National Aeronautics and Space Administration

Math Problems Related to

The Stars

Introduction

This book provides an assortment of mathematics problems that deal with a variety of aspects of the stars in the sky, from their brightness and evolution, to their physical properties and appearances. This includes their temperatures, classification, distances, clustering and relative sizes compared to our sun. Also included are examples of evolved stars, stars ejecting matter, pulsars, and supernovae.

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36	Advanced	Geometry	WISE: Exploring Power-law Functions Using WISE Data
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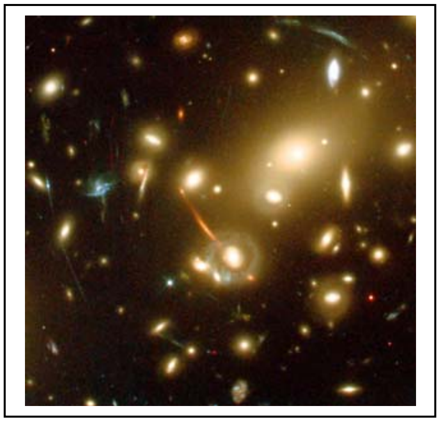
Groups, Clusters and Individuals



Astronomers study many different kinds of objects in space. Sometimes these objects contain many individuals that need to be tallied separately.

The cluster of galaxies called Abell-2218 is a rich collection of galaxies, as shown in the Hubble Space Telescope photo to the left.

1 – Astronomers observed the surface of the Sun on March 27, 2001 and counted 230 sunspots. If there were 10 sunspots in each group, about how many sunspot groups were on the Sun that day?

2 – Since 1950, astronomers have cataloged 35 individual galaxies within the group of galaxies containing the Milky Way. If this group is typical, and there are 200 galaxy groups within 100 million light years of the Milky Way, how many individual galaxies are present?

3 – The Milky Way galaxy has 158 satellite star clusters that orbit its in space. If each of these star clusters contains 100,000 stars, how many stars exist in these clusters?

4 – Astronomers have detected 430 planets orbiting 300 nearby stars. About how many planets orbit an average star in this sample?

5 – A cubic centimeter of gas in the Orion Nebula contains about 10 atoms of hydrogen and 4 atoms of helium. Hydrogen atoms contain one proton and one electron. Helium atoms contain 2 protons, 2 neutrons and 2 electrons. How many protons would you find in a single cubic centimeter of this gas? How many neutrons? How many electrons? What is the total number of protons, neutrons and electrons?

Answer Key

1 – Astronomers observed the surface of the sun on March 27, 2001 and counted 230 sunspots. If there were 10 sunspots in each group, about how many sunspot groups were on the sun that day?

Answer: 230 spots / 10 spots per group = **23 groups.**

2 – Since 1950, astronomers have cataloged 35 individual galaxies within the group of galaxies containing the Milky Way. If this group is typical, and there are 200 galaxy groups within 100 million light years of the Milky Way, how many individual galaxies are present?

Answer: 200 galaxy groups x 35 galaxies per group = **7000 galaxies**

3 – The Milky Way galaxy has 158 satellite star clusters that orbit its in space. If each of these star clusters contains 100,000 stars, how many stars exist in these clusters?

Answer: 158 clusters x 100,000 stars per cluster = **15,800,000 stars.**

4 – Astronomers have detected 430 planets orbiting 300 nearby stars. About how many planets orbit an average star in this sample?

Answer: 430 planets / 300 stars = 1.4 planets, or about **1 planet per star.**

5 – A cubic centimeter of gas in the Orion Nebula contains about 10 atoms of hydrogen and 4 atoms of helium. Hydrogen atoms contain one proton and one electron. Helium atoms contain 2 protons, 2 neutrons and 2 electrons. How many protons would you find in a single cubic centimeter of this gas? How many neutrons? How many electrons? What is the total number of protons, neutrons and electrons?

Answer: 10 atoms hydrogen + 4 atoms helium

10(1 proton) + 4(2 protons) = **18 protons**

10(0 neutrons) + 4(2 neutrons) = **8 neutrons**

10(1 electron) + 4(2 electrons) = **18 electrons.**

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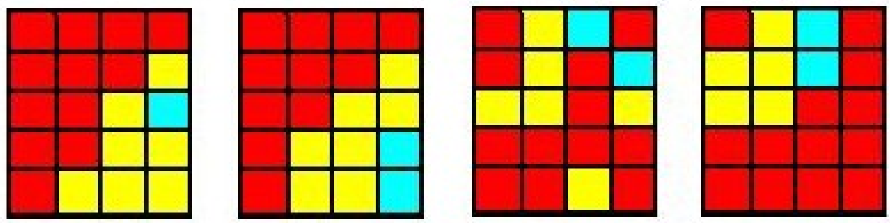
44 particles

Star Cluster Math



(a) (b) (c) (d)

Astronomers classify stars so that they can study their similarities and differences. A very common way to classify stars is by their temperature. This scale assigns a letter from the set [O, B, A, F, G, K, M] to represent stars with temperatures from 30,000 C (O-type) and 6,000 C (G-type), to 3,000 C (M-type).

Problem 1 – An astronomer studies a sample of stars in a cluster and identifies 6 as G-type like our Sun, 12 as M-type like Antares, and 2 stars as O-type like Rigel. Circle the pattern, above, that represents this sample.

Problem 2 – What fraction of the stars in the sample are G-type?

A) 6/9 B) 20/6 C) 6/20 D) 6/8

Problem 3 – What fraction of the G and M-type stars in the cluster are G-type?

A) 12/18 B) 6/12 C) 12/6 D) 6/18

Problem 4 – If you selected 2 stars randomly from this cluster, which calculation would give the probability that these would both be O-type stars?

A) 1/20 x 1/20 B) 2/20 x 1/20 C) 1/20 x 1/19 D) 2/20 x 1/19

Problem 5 – A second star cluster has a total of 2,040 stars. If the proportion of O, G and M-types stars is the same as in the first cluster, how many G-type stars would be present?

A) 612 B) 340 C) 1428 D) 680

Answer Key

1 – The boxes are colored red for M-type stars, yellow for G-type stars and blue for O-type stars. Count the boxes carefully. Only C) has the correct number of star boxes colored. **Answer: C)**

2 – There are 6 G-type stars in the cluster, which contains 20 stars, so the answer is C) 6/20. **Answer: C)**

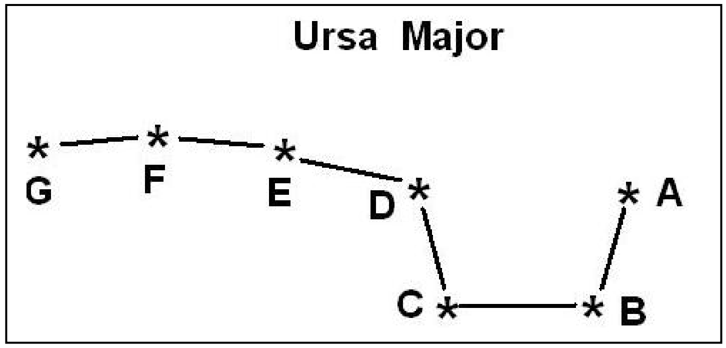
3 – There are a total of 18 G and M-type stars in the sample, and since only 6 are G-type, the correct fraction is D) 6/18. **Answer: D)**

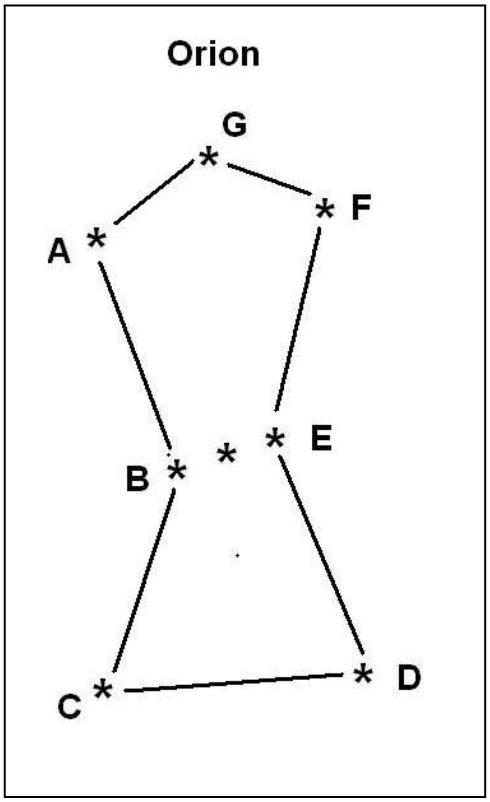
4 – There are 2 O-type stars in a sample of 20 stars. On the first draw, the probability is 2/20 that an O-type star will be selected. Now there are only 19 stars left and only 1 O-type, so the probability that the next star selected is an O-type star is now 1/19. The probability that both O-type stars are drawn in the first two draws is then D) 2/20 x 1/19. **Answer: D)**

5 – The correct answer is A) given by 2040 x 6/20 = 612 stars. The fraction 6/20 represents the proportion of 6 G-type stars out of the 20 stars in the first sample, and we are assuming that this proportion is the same for the second cluster. The incorrect answers come about by B) dividing 2040 by the number of G-type stars; C) multiplying 2040 by the fraction of stars that are O and M-class and; D) dividing 2040 by the number of star classes, which incorrectly assumes an equal number in each class. **Answer: A)**

Perimeters: Which constellation is the longest?

Star Segment	Length
AB	5 1/2
BC	8
CD	4 1/2
DE	5 1/3
EF	4 1/3
FG	6 2/3





Star Segment	Length
AB	10
BC	7
CD	8 1/4
DE	8
EF	6
FG	4 1/4
GA	5 1/2

Constellations form various kinds of irregular geometric figures, but we can study them by examining some of their basic properties. One of these is their perimeter.

Here are two well-known constellations, Ursa Major, this portion is also known as the Big Dipper in English-speaking countries, and Orion 'The Hunter'. On star charts they look like they are about the same size, but let's put this to a test. From Earth, we measure the distance between stars as they appear in the sky in terms of degrees. Let's measure the separations between the stars in degrees and calculate their perimeters.

Problem 1 - From the corresponding table above, calculate the total perimeter of Ursa Major by adding up the lengths of the star segments from AB to FG which are given in degrees.

Problem 2 - From the corresponding table to the left, calculate the total perimeter of Orion by adding up the lengths of the star segments from AB to GA which are given in degrees.

Problem 3 - Which constellation has the longest perimeter in degrees?

Problem 4 - What is the average distance in degrees between the stars along the perimeter of A) Ursa Major? B) Orion?

Problem 5 - In which constellation are the stars the farthest apart on average?

Problem 6 - Can you name another property of a constellation that could be interesting to study?

Answer Key

Problem 1 - From the corresponding table above, calculate the total perimeter of Ursa Major by adding up the lengths of the star segments from AB to FG which are given in degrees.

Answer: 5 1/2 + 8 + 4 1/2 + 5 1/3 + 4 1/3 + 6 2/3 = **34 1/3**

Problem 2 - From the corresponding table to the left, calculate the total perimeter of Orion by adding up the lengths of the star segments from AB to GA which are given in degrees.

Answer: 10 + 7 + 8 1/4 + 8 + 6 + 4 1/4 + 5 1/2 = **49**

Problem 3 - Which constellation has the longest perimeter in degrees?

Answer: **Orion.**

Problem 4 - What is the average distance in degrees between the stars along the perimeter of Ursa Major?

Answer: A) For Ursa Major, there are 6 segments so we have to divide the perimeter by 6 to find the average distance. 34 1/3 degrees / 6 = 34 1/3 x 1/6 = 103/3 x 1/6 = 103/18 = **5 13/18 degrees**

B) For Orion, there are 7 segments in Orion so the average length of a segment is 49 degrees/7 = 7 degrees

Problem 5 - In which constellation are the stars the farthest apart on average?
Answer: **Orion**

Problem 6 - Can you name another property of a constellation that could be interesting to study?

Answer: **Students may identify, for example, the total area of a constellation, the length of the largest star segment spanning the constellation (Such as AG in Ursa Major or GC in Orion), the number of obtuse angles.**

Name ________________________________

In ca 130 BC, Hipparchos created the first detailed star catalog of thousands of naked-eye stars. He ranked them according to their brightness with the brightest being assigned the First Rank, the next brightest the Second Rank, and so on. Today, astronomers use this same brightness scaling for stars which they call the apparent magnitude scale. As it turns out, the human eye can discern a brightness change of 2.5122-times, and that is why the star brightness rankings have such a specific pattern. For example, a star with a magnitude of +2.0 is 2.51 times brighter than a star of magnitude +3.0. The difference in brightness between a +6.0 magnitude star and a +3.0 star is 3.0 magnitudes, which corresponds to a brightness change of 2.512 x 2.512 x 2.512 = 15.9 times. A magnitude difference of 5.0 corresponds to a change of 100 times. Now, let's do some star math!! (Note; In the following problems we have stated star magnitudes to the nearest integer to simplify the math!)

Question 1) The star Sirius has a magnitude of -1.0 while the star Kruger 60B has a magnitude of +11.0. What is the magnitude difference between these stars? How many times fainter is Kruger 60B than Sirius?

Question 2) The apparent magnitudes of the sun and moon are -26.0 and -18.0. How many magnitudes brighter is the sun than the moon? How many magnitudes brighter is the sun than Kruger 60B? By what factor is Kruger 60B fainter then the sun?

Question 3) If the brightest object the human eye can observe (just short of instant blindness!) is the sun with a magnitude of -26.0, and if the faintest star we can see with the naked eye has a magnitude of +7.0, by what factor is the faintest object less discernable than the brightest object we can see? In other words, complete this sentence: "The sun is ……….. times brighter than the faintest star we can see with the naked-eye."

Question 4) The Palomar Sky Survey was a photographic survey of the northern sky performed during the 1940's. Nearly 1 billion objects have been identified in these photographs. The faintest stars have apparent magnitudes of +20.0. How many magnitudes fainter than the faintest naked-eye star (magnitude +7.0) are the stars in the Palomar Survey? By what factor are these stars fainter than naked-eye stars? (http://www-gsss.stsci.edu/gsc/gsc2/GSC2home.htm)

Question 5) In 1996, the Hubble Space Telescope photographed a small piece of the sky with a time-exposure lasting over 10 days. Although only 1,500 galaxies were detected, the faintest of these have a magnitude of +30.0, how many magnitudes fainter than the sun are these faint galaxies? By what factor are these galaxies fainter than the dimmest stare we can see with the human eye? (http://www.stsci.edu/ftp/science/hdf/hdf.html)

Question 6) The Sloan Digital Sky Survey is mapping ¼ of the entire sky. It has performed detailed studies on over 200 million stars and other objects. The faintest objects have magnitudes of +22.0. How much fainter are the galaxies in the Hubble Deep Field in terms of magnitudes? In terms of brightness? (http://www.sdss.org/tour/index.html)

Question 7) Compare the magnitude of the sun with the magnitude of the faintest objects astronomers have detected with the Hubble Space Telescope. How many magnitudes fainter are the Hubble objects than the sun? By what factor is the sun brighter than the faintest known astronomical objects?

Hint: use the relationships that a difference of 5 magnitudes is a factor of 100 in brightness. So, 8 magnitudes = 5 + 3 magnitudes or a factor of 100 x 2.512 x 2.512 x 2.512.

Question 1) The star Sirius has a magnitude of -1.0 while the star Kruger 60B has a magnitude of +11.0. What is the magnitude difference between these stars? How many times fainter is Kruger 60B than Sirius? Answer: a) +11 – (-1) = +12 magnitudes. B) 12 = 5 + 5 + 2 so the factors are 100 x 100 x 2.512 x 2.512 = 63,000 times fainter than Sirius.

Question 2) The apparent magnitudes of the sun and moon are -26.0 and -18.0. How many magnitudes brighter is the sun than the moon? How many magnitudes brighter is the sun than Kruger 60B? By what factor is Kruger 60B fainter then the sun? Answer: -26 – (-18) = -8 magnitudes or ' 8 magnitudes brighter' because the sign is negative. This is a factor of 100 x 2.512 x 2.512 x 2.512 = 1,600.

Question 3) If the brightest object the human eye can observe (just short of instant blindness!) is the sun with -26.0, and if the faintest star we can see with the naked eye is +7.0, by what factor is the faintest object less discernable than the brightest object we can see? In other words, complete this sentence: "The sun is ……….. times brighter than the faintest star we can see with the naked-eye." Answer: The magnitude difference is 26.0 + 7.0 = 33.0. The brightness difference is 33 = 5+5+5+5+5+5+3 or 100x100x100x100x100x100x2.512x2.512x2.512 = 16 trillion times. "The sun is 16 trillion times brighter than the faintest star we can see with the naked eye".

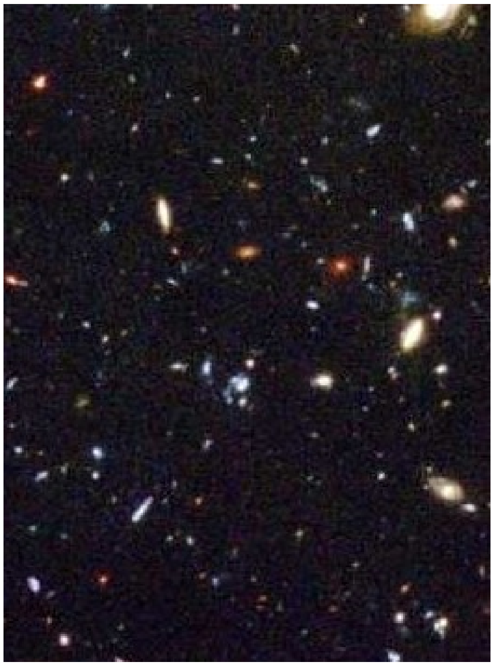
Question 4) The Palomar Sky Survey was a photographic survey of the northern sky performed during the 1940's. Nearly 1 billion objects have been identified in these photographs. The faintest stars have apparent magnitudes of +20.0. How many magnitudes fainter than the faintest naked-eye star (magnitude +7.0) are the stars in the Palomar Survey? By what factor are these stars fainter than naked-eye stars? (http://www-gsss.stsci.edu/gsc/gsc2/GSC2home.htm). Answer: 20-7 = 13 magnitudes fainter or a brightness of 100 x 100 x 2.512 x 2.512 x 2.512 = 160,000 times.

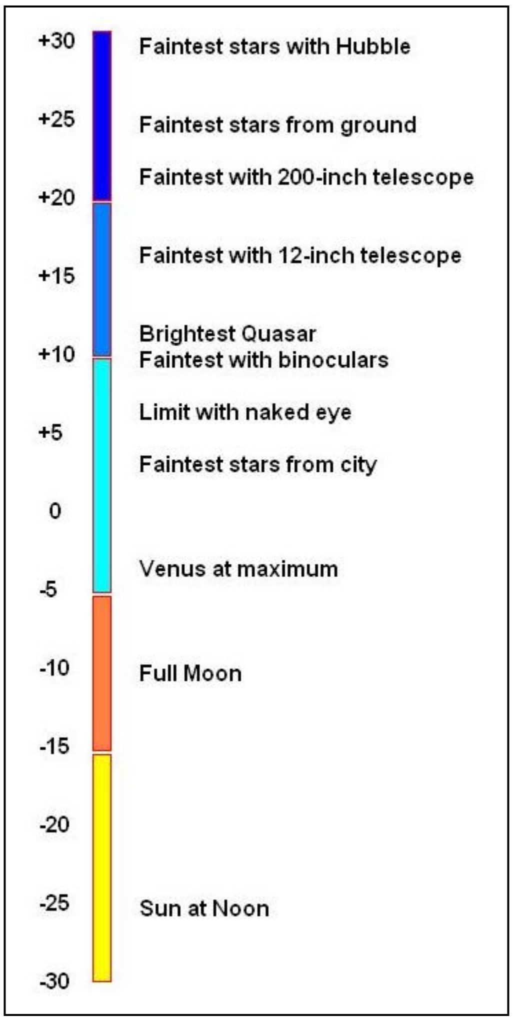
Question 5) In 1996, the Hubble Space Telescope photographed a small piece of the sky with a time-exposure lasting over 10 days. Although only 1,500 galaxies were detected ,the faintest of these have a magnitude of +30.0 ,how many magnitudes fainter than the sun are these faint galaxies? By what factor are these galaxies fainter than the dimmest stare we can see with the human eye? (http://www.stsci.edu/ftp/science/hdf/hdf.html) .Answer: 30 – 7 = 23 magnitudes or 5+5+5+5+3 magnitudes or a brightness of 100 x 100 x 100 x 100 x 2.512 x 2.512 x 2.512 = 1.6 billion times.

Question 6) The Sloan Digital Sky Survey is mapping ¼ of the entire sky. It has performed detailed studies on over 200 million stars and other objects. The faintest objects have magnitudes of +22.0. How much fainter are the galaxies in the Hubble Deep Field in terms of magnitudes? In terms of brightness? (http://www.sdss.org/tour/index.html) Answer: 30 – 22 = 8 magnitudes or a brightness of 100 x 2.512 x 2.512 x 2.512 = 1600.

Question 7) Compare the magnitude of the sun with the magnitude of the faintest objects astronomers have detected with the Hubble Space Telescope. How many magnitudes fainter are the Hubble objects than the sun? By what factor is the sun brighter than the faintest known astronomical objects? Answer: -26 – (30) = -56 magnitudes .The brightness factor is 100x100x100x100x100x100x100x100x100x100x100 x 2.512 = 2.5 x10^{22} times or 25,000 billion billion times fainter.

The Stellar Magnitude Scale





Astronomers measure the brightness of a star in the sky using a magnitude scale. On this scale, the brightest objects have the SMALLEST number and the faintest objects have the LARGEST numbers. It's a 'backwards' scale that astronomers inherited from the ancient Greek astronomer Hipparchus.

The image to the left taken by the Hubble Space Telescope shows hundreds f faint galaxies beyond the Milky Way. The faintest are of magnitude +25.0.

1 – At its brightest, the planet Venus has a magnitude of -4.6. The faintest star you can see with your eye has a magnitude of +7.2. How much brighter is Venus than the faintest visible star?

2 – The full moon has a magnitude of -12.6 while the brightness of the Sun is about -26.7. How many magnitudes fainter is the moon than the Sun?

3 – The faintest stars seen by astronomers with the Hubble Space Telescope are about +30.0. How much fainter are these stars than the Sun?

4 - Jupiter has a magnitude of –2.7 while its satellite, Callisto, has a magnitude of +5.7. How much fainter is the Callisto than Jupiter?

5 – Each step by 1 unit in magnitude equals a brightness change of 2.5 times. A star with a magnitude of +5.0 is 2.5 times fainter than a star with a magnitude of +4.0. Two stars that differ by 5.0 magnitudes are 100-times different in brightness. If Venus was observed to have a magnitude of +3.0 and the full moon had a magnitude of -12.0, how much brighter was the moon than Venus?

Answer Key

1 – At its brightest, the planet Venus has a magnitude of -4.6. The faintest star you can see with your eye has a magnitude of +7.2. How much brighter is Venus than the faintest visible star?

Answer: +7.2 – (-4.6) = +7.2 + 4.6 = **+11.8 magnitudes**

2 – The full moon has a magnitude of -12.6 while the brightness of the sun is about -26.7. How many magnitudes fainter is the moon than the sun?

Answer: -12.6 – (-26.7) = -12.6 + 26.7 = **+14.1 magnitudes fainter.**

3 – The faintest stars seen by astronomers with the Hubble Space Telescope is +30.0. How much fainter are these stars than the sun?

