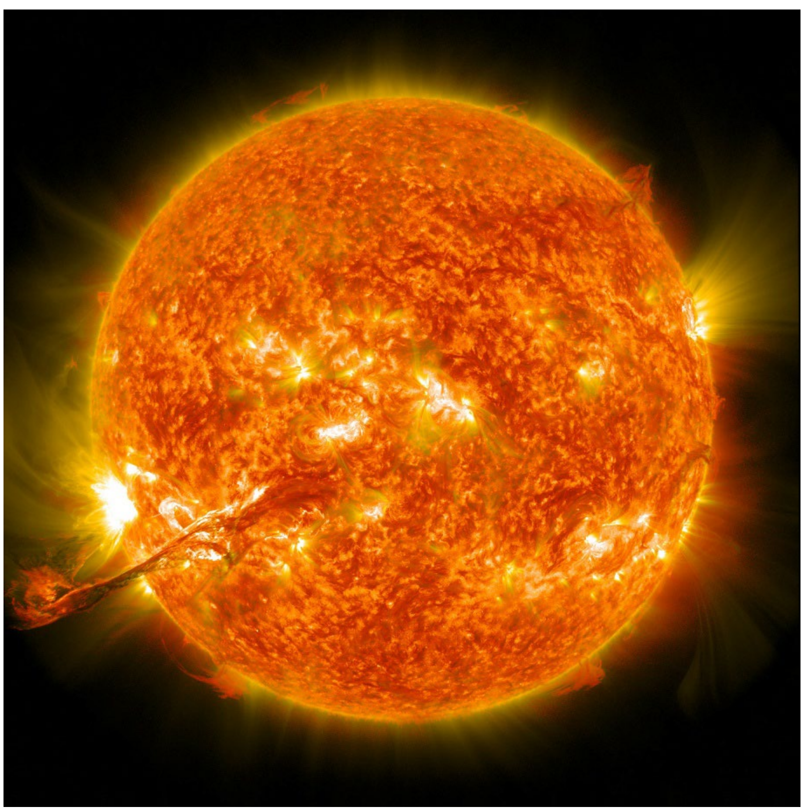
National Aeronautics and Space Administration

Math Problems Related to

Our Sun

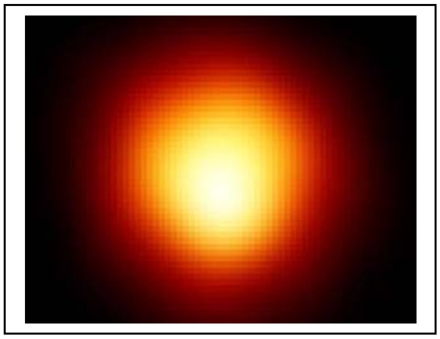


Introduction

This book provides an assortment of mathematics problems that deal with our sun. It is our closest star in the universe so from this vantage point we have learned a lot about how stars 'work'. The problems cover a broad range of topics, with a particular focus on what happens when our sun becomes a stormy star.

Page	Level	Math Skill	Topic
1	Beginning	Ratios	The Relative Sizes of the Sun and Stars-
2	Intermediate	Scale, Ratios, Measurement	Sunspots Close-up and Personal.
3	Intermediate	Scale, Ratios, Conversions	Hinode Sees a Sunspot Up Close.
4	Intermediate	Scale, Rates, Conversions	How fast does the sun rotate?
5	Intermediate	Scale, Ratios, Conversions	What are the mystery lines in this spectrum?
6	Intermediate	Arithmetic, Scale, Ratios	How big can sunspots get?
7	Intermediate	Scale, Ratios, Rates	Measuring the Speed of an Eruptive Prominence.
8	Intermediate	Scale, Ratios	Details on the Surface of the Sun Revealed by the SDO.
9	Intermediate	Percentage, Geometry	Changing Perspectives on the Sun's Diameter.
10	Intermediate	Geometry, Equations	The STEREO Spacecraft give a 360-degree View of the Sun.
11	Intermediate	Conversions	Energy at Home
12	Intermediate	Conversions	Exploring Temperature Change in Earth's Outer Crust
13	Intermediate	Data Analysis	Super-sized Sunspots and the Solar Cycle.
14	Advanced	Algebra I	Global Warming and the Suns Evolving Luminosity.
15	Advanced	Scientific Notation, Formulas	A Mathematical Model of the Sun.
16	Advanced	Calculus	The Internal Density and Mass of the Sun.
17	Advanced	Trigonometry	The Parker Solar Probe: Working with angular diameter.
18	Advanced	Equations	Exploring Energy and Temperature
19	Advanced	Rates	Exploring Temperature and States of Matter

The relative sizes of the sun and stars



Stars come in many sizes, but their true appearances are impossible to see without special telescopes. The image to the left was taken by the Hubble Space telescope and resolves the red supergiant star Betelgeuse so that its surface can be just barely seen. Follow the number clues below to compare the sizes of some other familiar stars!

Problem 1 - The sun's diameter if 10 times the diameter of Jupiter. If Jupiter is 11 times larger than Earth, how much larger than Earth is the Sun?

Problem 2 - Capella is three times larger than Regulus, and Regulus is twice as large as Sirius. How much larger is Capella than Sirius?

Problem 3 - Vega is 3/2 the size of Sirius, and Sirius is 1/12 the size of Polaris. How much larger is Polaris than Vega?

Problem 4 - Nunki is 1/10 the size of Rigel, and Rigel is 1/5 the size of Deneb. How large is Nunki compared to Deneb?

Problem 5 - Deneb is 1/8 the size of VY Canis Majoris, and VY Canis Majoris is 504 times the size of Regulus. How large is Deneb compared to Regulus?

Problem 6 - Aldebaran is 3 times the size of Capella, and Capella is twice the size of Polaris. How large is Aldebaran compared to Polaris?

Problem 7 - Antares is half the size of Mu Cephi. If Mu Cephi is 28 times as large as Rigel, and Rigel is 50 times as large as Alpha Centauri, how large is Antares compared to Alpha Centauri?

Problem 8 - The Sun is 1/4 the diameter of Regulus. How large is VY Canis Majoris compared to the Sun?

Inquiry: - Can you use the information and answers above to create a scale model drawing of the relative sizes of these stars compared to our Sun.

Answer Key

The relative sizes of some popular stars is given below, with the diameter of the sun = 1 and this corresponds to an actual physical diameter of 1.4 million kilometers.

Betelgeuse	440	Nunki	5	VY CMa	2016	Delta Bootis	11
Regulus	4	Alpha Cen	1	Rigel	50	Schedar	24
Sirius	2	Antares	700	Aldebaran	36	Capella	12
Vega	3	Mu Cephi	1400	Polaris	24	Deneb	252

Problem 1 - Sun/Jupiter = 10, Jupiter/Earth = 11 so Sun/Earth = 10 x 11 = **110 times.**

Problem 2 - Capella/ Regulus = 3.0, Regulus/Sirius = 2.0 so Capella/Sirius = 3 x 2 = **6 times.**

Problem 3 - Vega/Sirius = 3/2 Sirius/Polaris=1/12 so Vega/Polaris = 3/2 x 1/12 = **1/8 times**

Problem 4 - Nunki/Rigel = 1/10 Rigel/Deneb = 1/5 so Nunki/Deneb = 1/10 x 1/5 = **1/50.**

Problem 5 - Deneb/VY = 1/8 and VY/Regulus = 504 so Deneb/Regulus = 1/8 x 504 = **63 times**

Problem 6 - Aldebaran/Capella = 3 Capella/Polaris = 2 so Aldebaran/Polaris = 3 x 2 **= 6 times**.

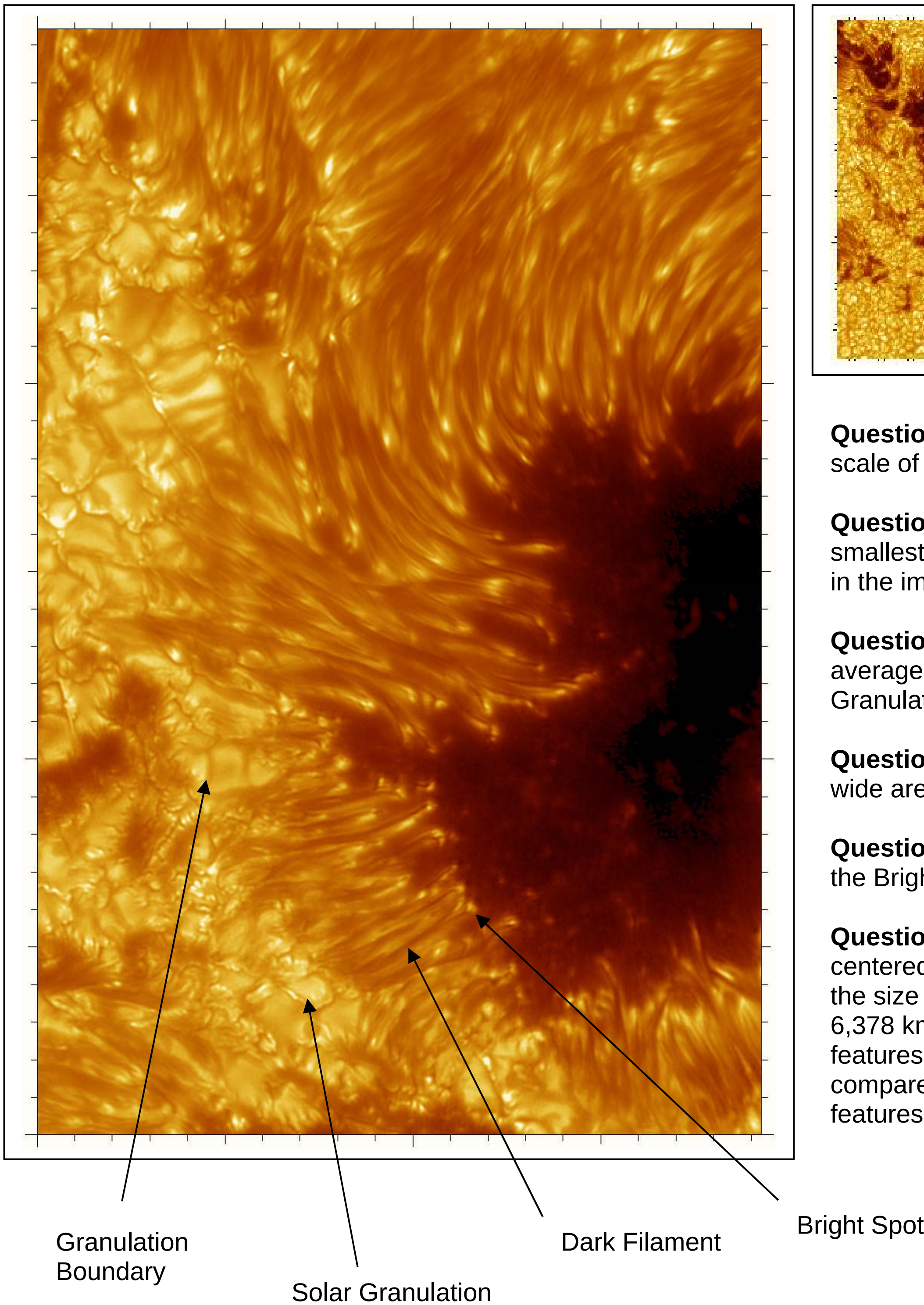
Problem 7 - Antares/Mu Cep = 1/2 Mu Cep/Rigel = 28 Rigel/Alpha Can = 50, then Antares/Alpha Can = 1/2 x 28 x 50 = **700 times**.

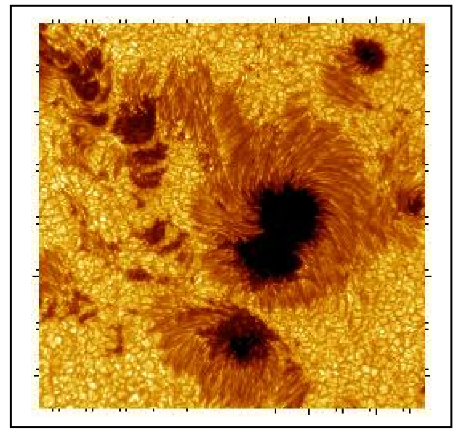
Problem 8 - Regulus/Sun = 4 but VY CMA/Regulus = 504 so VY Canis Majoris/Sun = 504 x 4 = **2016 times the sun's size!**

Inquiry: Students will use a compass and millimeter scale. If the diameter of the Sun is 1 millimeter, the diameter of the largest star VY Canis Majoris will be 2016 millimeters or about 2 meters!

