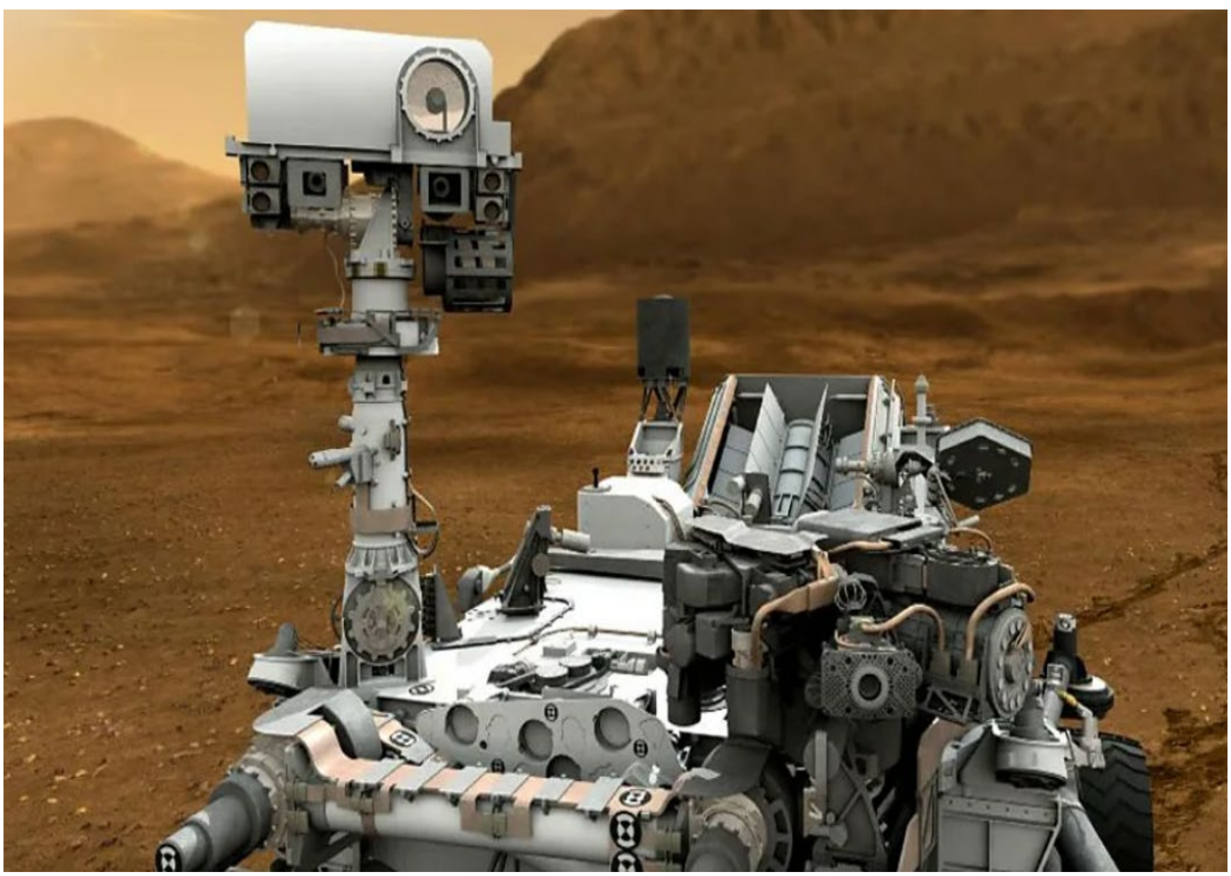
National Aeronautics and Space Administration

Math Problems Related to

Technology - I



Introduction

This book provides an assortment of 14 mathematics problems that cover aspects of technology such as telescopes, telemetry and solar energy.

It takes a lot of supplies to keep the International Space Station going!

On February 21, 2014, NASA asked commercial launch services, such as SpaceX, to figure out what they would charge to deliver cargo to the Station. NASA plans to buy this service for between $1 billion and $1.4 billion each year from 2017 to 2024.

Groups ‘did the math’ to figure out how many launches would be needed to deliver the required materials to the Station. On return flights, trash such as discarded equipment and garbage, will be brought back home.

For each year, NASA will need up to 16,750 kilograms of pressurized cargo delivered to the Station. This amount of cargo will fill about 30 lockers. Another 4,000 kg of unpressurized cargo will be included in each launch. This amount of cargo will fill about 8 lockers. There will be 5 flights per year.

Problem 1 – What is the total mass of the pressurized and unpressurized cargo?
A) per year? B) per flight?

Problem 2 – For each flight, 800 kg of fresh drinking water will be delivered to the Station. If 1 gallon of water has a mass of 3.8 kg, how many gallons of drinking water are transported to the Station every year? Round your answer to the nearest gallon.

Problem 3 – Each pressurized locker requires continuous power of 120 watts at 28 volts. Each unpressurized locker requires continuous power of 250 watts at 28 volts. What is the total electrical watts used by both the pressurized and unpressurized lockers for each flight?

Problem 4 – If it costs $4,500 per kg to place items in orbit, what is the cost for each launch?

NASA Seeks U.S. Industry Feedback on Options for Future Space Station Cargo Services
February 21, 2014
http://www.nasa.gov/content/nasa-seeks-us-industry-feedback-on-options-for-future-space-station-cargo-services/index.html

Problem 1 – What is the total mass of the pressurized and unpressurized cargo? A) per year? B) per flight?

Answer:

A) Pressurized cargo = 16,750 kg; Unpressurized cargo = 4,000 kg.
16,750 kg + 4,000 kg = **20,750 kg per year**.

B) There are 5 flights. Total mass (20,750 kg)/5= **4,150 kilograms**.

Problem 2 – If 1 gallon of water has a mass of 3.8 kg, how many gallons of drinking water are transported to the Station every year? Round your answer to the nearest gallon. rounded to the nearest gallon?

Answer: Each flight brings 800 kg to the Station. For 5 flights there would be 4000 kg of water. Then 4,000 kg x (1 gallon/3.8 kg) = **1053 gallons per flight**.

Problem 3 – What is the total electrical watts used by both the pressurized and unpressurized lockers for each flight?

Answer: There are 30 powered lockers brought to the Station each year with 120 watts/locker. The total power is 30 x 120 watts = 3,600 watts per year. There are 8 unpressurized lockers brought to the Station each year at 250 watts/locker for a total of 2,000 watts. The total per year is 5,600 watts in 5 flights or **1120 watts/flight**.

Problem 4 – If it costs $4,500 per kg to place items in orbit, what is the cost for each launch?

Answer: 4,150 kg cargo per launch x $4,500/kg = **$18,675,000 for the cargo**.

How Telescopes Work

You have probably seen a telescope before, and wondered how it works!

Telescopes are important in astronomy because they do two things extremely well. Their large lenses and mirrors can collect much more light than the human eye, which make it possible to see very faint things. This is called Light Gathering Ability. They also make distant things look much bigger than what the human eye can see so it is easier to study details. This is called magnification.

The human eye at night is a circle about 7 millimeters in diameter, called the pupil, which lets light pass through its lens and onto the retina. A telescope can have a main mirror or lens that can be many meters in diameter.