Answer: +30.0 – (-26.7) = +30.0 + 26.7 = **+56.7 magnitudes fainter.**

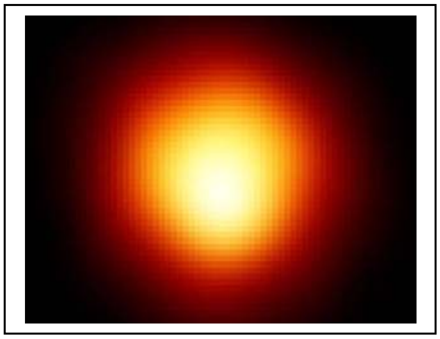
4 - Jupiter has a magnitude of –2.7 while its satellite, Callisto, has a magnitude of +5.7. How much fainter is the Callisto than Jupiter?

Answer: +5.7 – (-2.7) = +5.7 + 2.7 = **+8.4 magnitudes fainter than Jupiter.**

5 – Each step by 1 unit in magnitude equals a brightness change of 2.5 times. A star with a magnitude of +5.0 is 2.5 times fainter than a star with a magnitude of +4.0. Two stars that differ by 5.0 magnitudes are 100-times different in brightness. If Venus was observed to have a magnitude of +3.0 and the full moon had a magnitude of -12.0, how much brighter was the moon than Venus?

Answer: The magnitude difference between them is +15.0, since every 5 magnitudes is a factor of 100 fainter, +15.0 is equivalent to 100x100x100 = 1 million times, so the moon is **1 million times brighter** than Venus.

The relative sizes of the sun and stars



Stars come in many sizes, but their true appearances are impossible to see without special telescopes. The image to the left was taken by the Hubble Space telescope and resolves the red supergiant star Betelgeuse so that its surface can be just barely seen. Follow the number clues below to compare the sizes of some other familiar stars!

Problem 1 - The sun's diameter if 10 times the diameter of Jupiter. If Jupiter is 11 times larger than Earth, how much larger than Earth is the Sun?

Problem 2 - Capella is three times larger than Regulus, and Regulus is twice as large as Sirius. How much larger is Capella than Sirius?

Problem 3 - Vega is 3/2 the size of Sirius, and Sirius is 1/12 the size of Polaris. How much larger is Polaris than Vega?

Problem 4 - Nunki is 1/10 the size of Rigel, and Rigel is 1/5 the size of Deneb. How large is Nunki compared to Deneb?

Problem 5 - Deneb is 1/8 the size of VY Canis Majoris, and VY Canis Majoris is 504 times the size of Regulus. How large is Deneb compared to Regulus?

Problem 6 - Aldebaran is 3 times the size of Capella, and Capella is twice the size of Polaris. How large is Aldebaran compared to Polaris?

Problem 7 - Antares is half the size of Mu Cephi. If Mu Cephi is 28 times as large as Rigel, and Rigel is 50 times as large as Alpha Centauri, how large is Antares compared to Alpha Centauri?

Problem 8 - The Sun is 1/4 the diameter of Regulus. How large is VY Canis Majoris compared to the Sun?

Inquiry: - Can you use the information and answers above to create a scale model drawing of the relative sizes of these stars compared to our Sun.

Answer Key

The relative sizes of some popular stars is given below, with the diameter of the sun = 1 and this corresponds to an actual physical diameter of 1.4 million kilometers.

Betelgeuse	440	Nunki	5	VY CMa	2016	Delta Bootis	11
Regulus	4	Alpha Cen	1	Rigel	50	Schedar	24
Sirius	2	Antares	700	Aldebaran	36	Capella	12
Vega	3	Mu Cephi	1400	Polaris	24	Deneb	252

Problem 1 - Sun/Jupiter = 10, Jupiter/Earth = 11 so Sun/Earth = 10 x 11 = **110 times.**

Problem 2 - Capella/ Regulus = 3.0, Regulus/Sirius = 2.0 so Capella/Sirius = 3 x 2 = **6 times.**

Problem 3 - Vega/Sirius = 3/2 Sirius/Polaris=1/12 so Vega/Polaris = 3/2 x 1/12 = **1/8 times**

Problem 4 - Nunki/Rigel = 1/10 Rigel/Deneb = 1/5 so Nunki/Deneb = 1/10 x 1/5 = **1/50.**

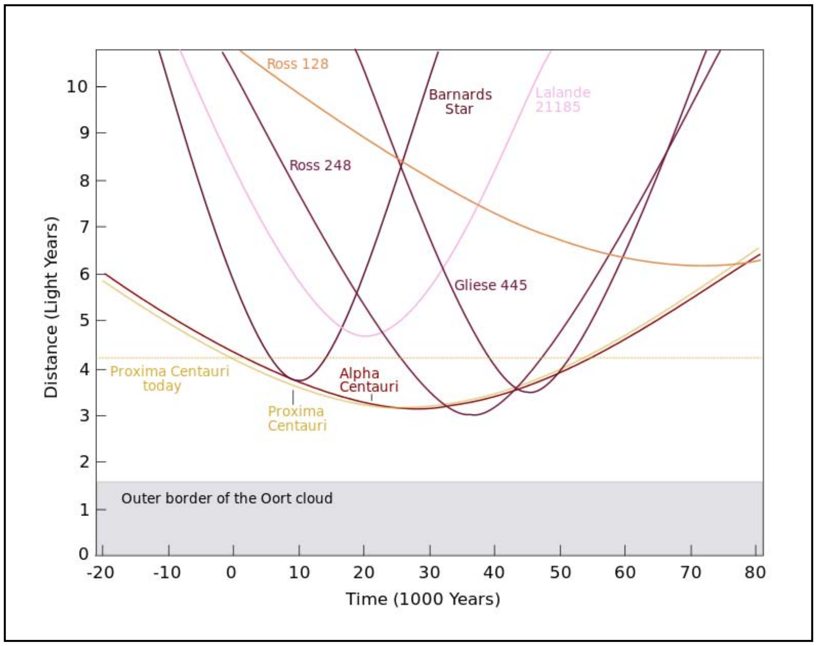
Problem 5 - Deneb/VY = 1/8 and VY/Regulus = 504 so Deneb/Regulus = 1/8 x 504 = **63 times**

Problem 6 - Aldebaran/Capella = 3 Capella/Polaris = 2 so Aldebaran/Polaris = 3 x 2 **= 6 times**.

Problem 7 - Antares/Mu Cep = 1/2 Mu Cep/Rigel = 28 Rigel/Alpha Can = 50, then Antares/Alpha Can = 1/2 x 28 x 50 = **700 times**.

Problem 8 - Regulus/Sun = 4 but VY CMA/Regulus = 504 so VY Canis Majoris/Sun = 504 x 4 = **2016 times the sun's size!**

Inquiry: Students will use a compass and millimeter scale. If the diameter of the Sun is 1 millimeter, the diameter of the largest star VY Canis Majoris will be 2016 millimeters or about 2 meters!



Stars don't stay in one place all the time, but move through space. Over time, our night sky will look very different than it does today. One thing that will change pretty soon is the answer to the question, 'Which star is the closest to our sun?' The figure shows the distances to several nearby stars from 20,000 years ago to 80,000 years in the future, based upon detailed studies of the speeds and current distances of these stars. The picture, by the way, is of the star Proxima Centauri taken by the Hubble Space Telescope. It is currently the closest star to our sun at a distance of 4.243 light years or 40.14 trillion kilometers (24.94 trillion miles).

Problem 1 – For the star Alpha Centauri in the graph, what does its curve represent?

Problem 2 – What will be the distances to the seven stars at a time 30,000 in the future?

Problem 3 – Create a timeline that gives the absolute minimum distances to each star over the next 80, 000 years.

Figure from Wikipedia – 'Proxima Centauri'
Matthews, R. A. J. (1994), "The Close Approach of Stars in the Solar Neighborhood", Quarterly Journal of the Royal Astronomical Society 35: 1–9

http://www.nasa.gov/content/goddard/hubbles-new-shot-of-proxima-centauri-our-nearest-neighbor/index.html
Hubble's New Shot of Proxima Centauri, our Nearest Neighbor
November 1, 2013

Problem 1 – For the star Alpha Centauri in the graph, what does its curve represent?

Answer: The student should mention that it is a mathematical model of the distance between the Sun and Alpha Centauri as it changes between 20,000 years ago and 80,000 into the future.

Problem 2 – What will be the distances to the seven stars at a time 30,000 in the future?

Answer: Students draw a line vertically from the point 'Time=30' and note the distances at which the curves for the stars cross this line:

Alpha Centauri	**D = 3.125 ly**
Proxima Centauri	**D = 3.188 ly**
Ross 248	**D = 3.500 ly**
Lalande 21185	**D = 5.625 ly**
Gliese 445	**D= 6.785 ly**
Ross 128	**D = 8.125 ly**
Barnards Star	**D = 10.00 ly**

Problem 3 – Create a timeline that gives the absolute minimum distances to each star over the next 80, 000 years.

Answer:

Barnards Star	**10,000**	**3.69 ly**
Lalande 21185	**21,000**	**4.69 ly**
Proxima Centauri	**25,000**	**3.13 ly**
Alpha Centauri	**30,000**	**3.13 ly**
Ross 248	**37,000**	**3.00 ly**
Gliese 445	**46,000**	**3.44 ly**
Ross 128	**75,000**	**6.13 ly**

A Pulsar Shot Out from a Supernova Explosion!

The Chandra X-ray Observatory has seen a fast-moving pulsar escaping from a supernova remnant while spewing out a record-breaking jet. This is, to date, the longest object observed in the Milky Way galaxy. The jet is nearly 37 light years long! The pulsar is 60 light years from the supernova remnant.

The supernova remnant, called SNR MSH 11-61A, is in the constellation of Carina, and located 23,000 light years from Earth.

Problem 1 – If one light year equals 9.5×10^{12} kilometers, how far is the pulsar from the supernova remnant?

Problem 2 – How long is the jet in kilometers?

Problem 3 – The supernova is estimated to have exploded about 9,000 years ago. How fast has the pulsar been traveling to get to its current location? Calculate this speed in a) km/sec and b) miles/hour.

Problem 4 – Our sun has a diameter of 1.4 million kilometers. The pulsar has a diameter of only 20 km, but carries twice the mass of our sun. Explain what would happen if the pulsar collided with a star like our sun.

NASA's Chandra Sees Runaway Pulsar Firing an Extraordinary Jet
February 18, 2014
http://www.nasa.gov/press/2014/february/nasas-chandra-sees-runaway-pulsar-firing-an-extraordinary-jet/index.html

Problem 1 – If one light year equals 9.5×10^{12} kilometers, how far is the pulsar from the supernova remnant?

Answer: $60 \times 9.5 \times 10^{12}$ km = **5.7×10^{14} km**.

Problem 2 – How long is the jet in kilometers?

Answer: $37 \times 9.5 \times 10^{12}$ km = **3.5×10^{14} km**.

Problem 3 – The supernova is estimated to have exploded about 9,000 years ago. How fast has the pulsar been traveling to get to its current location? Calculate this speed in a) km/sec and b) miles/hour.

Answer: 1 year = 3.1×10^{7} seconds. Based upon this, the travel time is 9,000 yrs. x 3.1×10^{7} sec/year = 2.8×10^{11} seconds.

A) From Problem 1, the distance is 5.7×10^{14} km, therefore the average speed is 5.7×10^{14} km/2.8×10^{11} sec = **2000 km/sec**.

B) 1 km = 0.62 miles
2000 km/sec x (0.62 miles/1 km) x (3600 sec/1hr) = **4.5 million miles/hour.**

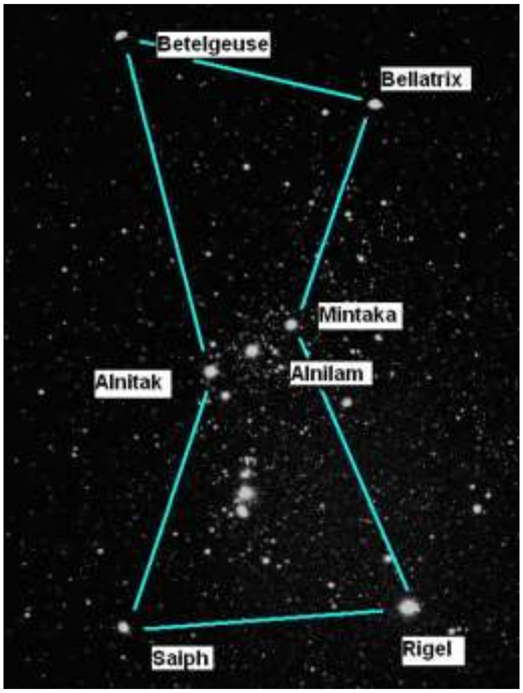
Problem 4 – Our sun has a diameter of 1.4 million kilometers. The pulsar has a diameter of only 20 km, but carries twice the mass of our sun. Explain what would happen if the pulsar collided with a star like our sun.

Answer: Traveling at 2000 km/sec, it would take the pulsar 12 minutes to travel through a star like our sun (1.4 million/2000 km/sec = 700 seconds or 12 minutes). Because the density of the pulsar is over one trillion times that of our sun, it would be like a bullet traveling through a cloud in the sky!

At a great distance, the sun would respond to the gravity of the pulsar and start to deform like a football with the long axis pointed close to the direction of the pulsar. The time it takes the pulsar to travel from Earth's orbit to the sun is only 150 million km/2000 km/sec = 21 hours. An object as huge as the sun would not have much time to deform before the encounter was over. The pulsar would shoot through the interior of the sun and exit the other side before our sun had much of a chance to change its shape. The friction of the pulsar against the gases in the sun would probably increase the temperature along the path by a few thousand degrees. This would not have much effect on the sun that has interior temperatures between 100,000 to 15 million degrees ^{o}C.

Constellations in 3D

A constellation is a pattern of stars that we see in the sky, but actually stars are spread out in space, and what we are seeing is only a geometric projection. This is much like the 2-dimensional photographs that we take, which only capture one perspective in how things actually look in space. For example, below is a photograph of the constellation Orion as it appears in the sky. The table gives the positions and distances to the brightest stars in Orion.



Star	R.A.	Dec.	Distance (parsecs)
Bellatrix	5:20	+6:16	77
Saiph	5:43	-9:42	198
Betelgeuse	5:50	+7:23	222
Rigel	5:10	-8:19	265
Mintaka	5:27	-0:37	380
Alnitak	5:36	-2:00	387
Alnilam	5:31	-1:16	600

1 - Cut out the photograph of Orion and glue it to a piece of stiff cardboard.

2 - Using a scale of 100 parsecs = 1 centimeter, cut 7 pipe cleaners to the lengths corresponding to the distances for each star in the table.

3 - On the back side of the cardboard, push the pipe cleaner through the cardboard at the location of each star so that the pipe cleaners stick out of the back of the card.

4 - Attach a small piece of round clay, or some other marker, to the end of each pipe cleaner to represent the corresponding star.

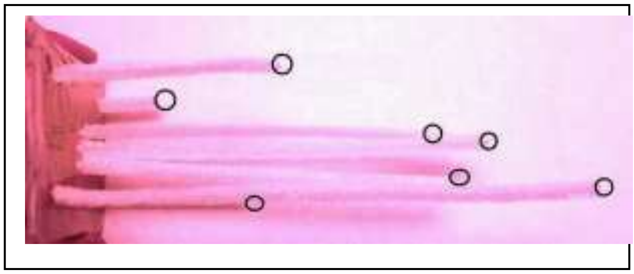
5 – With the cardboard viewed edge-on in your left hand, draw the locations of the stars on graph paper from the following perspectives. (You may also choose to take a digital photo and edit-out the pipe cleaners leaving only the stars in the picture!)

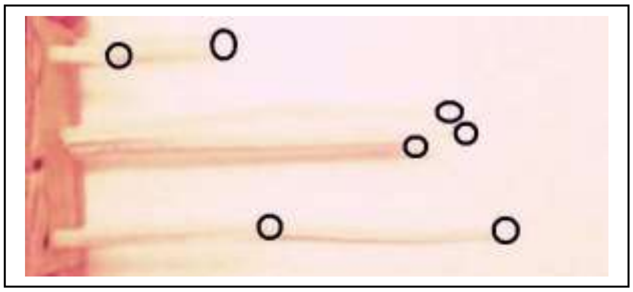
Problem 1 - Construct a bottom-view sketch by rotating the cardboard so that the bottom of the constellation faces you.

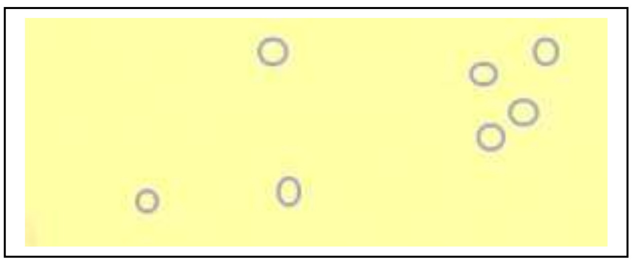
Problem 2 - Construct a side-view sketch.

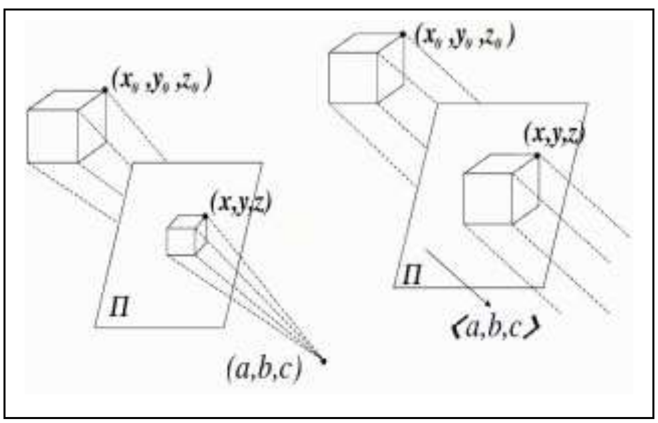
Problem 3 – Construct a top-view sketch

Answer Key









A digital camera was used to take these pictures, from top to bottom, for Problem 1, 2, and 3.

The stars were added in an image editing program.

Students will see that the placement of the stars changes as the viewer's orientation in space changes .This will happen because stars are located 'along the third dimension' at various distances from the viewer.

Orion is an interesting constellation because the three Belt Stars are at nearly the same distance from Earth, so from different orientations, they tend to stay together as a 3-star asterism.

Astronomers call this change of appearance a 'projection effect' because we are projecting the 3-d locations of stars in space onto a 2-d viewing screen.

This figure shows two kinds of projection effects. The first is called Perspective Projection because the lines converge at a point where the viewer is located. The second is called Orthographic Projection because the image on the projection plane is a one-for-one duplicate of the original.
(Courtesy Tom Farmer, Journal of Online Mathematics "Geometric Photo Manipulation")

Name ______________________________

This is the Pleiades star cluster. From resources at your library or the Internet, fill-in the following information:

1) Type of cluster ---------- Number of stars ----------
2) Alternate Names -----
3) Distance in light-years ----------
4) Right Ascension ------- Declination------
5) Constellation ---------
6) Diameter in light years ------
7) Diameter in arcminutes -----
8) Apparent visual magnitude ----------
9) How old is the star cluster? -----------
10) What kinds of stars can you find in the cluster? ------
11) What are some of the names of the stars? --------

From the photograph below, and the cluster's diameter light years, answer these questions:

12) How many stars are probably members of the cluster? --------
13) What is the average distance between the brightest stars? -----
14) What is the typical distance between the stars you counted in question 12? ------
15) Why do the stars have spikes? --------
16) Describe in 500 words why this star cluster is so interesting.

Star clusters up close

1) Type of cluster ---------- **Open, Galactic Cluster** Number of stars ----------**500**
2) Alternate Names -------- **Seven Sisters; Messier-45**
3) Distance in light-years ---------- **425 light years.**
4) Right Ascension ------- **3h 47m** Declination------ **+24d 00'**
5) Constellation --------- **Taurus**
6) Diameter in light years ------ **About 12 light years.**
7) Diameter in arcminutes ----- **110' or about 3 times the diameter of the full moon!**
8) Apparent visual magnitude ---------- **Between +5 and +6.**
9) How old is the star cluster? ----------- **About 100 million years.**
10) What kinds of stars can you find in the cluster? – **Mostly type-B main sequence**
11) What are some of the names of the stars? **Pleione, Atlas, Merope, Alcyone, Electra, Maia, Asterope, Taygeta, Celeano.**

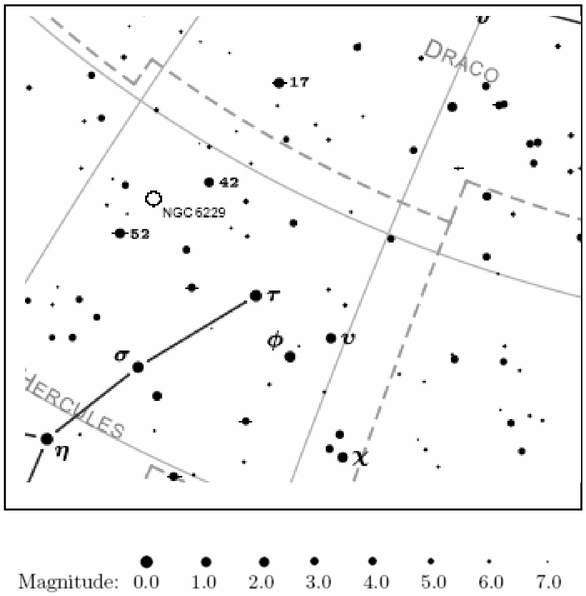
From the photograph below, and the cluster's diameter light years, the size of the cluster is about 120 mm which equals 12 light years for a scale of 10 millimeters to one light year.

12) How many stars in the photo are probably members of the cluster?
    Students should notice that there is a distinct gap between the bright stars and the faint stars in the photo. The faint stars are mostly background stars in the Milky Way unrelated to the cluster. By squinting at the photo, students should be able to find about 100-120 stars.
13) What is the average distance between the brightest stars? ----- **The seven bright stars are about 25 mm or 2.5 light years apart.**
14) What is the typical distance between the stars you counted in question 12? – **With a millimeter ruler, students can measure the spaces between a few dozen stars in the picture and find an average, or they can squint at the picture and use their ruler to estimate the answer. Answers between 5 and 10 mm are acceptable and equal 0.5 to 1 light year.**
15) Why do the stars have spikes? -- **Students may need to investigate this question by using books or the web. Generally, reflecting telescopes produce stellar spikes because the secondary mirror diffracts some of the starlight. The spikes are the four 'legs' used to support the smaller mirror inside the telescope. Refractors do not have spikes and produce round images. In no case do the round images suggest that the star is actually being resolved.**

16) Describe in 500 words why this star cluster is so interesting.

    Students will find many items on the web to form the basis for their essay including: The Pleiades cluster has a long history in mythology. There are many names for this cluster throughout many civilizations and languages. Astronomically, it is the closest open cluster to the sun. It is very young, and contains many stars that are 100 to 1000 times more luminous than the sun. This cluster will eventually fade away in about 250 million years as its brightest stars evolve and die. The cluster still contains the gas left over from its formation, which can be seen as the nebula surrounding the six brightest stars.

One of the earliest methods used by astronomers to study the Milky Way galaxy was to count stars. They would photograph a particular piece of the sky, and count the number of stars they found in a range of apparent magnitude bins. From these 'star gauging' histograms across the sky, they created mathematical models of the shape of the Milky Way that fit the collection of histograms. In this activity, you will learn how to construct a histogram for the stars in a star field, bin them according to their apparent magnitude, and answer a few questions.