Solar Surface Details

The sun is our nearest star. From Earth we can see its surface in great detail. The images below were taken with the 1-meter Swedish Vacuum Telescope on the island of La Palma, by astronomers at the Royal Swedish Academy of Sciences (http://www.astro.su.se/groups/solar/solar.html). The image to the right is a view of sunspots on July 15, 2002. The enlarged view to the left shows never-before seen details near the edge of the largest spot. Use a millimeter ruler, and the fact that the dimensions of the left image are 19,300 km x 29,500 km, to determine the scale of the photograph, and then answer the questions. See the arrows below to identify the various solar features mentioned in the questions.





Question 1 - What is the scale of the image in km/mm?

Question 2 – What is the smallest feature you can see in the image?

Question 3 – What is the average size of a Solar Granulation region?

Question 4 – How long and wide are the Dark Filaments?

Question 5 – How large are the Bright Spots?

Question 6 – Draw a circle centered on this picture that is the size of Earth (radius = 6,378 km). How big are the features you measured compared to familiar Earth features?

Question 1 - What is the scale of the image in km/mm? **Answer:** the image is about 108mm x 164mm so the scale is 19300/108 = 179 km/mm.

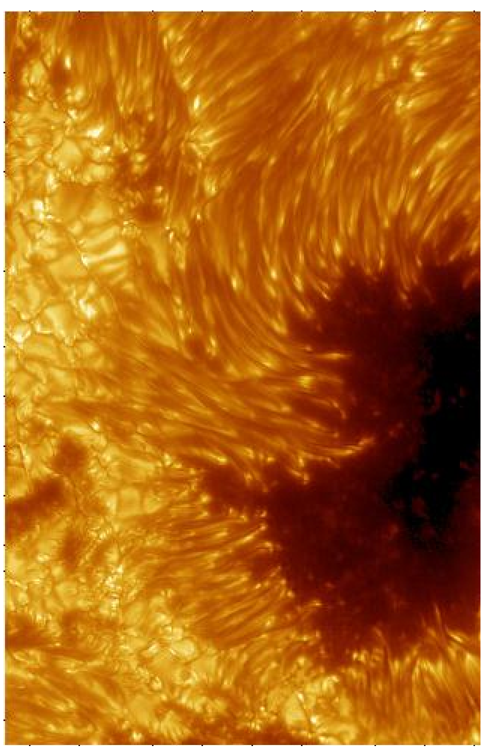
Question 2 – What is the smallest feature you can see in the image? **Answer:** Students should be able to find features, such as the Granulation Boundaries, that are only 0.5 mm across, or 0.5 x 179 = 90 km across.

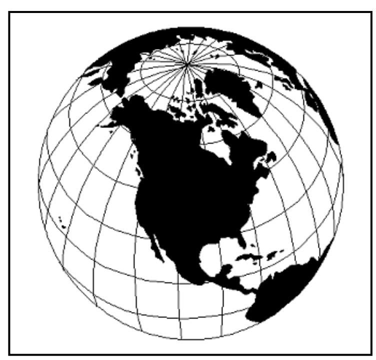
Question 3 – What is the average size of a Solar Granulation region? **Answer:** Students should measure several of the granulation regions. They are easier to see if you hold the image at arms length. Typical sizes are about 5 mm so that 5 x 179 is about 900 km across.

Question 4 – How long and wide are the Dark Filaments? **Answer:** Students should average together several measurements. Typical dimensions will be about 20mm x 2mm or 3,600 km long and about 360 km wide.

Question 5 – How large are the Bright Spots? **Answer:** Students should average several measurements and obtain values near 1 mm, for a size of about 180 km across.

Question 6 – Draw a circle centered on this picture that is the size of Earth (radius = 6,378 km). How big are the features you measured compared to familiar Earth features? **Answer:** See below.



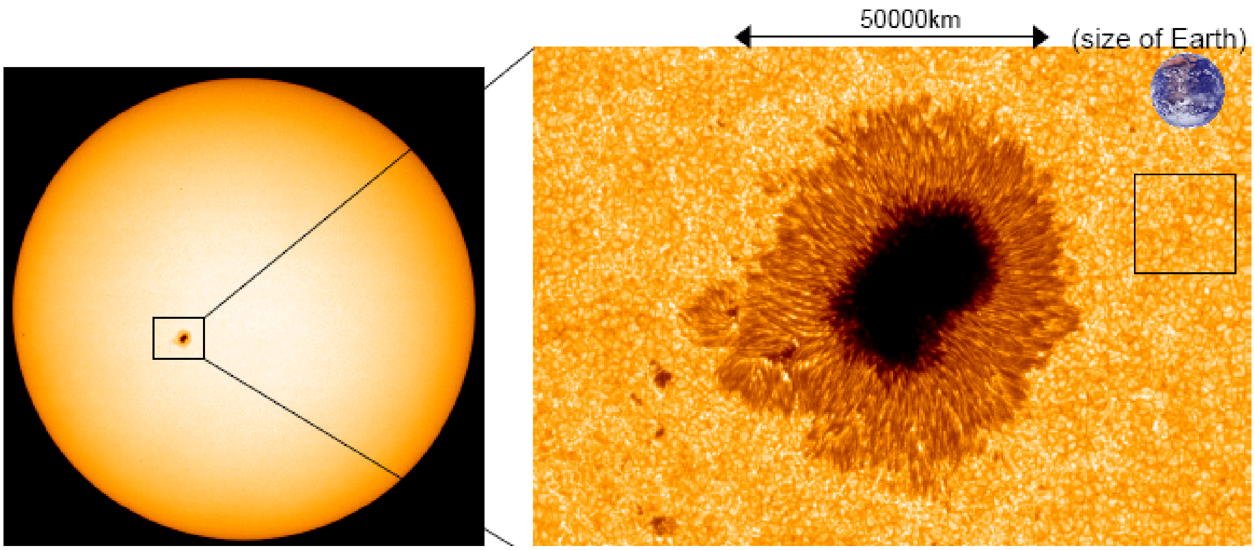


Granulation Region – Size of a large US state.

Bright Spot – Size of a small US state or Hawaii

Filament – As long as the USA, and as narrow as Baja California or Florida.

Hinode - Close-up of a Sunspot



After a successful launch on September 22, 2006 the Hinode solar observatory caught a glimpse of a large sunspot on November 4, 2006. An instrument called the Solar Optical Telescope (SOT) captured this image, showing sunspot details on the solar surface.

Problem 1 - Based on the distance between the arrow points, what is the scale of the image on the right in units of kilometers per millimeter?

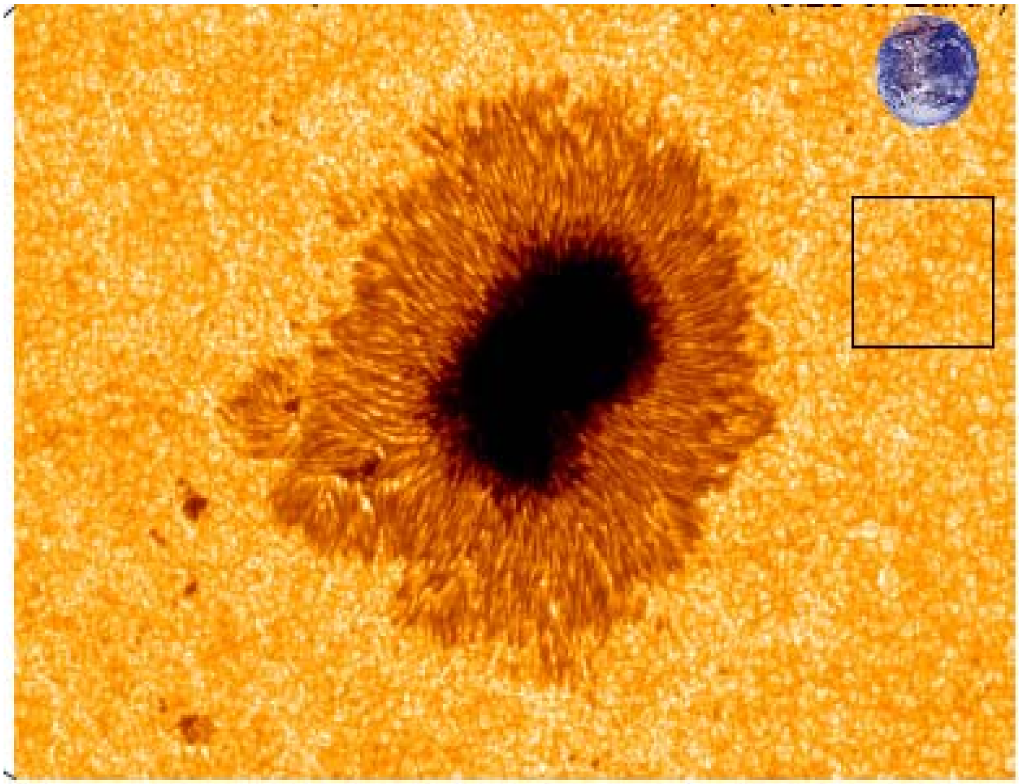
Problem 2 - What is the size of the smallest detail you can see in the image?

Problem 3 - Compared to familiar things on the surface of Earth, how big would the smallest feature in the solar image be?

Problem 4 - The gold-colored textured surface is the photosphere of the sun. The texturing is produced by heated gas that is flowing up to the surface from the hot interior of the sun. The convecting gases form cells, called granulations, at the surface, with upwelling gas flowing from the center of each cell, outwards to the cell boundary, where it cools and flows back down to deeper layers. What is the average size of a granulation cell within the square?

Problem 5 - Measure several granulation cells at different distances from the sunspot, and plot the average size you get versus distance from the spot center. Do granulation cells have about the same size near the sunspot, or do they tend to become larger or smaller as you approach the sunspot?

Answer Key:



Problem 1 - From the 40 millimeter length of the 50,000 km arrow marker, the scale of the image is 50,000 km/40 mm = 1250 kilometers per millimeter

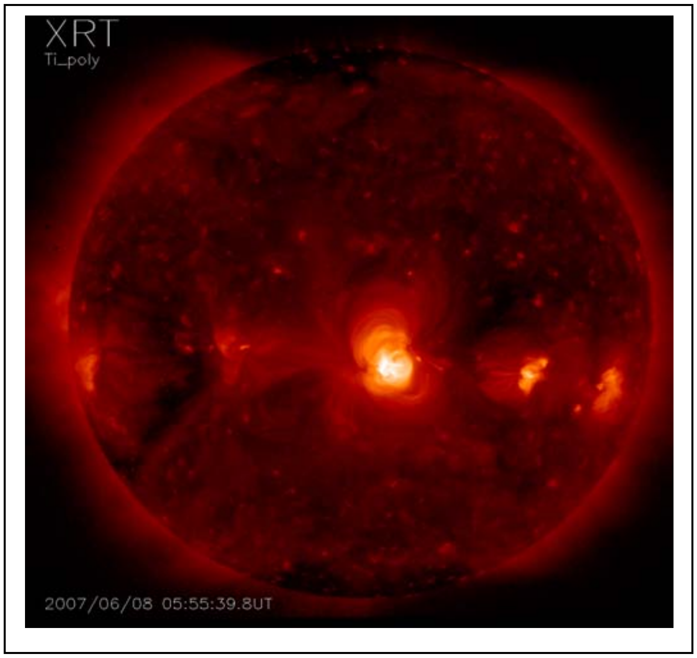
Problem 2 - Depending on the copy quality, the smallest detail is about 0.5 millimeters or 0.5 x 1250 = 625 kilometers across but details that are 1 or 2 mm across are also acceptable.