How do you figure out how much Light Gathering Ability a telescope has compared to the human eye? Just calculate the area of the two circles and form their ratio!

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of $\pi = 3.145$, what is the area of the human pupil in square millimeters?

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of p = 3.145, what is the area of the human pupil in square millimeters?

Answer: $A = 3.14\ (7/2)^2 =$ **0.78 mm^2**

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Answer: $A = 3.14\ (2400/2)^2 =$ **4,521,600 mm^2**

Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Answer: 4,521,600 / 0.78 = **5,796,923 times the human eye**

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

Answer: $100 = \pi R^2/0.78$ so $R^2 = 24.8$ and so R = 5.0 and **D = 10.0 millimeters**.


Early depiction of a 'Dutch telescope' from the "Emblemata of zinne-werck" (Middelburg, 1624) of the poet and statesman Johan de Brune (1588-1658). The print was engraved by Adriaen van de Venne, who, together with his brother Jan Pieters van de Venne, printed books not far from the original optical workshop of Hans Lipperhey.

Telescopes can magnify the sizes of distant objects so that the eye can see them more clearly. This is very handy for astronomers who want to study distant planets, stars and galaxies to figure out what they are!

A simple telescope, called a refractor, has two lenses. The large one collects the light from a distant object and amplifies it so that the image is much brighter than what the eye normally sees. This is called the Objective Lens, or for reflecting telescopes, the Objective Mirror. A second lens is placed at the focus of the Objective and provides the magnification you need to study the objects.

Both the Objective and the eye lens (called the Eyepiece) have their own focus points. The distance between the lens and this focus point is called the focal length. The magnification of the telescope is just the ratio of the Objective focal length to the eyepiece focal length!

$$M = \frac{\text{Objective focal length}}{\text{Eyepiece focal length}}$$

Note, the units for the focal lengths both have to be the same units…inches…millimeters….etc.

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 millimeters. What was the magnification of this telescope?

Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces? (1 inch = 25.4 mm)

Answer Key

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 millimeters. What was the magnification of this telescope?

Answer. M = 980/50 = **19.6 times**.

Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

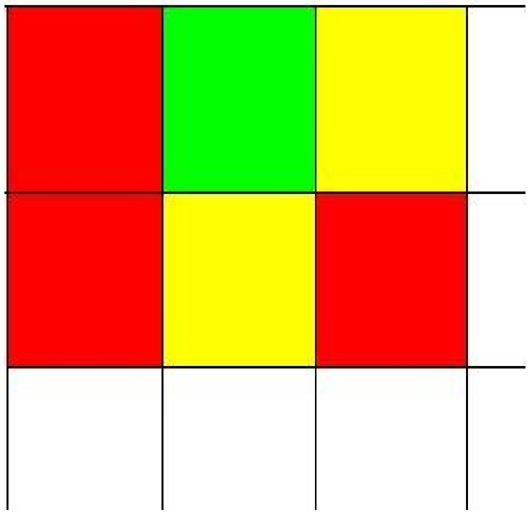
Answer: 52 meters = 52000 millimeters, then for a 50mm eyepiece, the magnification is M = 52000/50 = **1040 times**.

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces?

Answer: 157 inches x 25.4 mm/inch = 3988 mm, then

M = 3988/4 = 997x,
M = 3988/20 = 199 x
M = 3988/35 = 114 x

Correcting Bad Data Using Parity Bits



The first few pixels in a large image

Data is sent as a string of ‘1’s and ‘0’s which are then converted into useful numbers by computer programs. A common application is in digital imaging. Each pixel is represented as a ‘data word’ and the image is recovered by relating the value of the data word to an intensity or a particular color. In the sample image to the left, red is represented by the data word ‘10110011’, green is represented by ‘11100101’ and yellow by the word ‘00111000’, so the first three pixels would be transmitted as the ‘three word’ string ‘101100111110010100111000’. But what if one of those 1-s or 0-s was accidentally reversed? You would get a garbled string and an error in the color used in a particular pixel.

Since the beginning of the Computer Era, engineers have anticipated this problem by adding a ‘parity bit’ to each data word. The bit is ‘1’ if there are an even number of 1’s in the word, and’ 0’ if there is an odd number. In the data word for red ‘1011001**1**’ the last ‘**1**’ to the right is the parity bit.

When data is produced in space, it is protected by parity bits, which alert the scientists that a particular data word may have been corrupted by a cosmic ray accidentally altering one of the data bits in the word. For example, Data Word A ‘1110001**1**’ is valid but Data Word B ‘1111001**1**’ is not. There are five ‘1’s but instead of the parity bit being ‘0’ (‘1110001**0**’), it is ‘1’ which means Data Word B had one extra ‘1’ added somewhere. One way to recover the good data is to simply re-transmit data words several times and fill-in the bad data words with the good words from one of the other transmissions. For example:

Corrupted data string:	1011110**0**	101101**0**	1010101**1**	0011001**1**	1011101**0**
Good data string:	1011110**0**	100101**0**	1010101**1**	1011001**1**	1011101**0**

The second and fourth words have been corrupted, but because the string was re-transmitted twice, we were able to ‘flag’ the bad word and replace it with a good word with the correct parity bit. Cosmic rays often cause bad data in hundreds of data words in each picture, but because pictures are re-transmitted two or three times, the bad data can be eliminated and a corrected image created.

Problem: Below are two data strings that have been corrupted by cosmic ray glitches. Look through the data (a process called parsing) and use the right-most parity bit to identify all the bad data. Create a valid data string that has been ‘de-glitched’.

String 1:	10111010	11110101	10111100	11001011	00101101
	01010000	01111010	10001100	00110111	00100110
	01111000	11001101	10110111	11011010	11100001
	10001010	10001111	01110011	10010011	11001011

String 2:	10111010	01110101	10111100	11011011	10101101
	01011010	01111010	10001000	10110111	00100110
	11011000	11001101	10110101	11011010	11110001
	10001010	10011111	01110011	10010001	11001011

Answer Key:

Problem: Below are two data strings that have been corrupted by cosmic ray glitches. Look through the data (a process called parsing) and use the right-most parity bit to identify all the bad data. Create a valid data string that has been 'de-glitched'.

The highlighted data words are the corrupted ones.

String 1:	10111010	**11110101**	10111100	11001011	**00101101**
	01010000	01111010	10001100	00110111	00100110
	01111000	11001101	**10110111**	11011010	**11100001**
	10001010	10001111	01110011	**10010011**	11001011

String 2:	10111010	01110101	10111100	**11011011**	10101101
	01011010	01111010	**10001000**	**10110111**	**10100110**
	11011000	11001101	10110101	11011010	11110001
	10001010	**10011111**	01110011	10010001	**01001011**

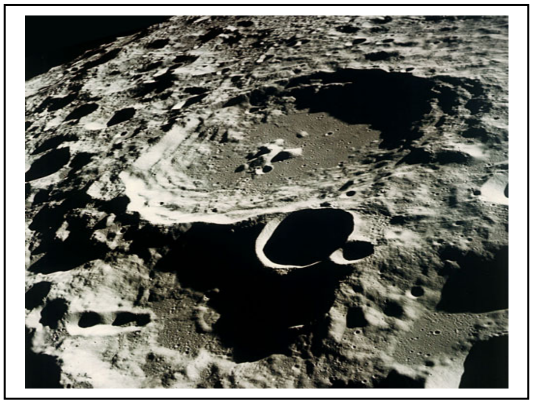
In the first string, 11110101 has a parity bit of '1' but it has an odd number of '1' so its parity should have been '0' if it were a valid word. Looking at the second string, we see that the word that appears at this location in the grid is '01110101' which has the correct parity bit. We can see that a glitch has changed the first '1' in String 2 to a '0' in the incorrect String 1.