Compare the size of each star in the chart with the legend using a millimeter ruler. Count the number of stars with the same magnitude, entering them in the table below.

Magnitude	Tally	Total
+0.0		
+1.0		
+2.0		
+3.0		
+4.0		
+5.0		
+6.0		
+7.0		

From the tabulated data, construct a histogram of the number of stars in each magnitude bin, versus the apparent magnitude of the star.

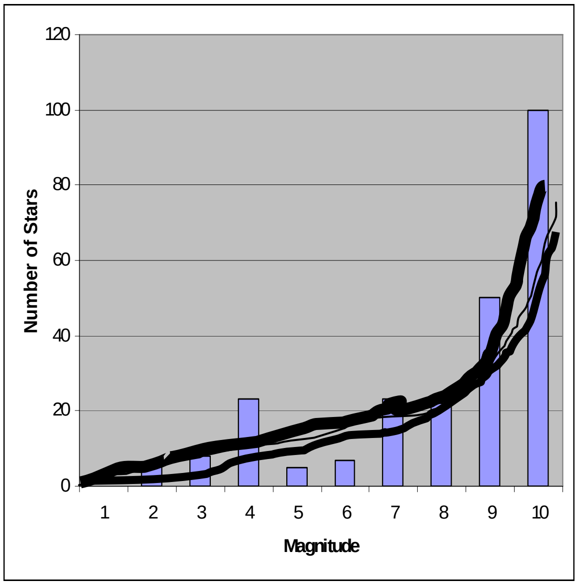
Question 1) How many stars are there in the Draco Field that are fainter than apparent magnitude +0.0?

Question 2) The area of the Draco Field is 17 degrees x 20 degrees. How many square degrees does the field cover?

Question 3) If the total area of the sky is 41,266 square degrees, about how many stars are there in the sky brighter than magnitude +7.0?

Question 4) Using the histogram, what would you predict as the number of stars with a magnitude of +8.0? +9.0?

Question 5) From your answer to Question 4, how many stars are there in the sky brighter than magnitude +8.0?



Magnitude	Tally	Total
0.0	0	0
1.0	I I I I I I	6
2.0	I I I I I I I I	8
3.0	I I I I I I I I I I I I I I I I I I I I I I I	23
4.0	I I I I I	5
5.0	I I I I I I I	7
6.0	I I I I I I I I I I I I I I I I I I I I I I I	23
7.0	I I I I I I I I I I I I I I I I I I I I I I	22

Question 1) How many stars are there in the Draco Field that are fainter than apparent magnitude 0.0? **Answer: Add the histogram bins for all of the magnitudes to get 6+8+23+5+7+23+22 = 139 stars in the Draco Field. Note, counting is a statistical process so have students average their numbers for each bin to get a 'class average' histogram. This also teaches students about averaging, and observer biases.**

Question 2) The area of the Draco Field is 17 degrees x 20 degrees. How many square degrees does the field cover? **Answer: The Draco Field covers an area of 340 square degrees.**

Question 3) If the total area of the sky is 41,266 square degrees, about how many stars are there in the sky brighter than magnitude 7.0? **Answer: There are 41,266/340 = 121.4 patches of the size of the Draco Field across the sky, so if the Draco star counts are typical of all these patches, there are 121.4 x 139 stars = 2,306 stars. Note: This is smaller that the commonly cited 6,000 stars because the Draco Field actually has fewer stars than the average sky patch.**

Question 4) Using the histogram, what would you predict as the number of stars with a magnitude of +8.0? +9.0? **Answer: Each student will extrapolate the trend seen in the bins in a slightly different way. There would be, perhaps 50, +8.0 stars and 100, +9.0 stars. The trend should be that there is a rapid increase in the number of faint stars.**

Question 5) From your answer to Question 4, how many stars are there in the sky brighter than magnitude +9.0? **Answer: Add up the estimate for the stars in the +8.0 and +9.0 bins to your answer to Question 1. Typical answers might be about 250 brighter than +9.0. Based on the answer to question 4, you can estimate about 121.4 x 250 or about 30,000 stars. This estimate will vary depending on how students decide to extrapolate the histogram from apparent magnitudes +5.0 to +6.0 to predict the fainter bin values.**

Spitzer Explores a Dusty Young Star

This is an image taken by the Spitzer Space telescope in the infrared part of the electromagnetic spectrum. Instead of seeing the light from stars, it sees mainly the light from heated dust grains in space, glowing with temperatures between 100 to 200 K degrees.

These bright young stars are found in a rosebud-shaped (and rose-colored) nebulosity known as NGC 7129. The star cluster and its associated nebula are located at a distance of 3300 light-years in the constellation Cepheus. A recent census of the cluster reveals the presence of 130 young stars. The stars formed from a massive cloud of gas and dust. Most stars in our Milky Way galaxy, including our own sun, are thought to form in such clusters.

Most of the infrared light comes from a deeply embedded proto-star called FIR-2, which produces 430 times the total power of the sun, with a temperature of about 35 K. We can use this information to estimate the mass of this nebula!

Problem 1 - A single dust grain is about 0.2 microns in diameter, and has a density of about 2 grams/cm^3. What is the total mass, in grams, of this dust grain if it has a spherical shape?

Problem 2 - What is the total power produced by this infrared source if the power from the sun is 3.8×10^{33} ergs/sec?

Problem 3 - A single dust grain, 0.2 microns in diameter, at a temperature of 35 K, emits about $7.0 \times 10^{--13}$ ergs/sec of power. How many dust grains are needed to produce the infrared power observed from FIR-2?

Problem 4 - From your answer to Problem 1 and 3, what is the total mass of dust grains involved in producing the infrared light from FIR-2; A) in grams? B) In units of the sun's mass which is 1.9×10^{33} grams?

Problem 5 - By mass, the interstellar medium consist of 99% gas and 1% dust grains. If the gas within FIR-2 has the same composition, what is the total mass of the interstellar medium within FIR-2 in Solar Masses?

Answer Key:

Problem 1 - A single dust grain is about 0.2 microns in diameter, and has a density of about 2 grams/cm^3. What is the total mass, in grams, of this dust grain if it has a spherical shape?

Answer: Radius = 1.0 microns,
Mass = Volume x Density
= 4/3 π R^3 x Density
= 4/3 x 3.14 x (0.1 x 10^{-4})^3 x 2.0
= 8.4 x 10^{-15} grams

Problem 2 - What is the total power produced by this infrared source if the power from the sun is 3.8 x 10^{33} ergs/sec?

Answer: Power = 430 x 3.8 x 10^{33} ergs/sec **= 1.6 x 10^{36} ergs/sec**

Problem 3 - A single dust grain, 0.2 microns in diameter, at a temperature of 35 K, emits about 7.0 x 10^{-13} ergs/sec of power. How many dust grains are needed to produce the infrared power observed from FIR-2?

Answer: Number = 1.6 x 10^{36} ergs/sec / 7.0 x 10^{-13} ergs/sec/dust grain
= 2.3 x 10^{48} dust grains

Problem 4 - From your answer to Problem 1 and 3, what is the total mass of dust grains involved in producing the infrared light from FIR-2; A) in grams? B) in units of the sun's mass which is 1.9 x 10^{33} grams?

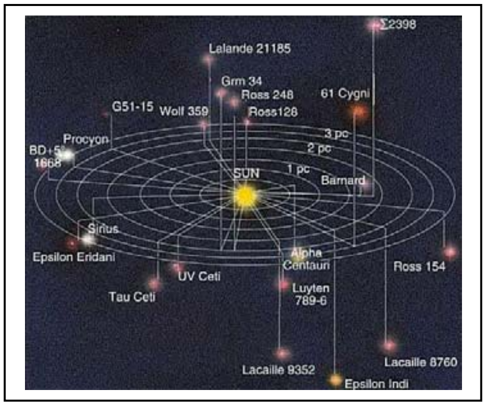
Answer: A) 8.3 x 10^{-15} grams / dust grain x 2.3 x 10^{48} dust grains **= 1.9 x 10^{34} grams**
B) 1.9 x 10^{34} grams / 1.9 x 10^{33} grams = **10.0 Solar Masses.**

Problem 5 - By mass, the interstellar medium consist of 99% gas and 1% dust grains. If the gas within FIR-2 has the same composition, what is the total mass of the interstellar medium within FIR-2 in solar Masses?

Answer - 100 x 10.0 = **1000 Solar Masses**.

The actual dust mass has been estimated as about 6 solar masses by C. Eiroa (AA 1998, v. 335, p. 243. and Muzerolle, et al, 2004, ApJ Suppl. V. 154, p.379.) The largest uncertainty in the problem is the size of the dust grain and the infrared radiation power emitted by the dust grain. This problem assumed that the dust grain size is typical of what is found for interstellar dust grains. But in the environment of the protostar, dust grain sizes are expected to vary due to grain growth. Also, the reflectivity (albedo) of the dust grain depends on its composition and size in a complex way.

The Solar Neighborhood within 17 light years.



There are 45 stars within 17 light years of the sun. It is very hard to appreciate just how big space is. By considering our neighborhood in the Milky Way, we can start to get a sense of scale.

Local star map courtesy NASA/JPL

The table below gives the names, distances and angles to 11 of the most well-known neighbors to the sun. Although stars are spread out in 3-dimensional space, we will compress these distances to their 2-dimensional equivalents. On a 2-dimensional grid, place a dot at the Origin to represent the Sun. With a metric ruler and a protractor, plot the stars on a piece of paper and label each star. Use a scale of 1 centimeter = 1 light year.

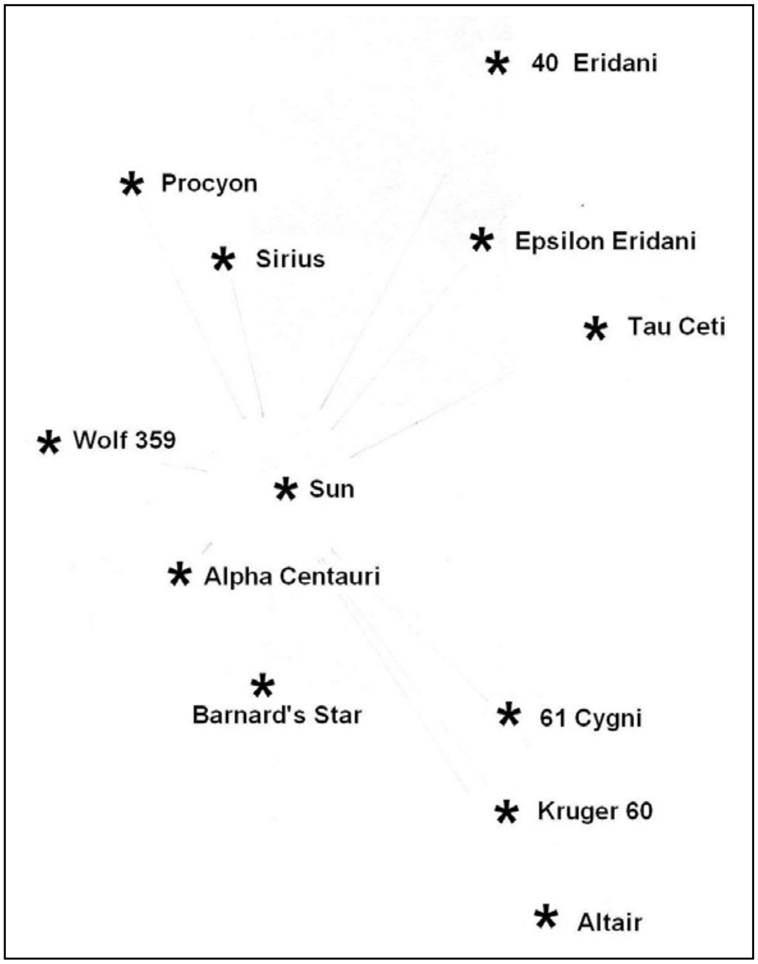
Name	Angle	Distance
Alpha Centauri	220	4.3 light years
Barnard's Star	270	5.9
Wolf 359	170	7.6
Sirius	100	8.6
Epsilon Eridani	50	10.7
61 Cygni	315	11.2
Procyon	115	11.4
Tau Ceti	25	11.9
Kruger 60	335	12.8
40 Eridani	60	15.9
Altair	300	16.6

Problem 1 - What is the distance between Sirius and Altair?

Problem 2 - What is the distance between Kruger 60 and Altair?

Problem 3 - Can you find a pair of stars that are closer to each other than either of them are to the Sun?

Problem 4 - If you were starting from Earth, what is the shortest journey you could make that would visit all of the stars in your map?

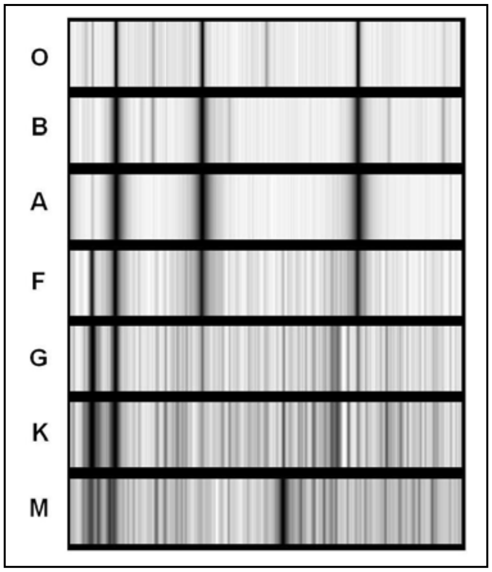


Problem 1 - At the scale of 1 cm = 1 light year, they are separated by about 24 centimeters or **24 light years.**

Problem 2 - They are separated by 7 centimeters or **7 light years.**

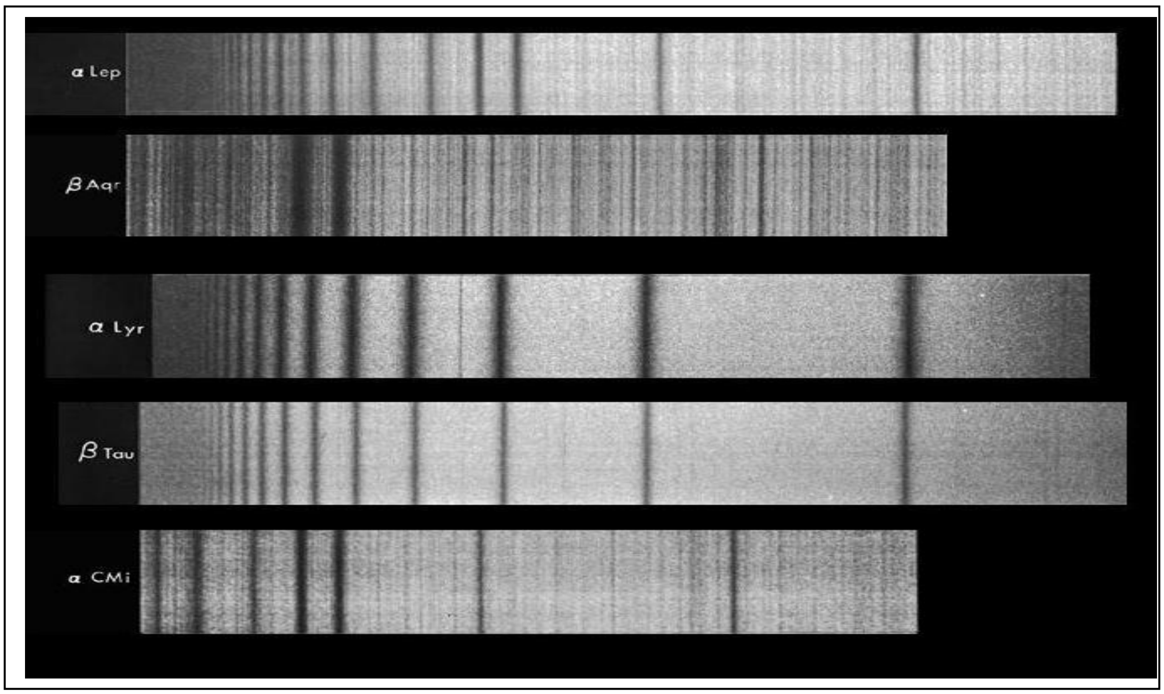
Problem 3 - For example: Procyon and Sirius are 4 cm apart or **4 light years.** 61 Cygni and Kruger 60 are 3 cm apart or **3 light years.**

Problem 4 - One path consists of Sun-Barnards Star-Altair-Kruger 60- 61 Cygni-Tau Ceti-Epsilon Eridani- 40 Eridani-Sirius-Procyon-Wolf 359-Alpha Centauri -Sun. The total distance is about **80 light years.**



The advent of the spectroscope in the 1800's allowed astronomers to study the temperatures and compositions of stars, and to classify stars according to their spectral similarities. At first, 26 classes were defined; one for each letter in the alphabet. But only 7 are actually major classes, and these survive today as the series 'O, B, A, F, G, K, M'. This series follows decreasing star temperatures from 30,000 K (O-type) to 3,000 K (M-type).

Images courtesy: Helmut Abt (NOAA).



Problem 1 - Sort the five stellar spectra according to their closest matches with the standard spectra at the top of the page. (Note, the spectra may not be to the same scale, aligned vertically, and may even be stretched!)

Problem 2 - The star α Lyr (Alpha Lyra) has a temperature of 10,000 K and β Aqr (Beta Aquarii) has a temperature of 5,000 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

Problem 3 - What spectral types appear to be missing from the sample of stars?

Problem 1 - This is designed to be a challenge! Student strategies should include looking for general similarities first. One obvious way to group the spectra in terms of the increasing (or decreasing!) number of spectral lines.

Alpha Lyr (Alpha Lyra) and Beta Tau (Beta Tauri) can be grouped together because of the strong lines that appear virtually alone in the spectra (these are hydrogen lines). The second grouping, Group 2, would include Alpha Lep (Alpha Leporis) and Alpha CMi (Alpha Canis Minoris) because they have a different pattern of strong lines than Group 1 but also the hint of many more faint lines in between. The last 'group' would be for Beta Aqr (Beta Aquarii) because it has two very strong lines close together (the two lines of the element calcium), but many more lines that fill up the spectrum and are stronger than in Group 2.

Group 1:

Vega (Alpha Lyra) - A type star
Alnath (Beta Tauri) - B type star

Group 2:

Arneb (Alpha Leporis) - F type star
Procyon (Alpha Canis Minoris) - F type star

Group 3:

Sadalsuud (Beta Aquarii) - G type star

Comparing Group 1, 2 and 3 with the standard chart, Group 1 consists of the B and A type stars, in particular Alpha Lyra has the thick lines of an A-type star, and Beta Tauri has the thinner lines of a B-type star; Group 2 is similar to the spectra of the F-type stars, and Group 3 is similar to the G-type stars.

Problem 2 - Vega has a temperature of 10,000 K and Beta Aquari has a temperature of 5,500 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

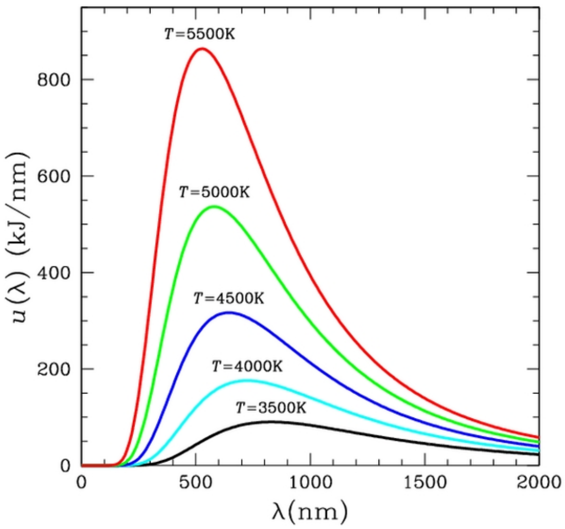
Answer: What you should notice is that, as the temperature of the star gets cooler, the number of atomic lines in this part of the spectrum (visible light) increases.

Problem 3 - What spectral types appear to be missing from the sample of stars?

Measuring Star Temperatures

Careful measurements of a star's light spectrum gives astronomers clues about its temperature. For example, incandescent bodies that have a red glow are 'cool' while bodies with a yellow or blue color are 'hot'. This can be made more precise by measuring very carefully exactly how much light a star produces at many different wavelengths.

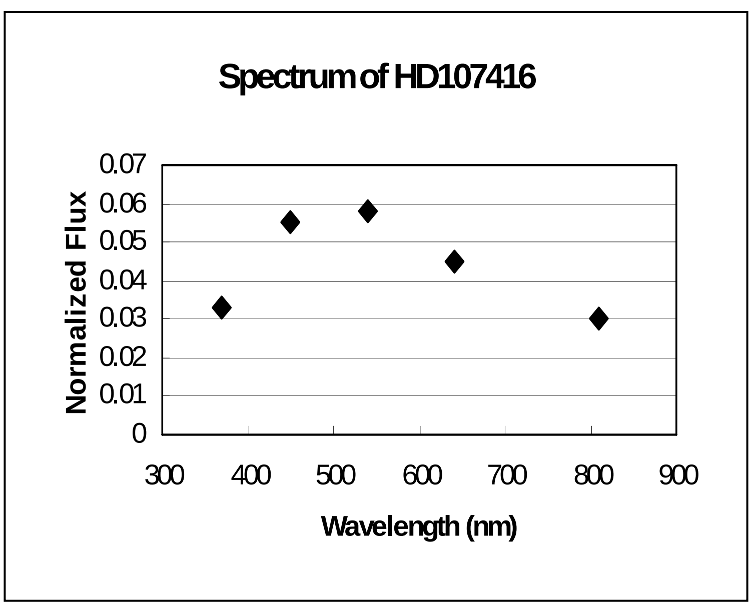
In 1900, physicist Max Planck worked out the mathematical details for how to exactly predict a body's spectrum once its temperature is known. The curve is therefore called a Planck 'black body' curve. It represents the brightness at different wavelengths of the light emitted from a perfectly absorbing 'black' body at a particular temperature.



From the mathematical properties of the Planck Curve, it is possible to determine a relationship between the temperature of the body and the wavelength where most of its light occurs - the peak in the curve. This relationship is called the Wein Displacement Law and looks like this:

$$\text{Temperature} = \frac{2897000 \text{ Kelvins}}{\text{Wavelength}}$$

Where the temperature will be in units of Kelvin degrees, and the wavelength will be in units of nanometers.



The lower plot shows measurements of the spectrum of the star HD107146. The horizontal axis is in units of nanometers (nm).

Problem 1 - Based on the overall shape of the curve, and the wavelength where most of the light is being emitted, use the Wein Displacement Law to determine the temperature of HD107146.

Problem 2 - What would be the peak wavelengths of the following stars in nanometers.

A) Antares .... 3,100 K
B) Zeta Orionis……..……. 30,000 K
C) Vega ………………….. 9,300 K
D) Regulus………………... 13,000 K
E) Canopus………………. 7,300 K
F) OTS-44 brown dwarf… 2,300 K
G) Sun……………………. 5,770 K

Answer Key:

Problem 1 - The peak of the curve is **near 500** nanometers. The temperature is 2897000 / 500 = **5,794 K**.

