a more rapid re-entry.

The curve below shows the predicted altitude for that last planned re-boost in 2009 of 18-km. NASA plans to use a robotic spacecraft after ca 2015 to allow a controlled re-entry for HST, but if that were not the case, it would re-enter the atmosphere sometime after 2020.



Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Answer: The Servicing Mission occurred in 2009. The upgrades and gyro repairs extend the satellite's operations by 5 more years, so if it re-enters after 2020 it will have about 6 years to go before uncontrolled re-entry.

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Answer: Use a millimeter ruler to determine the scale of the horizontal axis in weeks per millimeter. Mark the point on the curve that corresponds to a vertical value of 400 km. Draw a line to the horizontal axis and measure its distance from 2013 in millimeters. Convert this to weeks using the scale factor you calculated. The answer should be about 50 weeks.

"NASA's 23-year-old Hubble Space Telescope is still going strong, and agency officials said Tuesday (Jan. 8, 2013) they plan to operate it until its instruments finally give out, potentially for another six years at least.

After its final overhaul in 2009, the Hubble telescope was expected to last until at least 2015. Now, NASA officials say they are committed to keeping the iconic space observatory going as long as possible.

"Hubble will continue to operate as long as its systems are running well," Paul Hertz, director of the Astrophysics Division in NASA's Science Mission Directorate, said here at the 221st meeting of the American Astronomical Society. Hubble, like other long-running NASA missions such as the Spitzer Space Telescope, will be reviewed every two years to ensure that the mission is continuing to provide science worth the cost of operating it, Hertz added."

(Space.com, January 9, 2013.)

Story 1: On September 23, 1999 NASA lost the $125 million Mars Climate Orbiter spacecraft after a 286-day journey to Mars. Miscalculations due to the use of English units instead of metric units apparently sent the craft slowly off course -- 60 miles in all. Thrusters used to help point the spacecraft had, over the course of months, been fired incorrectly because data used to control the wheels were calculated in incorrect units. Lockheed Martin, which was performing the calculations, was sending thruster data in English units (pounds) to NASA, while NASA's navigation team was expecting metric units (Newtons).

Problem 1 - A solid rocket booster is ordered with the specification that it is to produce a total of 10 million pounds of thrust. If this number is mistaken for the thrust in Newtons, by how much, in pounds, will the thrust be in error? (1 pound = 4.5 Newtons)

Story 2: On January 26, 2004 at Tokyo Disneyland's Space Mountain, an axle broke on a roller coaster train mid-ride, causing it to derail. The cause was a part being the wrong size due to a conversion of the master plans in 1995 from English units to Metric units. In 2002, new axles were mistakenly ordered using the pre-1995 English specifications instead of the current Metric specifications.

Problem 2 - A bolt is ordered with a thread diameter of 1.25 inches. What is this diameter in millimeters? If the order was mistaken for 1.25 centimeters, by how many millimeters would the bolt be in error?

Story 3: On 23 July 1983, Air Canada Flight 143 ran completely out of fuel about halfway through its flight from Montreal to Edmonton. Fuel loading was miscalculated through misunderstanding of the recently adopted metric system. For the trip, the pilot calculated a fuel requirement of 22,300 kilograms. There were 7,682 liters already in the tanks.

Problem 3 - If a liter of jet fuel has a mass of 0.803 kilograms, how much fuel needed to be added for the trip?

Problem 1 - A solid rocket booster is ordered with the specification that it is to produce a total of 10 million pounds of thrust. If this number is mistaken for the thrust in Newtons, by how much, in pounds, will the thrust be in error? (1 pound = 4.5 Newtons)

Answer: 10,000,000 'Newtons' x (1 pound / 4.448 Newtons) = 2,200,000 pounds instead of 10 million pounds so the error is a 'missing' 7,800,000 pounds of thrust…an error that would definitely be noticed at launch!!!

Problem 2 - A bolt is ordered with a thread diameter of 1.25 inches .What is this diameter in millimeters? If the order was mistaken for 1.25 centimeters, by how many millimeters would the bolt be in error? Answer: 1- inch = 25.4 millimeters so 1.25 inches x (25.4 mm/ 1 inch) = 31.75 millimeters. Since 1.25 centimeters = 12.5 millimeters, the bolt would delivered 31.75 - 12.5 = 19.25 millimeters too small!

Problem 3 - In order to calculate how much more fuel had to be added, the crew needed to convert the quantity in the tanks, 7,682 liters, to a weight, subtract that figure from 22,300 kilograms, and convert the result back into a volume (liters).

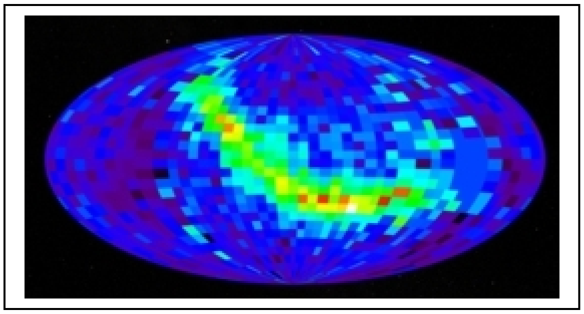
7,682 liters x (0.803 kilograms/ 1 liter)= 6,169 kg
22,300 kg – 6,169 kg = 16,131 kg
16,131 kg x (1 liter / 0.803 kilograms) = 20,088 liters of jet fuel.

Between the ground crew and flight crew, however, they arrived at an incorrect conversion factor of 1.77, the weight of a liter of jet fuel in pounds. This was the conversion factor provided on the refueller's paperwork and which had always been used for the rest of the airline's fleet. Their calculation produced:

7,682 liters x (1.77 pounds/liter) = 13,597 which they interpreted as kilograms but was actually the fuel mass in pounds! Then they continued the calculation:

22,300 kg – 13,597 'kg' = 8,703 kg
8,703 kg ÷ 1.77 = 4,916 liters ….so they were actually 15,172 liters short of fuel!

IBEX Creates an Unusual Image of the Sky!



NASA's IBEX satellite recently made headlines by creating a picture of the entire sky, not using light but by using cosmic particles called ENAs (Energetic Neutral Atoms). These fast-moving atoms flow through the solar system. Some of them reach Earth, where they can be captured by the IBEX satellite. By counting how many of these ENAs the satellite sees in different directions in the sky, IBEX can create a unique 'picture' of where ENAs are coming from in space.

The big surprise was that they were not coming from all over the sky as expected. They were also coming from a specific band of directions as we see in the image to the left. This image has the same kind of geometry as the map of the Earth below it! It is called a Mollweide Projection, except that instead of graphing geographic points on Earth, the IBEX image shows points in outer space!

	A	B	C	D	E
1					
2					
3					
4					
5					

Data String:
A5, E2, B2, D4, C3, A1, E4, C3, D4, B2, D4, B3, C4, E5, D5, D4, C2, D3, B1, E5, A2, C3, D5, C5, D4, E4, D3, C4, B4, D2, E3, C1, B5, A3, E1, A4, D1, B3, C2, E3

The IBEX satellite detected a series of particles entering its ENA instrument, and was able to determine the direction that each particle came from in the sky. The grid above shows a portion of the sky as a 5x5 grid with columns labeled by their letter and rows by their number. The data string to the right shows the detections of individual ENA particles with their direction indicated by their cell. 'A5, E2, B2…' means that the first ENA particle came from the direction of cell 'A5', the second from cell 'E2' and the third from cell 'B2' and so on. In some ways this process is like the 'call out' during a Bingo game, except that you keep track of the particle 'tokens' in each square to build a picture! Let's look at an example of constructing an ENA image.

Problem 1 - From the hypothetical data string, tally the number of particles detected in each sky cell in the grid. Select colors to represent the number of ENAs to create an 'image' of the sky in ENAs! How many particles were reported by this data string?

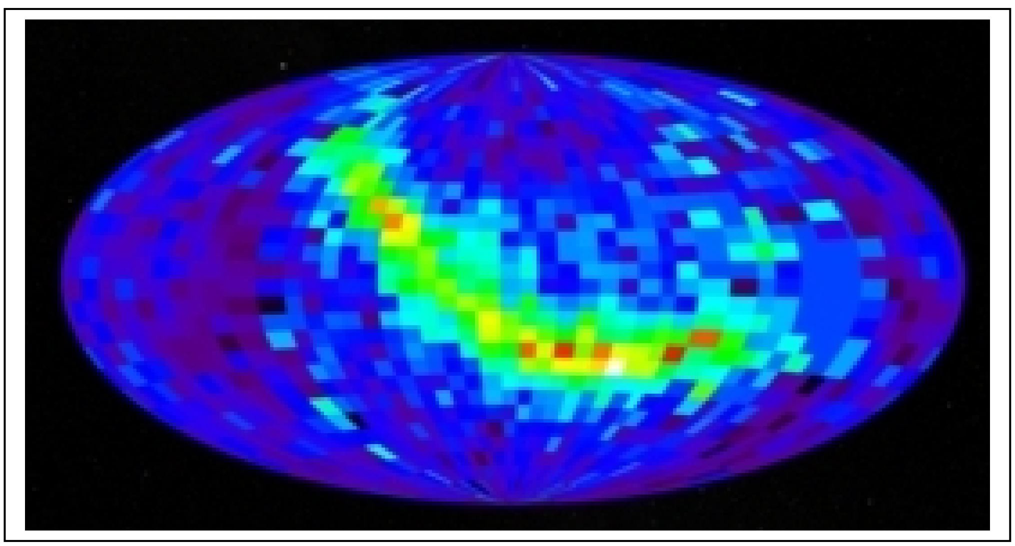
Problem 2 - Suppose that cell B2 is in the direction of the constellation Auriga, cell C3 is towards Taurus and cell D4 is towards Orion, from which constellation in the sky were most of the ENAs detected?

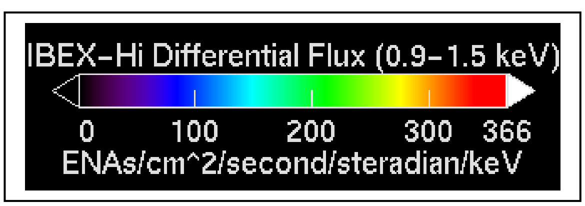
	A	B	C	D	E
1	1	1	1	1	1
2	1	2	2	1	1
3	1	2	3	2	2
4	1	1	2	5	2
5	1	1	1	2	2

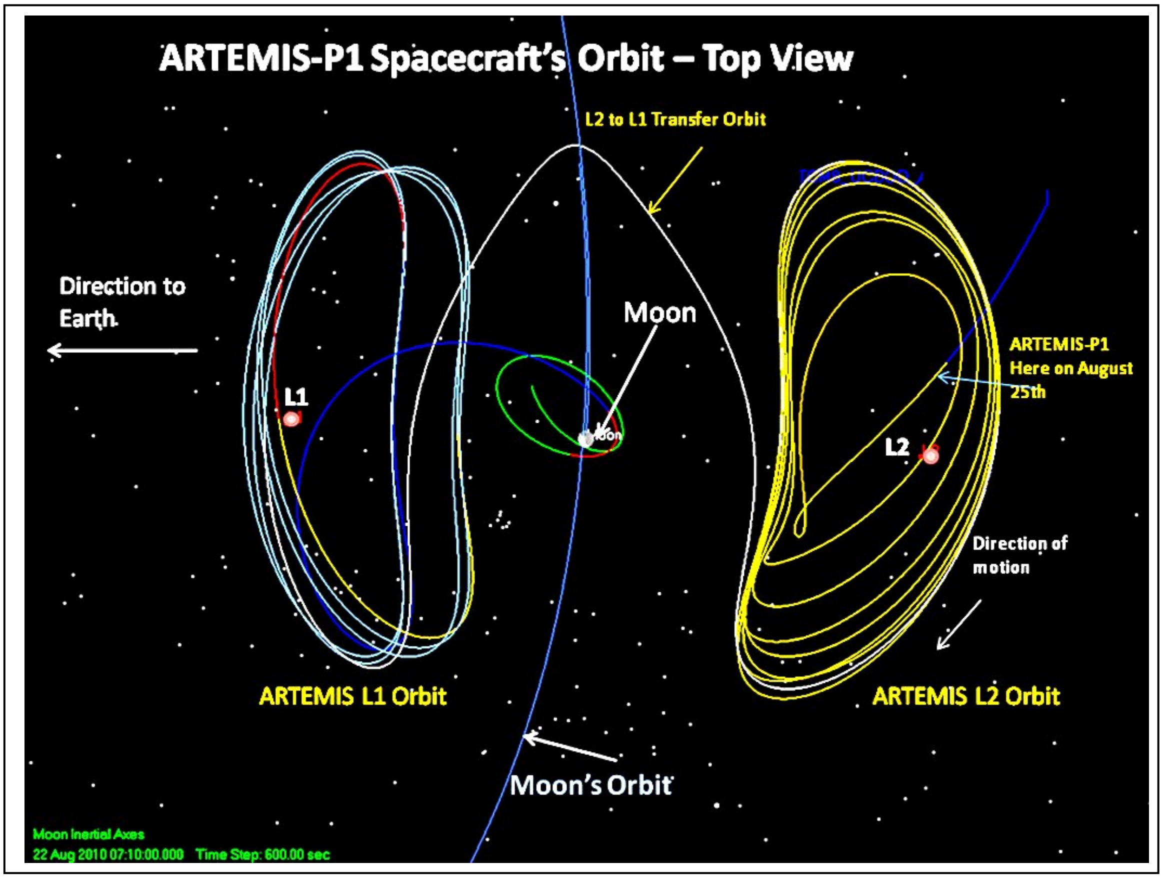
Problem 1 - Answer above. Students can select yellow = 5, orange=3, red=2 and blue=1 as an example and colorize the table. This is an example of using 'false color' to highlight data in order to reveal patterns. This technique is used by scientists, but should never be confused with the 'actual' color of an object. If you add up the number of ENAS in the data stream you get 40 ENAs being reported.

Problem 2 - The most ENAs recorded in any of the cells is '5' from Cell D4, which is in the direction of the constellation Orion.

The image below shows a close-up of actual IBEX data revealing the individual sky cells that make up the image. The color bar below shows the relationship between the color used, and the number of ENAs detected in each sky cell (called the particle flux).







The Artemis spacecraft, formerly P1 of the THEMIS constellation in Earth orbit, has been reprogrammed to journey to lunar orbit to continue making measurements of Earth's magnetic field. Once there, it will perform a complicated orbit between two 'invisible' points in space called the L1 and L2 Lagrange Points. These are locations within the combined gravitational forces of the Earth and Moon and spacecraft centrifugal forces are in near-equilibrium. The spacecraft can remain more or less at the same location with only minor use of its maneuvering rockets.

Problem 1 – The distance between L1 and L2 in the diagram above is 120,000 kilometers. Using a millimeter ruler, what is the scale of this figure in kilometers/millimeter?

Problem 2 – Using a piece of string and measuring to the nearest meter in length, about what is the total length of the spacecraft orbit shown in the figure in millions of kilometers?

Problem 3 – Assume that the average speed of the spacecraft in its travels is about 200 meters/sec. How long, in days, will it take the spacecraft to 'fly' the orbit pattern A) Around one of the Lagrange Points? B) Around the full orbital circuit shown above?

Problem 1 – The distance between L1 and L2 in the diagram above is 120,000 kilometers. Using a millimeter ruler, what is the scale of this figure in kilometers/millimeter?

Answer: The L1 and L2 points are 80 mm apart on the figure, so the scale is 120,000 km/80 mm = **1,500 km/mm**.

Problem 2 – Using a piece of string and measuring to the nearest meter in length, about what is the total length of the spacecraft orbit shown in the figure in millions of kilometers?

Answer: Depending on how students lay down the string along the indicated path the length should be about 2 meters. At the scale of the diagram, this is equivalent to **3 million kilometers.**

Problem 3 – Assume that the average speed of the spacecraft in its travels is about 200 meter/sec. How long, in days, will it take the spacecraft to 'fly' the orbit pattern A) Around one of the Lagrange points? B) Around the full orbital circuit shown above?

Answer:

A) The loop around L1 is about 150 mm, so the distance is about 225,000 kilometers. At a speed of 0.2 km/sec, a full loop will take about 1,100,000 seconds or since there are 86,400 seconds in 1 day, about **13 days**.

B) To travel 3 million kilometers at 0.2 km/sec will take about **170 days**.

A supercomputer consists of millions of identical 'processors' that work in parallel to perform thousands of trillions of calculations each second.

A supercomputer can perform a large number of calculations very fast compared to ordinary computers such as your desk or lap-top computer. This makes them very important for mathematically modeling complex systems such as the movement of millions of particles though space, the folding of complicated proteins, or the sequencing of DNA.

Let's have a look at one simple application to see how this works!

Imagine that you are following the movement of three stars through space, and that their distance from our sun in light years is given by the following three equations, where T is the elapsed time in millions of years:

$$d_1 = 3T + 2$$
$$d_2 = 4T - 2$$
$$d_3 = 8T + 3$$

Problem 1 - Using a calculator, A) evaluate by hand the values for d_1, d_2 and d_3 for the three time steps T= 1, 2 and 3. B) How long did it take you, in seconds, to perform the calculations? C) How many key strokes were involved? D) How many mathematical operations were involved?

Problem 2 - Suppose you wanted to follow the movement of 300 stars in a star cluster for three time steps. A) How many equations would be involved? B) How many keystrokes would you have to execute? C) How many mathematical operation would you have to perform? D) How long would the calculation take for three time steps? D) If N is the number of particles in your simulation, what is the general formula that gives the computation time in seconds for N particles?

Problem 3 - Suppose that you wrote a computer program to perform the calculations on a high-end laptop computer using a programming language such as FORTRAN, or C. With a 'clock speed' of 1 billion operations per second, how long would the computer take to evaluate the three particle positions after three time steps?

Problem 4 - A supercomputer not only performs an individual mathematical operation very fast, (3 million billion 'floating point operations per second' which is called 3 petaFLOPS) but it can perform many operations simultaneously by using parallel processing. In the example above, for each time step, suppose the computer might be built to use three 'processors', (one for each particle), to evaluate all three values for d in one step! How long would it take a 3 petaFLOP supercomputer to calculate three time steps for 10 million particles?

Problem 1 - Using a calculator, A) evaluate by hand the values for d_1, d_2 and d_3 for the three time steps T= 1, 2 and 3. B) How long did it take you, in seconds, to perform the calculations? C) How many key strokes were involved? D) How many mathematical operations were involved?

Answer: A) d1 = 5, 8 and 11. d2 = 2, 6 and 10; d3 = 11, 19, 27.

B) It takes about 5 seconds per evaluation, or **45 seconds for all three particles for 3 time intervals**, not including the time to write down the answer.

C) For one value: 3 x 1 + 1 = each entry is a key stroke. There are 3 timesteps x 3 particles x 6 keystrokes = **54 keystrokes** total.