By replacing the highlighted, corrupted data words with the uncorrupted values in the other string, we get the following de-glitched data words:

Corrected:	10111010	01110101	10111100	11001011	10101101
	01011010	01111010	10001100	00110111	00100110
	11011000	11001101	10110101	11011010	11110001
	10001010	10001111	01110011	10010001	11001011

The odd word is the first word in the third row. The first transmission says that it is '0111100**0**' and the second transmission says it is '1101100**0**' Both wrong words have a parity of '1' which means there is an even number of '1' in the first seven places in the data word. But the received parity bit says '**0**' which means there was supposed to be an odd number of '1's in the correct word. Examining these two words, we see that the first three digits are '011' and '110' so it looks like the first and third digits have been altered. Unfortunately, we can't tell what the correct string should have been. Because the rest of the word '11000' has an even parity, all we can tell about the first three digits is that they had an odd number of '1's so that the total parity of the complete word is '0' . This means the correct digits could have been '100', '010', '111', or '111', but we can't tell which of the three is the right one. That means that this data word remains damaged and can't be de-glitched even after the second transmission of the data strings.

Areas and Probabilities



There are many situations in astronomy where probability and area go hand in hand! The problems below can be modeled by using graph paper shaded to represent the cratered areas.

The moon's surface is heavily cratered, as the Apollo 11 photo to the left shows. The total area covered by them is more than 70% of the lunar surface!

1 – A 40km x 40km area of the Moon has 5 non-overlapping craters, each about 5km in radius. A) What fraction of this area is covered by craters? B) What is the percentage of the cratered area to the full area? C) Draw a square representing the surveyed region and shade the fraction covered by craters.

2 - During an 8-day period, 2 days were randomly taken off for vacation. A) What fraction of days are vacation days? B) What is the probability that Day-5 was a vacation day? C) Draw a square whose shaded area represents the fraction of vacation days.

3 – An asteroid capable of making a circular crater 40-km across impacts this same 40km x 40km area dead-center. About what is the probability that it will strike a crater that already exists in this region?

4 – During an 8-day period, 2 days were randomly taken of for vacation, however, during each 8-day period there were 4 consecutive days of rain that also happened randomly during this period of time. What is the probability that at least one of the rain days was a vacation day? (Hint: list all of the possible 8-day outcomes.)

Inquiry – How can you use your strategy in Problem 4 to answer the following question: An asteroid capable of making a circular crater 20-km across impacts this same 40km x 40km area dead-center. What is the probability that it will strike a crater that already exists in this region?

Answer Key

1 – Answer: A) The area of a crater with a circular shape, is $A = \pi\ (5km)^2 = 78.5\ km^2$, so 5 non-overlapping craters have a total area of 393 km^2. The lunar area is 40 km x 40km = 1600 km^2, so the fraction of cratered area is 393/1600 **= 0.25.**

B) The percentage cratered is 0.25 x 100% = 25%.

C) Students will shade-in 25% of the squares.

2 - Answer: A) 2 days/8 days = 0.25

B) .25 x 100% = 25%.

C) The square should have 25% of area shaded.

3 –Answer: The area of the impact would be $\pi\ (20km)^2 = 1240\ km^2$. The area of the full region is 1600 km^2. The difference in area is the amount of lunar surface not impacted and equals 360 km^2. Because the cratered area is 393 km^2 and is larger then the unimpacted area, the **probability is 100%** that at least some of the cratered area will be affected by the new crater.

4 – R = Rain days

R R R R X X X X
X R R R R X X X
X X R R R R X X
X X X R R R R X
X X X X R R R R

Vacation: There are 8 x 7/2 = **28 possibilities**

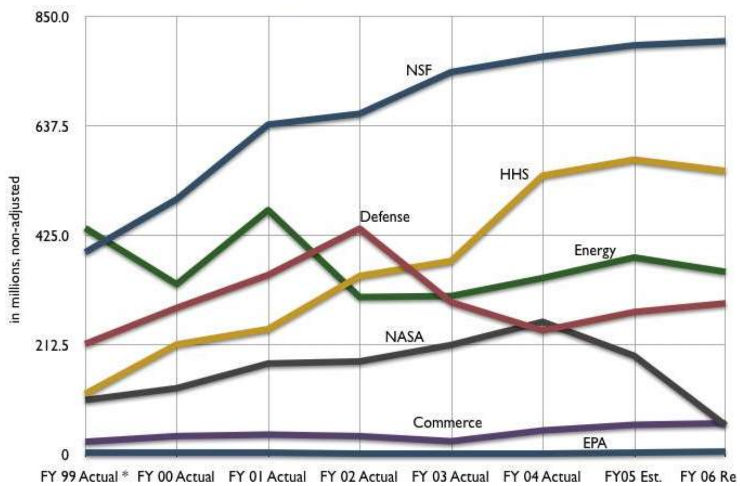
Rain 'area' = 50%. For each of the 5 possibilities for a rain period, there are 28 possibilities for a vacation series, which makes 5 x 28 = 140 combinations of rain and vacation. For a single rain pattern out of the 5, there are 28 vacation patterns, and of these, 50% will include at least one vacation day in the rain, because the other half of the days avoid the rain days entirely. So, out of the 140 combinations, 70 will include rain days and 70 will not, again reflecting the fact that the 'area' of the rain days is 50% of the total days.

Inquiry: The already cratered area is ¼ of the total. The area of the new impact is $\pi\ (10km)^2 = 314\ km^2$ which is 314/1600 = 1/5 of the total area. Draw a series of S=20 cells (like the 8-day pattern). The new crater represents 20x1/5 = 4 consecutive cells shaded. Work out all of the possible ways that 20 x 1/4 = 5 previously cratered areas can be distributed over the 20 days so that one of them falls within the 4 consecutive cells of the new crater. Example: X P X P X N N N N X X X X P X P X X P X

X = not cratered area, P = previously cratered area, N = new crater impact.

We are looking for: Probability = (The number of trials where a P is inside the 'N' region) / M , where M = the total number of possible trials. **Because there are 17 possibilities for the Ns, and 20!/ (15! 5!) = (20 x 19) / 2 = 190 possibilities for where the 5 Ps can go, m = 17 x 190 = 3230 . These do not all have to be worked out by hand to find the number of trials where a P is inside an N region in the series. You can also reduce the value of S so long as you keep the relative** areas between N, V and P the same.

The Dollars and Cents of Research



Scientific research costs money! You have to find money to pay your salary to work 40-hours a week. You also have to get money to pay for the trips you take to observatories, scientific conferences, to pay for your healthcare, retirement plans, and the rental for your office and laboratory space. Here are some problems that help you see how this all works!

Problem 1 – Professor Quark is a senior astronomer at PDQ University who makes \$50.00 an hour. If there are 2,000 work hours in a full year, what will be the astronomer's gross pay before taxes and other deductions, for the year?