Problem 2 - What would be the peak wavelengths of the following stars in Angstroms:

Answer:

A)	Antares occurs at…….	2897000/3100 K	=	934 nanometers
B)	Zeta Orionis is at……..	2897000/30,000 K	=	97 nanometers
C)	Vega …………………..	2897000/9,300 K	=	311 nanometers
D)	Regulus…………..……	2897000/13,000 K	=	223 nanometers
E)	Canopus……………….	2897000/7,300 K	=	397 nanometers
F)	OTS-44 brown dwarf…	2897000/2,300 K	=	1260 nanometers
G)	Sun…………………….	2897000/5770Ak	=	502 nanometers

Figure Credits:

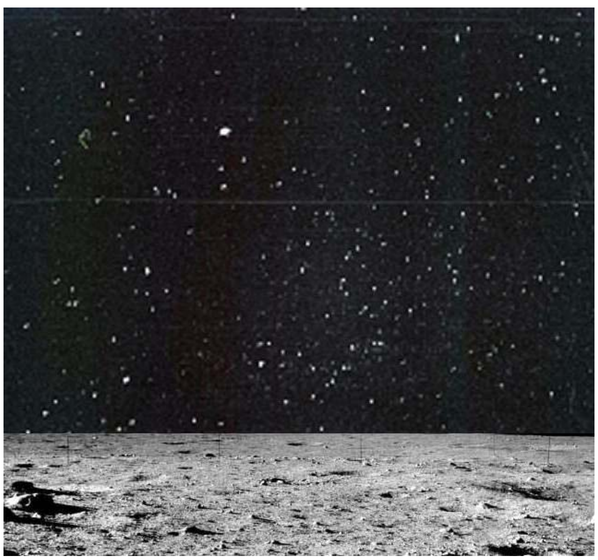
The spectrum of HD 107146 is adapted from a paper by Williams et al. published in the Astrophysical Journal, 2004 vol. 604 page 414. The graph of Planck curves is from Wikimedia and is copyright-free.

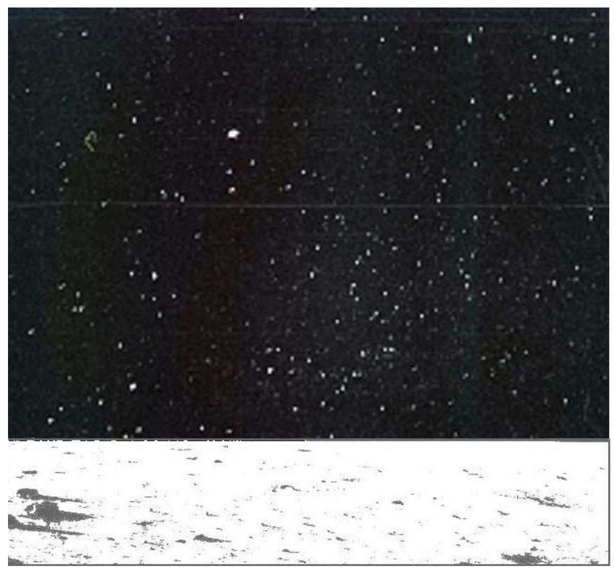
Where Did All the Stars Go?

Have you ever looked closely at NASA photographs from space and wondered where the stars went?

To the left is an Apollo-11 photo taken by astronauts on the surface of the moon. Notice the sky has no stars! The re-touched photo on the bottom-left gives an impression of the stars that a simple $100 camera would see if it used a 'timed exposure' of about 20 seconds. So why did the very expensive camera used by the Apollo-11 astronauts show not a single star?

The re-touched photo below shows what might happen to the lunar surface detail with a 20-second exposure.





A camera light meter measures the brightness of an object. Let's indicate brightness by the unit 'cents/second'. For example, a faint object might have a brightness of 10 cents/second while a bright object has 10,000 cents/second.

Problem 1 - If the stars in the Apollo photo have a brightness of 2.5 cents/sec, how many cents will be collected in a 20-second time-exposure?

Problem 2 - If the lunar surface has a brightness of 500 cents/second, how many cents will be collected in a 20-second exposure?

Problem 3 - If the lunar surface is scaled to a camera contrast setting of 100%, A) How bright, in cents, is a 1% contrast change? B) What contrast change do the stars represent?

Problem 4 - If the image is set to only record contrast changes of 1% or greater to bring out detail on the lunar surface, will the stars be visible? Explain.

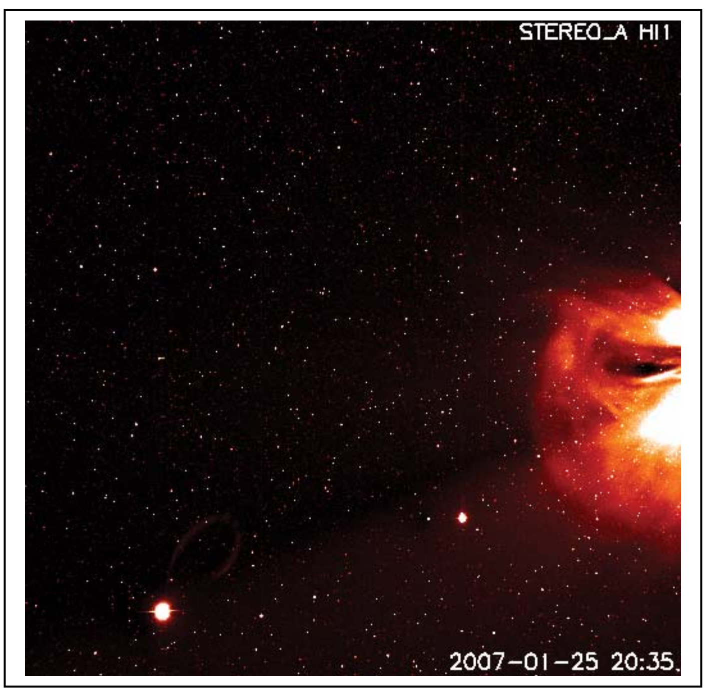
Answer Key

Problem 1 - If the stars in the Apollo photo have a brightness of 2.5 cents/sec, how many cents will be collected in a 20-second time-exposure? Answer: Multiply the rate at which the cents are accumulated (2.5 cents/sec) by the elapsed time (20 sec) to get 2.5 cents/sec x 20 sec **= 50 cents**.

Problem 2 - If the lunar surface has a brightness of 500 cents/second, how many cents will be collected in a 20-second exposure? Answer: Multiply the rate at which the cents are accumulated (500 cents/sec) by the elapsed time (20 sec) to get 500 cents/sec x 20 sec = **10,000 cents**.

Problem 3 - If the lunar surface is scaled to a camera contrast setting of 100%, A) how bright, in cents, is a 1% contrast change? Answer: Since 10,000 cents = 100%, that means that 1% will equal 10,000 cents/100 = 100 cents. B) What contrast change do the stars represent? Answer: The stars produce 50 cents, but in the same amount of time (20 seconds) the lunar landscape produces 10,000 cents, so the stars represent a contrast change of only (50 cents/10000 cents) x 100% **= 0.5 %.**

Problem 4 - If the image is set to only record contrast changes of 1% or greater to bring out detail on the lunar surface, will the stars be visible? Explain. Answer: **If 0% represents black and 100% represents white, the smallest intensity in the image (1%) will correspond to 100 cents, but the stars represent a contrast change of 0.5% or 50 cents, which is smaller than the 100 cents (1%) contrast that the image can register on this camera setting (20 seconds with 10,000 cents = 100%). So the stars will be replaced by 'black' and will be eliminated from the image so that scientists can study the landscape details on the lunar surface instead.**



This is an image taken by NASA's STEREO satellite of a coronal mass ejection (CME) from the sun. The gas is very faint, so the camera had to be designed to detect only faint light, rather than bright sources. Notice that the CME image has plenty of background stars that can in some cases be seen through the translucent gases. Had the camera been designed differently, there would have been no stars in the picture, and only the brightest portions of the CME would have been visible in the photograph.

Star Magnitudes and Multiplying Decimals

The brightness of a star is indicated by the Apparent Magnitude scale, which leads to some interesting math!

Rule 1: The larger the number, the fainter the star. For example, Procyon has a magnitude of +0.4 while Wolf-359 has a magnitude of +13.5, so Wolf-359 is fainter than Procyon.

Rule 2: Each difference, by one whole magnitude, represents a brightness change of 2.51 times. For example, the star Tau Ceti has a magnitude of +3 while Fomalhaut has a magnitude of +1. The brightness difference between them is +3 - (+1) = 2 magnitudes or a factor of 2.51 x 2.51 = 6.3 times.

Problem 1 - UV Ceti has a magnitude of +13.0 while Wolf-294 has a magnitude of +10.0. Which star is fainter, and by what factor?

Problem 2 - Sirius has a magnitude of -1 and Mintaka has a magnitude of +2, which star is faintest. What is the magnitude difference, and by what factor do they differ?

Problem 3 - Betelgeuse has a magnitude of +1 and 70 Ophiuchi has a magnitude of +6. What is the magnitude difference and by what factor do they differ?

Problem 4 - Capella has a magnitude of +0 and Barnard's Star has a magnitude of +9. What is the magnitude difference, and by what factor do they differ?

Problem 5 - Sort the stars in the table so that the brightest star appears first, and the faintest star appears last.

Star	Apparent Magnitude
Ross-47	+11.6
Antares	+1.0
Alpha Centauri	-0.1
36 Ophichi	+5.1
Beta Hydra	+2.7
Rigel	+0.1
Eta Cassiopeia	+3.5
Sirius	-1.5
Wolf-359	+13.5
Kruger-60	+9.9

Answer Key

Problem 1 - UV Ceti has a magnitude of +13.0 while Wolf-294 has a magnitude of +10.0. Which star is fainter, and by what factor?
Answer: UV Ceti has the larger apparent magnitude so it is the fainter star. They differ by +13 - +10 = +3 magnitudes, which is a factor of 2.51 x 2.51 x 2.51 = **15.8 times.**

Problem 2 - Sirius has a magnitude of -1 and Mintaka has a magnitude of +2, which star is faintest. What is the magnitude difference, and by what factor do they differ?
Answer: Mintaka has the larger apparent magnitude so it is the fainter star. They differ by +2 - (-1) = +3 magnitudes, which is a factor of 2.51 x 2.51 x 2.51 = **15.8 times.**

Problem 3 - Betelgeuse has a magnitude of +1 and 70 Ophiuchi has a magnitude of +6. What is the magnitude difference and by what factor do they differ?
Answer: 70 Ophichi has the larger apparent magnitude so it is the fainter star. They differ by +6 - (+1) = +5 magnitudes, which is a factor of 2.51 x 2.51 x 2.51 x 2.51 x 2.51 **= 100 times**.

Problem 4 - Capella has a magnitude of +0 and Barnard's Star has a magnitude of +9. What is the magnitude difference, and by what factor do they differ?
Answer: Barnard's Star has the larger apparent magnitude so it is the fainter star. They differ by +9 - (+0) = +9 magnitudes, which is a factor of 2.51 x 2.51 x 2.51x 2.51 x 2.51 x 2.51 x 2.51 x 2.51 x 2.51 = **3,950 times.**

Problem 5 - Sort the stars in the table so that the brightest star appears first, and the faintest star appears last.

Star	Apparent Magnitude
Sirius	-1.5
Alpha Centauri	-0.1
Rigel	+0.1
Antares	+1.0
Beta Hydra	+2.7
Eta Cassiopeia	+3.5
36 Ophichi	+5.1
Kruger-60	+9.9
Ross-47	+11.6
Wolf-359	+13.5

Pulsars and Simple Equations

The Crab Nebula is all that remains of a supernova explosion 900 years ago. At the center of this picture, taken by the Hubble Space Telescope, is a rapidly spinning pulsar which flashes 30 times a second as it spins.

A pulsar is a rapidly spinning star. It's about the same size as Earth, but it contains as much mass as an entire normal star like the sun.

When they are formed, they spin at an unimaginable pace: nearly 30 times every second. As they grow older, they slow down.

Astronomers have measured the spinning of two pulsars: The Crab Nebula pulsar, and AP 2016+28. They used this data to create two simple equations that predict the pulsar's spin rates in the future.

Crab Nebula Pulsar: **P = 0.033 + 0.000013 T**

AP 2016+28 Pulsar: **P = 0.558 + 0.0000000047 T**

P is the time, in seconds, it takes the pulsar to spin once-around on its axis. T is the number of years since today.

Problem 1: Evaluate each equation for P for a time that is 10,000 years in the future. How fast are the two pulsars spinning at that time?

Problem 2: How long will it take the Crab Pulsar to slow to a period exactly twice its current period of 0.033 seconds?

Problem 3: How long will it take Pulsar AP 2016+28 to slow to a period of 1.116 seconds (exactly twice its current period of 0.558 seconds)?

Problem 4: How many years ago was the pulsar AP 2016+28 spinning at the same rate as the Crab Pulsar?

Problem 5: How long will it take each pulsar to slow to a period of exactly 2.0 seconds?

Problem 6: In how many years from now will the two pulsars be spinning at exactly the same rates?

Answer Key

Crab Nebula Pulsar: **P = 0.033 + 0.000013 T**

AP 2016+28 Pulsar: **P = 0.558 + 0.0000000047 T**

Problem 1: Evaluate each equation for P for a time that is 10,000 years in the future. How fast are the two pulsars spinning at that time?

Crab: P = 0.033 + 0.000013 (10,000 years) = 0.033 + 0.13 = **0.163 seconds.**
AP 2016+28 Pulsar: P = 0.558 + 0.0000000047 (10,000) = 0.558 + 0.000047 = **0.558047 seconds**.

Problem 2: How long will it take the Crab Pulsar to slow to a period exactly twice its current period of 0.033 seconds?

P = 2 x 0.033 = 0.066 seconds. So 0.066 = 0.033 + 0.000013 T, and solving for T we get T = (0.033/0.000013) = **2,500 years from now**.

Problem 3: How long will it take Pulsar AP 2016+28 to slow to a period exactly twice its current period of 0.558 seconds?
P = 2 x 0.558 = 1.116 seconds. So 1.116 = 0.558 + 0.0000000047 T, and solving for T we get T = (0.558/0.0000000047) = **119 million years from now**.

Problem 4: How many years ago, was the pulsar AP 2016+28 spinning at the same rate as the Crab Pulsar?
P = 0.033 seconds = 0.558 + 0.0000000047 T, so
0.033 - 0.558 = 0.0000000047 T
-0.525 = 0.0000000047 T
-0.525/0.0000000047 = T and so **T = 112 million years ago**.
Note that the negative sign for T means that the time was before today (year-zero).

Problem 5: How long will it take each pulsar to slow to a period of exactly 2.0 seconds?
Crab Pulsar: 2.0 = 0.033 + 0.000013 T so T = **151,000 years**.
AP 2016+28: 2.0 = 0.558 + 0.0000000047 T so T = **307 million years**

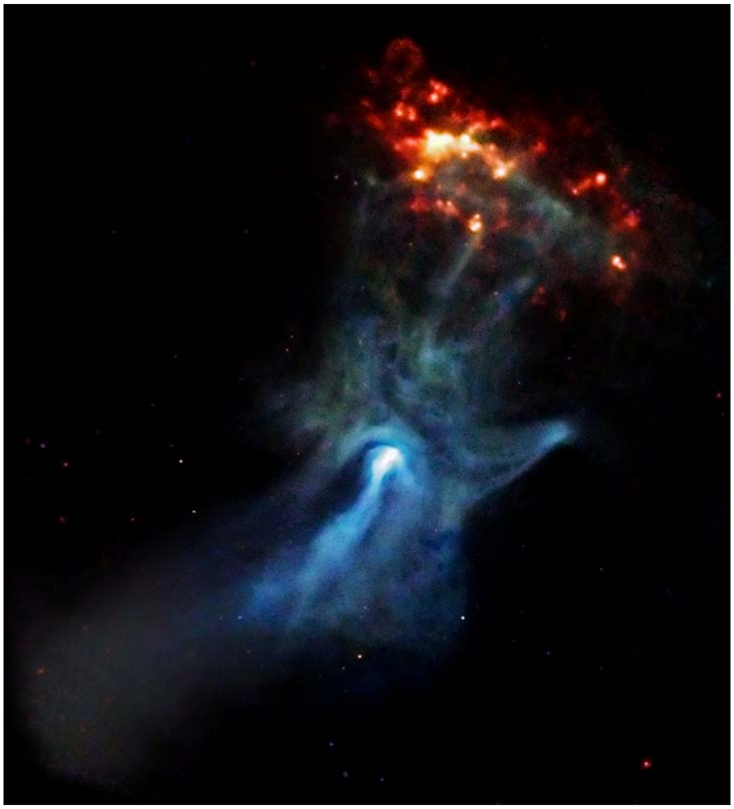
Problem 6: In how many years from now A) will the two pulsars be spinning at exactly the same rates? B) What will be their spin rates?

This requires that students set the equation for P in the Crab Nebula Pulsar to the P in the equation for AP 2016+28, and solve for T.

0.558 + 0.0000000047 T = 0.033 + 0.000013 T
0.558 - 0.033 = 0.000013 T - 0.0000000047 T
0.525 = 0.0000129953 T
T = 0.525/0.0000129953 so T = **+ 40,400 years from now.**

B) P = 0.033 + 0.000013 (40400) = **0.558 seconds**.

The Hand of Chandra!



A small, dense object only twelve miles in diameter is responsible for this beautiful X-ray nebula that spans 150 light years and resembles a human hand!

At the center of this image made by NASA's Chandra X-ray Observatory is a very young and powerful pulsar known as PSR B1509-58.

The pulsar is a rapidly spinning neutron star which is spewing energy out into the space around it to create complex and intriguing structures, including one that resembles a large cosmic hand.

Astronomers think that the pulsar and its nebula is about 1,700 years old, and is located about 17,000 light years away (e.g. 5,200 parsecs). Finger-like structures extend to the north, apparently energizing knots of material in a neighboring gas cloud known as RCW 89. The transfer of energy from the wind to these knots makes them glow brightly in X-rays (orange and red features to the upper right).

Problem 1 - This field of view is 19 arcminutes across. Using similar triangles and proportions, if 1 arcminute at a distance of 1,000 parsecs equals a length of 0.3 parsecs how wide is the image in parsecs?

Problem 2 - Measure the width of this image with a millimeter ruler. What is the scale of this image in parsecs per millimeter? How far, in light years, is the bright spot in the 'palm' where the pulsar is located, from the center of the ring-like knots in RCW 89? (1 parsec = 3.26 light years). Round your answer to the nearest light year.

Problem 3 - If the speed of the plasma is 10,000 km/sec, how many years did it take for the plasma to reach RCW-89 if 1 light year = 9.5×10^{12} kilometers, and there are 3.1×10^{7} seconds in 1 year?

Answer Key

Problem 1 - This field of view is 19 arcminutes across. Using similar triangles and proportions, if 1 arcminute at a distance of 1,000 parsecs equals a length of 0.3 parsecs how wide is the image in parsecs?

Answer: If 0.3 parsecs seen from a distance of 1,000 parsecs subtends an angle of 1 arcminute, then 1 arcminute at a distance of 5,200 parsecs will subtend

$$\frac{X}{0.3 \text{ parsecs}} = \frac{5{,}200 \text{ parsecs}}{1{,}000 \text{ parsecs}}$$

so that X = 0.3 x (5200/1000) = 1.6 parsecs. Then 19 arcminutes will subtend

$$\frac{X}{1.6 \text{ parsecs}} = \frac{19 \text{ arcminutes}}{1 \text{ arcminute}}$$

so that X = 1.6 x (19/1) = 30.4 parsecs.

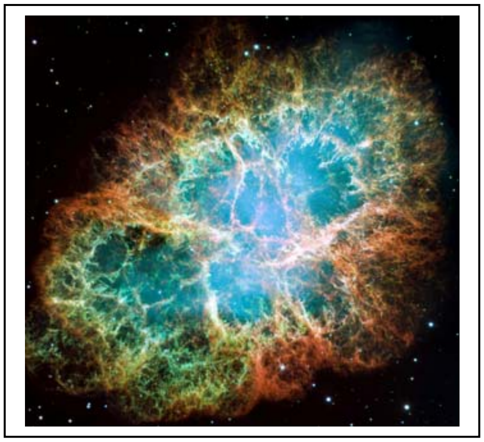
Problem 2 - Measure the width of this image with a millimeter ruler. What is the scale of this image in parsecs per millimeter? How far, in light years, is the center of the 'palm' where the pulsar is located, from the ring-like knots in RCW 89? (1 parsec = 3.26 light years).

Answer: The width is about 96 millimeters. The scale is then 30.4 parsecs/96 millimeters or 0.32 parsecs/millimeter. The distance from the bright spot to the ring of knots is about 40 millimeters. At the scale of the image, this equals 40 mm x 0.32 pc/mm = 12.8 parsecs. Converting this to light years we get 12.8 parsecs x (3.26 light years/parsec) or 12.8 x 3.26 = 41.728 light years, which is rounded to 42 light years.

Problem 3 - If the speed of the plasma is 10,000 km/sec, how many years did it take for the plasma to reach RCW-89?

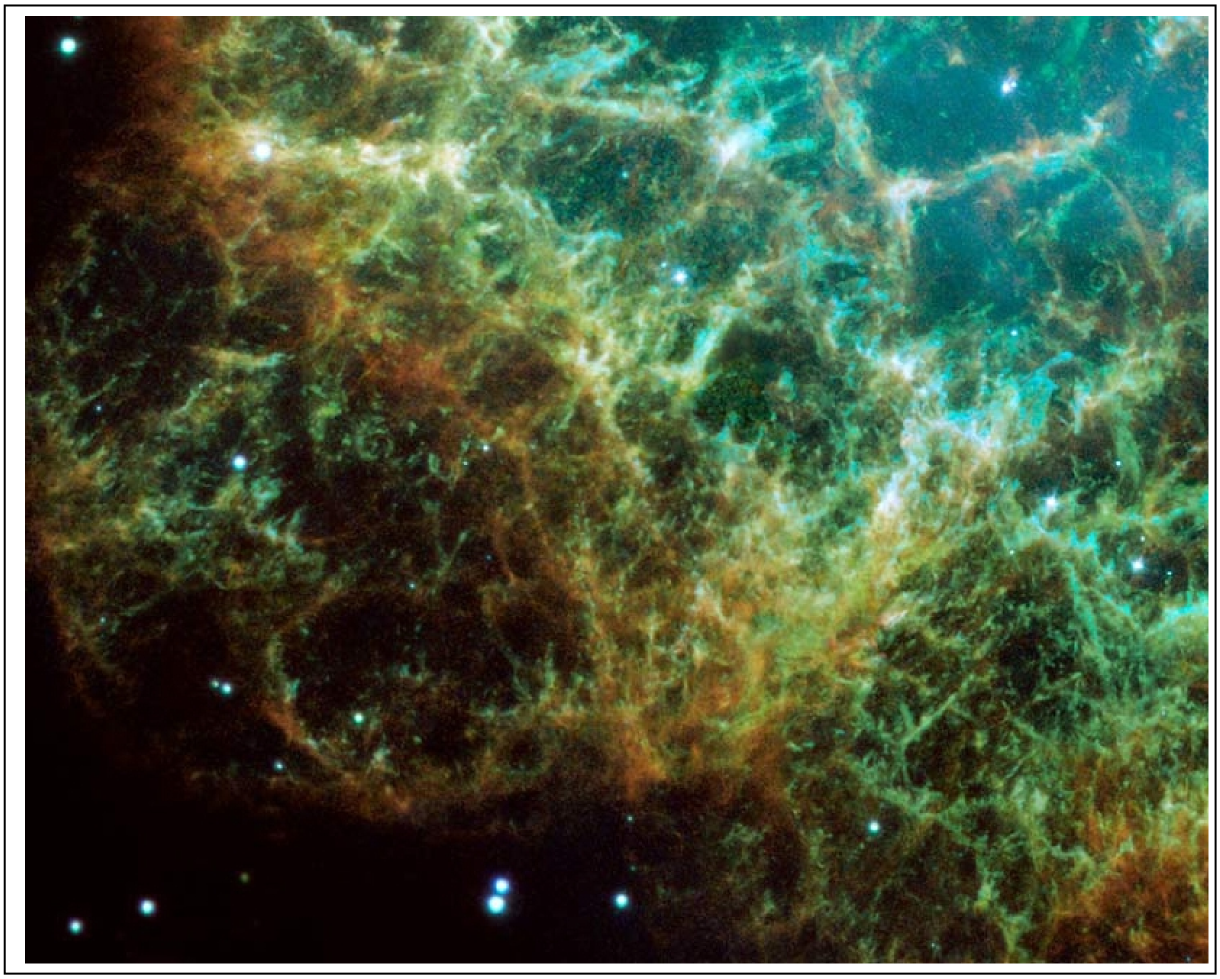
Answer: Time = distance/speed. First convert light years to kilometers: 42 light years x (9.5 x 10^{12} kilometers/light year) = 4.0 x 10^{14} kilometers. Then divide this by the speed to get Time = 4.0 x 10^{14} kilometers / (10,000 km/s) = 4.0 x 10^{10} seconds. Converting this to years: 4.0 x 10^{10} seconds x (1 year/3.1 x 10^{7} seconds) = 1,300 years.