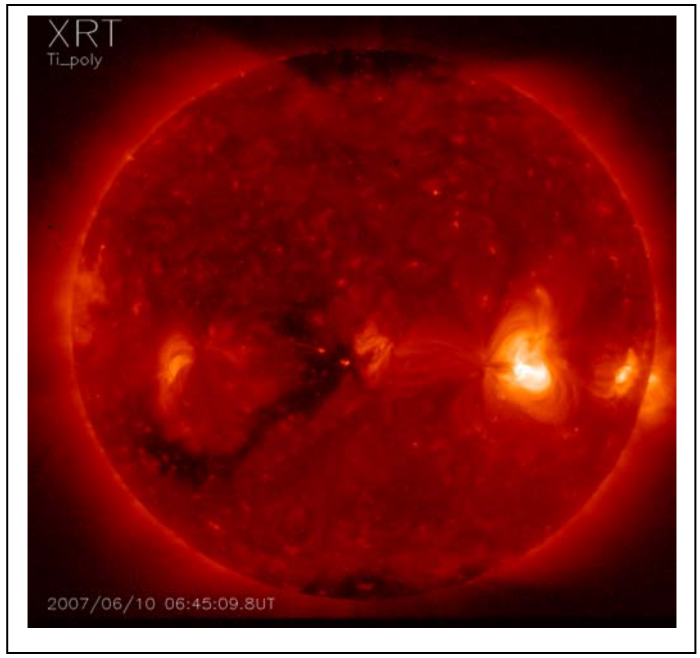
Problem 3 - Similar features on Earth would be continents like Greenland (1,800 km) or England (700 km).

Problem 4 - Measure about 5 cells to get: 1.5 mm, 1.0 mm, 0.8mm, 1.2mm and 1.4 mm. The average is about 1.2 mm, so the average size is (1.2) x 1250km = 1,500 km.

Problem 5 - Students should measure about 5 granulation cells in three groups; Group 1 should be far from the center of the spot. Group 3 should be as close to the outer, tan-colored, 'penumbra' of the spot as possible, and Group 2 should be about half-way in between Group 1 and 3. The average granulation sizes do not change significantly.

How fast does the sun spin?





The sun, like many other celestial bodies, spins around on an axis that passes through its center. The rotation of the sun, together with the turbulent motion of the sun's outer surface, work together to create magnetic forces. These forces give rise to sunspots, prominences, solar flares and ejections of matter from the solar surface.

Astronomers can study the rotation of stars in the sky by using an instrument called a spectroscope. What they have discovered is that the speed of a star's rotation depends on its age and its mass. Young stars rotate faster than old stars, and massive stars tend to rotate faster than low-mass stars. Large stars like supergiants, rotate hardly at all because they are so enormous they reach almost to the orbit of Jupiter. On the other hand, very compact neutron stars rotate 30 times each second and are only 40 kilometers across.

The X-ray telescope on the Hinode satellite creates movies of the rotating sun, and makes it easy to see this motion. A sequence of these images is shown on the left taken on June 8, 2007 (Left); June 10 2007 (Right) at around 06:00 UT.

Although the sun is a sphere, it appears as a flat disk in these pictures when in fact the near-side surface of the sun is bulging out of the page at you! We are going to neglect this distortion and estimate how many days it takes the sun to spin once around on its axis.

The radius of the sun is 696,000 kilometers.

Problem 1 - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

Problem 2 – About how many days does it take to rotate once at the equator?

Inquiry Question: What geometric factor produces the largest uncertainty in your estimate, and can you come up with a method to minimize it to get a more accurate rotation period?

Answer Key:

Problem 1 - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

First, from the diameter of the sun's disk, calculate the image scale of each picture in kilometers per millimeter.

Diameter= 76 mm. so radius = 38 mm. Scale = (696,000 km)/38 mm = 18,400 km/mm

Then, find the center of the sun disk, and using this as a reference, place the millimeter ruler parallel to the sun's equator, measure the distance to the very bright 'active region' to the right of the center in each picture. Convert the millimeter measure into kilometers using the image scale.

Picture 1: June 8 distance = 4 mm d = 4 mm (18,400 km/mm) = 74,000 km
Picture 2; June 10 distance = 22 mm d = 22 mm (18,400 km/mm) = 404,000 km

Calculate the average distance traveled between June 8 and June 10.

Distance = (404,000 - 74,000) = 330,000 km

Divide this distance by the number of elapsed days (2 days)…………….. 165,000 km/day

Convert this to kilometers per hour………………………………………….. 6,875 km/hour

Convert this to kilometers per second……………………………………….. 1.9 km/sec

Convert this to miles per hour ……………………………………………… 4,400 miles/hour

Problem 2 – About how many days does it take to rotate once at the equator?

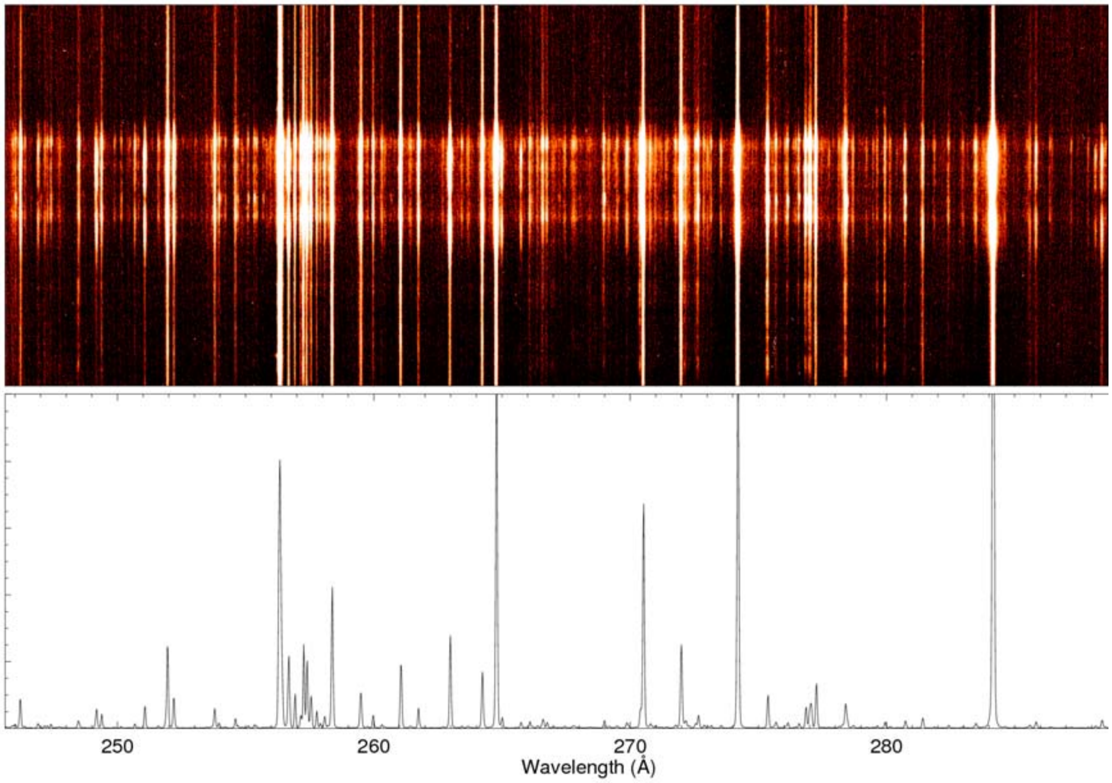
The circumference of the sun is 2 π (696,000 km) = 4,400,000 kilometers.

The equatorial speed is 66,000 km/day so the number of days equals 4,400,000/165,000 = **26.6 days.**

Inquiry Question:
Because the sun is a sphere, measuring the distance of the spot from the center of the sun on June 10 gives a distorted linear measure due to foreshortening.

The sun has rotated about 20 degrees during the 2 days, so that means a full rotation would take about (365/20) x 2 days = 36.5 days which is closer to the equatorial speed of the sun of 35 days.

Find the mystery lines...if you can!



This is an image (top) from the Hinode satellite's Extreme Ultraviolet Spectrometer (EIS). The figure (below) is a series of atomic spectral lines from known ions. Each line in the bottom graph has an intensity that is indicated by its length along the vertical axis of the figure. The table to the right gives the wavelengths of some spectral lines that fall within the wavelength range of the figure.

Table 1: List of ionic lines

He II	256.32 A
Fe XVI	262.98
S X	264.23
Fe XIV	264.79
Si VII	275.35
Fe XV	284.16

Note: Wavelengths in Angstrom (A) units.
1 A = 10^{-8} cm.

Problem 1) What is the scale of the horizontal axis in Angstroms/millimeter?

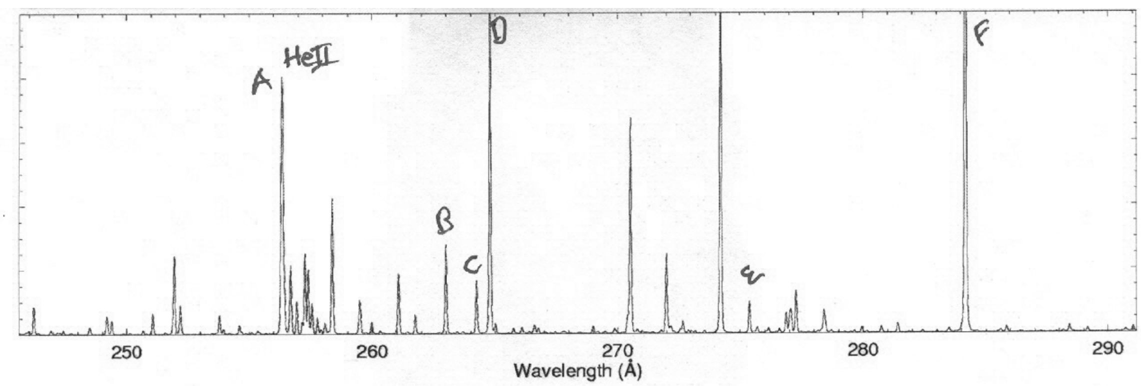
Problem 2) Using your answer to Problem 1, A) Match up the tabulated lines with the lines shown in the above figure. *(Hint: Use the '250' mark and calculate the number of millimeters to the He II line at 256.32 A (21.8 mm) and convert to Angstrom units using your answer to Problem 1. Then add this to '250' to get the wavelength of the He II line.)*

Problem 3) Compare the identified lines in the figure, with the solar spectrum from the EIS instrument in the top illustration. Which lines can you match up?

Problem 4) What percentage of lines in the Hinode solar data are not identified?

Inquiry problem: Can you find any resources online that help you identify some of the missing lines?

Answer Key:



Problem 1) What is the scale of the horizontal axis in Angstroms/millimeter?

Answer: (290 - 250) A/138 millimeters = 0.29 Angstroms / millimeter.

Problem 2) Using your answer to Problem 1, A) Match up the tabulated lines with the lines shown in the above figure.

A	He II	(256.32 - 250) / 0.29 = 21.8 mm
B	Fe XII	(262.98 - 250)/0.29 = 44.8 mm
C	S X	(264.23-250)/0.29 = 49.0 mm
D	Fe XIV	(264.79 - 250) / 0.29 = 51.0 mm
E	Si VII	(275.35 - 250) / 0.29 = 87.4 mm
F	Fe XV	(284.16 - 250) / 0.29 = 117.8 mm

Problem 3) Compare the identified lines in the figure, with the solar spectrum from the EIS instrument in the top illustration. Which lines can you match up?

Answer: All of the known lines in the table can be matched up as shown in the above diagram.

Problem 4) What percentage of lines in the Hinode solar data are not identified?

Answer: There are about 127 lines in the top Hinode spectrum, but only 6 lines are known from the table, so (121/127) x 100 = 95% of the lines in the Hinode spectrum are unknown.

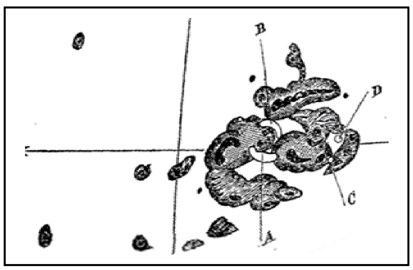
Inquiry problem: Can you find any resources online that help you identify some of the missing lines?

Monster Sunspots!

Sunspots have been observed for thousands of years because, from time to time, the sun produces spots that are so large they can be seen from Earth with the naked eye…with the proper protection. Ancient observers would look at the sun near sunrise or sunset when Earth's atmosphere provided enough shielding to very briefly look at the sun for a few minutes. Astronomers keep track of these large 'super spots' because they often produce violent solar storms as their magnetic fields become tangled up into complex shapes.

Below are sketches and photographs of some large sunspots that have been observed during the last 150 years. They have been reproduced at scales that make it easy to study their details, but do not show how big they are compared to each other.