Problem 2 – PDQ University charges each astronomer an additional 50% of the astronomer's salary to cover the rental of office space, medical benefits, and retirement benefits. How much extra money in addition to his salary will Prof. Quark have to pay PDQ University to conduct his research?

Problem 3 - Prof. Quark needs to buy a new computer 'work station' to conduct his research. He can choose System A for \$5,000, which has 200 gigabytes of hard drive space, and operates at 5 megahertz, or he can buy a cheaper System B for \$1,000, which has a 1,000 gigabyte hard drive and runs five times slower at 1 megahertz. Which system do you think he should buy? Why?

Problem 4 - Prof. Quark expects to take several trips each year to conferences in France, China and Canada. These trips will cost a total of \$9,000. The university adds on an additional cost to this expense to cover their setting up the travel arrangements and handling all of the accounting. They charge the researcher 30% of the researcher's cost to do this. What is the total cost to the researcher for the travel expenses?

Problem 5 – Prof. Quark plans to support his research by applying to the Long-Term Space Astrophysics research grant program at NASA. What is the total amount of money he has to apply for if he wants to get fully supported for one year?

Problem 6 – PDQ University will support Prof. Quark for ¾ of his labor in Problem 2 if he agrees to teach college courses and train graduate students. He will have to get external support for the balance of his research support. How much money will he have to ask NASA for in a grant proposal to support all of his research for one year?

Answer Key

Problem 1 – Professor Quark is a senior astronomer at PDQ University who makes $50.00 an hour. If there are 2,000 work hours in a full year, what will be the astronomer's gross pay before taxes and other deductions, for the year?
Answer: 2,000 hours x $50.00/hour = **$100,000**

Problem 2 – PDQ University charges each astronomer an additional 50% of the astronomer's salary to cover the rental of office space, medical benefits, and retirement benefits. How much extra money in addition to his salary will Prof. Quark have to pay PDQ University to conduct his research?
Answer: $100,000 x 50%/100% = $50,000

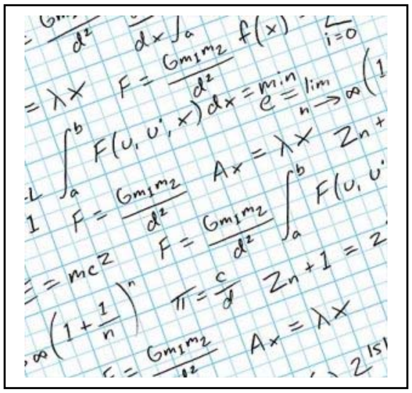
Problem 3 - Prof. Quark needs to buy a new computer 'work station' to conduct his research. He can choose System A for $5,000, which has 200 gigabytes of hard drive space, and operates at 5 megaHertz, or he can buy a cheaper System B for $1,000, which has a 1,000 gigabyte hard drive and runs five times slower at 1 megahertz. Which system do you think he should buy? Why?
Answer: He should buy the faster, but more expensive, computer. His time is worth $50.00 an hour, so if he has to wait around for a slower computer to process data, over time, he will be wasting money.

Problem 4 - Prof. Quark expects to take several trips each year to conferences in France, China and Canada. These trips will cost a total of $9,000. The university adds on an additional cost to this expense to cover their setting up the travel arrangements and handling all of the accounting. They charge the researcher 30% of the researcher's cost to do this. What is the total cost to the researcher for the travel expenses?
Answer: $9,000 x 30%/100% = $2,700. His total Travel Expenses will be $9,000 + $2,700 = $11,700.

Problem 5 – Prof. Quark plans to support his research by applying to the Long-Term Space Astrophysics research grant program at NASA. What is the total amount of money he has to apply for if he wants to get fully supported for one year?
Answer: From the answers to the first four problems, his total budget will be $100,000 + $50,000 + $5,000 + $11,700 or $166,700.

Problem 6 – PDQ University will support Prof. Quark for ¾ of his labor in Problem 2 if he agrees to teach college courses and train graduate students. He will have to get external support for the balance of his research support. How much money will he have to ask NASA for in a grant proposal to support all of his research for one year?
Answer: ¾ x $150,000 = $100,000 then $166,700 - $100,000 so the amount he will need from the grant is **$66,700.**

Number Sentence Puzzles 5



Scientific research has a lot in common with solving number puzzles like SODUKO in order to figure out what stars or planets are doing in space. Use your puzzle-solving ability to figure out what event is described by the following number sentences!

1 - Which story matches the sentence 23 – 10 + 6 = 19 ?

A) An astronomer discovers 23 quasars on one photograph, 10 quasars on a second photograph, and 6 additional quasars on a third photograph. How many quasars did she identify?

B) An astronomer spots 23 solar flares on Tuesday and 6 solar flares on Thursday, then decides that 10 of the solar flares were not real. How many real flares did he see?

C) An astronomer counts a total of 19 lunar craters, and classifies 23 of them as asteroid impacts, 6 of them as volcanic calderas and 10 of them as meteor impacts.

2 - Which story matches the sentence 145 + N = 375 ?

A) Two astronomers combined their databases of planets. They observe a total of 375. If one astronomer contributed 145 planets, how many did the second astronomer contribute?

B) The temperature of an asteroid's interior changes by 375 degrees between the center and the surface. If the surface temperature is 145 degrees Centigrade, what is the interior temperature of the asteroid?

C) The width of Saturn's rings is 375 megameters. If the ring system starts at a distance of 145 megameters from Saturn's outer atmosphere, what is the distance to the outer edge of the ring system?

3 - Two astronomers combined their catalogs of cosmic gamma-ray bursts. There were 287 and 598 cataloged by each astronomer with 65 events in common. How many unique events are in the combined catalog?

A) (287 - 65) + (598 - 65) = M
B) 287 + (598 – 65) = M
C) 287 + 598 = M
D) (287 + 65) + (598 + 65) = M

Answer Key

1 - Which story matches the sentence 23 – 10 + 6 = 19 ?

A) An astronomer discovers 23 quasars on one photograph, 10 quasars on a second photograph, and 6 additional quasars on a third photograph. How many quasars did she identify? **Answer: No. This would be the sentence 23 + 10 + 6 = N**

B) An astronomer spots 23 solar flares on Tuesday and 6 solar flares on Thursday, then decides that 10 of the solar flares were not real. How many real flares did he see? **Answer: Yes this is correct.**

C) An astronomer counts a total of 19 craters, and classifies 23 of them as asteroid impacts, 6 of them as volcanic calderas and 10 of them as meteor impacts. **Answer: No. This is the sentence 23 + 6 + 10 = 39.**

2 - Which story matches the sentence 145 + N = 375 ?

A) Two astronomers combined their databases of planets. They observe a total of 375. If one astronomer contributed 145 planets, how many did the second astronomer contribute? **Yes. This is correct.**

B) The temperature of an asteroid's interior changes by 375 degrees between the center and the surface. If the surface temperature is 145 degrees Centigrade, what is the core temperature of the asteroid? **Answer: No. This is the sentence 145 + 375 = N**

C) The width of Saturn's rings is 375 megameters. If the ring system starts at a distance of 145 megameters from Saturn's outer atmosphere, what is the distance to the outer edge of the ring system**? Answer: No. This is the sentence 145 + 375 = N**

3 - Two astronomers combined their catalogs of cosmic gamma-ray bursts. There were 287 and 598 cataloged by each astronomer with 65 events in common. How many unique events are in the combined catalog?