Details from an exploding star



These dramatic images of the Crab Nebula were taken in 2005 by the Hubble Space Telescope. The image on the left has a scale of 0.2 light years/millimeter. The enlargement below has a scale of 0.025 light years/millimeter.

The star that produced this nebula exploded as a supernova in the year 1054 AD, and the expanding gas has been traveling outwards ever since.



Problem 1 – From the information given, what is the average speed of the expanding gas cloud in kilometers/hour? (Note that 1 light year = 62,000 Astronomical Units, and 1 Astronomical Unit = 150 million kilometers, also 1 year = 8760 hours).

Problem 2 – How large are the smallest clumps of the gas in the expanding cloud?

Problem 3 – Draw a sketch, to scale, of the diameter of the solar system (80 Astronomical Units) compared to the size of two or three of the smallest gas clumps.

Answer Key

Problem 1 – From the information given, what is the average speed of the expanding gas cloud in kilometers/hour? (Note that 1 light year = 62,000 Astronomical Units, and 1 Astronomical Unit = 150 million kilometers, also 1 year = 8760 hours).

Answer: The fastest gas will have traveled the farthest distance in the picture, so we will measure the longest dimension of the nebula. From the upper image, the largest diameter of the nebula is about 66 millimeters, so the radius is 33 millimeters. At a scale of 0.2 light years/mm, this equals 33 x 0.2 = 6.5 light years. From the conversion information, this equals 6.5 x (62,000 AU/ly) x (150 million km/AU) = 60 trillion kilometers. The time taken to travel this distance is the number of years between 1054 and 2005, which is 951 years. This equals 951 years x (8760 hours/yr) = 8,331,000 hours. The speed average is then 60 trillion km/8,331,000 hours = 7,300,000 kilometers/hour.

Problem 2 – How large are the smallest clumps of the gas in the expanding cloud?
Answer: In the larger image, students will find filaments and condensations that are about 0.2 millimeters across, which corresponds to 0.2 x 0.025 light years/mm = 0.005 light years across. In terms of Astronomical Units, this equals 0.005 Light years x 62,000 AU/ly) = 310 AU.

Problem 3 – Draw a sketch, to scale, of the diameter of the solar system (80 Astronomical Units) compared to the size of two or three of the smallest gas clumps.
Answer: Creating a scaled image is a bit of a challenge. You want the scale to accommodate 1) enough resolution that you can comfortably draw the smallest object you want to represent, and 2) include a full rendition of the largest object you want to represent. For common 8.5 x 11-inch paper, a scale of 5 AU per millimeter would cover the entire solar system (Diameter of 16 millimeters) and a Crab nebula globule (Diameter of 310 AU = 62 millimeters). Students may color the image with a range of colors suggested by the Hubble photo and include several globules and filaments of the proper scale in the field.

How many stars are there?

On a clear night in the city you might be able to see a few hundred stars. In the country, far away from city lights, perhaps 5000 can be seen. Telescopes can see literally millions of stars. But how do we accurately count them? This exercise will show you the basic method!

This image was taken by the 2MASS sky survey. It is a field that measures 9.0 arcminutes on a side.

Problem 1 – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells.

Problem 2 – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell.

Problem 3 – A square degree measures 60 arcminutes x 60 arcminutes in area. The full sky has an area of 41,253 square degrees. What are the total number of stars in A) one square degree of the sky; B) the number of stars in the entire sky.

Problem 4 – Why do you think we needed to average the numbers in Problem 2?

Answer Key

Problem 1 – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells. Answer: An example is shown below.

Problem 2 – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell. Answer: Using the cells in the top row you may get: 159, 154 and 168. The average is 481/3 = 160 stars.

Problem 3 – Use the information in the text to convert your answer into the total number of stars in one square degree of the sky.

Answer: A) The answer from 3 is the number of stars in one cell. The area of that cell is 3 x 3 = 9 square arcminutes. One degree contains 60 arcminutes, so a square degree contains 60 x 60 = 3600 square arcminutes. Your estimated number of stars in one square degree is then 3600/9 = 40 times the number of stars you counted in one average one cell. For the answer to Problem 2, the number in a square degree would be 160 X 40 = 6,400 stars.

B) The text says that there are 41,253 square degrees in the full sky, so from your answer to Problem 3, you can convert this into the total number of stars in the sky by multiplying the answer by 41,253 to get 6,400 x 41,253 = 264,000,000 stars!

Problem 4 – Why do you think we needed to average the numbers in Problem 2? Answer: Because stars are not evenly spread across the sky, so you need to figure out the average number of stars.

Death Stars


Artist rendering courtesy NASA/G. Bacon (STScI)

Our sun is an active star that produces a variety of storms, such as solar flares and coronal mass ejections. Typically, these explosions of matter and radiation are harmless to Earth and its living systems, thanks to our great distance from the sun, a thick atmosphere, and a strong magnetic field. The most intense solar flares rarely exceed about 10^{21} Watts and last for an hour, which is small compared to the sun's luminosity of 3.8×10^{26} Watts.

A long-term survey with the Hubble Space Telescope of 215,000 red dwarf stars for 7 days each revealed 100 'solar' flares during this time. Red dwarf stars are about 1/20000 times as luminous as our sun. Average flare durations were about 15 minutes, and occasionally exceeded 2.0×10^{21} Watts.

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how many years would inhabitants on each planet have to wait between solar flares?

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Answer: $P = 100\% \times (1.0 \times 10^{21}$ Watts$)/(3.8 \times 10^{26}$ Watts$)$

P = 0.00026 %

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Answer: The average luminosity of a red dwarf star is stated as 1/20000 times our sun's luminosity, which is 3.8×10^{26} Watts, so the red dwarf star luminosity is about 1.9×10^{22} Watts.

$P = 100\% \times (2.0 \times 10^{21}$ Watts$)/(1.9 \times 10^{22}$ Watts$)$

P = 10 %

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Answer: We have two samples: N1 = 215,000 stars for 7 days each producing 100 flares. N2 = 1000 stars for 480 hours each, producing x flares.

From the first survey, we calculate a rate of flaring per star per day:

Rate = 100 flares x (1/215,000 stars) x (1 / 7days)

= 0.000066 flares/star/day

Now we multiply this rate by the size of our current sample to get the number of flares to be seen in the 480-hour (20-day) period.

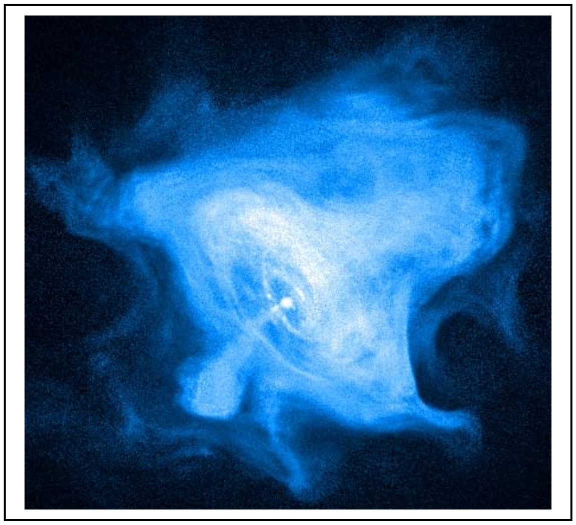
N = 0.000066 flares/star/day) x (1000 stars) x (20 days)

= 1.32 or **1 flare event**.

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how long would inhabitants on each planet have to wait between solar flares?

Answer: The Flare rate was 0.000066 flares/star/day. For 1 star, we have T = 0.000066 flares/star/day x (1 star) = 0.000066 flares/day so the time between flares (days/flare) is just T = 1/0.000066 = 15,151 days. Since there are 365 days/year, this is about **42 years between flares on the average**.

The Crab Nebula - Exploring a pulsar up close!



The Crab Nebula is all that remains of a massive star that exploded as a supernova, and was seen by humans for the first time in 1054 AD. Located 6000 light years away, it is still being powered by a rapidly-spinning neutron star.

The electromagnetic energy from this spinning, magnetized object (also called a pulsar) produces the amazing high-energy cloud of particles now seen clearly by the NASA, Chandra Observatory in the image to the left.

This image gives the first clear view of the faint boundary of the Crab Nebula's X-ray-emitting pulsar wind nebula. The combination of rapid rotating and strong magnetic field generates an intense electromagnetic field that creates jets of matter and anti-matter moving away from the north and south poles of the pulsar, and an intense wind flowing out in the equatorial direction.

The inner X-ray ring is thought to be a shock wave that marks the boundary between the surrounding nebula and the flow of matter from the pulsar. Energetic electrons and positrons (a form of antimatter) move outward from this ring to produce an X-ray glow that Chandra sees as a ghostly cloud in the image above.

Problem 1 - The width of this image is about 5 light years. If the elliptical ring near the center is actually a circular ring seen at a tilted angle, what is the radius of this ting in: A) light years? B) kilometers? (Note: 1 light year = 5.9 trillion kilometers).

Problem 2 - The high-energy particles that make-up the ring were created near the neutron star at the center of the ring. If they are traveling at a speed of 95% the speed of light, to the nearest day, how many days did it take for the particles to reach the edge of the ring? (Speed of light = 300,000 km/s)

Problem 3 - Suppose the pulsar ejected the particles and was visible to astronomers on Earth as a burst of light from the central neutron star 'dot'. If the astronomers wanted to see the high-energy particles from this ejection reach the ring and change its shape, how long would they have to wait for the ring to change after seeing the burst of light?

Problem 1 - The width of this image is about 5 light years. If the elliptical ring near the center is actually a circular ring seen at a tilted angle, what is the radius of this ting in: A) light years? B) kilometers? (Note: 1 light year = 5.9 trillion kilometers).

Answer: the scale of this image can be found using a millimeter ruler. When printed, the image is about 70 mm. The scale is then 5 ly/70mm = 0.071 ly/mm. The radius of the ring will be the maximum radius of the elliptical ring, which you can see by drawing a circle on a piece of paper and tilting it so it looks like an ellipse. On the image, the length of the major axis of the ellipse is 10 mm, so the radius of the circle is 5 mm.
A) Using the scale of the image we get 5 mm x 0.071 ly/mm = **0.36 light years.**
B) The radius in kilometers is just 0.36 ly x 5.9 trillion km/1 ly = **2.1 trillion km.**

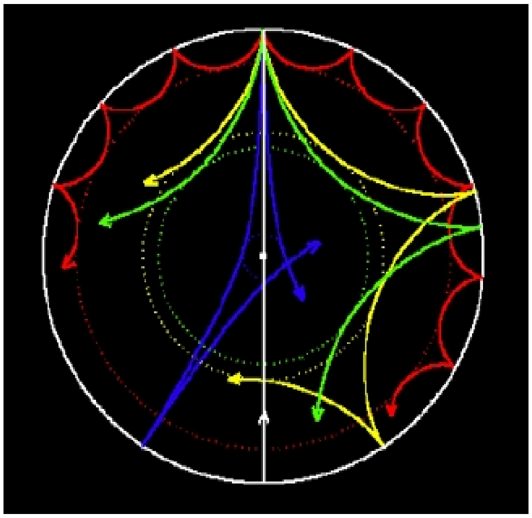
Problem 2 - The high-energy particles that make-up the ring were created near the neutron star at the center of the ring. If they are traveling at a speed of 95% the speed of light, to the nearest day, how many days did it take for the particles to reach the edge of the ring? (Speed of light = 300,000 km/s)

Answer: Time = distance/speed, so for s = 0.95x300,000 km/s = 285,000 km/s, and d = 2.1 trillion km, we get T = 2,100,000,000,000 / 285,000 = 7,368,421 seconds. Converting to days: 7,368,421 seconds x (1 hour/3600 sec) x (1 day/24 hours) = 85.28 days. To the nearest day, this is 85 days.

Problem 3 - Suppose the pulsar ejected the particles and was visible to astronomers on Earth as a burst of light from the central neutron star 'dot'. If the astronomers wanted to see the high-energy particles from this ejection reach the ring and change its shape, how long would they have to wait for the ring to change after seeing the burst of light?

Answer; They would have to wait 85 days after seeing the burst of light because light travels faster than the matter in the particles.

Note: Another way to appreciate how much faster light travels, calculate the number of days it would take for the pulse of light to reach the ring, compared to the 85 days taken by the particles. The light pulse would take 2.1 trillion km/300,000 km/s = 7 million seconds or about 81 days. So astronomers would have to wait 81 days to see whether the light pulse affects the ring, and then another 4 days for the particles to arrive.



Sound waves from turbulent activity on the star's Surface are reflected through the interior along a variety of paths. The surface layer, producing the light we see, moves up and down in complex ways that cause brightness changes over many different time scales from seconds to minutes.

One of the hallmarks of a truly successful space mission is that, not only does it meet all of its planned scientific goals for which it was designed, but that it contributes to advancements in science well-beyond the perimeter of its own specialized research areas.

Recently, the Kepler satellite, which was designed to detect Earth-sized planets orbiting other stars, has made a series of discoveries that have revolutionized the subject of stellar structure and evolution.

Using the minute brightness changes that indicate sound waves traveling in the interiors of giant stars, Kepler data has now confirmed a fundamental hypothesis of stellar evolution proposed by physicist George Gamow nearly 75 years ago!

As stars no longer have enough hydrogen fuel in their cores to sustain them, they switch to fusing hydrogen into helium in a thin shell just outside the core. This shell-burning phase is the prelude to the star becoming a red giant or red supergiant star, depending on its mass. By measuring the brightness changes in many of these red giant stars, Kepler data has been used to probe the interior structure of these stars, revealing just such a shell-burning zone! The sound waves that bounce from the surface to the inner shell are reflected back to the surface where that cause minute brightness changes over time. These 'seismic waves' can be modeled to reveal the location, thickness and depth of the shell-burning zones.

Problem 1 - Draw a circle to represent a star's surface. Draw a series of semi-circles of different radii similar to the ones in the diagram above, but where the sequence of circles along the circumference have the same ending point as its starting point. This represents a 'standing wave' on the surface. The point on the surface where the arc meets the surface is called the 'node'. How many nodes do you count for circles of the different radii that you create?

Problem 2 - If the location of the shell-burning zone outer edge is half-way to the center of the star, for the arc that just touches the shell-zone, how far apart will the nodes be on the surface of the star, if the radius of the star is 5 million kilometers?

Problem 3 - If the speed of the wave is 1,000 km/sec, how many seconds elapse in the travel between the nodes, and what would you see in the light from this star?

Problem 1 - Draw a circle to represent a star's surface. Draw a series of semi-circles of different radii similar to the ones in the diagram above, but where the sequence of circles along the circumference have the same ending point as its starting point. This represents a 'standing wave' on the surface. The point on the surface where the arc meets the surface is called the 'node'. How many nodes do you count for circles of the different radii that you create?

Answer: For example, divide the circle into four equal quadrants. Place the compass in the upper right quadrant at a position on the circumference that is half-way between the two points where the horizontal and vertical lines intersect the circumference. Set your compass radius so that it touches one of these points, then draw the interior arc. Repeat this for the other three quadrants. Divide the circle into five equal parts and repeat this construction process. The number of nodes for each compass radius length are shown in the table below. The arc radius is given in terms of the length of the radius of the circle taken as R=1.0.

Nodes	Number of arcs	Radius of arc
2	**2**	**1.414**
3	**3**	**1.000**
4	**4**	**0.765**
5	**5**	**0.618**
6	**6**	**0.518**
7	**7**	**0.445**
8	**8**	**0.390**

Students may prove that, if the radius of the circle is defined as R, and the number of nodes on the circumference is N, the general formula for the arc radius is just

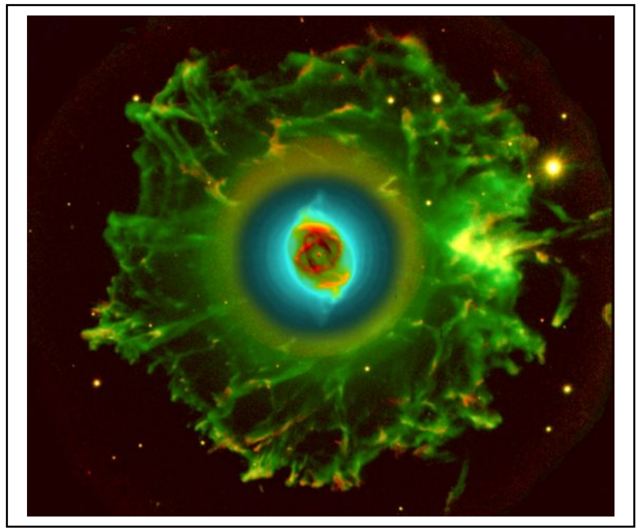
$$d = R\sqrt{2 - 2\cos\left(\frac{\pi}{N}\right)}$$

Problem 2 - If the location of the shell-burning zone outer edge is half-way to the center of the star, for the arc that just touches the shell-zone, how far apart will the nodes be on the surface of the star, if the radius of the star is 5 million kilometers?

Answer: If the shells outer edge is half way to the center of the star, its radial location is 0.500 in units of the star's radius. The closest wave arc that come close to this is for the Node=6 arc with R = 0.518. The full circumference of the star's circle is 2π, so the nodes are located along the circumference a distance of $2\pi/N = 2\pi/6$ or $\pi/3$ apart in multiples of the radius of the star. Since the star's radius is 5 million km, the nodes are located (3.141) x (0.333) x (5million km) = **5.22 million km apart**.

Problem 3 - If the speed of the wave is 1,000 km/sec, how many seconds elapse in the travel between the nodes, and what would you see in the light from this star? Answer: The wave travels along the arc between the node points, whose radius is 5.2 million km, and length is 3.141 x (5.22 million km) = 16.4 million km.
T = 16.4 million km/(1000 km/s) = 16,400 seconds or about 4.6 hours.
The light from the star may 'flicker' slightly in brightness with a period of 4.6 hours.

The Spectacular Cats Eye Planetary Nebula



Thousands of years ago, a star reached the end of its life and began to eject its outer layers forming one of the most complex 'planetary nebulae' in the sky. Known as NGC 6543, it is located 3,300 light years from Earth in the constellation Draco. Its stunning appearance has been known to astronomers for a century as the Cat's Eye Nebula. Detailed telescopic studies of this nebula by astronomers using the Hubble Space Telescope (top left) and recently the Nordic Optical Telescope (top right) have uncovered a wealth of detail caused by the ejected gas and dust from the star (white spot at the center of the Hubble image). Careful measurements allow us to estimate how this nebula has evolved over the centuries.

Problem 1 – The gases that make up the bright central nebula are traveling at 16.4 km/sec. If the radius of the nebula is about 0.25 light years. If one light year = 9.3×10^{12} km, about how long did it take, in years to 2 significant figures, for the nebular shell to form?

Problem 2 – The eleven equally-spaced outer dust shells that have been detected in the Hubble image extend to a maximum radius of 2.5 light years. If the expansion speeds of the shells is 16.4 km/sec, and to two significant figures, how long ago was the outermost shell ejected? About how often did the central star eject these shells?

Problem 3 – The dying star was ejecting about 20 trillion tons of matter every second (2×10^{16} kg). If the estimated mass of the star was 5.0 times our sun, about how many percent of the stars mass was ejected to make the planetary nebula with the age calculated in Problem 2? (1 solar mass = 3.0×10^{30} kg).

http://www.nasa.gov/mission_pages/hubble/science/dark-matter-survey.html
Ambitious Hubble Survey Obtaining New Dark Matter Census
Oct 13, 2011

http://apod.nasa.gov/apod/ap020904.html

HST ACS image http://w3.iaa.es/xpn/pn_files/NGC6543/optical.html

Problem 1 – The gases that make up the bright central nebula are traveling at 16.4 km/sec. If the radius of the nebula is about 0.25 light years. If one light year = 9.3×10^{12} km, about how long did it take, in years, for the nebular shell to form?

Answer: Time = Distance/speed, so
Time = $0.25 \times 9.3 \times 10^{12}$ km/16.4 and
Time = 1.4×10^{11} seconds. x (1 year / 3.1×10^{7} sec)
Time = 4,500 years.

Problem 2 – The eleven equally-spaced outer dust shells that have been detected in the Hubble image extend to a maximum radius of 2.5 light years. If the expansion speeds of the shells is 16.4 km/sec, how long ago was the outermost shell ejected? About how often did the central star eject these shells?

Answer:

Time = Distance/speed
= $2.5 \times 9.3 \times 10^{12}$ km / (16.4 km/s)
= **46,000 years**.

The shell spacing is about 2.5 light years/(11 shells) so the ejection interval is 46,000 years/11 shells= **4,200 years per shell ejection event**.

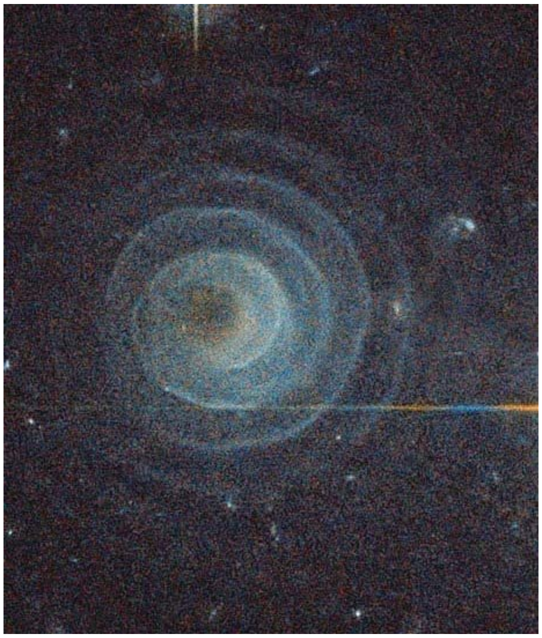
The dying star ejected its first detectable shell about 46,000 years ago, and every 4,000 years would eject a new shell until about 4,500 years ago when it ejected a massive cloud of gas and dust in a giant explosion.

Problem 3 – The dying star was ejecting about 20 trillion tons of matter every second (2×10^{16} kg/sec). If the estimated mass of the star was 5.0 times our sun, about how many percent of the stars mass was ejected to make the planetary nebula with the age calculated in Problem 2? (1 solar mass = 3.0×10^{30} kg).