D) There are only 2 operations per evaluation 'x' and '+' or '-' so a total of **18 operations.**

Problem 2 - Suppose you wanted to follow the movement of 300 stars in a star cluster for three timesteps. A) How many equations would be involved? B) How many keystrokes would you have to execute? C) How many mathematical operation would you have to perform? D) How long would the calculation take for three time steps? D) If N is the number of particles in your simulation, what is the general formula that gives the computation time in seconds for N particles? E) How long, in hours ,will it take to calculate the position of 300 particles for three timesteps?

Answer: A) There would be **300 equations** for ; $d_{\mathbf{1}}$, $d_{\mathbf{2}}$, $d_{\mathbf{3}}$, … $d_{\mathbf{300}}$.

B) 6 keystrokes x 3 timesteps x 300 equations = **5400 keystrokes**

C) 2 operations x 3 timesteps x 300 equations = **1800 operations**

D) From your answer to Problem 1A it takes about 5 seconds to calculate one time step for one particle, so T = 3 timesteps x 5 seconds x N or **T = 15 N seconds**.

E) T = 15 x 300 = 4500 seconds or **1.25 hours**.

Problem 3 - Suppose that you wrote a computer program to perform the calculations on a high-end laptop computer using a programming language such as FORTRAN, or C. With a 'clock speed' of 1 billion operations per second, how long would the computer take to evaluate 10 million particle positions after three time steps?

Answer: T = 3 timesteps x (2 operations/timestep) x (1 second/1 billion operations) x 10 million particles

= **0.06 seconds**.

Problem 4 - A supercomputer not only performs an individual mathematical operation very fast, (3 million billion 'floating point operations per second' which is called 3 petaFLOPS) but it can perform many operations simultaneously by using parallel processing. In the example above, for each time step, suppose the computer might be built to use three 'processors', (one for each particle), to evaluate all three values for d in one step! How long would it take a 3 petaFLOP supercomputer to calculate three time steps for 10 million particles?

Answer: The supercomputer would use 10 million 'parallel processors' so that

T = 3 timesteps x (2 operations/timestep) x (1 second/3 x 10^{12} operations)

= **2 x 10^{-12} seconds**.

The Advanced Land Imager (ALI) on NASA's Earth Observing-1 (EO-1) satellite captured this true-color image on February 15, 2010. A network of bright rectangles of varying shades of green contrasts with surroundings of gray, beige, tan, and rust typical of the arid conditions prevailing in Namibia. The Orange River serves as part of the border between Namibia and the Republic of South Africa. Along the banks of this river, roughly 100 kilometers (60 miles) inland from where the river empties into the Atlantic Ocean, irrigation projects take advantage of water from the river and soils from the floodplains to grow produce, mostly grapes, turning parts of a normally earth-toned landscape emerald green. (North is indicated by arrow)

Problem 1 - Assume that each one of the square cultivated areas is about 300 meters on a side. How large is a single cultivated area in square meters and in acres? (Note: 1 acre = 4,047 square meters)

Problem 2 - What is the total area under cultivation in this region in square kilometers and in acres?

Problem 3 - If a single grape plant covers 4 square meter of land and requires about 5 gallons of water per day, how many grape plants are being supported in this cultivated area, and how many gallons of water per day do they require from the Orange River?

Problem 1 - Each one of the square cultivated areas is about 300 meters on a side. What is the total area of a single cultivated area in square meters and in acres if 1 acre equals 4,047 square meters?

Answer: Since each plot of land is assumed to be square, the area of a single plot is just A = (300 m) x (300 m) = **90,000 square meters**. Then since 1 acre equals 4,067 square meters, we have:

90,000 meters^2 x (1 acre / 4,067 m^2) = **22.1 acres**.

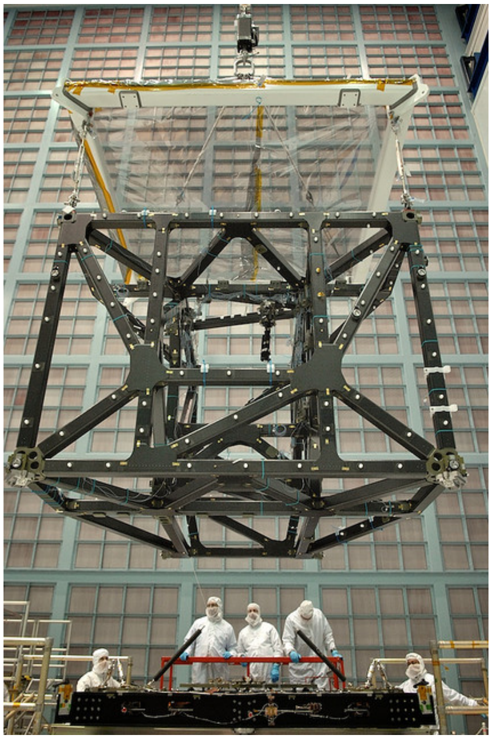
Problem 2 - What is the total area under cultivation in this region in square kilometers and in acres?

Answer: Carefully count the number of green squares in the picture. Count the partial square areas as best as possible. Answers should be near 103. Then multiply by the area of one square determined from Problem 1 to get A = 90,000 m^2 x (103) = 9.3 million square meters. In terms of square kilometers, 1 km^2 = 1 million square meters, so the area is **9.3 km^2**.

In terms of acres, 22.1 acres/square x (103 squares) = **2276.3 acres**. To three significant figures, the answer is also **2280 acres**.

Problem 3 - If a single grape plant covers 4 square meter of land and requires about 5 gallons of water per day, how many grape plants are being supported in this cultivated area, and how many gallons of water per day do they require from the Orange River?

Answer: The total cultivated area is 9.3 million square meters, which represents about N = 9.3 million meters^2 x (1 plant/4 meters^2) = 2.3 million plants. Since each plant requires 5 gallons per day, the total daily irrigation draw from the Orange River would be about 2.3 million plants x (5 gallons / 1 plant) = **11.5 million gallons per day.**



The Webb Space Telescope, to be launched in 2014, will allow astronomers to see the first generations of stars that formed in the universe.

The ISIM, or the Integrated Science Instrument Module Flight Structure, will serve as the structural "heart" of the James Webb Space Telescope. It will serve as the cage in which the many sensitive instruments will analyze the ancient starlight from the distant universe.

NASA engineers recently tested the ISIM structure by exposing it to extreme cryogenic temperatures, proving that the structure will remain stable when exposed to the harsh environment of space.

The ISIM is 1.92 meters tall and has a mass of 193 kilograms. Engineers invented a new material so that the ISIM can support more than four times its own mass in sensitive scientific equipment. It also has to keep the geometry of this equipment exactly right under cold space conditions. The heating and cooling of the ISIM in space causes expansion and contraction, which can ruin the scientific measurements. The latest cold tests by NASA engineers proved that the ISIM does not change its shape by more than 150 microns as it is chilled from room-temperature (300 K) to near-absolute zero (20 K). This is very good news!

To better see how little the ISIM changed its dimensions, suppose that you created a scale model of this structure and made it the same size as a 10-story building 30 meters tall.

Problem 1 –On this scale, how much would your model of the ISIM have changed its dimensions?

Problem 2 – Engineers prefer to work in English units. If the density of the composite materials used in this structure is 0.06 pounds/inch^3, what is the density in metric units of kilograms/meter^3 if 1 pound = 0.45 kg, and 1 meter = 39.4 inches?

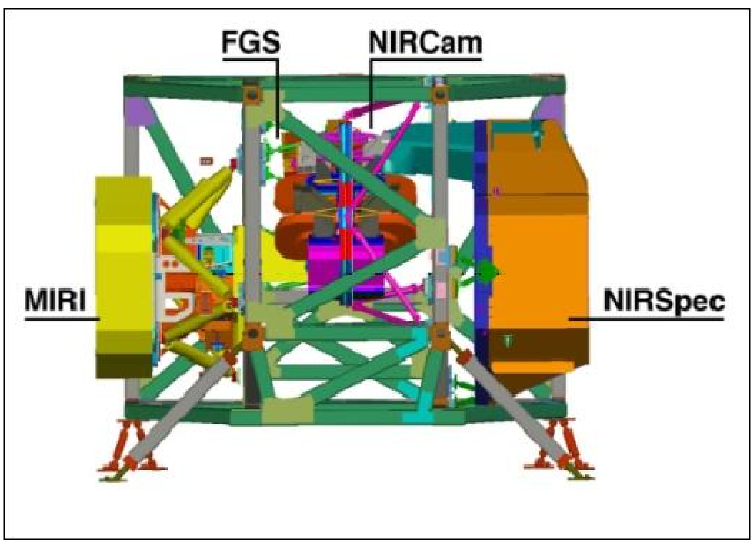
Problem 1 – On this scale, how much would your model of the ISIM have changed its dimensions?

Answer: The true size of the ISIM is 1.92 meters tall, so the new scaled model is a factor of 30 meters / 1.92 meters = 15.6 times larger that the actual ISIM module. The original ISIM module changed its dimensions during cooling by 150 microns, so in the new scaled model the dimensions would have changed by 15.6 x 150 microns = 2340 microns. In terms of meters, this is just 2340 microns x (1 millimeter/1000 microns) = **2.3 millimeter**. On the scale of the 10-story building, the structure would have changed by about the thickness of a dime!

Problem 2 – Engineers prefer to work in English units. If the density of the composite materials used in this structure is 0.06 pounds/inch^3, what is the density in metric units of kilograms/meter^3 if 1 pound = 0.45 kg, and 1 meter = 39.4 inches?

Answer: 0.06 pounds/inch^3 x (0.45 kg/1 pound) x (39.4 inches/1 meter)^3 = **1651 kg/m^3**
Note: For the correct number of significant figures (0.29 and 0.45 have 2 which is the minimum number) the answer would be **1700 kg/m^3** . By comparison, water has a density of 1000 kg/m^3.

The diagram below shows how the ISIM module will be loaded with scientific equipment to measure the faint starlight collected by the Webb Space Telescope in 2014. The instruments are very much like 'high-tech' digital cameras that can precisely measure the brightness of the light falling onto the millions of pixels in each camera system. Astronomers will use these images to discover the first generation of stars and galaxies born in our universe over 13 billion years ago.



The Space Shuttle – Fly me to the moon?

A common misconception shared by many students, and perhaps some members of the public, is that the Space Shuttle could have been used to travel to the Moon.

A few simple math problems will show that this could never have been a possibility, even if engineers had decided to 'upgrade' the Shuttle. Here are some basic facts:

Lunar distance : 384,000 km

Shuttle Orbital Speed : 17,500 miles / hr

Maximum cargo mass : 55,000 pounds

Astrodynamicists are physicists and engineers who calculate the trajectories and orbits for spacecraft throughout the solar system. Near Earth, we can think of successive orbits at increasing distance from Earth as representing a ladder. A spacecraft needs to expend more energy the higher up the ladder its orbit is from the surface of earth. This means that more distant orbits require larger launch vehicles and longer 'burn' times to get to them, than orbits close to Earth. Astrodynamicists think of the process of changing from one orbit to the other as a series of speed (velocity) changes. For example, the orbit of the Space Shuttle at an altitude of 380 km has an orbital speed of 7.68 km/s. The more distant orbits of the commercial geostationary communications satellites, located 35,800 km from Earth's surface, represent an orbital speed of 3.07 km/sec.

You might think that, since the GEO satellite orbit speed is slower than the Space Shuttle, all you have to do is 'slow down' the Space Shuttle by decreasing its kinetic energy, and it will move out to GEO satellite orbits. In fact, because of the way that kinetic energy changes in a gravitational field, you actually have to <u>increase</u> the kinetic energy of the Space Shuttle to make this orbit change. This is done by turning on its rockets for enough time to move it outwards from Earth. Once its orbit speed has dropped 4.61 km/s to the new value of 3.07 km/s, it will find itself at GEO orbit altitude.

Problem 1 – The Moon is 'located' at an orbit speed of 1.0 km/sec. What must be the Space Shuttle speed change to reach lunar orbit?

Problem 2 – When the Shuttle Orbital Maneuvering System (OMS) is turned on, it can cause a speed change of 0.6 meter/s for every second that the engines are burning. How many seconds would the OMS have to remain on in order for the Space Shuttle to build up the necessary velocity change to reach the Moon?

Problem 3 – The OMS can produce a maximum delta-V of 1,000 m/s before consuming all of its 9,700 pounds of fuel. How many pounds of fuel will the OMS have to expend to get to the Moon?

Problem 4 – Does the Space Shuttle have the payload capacity to carry enough extra fuel for this one-way trip?

Answer Key

Problem 1 – The Moon is 'located' at an orbit speed of 1.0 km/sec. What must be the Space Shuttle speed change to reach lunar orbit?

Answer: Delta-V = 1.0 – 7.68 = **- 6.68 km/sec**.

Problem 2 – When the Shuttle Orbital Maneuvering System (OMS) is turned on, it can cause a speed change of 0.6 meter/s for every second that the engines are burning. How many seconds would the OMS have to remain on in order for the Space Shuttle to build up the necessary velocity change to reach the Moon?

Answer: The delta-V to get to the moon is 6,680 meters/sec. The OMS can produce 0.6 m/s every second, so the total burn time would have to be T = 6,680 / 0.6 or about **11,000 seconds or 3 hours of continuous thrust.**

Problem 3 – The OMS can produce a maximum delta-V of 1,000 m/s before consuming all of its 9,700 pounds of fuel. How many pounds of fuel will the OMS have to expend to get to the Moon?

Answer: We need a total delta-V of 6,680 m/s, so 9,700 pounds x (6,680/1,000) = **64,796 pounds of fuel.**

Problem 4 – Does the Space Shuttle have the payload capacity to carry enough extra fuel for this one-way trip?

Answer: The maximum cargo mass is 55,000 pounds, so the OMS fuel would require 64,796 / 55,000 = 1.2 times the maximum load of the Space Shuttle cargo bay. **No, the Shuttle does not have enough capacity to lift all of the required OMS fuel to Earth orbit.**

This pair of images shows the historic launch of the Space Shuttle Atlantis (STS-135) on July 8, 2011 at 11:29 a.m. EDT, from launch pad 39A at the NASA Cape Canaveral Space Center. It shows part of the exhaust plume cloud ejected by the shuttle engines as they were ignited. This vapor plume is created when the exhaust gases from the rocket engines interact with the Sound Suppression Water System (SSWS).

The bottom image was taken at 11:29:14.0 a.m. EDT, and the top image was obtained at 11:29:15.0 a.m. EDT. The length of the space shuttle is 37 meters from its pointed top end to the base of its rocket nozzles.

The speed of the SSWS plume can be estimated from the clues in the two images.

Problem 1 - What is the scale of each image in meters/millimeter?

Problem 2 - How far did the SSWS plume travel in the time interval between the two images?

Problem 3 - What is the speed of the SSWS plume in A) meters/sec? B) kilometers/hr and C) miles per hour?

Problem 4 - Assuming that you could survive the noise of the rocket motors, would you be able to out-run the SSWS plume if you ran as fast as Olympic sprinter Usain Bolt in the 2008, 100-meter race whose time was 9.7 seconds?

Problem 1 - What is the scale of each image in meters/millimeter?
Answer: The shuttle measures about 12 millimeters so the scale is 37 meters/12 mm = **3 meters/mm**.

Problem 2 - How far did the SSWS plume travel in the time interval between the two images?
Answer: Students can measure the distance from the tip of the plume to the right-hand edge of the image to get 14mm (top) and 33mm (bottom) for a difference of 19 mm. Multiplying by the scale of 3 m/mm we get **57 meters**.

Problem 3 - What is the speed of the SSWS plume in A) meters/sec? B) kilometers/hr/ C) miles per hour?

Answer: The time between the two images is 1 second, then

A) 57 meters/1 sec = **57 m/sec**.
B) 57 m/s x (1 km/1000 m) x (3600 s/1 hr) = **205 km/hr**.
C) 205 km/hr x 0.62 miles / 1 km) = **127 mph**.

Problem 4 - Assuming that you could survive the noise of the rocket motors, would you be able to out-run the SSWS plume if you ran as fast as Olympic sprinter Usain Bolt in the 2008, 100-meter race whose time was 9.7 seconds?

Answer: Usains speed was 100 meters/9.7 sec = 10.3 meters/sec which is <u>much slower</u> than the 57 m/sec plume speed.

Saturn V Rocket Launch Speed and Height

Time (Sec)	Altitude (m)	Speed (m/s)
0	0	0
1	2	3
2	4	4
3	13	7
4	25	10
5	32	12
6	48	15
7	55	16
8	83	20
9	103	23
10	126	25
11	148	27
12	179	31
13	217	34
14	242	36
15	269	39
16	321	43
17	367	46
18	406	49
19	465	53
20	510	56

The Saturn V rocket carrying the Apollo-11 astronauts to the moon was launched from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT) from launch pad 39A. It weighed 2,766,913 kg just before launch, and was 102 meters tall. The gantry was 106 meters tall. See the YouTube Video of the launch at
http://www.youtube.com/watch?v=F0Yd-GxJ_QM&feature=related

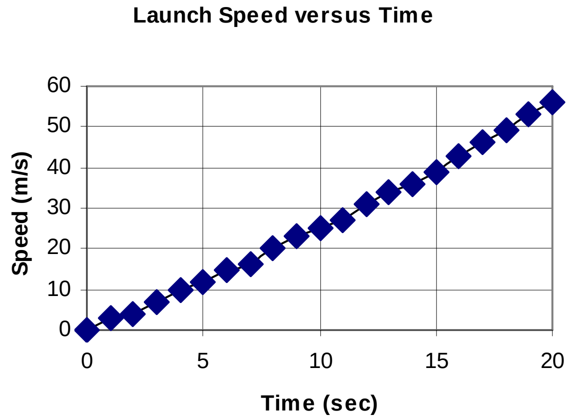
Problem 1 - Graph the speed of the rocket during the first 20 seconds after launch.

Problem 2 - Graph the height of the rocket above the launch pad during the first 20 seconds after launch.

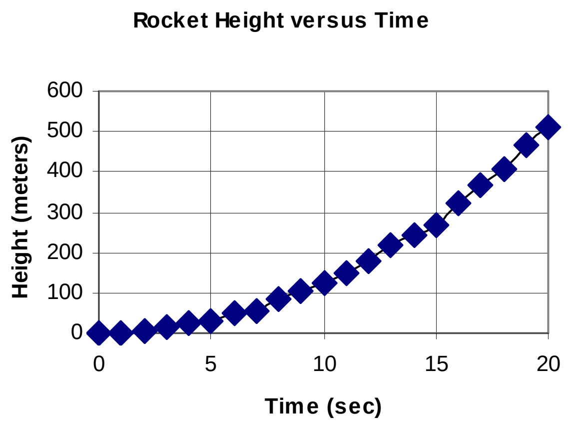
Problem 3 - At what time did the bottom of the rocket just clear the top of the launch gantry?