A) (287 - 65) + (598 - 65) = M **No. This eliminates the common bursts from the final catalog.**

B) 287 + (598 – 65) = M **Yes. This is correct.**

C) 287 + 598 = M - **No. This is just the sum of the two catalogs including duplications.**

D) (287 + 65) + (598 + 65) = **M** **No. The duplications are added to each not subtracted.**

Answer B is correct. Among the 287 + 598 gamma ray bursts in the two catalogs, there are 65 that are in common. Starting with the full catalog provided by one astronomer (287), we add only the non-duplicated bursts in the second astronomers catalog (598-65).

The STEREO Mission: getting the message across

In 2007, NASA launched two satellites, STEREO-A and STEREO-B that were to slowly drift away from Earth in opposite directions, taking 'stereo' images of the Sun as they went. Getting the information and images back to Earth posed a challenge because the farther away they drifted, the weaker the signal got. The table below gives the distance of STEREO-A from Earth. The satellite used a 10-watt transmitter operating at a frequency of 2,300 Megahertz .This, by the way, is about 10-times the frequency of a normal UHF TV station.

Date	Distance (Million km)	Intensity ($\times 10^{-24}$ W/m^2)	Received Power ($\times 10^{-20}$ Watts)
June-2007	24.6	1316	**126**
August-2007	40.3	490	
October-2007	51.0	306	
December-2007	55.2	261	
February-2008	58.4	234	
April-2008	67.0	177	
June-2008	81.9	119	
August-2008	97.6	84	
October-2008	107.3	69	
December-2008	111.2	64	

Problem 1 - How much weaker was the radio signal from STEREO-A in December 2008 than in June 2007?

Problem 2 - The Deep Space network radio dish has a diameter of 70-meters. In Column 4 calculate how many watts the dish collected from the signals sent during each month.

Problem 3 - If the satellite sends one bit of data ('1' or '0') every 5 seconds, how much energy is detected (in Joules) per bit sent in A)June 2007? and B) December 2008? (1 watt = 1 Joule per second).

Problem 4 - Suppose the receiver cannot detect less radio energy less than 2 x 10^{-25} Joules each bit. What is the largest number of bits that can be detected each second in A) June 2007 and B) December 2008?

Problem 5 - As a spacecraft gets further and further away from Earth, what kinds of strategies do engineers and scientists have to use to get their data back to Earth?

Answer Key

Problem 1 - How much weaker was the radio signal from STEREO-A in December 2008 than in June 2007? **Answer: The signal was 126/6 = 21 times weaker in December than in June.**

Problem 2 - Answer:

Date	Distance (Million km)	Intensity ($x10^{-24}$ W/m^2)	Receiver Power ($x10^{-20}$ Watts)
June-2007	24.6	1316	126
August-2007	40.3	490	**47**
October-2007	51.0	306	**29**
December-2007	55.2	261	**25**
February-2008	58.4	234	**22**
April-2008	67.0	177	**17**
June-2008	81.9	119	**11**
August-2008	97.6	84	**8**
October-2008	107.3	69	**7**
December-2008	111.2	64	**6**

Problem 3 - If the satellite sends one bit of data ('1' or '0') every 5 seconds, how much energy is detected (in Joules) per bit sent in A)June 2007? and B)December 2008? (1 watt = 1 Joule per second). Answer: A) 1.26 x 10^{-18} Joules/sec x 5 sec = **6.3 x 10^{-18} Joules**. B) 6 x 10^{-20} Joules/sec x 5 sec = **3.0 x 10^{-19} Joules.**

Problem 4 - Suppose the receiver cannot detect less radio energy less than 2 x 10^{-25} Joules each bit. What is the largest number of bits that can be detected each second in A) June 2007 and B) December 2008? Answer: A) 1.26 x 10^{-18} Joules/sec / (2.0 x 10^{-25} Joules/bit) = **6.3 million bits/sec**. B) 6 x 10^{-20} Joules/sec / (2.0 x 10^{-25} Joules/bit) = **300,000 bits/sec**.

Problem 5 - Answer: **Here are just a few possibilities: 1) They can use larger radio dishes to increase the signal strength at the receiver. 2) They can slow down the data rates so that the energy per bit is larger making the data more easily detectable. 3) They can design the spacecraft with a more powerful transmitter. 4) They can do all of the above.**

Equations with one variable I

$$T_F = 9/5\ T_C + 32$$

Calculations involving a single variable come up in many different ways in astronomy, like the popular one to the left for converting centigrade degrees (Tc) into Fahrenheit degrees (Tf). Here are some more examples.

Problem 1 – To make the data easier to analyze, an image is shifted by X pixels to the right from a stating location of 326. Find the value of X if the new location is 1436 by solving 326 + X = 1436.

Problem 2 – The temperature, T, of a sunspot is 2,000 C degrees cooler than the Sun's surface. If the surface temperature is 6,100 C, solve the equation for the sunspot temperature if T + 2,000 = 6,100.

Problem 3 – The radius, R (in kilometers) of a black hole is given by the formula R = 2.9 M, where M is the mass of the black hole in multiples of the Sun's mass. If an astronomer detects a black hole with a radius of 18.5 kilometers, solve the equation 18.5 = 2.9M for M to find the black hole's mass.

Problem 4 – The sunspot cycle lasts 11 years. If the peak of the cycle occurred in 1858, and 2001 solve the equation 2001 = 1858 + 11X to find the number of cycles, X, that have elapsed between the two years.

Answer Key

1 – To make the data easier to analyze, an image is shifted by X pixels to the right from a stating location of 326. Find the value of X if the new location is 1436 by solving 326 + X = 1436.
Answer: X = 1438 – 326 so X = 1112.

2 – The temperature, T, of a sunspot is 2,000 C degrees cooler than the Sun's surface. If the surface temperature is 6,100 C, solve the equation for the sunspot temperature if T + 2,000 = 6,100.
Answer: T = 6,100 – 2,000 so T = 4,100 C.

3 – The radius, R (in kilometers) of a black hole is given by the formula R = 2.9 M, where M is the mass of the black hole in multiples of the Sun's mass. If an astronomer detects a black hole with a radius of 18.5 kilometers, solve the equation 18.5 = 2.9M for M to find the black hole's mass.
Answer: 18.5 = 2.9M so M = 8.5/2.9 so M = 6.4 times the mass of the sun.

4 – The sunspot cycle lasts 11 years. If the peak of the cycle occurred in 1858, and 2001 solve the equation 2001 = 1858 + 11X to find the number of cycles, X, that have elapsed between the two years.
Answer: 2001-1858 = 11X; 143 = 11X; X = 143/11 so X = 13

ISS - Orbit Altitude Changes

The International Space Station is a 400-ton, $160 billion platform that supports an international team of 3-5 astronauts for tours of duty lasting up to 6 months at a time. Like all satellites that orbit close to Earth, the atmosphere causes the ISS orbit to decay steadily every day, so the ISS has to be 're-boosted' every few months to prevent it from burning up in the atmosphere.