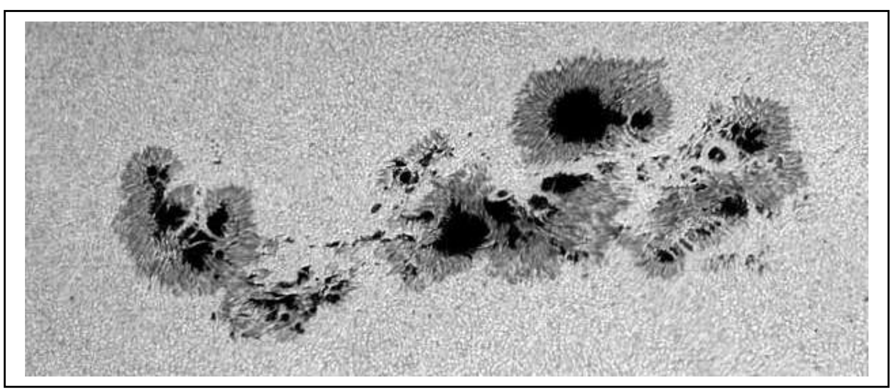
Problem: By using a millimeter ruler, use the indicated scales for each image to compute the physical sizes of the three sunspots in kilometers. Can you sort them by their true physical size?

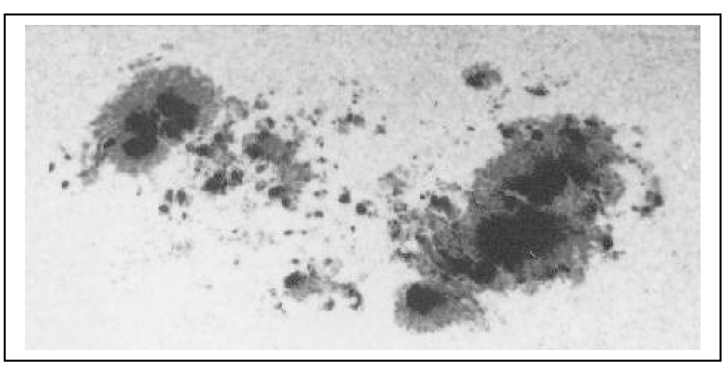


Top is the sunspot drawn by Richard Carrington on August 28, 1859 at a scale of 5,700 kilometers/mm.

Middle is a photograph of a sunspot seen on March 29, 2000 at a scale of 23,500 kilometers/cm.

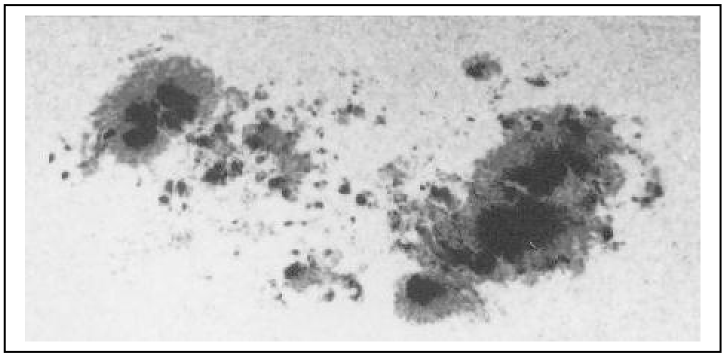
Bottom is a sunspot seen on April 8, 1947 at a scale of 100,000 kilometers/inch



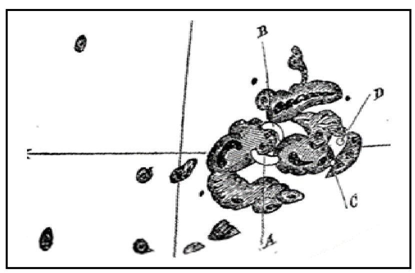


Answer Key:

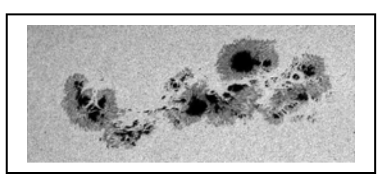
Images ordered from largest to smallest and to scale:



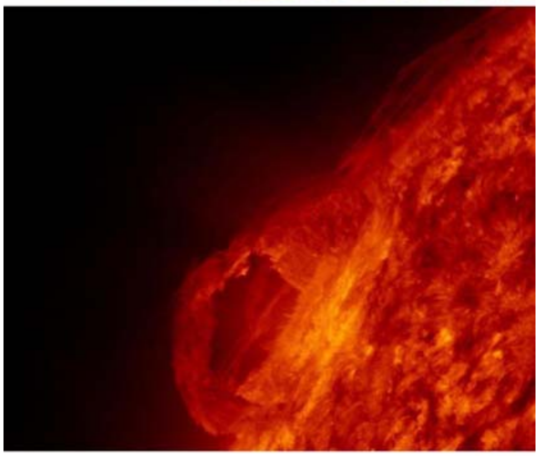
Sunspot seen on April 8, 1947 reproduced at a scale of 100,000 kilometers/inch. The linear extent on the page is 7 centimeters, so the length in inches is 7 / 2.5 = 2.8 inches. The true length is then 2.8 x 100,000 = <u>280,000 kilometers.</u>

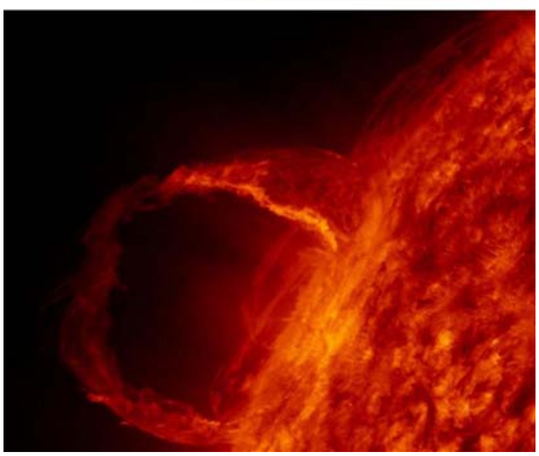


The sunspot drawn by Richard Carrington on August 28, 1859 at a scale of 5,700 kilometers/mm. With a ruler, the distance from the left to the right of the group is about 40 millimeters, so the true length is about 40 x 5,700 = <u>228,000 kilometers.</u>



A photograph of a sunspot seen on March 29, 2000 at a scale of 23,500 kilometers/cm. The length of the spot is 90 millimeters or 9 centimeters. The true length is then 211,500 kilometers.





On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. Among them was a sequence of images taken on March 30, showing an eruptive prominence ejecting millions of tons of plasma into space. The three images to the left show selected scenes from the first 'high definition' movie of this event. The top image was taken at 17:50:49, the middle image at 18:02:09 and the bottom image at 18:13:29.

Problem 1 – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Problem 2 – If the Earth were represented by a disk the size of a penny (10 millimeters), on this same scale how big was the loop of the eruptive prominence in the bottom image if the radius of Earth is 6,378 kilometers?

Problem 3 – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

For additional views of this prominence, see the NASA/SDO movies at:

http://svs.gsfc.nasa.gov/vis/a000000/a003600/a003693/index.html

or to read the Press Release:

http://www.nasa.gov/mission_pages/sdo/news/first-light.html

Problem 1 – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The width is 70 millimeters so the scale is 300,000 km/70 mm **= 4,300 km/mm**

Problem 2 – If the Earth were represented by a disk the size of a penny (10 millimeters), on this same scale how big was the loop of the eruptive prominence in the bottom image if the radius of Earth is 6,378 kilometers?

Answer: **The diameter of the loop is about 35 millimeters or 35mm x 4300km/mm = 150,000 km. The diameter of Earth is 13,000 km, so the loop is 12 times the diameter of Earth. At the scale of the penny, 13 penny/Earth's can fit across a scaled drawing of the loop.**

Problem 3 – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

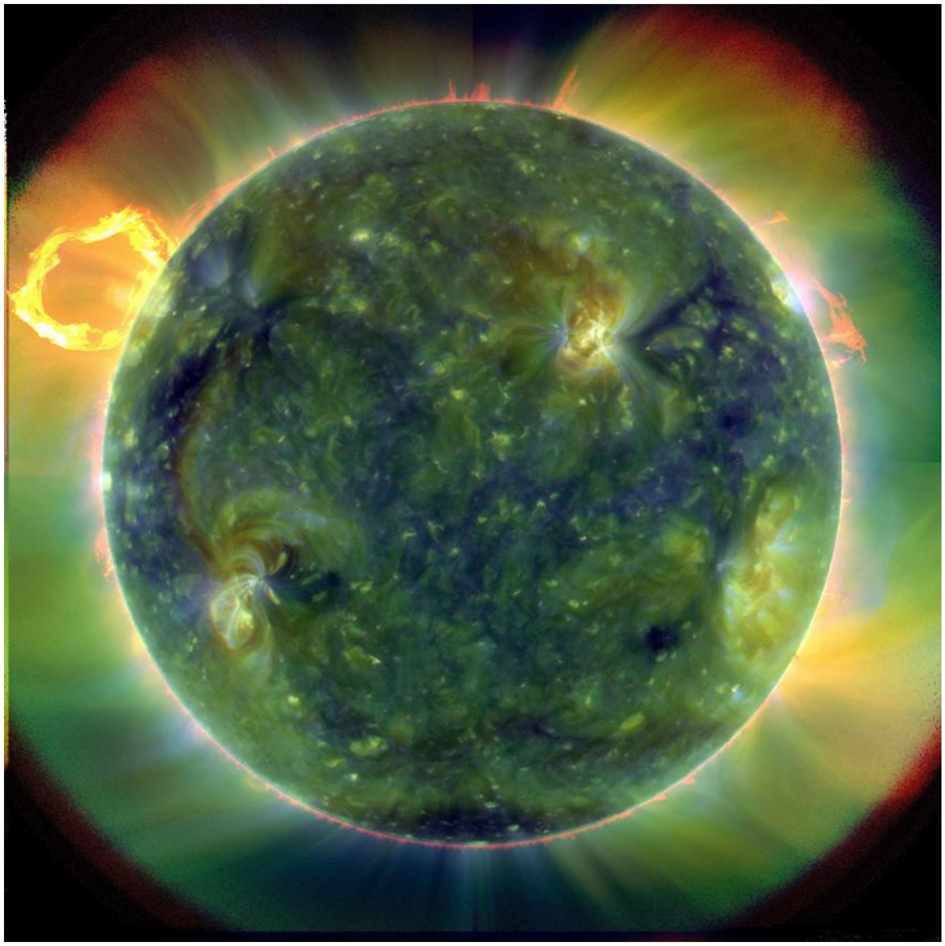
Answer: Speed = distance traveled / time elapsed.
In the bottom image, draw a straight line from the lower right corner THROUGH the peak of the coronal loop. Now draw this same line at the same angle on the other two images. With a millimeter ruler, measure the distance along the line from the lower right corner to the edge of the loop along the line. Example:
Top: 47 mm ;
Middle: 52 mm,
Bottom: 67 mm.
The loop has moved 67mm – 47mm = 20 millimeters. At the scale of the image this equals 20mm x 4,300 km/mm so D = 86,000 km.

The time between the bottom and top images is 18:13:29 – 17:50:49 or 22minutes and 40 seconds or 1360 seconds.

A) The average speed of the loop is then S = 86,000 km/1360 sec = **63 km/sec**.

B) 63 km/sec x 3600 sec/hr = **227,000 km/hour**.

C) 227,000 km/hr x 0.62 miles/km = **140,000 miles/hour**.



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. The image above shows a full-disk, multi-wavelength, extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Black indicates very low temperatures near 10,000 K close to the solar surface (photosphere). Reds are relatively cool plasma heated to 60,000 Kelvin (100,000 F); blues, greens and white are hotter plasma with temperatures greater than 1 million Kelvin (2,000,000 F).

Problem 1 – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Problem 2 – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

Problem 3 – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Problem 1 – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The diameter of the Sun is 98 millimeters, so the scale is 1,380,000 km/98 mm = **14,000 km/mm**.