Problem 4 - How fast was the Saturn V traveling at the time the rocket engines just cleared the top of the gantry in A) meters/second? B) miles/hour?

Problem 1 - Graph the speed of the rocket during the first 20 seconds after launch.



Problem 2 - Graph the height of the rocket above the launch pad during the first 20 seconds after launch.



Problem 3 - At what time did the bottom of the rocket just clear the top of the launch gantry?

Answer: The gantry is 106 meters tall. The table, or graph, shows that at about **9 seconds**, the Saturn V rocket has an altitude of 103 meters, which is close to the gantry height.

Problem 4 - How fast was the Saturn V traveling at the time the rocket engines just cleared the top of the gantry in A) meters/second? B) miles/hour?

Answer: A) At 9 seconds, the rocket was traveling at about 23 meters/sec.
B) Converting 23 meters/sec to miles/hr:

23 meters/sec x (1 km / 1000 meters) x (0.62 miles/1 km) x (3600 sec/1 hr) = **51 miles per hour**.

Space Shuttle Endeavor - Last Flight

Time	Altitude (meters)	Speed (m/s)
0	0	0
1	3	5
2	7	12
3	17	17
4	42	24
5	69	30
6	94	36
7	134	42
8	186	50
9	247	59
10	283	63
11	334	69
12	429	79
13	523	87
14	614	95
15	665	100
16	755	106
17	888	117
18	1019	126
19	1158	136
20	1334	148

The final launch of NASA's space shuttle Endeavor (STS-134) occurred on May 16, 2011 at 8:56:28 a.m. EDT from launch pad 39A. The image above was taken 17 seconds after launch. See the launch video on YouTube at **http://www.youtube.com/watch?v=ShRa2RG2KDI**

This historic flight was watched by millions of people world-wide. The table above shows the speed and altitude data for the first 20 seconds after launch. The combined fuel tanks and Orbiter had a mass of 2,052,443 kg at launch. The launch gantry had a height of 106 meters.

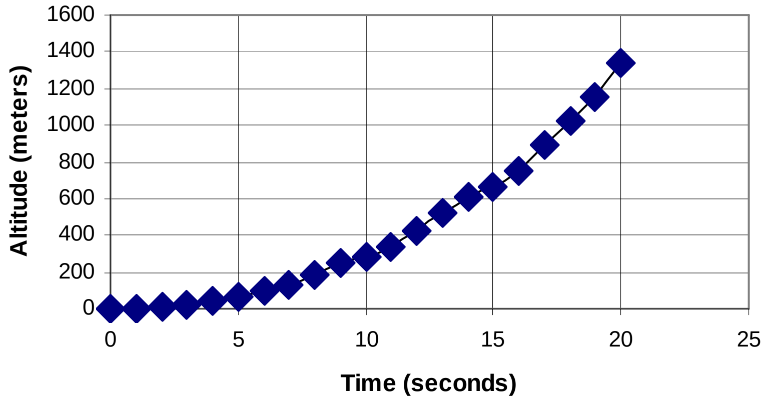
Problem 1 - Plot the altitude of Endeavor Shuttle versus time during the first 20 seconds of launch.

Problem 2 - Plot the speed of the Endeavor Shuttle versus time during the first 20 seconds of launch.

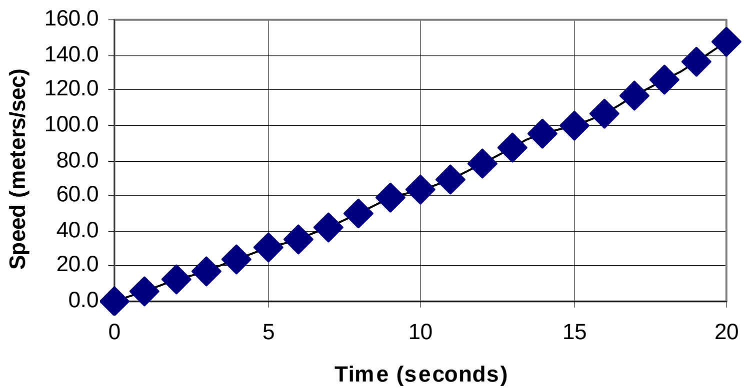
Problem 3 - About what is the speed of the Shuttle when it clears the gantry in A) meters/sec/ B) miles per hour?

Problem 4 - What is the average acceleration of the shuttle during its first 20 seconds of flight?

Problem 1 - Plot the altitude of Endeavor Shuttle versus time during the first 20 seconds of launch.



Problem 2 - Plot the speed of the Endeavor Shuttle versus time during the first 20 seconds of launch.



Problem 3 - About what is the speed of the Shuttle when it clears the gantry in A) meters/sec/ B) miles per hour?

Answer: A) The gantry height is 106 meters, so from Problem 1, we estimate that this speed occurred about 6 seconds after launch. The speed at this time is about **S=36 m/s.** B)
S = 36 m/s x (3600 s/1 hr) x (1 km/1000 meters) x (0.62 miles/km) = **80 mph**.

Problem 4 - What is the average acceleration of the shuttle during its first 20 seconds of flight?

Answer: Acceleration = velocity change/time, so between T=0 and T = 20 sec, the speed changed from 0 m/s to 148 m/s so A = 148/20 sec = **7.4 m/sec/sec**.

Approximately 24,300 tiles were installed on each space shuttle and each tile was designed to survive 100 trips to space and back. Varying in thickness from one inch (2.54 cm) to five inches (12.7 cm) depending on the heating they will be subjected to, the tiles collectively protected the orbiter from temperatures as high as 2,300 degrees Fahrenheit during its reentry into the Earth's atmosphere.

The silica tile material is referred to as LI-900. They insulate heat so well that tiles can be held bare-handed on one side even while the opposite side is still red hot. Educators can demonstrate that ability in the classroom, substituting a blow torch for the re-entry-generated heating.

LI-900 has a density of 9 pounds per cubic foot (144.2 kg/m^3). It is made from pure silica glass fibers, but 94% of the volume of each tile is pure air, making each tile incredibly light and strong!

Problem 1 – If the dimensions of an average tile are 15cm x 15 cm x 6cm, what is the total volume of the Space Shuttle heat shield provided by the 24,300 tiles in cubic meters?

Problem 2 – About what is the mass, in grams, of one average tile?

Problem 3 – What is the total mass of the Space Shuttle heat shield in

A) kilograms?

B) pounds ? (1 pound = 0.453 kg)

Problem 1 – If the dimensions of an average tile are 15cm x 15 cm x 6cm, what is the total volume of the Space Shuttle heat shield provided by the 24,300 tiles in cubic meters?

Answer: A single average tile has a volume of
V = 0.15 m x 0.15m x 0.06 m
= 0.00135 meters^3,

so the total volume occupied by 24,300 tiles is about

V = 24,300 x 0.00135 = **32.8 cubic meters.**

Problem 2 – About what is the mass, in grams, of one average tile?

Answer: Mass = Volume x Density
= 0.00135 m^3 x 144.2 kg/m^3
= 0.195 kilograms.

Since 1 kilogram = 1000 grams, we have a mass per tile of about **195 grams**.

Problem 3 – What is the total mass of the Space Shuttle heat shield in A) kilograms? B) pounds is 1 pound = 0.453 kg?

Answer: A) Mass = volume x density
= 32.8 cubic meters x 144.2 kg/m^3 = **4,730 kg**.

B) 4,730 kg x (1 pound / 0.453 kg) = 10,441 pounds (or about 5 tons!)

Note to Teachers: **Free Space Shuttle Tiles for your Classroom!**

You can still get your own free NASA tiles from the Space Shuttle program!!! Schools may request a tile at the "NASA Space Programs - Historic Artifacts Prescreening" Web site. **http://gsaxcess.gov/NASAWel.htm**

Once at the site, go to the "NASA Artifacts Prescreening Register" block of information to register and receive your login ID and password.

There is no charge for the Shuttle Tiles and Space Food Kits . However, the recipient is responsible for the shipping and handling fee of $23.40 for Shuttle Tiles and $28.03 for the Space Food Kits. Payment must be made to the shipping agent with a credit card via a web link provided in the module.

A Simple Gauge in a Fuel Tank - I

This is a photo of the Space Shuttle main fuel tank just after being jettisoned at an altitude of 50 miles. The liquid hydrogen is approximately shaped like a cylinder with a diameter of 8.4 meters and a length of 29.6 meters.

The formula for the volume of a cylinder is given by

$$V = \pi R^2 h$$

where **R** is the radius of the cylinder and **h** is its length.

Problem 1 – To two significant figures, what is the volume of the fuel tank in:

A) Cubic meters?

B) Cubic centimeters?

C) Liters (1 liter = 1000 cm^3)?

D) Gallons? (1 liter = 0.26 gallons)

Problem 2 – For safety, engineers want to install a gauge that will indicate when there is only 1/8 of the original fuel volume remaining in the tank. How many meters from the bottom of the tank must the gauge be located to be triggered?

Problem 3 – The rate at which fuel is burned by the Space Shuttle engines is about 1000 gallons per second. From ignition, how many minutes (to the nearest tenth) will it take for the fuel to reach the level of the fuel gauge?

Problem 1 – What is the volume of the fuel tank for R = 8.4/2 = 4.2 meters and h = 29.6 meters.

A) $V = 3.14\,(4.2)^2\,(29.6) = 1639$ **= 1600 meters^3**

B) $V = 1600 \times (100\text{ cm} / 1\text{ m})^3 =$ **1.6 billion cm^3**

C) V = **1.6 million liters**

D) V = 1.6 million liters x (0.26 gallons/1 liter)
= 416,000 = **420,000 gallons**

Problem 2 – For safety, engineers want to install a gauge that will indicate when there is only 1/8 of the original fuel volume remaining in the tank. How many meters from the bottom of the tank must the gauge be located to be triggered?

Answer: Method 1: V = 1/8 (1600) = 200 meters^3. Then since R = 4.2 meters, from the formula we get

$200 = 3.14(4.2)^2\,h$ and solving for h we get **3.6 meters** from the bottom of the tank.

Method 2: From the formula for the cylinder, since R remains the same, only h will vary and so the gauge will be located 1/8(29.6 meters) = **3.7 meters** from the bottom of the tank.

Problem 3 – The rate at which fuel is burned by the Space Shuttle engines is about 1000 gallons per second. From ignition, how many minutes (to the nearest tenth) will it take for the fuel to reach the level of the fuel gauge?

Answer: The tank must lose 7/8 of its original volume in gallons which equals 7/8 (420,000) = 367,500 gallons. At a rate of 1000 gallons per second, this will take 367.5 seconds or **6.1 minutes**.

Apollo 17 - Launch from the Moon's Surface

On December 14, 1972 at 10:54:37 p.m. GMT, Astronauts Eugene Cernan and Harrison Schmidt blasted off from the lunar surface in the Lunar Module (LM). The launch was recorded by a camera left behind at the landing site in the Taurus-Litrow region. A sequence of images from this recording is shown to the left.

The sequence of images runs from the top to the bottom. The top image was taken at 10:54:37.00 p.m. and the bottom image was taken at 4.9 seconds later at 10:54:41.87 p.m. The width of the LM is 4.3 meters. See the YouTube video of the LM launch at
http://www.youtube.com/watch?v=iziumcklDbM&feature=related

Table of LM Heights and Times

Image	Time (seconds)	Height (meters)	Speed (m/s)
1	0	0	
2	1.8	2	
3	2.3	6	
4	2.9	10	
5	3.2	15	
6	3.7	18	
7	4.9	21	

Problem 1 - What is the average speed of the LM during the 4.9 seconds covered by this image sequence?

Problem 2 - What are the average speeds of the LM between Image 1 and A) Image 2? B) Image 3? C) Image 4? D) Image 5? E) Image 6? F) Image 7? Enter these speeds in the above table.

Problem 1 - What is the average speed of the LM during the 4.9 seconds covered by this image sequence?

Answer: The distance traveled was 21.0 meters, which took 4.9 seconds, so the average speed was S = 21.0 m/4.9s so **S = +4.3 meters/sec**.

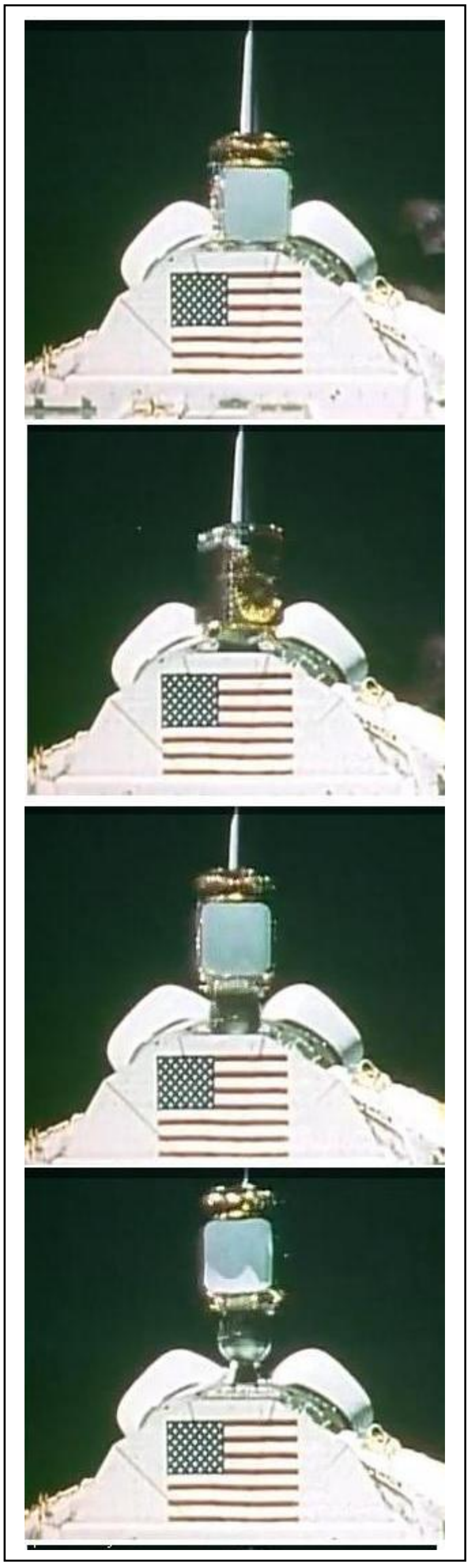
Problem 2 - What are the average speeds of the LM between Image 1 and A) Image 2? B) Image 3? C) Image 4? D) Image 5? E) Image 6? F) Image 7? Enter these speeds in the table.

Answer: A) distance traveled = 2 meters, time = 1.8 seconds, so speed = 2/1.8 = **1.1 meters/sec**.
B) distance traveled = 6 meters, time = 2.3 seconds, so speed = 6/2.3 = **2.6 meters/sec.**
C) distance traveled = 10 meters, time = 2.9 seconds, so speed = 10 / 2.9 s = **3.4 meters/sec**.
D) distance traveled = 15.0 meters, time = 3.2 seconds, so speed = 15m / 3.2s = **4.7 meters/sec**.
E) distance traveled = 18.0 meters, time = 3.7 seconds, so speed = 18.0 m / 3.7 s = **4.9 meters/sec**.
F) distance traveled = 21.0 meters, time = 4.9 seconds, so speed = 21 m / 4.9 s = **4.3 meters/sec**.

Table of LM Heights and Times

Image	Time (seconds)	Height (meters)	Speed (m/s)
1	0	0	
2	1.8	2	1.1
3	2.3	6	2.6
4	2.9	10	3.4
5	3.2	15	4.7
6	3.7	18	4.9
7	4.9	21	4.3

Note: These speeds are approximate due to the quality of the video images, which had no time stamps to verify when the individual frames were taken. The times and heights were estimated from an approximate analysis of the video sequence.



This sequence of images shows the launch of the INSAT-1B Indian communications satellite from the Challenger Space Shuttle cargo bay (STS-8) on August 30, 1983.

The sequence begins at the top frame and progresses downwards to the last frame showing the launch. The image times in seconds from top to bottom are: 36.3, 37.0, 37.7, and 39.2. The blue rectangular panel is 2 meters tall and is located on only one side of the satellite. The satellite is launched in a spinning mode to help stabilize it as it orbits Earth.

See the NASA video at the National Space Society:
http://www.nss.org/resources/library/shuttlevideos/shuttle08.htm

Table of Image Metrology

Image	Time (sec)	Height (meters)	Speed (m/s)
1	36.3	0.9	
2	37.0	1.5	
3	37.7	2.4	
4	39.2	3.9	

Problem 1 - From the table above, determine the average speed in meters/sec between: A) Image 1 and 2; B) Image 2 and 3; C) Image 3 and 4.

Problem 2 - What is the average speed of the satellite between Image 1 and Image 4?

Problem 3 - What is the rotation period of the satellite in seconds?

Problem 4 - How many revolutions per minute (RPM) was the satellite spinning after it was launched?

Problem 1 - From the table above, determine the average speed in meters/sec between: A) Image 1 and 2; B) Image 2 and 3; C) Image 3 and 4.

Table of Image Metrology

Image	Time (sec)	Height (meters)	Speed (m/s)
1	36.3	0.9	
2	37.0	1.5	**0.9**
3	37.7	2.4	**1.3**
4	39.2	3.9	**1.0**

Answer: Speed = distance/time. The time interval between Image 1 and 2 is 37.0 - 36.3 = 0.7 seconds. During this time, the satellite moved vertically a distance of 1.5 meters - 0.9 meters = 0.6 meters, so the speed was 0.6 meters / 0.7 seconds = 0.9 meters/sec. Similarly the answers for the other 2 time intervals are shown in the above table.

Problem 2 - What is the average speed of the satellite between Image 1 and Image 4?

Answer: Distance = 3.9 - 0.9 = 3.0 meters. Time interval = 39.2 - 36.3 = 2.9 seconds, so speed = **1.0 meters/sec**.

Problem 3 - What is the rotation period of the satellite in seconds?

Answer: We see from Image 3 and 4 that the satellite has made one full revolution. The time interval between the two frames is 39.2 - 37.7 = 1.5 seconds, so the period is **1.5 seconds**.

Problem 4 - How many revolutions per minute (RPM) was the satellite spinning after it was launched?

Answer: There are 60 seconds in 1 minute, so the RPM is just 60 seconds / 1.5 = 40 RPM.

1 revolution / 1.5 seconds x (60 seconds/1 minute) = **40 revolutions/sec**

The table below provides the altitude, range and times for the Space Shuttle Atlantis after its launch at 11:29:00 a.m. EDT from NASA's Cape Canaveral Space Center, Launch Pad 39A.