Problem 1 - Based on the following information, what is the altitude of the ISS by April 2009?

"In January, the altitude was 340 kilometers. By March it has lost 8 kilometers before the Progress-59 supply ship raised its altitude by 5 kilometers. In May, the ISS lost 4 1/2 kilometers and was re-boosted by the Progess-60 supply ship by 5 1/2 kilometers. Again the ISS continued to lose altitude by 5 1/2 kilometers by July when the Progress-61 supply ship raised its orbit by 9 1/2 kilometers. The ISS altitude then fell by 3 kilometers by October when the Soyuz TMA-11 mission re-boosted the station by 5 kilometers. The ISS continued to lose altitude until late December, 2007 when it had lost a total of 8 1/2 kilometers since its last re-boost by Soyuz. Since December 2007, the total of all the declines and re-boosts added up to a net change of + 11 1/2 kilometers by April 2009. "

Problem 1 - Based on the following information, what is the altitude of the ISS by April 2009?

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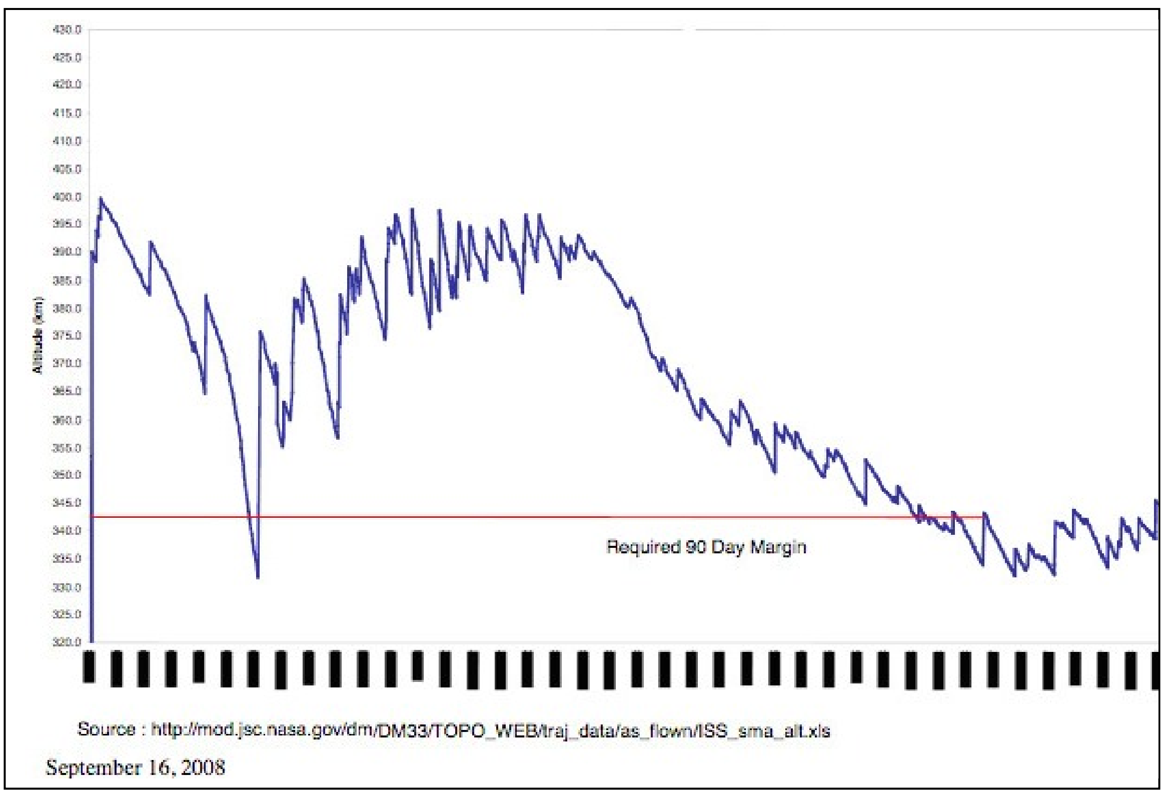
Answer: 340 kilometers - 8 kilometers + 5 kilometers - 4 1/2 kilometers + 5 1/2 kilometers - 5 1/2 kilometers + 9 1/2 kilometers - 3 kilometers + 5 kilometers - 8 1/2 kilometers + 11 1/2 kilometers = 347 kilometers.

Note to Teacher: The figure below shows the altitude changes between November 1998 and July 2008.



Tracking a Sea Turtle from a Satellite

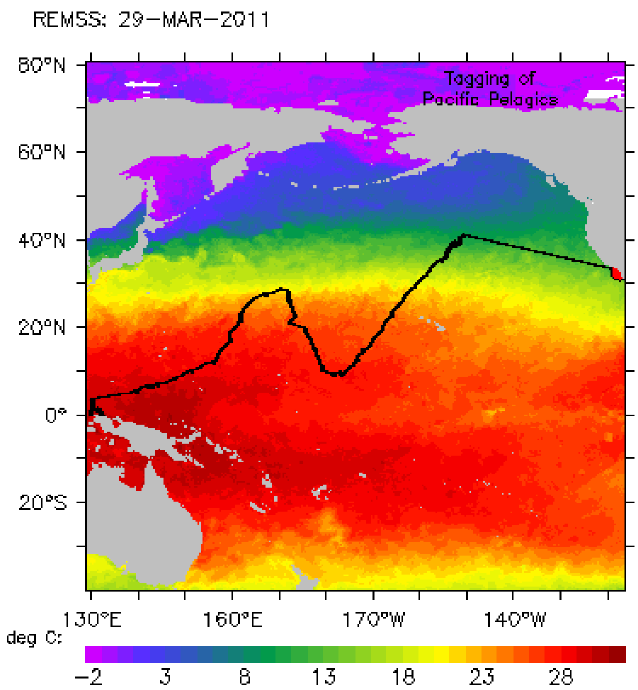
	Day Number	Latitude	Longitude	Distance (km)
1	0	+5° North	130° East	0
2	249	+10° North	155° East	2813
3	488	+30° North	170° East	2707
4	705	+10° North	180° East	2446
5	1099	+40° North	150° West	4453
6	1347	+32° North	120° West	2831

Plot the points listed in the table above to track the sea turtle from the start of its journey to its end. Use your map to answer these questions:

Question 1 - In what part of the world did it begin its trip?
Question 2 - Where did the turtle end up?
Question 3 - How many total kilometers did the turtle travel?