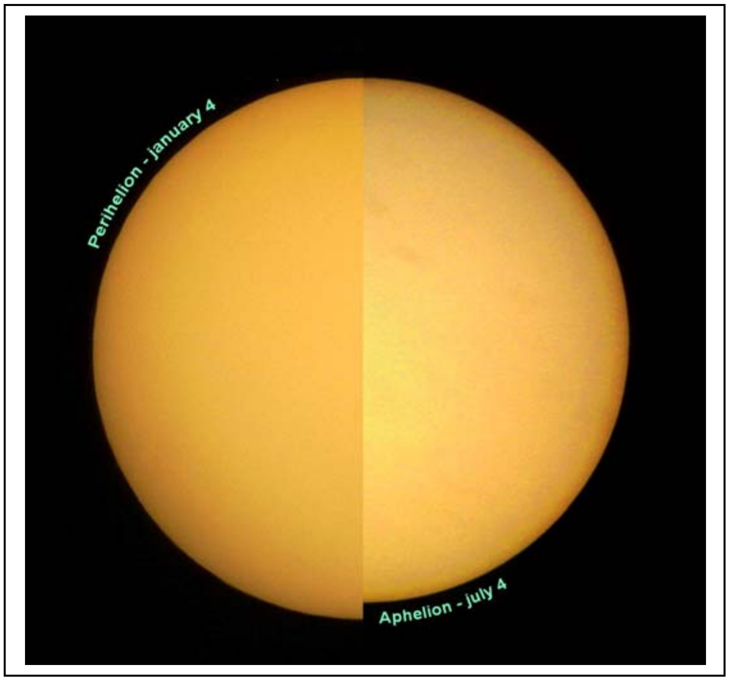
Problem 2 – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

Answer: **Students should see numerous bright points freckling the surface, the smallest of these are about 0.5 mm across or 7,000 km. This is slightly larger than ½ the diameter of Earth.**

Problem 3 – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Answer: **There are large irregular blotches all across the disk of the sun that are dark blue-black. These are regions where thee is little of the hot coronal gas and only the 'cold' photosphere can be seen. The hottest gas seems to reside in the corona, and in the very small point-like 'microflare' regions that are generally no larger than the size of Earth**.

Note: Microflares were first observed, clearly, by the Hinode satellite between 2007-2009. Some solar physicists believe that these microflares, which erupt violently, are ejecting hot plasma that eventually ends up in the corona to replenish it. Because the corona never disappears, these microflares happen all the time no matter what part of the sunspot cycle is occurring.



Earth's orbit is not a perfect circle centered on the sun, but an ellipse! Because of this, in January, Earth is slightly closer to the sun than in July. This means that the sun will actually appear to have a bigger disk in the sky in January than in July…but the difference is impossible to see with the eye, even if you could do so safely!

The figure above shows the sun's disk taken by the SOHO satellite. The left side shows the disk on January 4 and the right side shows the disk on July 4, 2009. As you can see, the diameter of the sun appears to change slightly between these two months.

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure?

Problem 2 - By what percentage did the diameter of the Sun change between January and July compared to its average diameter?

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in July compared to January?

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure?

Answer: Using a millimeter ruler, and measuring vertically along the join between the two images, the left-hand, January, image is 72 millimeters in diameter, while the right-hand image is 69 millimeters in diameter. The average of these two is (72 + 69)/2 = **70.5 millimeters.**

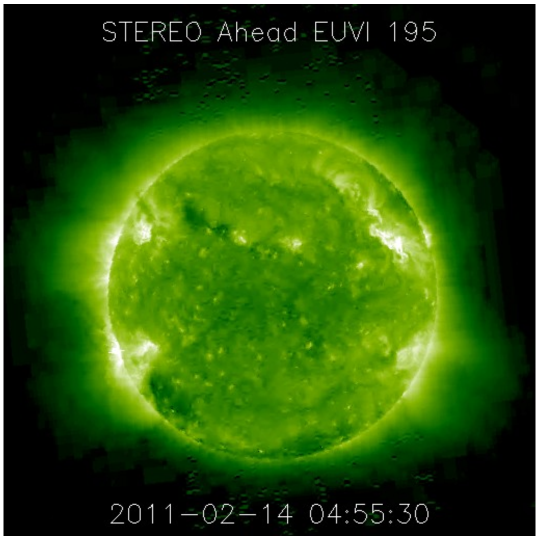
Problem 2 - By what percentage did the diameter of the Sun change between January and July compared to its average diameter?

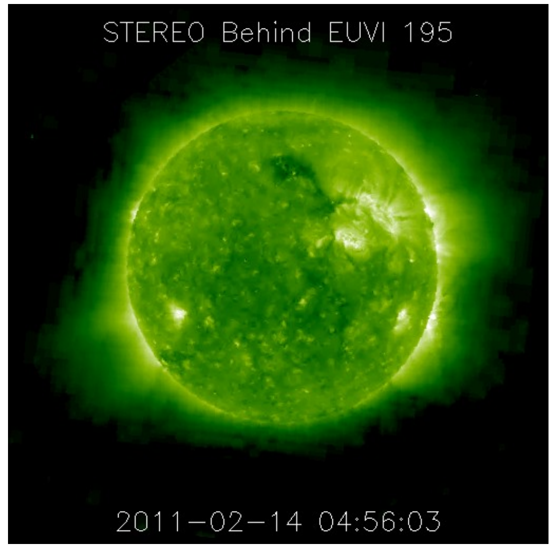
Answer: In January the moon was larger then the average diameter by 100% x (72 - 70.5)/70.5 = **2.1 %.** In July it was smaller then the average diameter by 100% x (70.5 - 69)/70.5 = **2.1%.**

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in July compared to January?

Answer: The diameter of the sun appeared to change by 2.1% + 2.1% = 4.2% between January and July. Because the apparent size of an object is inversely related to its distance (i.e. the farther away it is the smaller it appears), this 4.2% change in apparent size occurred because of a 4.2% change in the distance between Earth and the Sun, so since 0.042 x 149,600,000 km = 6,280,000 kilometers, the change in the Sun's apparent diameter reflects the 6,280,000 kilometer change in earth's distance between January and July. The Earth is **6,280,000 kilometers closer to the Sun in July than in January.**

STEREO Spacecraft give 360^o Solar View





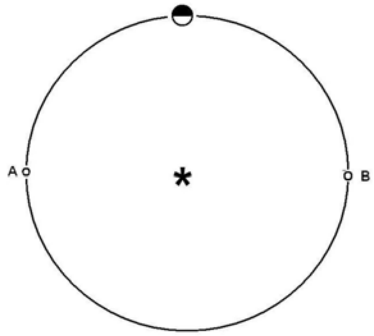
This remarkable pair of images of the sun was taken on February 14, 2011 by NASA's STEREO A (left) and B (right) spacecraft. The spacecraft are close to Earth's orbit, however as viewed looking down on Earth's orbit with the sun at the center, STEREO-A is 90-degrees counter-clockwise of Earth's orbit location, and STEREO-B is 90-degrees clockwise. This means that when the two images above are combined, the entire 360-degree span of the solar surface can be seen at the same time, making this an historical moment for Humanity.

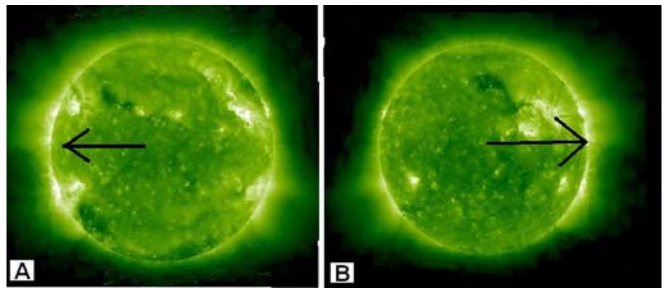
Problem 1 – From the information in the text, draw a diagram that shows the location of the sun, Earth's orbital path (assume it is a circle, whose plane passes through the equator of the sun) and the locations of the STEREO spacecraft. In each of the two images, draw an arrow that points in the direction of Earth.

Problem 2 – The images were taken in the light of iron atoms heated to 1.1 million degrees. The bright (white) features are solar 'active regions' that correspond to the locations of sunspots. At the time the images were taken, what active regions in the above images: A) Could be observed from Earth? B) Could not be observed from Earth?

Problem 3 – Relative to the location of Earth in its orbit, the STEREO-A and B spacecraft move 22 degrees farther from Earth each year. On your diagram from Problem 1, about what will be the positions of the spacecraft along Earth's orbit in February of A) 2013? B) 2015? C) 2017? and D) 2022?

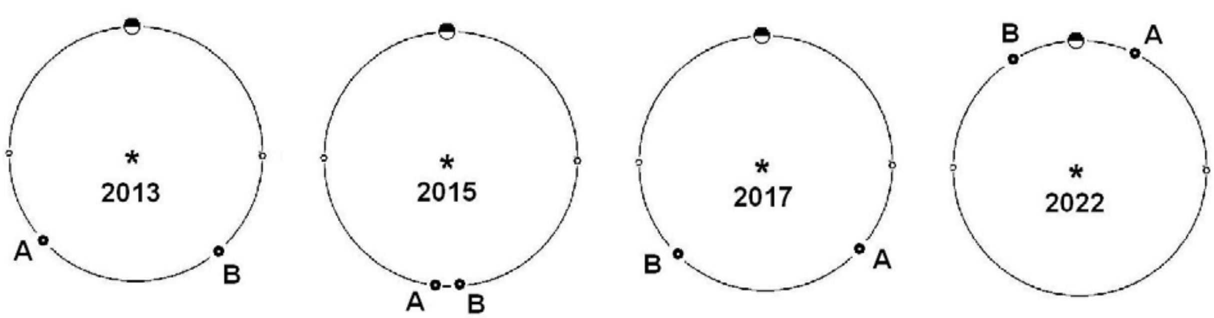
Problem 1 – From the information in the text, draw a diagram that shows the location of the sun, Earth's orbital path (assume it is a circle whose plane passes through the equator of the sun) and the locations of the STEREO spacecraft. In each of the two images, draw an arrow that points in the direction of Earth.





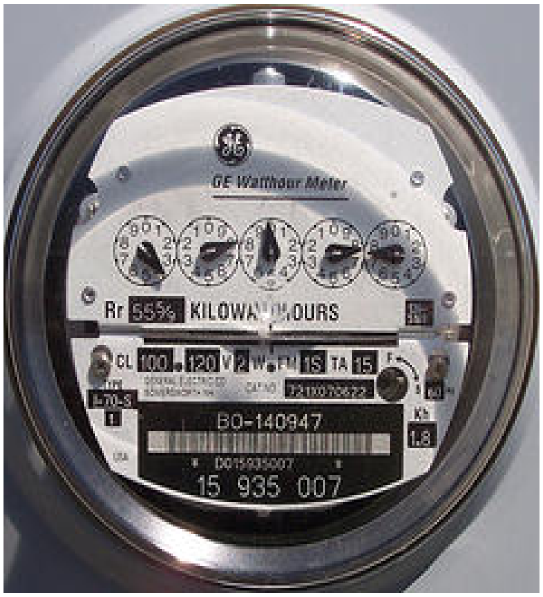
Problem 2 – The images were taken in the light of iron atoms heated to 1.1 million degrees. The bright (white) features are solar 'active regions' that correspond to the locations of sunspots. At the time the images were taken, what active regions in the above images: A) Could be observed from Earth? B) Could not be observed from Earth? **Answer A) Features on the left half of Image A and the right half of Image B. B) Features on the right half of Image A and the left half of Image B.**

Problem 3 – Relative to the location of Earth in its orbit, the STEREO-A and B spacecraft move 22 degrees farther from Earth each year. On your diagram from Problem 1, about what will be the positions of the spacecraft along Earth's orbit in February of A) 2013? B) 2015? C) 2017? and D) 2022? Answer: **If the satellites move 22 degrees along Earth's orbit each year, then for A) 2013, Spacecraft A will be (2013 - 2011) x 22 = 44 degrees counter-clockwise of its February 2011 position. Spacecraft B will be 44 degrees clockwise of its February 2011 position. B) For 2015, Spacecraft A = (2015 - 2011) x 22 = 88 degrees CCW; Spacecraft B = (2015 - 2011) x 22 = 88 degrees CW C) For 2017, Spacecraft A = (2017 - 2011) x 22 = 132 degrees CCW; Spacecraft B = (2017 - 2011) x 22 = 132 degrees CW; For 2022, Spacecraft A = (2022 - 2011) x 22 = 242 degrees CCW; Spacecraft B = (2022 - 2011) x 22 = 242 degrees CW. See diagram below**: - Angles approximate at this scale



Note: The spacecraft were launched in October 2006, so the time for the spacecraft to drift back to earth's vicinity will be 360/22 = 16 1/3 years after October 2006 or about February 2023.