Time (minutes)	Altitude (km)	Range (km)
0	0	0
0.7	6	5
1.2	17	10
1.5	27	20
2.5	61	80
2.7	67	104
3.0	76	126
3.3	84	172
3.8	94	224
4.3	101	297
5.1	107	413
5.4	108	486
6.0	108	600
6.4	107	821
7.0	105	1040
7.6	103	1245
8.0	103	1474
8.6	106	1859
9.0	108	2006

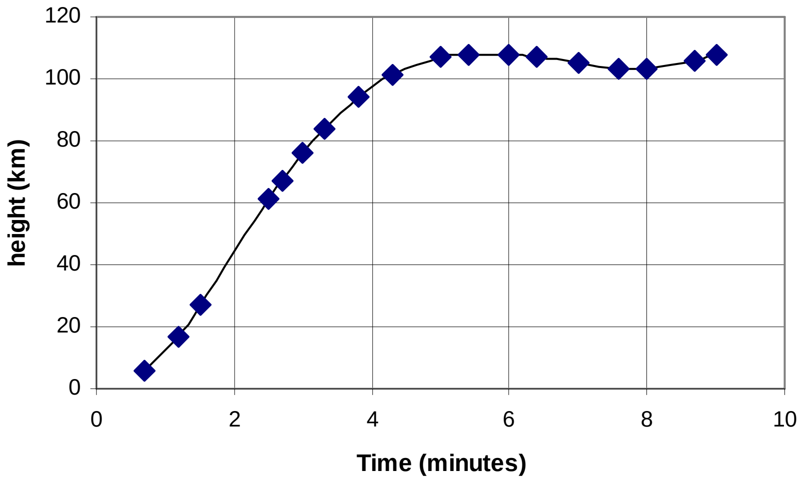
Problem 1 - Plot the altitude versus time for the launch.

Problem 2 - Plot the down-range distance versus time for this launch.

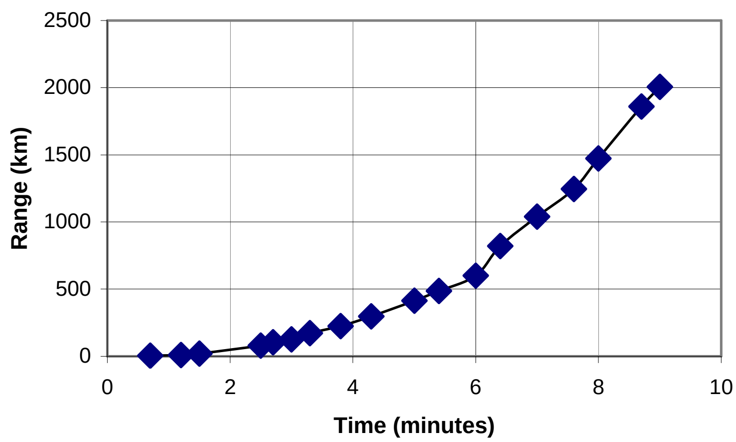
Problem 3 - The actual distance traveled by the Shuttle can be found using the Pythagorean Theorem where the hypotenuse of the right triangle is formed from the distance traveled in altitude (vertical 'y' direction) and the distance traveled in range (horizontal 'x' direction). How far did Atlantis travel between 8.0 and 9.0 minutes after launch?

Problem 4 - What was the average speed of Atlantis between 8.0 and 9.0 minutes after the launch in: A) kilometers/minute? B) miles per hour?

Problem 1 - Plot the altitude versus time for the launch.



Problem 2 - Plot the down-range distance versus time for this launch.



Problem 3 - The actual distance traveled by the Shuttle can be found using the Pythagorean Theorem where the hypotenuse of the right triangle is formed from the distance traveled in altitude (vertical 'y' direction) and the distance traveled in range (horizontal 'x' direction). How far did Atlantis travel between 8.0 and 9.0 minutes after launch?
Answer: Y = altitude difference = 108 - 103 = 5 kilometers .Y = range difference = 2006 - 1474 = 532 km, so the distance traveled = $(5^2 + 532^2)^{1/2}$ = **532 km**.

Problem 4 - What was the average speed of Atlantis between 8.0 and 9.0 minutes after the launch in: A) kilometers/minute? B) miles per hour?

Answer: A) speed = distance/time so speed = 532 km/1 minute, speed = **532 km/minute**.
B) 532 km/minute x (60 minutes/1 hr) x (0.62 miles / 1 km) so **speed = 19,790 miles/hr**.

The data were obtained from the GOOGLE Earth tracking data using the application file available at: **http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/shuttle_google_earth.html**

Space Shuttle Atlantis (STS-135) Launch Speed

This sequence of images shows the historic launch of the Space Shuttle Atlantis (STS-135) on July 8, 2011 at 11:29 a.m. EDT, from launch pad 39A at the NASA Cape Canaveral Space Center.

From bottom to top, the image times are 11:29:15.0, 11:29:16.0, 11:29:17.0, 11:29:18.0, and 11:29:19.0. The length of the space shuttle orbiter (not the red fuel tank) is 37 meters.

The launch sequence can be seen in the video located at:

Problem 1 - Using a millimeter ruler, what is the scale of an individual image in meters/mm?

Problem 2 - Measure the height in meters between the tip of the red shuttle fuel tank and a fixed location near the bottom of each frame.

Problem 3 - Graph the height of the fuel tank versus elapsed time beginning at T=0 in the bottom (first) image.

Problem 4 - About what was the average speed of the Shuttle in the top image in A) meters/sec? B) miles per hour?

Problem 1 - Using a millimeter ruler, what is the scale of an individual image in meters/mm?

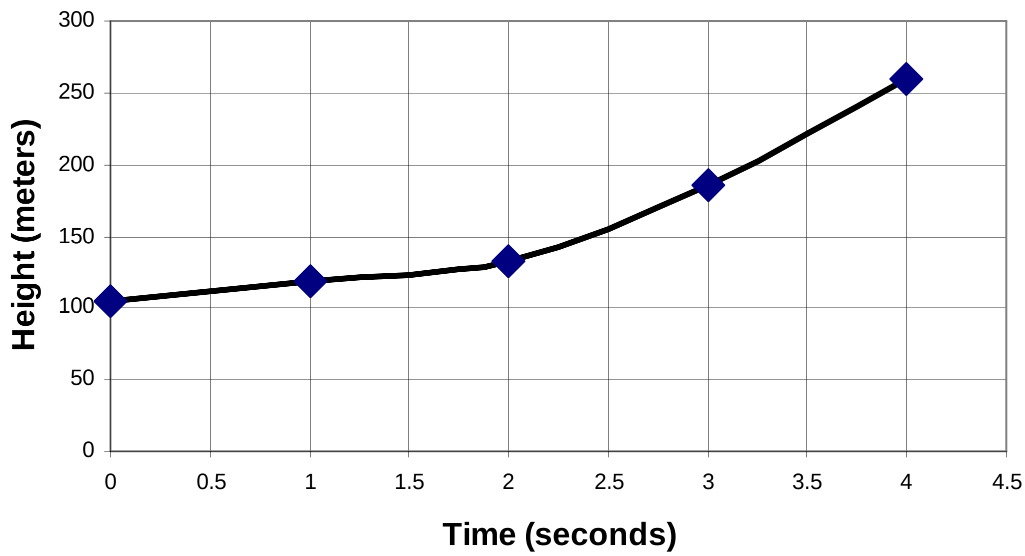
Answer: When this page is reproduced at normal scale, the length of the Orbiter is about 5 millimeters ,which corresponds to 37 meters, so the scale is 37/5 = **7.4 meters/mm.**

Problem 2 - Measure the height in meters between the tip of the red shuttle fuel tank and a fixed location near the bottom of each frame.

Answer; Students need to select a feature on the ground that is visible in each of the frames .One such feature is the top of the black rectangle 'status board' directly below the launch gantry. Measured from the top of this board to the tip of the red fuel tank, the values are as follows: 14 mm, 16mm, 18mm, 25mm, 35mm which correspond to heights of 104m ,118m, 133m, 185m and 259m.

Problem 3 - Graph the height of the fuel tank versus elapsed time beginning at T=0 in the bottom (first) image.

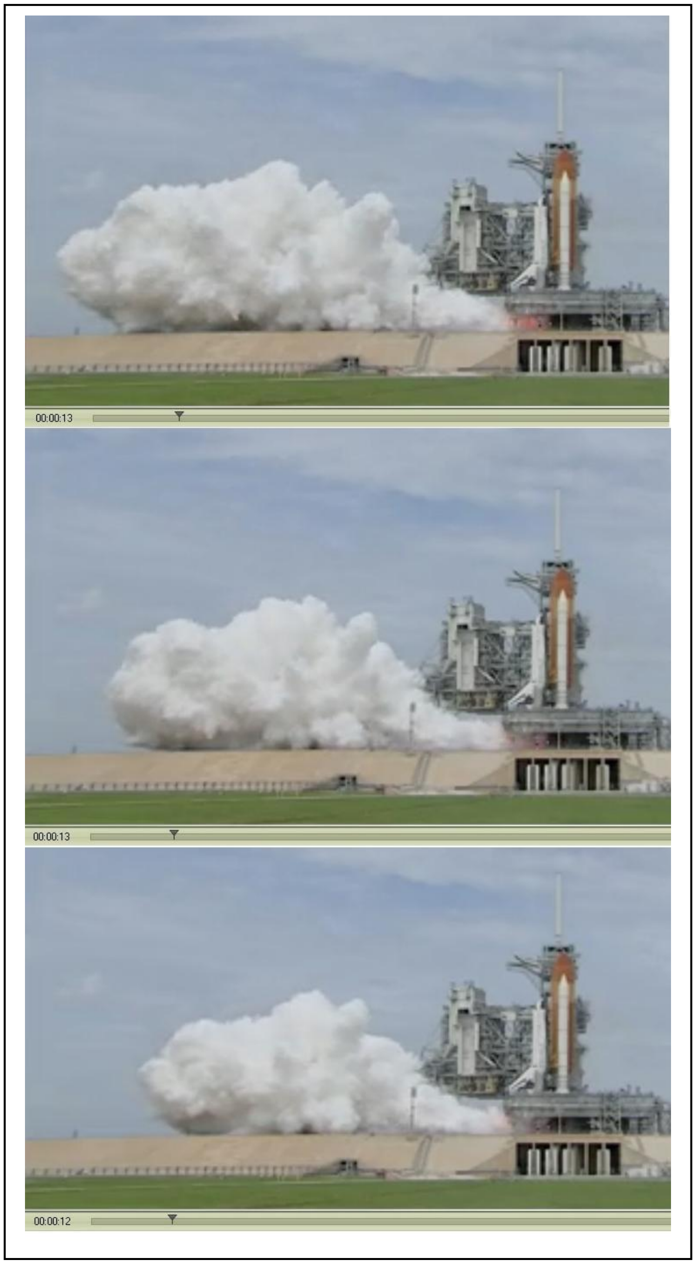
Answer: The elapsed times for each of the frames are 0s, 1s, 2s, 3s and 4s. The height graph is as follows:



Problem 4 - About what was the average speed of the Shuttle in the top image?

Answer: A) Between 3 and 4 seconds, the height changed from 185 to 259 meters, so the speed was about (259-185)/ (4 - 3) = **74 meters/sec** B) about **165 mph**.

Space Shuttle Atlantis (STS-135) - Plume speed



This sequence of images shows the historic launch of the Space Shuttle Atlantis (STS-135) on July 8, 2011 at 11:29 a.m. EDT, from launch pad 39A at the NASA Cape Canaveral Space Center.

From bottom to top, the image times are 11:29:12.0, 11:29:12.5, and 11:29:13.0. The length of the space shuttle is 37 meters from its pointed top end to the base of its rocket nozzles.

A rocket moves forward by throwing mass out its rocket engines as fast as possible. It does NOT move forward by 'pushing against' the ground as a popular misconception might suggest.

By ejecting thousands of pounds of gas every second, a rocket motor produces the thrust needed to lift a payload and move it in the opposite direction to its exhaust.

The plume of gas is ejected at high speed from the Shuttle main engines and makes a right-angle turn as it is vented horizontally across the gantry platform. The vented gas seen in the sequence to the left is the plume created by the Sound Supression Water System (SSWS).

Problem 1 - Using a millimeter ruler, what is the scale of each image in meters/mm?

Problem 2 - How far did the leading edge of the SSWS plume travel between the top and bottom images?

Problem 3 - What was the speed of the SSWS plume in A) meters/sec? B) kilometers/hr? C) miles/hr?

Problem 1 - Using a millimeter ruler, what is the scale of each image in meters/mm?

Answer: The Shuttle measures about 13 millimeters in length on an ordinary reproduction of this 8.5 x 11-inch page, so the scale is **2.8 meters/mm**.

Problem 2 - How far did the leading edge of the SSWS plume travel between the top and bottom images?

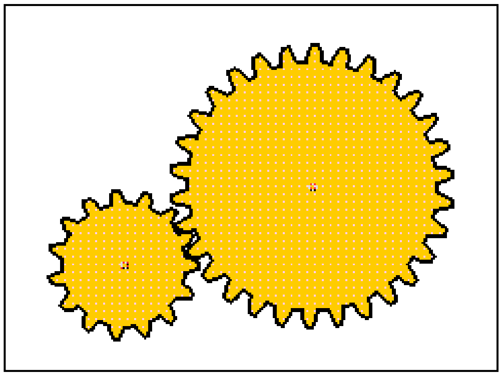
Answer: For example, students might measure the distance from the tip of the cloud and the left-edge of the image. Top Image = 5 mm. Bottom image = 15 mm ,so the distance traveled is 10 millimeters or from the scale factor, 10 x 2.8 = **28 meters**.

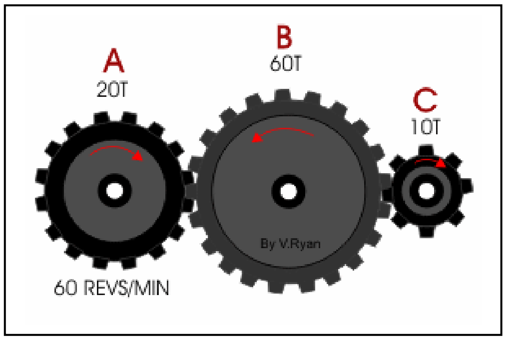
Problem 3 - What was the speed of the SSWS plume in A) meters/sec? B) kilometers/hr? C) miles/hr?

Answer: A) The time difference between the top and bottom images is 11:29:13.0 - 11:29:12.0 = 1 second. The average speed would be 28 meters/1 sec = **28 meters/sec.**

B) 28 meters/sec x (1 km/1000 meters) x (3600 sec / 1 hr) = **100 km/hr**.

C) 100 km/hr x (0.62 miles / 1 km) = **62 miles per hour**.





Gears are a common mechanical object used to change the rate of rotation of one shaft into a faster or slower rotation of a second shaft.

The top diagram shows two gears on two different shafts. The large gear has 30 teeth and the small gear has 15 teeth. As the small gear rolls along the circumference of the large gear, the shaft of the small gear completes 30/15 = 2 rotations for every 1 rotation of the large gear shaft. This is called a gear reduction, or gear ratio of 1:2.

The bottom diagram shows a more complicated gear 'chain' with 3 shafts. Gear A has 20 teeth, Gear B has 60 teeth and Gear C has 10 teeth. Gear A is turning at a rate of 60 revolutions per minute.

Problem 1 – When Gear A completes one revolution, how many revolutions does Gear B make?

Problem 2 – When Gear B makes one complete revolution, how many revolutions does Gear C make?

Problem 3 – What is the chain of reductions (example 1/2 x 1/3 x 4/6) that lets you calculate how many times Gear C rotates in revolutions per minute?

Problem 4 - An astronomer wants to build a 'clock drive' that will allow his telescope to keep up with the rotation of Earth so that stars will not move as he is looking at the sky with his telescope. To do this, the polar axis of the telescope mounting has to rotate exactly once each day, which lasts about 1437 minutes. He has a motor whose shaft rotates once every minute. The astronomer has the following gears with the indicated number of teeth: 1395, 309, 20 and 15. What combination will give him a speed reduction close to one shaft rotation every 1437 minutes?

Problem 1 – When Gear A completes one revolution, how many revolutions does Gear B make?

Answer: 20/60 = 1/3

Problem 2 – When Gear B makes one complete revolution, how many revolutions does Gear C make?

Answer: 60/10 = 6

Problem 3 – What is the chain of reductions (example 1/2 x 1/3 x 4/6) that lets you calculate how many times Gear C rotates in revolutions per minute?

Answer: 60 rev/min x (20/60) x (60/10) = 120 revolutions/minute

Problem 4 - An astronomer wants to build a 'clock drive' that will allow his telescope to keep up with the rotation of Earth so that stars will not move as he is looking at the sky with his telescope. To do this, the polar axis of the telescope mounting has to rotate exactly once each day, which lasts about 1437 minutes. He has a motor whose shaft rotates once every minute. The astronomer has the following gears with the indicated number of teeth: 1395, 309, 20 and 15. What combination will give him a speed reduction close to one shaft rotation every 1437 minutes?

Answer: The astronomer needs a gear reduction of **1:1437**

(309/15) x (1395/20) = (20.6)(69.75) = **1436.85 or 1437**.

So the gear reductions look like this:

1 revolution/minute x (15/309) x (20/1395) = 1 rpm x (1/1437) = 1 revolution/day.

On March 1, 2013, Space Exploration Technologies (SpaceX) launched the Dragon Supply Capsule on a Falcon 9 booster. SpaceX 2 is the second commercial resupply mission to the International Space Station.

Dragon delivered about 1,268 pounds (575 kilograms) of supplies to support continuing space station research experiments and will return with about 2,668 pounds (1,210 kilograms) of science samples from human research, biology and biotechnology studies, physical science investigations, and education activities.

According to the launch video narration, while the first stage engines were operating before Main Engine Cut-Off (called MECO), the rocket position and speed were given by the values in the table below:

Time Min:Sec	Altitude (km)	Down Range (km)	Speed (km/s)
2:20	30	23	1.0
2:45	51	59	1.8

Problem 1 - At what average speed was the altitude changing over this time interval?

Problem 2 - At what average speed was the down-range distance changing over this time interval?

Problem 3 - At what rate was the rocket accelerating over this time interval?

Problem 4 - What was the distance from the launch pad to the rocket at each time?

Problem 5 - At what average speed was the distance increasing over this time interval?

Problem 1 - At what average speed was the altitude changing over this time interval?

Answer: Speed = distance/time. Time = 2:45-2:20 = 25 seconds. Distance = 51-30 = 21 km, so speed = 21 km/25 sec = **0.84 km/sec**.

Problem 2 - At what average speed was the down-range distance changing over this time interval?

Answer: Time = 2:45-2:20 = 25 seconds. Distance = 59-23 = 36 km, so speed = 36 km/25 sec = **1.4 km/sec.**

Problem 3 - At what rate was the rocket accelerating over this time interval?

Answer: Acceleration = speed change/time, so

A = (1.8 km/s - 1.0 km/s)/(2:45-2:20)

= + 0.8 km/s/ (25 sec)

= 0.032 km/sec/sec

or 32 m/sec2. This is about 3-Gs of acceleration felt by the astronauts.