Note to teacher: The data was obtained from the Census of Marine Life's Tagging of Pacific Predators (TOPP) website at http://www.topp.org/

The data is for a Leatherback Sea Turtle Tag Number 2607020, PTT number 23662, between July 22, 2007 to March 30, 2011. The actual data is shown in the image below, which superimposes the track onto a satellite map of the ocean temperature in degrees Celsius (see color bar).
http://las.pfeg.noaa.gov/TOPP_recent/TOPP_tracks180.html?species=0&zone=10



Suggested answers:

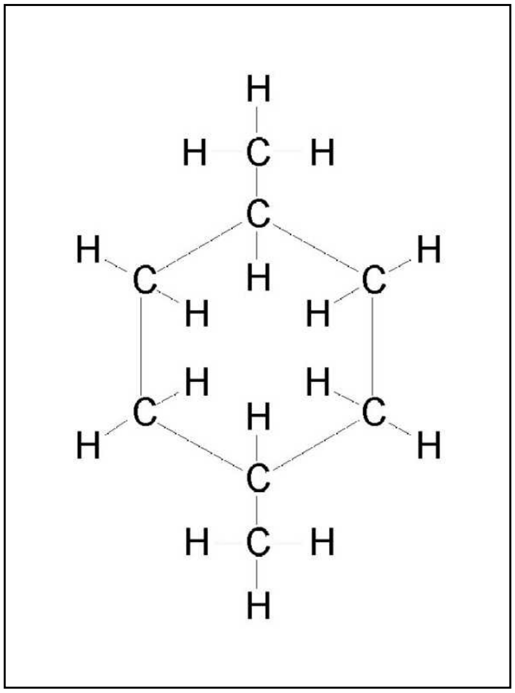
Question 1 - In what part of the world did it begin its trip? - **New Guinea**

Question 2 - Where did it end up? **California**

Question 3 - How many total kilometers did it travel? (Hint: use a piece of string on a globe). 2813 + 2707 + 2446 + 4453 + 2831 = **15,250 kilometers**.

Extension Problem: Students can be asked to estimate the speed of the turtle in kilometers per day (about 11.3 km/d) or other units (miles per hour or miles per year etc..) where 1 km = 0.62 miles.
MPH = 11.3 km/day x (1 day / 24 h) x (1 mile / 0.62 km) = 0.76 mph.
Miles per year = 0.76 mph x (24 h/ 1 day) x (365 days/yr) = 6,658 miles/year.

Parts Per Hundred (pph)



A common way of describing the various components of a population of objects is by the number of parts that they represent for every 100, 1000 or 1 million items that are sampled.

For instance, if there was a bag of 100 balls: 5 were red and 95 were white, you would say that the red balls represented 5 parts per hundred (5 pph) of the sample.

This also means that if you had a bag with 300 balls in the same proportions of red and white balls, the red balls would still be '5 parts per hundred' even though there are now 15 red balls in the sample (15/300 = 5/100 = 5 pph).

For each of the situations below, calculate the parts-per-hundred (pph) for each sample.

Problem 1 - Your current age compared to 1 century

Problem 2 - 10 cubic centimeter (10 cc) of food coloring blended into 1 liter (1000 cc) of water.

Problem 3 - The 4 brightest stars in the Pleiades star cluster, compared to the total population of the cluster consisting of 200 stars.

Problem 4 - One day compared to one month (30 days)

Problem 5 - Five percent of anything.

Problem 6 - The figure above shows the atoms of hydrogen (H) and carbon (C) in the molecule of dimethylcyclohexane. What is the pph of the carbon atoms in this molecule?

Answer Key

Problem 1 - Your current age compared to 1 century

Answer: If your current age is 14 years, then compared to 100 years, your lifetime is 14 pph of a century.

Problem 2 - 1 cubic centimeter (10 cc) of food coloring blended into 1 liter (1000 cc) of water.

Answer: 10 cc / 1000 cc is the same as 1 /100 so the food coloring is 1 pph of a liter.

Problem 3 - The 4 brightest stars in the Pleiades star cluster, compared to the total population of the cluster consisting of 200 stars.

Answer: 4 stars / 200 members = 2 / 100 = 2 pph of the cluster stars.

Problem 4 - One day compared to one month (30 days)

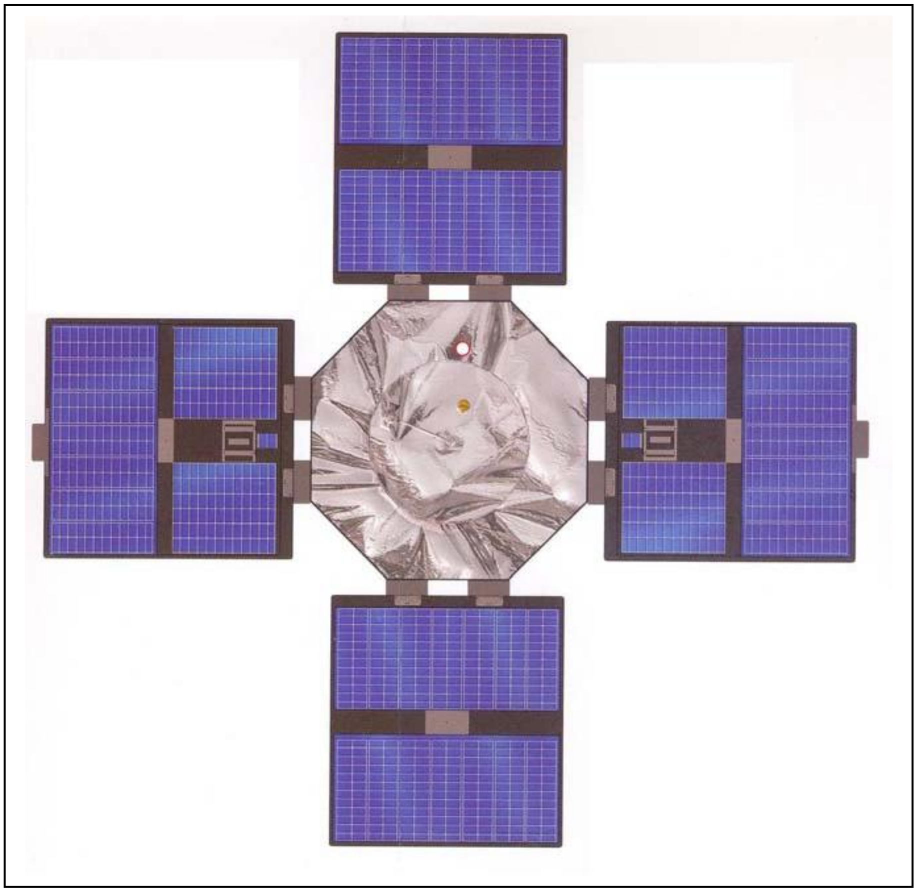
Answer: 1 day / 30 days = 0.0333 so 0.033 x 100 = 3.3 pph of a month.

Problem 5 - Five percent of anything.

Answer: 5 % = 5 / 100 = 5 pph of anything.

Problem 6 - The figure above shows the atoms of hydrogen (H) and carbon (C) in the molecule of dimethylcyclohexane. What is the pph of the carbon atoms in this molecule?

Answer: There area total of 24 atoms in this molecule. There are a total of 8 carbon atoms, so the fraction is 8/24 = 1/3 which equals 0.333 x 100 = 33 pph of the total atoms.



NASA's twin Van Allen Probes spacecraft will be launched in 2012. The figure above shows the octagonal spacecraft body and the location of the surrounding four solar panel 'wings' that provide power to the spacecraft instruments. The small blue rectangles within each of the four solar panels show the location of the solar cells used to power the satellite. As the spacecraft orbits Earth, the four solar panels continuously face the sun to provide constant power.

Problem 1 – Using a millimeter ruler to measure the (silver) octagonal satellite body in the above figure, and the fact that the actual top-to-bottom height of the octagon is 2.0 meters, what is the scale of this figure in centimeters/millimeter?