Energy in the Home



Every month, we get the Bad News from our local electrical company. A bill comes in the mail saying that you used 900 Kilowatt Hours (kWh) of electricity last month, and that will cost you $100.00! *What is this all about?*

Definition: 1 kiloWatt hour is a unit of energy determined by multiplying the electrical power, in kilowatts, by the number of hours of use.

Example: A 100-watt lamp is left on all day. E = 0.1 kilowatts x 24-hours = 2.4 kWh. Note: At 11-cents per kWh, this costs you 2.4 x 11 = 26 cents!

Problem 1 – You and your sister fired-up your two computers at 3:00 PM, and finished your homework at 9:00 PM, but you forgot to turn them off before going to bed. At 7:00 AM, they were finally shut off after being on all night. If this happened each school day in the month (25 days):

A) How many kilowatt hours did it cost to run the computers this way for 25 days?

B) How many kilowatt hours were wasted?

C) If each computer runs at 350 watts, and if electricity costs 11-cents per kilowatt hour, how much did this waste cost each month?

D) How many additional songs can you buy with iTunes for the wasted money each month?

Problem 2 – The Tevatron 'atom smasher' at Fermilab in Batavia, Illinois collides particles together at nearly the speed of light to explore the innermost structure of matter. When operating, the accelerator requires 70 megaWatts of electricity – about the same as the power consumption of the entire town of Batavia (population: 27,000). If an experiment, from start to finish, lasts 24 hours:

A) What is the Tevatron's electricity consumption in kilowatt hours?

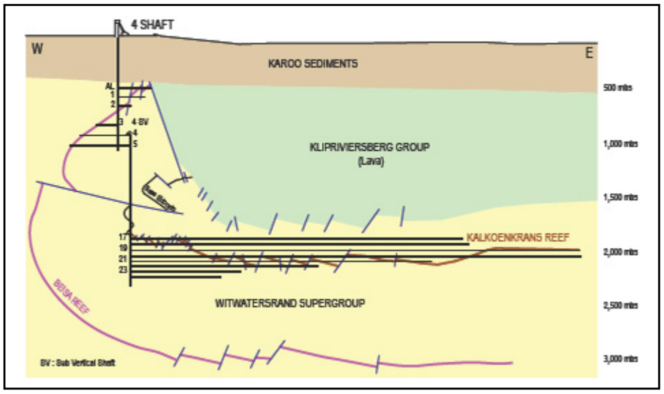
B) At $0.11 per kilowatt hour, how much does one experiment cost to run?

Answer Key

Problem 1 – You and your sister fired-up your two computers at 3:00 PM, and finished your homework at 9:00 PM, but you forgot to turn them off before going to bed. At 7:00 AM, they were finally shut off after being on all night. If this happened each school day in the month (25 days) A) How many kilowatt hours did it cost to run the computers this way for 25 days? Answer: The computers were turned off between 7:00 AM and 3:00 PM which is 8-hours, so the total time they stayed on each day is 24 – 8 = 16 hours. Over 25 days, this comes to 25 x 16 = 400 hours for each computer, or 800 hours for both. B) How many kilowatt hours were wasted? Answer: The computers were used between 3:00 PM and 9:00 PM so this is 6-hours out of the 16, so the wasted time was 10 hours each day per computer, or 2 x 10 x 25 = 500-hours of wasted 'ON' time. C) If each computer runs at 350 watts, and if electricity costs 11-cents per kilowatt hour, how much did this waste cost each month? Answer: The wasted time = 500 hours, and the total wattage of the two computers is 700 watts, so the number of kilowatt hours is 0.7 kilowatts x 500 hours = 350 kilowatt hours. At $.11 per kilowatt hours, this becomes 350 x 0.11 = $38.50. D) How many additional songs can you buy with iTunes for the wasted money each month? Answer: At $0.99 per song, you can buy $38.50/.99 = 38 songs!

Problem 2 – The Tevatron 'Atom Smasher' at Fermilab in Batavia, Illinois collides particles together at nearly the speed of light to explore the innermost structure of matter. When operating, the accelerator requires 70 megaWatts of electricity – about the same as the power consumption of the entire town of Batavia (population: 27,000). If an experiment, from start-up to finish, lasts 24 hours, what is the Tevatron's electricity consumption in kilowatt hours? Answer: 70 megaWatts x (1,000 kiloWatts/1 megaWatt) = 70,000 kiloWatts. Then 70,000 kiloWatts x 24 hours = 1,680,000 kiloWatt hours. B) At $0.11 per kilowatt, how much does one experiment cost to run? Answer: 1,680,000 kiloWatts x $0.11 / kilowatt = $184,800.00

Below is a picture of Fermilab's Tevatron accelerator. The ring has a diameter of 2 kilometers.



As miners on Earth dig deeper mines, they have noticed that the temperature of rocks gets higher. This tells scientists that the interior of Earth is much hotter than its surface. Exactly how much hotter depends on how deep into Earth you travel.

The Beatrix Mine is located in the extreme southern Witwatersrand Basin in South Africa. The basin, created by an ancient asteroid impact over 3 billion years ago, is the world's largest gold deposit. To get to the richest veins of gold, miners have to dig 2,200 meters below ground level.

A measure of how fast the temperature increases for a given depth is called the geothermal gradient. Gradient is a mathematical term that just means how fast one number (temperature in degrees) changes as you move a certain distance (depth in meters). The geothermal gradient for the Beatrix Gold Mine is about $+29^{o}$C/km. This means that for every kilometer of depth, the temperature of the rock increases by 29^{o} Celsius ($+52^{o}$F).

Problem 1 – On a summer day at the mine's surface, the temperature is about 90^{o}F (32^{o}C). What temperature in Celsius and Fahrenheit will a miner experience at the bottom of Shaft 4 at a depth of 2,200 meters?

Problem 2 – A geologist measures the geothermal gradient of $+0.02^{o}$C/meter at one location in Africa, and $+29^{o}$C/km at another location in Texas. Which location has the fastest temperature change with depth?

Problem 3 – You want to dig a basement floor to your home so that the lowest floor is always at a temperature of 60^{o}F. It is known that at a depth of 3 meters, the ground is always at a temperature of 13^{o}C (55^{o}F) year-round because the soil above acts as an insulator. How deep must you excavate to reach the proper depth if you are in a volcanically-active area where the geothermal gradient is $+35^{o}$C/km?

Problem 4 - One of the deepest laboratories in the world is in the small town of Soudan, Minnesota. It is a physics research facility and is at a depth of 690 meters (2,263 feet) below the surface. It is operated by the University of Minnesota. If the geothermal gradient is $+25^{o}$C/km, how warm are the walls of this laboratory if the average surface temperature is $+55^{o}$F ($+13^{o}$C)?

The temperature gradient in the Beatrix Mine is given by T. C. Onstott, D. P. Moser, M. F. DeFlaun, L. M. Pratt, and B. Sherwood Lollar, Abstr. 101st Gen. Meet. Am. Soc. Microbiol., p. 515, 2001 in their study of bacteria living in the crust at high temperatures. (http://www.ncbi.nlm.nih.gov/pmc/articles/PMC93369

*"The geothermal gradients were found to vary among the four mines and appeared to be correlated with the amounts of water moving through fractures or along dyke contacts. At East Driefontein only a few slowly flowing boreholes were noted at the time of this study and the geothermal gradient is 9°C km−1, whereas at West Driefontein, Kloof, and **Beatrix fissure** water emanating from boreholes is common and the geothermal gradients are 13, 16, and **29°C km−1**, respectively"*

Problem 1 – On a summer day at the mine's surface, the temperature is about 90°F (32°C). What temperature in Celsius and Fahrenheit will a miner experience at the bottom of Shaft 4 at a depth of 2,200 meters?

Answer: The gradient is +29°C/km so the increase in temperature will be +29°C/km x (2.2km) = +64°C, and so adding this to the surface temperature you get +32°C +64°C = +96°C. In terms of Fahrenheit we have (+96°C x 9/5) + 32°F =+205° F. That means you can use the mined rocks to boil water, and that makes them very dangerous to the miners.

Problem 2 – A geologist measures the geothermal gradient of +0.02°C/meter at one location in Africa, and +29°C/km at another location in Texas. Which location has the fastest temperature change with depth?

Answer: 1 km = 1000 meters so we can convert to the same units: +0.02°C/meter x (1000 meters/1km) = +20°C/km. The location in Africa has a faster increase of temperature with depth than Texas.

Problem 3 – You want to dig a basement floor to your home so that the lowest floor is always at a temperature of 60°F (16°C). It is known that at a depth of 3 meters, the ground is always at a temperature of 13°C (55°F) year-round because the soil above acts as an insulator. How deep must you excavate to reach the proper depth if you are in a volcanically-active area where the geothermal gradient is +35°C/km?

Answer: First you need to excavate to 3 meters depth (about 9 feet) where you start with a temperature of 13°C. You need an additional 3°C of warming. The geothermal gradient is +35°C/km or +0.35°C/meter. To get an additional 3°C of heating you need to dig an additional +3°C/(+0.35°C/m) = 8.6 meters for a total depth of 11.6 meters (or 35 feet).

Problem 4 - One of the deepest laboratories in the world is in the small town of Soudan, Minnesota. It is a physics research facility and is at a depth of 690 meters (2,263 feet) below the surface. It is operated by the University of Minnesota. If the geothermal gradient is +25°C/km, how warm are the walls of this laboratory if the average surface temperature is +55°F (+13°C)?

Answer: T = +13°C + 0.69km x (+25°C/km) = **+30°C (or +86°F)**

Sunspots are some of the most interesting, and longest studied, phenomena on the sun's surface. They were known to ancient Chinese observers over 2000 years ago. Below is a list of the largest sunspots seen since 1859 when careful measurements were first made of their sizes. Their sizes are given in terms of the area of the solar hemisphere facing earth. For example, '3600' means 3600 millionths of the solar area or (3600/1000000). On this scale, the area of Earth is '169'. All of these spots were large enough to be seen with the naked eye when proper (and safe!) viewing glasses were used.

Date	Size	Earths
February 10, 1917	3600	**21.3**
January 25, 1926	3700	
January 18, 1938	3650	
February 6, 1946	5250	
July 27, 1946	4700	
March 10, 1947	4650	
April 7, 1947	6150	
May 16, 1951	4850	
Nov. 14, 1970	3500	
August 23, 1971	3500	
October 30, 1972	4120	
Nov. 11, 1980	3820	
July 28, 1981	3800	

Date	Size	Earths
October 14, 1981	4180	
October 19, 1981	4500	
February 10, 1982	3800	
June 18, 1982	4400	
July 15, 1982	4900	
April 28, 1984	5400	
May 13, 1984	3700	
March 13, 1989	5230	
September 5, 1989	3500	

Years of Peak Sunspot Activity:

Sunspot Cycle	Peak Year
14	1906
15	1917
16	1928
17	1937
18	1947
19	1958
20	1968
21	1979
22	1989

Question 1: The peaks of the 11-year sunspot cycle during the 20th century occurred during the years shown to the left. On average, how close to sunspot maximum do the largest spots occur?

Question 2: From this sample, do more of these spots happen in the 5 years before sunspot maximum, or within 5 years after sunspot maximum?

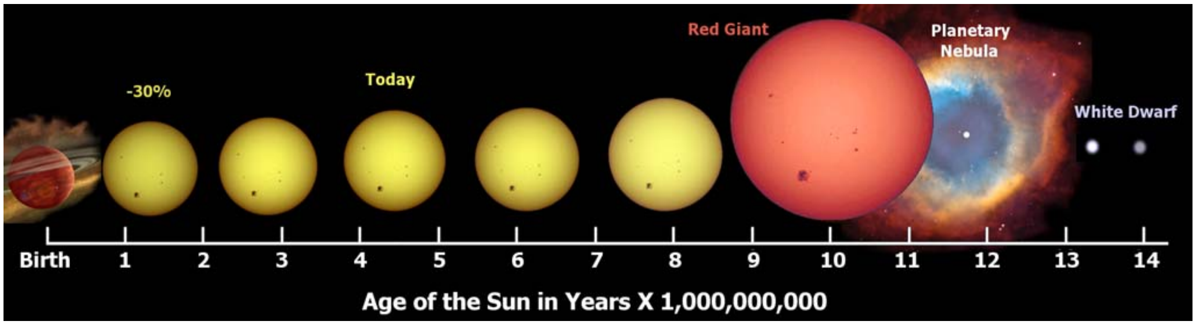
Question 3: Convert the sunspot areas into an equivalent area of the Earth. (Note the first one has been done as an example). What is the average large spot size in terms of Earth?