Problem 4 - What was the distance from the launch pad to the rocket at each time?

Answer: Use the Pythagorean Theorem

At 2:20 $d = (30^2 + 23^2)^{1/2}$ = **38 km**

At 2:45 $d = (51^2 + 59^2)^{1/2}$ = **78 km**

Problem 5 - At what average speed was the distance increasing over this time interval?

Answer: From the distance calculation, speed = (78 - 38)/25 sec = **1.6 km/s**.

An image from an instrument aboard NASA's Landsat Data Continuity Mission or LDCM satellite may look like a typical black-and-white image of a dramatic landscape, but it tells a story of temperature. The dark waters of the Salton Sea are shown in the semi-circle on the left-hand edge of the image. Crops create a checkerboard pattern stretching south to the Mexican border.

The size of this image is 26 km wide and 17 km tall. Each green square represents a planted crop measuring 160 meters on a side and an area of about 6 acres.

Problem 1 - What percentage of the total area of this image is occupied by planted crops?

Problem 2 – What percentage of all the farmed areas actually have growing crops?

Problem 3 – The annual rain fall is about 3 inches per year (0.076 meters/yr). If one gallon of water has a volume of 0.0038 meters^3, how many gallons of water fall on the planted crop area each year?

New NASA Satellite Takes the Salton Sea's Temperature
April 22,2013
http://www.nasa.gov/mission_pages/landsat/news/salton-sea.html

Problem 1 - What percentage of the total area of this image is occupied by planted crops?

Answer: The total area of this image is 26 km x 17 km = 442 km^2.
Students should count the number of green squares to tally the number of planted areas. A typical number would be about 50, so the total planted area is 50 x 0.16 km x 0.16 km = 1.3 km^2. The percentage of the total area is then 100% x 1.3/442 = **0.3 %.**

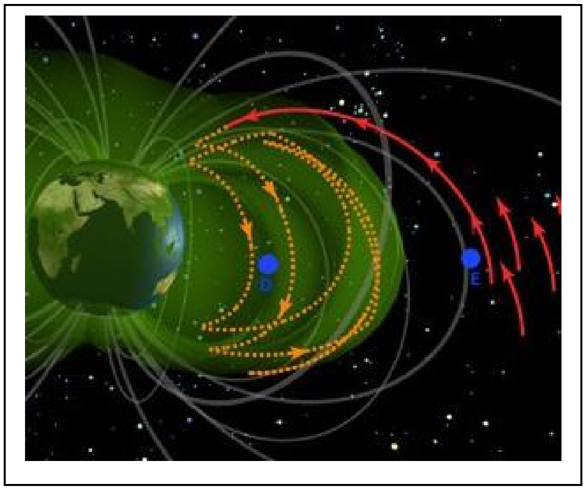
Problem 2 – What percentage of all the farmed areas actually have growing crops?

Answer: This is a bit more difficult because students have to count all of the square patches that they can see in the image, not just the green ones. A typical answer would be about 100 patches, so the total number of green + brown patches is about 150, and so the percentage of the planted areas is 100% x 50/150 = **33% or 1/3**.

Problem 3 – The annual rain fall is about 3 inches per year (0.076 meters/yr). If one gallon of water has a volume of 0.0038 $meters^3$, how many gallons of water fall on the planted crop area each year?

Answer:

From Problem 1, the total planted area is 1.3 km^2 or 1.3×10^6 $meters^2$. If the rain covers a depth of 0.076 meters each year, the rain volume is just 1.3×10^6 x 0.076 = 98800 cubic meters. This equals 98800 $meters^3$ x (1 gallon/0.0038 m^3) = **26 million gallons each year.**



Amateur radio operators have been hearing this sound for decades, especially at 'dawn'. It is an eerie sound, like a chorus of birds chirping, so it was called Dawn Chorus.

This sound cannot be heard with ordinary ears even though it is in the right frequency range. Because it is a radio wave, you need a radio receiver to hear it.

Space physicists have tried to understand what produces this electromagnetic 'sound wave', but whatever is producing it is occurring somewhere in the Van Allen belts high above Earth.

During its 60-day checkout phase, the twin Van Allen Probes satellites captured chorus waves close to where they are being produced in the Van Allen belts. As the satellites continue to take more data, scientists hope to be able to triangulate the location of these waves to their place of origin. This will provide scientists with a HUGE clue about what is causing them in the first place.

Let's have a look at how they will 'triangulate' the chorus position in space using simple graphing techniques, a compass and the Pythagorean Theorem!

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0].

Problem 2 – If 1 unit on the graph equals the radius of Earth, what is the domain and range of the graph in kilometers?

Problem 3 - What is the separation between the Van Allen Probes spacecraft in kilometers? (Hint: Use either the 2-point distance formula or a ruler!).

Problem 4 – From the location of VAP-A, the distance to the chorus source was found to be 12,800 km. From VAP-B it was 19,200 km. What point inside the orbits of the spacecraft is consistent with these measurements?

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0]. Answer: **See figure below**

Problem 2 – If 1 unit on the graph equals the radius of Earth, what is the domain and range of the graph in kilometers?

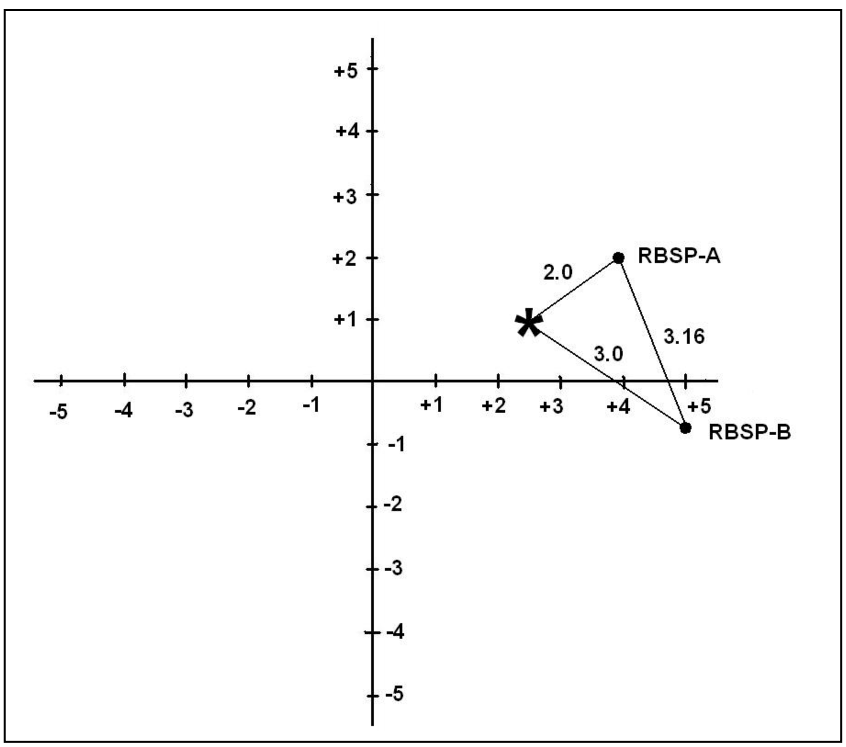
Answer: 5 units = 5 x 6400km = 32,000km.
X domain [-32,000 km, + 32,000 km], Y range [-32,000 km, +32,000 km].

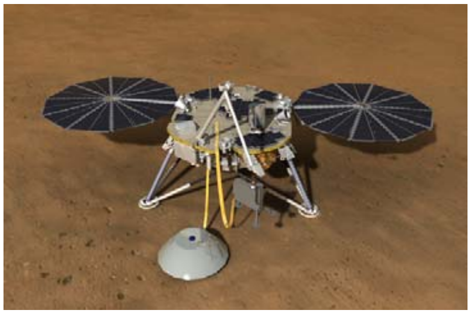
Problem 3 - What is the separation between the Van Allen Probes spacecraft in kilometers? (Hint: Use either the 2-point distance formula or a ruler!).

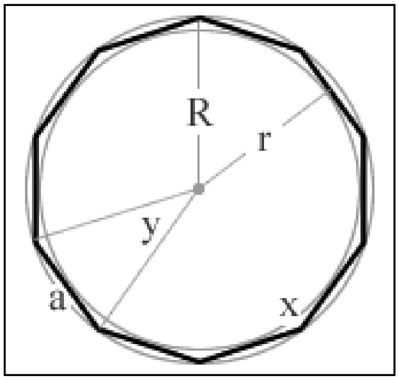
Answer: $d^2 = (5.0-4.0)^2 + (-1.0-(-2.0)^2$, $d^2 = 10.0$ so d= 3.16 units. In kilometers this is 3.16 x 6400 = **20,224 kilometers**.

Problem 4 – From the location of VAP-A, the distance to the chorus source was found to be 12,800 km. From VAP-B it was 19,200 km. What point inside the orbits of the spacecraft is consistent with these measurements?

Answer: 12,800 km/ 6,400km = 2.0 units. 19,200 km/6400 km = 3.0 units. Pt (+2.5, +1.0) or **(+16000 km, +6400 km)**







NASA's new mission to Mars called InSight will be launched in March, 2016. It will land on September 20, 2016 in a region of Mars located near the equator and deploy a seismographic station to study the interior of Mars.

To provide the electricity it needs, the lander will deploy two solar panels, each shaped like a regular, 10-sided polygon called a decagon.

In a regular decagon, the lengths of each of the 10 sides, a, are equal. For the two InSight lander solar panels:

$a = 0.62$ meters,
$r = 0.95$ meters,
$R = 1.0$ meters.

Problem 1 – What is the measure of the interior angle, y for a regular decagon?

Problem 2 – An isosceles triangle is formed by the base a and side length R. What is the length, r, in terms of a and R?

Problem 3 – What is the area of the isosceles triangle in Problem 2?

Problem 4 – What is the area of the regular decagon in terms of a and r?

Problem 5 - Calculate the area of one InSight solar panel in meter^2.

Problem 6 - What is the estimated area of one solar panel by using the inscribed circle with a radius of r and the circumscribed circle with a radius R?

Problem 7 – To two significant figures, if the solar panels produce 75 watts/m^2 of electricity at the distance of Mars from the sun, what is the total power produced by the two solar panels using either area method?

Problem 1 – What is the measure of the interior angle, y for a regular decagon?

Answer: $y = 360/10 = \mathbf{36^o}$.

Problem 2 – An isosceles triangle is formed by the base a and side length R. What is the length, r, in terms of a and R?

Answer: The segment with the length, r, is called the apothem and is the perpendicular bisector of the side with the length a, so from the Pythagorean Theorem we get $\mathbf{r = (R^2 - (a/2)^2)^{1/2}}$.

Note for the InSight dimensions: $0.95 = (1 - 0.096)^{1/2}$

Problem 3 – What is the area of the isosceles triangle in Problem 2?

Answer: $A = 2 \times ½ (a/2) \times r$ so $\mathbf{A = ar/2}$

For the InSight solar panel: $A = 0.62 \times 0.95/2 = 0.29\ m^2$.

Problem 4 – What is the area of the regular decagon in terms of a and r?

Answer: $A = 10 \times (ar/2)$ so $\mathbf{A = 5ar}$.

Problem 5 - Calculate the area of one InSight solar panel in $meter^2$.

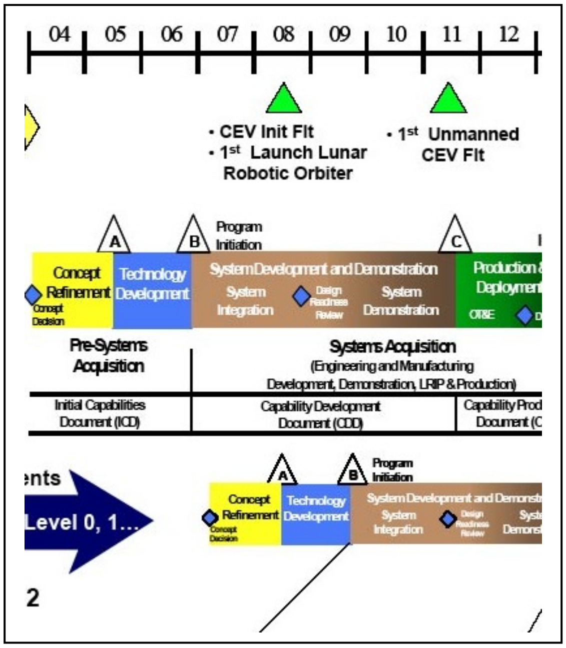
Answer: For the Insight solar panel, $A = 5 (0.62)(0.95) = \mathbf{2.95\ m^2}$.

Problem 6 - What is the estimated area of one solar panel by using the inscribed circle with a radius of r and the circumscribed circle with a radius R?

Answer: Take the average areas of the inscribed and circumscribed circles to get $A = 0.5 \pi (R^2 + r^2)$. For Insight, $A = 0.5 \times 3.141 \times (1 + 0.90) = \mathbf{2.98\ m^2}$.

Problem 7 – To two significant figures, if the solar panels produce 75 $watts/m^2$ of electricity at Mars, what is the total power produced by the two solar panels using either area method?

Answer: To 2 SF, the areas are both $3.0\ m^2$, so $P = 2$ panels $\times$ $75\ w/m^2 \times 3.0\ m^2$ = **450 watts**.



NASA's new mission to Mars called InSight will be launched in March, 2016. It will land in a region of Mars located near the equator and deploy a seismographic station to study the interior of Mars.

From the time a mission is imagined to the time it actually launches, many events have to be scheduled so that everything is built, tested, and delivered in time for the launch of the mission. The date and time that a mission lunches is very carefully determined and called a Launch Window. Sometimes it is only few hours long on a specific date that is many years in the future. The InSight mission must land on Mars on September 20, 2016.

To make sure that all of the thousands of components to a spacecraft are built, tested and delivered on time, Mission Planners develop very detailed schedules that show all of the components, their state of construction, and when the many critical tests and 'milestones' have to be reached to keep a mission on time for launch.

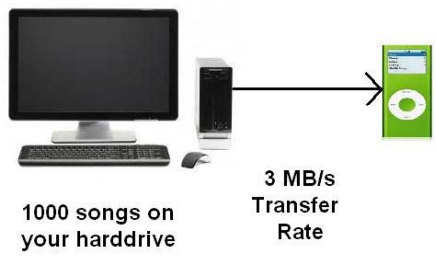
Believe it or not, you do this kind of planning yourself and probably do not even know it!

Imagine that you are taking a vacation to visit a family member, and that you are flying on a jet plane. Your parents have already booked your reservations for August 5 and the plane will leave the gate at exactly 11:35 AM. Allow exactly 2:00 to travel from your home, park your car, pass through Airport Security and walk to the gate. You have to arrive at the gate 45 minutes before the flight leaves to check your baggage and to board the flight.

Problem – Create a timeline for each person in your family that includes waking up on August 5, packing bags and loading them in the family car, eating breakfast, taking showers and other bathroom preparations. Oh…suppose, also, that you only have one bathroom to share, and that no two family members took exactly the same amount of time to do each of their tasks!

Problem – Students timelines will vary. Students should organize each person's activities as a row on the table, and time progressing from left to right along the columns of the table as shown in the example below. The times denote the start of each event.

	7:15	7:30	7:45	8:15	8:30	9:20	9:34	9:35	11:35
Person 1	Wake up	Get dressed	Eat breakfast	Use bathroom	Pack bags	Load car	Fasten seatbelt	Car Leaves Home	Flight Leaves
Person 2	Sleeping	Wake up	Use bathroom	Get dressed	Pack bags	Load car	Fasten seatbelt	Car Leaves Home	Flight Leaves



Transferring digital data from place to place takes time. Like water flowing into a lake, the faster it flows the more rapidly the lake fills up and overflows. With computer data, we have a similar problem. You have probably had to do this yourself many times. Each time you copy your playlist from your PC to your portable music player, you will have to wait a certain length of time. The transfer rate is fixed, so the more songs you want to transfer the longer you have to wait. Here's how this works!

Problem 1 - Suppose you want to transfer 1000 songs from your PC collection to your music Player. Each 4-minute song takes up 4 megabytes on the PC, and the cable link from your computer to your Player can handle a transfer rate of 3 million bytes/second. How many minutes does it take to transfer all your songs to the Player?

Imagine a lake fed by one large slow-moving river that brings water to it, and a second small, fast-moving river that takes water from the lake. If the rates at which the water enters and leaves the lake are not in step, the lake's water level will overflow. The InSight lander has a similar problem. It is gathering data at one rate, but transmitting it to Earth at another rate. We don't want to lose any of the data, so the data has to be stored in a memory device called a buffer.

InSight has two instruments that generate constant streams of digital data. The SEIS seismometer produces 48 megabytes/hr and the HP3 produces 2 megabytes/hr. This data is stored in a 500 megabyte buffer. Every 2 hours, the data in the buffer is transmitted to Earth at a rate of 4 megabytes/sec.

Problem 2 - How long will it take to fill up the buffer with data?

Problem 3 - How long will be required to transmit the buffer data to Earth during each 2-hour transmission cycle?

Problem 4 – The receiver on Earth can be scheduled to contact the Lander as often as once every 2 hours. How large a buffer would you need so that you could gather as much data as 4 megabytes/sec over 2 hours? How long does it take the instruments to gather this much data?

Problem 1 - You want to transfer 1000 songs from your PC collection to your music Player. Each 4-minute song takes up 4 megabytes, and the cable link from your computer to your Player can handle a transfer rate of 3 million bytes/second. How many minutes does it take to transfer all your songs?

Answer: 4000 megabytes x (1 second/ 3 megabytes) = 1333 seconds or about **22 minutes**.

The InSight lander has two instruments that generate constant streams of digital data. The SEIS seismometer produces 48 megabytes/hr and the HP3 produces 2 megabytes/hr which is stored in a 500 megabyte digital memory called a buffer. Every 2 hours, the data in the buffer is transmitted to Earth at a rate of 4 megabytes/sec.

Problem 2 - How long will it take to fill up the buffer with data?

Answer: The data enters the buffer at 50 megabytes/hr and the buffer contains 500 megabytes, so it can store data for 500 MBytes/(50Mbytes/hr) = **10 hours**.

Problem 3 - How long will be required to transmit the buffer data to Earth during each 2-hour transmission cycle?

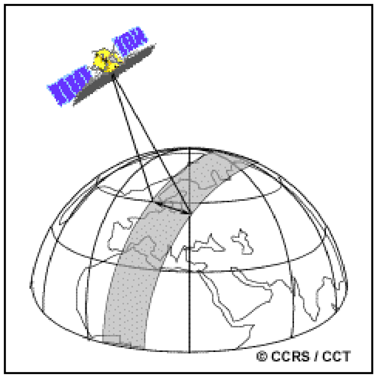
Answer: In 2 hours at a data rate of 50 megabytes/hr you have 100 megabytes stored in the buffer. At a transmission rate of 4 megabytes/sec it takes 100 Mbytes/(4 Mbytes/sec) = **25 seconds** to transmit the100 megabytes from the buffer to Earth.