Problem 2 –What is the total area of the 10 solar cells in square-meters?

Problem 3 – The amount of electrical power generated by a solar panel is 0.0077 watts/cm^2. What is the total power generated by the four solar panels on one Van Allen Probes satellite to the nearest hundred watts?

Problem 1 – Using a millimeter ruler to measure the (silver) octagonal satellite body in the above figure, and the fact that the actual top-to-bottom height of the octagon is 2.0 meters, what is the scale of this figure in centimeters/millimeter?

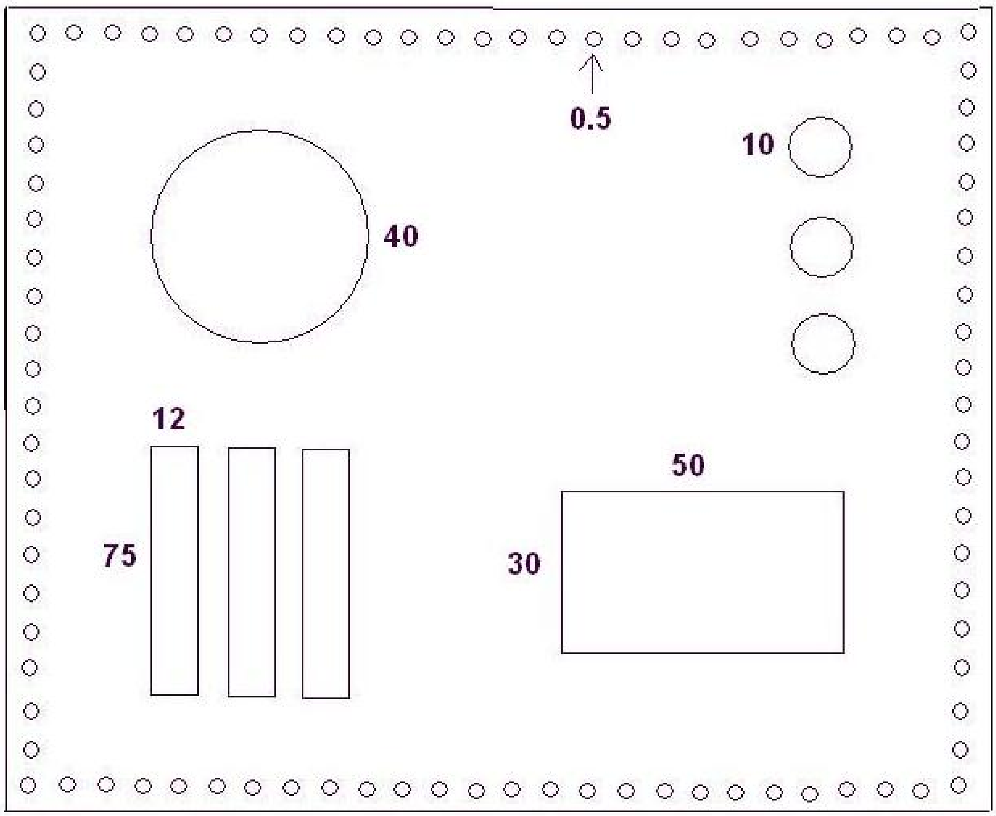
Answer: If you print this problem on a standard '8.5x11' page, the top-to-bottom length is 37 millimeters. This corresponds to 2 meters or 200 cm, so the scale is 200 cm/37mm = **5.4 cm/mm**.

Problem 2 –What is the total area of the 10 solar cells to the nearest tenth of a square-meter?

Answer: The 5 large rectangles have dimensions of 29mm x 13mm, and the 5 small rectangles measure 13mm x 12mm, so their actual dimensions are 157cm x 70 cm, and 70cm x 65cm. The total area is 5(157x70) + 5(70x65) = 77,700 cm^2. Since 1 meter = 100 cm, the area in square-meters is just 77700 cm^2 x (1 m/100 cm)(1m/100cm) = 7.77 $meters^2$, or to the nearest tenth of a square-meter we get **7.8 $meters^2$.**

Problem 3 – The amount of electrical power generated by a solar panel is 0.0077 $watts/cm^2$. What is the total power generated by the four solar panels on one RBSP satellite to the nearest hundred watts?

Answer: In square centimeters, the total area of the solar panels is 78,000 cm^2. The electrical power produced is then P = 0.0077 $watts/cm^2$ x (78000 cm^2) = **600 watts**.



This is a diagram of a panel from a spacecraft showing all of the openings. All units are in centimeters. The panel is 190 centimeters wide and 150 centimeters tall. The diameters of each circular opening is also given. The small holes around the circumference are for the screws that fasten the panels together.

Problem 1 - To the nearest square-centimeter, what is the total area of the panel in square-centimeters before the openings were made?

Problem 2 - To the nearest square-centimeter, what is the total area of all of the openings in the panel? (Use π = 3.1415)

Problem 3 - To the nearest square-centimeter, what is the area of the panel after the openings were made?

Problem 4 - The panel is 1.5 centimeters thick. To the nearest cubic-centimeter, what is the volume of the finished panel?

Problem 5 - To the nearest cubic-centimeter, what is the volume of the material that was removed to make the holes?

Problem 6 - If the density of the aluminum in the panel is 2.7 grams/cm^3, to the nearest tenth of a kilogram, what is the mass of the finished panel?

Problem 7 - To the nearest tenth of a kilogram, how many grams of aluminum were removed to make all of the openings?

Problem 1 - To the nearest square-centimeter, what is the total area of the panel in square-centimeters before the openings were made?

Answer: $A = w \times h$ so $A = 190 \times 150 =$ **28,500 cm^2**

Problem 2 - To the nearest square-centimeter, what is the total area of all of the openings in the panel?

Answer:

$A = (30 \times 50) + 3\,(12 \times 75) + 1\,(3.1415)(40/2)^2 + 3 \times (3.1415)(10/2)^2 + 90x(3.1415)(0.5/2)^2$

$A = 1500 + 2700 + 1256.6000 + 235.6125 + 17.6709$

$A = 5709.8834$

A = 5710 cm^2.

Problem 3 - To the nearest square-centimeter, what is the area of the panel after the openings were made?

Answer: 28,500 - 5710 = **22,790 cm^2**

Problem 4 - The panel is 1.5 centimeters thick. To the nearest cubic-centimeter, what is the volume of the finished panel?
Answer: Volume = Area x Thickness
= 22,790 cm^2 x 1.5 cm
= **34,185 cm^3**

Problem 5 - To the nearest cubic-centimeter, what is the volume of the material that was removed to make the holes?

Answer: Volume = 5710 cm^2 x 1.5 cm = **8,565 cm^3**

Problem 6 - If the density of the aluminum in the panel is 2.7 grams/cm^3, to the nearest tenth of a kilogram, what is the mass of the finished panel?

Answer: Mass = Density x volume
= 2.7 grams/cm^3 x 34,185 cm^3
= 92,299.5 grams
= **92.3 kilograms**

Problem 7 - To the nearest tenth of a kilogram, how many grams of aluminum were removed to make all of the openings?

Answer: Mass = 2.7 grams/cm^3 x 8,565 cm^3
= 23,125.5 grams
= **23.1 kilograms**