Date	Size	Area In Earths	Nearest Sunspot Max. Year	Difference In Years
February 10, 1917	3600	21.3	1917	0
January 25, 1926	3700	21.9	1928	-2
January 18, 1938	3650	21.6	1937	+1
February 6, 1946	5250	31.1	1947	-1
July 27, 1946	4700	27.8	1947	-1
March 10, 1947	4650	27.5	1947	0
April 7, 1947	6150	36.4	1947	0
May 16, 1951	4850	28.7	1951	0
Nov. 14, 1970	3500	20.7	1968	+2
August 23, 1971	3500	20.7	1968	+3
October 30, 1972	4120	24.4	1968	+4
Nov. 11, 1980	3820	22.6	1979	+1
July 28, 1981	3800	22.5	1979	+2
October 14, 1981	4180	24.7	1979	+2
October 19, 1981	4500	26.6	1979	+2
February 10, 1982	3800	22.5	1979	+3
June 18, 1982	4400	26.0	1979	+3
July 15, 1982	4900	29.0	1979	+3
April 28, 1984	5400	32.0	1989	-5
May 13, 1984	3700	21.9	1989	-5
March 13, 1989	5230	31.0	1989	0
September 5, 1989	3500	20.7	1989	0

Question 1: On average, how close to sunspot maximum do the largest spots occur? **Answer**: Find the average of the differences in last column = 0 years. So, the largest sunspots occur, on average, close to the peak of the sunspot cycle.

Question 2: For this sample, do more of these spots happen in the 5 years before sunspot maximum, or within 5 years after sunspot maximum? **Answer**: More happen after the peak. Five happen before the peak (negative differences) and 11 happen after the peak (Positive differences).

Question 3: What is the average large spot size in terms of Earth? **Answer:** Average = (561.6/22) = 25.5 Earth Areas.



Our sun was formed 4.6 billion years ago. Since then it has been steadily increasing its brightness. This normal change is understood by astronomers who have created detailed mathematical models of the sun's complex interior. They have considered the nuclear physics that causes its heating and energy, gravitational forces that compress its dense core, and how the balance between these processes change in time. The diagram above shows the major stages in our sun's evolution from birth to end-of-life after 14 billion years. A simple formula describes how the power of our sun changes over time:

$$L = \frac{L_0}{1 + \frac{2}{5}(1 - x)}$$

where $x = t/t_0$ t_0 = 4.6 billion yrs, L_0 = 1.0 for the luminosity of the sun today.

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years

Problem 2 – By what percentage will L increase when it is 2 billion years older than it is today?

Problem 3 – A simple formula for the temperature, in kelvins, of Earth is given by:

$$T = 284[(1 - A)L]^{\frac{1}{4}}$$

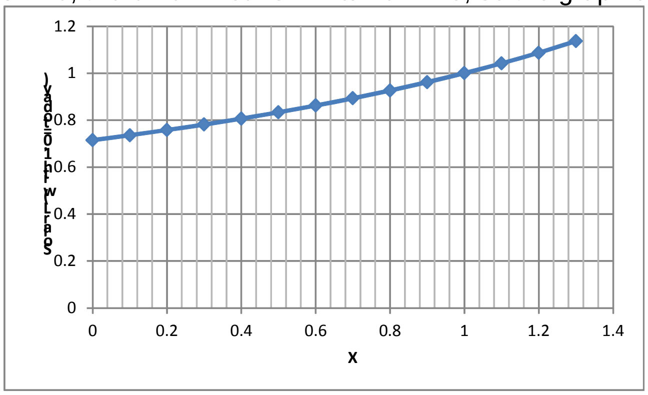
where L is the solar luminosity (today L=1.0), and A is the surface albedo, which is a number between 0 and 1, where asphalt is A=0 and A=1.0 is a perfect mirror A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Problem 4 - Combine the two formulae above to define a new formula that gives Earth's temperature in kelvins only as a function of time, t, and albedo, A.

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 150° F (339 kelvins)? (Note: it is currently 60° F)

$$L = \frac{L_0}{1 + \frac{2}{5}(1-x)}$$

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years.
Answer: t = 0 means X=0, t=6 billion means x = 6/4.6 = 1.3, so the graph domain is [0,1.3]



Problem 2 - By what percentage will L increase when it is 2 billion years older than it is today?

Answer: X = (4.6 + 2.0)/4.6 = 1.43, then L = 1 / (1+0.4(1-1.43)) so L = 1.20 this is **20% brighter** than today.

Problem 3 – A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Answer: A) L = 1.0 today so T = $284((1-0.4)\times1.0)^{1/4}$ = **250 kelvins** (or -23^{o} Celsius)
B) L = 1.05 so T = $284(0.6\times1.05)^{1/4}$ = **253 kelvins** (or -20^{o} Celsius)

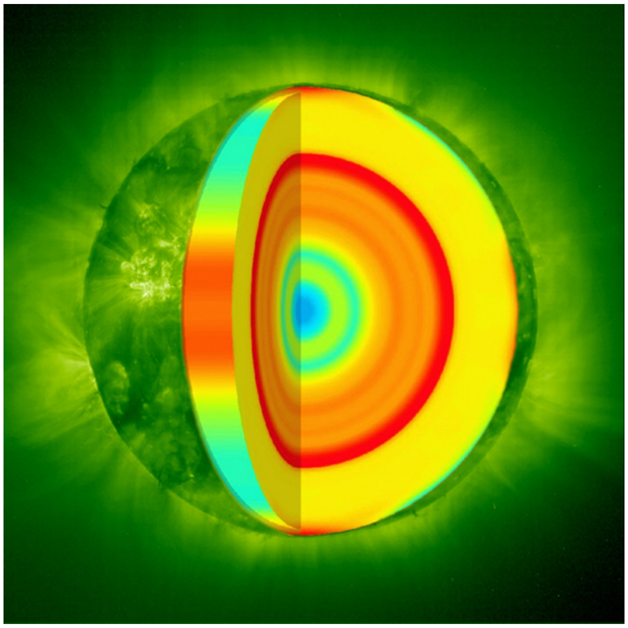
Problem 4 - Combine the formulae for L(x) and T to define a new formula, T(x,A) that gives Earth's temperature only as a function of time, x, and albedo, A, and assumes that L_0=1.0 today.

$$T(x,A) = 425\left(\frac{1-A}{7-2x}\right)^{\frac{1}{4}}$$

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 100^{o} F (310 kelvins)? (Note: it is currently 60^{o} F)

Answer: $310 = 425\,(0.4/(7-2x))^{1/4}$
$0.4\,(425/310)^4 = 7-2x$
1.4 = 7 – 2x
2x = 5.6 so x = 2.8
and the age of the sun will be t = 2.8 x 4.6 billion = **12.9 billion years**.
This occurs 12.9 – 4.6 = 8.3 billion years in the future.

A Mathematical Model of the Sun



Once an astronomer knows the radius and mass of a planet or star, one of the first things they might do is to try to figure out what the inside looks like.

Gravity would tend to compress a large body so that its deep interior was at a higher density than its surface layers. Imagine that the sun consisted of two regions, one of high density (the core shown in red) and a second of low density (the outer layers shown in yellow). All we know about the sun is its radius (696,000 km) and its mass (2.0×10^{30} kilograms). What we would like to estimate is how dense these two regions would be, in order for the final object to have the volume of the sun and its total observed mass. We can use the formula for the volume of a sphere, and the relationship between density, volume and mass to create such a model.

Suppose the inner zone has a radius of 417,000 km. The volume of this 'core' is

$$Vc = \frac{4}{3} \pi \, (417{,}000{,}000 \text{ m})^3 = 3.0 \times 10^{26} \text{ m}^3$$

The volume of the entire sun is

$$V^* = \frac{4}{3} \pi \, (696{,}000{,}000 \text{ m})^3 = 1.4 \times 10^{27} \text{ m}^3$$

So the outer shell zone has a volume of

$$Vs = V^* - Vc$$

So, $Vs = 1.1 \times 10^{27} \text{ km}^3$

Mass = Density x Volume, so if we estimate the density of the sun's core as $Dc = 100{,}000 \text{ kg/m}^3$, and the outer shell as $Ds = 10{,}000 \text{ kg/m}^3$, the masses of the two parts are:

$$Mc = 100{,}000 \times (3.0 \times 10^{26} \text{ m}^3) = 3.0 \times 10^{31} \text{ kg}$$

$$Ms = 10{,}000 \times (1.1 \times 10^{27} \text{ m}^3) = 1.1 \times 10^{31} \text{ kg}$$

So the total solar mass in our model would be:

$$M^* = Mc + Ms = 4.1 \times 10^{31} \text{ kg}$$

This is about 20 times more massive than the sun actually is!

Inquiry Problem.

With the help of an Excel spreadsheet, program the spreadsheet so that you can adjust the radius of the core and calculate the volume of the core and shell zones. Then create a formula so that you can enter various choices for the densities of the core and shell zones and then sum-up the total mass of your mathematical model for the sun.

What are some possible ranges for the core radius, and zone densities (in grams per cubic centimeter) that give about the right mass for the modeled sun? How do these compare with what you find from searching the web literature? How could you improve your mathematical model to make it more accurate? Show how you would apply this method to modeling the interior of the Earth, or the planet Jupiter?

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With the help of an Excel spreadsheet, program the spreadsheet so that you can adjust the radius of the core and calculate the volume of the core and shell zones. Then create a formula so that you can enter various choices for the densities of the core and shell zones and then sum-up the total mass of your mathematical model for the sun.

Notes: The big challenge for students who haven't used the volume formula is how to work with it for large objects…requiring the use of scientific notation as well as learning how to use the volume formula. Students will program a spreadsheet in whatever way works best for them, although it is very helpful to lay out the page in an orderly manner with appropriate column labels. Students, for example, can select ranges for the quantities, and then generate several hundred models by just copying the formula into several hundred rows. Those familiar with spreadsheets will know how to do this ,and it saves entering each number by hand…which would just as easily have been done with a calculator and does not take advantage of spreadsheet techniques.

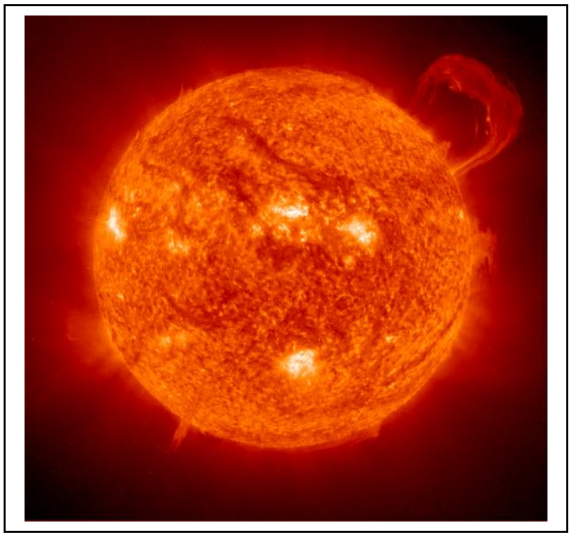
What are some possible ranges for the core radius, and zone densities (in grams per cubic centimeter) that give about the right mass for the modeled sun? How do these compare with what you find from searching the web literature? How could you improve your mathematical model to make it more accurate? Show how you would apply this method to modeling the interior of the Earth, or the planet Jupiter?

Notes: Most solar models suggest a core density near 160 grams/cc, and an outer 'convection zone' density of about 0.1 grams/cc. The average value for the shell radius is about 0.7 x the solar radius. Students will encounter, using GOOGLE, many pages on the solar interior, but the best key words are things like 'solar core density' and 'convection zone density'. These kinds of interior models are best improver by 1) using more shells to divide the interior of the body, and 2) developing a mathematical model of how the density should change from zone to zone. Does the density increase linearly as you go from the surface to the core, or does it follow some other mathematical function? Students may elect to also test various models for the interior density change such as

$$D = \frac{100 \text{ gm/cc}}{R} \qquad \text{or even} \qquad 100 \text{ gm/cc} \times e^{-R/R^*}$$

This method can be applied to any spherical body (planet, asteroid, etc) for which you know a radius and mass ahead of time.

Alternately, if you know a planet's mass and its composition (this gives an average density), then you can estimate its radius!