Problem 4 – The receiver on Earth can be scheduled to contact the Lander as often as once every 2 hours. How large a buffer would you need so that you could gather as much data as 4 megabytes/sec over 2 hours? How long does it take the instruments to gather this much data?

Answer: 2 hours equals 2 x 3600 = 7200 seconds. At a transmission rate of 4 Mbytes/sec, this equals 7200 sec x 4 Mbytes/sec = 28,800 Megabytes or **28.8 Gigabytes.**

The instruments gather 50 Megabytes/hour, so it would take them 28,800 Megabytes/(50 MB/hr) = 576 hours or 24 days to gather this much data.

What this says is that if you had a buffer this large (28.8 gigabytes) it could store 24 days of data from InSight and only take 2 hours to transmit to Earth. The danger of waiting so long to transmit data (every 24 days) is that something could happen to the lander and you would lose all this data! That's why scientists try to download their data as often as possible.



When a satellite looks down on the Earth from space, it only sees a small part of the surface at a time during each orbit. But after many orbits, it eventually sees all of the surface so that a full map can be put together.

This figure shows one 'swath' of images seen by the satellite during part of its orbit.

Problem 1 – A satellite at an altitude of 500 km above the surface orbits Earth from pole to pole once every 90 minutes. How many times does it pass across the Equator?

Problem 2 – If the satellite passes across the Equator going north to south during the first half of its orbit, what direction does it pass across the equator during the second half of the orbit?

Problem 3 – How many times does the satellite pass over the Equator every day if 1 day = 24 hours?

Problem 4 - If the satellite can view a swath of Earth's surface that is 2500 km wide, how many orbits will it take to view all of the Equatorial areas of Earth, if the radius of Earth is 6378 km?

Problem 5 – Draw a scaled diagram of a satellite located 500 km above a flat surface. Draw a vertical line from the satellite perpendicular to the surface. Complete the right triangle by drawing a hypotenuse from the satellite to the surface with a base length of 2500 km/2 = 1250 km. What is the measure of the angle from the satellite between the perpendicular segment and the hypotenuse? (Use a protractor, or use the property of tangents.)

Problem 6 – The distance to the horizon of a planet is given by the formula $D = (2Rh)^{1/2}$ where R is the planet's radius in km, and h is the height of the satellite above the ground in km. For the satellite problem above, what is the maximum swath width that can be viewed by the satellite?

Problem 1 – A satellite at an altitude of 500 km above the surface orbits Earth from pole to pole once every 90 minutes. How many times does it pass across the Equator?

Answer: It will cross the Equator exactly twice per orbit.

Problem 2 – If the satellite passes across the Equator going north to south during the first half of its orbit, what direction does it pass across the equator during the second half of the orbit?

Answer: The satellite will travel south to north across the equator during the second half of the orbit.

Problem 3 – How many times does the satellite pass over the Equator every day if 1 day = 24 hours?

Answer: 90 minutes / 24 hours = 16 orbits. During each orbit it passes across the equator twice, so in one day it passes across the equator 2 x 16 = 32 times.

Problem 4 - If the satellite can view a swath of Earth's surface that is 2500 km wide, how many orbits will it take to view all of the Equatorial areas of Earth, if the radius of Earth is 6378 km?

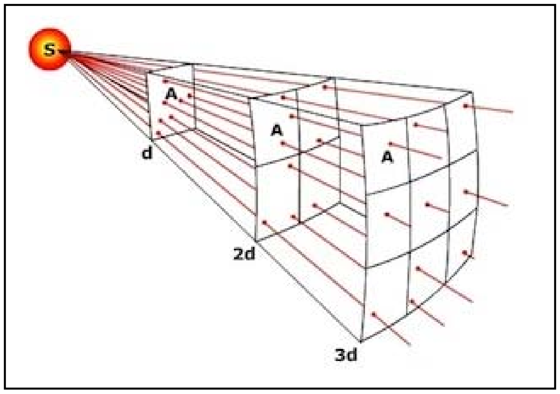
Answer: The circumference of Earth is given by $C = 2 \pi R$ so $C = 2 (3.141 \times 6378 \text{ km}) =$ 40,066 km. The number of swaths is given by 40066/2500 = 16. But each orbit counts for two equatorial swaths, so we only need 8 orbits to provide 16 swaths at the equator, and this will cover all of the equatorial area when the swath images are placed exactly side by side.

Problem 5 – Draw a scaled diagram of a satellite located 500 km above a flat surface. Draw a vertical line from the satellite perpendicular to the surface. Complete the right triangle by drawing a hypotenuse from the satellite to the surface with a base length of 2500 km/2 = 1250 km. What is the measure of the angle from the satellite between the perpendicular segment and the hypotenuse? (Use a protractor, or use the property of tangents.)

Answer: Tan(theta) = 1250/500 = 2.5, so **Theta = 68^o**, and the full swath viewing angle is 2 x 68 = 136^o.

Problem 6 – The distance to the horizon of a planet is given by the formula D = (2Rh)1/2 where R is the planet's radius in km, and h is the height of the satellite above the ground in km. For the satellite problem above, what is the maximum swath width that can be viewed by the satellite? D = (2 x 500 x 6378)1/2 = **2525 km**, so the swath is about equal to the maximum area of the Earth that can be viewed from horizon to horizon at the altitude of the satellite.

Exploring Light Brightness and the Inverse-Square Law



When we view a source of light at different distances, the light gets fainter as the distance increases. The figure to the left shows why this happens. At a distance of '2d', the same amount of light energy passes through 4 times the area that it did at the closer distance, so its brightness will 4 times fainter. This is called the **Inverse-Square Law**. Can you explain why it has this name?

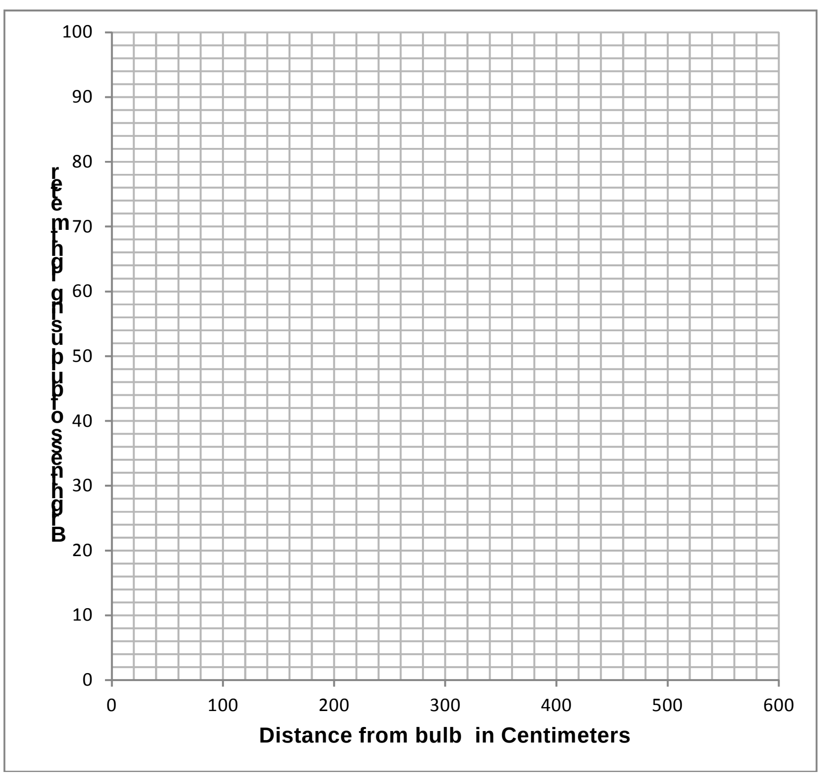
Our robotic light meter can be used to test the Inverse-Square law for an unknown light source by comparing its distance from the source with the light intensity its meter registers. Here's how we do this.

Step 1 – In a darkened room, turn on a bare 40-watt light bulb on the floor. Position the rover so that the light meter just registers '99' before changing to '98'. Measure this distance from the sensor to the bulb in centimeters. Enter the reading and the distance into a data table.

Step 2 – Move the rover directly away from the bulb until the light meter reads '50' and measure this distance. Record these numbers in your data table.

Step 3 – Repeat Step 2 for light readings of '25', '12' , and ''6'.

Step 4 - Graph your data below and connect the dots with a smooth curve.



Problem 1– What would you predict as the distance where the light reading is '1'?

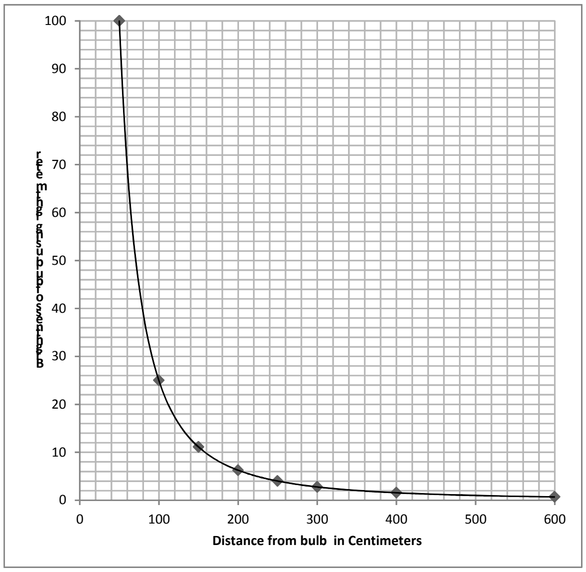
Problem 2 – What would happen if you used a 80-watt bulb instead of the 40-watt bulb?

Problem 3 – Write a mathematical model for the brightness, B, of a light source, the distance, D, and the wattage, W, of the light source, if for 40 watts you get B=100 at a distance of 50 cm. What do you predict the brightness will be at 800 cm for a 250 watt bulb?

This is called the Inverse-Square Law. Can you explain why it has this name?

Answer: The same number of light rays go through 4 times the area A at twice the distance, 9 times the area of A at three times the distance, and since $4 = 2^2$ and $9 = 3^2$ the brightness decreases by $1/2^2 = 1/4$ at twice the distance and $1/3^2 = 1/9$ at three times the distance, which follows a $1/d^2$ or Inverse-Square model.

Let's suppose that the light meter data follows the curve graphed below, where we measured '99' at a distance of 50 cm:



Problem 1– What would you predict as the distance where the light reading is '1'?

Answer: Using the hypothetical graph above, we have to reduce the light intensity by 1/100 of its original level at 50 cm. Because $1/100 = 1/10^2$, we need to move to 10 times the distance or 10 x 50 cm = 500 cm.

Problem 2 – What would happen if you used a 80-watt bulb instead of the 40-watt bulb?

Answer: You can test what happens using a brighter bulb. Because there is twice as much light at every distance, the curve drawn above **moves upwards** by a factor of 80/40 = 2 times, so at a distance of 100 cm the light meter would read 25x2 = 50.

Problem 3 – Write a mathematical model for the brightness, B, of a light source, the distance, D, and the wattage, W, of the light source if for 40 watts you get B=100 at a distance of 50 cm. What do you predict the brightness will be at 800 cm for a 250 watt bulb?

Answer: $B = 100 \times \dfrac{(W/40)}{(D/50)^2}$ and so $B = 100 \times (250/40) / (800/50)^2 =$ **2.4 units**

This sequence of images was taken of the launch of the Juno spacecraft on August 5, 2011 from Cape Canaveral. The images were taken, from left to right, at T+21, T+23 and T+25 seconds after launch, which occurred at 12:25:00 pm EDT. The original video can be found on *YouTube*. The distance from the base of the Atlas-Centaur rocket to its top is 45 meters (148 feet). As the video was produced, the camera zoomed-out between the T+21 image and the T+23 image. Both the T+23 and T+25 images were taken at exactly the same zoom scale.

Problem 1 - From the information given, find the speed of the rocket in meters/sec and kilometers/hr between A) 21 and 23 seconds after launch and B) 23 to 25 seconds after launch.

Problem 2 - What is the average acceleration of the rocket in meters/sec^2 between 21 and 25 seconds after launch?

Problem 3 - At the average acceleration of this rocket, about when will it be traveling faster then the speed of sound (Mach 1) which is 340 meters/sec?

Problem 1 - From the information given, find the speed of the rocket in meters/sec and kilometers/hr between A) 21 and 23 seconds after launch and B) 23 to 25 seconds after launch.
Answer: Students will need to determine the scale of each image by using a millimeter ruler to measure the length of the rocket body, which is known to be 45 meters. When printed using a regular laser printer, the lengths of the rockets are about 21) 5.5mm 23) 4.0 mm and 25) 3.0 mm
The image scales are therefore 8.2 meters/mm, 11.3 meters/mm and 15 meters/mm

To measure speed, all we need to do is measure the height of the bottom of the rocket vertically from a well-defined point near the bottom of the image away from the exhaust cloud. The horizontal band of water just below the exhaust plume provides a good reference. Using the millimeter ruler we get 21) 35 mm 23) 36 mm and 25) 43 mm Converting this in to meters using the three scales we get 21) 287 meters 23) 407 meters and 25) 645 meters

Speed: 21 to 23 seconds; s1 = (407-287) / 2 sec so s1 = **60 meters/sec**
23 to 25 seconds: s2 = (645 - 407)/ 2 sec so s2 = **119 meters/sec**

In km/h we get s1 = **216 km/hour** and s2 = **428 km/hr**.

Students estimates will vary depending on the method and measuring accuracy used.

Problem 2 - What is the average acceleration of the rocket in meters/sec^2 between 21 and 25 seconds after launch?

Answer: acceleration = difference in speed/difference in time so
Acc = (119 meters/sec - 60 meters/sec) / (4 seconds)
= **15 meters/sec^2**

Problem 3 - At the average acceleration of this rocket, about when will it be traveling faster then the speed of sound (Mach 1) which is 340 meters/sec?

Answer: speed = initial speed + acceleration x time

340 = 119 + 15 x T

So **T = 15 seconds after the initial speed** of 119 m/s was reached. This occurs about T = 25 sec + 15 sec = **40 seconds after launch**.

According to actual flight information, Mach 1 was reached a bit later at T+ 51 sec.

The Juno spacecraft was launched on August 5, 2011 on a 5 year journey to Jupiter. This image was taken 120 seconds after launch and shows one of the solid rocket boosters being jettisoned. The camera is on the Atlas booster and looks down on the engines and the distant arc of a cloudy Earth. Scenes from the launch can be found on *YouTube*, and show a dazzling launch from multiple viewing locations on Earth and in space.

During the launch, and the boost to orbit, the altitude of the rocket changes continuously as the engines provide thrust, eventually lifting the entire payload into orbit at a planned altitude of 420 kilometers (261 miles). A short table of the rocket's altitude and times is provided below.

Elapsed Time (seconds)	Altitude (miles)	Altitude (kilometers)
160	46	
192	60	
268	81	
274	83	
315	95	
319	97	
339	112	

Problem 1 - American engineers use English units for all measurements including the details of the rocket launch where 1 mile = 1.6 kilometers. Use this information to complete the table above in metric units rounded to the nearest kilometer.

Problem 2 - From the tabular data, graph the altitude of the rocket in time.

Problem 3 - From the data, find the function A(t) that predicts the altitude of the rocket at future times. The function will be of the form

$$A(t) = a + b\ln(t)$$

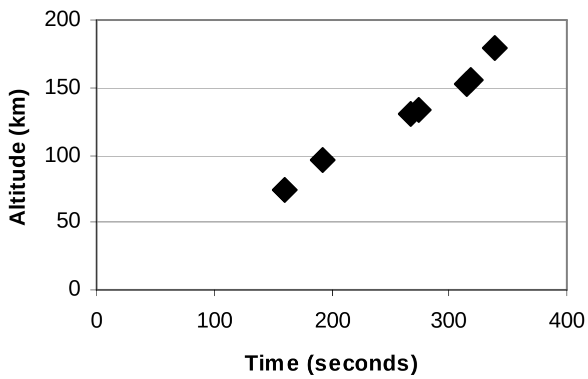
Find the constants a and b for A(t) in kilometers and A(t) in miles.

Problem 4 - How many seconds after launch will it take for the rocket to reach orbit altitude at 420 kilometers?

Problem 1 - American engineers use English units for all measurements including the details of the rocket launch where 1 mile =1.6 kilometers. Use this information to complete the table above in metric units rounded to the nearest kilometer.

Elapsed Time (seconds)	Altitude (miles)	**Altitude (kilometers)**
160	46	**74**
192	60	**96**
268	81	**130**
274	83	**133**
315	95	**152**
319	97	**155**
339	112	**179**

Problem 2 - From the tabular data, graph the altitude of the rocket in time.



Problem 3 - From the data, find the function A(t) that predicts the altitude of the rocket at future times. The function will be of the form

$$A(t) = a + b\ln(t)$$ Find the constants a and b.

Answer: Choose (160,74) and (319,155) then
74 = a + 5.1b and 155 = a + 5.8b
Solve by substitution: a = 74 - 5.1b then 155 =(74-5.1b) + 5.8b
So 81 = 0.7b and b = 116. Then a = -518
So with A(t) in kilometers, and t in seconds, we have:

$$A(t) = -518 + 116\ln(t)$$

In miles this becomes $A(t) = -324 + 73\ln(t)$

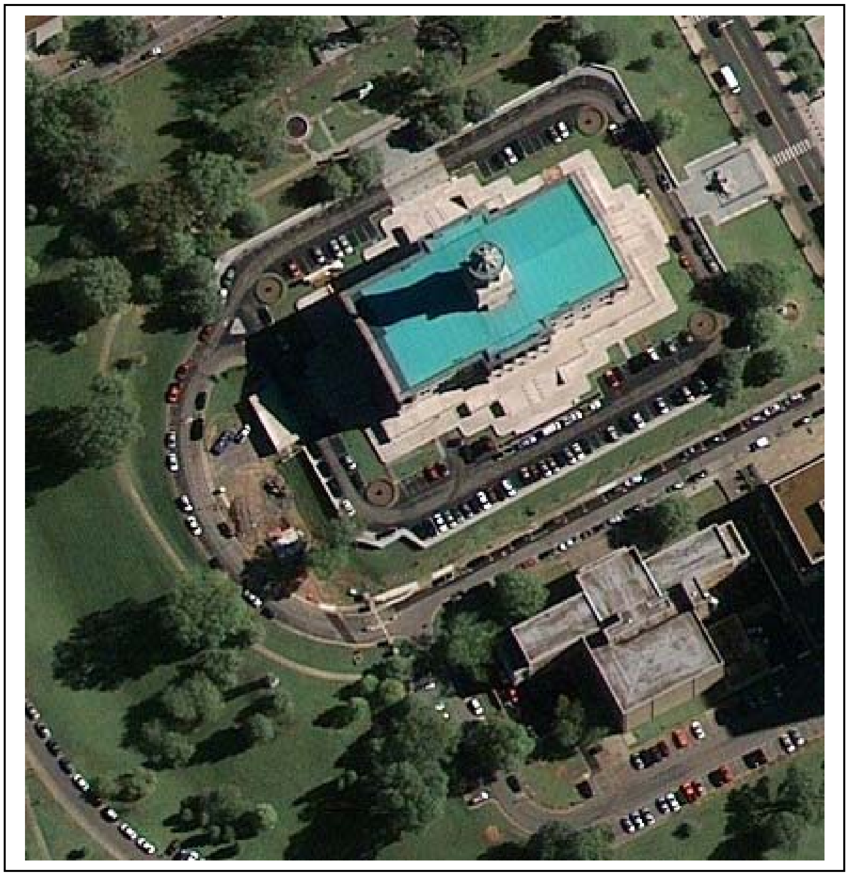
Problem 4 - How many seconds after launch will it take for the rocket to reach orbit altitude at 420 kilometers?