Answer:

Mass = (2×10^{16} kg/sec) x 46000 years x (3.1×10^{7} sec / 1 year)
= 2.8×10^{28} kg

Fraction = 2.8×10^{28} kg / ($5.0 \times 3.0 \times 10^{30}$ kg) = 0.0018
Percentage = 100% x 0.0018 = **0.18 %.**



Courtesy Advanced Camera/ HST (2010)

This remarkable picture captures the formation of an unusual spiral nebula around the star LL Pegasi located 3,000 light years from the sun in the constellation of Pegasus (the Winged Horse). The spiral is made the same way that an 'arc' of water is produced by a revolving water sprinkler on your lawn.

The picture shows a thin spiral pattern winding around the star, which is itself hidden behind a cloud of thick dust at the center. The spiral is thought to arise because LL Pegasi is a binary system, with the star that is losing material and a companion star orbiting each other. The spacing between layers in the spiral is equal to the 800-year orbit period of the binary.

Problem 1 – If the expansion speed is 50,000 km/hour, how far does the gas travel every year?

Problem 2 - Draw a line from the center of the spiral to the edge of the picture in any direction. How far apart will the successive 'shells' of gas be along this axis?

Problem 3 - Find a direction where the number of shells is a maximum. How many years did it take for the gas to reach the farthest shell you can see?

Problem 4 - Suppose that the brightness of each shell decreases according to the function $B = 128\ N^{-3/2}$ where N is the shell number. If the sky has a brightness of B=2 in these units, what is the maximum number of shells that you could count in a picture?

Problem 5 - About how long ago was the last detectable shell in Problem 4 emitted by the binary system?

Problem 1 – If the expansion speed is 50,000 km/hour, how far does the gas travel every year?

Answer: 50,000 km/hour x (24 hours / 1 day) x (365 days / 1 year) = **438 million km**.

Problem 2 - Draw a line from the center of the spiral to the edge of the picture in any direction. How far apart will the successive 'shells' of gas be along this axis?

Answer; the gas travels 438 million km/yr x 800 yrs = **350 billion kilometers**.

Problem 3 - Find a direction where the number of shells is a maximum. How many years did it take for the gas to reach the farthest shell you can see?

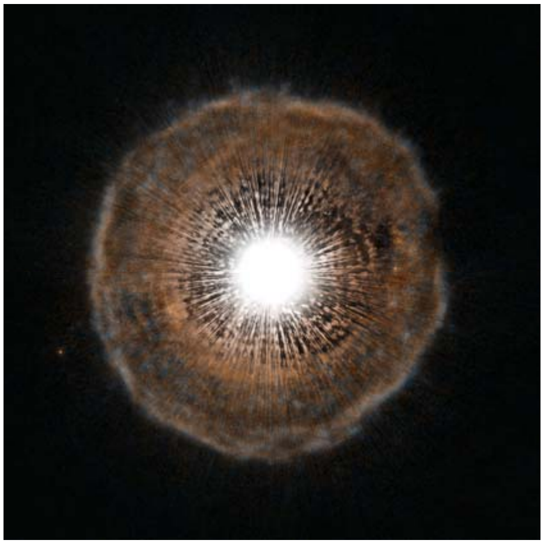
Answer; If students count 6 shells, then 6 x 800 years = **4800 years**.

Problem 4 - Suppose that the brightness of each shell decreases according to the function $B = 128\ N^{-3/2}$ where N is the shell number. If the sky has a brightness of B=2 in these units, what is the maximum number of shells that you could count in a picture?

Answer $N^{3/2} = 128/2$ so $N^{3/2} = 2^6$ and $N = (2^6)^{2/3}$ so $\mathbf{N = 16}$

Problem 5 - About how long ago was the last detectable shell in Problem 4 emitted by the binary system?

Answer: 800 years x (16) = **12,800 years ago**!



This dramatic image taken by the Hubble Space Telescope reveals details in the shell of gas ejected by the star U Camelopardalis thousands of years ago.

Located 1,400 light years from our sun, the shell is expanding at a speed of about 25 km/sec, and its outer edge is about 500 billion km from the central star, whose image has been greatly over exposed making it seem huge in this image!

Although it looks impressive, the amount of mass in this shell is actually quite small. It is only about 1/10 the mass of our own planet Earth!

Problem 1 - The radius of the shell is 500 billion kilometers, and the estimated speed is about 25 km/sec. How many years did it take for the shell to get this big if 1 year = 31 million seconds?

Problem 2 – The mass of Earth is 6.0×10^{24} kg. The mass of a hydrogen atom is 1.6×10^{-27} kg. If the entire mass of the shell were evenly spread out in a sphere with the shell's radius, how many hydrogen atoms would you expect to find in a cubic meter of this shell to the nearest 10,000 atoms?

Problem 1 - The radius of the shell is 500 billion kilometers, and the estimated speed is about 25 km/sec. How many years did it take for the shell to get this big if 1 year = 31 million seconds?

Answer: Time = distance/speed
= 500 billion km / 25 km/sec
= 20 billion seconds

Since 1 year = 31 million seconds, Time = 20 billion/31 million = **645 years**.

Problem 2 – The mass of Earth is 6.0×10^{24} kg. The mass of a hydrogen atom is 1.6×10^{-27} kg. If the entire mass of the shell were evenly spread out in a sphere with the shell's radius, how many hydrogen atoms would you expect to find in a cubic meter of this shell to the nearest 10 million atoms?

Answer: Convert all lengths to meters: 5×10^{11} km x (1000 m/1km) = 5.0×10^{14} meters.

Volume = $4/3\ \pi\ R^3$ so V = 1.333 x 3.141 x $(5.0\times10^{14})^3 = 5.2\times10^{44}$ meters^3.

Mass of shell is 1/10 Earth = 0.1 x 6.0×10^{24}kg = 6.0×10^{23} kg.

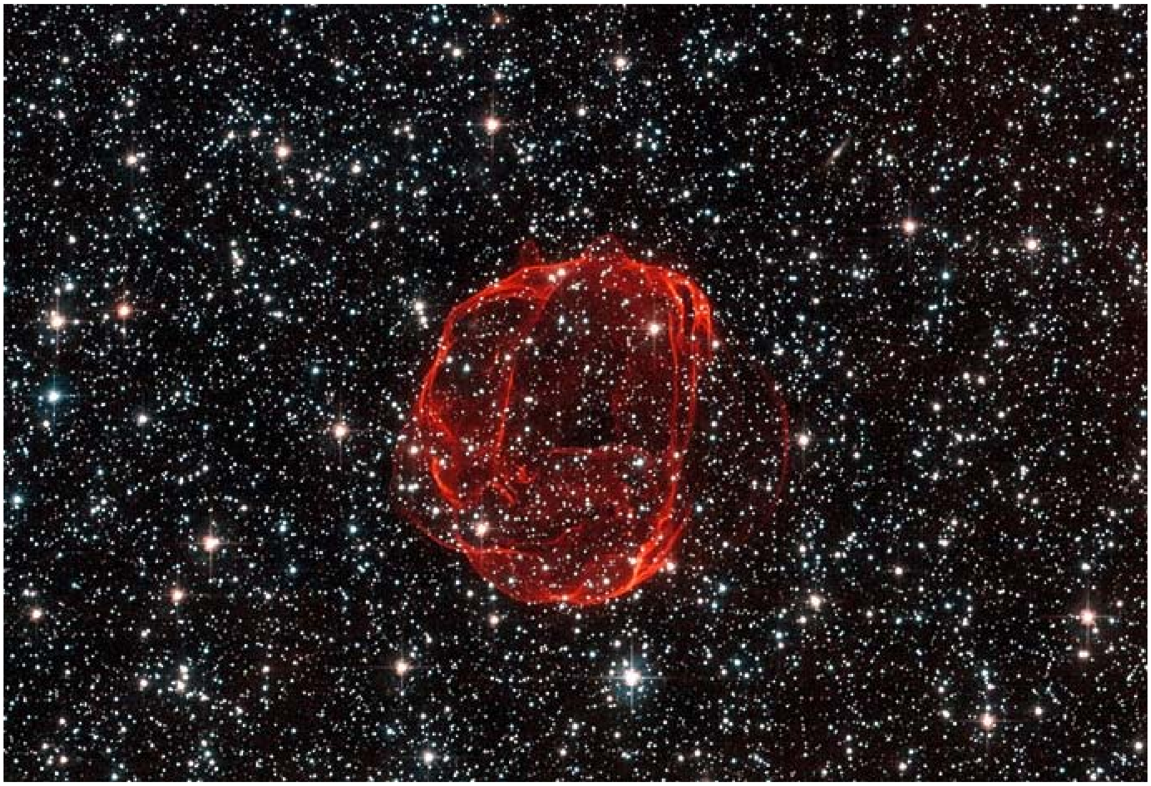
Density = mass/volume
= 6.0×10^{23} kg/5.2×10^{44} m^3
= 1.1×10^{-21} kg/m^3

Since 1 hydrogen atom = 2.0×10^{-27} kg, the density corresponds to

N = 1.1×10^{-21} kg x (1 atom/2.0×10^{-27} kg) = 576,923 atoms/meter^3

Rounded to the nearest 10,000 atoms we get **580,000 atoms**.

A Distant Supernova Remnant Discovered



These delicate wisps of gas make up an object known as supernova remnant SNR 0519. The thin, blood-red shells are actually the remnants from when an unstable star exploded violently as a supernova around 600 years ago. SNR 0519 is located over 150,000 light-years from Earth in the southern constellation of Dorado, a constellation that also contains most of our neighboring galaxy, which is called the Large Magellanic Cloud. One light year equals about 10 trillion kilometers.

Problem 1 – The diameter of this supernova remnant shell is about 24 light years. The distance from our sun to the nearest star Alpha Centauri is 4.3 light years. If the sun were placed at the center of the supernova shell, about where would Alpha Centauri be at the same scale?

Problem 2 – The star that produced this shell exploded 600 years ago. If there are about 30 million seconds in one year, how fast was the shell traveling in kilometers/second expressed A) as a simplified fraction? B) As a decimal number?

http://www.nasa.gov/mission_pages/hubble/science/snr-0519.html

Hubble Sees the Remains of a Star Gone Supernova

May 3, 2013

Problem 1 – The diameter of this supernova remnant shell is about 24 light years. The distance from our sun to the nearest star Alpha Centauri is 4.3 light years. If the sun were placed at the center of the supernova shell, about where would Alpha Centauri be at the same scale?

Answer: **About 1/3 of the way from the center to the edge of the shell**.

Problem 2 – The star that produced this shell exploded 600 years ago. If there are about 30 million seconds in one year, how fast was the shell traveling in kilometers/second expressed A) as a simplified fraction? B) As a decimal number?

Answer: The center of the shell is 12 light years from the edge, so the distance traveled in 600 years is 12 x 10 trillion km, or 120 trillion kilometers.

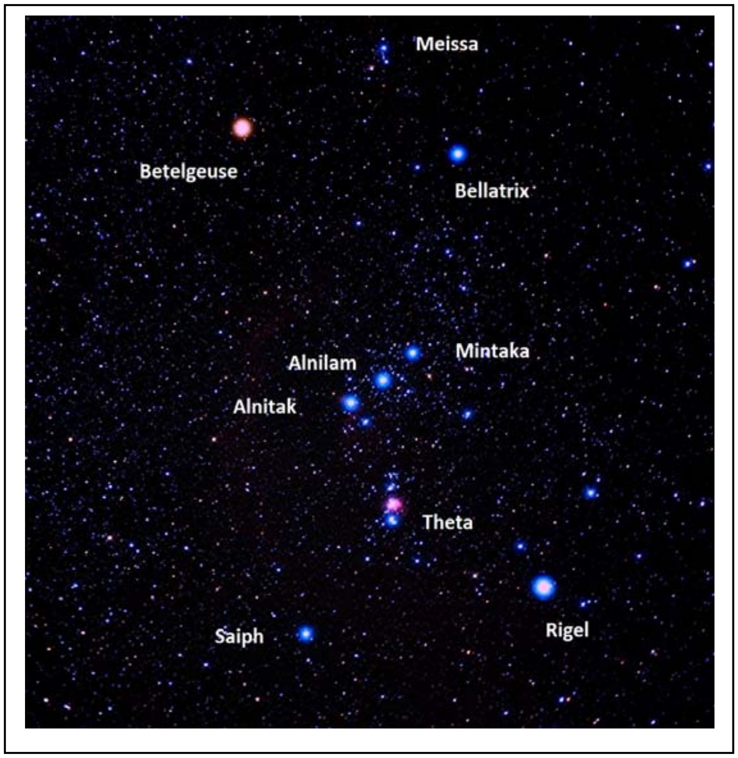
Since 600 years equals 600x30 million seconds = 18 billion seconds,

the shell travels 120 trillion km / (18 billion seconds)
= 120,000 billion/18 billion
= 120,000/18

A) As a simplified fraction $\mathbf{\dfrac{20000}{3}}$ **kilometers per second**

B) As a decimal: 120000/18 = **6,666 kilometers per second**.

This speed is about 100 times faster than the Space Station orbiting Earth.



Next to the Big Dipper (Ursa Major) and Scorpius, the Orion is the most widely recognized of all the 89 constellations in the sky. It is also one of the oldest known to humans. The Ancient Egyptians called it Osiris as long ago as 2000 BC!

The brilliant stars that make up this rectangular star pattern seem to be close-by because they are so bright, but in fact they are very far away. Astronomers measure distances using a unit called the **light year**, which equals about 9.5 trillion kilometers, or 63,240 times the distance from Earth to the Sun!

Problem 1 – Light travels at a speed of 300,000 km/sec. How long does it take light to travel:

A) To the moon at a distance of 380,000 km?
B) To the sun at a distance of 150 million km?
C) To Neptune at a distance of 4.5 billion kilometers?
D) To the star Alpha Centauri at a distance of 41 trillion km?

Problem 2 – How far does light travel in one year, if 1 Earth year = 31,000,000 seconds? (Note: Astronomers call this distance one light year.)

Problem 3 – The bright star in Orion called Betelgeuse is located 650 light years from Earth. What is this distance in kilometers? Write your answer using words like thousand or trillion where appropriate, and round the answer to the nearest 1000 trillion.

Problem 4 – Betelgeuse is expected to blow up as a supernova sometime in the next 1 million years. Suppose this happened in the year 3000 AD. In what year would someone on earth see this explosion?

Problem 5 – The star Bellatrix is located 300 light years from Earth and closer to Betelgeuse. In what year would colonists there first see Betelgeuse explode?

Problem 1 – Light travels at a speed of 300,000 km/sec. How long does it take light to travel: A)To the moon at a distance of 380,000 km?; B) To the sun at a distance of 150 million km? C) To Neptune at a distance of 4.5 billion kilometers? D) To the star Alpha Centauri at a distance of 41 trillion km?

Answer: Time = Distance/speed, so

A) Time = 380,000 km/300,000 = **1.3 seconds**. B) Time = 150,000,000/300,000 = 500 seconds or **8 1/3 minutes**. C) Time = 4,500,000,000 km/300,000 = 15,000 seconds or **4 1/6 hours**. D) 41,000,000,000,000 km/300,000 = 136,666,666 seconds or **4 1/3 years.**

Problem 2 – How far does light travel in one year, if 1 Earth year = 31,000,000 seconds? (Note: Astronomers call this distance one light year.)

Answer: Distance = speed x Time,
Distance = 300,000 km/s x 31,000,000 sec
Distance = **9.3 trillion kilometers**.

Problem 3 – The bright star in Orion called Betelgeuse is located 650 light years from Earth. What is this distance in kilometers? Write your answer using words like thousand or trillion where appropriate, and round the answer to the nearest 1000 trillion.

Answer: 1 light year is 9.3 trillion kilometers, so multiply this by 650 to get the distance to Betelgeuse of 650 x 9.3 = 6045 trillion kilometers, which we can re-write as **6 thousand trillion kilometers**. The actual name for one thousand trillion is one quadrillion, so we might also write Betelgeuse's distance as 6 quadrillion kilometers.

Problem 4 – Betelgeuse is expected to blow up as a supernova sometime in the next 1 million years. Suppose this happened in the year 3000 AD. In what year would someone on earth see this explosion?

Answer: Betelgeuse is 650 light years from Earth so it takes light 650 years to reach us .If the explosion happened in the Year 3000 AD, then we will see the light arrive in the year **3650 AD**, 650 years AFTER the event occurred.

Problem 5 – The star Bellatrix is located 300 light years from Earth and closer to Betelgeuse. In what year would colonists there first see Betelgeuse explode?

Answer: Light from the explosion has to travel from Betelgeuse to Bellatrix, which is a distance of 650-300 = 350 light years. That means that colonists near Bellatrix will see the event in the year 3000 AD + 350 = **3350 AD**, or about 300 years <u>sooner</u> than the most distant earthlings!

Exploring Two Nearby Stars to the Sun.

The Hubble Space Telescope recently took a picture of the currently nearest star to our sun, Proxima Centauri, which is at a distance of 4.243 light years (1 light year = 9.5 trillion km). The orbits of several other stars near the sun are also known, and during the next 80,000 they will replace Proxima Centauri as the nearest star to our sun.

For this problem we will study the two stars Ross 128 and Gliese 445.

The quadratic equations that approximate the distance to each star from our sun in light years are given by:

Gliese 445 : $D = 0.0104\,T^2 - 0.942\,T + 25.382$

Ross 128 : $D = 0.0007\,T^2 - 0.1197\,T + 11.003$

where T is the number of years from the current year in multiples of 1000 years.

Problem 1 – What are the distance ranges for each star over the time interval from 10,000 to 70,000 in the future?

Problem 2 – For what values of T, in years, will the distances be identical?

Problem 3 – What will be the distances to the stars when their distances are identical?

Problem 4 – When will the two stars be exactly 3.00 light years apart sometime over the time range from 30,000 to 80,000 years from now?

Gliese 445 : $D = 0.0104T^2 - 0.942T + 25.382$
Ross 128 : $D = 0.0007T^2 - 0.1197T + 11.003$

Problem 1 – What are the distance ranges for each star over the time interval from 10,000 to 70,000 in the future?

Answer: Gliese 445: T=10 so D = 17.00 Ly; T = 70 so D = 10.40 ly **[10.40 , 17.00]**
Ross 128: T=10, so D = 9.88 ly ; T=70 so D = 6.05 ly **[6.05 , 9.88]**

Problem 2 – For what values of T, in years, will the distances be identical?

Answer: Set the two equations equal to each other and solve the resulting quadratic equation for its two roots using the Quadratic Formula.

$0 = (0.0104-0.0007)T^2 + (-0.942 + 0.1197)T + (25.382 – 11.003)$

So the coefficients are a = +0.0097 b= -0.822 and c = +14.379

The roots are $T = [–b +/- (b^2-4ac)^{1/2}]/ 2a$ so
$T = 42.37 +/- 51.54 (0.6757-0.5579)^{1/2}$

T1 = 24,700 years and T2 = 60,000 years.

Problem 3 – To the nearest tenth of a light year what will be the distances to the stars when their distances are identical?

Answer: At 24,700 years, T = 24.7 so D = **8.5 light years**
60,000 years, T=60 so D = **6.3 light years**

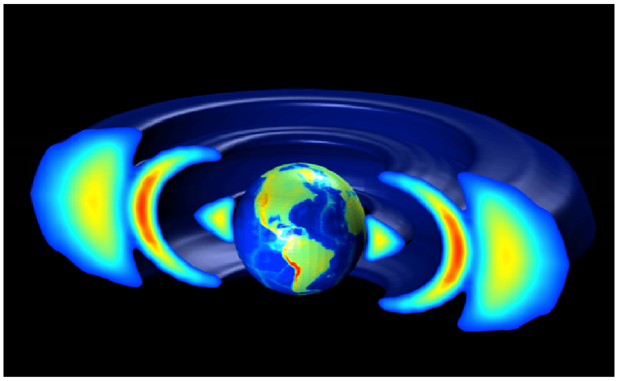
Problem 4 – When will the two stars be exactly 3.00 light years apart sometime over the time range from 30,000 to 80,000 years from now?

Answer: We want the difference between the two quadratic equations to be 10.0

$3.0 = 0.0097T^2 – 0.822T + 14.379$ so we solve $0 = 0.0097T^2 – 0.822T + 11.379$

$T = [0.822 +/- (0.6757 – 0.4415)^{1/2}] / 0.0194$
$T = 42.37 +/- 51.54(0.4839)$
T1 = 67.3
T2 = 17.4

So the only solution in the required time interval is **67,300** years from now.



(Image credit: Yuri Shprits/UCLA)

The Van Allen Probes spacecraft travel in an elliptical orbit through the Van Allen belts. Soon after launch, they detected a third radiation belt shown by the middle crescent in the figure to the left.

In this problem, we predict when the spacecraft will encounter this third belt along their orbit, so that scientists can schedule observations of this new region.

The equation for the orbit of the spacecraft is given in Standard Form by

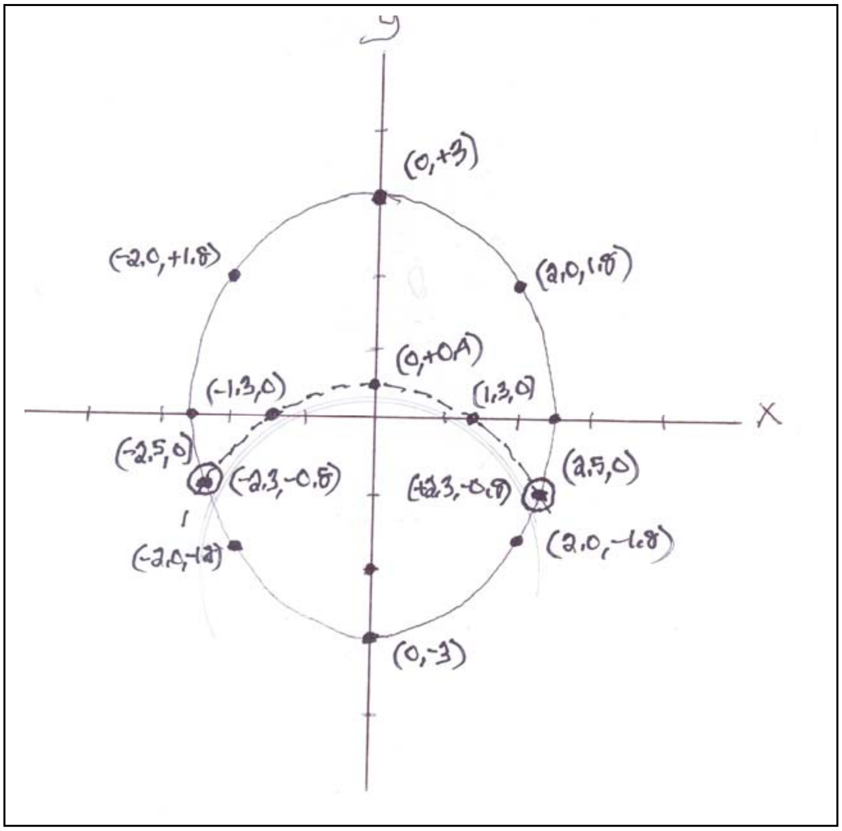
$$1 = \frac{x^2}{6.25} + \frac{y^2}{9.0}$$

The equation for the location of the third Van Allen belt projected into the plane of the elliptical orbit and concentric with the orbit focus centered on Earth (0,-2) is given by

$$5.8 = x^2 + (y+2)^2$$

Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth.

Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth.



Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth. Answer: See plot above (-2.3, -0.8), (+2.3, -0.8).

Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth. Answer: Expand out all terms and simplify

Equation of circle: $5.8 = x^2 + y^2 + 4y + 4.0$ so
$1.8 = x^2 + y^2 + 4y$ Equation of ellipse: $9.0 = 1.54x^2 + y^2$

Eliminate x^2 in equation of circle by using equation of ellipse solved for x^2.

$1.8 = \mathbf{(9.0 - y^2)/1.54} + y^2 + 4y$ $1.8 - 5.84 = -0.65y^2 + y^2 + 4y$ then $4.0 = 0.35y^2 - 4y$

Solve the quadratic equation in y for its two roots: $0 = 0.35y^2 - 4y - 4.0$ A = +0.35, b = -4.0 c = -4.0 then

$Y = [4.0 \pm (16+5.6)^{1/2}]/0.70$ so Y1 = +12.3 Y2 = -0.86

Then from the equation for the ellipse we get the x coordinates:

Y1 = +12.3 : $X1 = \pm ((9.0 - (12.3)^2)/1.54)^{1/2}$ x1 = <u>imaginary solution</u>! (+/- 9.6 i)

Y2 = -0.86 : $X2 = \pm ((9.0 - (0.86)^2)/1.54)^{1/2}$ = +/- 2.30

Rounded to the nearest tenth we have +/- 2.8 so the real coordinates are **(+2.3, -0.9) and (-2.3, -0.9)**

Stars are spread out through space at many different distances from our own Sun and from each other. In this problem, you will calculate the distances between some familiar stars using the 3-dimensional distance formula in Cartesian coordinates. Our own Sun is at the origin of this coordinate system, and all distances are given in light-years. The distance formula is given by:

$$d = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}$$

Star	Distance from Sun	Constellation	X	Y	Z	Distance from Polaris
Sun			0.0	0.0	0.0	
Sirius			-3.4	-3.1	7.3	
Alpha Centauri			-1.8	0.0	3.9	
Wolf 359			4.0	4.3	5.1	
Procyon			-0.9	5.6	-9.9	
Polaris			99.6	28.2	376.0	0.0
Arcturus			32.8	9.1	11.8	
Tau Ceti			-6.9	-8.6	2.5	
HD 209458			-94.1	-120.5	5.2	
Zubenelgenubi			64.6	-22.0	23.0	

Question 1: Within which constellations are these stars located?

Question 2: What are the distances of these stars from the Sun in light-years?

Question 3: If you moved to the North Star, Polaris, how far would the Sun and other stars be from you? Enter the answer in the table above.

Question 4: Which of these stars is the closest to Polaris?

Question 5: What does your answer to Question 4 tell you about the stars you see in the sky from Earth?

Star	Distance from Sun	Constellation	X	Y	Z	Distance from Polaris
Sun	**0.0**	**None**	0.0	0.0	0.0	**390**
Sirius	**8.68**	**Canis Major**	-3.4	-3.1	7.3	**384**
Alpha Centauri	**4.34**	**Cantaurus**	-1.8	0.0	3.9	**387**
Wolf 359	**7.8**	**Leo**	4.0	4.3	5.1	**384**
Procyon	**11.45**	**Canis Minor**	-0.9	5.6	-9.9	**399**
Polaris	**390**	**Ursa Minor**	99.6	28.2	376.0	**0**
Arcturus	**36**	**Bootes**	32.8	9.1	11.8	**371**
Tau Ceti	**11.35**	**Cetus**	-6.9	-8.6	2.5	**390**
HD 209458	**153**	**Pegasus**	-94.1	-120.5	5.2	**444**
Zubenelgenubi	**72**	**Libra**	64.6	-22.0	23.0	**358**

Question 1: Within which constellations are these stars located? **Answer:** Students may use books or GOOGLE to enter the answers in the table.

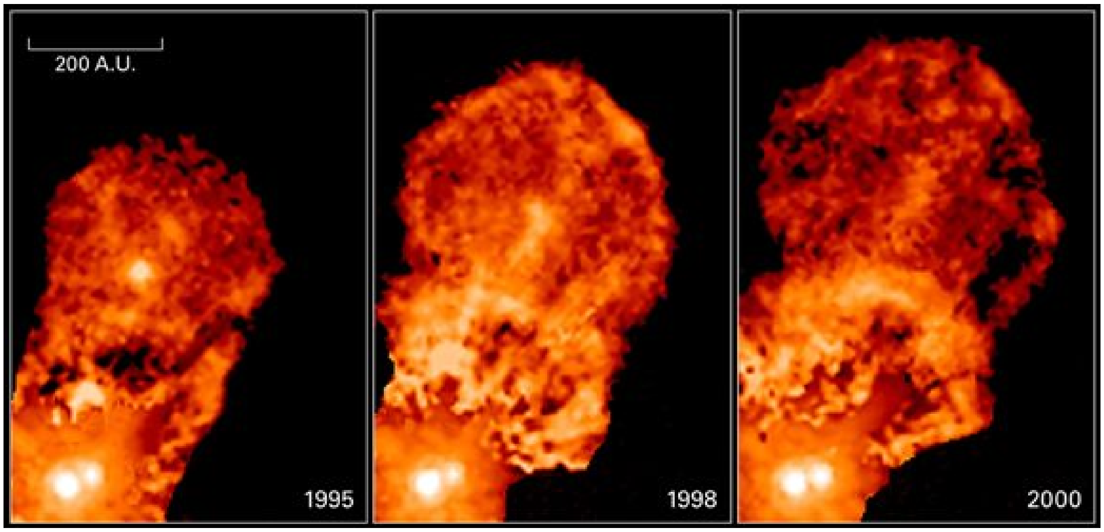
Question 2: What are the distances of these stars from the Sun in light-years? **Answer:** Use the formula provided with X1=0, y1=0 and z1 = 0. Example for Sirius where x2 = -3.4, y2 = -3.1 and z2=7.3 yields, D = ((-3.4)^2 + (-3.1)^2 + (7.3)^2)^0.5 = 8.7 light-years.

Question 3: If you moved to the North Star, Polaris, how far would the Sun and other stars be from you? Enter the answer in the table. **Answer:** To do this, students select the new origin at Polaris and fix x1 = **99.6**, y1=**28.2** and z1 = **376.0** in the distance formula. They then insert the X, Y and Z coordinates for the other stars and compute the distance. Example, for the Sun, the distance will be 390 light years, because that is how far Polaris is from the Sun. For HD 209458, the distance formula gives D = ((-94.1 – **99.6**)^2 + (-120.5 – **28.2**)^2 + (5.2 – 376)^2)^0.5 = (37519 + 22111 + 137492)^0.5 = 444 light years.

Question 4: Which of these stars is the closest to Polaris? **Answer:** Zubenelgenubi!

Question 5: What does your answer to Question 4 tell you about the stars you see in the sky from Earth? **Answer:** This is a great lesson in 3-d space visualization. Even though Polaris and HD 209458 are close in the sky as viewed from Earth (they are in Ursa Minor and Pegasus respectively as a star chart will show) they are actually the farthest apart of any two stars in this list.

XZ Tauri and the Super CME!



These pictures were taken by the Hubble Space Telescope between 1995 and 2000. They show a time sequence that captures the explosion of matter from the star XZ Tauri in the constellation Taurus. The star is located 450 light years from the sun. This star is less than one million years old, and is probably similar to what our own sun was like at the same age.

Today, our sun still ejects gas in events called Coronal Mass Ejections (CMEs) but involve far less matter ejected into space.

Problem 1 - Using a millimeter ruler, and the fact that 1 A.U. = 147 million kilometers what is the scale of these images in kilometers per millimeter?

Problem 2 - A) How many millimeters did the XZ Tauri cloud travel between 1995 and 2000? B) How many kilometers did it travel?

Problem 3 - What was the average speed of the cloud between 1995 - 2000 in; A) kilometers per day; B) kilometers per hour? C) kilometers per second?

Extra for Experts:

Problem 4 - The estimated total mass of this 'round' cloud was 1.1×10^{25} grams. Assuming it filled a spherical volume with a volume $V = 4/3\, \pi\, R^3$ A) What was the volume of this cloud in 1998 in cubic centimeters? B) What was the density of this cloud in grams per cubic centimeter? C) If one hydrogen atom has a mass of 1.6×10^{-24} grams, how many hydrogen atoms per cubic centimeter were present in the star's interplanetary space?

Answer Key:

Problem 1 - Using a millimeter ruler, and the fact that 1 A.U. = 147 million kilometers what is the scale of these images in kilometers per millimeter?

Answer - The 200 AU image scale is 17.5 millimeters long. 200 AU = 200 x 147 million kilometers, so the scale is 200 x 147 million km/17.5 mm = 1,680 million km/mm

Problem 2 - A) How many millimeters did the XZ Tauri cloud travel between 1995 and 2000? B) How many kilometers did it travel?

Answer A) Depending on how you measure, the cloud traveled about 12 millimeters. B) 12 millimeters x 1,680 million km/mm = 20,160 million kilometers.

Problem 3 - What was the average speed of the cloud between 1995 - 2000 in; A) kilometers per day; B) kilometers per hour? C) kilometers per second?

Answer: A) 2000-1995=Five years = 365 x 5 = 1825 days, so it traveled 20,160 million km/1825 days = 11.0 million km/day. B) 11.0 million km / 24 hours = 458,000 km/hour. C) 458,000 km/3600 seconds = 127 kilometers/second.

Extra for Experts:

Problem 4 - The estimated total mass of this 'round' cloud was 1.1×10^{25} grams. Assuming it filled a spherical volume

A) What was the volume of this cloud in 1998 in cubic centimeters?

Answer - The diameter of the cloud in the middle image is 38 mm or 38 x 1,680 million km = 63,840 million km. The radius is 31,920 million km. This equals a radius of $31{,}920 \times 10^{6}$ km x 1.0×10^{5} cm/km = 3.2×10^{15} cm. The volume of a sphere is $V = 4/3\, \pi R^3$, so the volume of the cloud is $4/3\, \pi\, (3.2 \times 10^{15}\ \text{cm})^3 = 1.4 \times 10^{47}$ cubic centimeters.

B) What was the density of this cloud in grams per cubic centimeter?
Answer: Density = Mass / volume = 1.1×10^{25} grams / 1.4×10^{47} cubic centimeters = 7.8×10^{-23} grams/cc

C) If one hydrogen atom has a mass of 1.6×10^{-24} grams, how many hydrogen atoms per cubic centimeter were present in the star's interplanetary space?
Answer = 7.8×10^{-23} grams/cc / (1.6×10^{-24} grams) = 50 hydrogen atoms/cc.

One of the first things that amateur astronomers do with a camera is to point it at the North Celestial Pole (near Polaris the North Star) and take a time-exposure lasting minutes or hours. The Earth's rotation causes the stars to form long semi-circular trails.

Problem 1 - Which stars in the photo are nearest the NCP?

Problem 2 - The width of the image is 36.0 degrees. What is the scale of the image in arcminutes/millimeter?

Problem 3 - Using your method of choice, identify the location of the NCP as accurately as possible.

Problem 4 - Which star do you think is Polaris in the photograph, and how far is it from your location for the NCP?

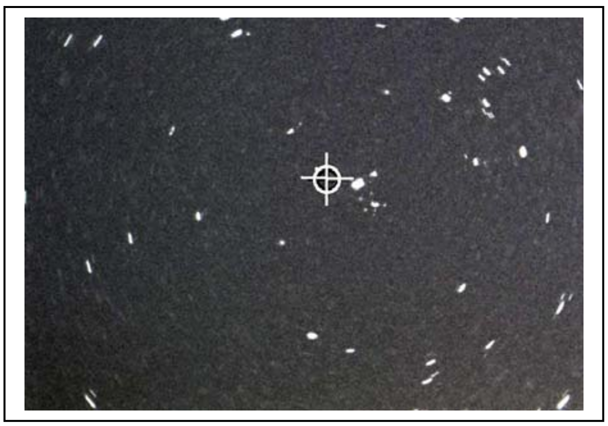
Problem 5 - What features in the photograph are not stars or planets? Explain your reasoning.

Problem 6 - From the information in the photograph, to the nearest minute of time, how long was the camera shutter left open to make this 'star trail' photograph?

Answer Key

Problem 1 - As Earth rotates, the stars will be trailed into arcs of circles that are exactly concentric with the North Celestial Pole. **Look for a region where the star arcs nearly vanish.** This region is in the lower left-hand corner of the picture, which is enlarged and shown on the left.

Problem 2 - The width of the picture is 160 mm, which corresponds to 36.0 degrees, so the scale is 0.225 degrees/mm. Since 1 degree = 60 arcminutes, this is equivalent to 0.225 x 60 = **13.5 arcminutes/mm.**



Problem 3 - Use a compass to place trial pivot points within the region, and test by matching the arc drawn by the compass with the arcs of several stars that are far from this region. **By trial and error, students should be able to get points that are close to the one indicated in the image by the circle and cross.**

Problem 4 - The bright star just to the right of the cross. **The distance is about 3 mm or 40 arcminutes (0.7 degrees).**

Problem 5 - What features in the photograph are not stars or planets? Explain your reasoning. Answer: **Stars and planets do not share Earth's rotation so they will leave arcs concentric with the NCP in the photograph. Objects on Earth, or in the camera, will remain in the same fixed spot in the picture because they are not moving.**

Problem 6 - From the information in the photograph, how long was the camera shutter left open to make this 'star trail' photograph? Answer: A full 360-degree arc would represent 24 hours. We need to measure the length of an arc in degrees, then determine how far it is from the NCP in degrees, and from the ratio of the arc length to the circumference of the complete circle we get the fraction of a 24-hour period that the trail represents. Selecting one star at random in the picture, like the bright one at the far-right edge of the picture (the star Delta Cassiopeia), it is located 133 mm from the NCP, and its arc is 6 mm long. This represents a radius of 29.9 degrees and a length of 1.35 degrees. The circumference is 2 x 3.141 x 29.9 = 187.8 degrees. The arc is the fraction 1.35/187.8 = 0.00719 of full circle or 24 hours x 0.00719 = 0.17 of an hour or **10 minutes.**

How many stars are in the sky?

One of the very first things that astronomers studied was the number of stars in the sky. From this, they hoped to get a mathematical picture of the shape and extent of the entire Milky Way galaxy. This is perhaps why some cartoons of 'astronomers' often have them sitting at a telescope and tallying stars on a sheet of paper! Naked-eye counts usually number a few thousand, but with increasingly powerful telescopes, fainter stars can be seen and counted, too.

Over the decades, 'star count' sophisticated models have been created, and rendered into approximate mathematical functions that let us explore what we see in the sky. One such approximation, which gives the average number of stars in the sky, is shown below:

$$\mathrm{Log}_{10}N(m) = -0.0003\ m^3 + 0.0019\ m^2 + 0.484\ m - 3.82$$

This polynomial is valid over the range [+4.0, +25.0] and gives the Log_{10} of the total number of stars per square degree fainter than an apparent magnitude of m. For example, at an apparent magnitude of +6.0, which is the limit of vision for most people, the function predicts that $\mathrm{Log}_{10}N(6) = -0.912$ so that there are $10^{-0.912} = 0.12$ stars per square degree of the sky. Because the full sky area is 41,253 square degrees, there are about 5,077 stars brighter than, or equal to, this magnitude.

Problem 1 - A small telescope can detect stars as faint as magnitude +10. If the human eye-limit is +6 magnitudes, how many more stars can the telescope see than the human eye?

Problem 2 - The Hubble Space Telescope can see stars as faint as magnitude +25. About how many stars can the telescope see in an area of the sky the size of the full moon (1/4 square degree)?

Problem 3 - A photograph is taken of a faint star cluster that has an area of 1 square degree. If the astronomer counts 5,237 stars in this area of the sky with magnitudes in the range from +11 to +15, how many of these stars are actually related to the star cluster?

Problem 1 - Answer: From the example, there are 0.12 stars per square degree brighter than +6.0

$$\mathrm{Log}_{10}N(+10) = -0.0003\ (10)^3 + 0.0019\ (10)^2 + 0.484\ (10) - 3.82$$
$$= -0.3 + 0.19 + 4.84 - 3.82$$
$$= +0.55$$

So there are $10^{.55}$ = 3.55 stars per square degree brighter than +10. Converting this to total stars across the sky (area = 41,253 square degrees) we get 5,077 stars brighter than +6 and 146,448 stars brighter than +10. The number of additional stars that the small telescope will see is then 146,448 - 5,077 = **141,371 stars.**

Problem 2 - Answer: $\mathrm{Log}_{10}N(25) = -0.0003\ (25)^3 + 0.0019\ (25)^2 + 0.484\ (25) - 3.82$
$= +4.78$

So the number of stars per square degree is $10^{+4.78}$ = 60,256. For an area of the sky equal to 1/4 square degree we get (60,256) x (0.25) = **15,064 stars**.

Problem 3 - Answer: $\mathrm{Log}_{10}N(15)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+15. $\mathrm{Log}_{10}N(10)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+10. The difference between these two will be the number of stars with magnitudes of +11, +12, +13, +15, which is the number of stars in the sky per square degree, in the magnitude range of the star cluster. We then subtract this number from 5,237 to get the number of stars actually in the cluster.

$\mathrm{Log}_{10}N(15) = +2.86$ corresponds to $10^{+2.86}$ = 716.1 stars/square degree
$\mathrm{Log}_{10}N(11) = +1.33$ corresponds to $10^{+1.33}$ = 21.6 stars/square degree

So the difference between these is 716.1-21.6 = 694.5 stars/deg^2. The star cluster area is 5 square degrees, so we have 694.5 stars/deg^2 x (5 square degrees) = 3,473 stars that we expect to find in the sky in the magnitude range of the star cluster. Since the astronomer counted 5,237 stars in the star cluster field, that means that 3,473 of these stars are probably just part of the general sky population of stars, while only 5,237 - 3,473 = **1,764 stars are actually members of the cluster** itself.

Note to Teacher: Show that the students have to evaluate N first before taking the difference because $\mathrm{Log}_{10}N(15) - \mathrm{Log}_{10}N(11)$ is not the same as N(15) - N(10).

The Wide-field Infrared Survey Experiment (WISE) recently took its first photo of a test field in the constellation Carina to check out its instruments.

WISE is based on a 40-cm (16-inch) telescope designed to detect radiation at four wavelengths: 3.4, 4.6, 12 and 22 microns. The telescope is kept cold using solid hydrogen at a temperature of -438 F (12 Kelvin), and will be able to function for about 10 months in space until the hydrogen evaporates. In this time, WISE will take over a million pictures of the whole sky, revealing hundreds of millions of stars, galaxies, asteroids and other objects that shine brightly in the infrared spectrum.

The image to the left, measures 47arcminutes x 23 arcminutes (60 arcmin equals one angular degree). It is a portion of the WISE 'First Light' image near the bright star V482 Carinae seen to the right. The full moon would cover a square 30 arcminutes on a side.

Astronomers not only study individual stars in a field like this, but also count the stars in each brightness interval in order to mathematically model how stars are distributed in the Milky Way. Such models help us understand the shape and history of

Problem 1 - The bright star seen in this field is V482 Carinae. It has a stellar brightness of +6.0. If the area of this field has dimensions of 0.8 degrees x 0.4 degrees, how many stars as bright as V482 are present, on average, A) per square degree of the sky? B) Across the entire sky if its area is 41,253 deg^2?

Problem 2 - The function A(m) represents the total number of stars per deg^2 counted in the stellar brightness (called 'magnitude') bin from m-1/2 to m+1/2 centered on m. The function has the units of 'stars/deg^2/magnitude'. If $A(m)= 3m^{+3.5}$, how many stars would be counted, on average, in: A) a field that is the size of the full moon, (area = 0.25 deg^2) at a magnitude of m=+12.0? B) the same field and with a magnitude bin +2.3 in size?

Problem 3 - Near the wavelength of 3.4 microns being explored by WISE, an astronomer estimated from the Spitzer Infrared Observatory 'First Light' survey that at this wavelength, $A(m) = 2.4 \times 10^{-6}\ m^{+7.4}$ stars/magnitude/deg^2. Suppose that the faintest star detectable by WISE has a magnitude of m = +15. Using the method of integration, how many total number of stars would WISE be able to detect in a field equal to the WISE survey area of 0.64 deg^2, and that are fainter than V482 Carina?

Problem 1 - The bright star seen in this field is V482 Carinae. It has a stellar brightness of +6.0. If the area of this field has dimensions of 0.8 degrees x 0.4 degrees, how many stars as bright as V482 are present, on average, A) per square degree of the sky? B) Across the entire sky if its area is 41,253 deg^2?

Answer: A) The area of the field is 0.8 x 0.4 = 0.32 deg^2 and since there is only one star with a brightness of +6 in this area, A(+6.0) = 1 star / 0.32 deg^2 = **3.1 stars/deg^2**. B) Across the entire sky, N = 3.1 stars/deg^2 x 41,253 deg^2 gives **N = 128,000 stars**.

Problem 2 - The function A(m) represents the total number of stars per deg^2 counted in the stellar brightness (called 'magnitude') bin from m-1/2 to m+1/2 centered on m. The function has the units of 'stars/deg^2/magnitude'. If A(m)= $3m^{+3.5}$ stars/deg^2/magnitude how many stars would be counted, on average, in A) a field that is the size of the full moon, (area = 0.25 deg^2) at a magnitude of m=+12.0? B) the same field and with a magnitude bin that is +2.3 in size?

Answer: A) First A(12.0) = 3 $(12)^{+3.5}$ = 18,000 stars/deg^2/magnitude. Then for an area of 0.25 deg^2 we have N(m) = A(m) x (0.25 deg^2) = 18,000 x 0.25 = **4,500 stars/magnitude**.
B) The bin width is now 2.3, so N = 4,500 stars/magnitude x (2.3 magnitudes) = **10,000 stars**.

Problem 3 - Near the wavelength of 3.4 microns being explored by WISE, an astronomer estimated from the Spitzer Infrared Observatory 'First Light' survey that at this wavelength, A(m) = $2.4 \times 10^{-6}\ m^{+7.4}$ stars/magnitude/deg^2. Suppose that the faintest star detectable by WISE has a magnitude of m = +15. Using the method of integration, how many total number of stars would WISE be able to detect in a field equal to the WISE survey area of 0.64 deg^2, and that are fainter than V482 Carina?

Answer: The integral to perform is:

$$N = \int_{+6}^{+15} 2.4x10^{-6} m^{+7.4} dm \qquad \text{which yields:} \qquad N = \frac{2.4x10^{-6}}{8.4}\left[(15)^{8.4} - (6)^{8.4}\right]$$

so N = 2,162 stars/deg^2.

Since the field in question has an area of 0.64 deg^2, the prediction is that there would be N = 2,164 stars/deg^2 x (0.64 deg^2) = 1,385 stars in an average star field of this size.

Note: The WISE satellite's First Light star field near V482 Carinae was close to the plane of the Milky Way, so the estimated 3,000 stars reportedly seen in its field is higher than what one may estimate from functions that model average numbers. Students may experiment with other choices for the power-law exponent, n, to see how sensitive the predictions are to the choice made, and the number of stars seen in the WISE image. Challenge them to find a means for excluding 0 < n < 3, or N > 8 as possibilities.


Life-size model of the Webb Space Telescope.