Detailed mathematical models of the interior of the sun are based on astronomical observations and our knowledge of the physics of stars. These models allow us to explore many aspects of how the sun 'works' that are permanently hidden from view.

The Standard Model of the sun, created by astrophysicists during the last 50 years, allows us to investigate many separate properties. One of these is the density of the heated gas throughout the interior. The function below gives a best-fit formula, D(x) for the density (in grams/cm^3) from the core (x=0) to the surface (x=1) and points in-between.

$$\boxed{D(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155}$$

For example, at a radius 30% of the way to the surface, x = 0.3 and so D(x=0.3) = 14.5 grams/cm^3.

Problem 1 - What is the estimated core density of the sun?

Problem 2 - To the nearest 1% of the radius of the sun, at what radius does the density of the sun fall to 50% of its core density at x=0? (Hint: Use a graphing calculator and estimate x to 0.01)

Problem 3 - What is the estimated density of the sun near its surface at x=0.9 using this polynomial approximation?

Problem 4 - Integrate D(x) throughout the volume of the solar interior to estimate the total mass of the sun in grams. (Use the volume element dV = $4\pi x^2$ dx, and use the fact that for x=1, the physical radius of the sun is 6.9×10^{10} centimeters.)

Problem 1 - Answer; At the core, x=0, do D(0) = 155 grams/cm^3.

Problem 2 - Answer: We want D(x) = 155/2 = 77.5 gm/cm3. Use a graphing calculator, or an Excell spreadsheet, to plot D(x) and slide the cursor along the curve until D(x) = 77.5 .Then read out the value of x. The relevant portion of D(x) is shown in the table below:

X	D(x)
0.08	94.87
0.09	88.77
0.1	82.96
0.11	**77.43**
0.12	72.16
0.13	67.16
0.14	62.41

Problem 3 - Answer: At x=0.9 (i.e. a distance of 90% of the radius of the sun from the center).
D(0.9) = 519(0.9)^4 - 1630(0.9)^3 + 1844(0.9)^2 -889(0.9) + 155
D(0.9) = 340.516 - 1188.27 + 1493.64 - 800.10 + 155.00
D(0.9) = 0.786 gm/cm^3.

Problem 4 - Integrate D(x) throughout the volume of the solar interior to estimate the total mass of the sun in grams. (Use the volume element dV = 4πx^2 dx and use the fact that for x=1, the physical radius of the sun is 6.9 x 10^{10} centimeters.). Answer:

$$M = \int_0^1 [519x^4 - 1630x^3 + 1844x^2 - 889x + 155] 4\pi x^2 dx$$

$$M(x) = 4\pi[\frac{519}{7}x^7 - \frac{1630}{6}x^6 + \frac{1844}{5}x^5 - \frac{889}{4}x^4 + \frac{155}{3}x^3]$$

$$M(1) = 4(3.14)[74.14 - 271.67 + 368.80 - 222.25 + 51.67]$$

$$M(1) = 4(3.14)[0.69]$$

$$M(1) = 8.71$$

Since x=1 is a physical distance of R = 6.9 x 10^{10} centimeters, we have to multiply M(1) by (6.9 x 10^{10})^3 to get M = 8.29 x (3.28 x 10^{32} cm^3) or M = 2.7 x 10^{33} grams. The actual astronomical value is 1.98 x 10^{33} grams. The polynomial approximation for the internal density is a close, but not exact, estimate.

The Parker Solar Probe mission will be launched in 2018 for a rendezvous with the sun in 2024. To lose enough energy to reach the sun, the spacecraft will make seven fly-bys of Venus.

The 480 kg spacecraft, costing $740 million, will approach the sun to a distance of 5.8 million km. Protected by the heat shield will be five instruments that will peak over the edge of the heat shield and measure the particles and radiation fields in the outer solar corona.

At the closest approach distance, the sun will occupy a much larger area of the sky than what it does at the distance of Earth.

The angular size of an object as it appears to a viewer depends on the actual physical size of the object, and its distance from the viewer. Although physical size and diameter are usually measured in terms of meters or kilometers, the angular diameter of an object is measured in terms of degrees or radians. The angular size, t, depends on its actual size d and its distance r according to the simple formula

$$\theta = 2arcTan\left(\frac{d}{2R}\right)$$

For example, the angular size of your thumb with your arm fully extended for d = 2 centimeters and r = 60 cm is just θ = 2 arcTan(0.016) so θ = 1.8 degrees.

Problem 1 – At the distance of Earth, 147 million kilometers, what is the angular diameter of the sun whose physical diameter is 1.4 million kilometers?

Problem 2 – At the distance of closest approach to the solar surface Solar Probe Plus will be at a distance of 5.8 million kilometers from the solar surface. What will be the angular diameter of the sun if its physical diameter is 1.4 million kilometers?

Problem 3 – A DVD disk measures 12 cm in diameter. How far from your eyes will you have to hold the DVD disk in order for it to subtend the same angular size as the disk of the sun viewed by Solar Probe Plus at its closest distance?

Problem 1 – At the distance of Earth, 147 million kilometers, what is the angular diameter of the sun whose physical diameter is 1.4 million kilometers?

Answer: $\theta = 2arcTan\left(\frac{1.4million}{2x147million}\right)$

θ = 2 arctan(0.0048)

so θ **= 0.55 degrees**.

Problem 2 – At the distance of closest approach to the solar surface Solar Probe Plus will be at a distance of 5.8 million kilometers from the solar surface. What will be the angular diameter of the sun if its physical diameter is 1.4 million kilometers?

Answer: θ = 2 arctan (1.4/(2 x 5.8)) so θ **= 14 degrees**.

Problem 3 – A DVD disk measures 12 cm in diameter. How far from your eyes will you have to hold the DVD disk in order for it to subtend the same angular size as the disk of the sun viewed by Solar Probe Plus at its closest distance?

Answer: The desired angular diameter of the sun is θ = 14 degrees, and for the DVD disk we have d = 12 cm, so we need to solve the formula for r.

Tan (θ/2) = d/2r so

2r = d / tan(θ/2) then

r = 6 cm / tan(7) yields

r = **49 cm. This is about 18 inches**.

Note: From the vantage point of the Solar Probe Plus spacecraft, the sun's disk at a temperature of 5570 K is much larger (14 degrees) than it appears in Earth's sky (0.5 degrees), and this results in the spacecraft heat shield being heated to over 1600 K.

For more mission details, visit:

http://solarprobe.jhuapl.edu/

http://www.nasa.gov/topics/solarsystem/sunearthsystem/main/solarprobeplus.html

In the figure to the left, the first column represents gas particles with little energy. A thermometer placed in contact with this group of particles would indicate a very low temperature. The column to the right represents particles with a high enough speed and energy to spread out inside the column. A thermometer placed in this group would show a high temperature.

When the state of matter changes its phase, the temperature and energy of matter also changes. At low temperature and energy we have a solid phase. At a medium temperature and energy we have a liquid phase, and at a high temperature and energy we have a gaseous phase.

A simple formula gives us the average speed, V, of water molecules in meters per second (m/s) for a given temperature in degrees Celsius, T:

$$V^2 = 1380(273+T)$$

Problem 1 – What is the speed of an average water molecule near A) the freezing point of water at 0^{o} C? B) The boiling point of water at 100^{o} C?

Problem 2 - The kinetic energy in Joules for all of the water molecules in a gallon of water, which has a mass of about M = 4.0 kilograms, and an average molecule speed of V in meters/sec, is given by the formula:

$$K.E. = \frac{1}{2}MV^2$$

To the nearest Joule, what is the kinetic energy of a 1 gallon of water at the temperatures given in Problem 1?

Problem 3 - If you heated the one gallon of water from 0^{o}C to 100^{o}C, how much 'thermal' energy would you have to add?

Problem 1 - What is the speed of an average water molecule near A) the freezing point of water at 0^o C? B) The boiling point of water at 100^o C?

Answer: From the formula:

A) V = $(1380(273+(+0))^{1/2}$ = $(376740)^{1/2}$ = 614 meters/sec.

B) V = $(1380(273+(100))^{1/2}$ = $(514740)^{1/2}$ = 717 meters/sec.

Problem 2 - The kinetic energy in Joules for all of the water molecules in a gallon of water, which has a mass of about M = 4 kilograms, and an average molecule speed of V in meters/sec, is given by the formula:

$$K.E. = \frac{1}{2}MV^2$$

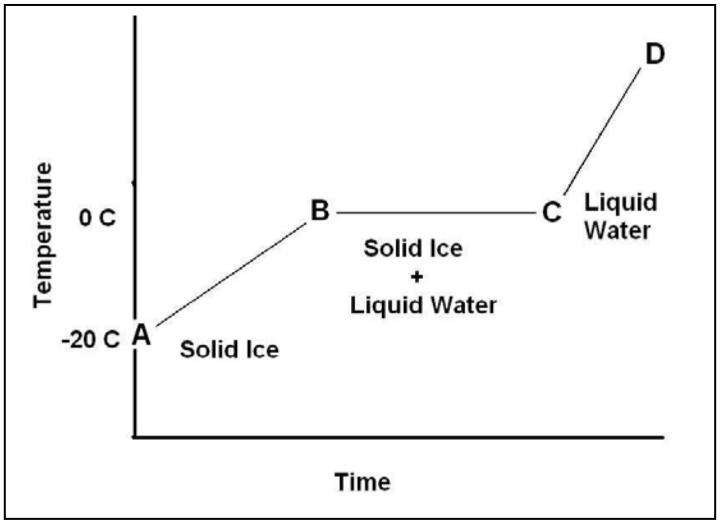
To the nearest Joule, what is the kinetic energy of a 1 gallon of water at the temperatures given in Problem 1?

Answer: A) For $+0^OC$, we calculated an average speed of 614 m/s, so the kinetic energy of the water is KE = 1/2 $(4.0)(614)^2$ = **753,992 Joules**.
B) For 100^OC we have V = 717 m/s, so KE = 1/2$(4.0)(717)^2$ = **1,028,178 Joules.**

Problem 3 - If you heated the one gallon of water from 0^OC to 100^OC, how much 'thermal' energy would you have to add?

Answer: You have to add the difference in energy (1,028,178 – 753,992) = **274,186 Joules** to heat the gallon of water to its boiling point at 100^o C.

Note: A typical hotplate at a temperature of 400 C generates about 1000 Joules/second, so to heat the gallon of water to make it boil would take about 274186/4000 or about 4 minutes at this hotplate setting.



As energy is added to solid matter, it changes its state. The figure to the left shows what happens to water as it changes from solid ice (A to B), to a mixture of cold water and 'ice cubes' (B to C) and then finally to pure liquid water (C to D).

The energy required to change a kilogram of solid ice by one degree Celsius is called the **Specific Heat**. The energy needed to change a kilogram of solid ice at 0^{o}C into 100% liquid water at 0^{o}C is called the **Latent Heat of Fusion**.

Problem 1 - The Specific Heat of ice is 2090 Joules/kg C. How many Joules of energy do you need to raise the temperature of 1 kg of ice from -20^{o}C to 0^{o}C along the path from A to B on the graph?

Problem 2 - The Latent Heat of Fusion for water is 333 Joules/gram. How many Joules of energy do you need to melt all the ice into a pure liquid along the path from B to C on the graph?

Problem 3 - The Specific Heat of liquid water is 4180 Joules/kg C. How much energy is needed to raise the temperature of 100 grams of liquid water to $+60^{o}$C for a nice warm cup of tea along the path from C to D in the graph?

Problem 1 - The Specific Heat of ice is 2090 Joules/kg C. How many Joules of energy do you need to raise the temperature of 1 kg of ice from -20^{o}C to 0^{o}C along the path from A to B on the graph?

Answer: The temperature difference is 20^{o}C, so for 1 kg of ice we need 2090 Joules/kgC x (1 kg) x (20^{o}C) = **41,840 Joules**.

Problem 2 - The Latent Heat of Fusion for water is 333 Joules/gram. How many Joules of energy do you need to melt all the ice into a pure liquid along the path from B to C on the graph?

Answer: For 1 kilogram of ice ,which equals 1000 grams, we need 333 Joules/gram x 1000 grams = **333,000 Joules**.

Problem 3 - The Specific Heat of liquid water is 4180 Joules/kg C. How much energy is needed to raise the temperature of 100 grams of liquid water to +60oC for a nice warm cup of tea along the path from C to D in the graph?

Answer: 4180 Joules/kgC x 0.1 kg = **418 Joules**.