Answer: $420 = -518 + 116\ln(t)$ so t = **3,248 seconds or about 54 minutes**.

Note: The actual time is about 3,229 seconds.

A High-Resolution Satellite Photo

The Lunar Reconnaissance Orbiter (LRO) will take photographs of the lunar surface at a resolution of 0.5 meters per pixel. The 425x425 pixel image below (Copyright © 2009 GeoEye) was taken of the Tennessee Court House from the GeoEye-1 satellite with a width of about 212 meters.



Problem 1 - What is the scale of the image in: A) meters per millimeter? B) meters per pixel?

Problem 2 - How does the resolution of the expected LRO images compare with the resolution of the above satellite photo?

Problem 3 - What are the smallest features you can easily identify in the above photo?

Problem 4 - From the length of the shadows, what would you estimate as the elevation of the sun above the horizon?

Answer Key

Problem 1 - What is the scale of the image in: A) meters per millimeter? B) meters per pixel? Answer: A millimeter ruler would indicate a width of 105 millimeters, so the scale is A) 212 meters/105 mm = 2 meters/millimeter and B) 212 meters/425 pixels = 0.5 meters/pixel.

Problem 2 - How does the resolution of the expected LRO images compare with the resolution of the above satellite photo? Answer: The resolutions are identical and both equal to 0.5 meters/pixel, so we should be able to see about the same kinds of details at the lunar surface with LRO.

Problem 3 - What are the smallest features you can easily identify in the above photo?
Answer: With a millimeter ruler you can determine that the smallest features are comfortably about 0.5 millimeters across or 1 meter. Examples would include the parked cars, the widths of the various footpaths, and the lane and street markings stripes. Some of the smaller spots may be the shadows of people!

Problem 4 - From the length of the shadows, what would you estimate as the elevation of the sun above the horizon?

Answer: This is a challenging problem for students because they first need to estimate what the height of the object is that is casting the shadow. From this they can construct a triangle and determine the sun angle, or use trigonometry.

For example, suppose that the large tree in the lower left corner of the picture has a height of 50-feet (17 meters). We can measure the shadow length with a millimeter ruler to get 20 millimeters or 20 mm x 2 M/mm = 40 meters. Then tan(theta) = 17 meters/40 eters = 0.45, and so theta = 24 degrees above the horizon.

This problem shows students one of the problems encountered when studying photos like this. We can easily measure the widths of objects, but measuring their heights can be challenging, especially when the objects in question are unfamiliar.

Satellite technology is everywhere! Right now, there are over 1587 working satellites orbiting Earth. They represent over $160 billion in assets to the world's economy. In the United States alone, satellites and the many services they provide produce over $225 billion every year. But satellites do not work forever. Typically they have to be replaced every 10 to 15 years as new services are created, and better technology is developed. Satellites in the lowest orbits, called Low Earth Orbit (LEO) orbit between 300 to 1000 kilometers above the ground. Because Earth's atmosphere extends hundreds of kilometers into space, LEO satellites eventually experience enough frictional drag from the atmosphere that at altitudes below 300 km, they fall back to Earth and burn up. The table below gives the number of LEO satellites that re-entered Earth's atmosphere, and the average sunspot number, for each year since 1969.

Year	Sunspots	Satellites
2004	43	19
2003	66	31
2002	109	38
2001	123	41
2000	124	37
1999	96	25
1998	62	30
1997	20	21
1996	8	22
1995	18	20
1994	31	17
1993	54	28
1992	93	41
1991	144	40
1990	145	30
1989	162	45
1988	101	33
1987	29	13
1986	11	16
1985	16	17
1984	43	14
1983	65	28
1982	115	19
1981	146	32
1980	149	41
1979	145	42
1978	87	33
1977	26	18
1976	12	16
1975	14	15
1974	32	21
1973	37	14
1972	67	12
1971	66	19
1970	107	25
1969	105	26

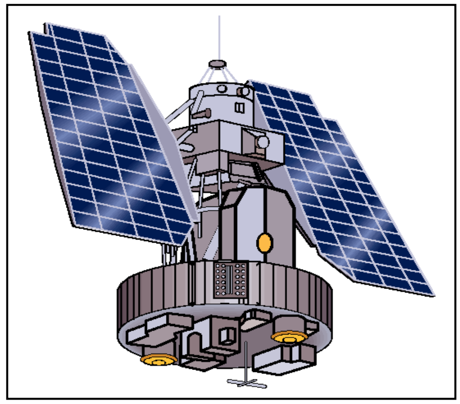


Figure: A typical weather satellite

Question 1: On the same graph, plot the number of sunspots and decayed satellites (vertical axis) for each year (horizontal axis). During what years did the peaks in the sunspots occur?

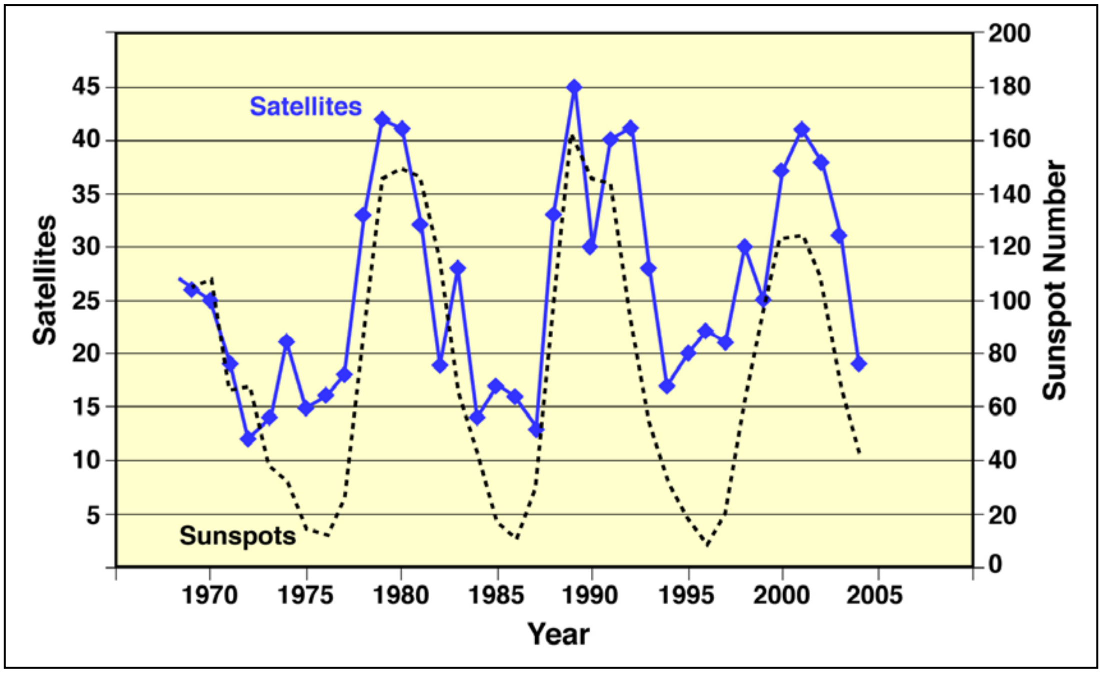
Question 2: When did the peaks in the satellite re-entries occur?

Question 3: Is there a correlation between the two sets of data?

Question 4: If you are a satellite operator, should you be concerned about the sunspot cycle?

Question 5: Do some research on the topic of how the sun affects Earth. Can you come up with at least two ways that the sun could affect a satellite's orbit?

Question 6: Can you list some different ways that you rely on satellites, or satellite technology, during a typical week?



Question 1: On the same graph, plot the number of sunspots (divided by 4) and decayed satellites for each year. During what years did the peaks in the sunspots occur? **Answer:** From the graph or the table, the 'sunspot maximum' years were 2000, 1989, 1980 and 1970.

Question 2: When did the peaks in the satellite re-entries occur? **Answer:** The major peaks occurred during the years 2001, 1989, 1979 and 1969.

Question 3: Is there a correlation between the two sets of data? **Answer:** A scientist analyzing the two plots 'by eye' would be impressed that there were increases in the satellite decays that occurred within a year or so of the sunspot maxima years. This is more easy to see if you subtract the overall 'trend' line which is increasing from about 10 satellites in 1970 to 20 satellites in 2004. What remains is a pretty convincing correlation between sunspots numbers and satellite re-entries.

Question 4: If you were a satellite operator, should you be concerned about the sunspot cycle? **Answer:** Yes, because for LEO satellites there seems to be a good correlation between satellite re-entries near the times of sunspot maxima.

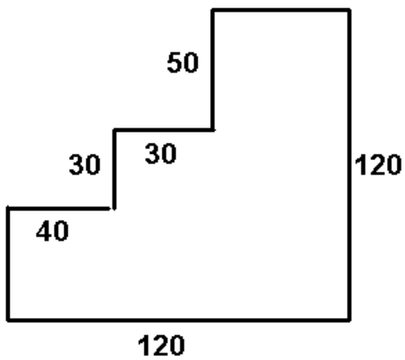
Question 5: Do some research on the topic of how the sun affects Earth. Can you come up with at least two ways that the sun could affect a satellite's orbit? **Answer:** The answers may vary, but as a guide, space physicists generally believe that during sunspot maxima, the sun's produces more X-rays and ultraviolet light, which heat Earth's upper atmosphere. This causes the atmosphere to expand into space. LEO satellites then experience more friction with the atmosphere, causing their orbits to decay and eventually causing the satellite to burn-up. There are also more 'solar storms', flares and CMEs during sunspot maximum, than during sunspot minimum. These storms affect satellites in space, causing loss of data or operation, and can also cause electrical blackouts and other power problems.

Question 6: Can you list some different ways that you rely on satellites, or satellite technology, during a typical week? **Answer:** Satellite TV, ATM banking transactions, credit card purchases, paying for gas at the gas pump, weather forecasts, GPS positions from your automobile, news reports from overseas, airline traffic management, tsunami reports in the Pacific Basin, long distance telephone calls, internet connections to pages overseas.

The outside surface of the NASA, IMAGE satellite is covered with solar cells (black) to collect sunlight and generate electricity for its many instruments and systems.

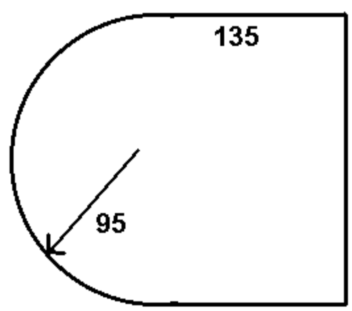
Solar Electricity.

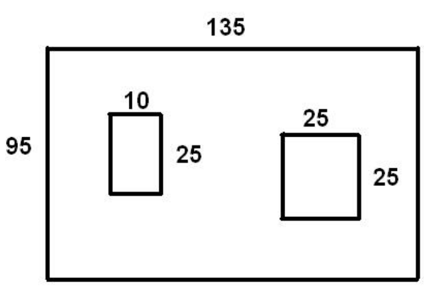
Most satellites depend on sunlight converted into electricity and stored in batteries, to run their instruments. Typical research satellites operate with only 200 to 800 watts of electricity generated by sunlight and 'solar cells'. Solar cells can be attached directly to the outer surface of a satellite, or can be found on 'solar panels' that the satellite deploys after it reaches its orbit. Engineers decide how many solar cells are needed by finding out how much power the satellite needs to operate, and then figuring out what the satellite's surface area is. If the satellite is not big enough, additional solar panels may be needed to supply the electricity. In this exercise, you will calculate the perimeter of the satellite area in centimeters, and the area, in square centimeters, of several kinds of satellite surfaces, and decide if there is enough available surface to supply the electrical needs of the satellite. All measurements in the diagrams are in centimeters. Some dimensions are missing.



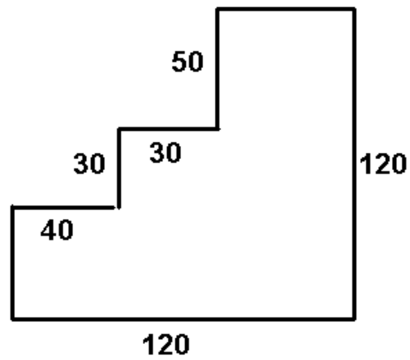
Question 1 – What is the perimeter? The area? The solar cells can provide 0.03 watts per square centimeter, and the satellite needs 257 watts. Is there enough surface area in the figure to the left to meet the electrical needs?

Question 2 - What is the perimeter? The area? The solar cells can provide 0.03 watts per square centimeter, and the satellite needs 957 watts. Is there enough surface area in the figure to the right to meet the electrical needs?





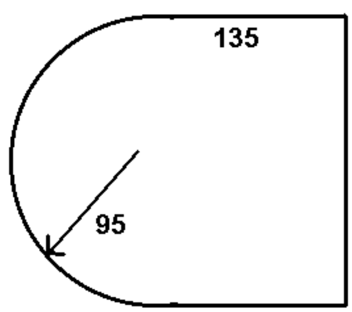
Question 3 - What is the outside perimeter? The area with the two holes removed? The solar cells can provide 0.03 watts per square centimeter, and the satellite needs 759 watts. Is there enough surface area in the figure to the left to meet the electrical needs?



Answer 1 a) The missing side can be reconstructed from the other measurements. The perimeter is P = (120+ 120+ 50 + 50 + 30 + 30 + 40 + 40) =
Perimeter = 480 centimeters.

B) The area can be found from breaking the figure into rectangular sections from right to left:
Area = (120 x 50) + (30 x 70) + (40 x 40) =
6000 + 2100 + 1600 = **9,700 square cm**.

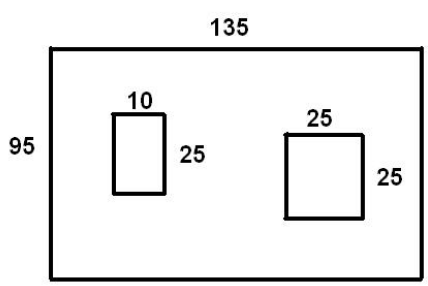
C) The solar cells produce 0.03 watts per square cm, so the power available is 9,700 x 0.03 = **291 watts**. This is greater than the minimum satellite requirement of 257 watts.



Answer 2 a) From the radius of the half-circle, the rectangle has a length of 2 x 95 = 190 cm, so the sum of the three sides is 460 cm. The circumference of the semi-circle is (3.14) x 95 = 298.3 cm. **Perimeter = 758.3 centimeters.**

B) The area can be found from breaking the figure into a rectangle and a semi-circle.
Area = (135 x 190) + ½ (3.14) $(95)^2$ =
25,650 + 14,169 = **39,819 square cm**.

C) The solar cells produce 0.03 watts per square cm, so the power available is 39819 x 0.03 = **1194 watts**. This more than the minimum satellite requirement of 957 watts.



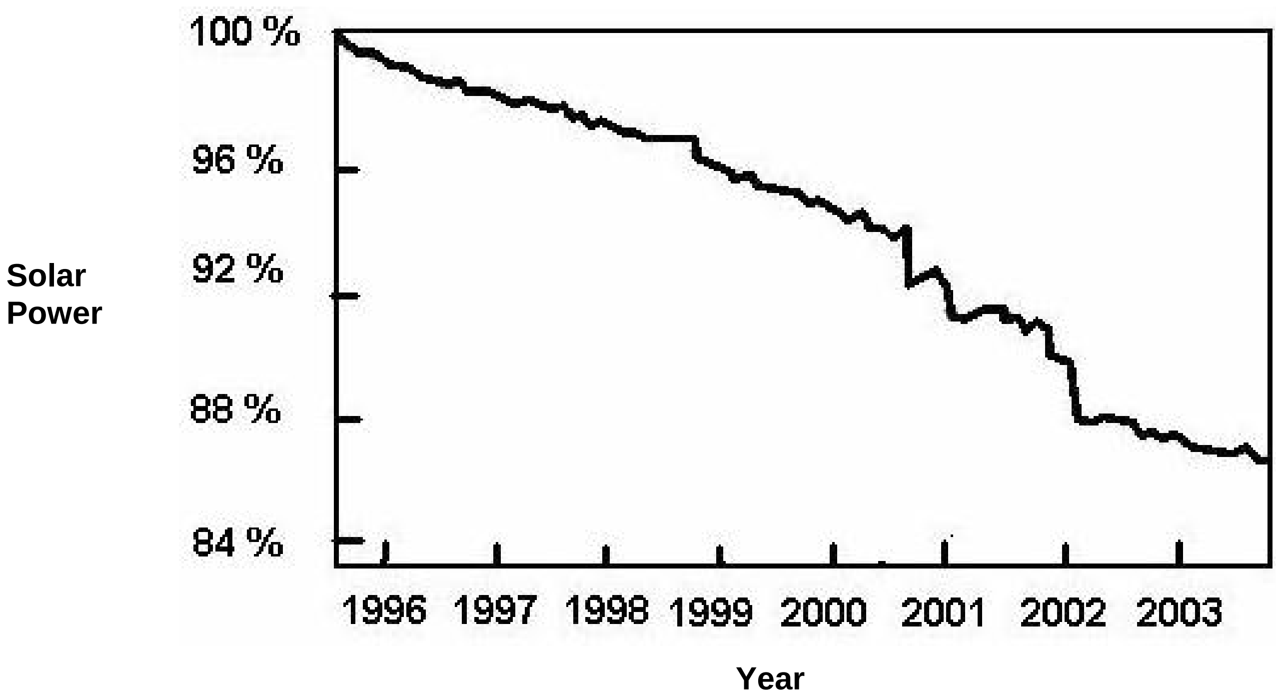
Answer 3 – a) The perimeter is P = (135 + 135 + 95 + 95) **= 460 centimeters.**

B) Calculate the area of the large rectangle and subtract the two interior holes: 135x95 - 10x25 - 25x25
12825 - 250 - 625 = **11,950 square cm**.

C) The solar cells produce 0.03 watts per square cm, so the power available is 11950 x 0.03 = **358 watts**. This less than the minimum satellite requirement of 759 watts.

Name ________________________________

Soon after the first satellites were placed into orbit, engineers began to notice that the electrial power needed to operate the satellites was slowly declining. Very careful studies of the way that solar cells worked in space soon came up with a culprit: Cosmic rays! Cosmic rays are very high-speed ions, protons and electrons that travel through space. Some are produced by the sun during solar flares, while others come from space beyond the solar systems itself. Cosmic rays are deflected by strong magnetic fields. Fortunately, Earth has a very strong magnetic field that shields us from most of these cosmic rays, but enough get through that they collide with solar panels and solar cells on satellites orbiting Earth. Over time, these fast-moving particles cause changes in solar cells, making them less able to generate the electricity they are supposed to when sunlight falls on them. Thanks to decades of study, engineers can make very accurate models of these cosmic rays effects and incorporate them into the design of satellite power systems. Below is an actual graph prepared by Dr. Paul Brekke, the Project Scientist for the Solar Heliospheric Observatory. It shows how fast the satellite's solar arrays have changed their power output since 1995.

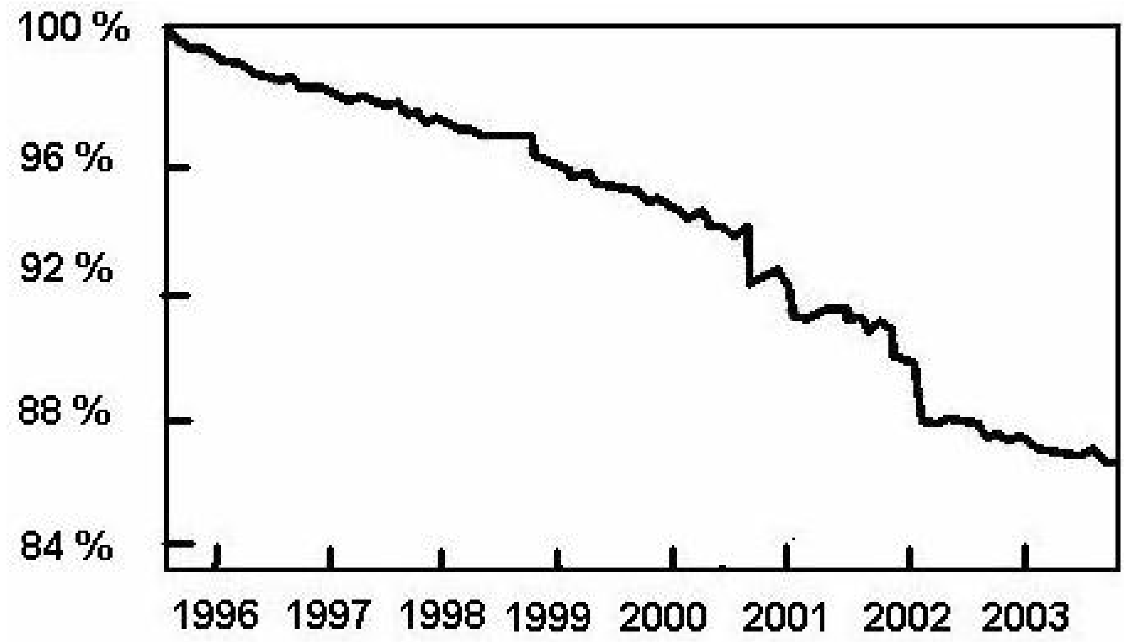


Question 1: By how much has the SOHO satellite power declined by 2002?

Question 2: Satellites often remain usable for 15 years. By what percentage will the electricity from the SOHO solar panels have declined by that time?

Question 3: The instruments on a satellite require 850 watts of power to operate. If a scientist wants her instrument to continue working for 15 years, how large should the satellite power supply be at the time of launch?

Question 4: The surface of a satellite has an area of 1000 square centimeters. The solar cells can generate about 0.03 watts per square centimeter. A) What is the total power available to the satellite by covering its surface with solar cells? B) If the satellite is to last 15 years, what is the maximum power that the instruments can draw and still work after 15 years?



Question 1: By how much has the SOHO satellite power declined by 2002?
Answer: From the plot, the power has declined to 88% of its original 100% at launch.

Question 2: Satellites often remain usable for 15 years. By what percentage will the electricity from the SOHO solar panels have declined by that time? **Answer:** From the previous answer, the decline was 12% in 6 years, so after 15 years the decline will be (15/6) x 12% = 30%.

Question 3: The instruments on a satellite require 850 watts of power to operate. If a scientist wants her instrument to continue working for 15 years, how large should the satellite power supply be at the time of launch? **Answer:** From the previous answer, if the experiments need 850 watts to operate after 15 years, the solar panels have to be designed to produce 30% more than 850 watts at the start of the mission or (1.30) x 850 watts = 1105 watts.

Question 4: The surface of a satellite has an area of 1000 square centimeters. The solar cells can generate about 0.03 watts per square centimeter. A) What is the total power available to the satellite by covering its surface with solar cells? B) If the satellite is to last 15 years, what is the maximum power that the instruments can draw and still work after 15 years?
Answer:
A) The total power will be 3000 square cm x 0.03 watts per square cm = 90 watts.
B) Because the solar panels will degrade by 30% during the 15 years, this means that after 15 years you will only have 90 watts x 0.70 = 63 watts to run your instruments!!
Scientists typically take advantage of the surplus of energy at the start of a mission to run the most energy-consuming instruments, then shut them down one at a time until the satellite finally stops being a useful scientific instrument.

A modern home computer has a hard drive whose capacity is typically about 500 gigabytes. A song that you download typically has a size of about 10 megabytes, and if you have a fast internet connection you can usually download at a rate of about 1 megabytes/sec. At that rate it takes about 10 seconds to download a song, and if your entire hard drive were available to store songs on your playlist, you could accommodate about 50000 songs!

The Van Allen Probes spacecraft instruments will be generating data that has to first be stored onboard the spacecraft, then at a specific time in the orbit, transmitted to Earth before the next round of data has to be stored on top of the old data.

The spacecraft engineers have worked with the scientists to determine how often the scientists want to store their measurements on each satellite. The average rate is 9,000 bytes/second. Each satellite will download the data once every orbit when the satellite is closest to Earth (perigee). The ground station has a busy schedule working with other satellites so each of the two Van Allen Probes satellites will only have 10 minutes every orbit to download all of its stored data. The orbit period is 9 hours.

Problem 1 – How many megabytes of data do both of the spacecraft collect after one orbit?

Problem 2 – At what rate in bytes/second will the data have to be downloaded to the ground station?

Problem 3 – The mission is being supported by NASA to last two years during its first operations cycle. How much data will both of the Van Allen Probes satellites accumulate during its first 2-year period in :

A) DVD disks (1 DVD = 4 gigabytes)

B) Songs (1 song = 10 megabytes).

Problem 1 – How many megabytes of data do both of the spacecraft collect after one orbit?

Answer: Time = 9 hours x (3600 seconds / 1 hr) = 32,400 seconds.
Total data = 9,000 bytes/sec x 32400 seconds = 291,600,000 bytes or 291.6 megabytes per spacecraft and **583.2 megabytes**/orbit for two spacecraft combined.

Problem 2 – At what rate in bytes/second will the data have to be downloaded to the ground station?

Answer: 583.2 megabytes have to be downloaded within 10 minutes or 600 seconds so the rate will be R = 583,200,000 bytes / 600 seconds = **972,000 bytes/second**.

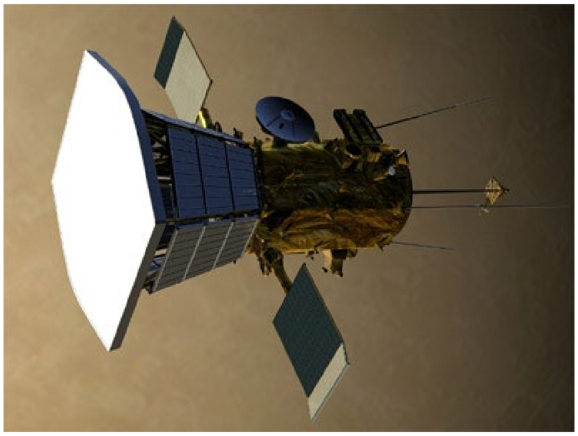
Problem 3 – The mission is being supported by NASA to last two years during its first operations cycle. How much data will both of the Van Allen Probes satellites accumulate during its first 2-year period A) In DVD disks (1 DVD = 4 gigabytes) B) Songs (1 song = 10 megabytes).

Answer: A) 1 year = 365 days x 24 hours/day x 3600 seconds/hr = 31,536,000 seconds.
The data rate for two satellites is 18000 bytes/second, so in 2 years the satellites will accumulate 18000 x 31536000 = 567.6 billion bytes or 567.6 gigabytes.

A single DVD stores 4 gigabytes so you will need 567.6/4 = 141.9 or **141 DVDs**.

B) Number of songs = 567,600 megabytes / 10 megabytes = 56,760 or **56,760 song**s.

Exploring the ***Parker Solar Probe*** Heat Shield



The NASA, Parker Solar Probe mission to be launched in 2017 will travel to a location very close to our sun. To prevent its delicate scientific instruments from over-heating, the entire spacecraft will 'hide' behind a heat shield like the one shown in the figure to the left. Also called the Thermal Protection System, it is roughly rectangular in shape with dimensions of 2.4 meters x 3.1 meters.

Problem 1 – If the thickness of the heat shield is 17 centimeters, what is the total volume of the heat shield in cubic centimeters?

Problem 2 – The heat shield is made from a special carbon-composite material with a density of 0.054 grams/cm^3 (54 kg/m^3). If the density of water is 1 gm/cm^3 (1000 kg/m^3) will the heat shield sink or float in a swimming pool?

Problem 3 – If one cubic centimeter of the heat shield has a mass of 0.054 grams, what is the total mass of the heat shield in kilograms?

Answer Key

Problem 1 – If the thickness of the heat shield is 17 centimeters, what is the total volume of the heat shield in cubic centimeters rounded to two digits?

Answer: First convert all units to centimeters:
2.4 meters x (100 cm/1 meter) = 240 centimeters
3.1 meters x (100 cm/1 meter) = 3410 centimeters

Volume of a rectangular solid is V = L x W x H

= 240 x 310 x 17 = **1,300,000 cm^3**.

Problem 2 – The heat shield is made from a special carbon-composite material with a density of 0.054 grams/cm^3 (54 kg/m^3). If the density of water is 1 gm/cm^3 (1000 kg/m^3) will the heat shield sink or float in a swimming pool?

Answer: **The shield has less density than water so it will float.**

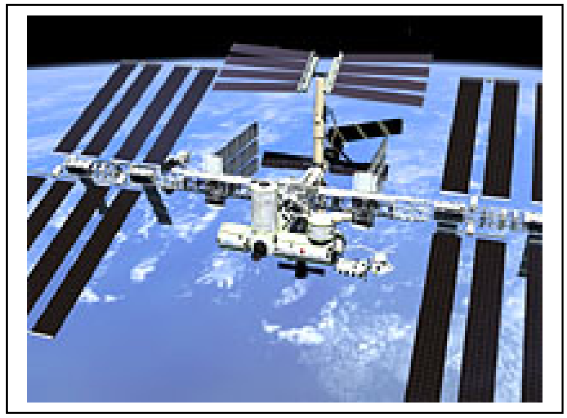
Problem 3 – If one cubic centimeter of the heat shield has a mass of 0.054 grams, what is the total mass of the heat shield in kilograms?

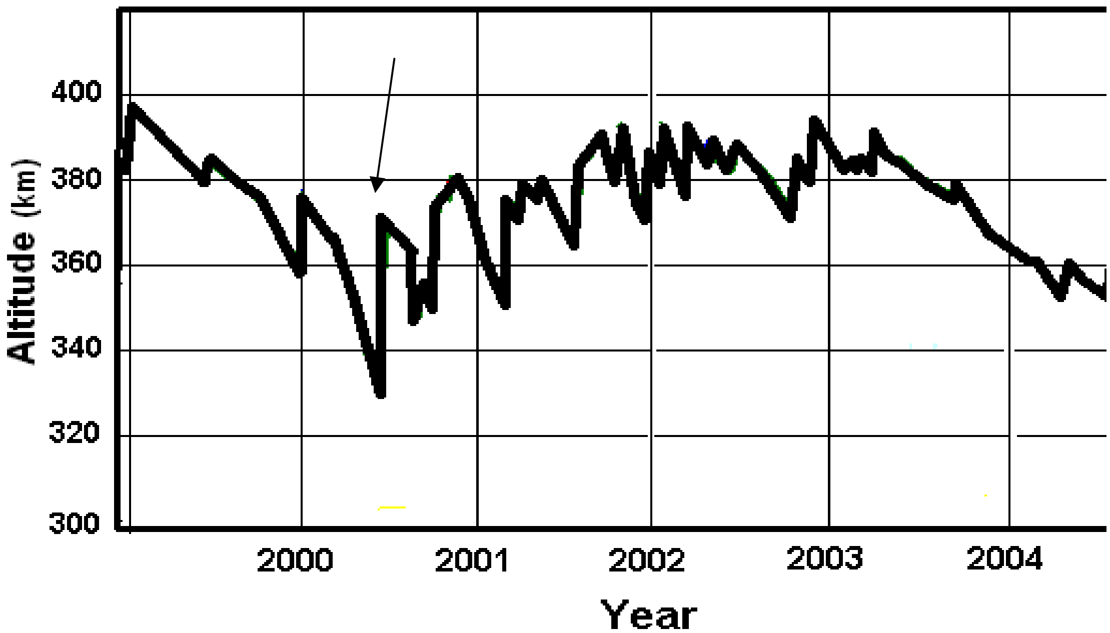
Answer: 0.054 grams/1 cm^3 x 1,300,000 cm^3
= 70,200 grams

70,200 grams x (1 kg/1000 gm) = **70.2 kilograms**

Name ________________________________

The first pieces of the International Space Station were launched in 1998, and the 'ISS' has been continuously occupied by a crew of 3 to 4 astronauts since 1999. At completion, it will have cost $100 billion, and will have a mass of 187 metric tons. It requires 110 kiloWatts to operate (equal to 55 average houses). One interesting thing about the ISS is that its orbit is constantly decaying because of aerodynamic drag with Earth's upper atmosphere. The figure below shows the altitude changes from 2000 to 2005.





The Space Shuttle and Russian Prognos vehicles periodically re-boost the ISS to higher altitudes. These appear as sudden changes in altitude from a low altitude to a higher one. One of these is identified with an arrow.

Question 1) How many other re-boosts can you identify in the graph?

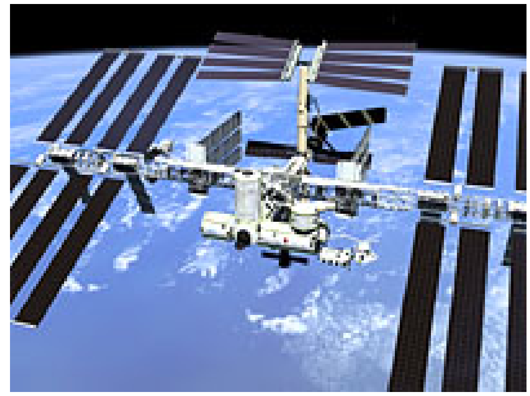
Question 2) What was the largest change in altitude by a re-boost?

Question 3) How fast was the altitude decreasing before the re-boost in 2000 and in 2004?

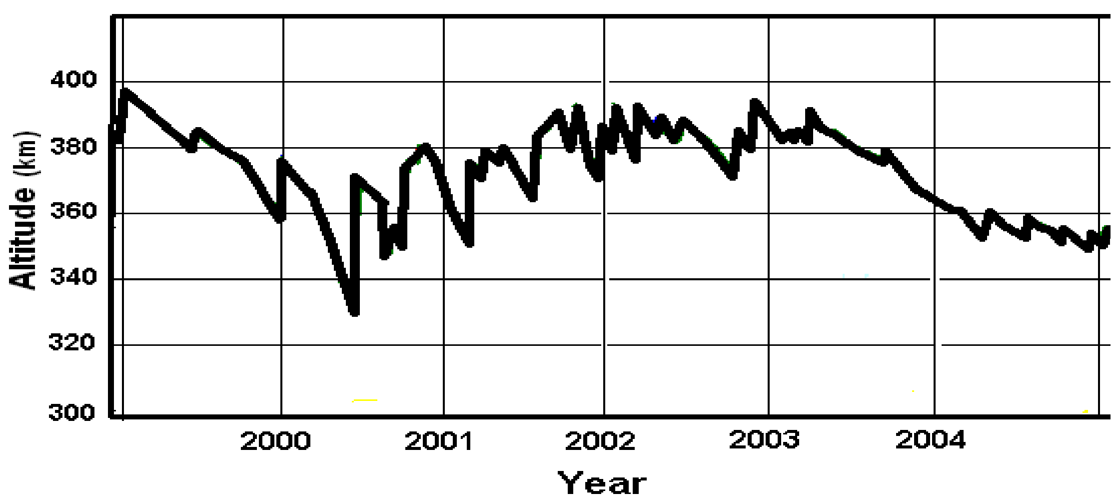
Question 4) Why do you think the orbit altitudes were changing faster near 2000 than 2004?

Question 5) From your answer to Question 3, how long would it take the ISS to burn-up at an altitude of 150 km if no re-boost had been performed?

The first pieces of the International Space Station were launched in 1998, and the 'ISS' has been continuously occupied by a crew of 3 to 4 astronauts since 1999. At completion, it will have cost $100 billion, and will have a mass of 187 metric tons. It requires 110 kiloWatts to operate (equal to 55 average houses). It is scheduled to be retired from service in 2010. One interesting thing about the ISS is that its orbit is constantly decaying because of aerodynamic drag with Earth's upper atmosphere.



The figure below shows its altitude from 1999 to 2005. The sharp vertical 'jags' near '2000' and '2000.5' are orbit re-boosts provided by the Space Shuttle to move the ISS to higher altitudes. In this activity, students will calculate the orbit decay rates from this figure, and investigate why the decay rate (slope) is higher at some times (i.e. 2000) than at other times (i.e. 2004). They will also be asked to predict how long it would take the ISS to re-enter the atmosphere without periodic orbit re-boosts.



Question 1) About 16, but you can check by visiting
Question 2) Around 2000.4 the change in altitude was about 40 km!
Question 3) In 2000 about 400 meters per day (0.4 km/day). In 2004 about 80 meters per day (0.08 km/day). Students should make measurements during each year in the plot.
Question 4) This is a good 'inquiry' problem. Solar activity heats upper atmosphere and causes increased drag. Compare the orbit decay rates for each year with the average sunspot numbers for each year.
Question 5) In 2000, it would reach an altitude of 150 km in about (330—150)/0.4 = 450 Days. In 2004 it would take (330-150)/0.08 = 2250 days. This is why there were more, and larger, orbit re-boosts in 1999-2001 than at other times.