In 2018, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, θ(R), for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on its surface is given by

$$T(R) = \frac{250}{\sqrt{R}} \text{ Kelvins}$$

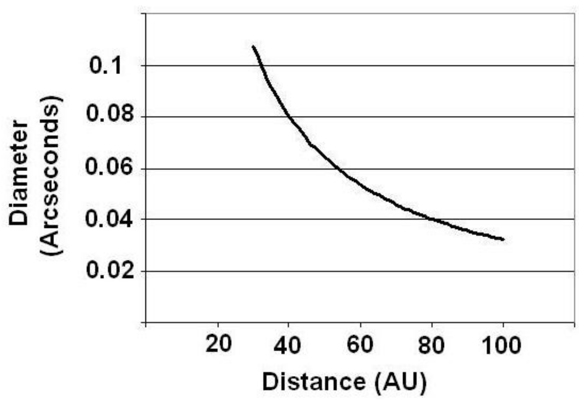
where R is the distance from the sun in AU. Create a graph that shows T(R) vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

Problem 3 - A body in other outer solar system with an angular size θ(R) emits most of its light energy in the infrared and has a temperature given by T(R) in Kelvins. Its brightness in units of Janskys, F, at a wavelength of 20 microns will be given by:

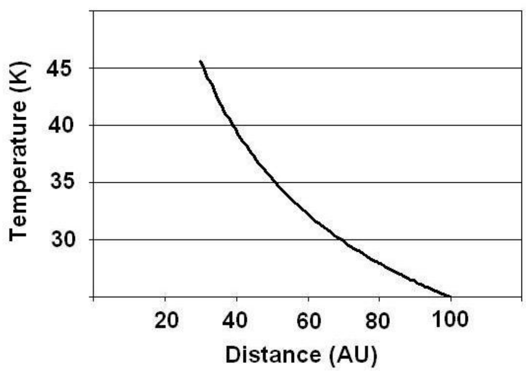
$$F(T) = \frac{120000}{(e^x - 1)}\theta(R)^2 \quad \text{Janskys} \qquad \text{where} \qquad x = \frac{720}{T(R)}$$

From the formula for θ(R) and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

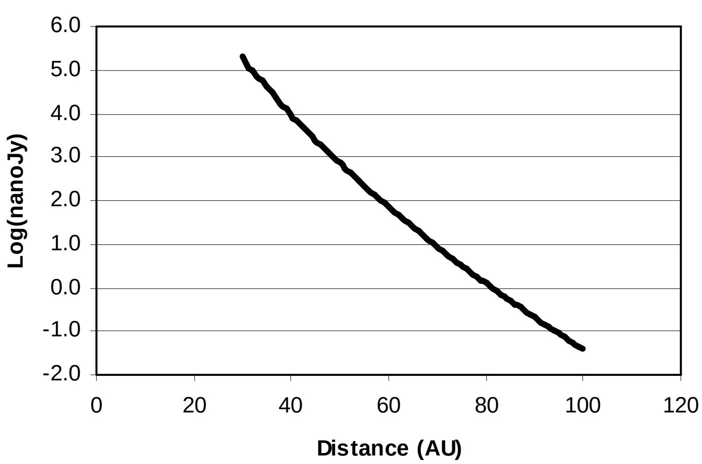
Problem 1 - Answer: At 90 AU, the disk of a Pluto-sized body will be 0.035 arcseconds in diameter.

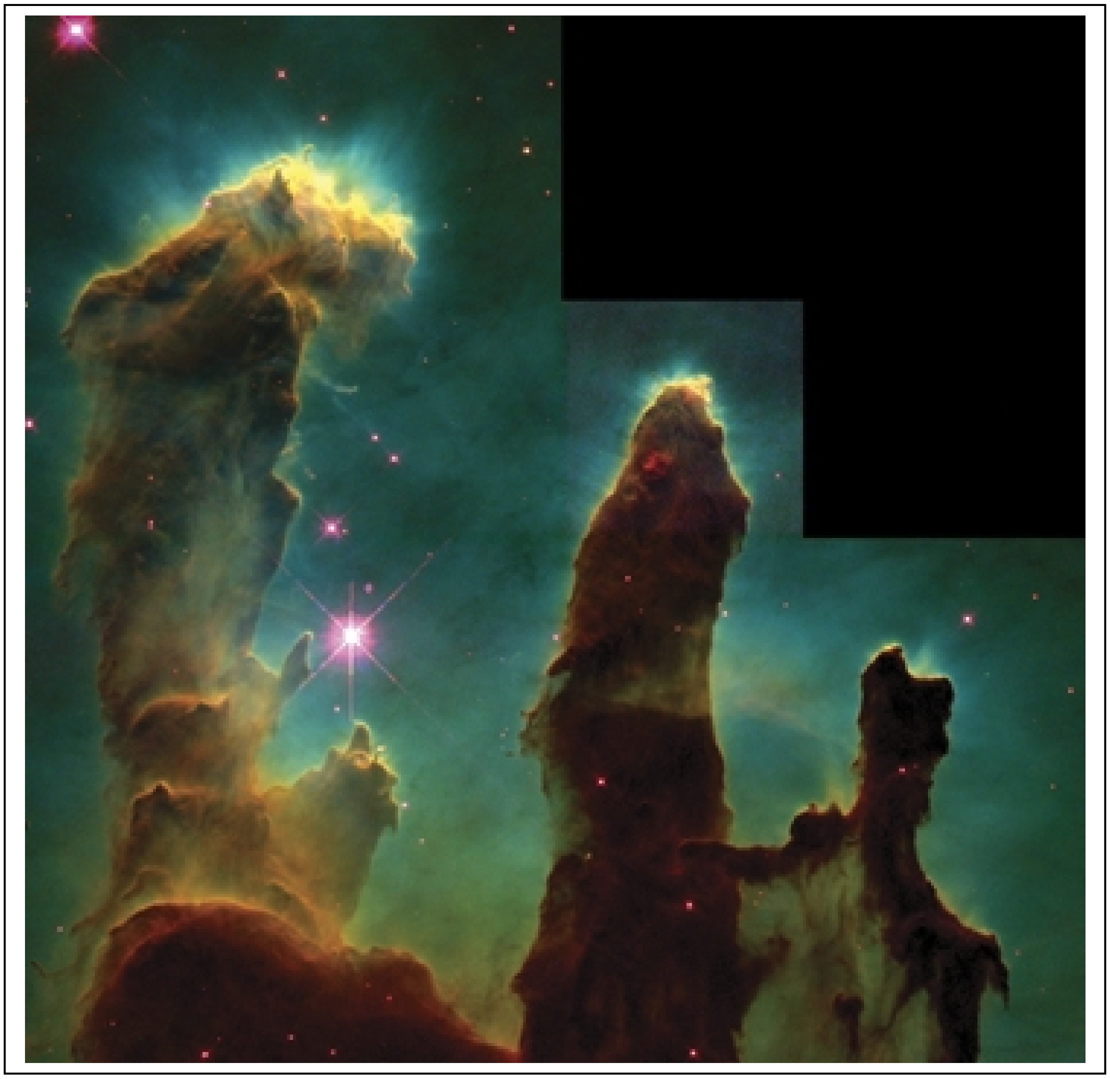


Problem 2 Answer: At 90 AU, the predicted temperature will be about 28 K.



Problem 3 - Answer: At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about 40 AU. More accurate estimates using more realistic emission properties for Pluto suggest 90 AUs as an actual limit.





The Hubble Space Telescope took the image of the Eagle Nebula (M-16). This star-forming region is in the constellation Serpens, and located 6,500 light years from Earth. It is only about 6 million years old, and the dense clouds of interstellar gas are still collapsing to form new stars. This image is 2.5 arcminutes across.

Problem 1 – If an angular size of 200 arcseconds corresponds to 1 light year at a distance of 1000 light years. What is the size of this field at the distance of the nebula?

Problem 2 – What is the scale of this image in light years/millimeter?

Problem 3 – Our Solar System is about 1/400 of a light year across. How big is it, in millimeters, at the scale of this photo?

Problem 4 - How many times the size of our solar system is the smallest nebula feature you can see in the photo?

Answer Key

Problem 1 – If an angular size of 200 arcseconds corresponds to 1 light years at a distance of 1000 light years. What is the size of this field at the distance of the nebula?

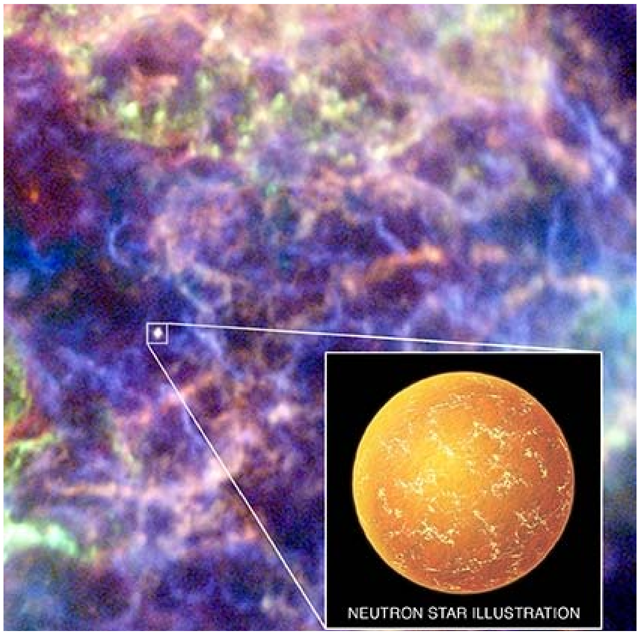
Answer: First we have to find out the scale of the image at the distance of the nebula. The field is stated to be 2.5 arcminutes across. At the distance of the nebula, 6,500 light years, the scale would be 200 arcseconds = 6,500/1000 x 1 light year = 6.5 light years.

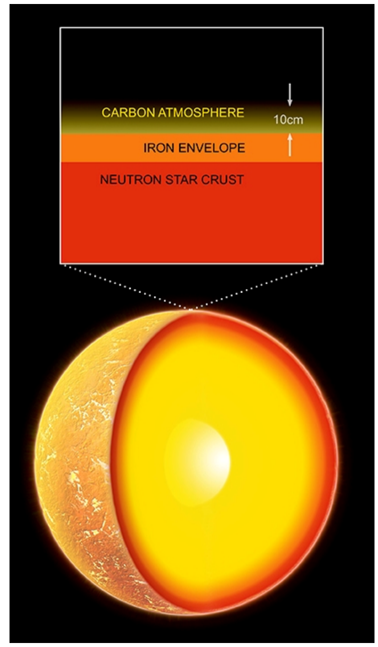
Since 1 arcminute = 60 arcseconds, by converting units we have 2.5 arcminutes x (60 arcseconds/arcminutes) x (6.5 ly/200 arcseconds) = 4.9 light years. The field is 4.9 x 4.9 light years in size.

Problem 2 – What is the scale of this image in light years/millimeter? Answer: The Hubble image is 140 millimeters across. Since this equals 4.9 light years, the scale is 4.9 ly/140mm = 0.035 light years/millimeter.

Problem 3 – Our Solar System is about 1/400 of a light year across. How big is it, in millimeters, at the scale of this photo? Answer: At the scale of the photo ,1/400 of a light years = 0.0025 light years = 0.0025/.035 = 0.07 millimeters. This is about the same size as the 'period' at the end of this sentence .

Problem 4 - How many times the size of our solar system is the smallest nebula feature you can see in the photo? Answer: Some features are about 0.2 millimeters in size, which equals 0.2 mm/0.07 = 2.8 times the diameter of our solar system!





NASA Press Release – “The Chandra X-ray Observatory image to the left shows the central region of the supernova remnant Cassiopeia A. This interstellar cloud 14 light years across, is all that remains of a massive star that exploded 330 years ago. A careful analysis of the X-ray data has revealed that the dense neutron star left behind by the supernova has a thin carbon atmosphere as shown in the figure to the right. The neutron star is only 14 miles (23 kilometers) in diameter, and is as dense as an atomic nucleus (100 trillion gm/cc). The atmosphere is only about four inches (10 cm) thick, has a density similar to diamond (3.5 gm/cc), and a temperature of nearly 2 million Kelvin. The surface gravity on the neutron star is 100 billion times stronger than on Earth, which causes the atmosphere to be incrredibly thing even with such a high temperature.”

How much carbon is there?

Problem 1 – What are the facts that we know about the atmosphere from the news announcement, and what combination of facts will help us estimate the atmosphere’s mass?

Problem 2 – If the volume of a thin spherical shell is $V = 4 \pi R^2 h$ where R is the radius of the sphere and h is the thickness of the shell, what other formula do you need to calculate the atmosphere’s mass?

Problem 3 – What is your estimate for the mass of the carbon atmosphere in A) kilograms? B) metric tons? C) Earth Atmosphere masses (Ae) where 1 Ae = 5.1×10^{18} kg? (Provide answers to two significant figures)

Problem 1 – What are the facts that we know about the atmosphere from the news announcement, and what combination of facts will help us estimate the atmosphere's mass?

Answer: The facts are as follows, with the facts that help estimate the atmosphere's mass indicated in bold face:

1... The interstellar cloud is 14 light years across,

2… The supernova exploded 330 years ago.

3… **The neutron star is only 14 miles (23 kilometers) in diameter**.

4… The neutron star is as dense as an atomic nucleus (100 trillion gm/cc).

5… **The atmosphere is only about four inches (10 cm) thick,**

6… **The atmosphere has a density similar to diamond (3.5 gm/cc),**

7… The atmosphere has a temperature of nearly 2 million Kelvin.

8… The surface gravity on the neutron star is 100 billion times stronger than on Earth.

Problem 2 – If the volume of a thin spherical shell is $V = 4 \pi R^2 h$ where R is the radius of the sphere and h is the thickness of the shell, what other formula do you need to calculate the atmosphere's mass?

Answer: The formula gives the volume occupied by the atmosphere, so you need a relationship that relates volume to mass: **Mass = Density x Volume.**

Problem 3 – What is your estimate for the mass of the carbon atmosphere in A) kilograms? B) metric tons? C) Earth Atmosphere masses (Ae) where 1 Ae = 5.1×10^{18} kg? (Provide answers to two significant figures)

Answer: Convert all measures to centimeters, so the neutron star diameter is 23 kilometers x (100,000 cm/1 km) = 2.3×10^6 cm and its radius is 1.1×10^6 cm. . The volume of the atmosphere is

V(shell) = $4 \times 3.14 \times (1.1 \times 10^6 \text{ cm})^2 (10 \text{ cm})$
$= 1.5 \times 10^{14} \text{ cm}^3$

A) Mass = $(3.5 \text{ grams/cm}^3) \times 1.5 \times 10^{14} \text{ cm}^3$
$= 5.3 \times 10^{14}$ grams
$= 5.3 \times 10^{14}$ grams x (1 kg/1000 grams) = **5.3×10^{11} kilograms**

B) 5.3×10^{11} kilograms x (1 tons/1000 kilograms) = **5.3×10^8 tons (also 0.53 gigatons)**

C) 5.3×10^{11} kilograms x (1 Ae/ 5.1×10^{18} kilograms) = **1.0×10^{-7} Ae**

The Most Massive Stars Known

Star	Mass (Sun= 1)	Diameter (Sun=1)	Luminosity (Sun=1)
R136a1	350	35	10,000,000
Eta Carina	250	195	5,000,000
Peony Nebula	175	100	3,200,000
Pistol Star	150	340	6,300,000
HD269810	150	19	2,200,000
LBV1806	130	200	10,000,000
HD93129a	120	25	5,500,000
HD93250	118	18	5,000,000
S Doradus	100	380	600,000

The picture above shows the Tarantula Nebula in the Large Magellanic Cloud, located 165,000 light years from Earth. Astronomers using data from the Hubble Space Telescope and the European Space Observatory's Very Large Telescope in Chile have recently determined that it harbors the most massive known star in our 'corner' of the universe which they call R136a1. The table lists some of the most massive known stars as of July, 2010.

Problem 1 - From the indicated sizes relative to the Sun, create a scale model that shows the relative sizes of these stars compared to the Sun, assuming that in the scaled model the solar disk has a diameter of 1 millimeter.

Problem 2 - The predicted lifespan of a star depends on its luminosity according to the formula

$$T = \frac{10 \text{ billion years}}{M^{2.5}}$$

For the Sun, M = 1 and so its lifespan is about 10 billion years. A star with 10 times the mass of our Sun, M=10, will last about 30 million years! From the table above, what are the predicted life spans for these 'hypergiant' stars to two significant figures?

Problem 3 - How many generations of a 100 solar-mass hypergiant could pass during the life span of a single star like our own Sun?

Problem 1 - From the indicated sizes relative to the Sun, create a scale model that shows the relative sizes of these stars compared to the Sun, assuming that in the scaled model the solar disk has a diameter of 1 millimeter.

Answer: **Example of calculation: R136a1 diameter = 700x sun, so 700 x 1 mm = 0.7 meters in diameter!**

Problem 2 - The predicted lifespan of a star depends on its luminosity according to the formula

$$T = \frac{10 \text{ billion years}}{M^{2.5}}$$

For the Sun, M = 1 and so its lifespan is about 10 billion years. A star with 10 times the mass of our Sun, M=10, will last about 30 million years! From the table above, what are the predicted lifespans for these 'hypergiant' stars?

Answer:

Star	Mass (Sun=1)	Diameter (Sun=1)	Luminosity (Sun=1)	Lifespan (years)
R136a1	350	35	10,000,000	**4,400**
Eta Carina	250	195	5,000,000	**10,000**
Peony Nebula	175	100	3,200,000	**25,000**
Pistol Star	150	340	6,300,000	**36,000**
HD269810	150	19	2,200,000	**36,000**
LBV1806	130	200	10,000,000	**52,000**
HD93129a	120	25	5,500,000	**63,000**
HD93250	118	18	5,000,000	**66,000**
S Doradus	100	380	600,000	**100,000**

Problem 3 - How many generations of a 100 solar-mass hypergiant could pass during the life span of a single star like our own Sun?

Answer: Life span of our sun is 10 billion years. Life span of a 100 solar-mass hypergiant is 100,000 years, so about 10 billion/100,000 = **100,000 generations** could come and go.

Celestial Fireworks Near Cluster NGC-3603

This young star cluster, barely 1 million years old, is still surrounded by the clouds of interstellar gas and dust from which it formed. The nebula, located 20,000 light-years away in the constellation Carina, contains a central cluster of 50, huge, hot stars.

The image to the left was taken by the Hubble Space Telescope, and is 20 light years across.

The massive, hot stars in this cluster emit nearly 1/3 of their light at ultraviolet wavelengths and shorter. A single ultraviolet photon, when it encounters a hydrogen atom, can fully ionize the atom so that the lone electron is stripped away from the hydrogen's proton nucleus. These stars produce so many ultraviolet photons that they can ionize the entire gas cloud that surrounds them to a distance of many light years.

Problem 1 - A star with a temperature of 40,000 K emits about 6×10^{49} ultraviolet photons each second. The density of the hydrogen gas surrounding the star is about 100 atoms/cm^3. How many hydrogen atoms exist in a volume of space with a radius of 3 light years? (1 ly = 9.5×10^{17} centimeters.)

Problem 2 - The size of an ionized region is determined by the balance between the rate at which ultraviolet photons are ionizing the hydrogen atoms, and the rate at which the electrons 'recombine' with the protons to re-form the hydrogen atom. A formula that determines the radius of an ionization region is given by

$$R^3 = 2.6x10^{11} \frac{P}{n^2}$$

If P = 6.0×10^{49} photons/sec and n = 100 atoms/cm^3, what is the radius of this nebula in light years?

Image credit: NASA, ESA, R. O'Connell (University of Virginia), F. Paresce (National Institute for Astrophysics, Bologna, Italy), E. Young (Universities Space Research Association/Ames Research Center), the WFC3 Science Oversight Committee, and the Hubble Heritage Team (STScI/AURA)

Problem 1 - A star with a temperature of 40,000 K emits 6.0×10^{49} ultraviolet photons each second. The density of the hydrogen gas surrounding the star is about 100 atoms/cm^3. How many hydrogen atoms exist in a volume of space with a radius of 12 light years? (1 ly = 9.5×10^{17} centimeters.)

Answer: R = $12 \times 9.5 \times 10^{17}$ cm = 1.1×10^{19} centimeters. V = $4/3 \pi R^3$, so V = $1.66(3.14)(1.1 \times 10^{19})^3 = 5.6 \times 10^{57}$ cm^3. The number of hydrogen atoms is just N = 100 atoms/cc x 5.6×10^{57} cm^3 = **5.6×10^{59} atoms**.

Problem 2 - The size of an ionized region is determined by the balance between the rate at which ultraviolet photons are ionizing the hydrogen atoms, and the rate at which the electrons 'recombine' with the protons to re-form the hydrogen atom. A formula that determines the radius of an ionization region is given by

$$R^3 = 2.6x10^{11} \frac{P}{n^2}$$

If P = 6.0×10^{49} photons/sec and n = 100 atoms/cm^3, what is the radius of this nebula in light years?

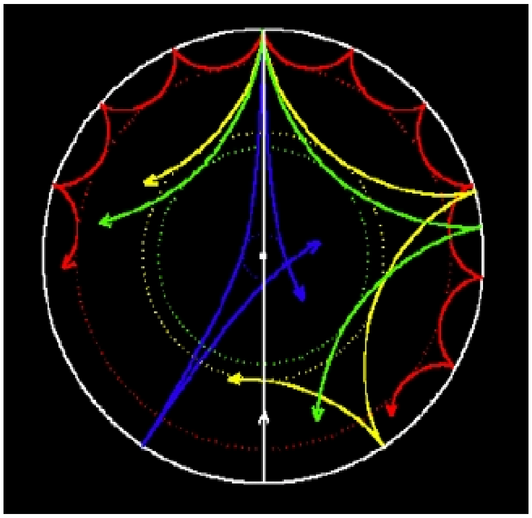
Answer: $$R^3 = 2.6x10^{11} \frac{(6.0x10^{49})}{100^2}$$ so

$R^3 = 1.6 \times 10^{57}$

$R = 1.1 \times 10^{19}$ cm.

Then R = 1.1×10^{19} cm x (1 ly/9.5×10^{17} cm) = **12 light years**.

Note: NGC-3603 contains 50 of these stars. At the above gas density, this cluster can form a nebula with a radius of at least 12 light years x $50^{1/3}$ = 44 light years, which is quite a bit larger than the scale of the Hubble image.



Sound waves from turbulent activity on the star's Surface are reflected through the interior along a variety of paths. The surface layer, producing the light we see, moves up and down in complex ways that cause brightness changes over many different time scales from seconds to minutes.

One of the hallmarks of a truly successful space mission is that, not only does it meet all of its planned scientific goals for which it was designed, but that it contributes to advancements in science well-beyond the perimeter of its own specialized research areas.

Recently, the Kepler satellite, which was designed to detect Earth-sized planets orbiting other stars, has made a series of discoveries that have revolutionized the subject of stellar structure and evolution.

Using the minute brightness changes that indicate sound waves traveling in the interiors of giant stars, Kepler data has now confirmed a fundamental hypothesis of stellar evolution proposed by physicist George Gamow nearly 75 years ago!

As stars no longer have enough hydrogen fuel in their cores to sustain them, they switch to fusing hydrogen into helium in a thin shell just outside the core. This shell-burning phase is the prelude to the star becoming a red giant or red supergiant star, depending on its mass. By measuring the brightness changes in many of these red giant stars, Kepler data has been used to probe the interior structure of these stars, revealing just such a shell-burning zone! The sound waves that bounce from the surface to the inner shell are reflected back to the surface where that cause minute brightness changes over time. These 'seismic waves' can be modeled to reveal the location, thickness and depth of the shell-burning zones.

Problem 1 - Draw a circle to represent a star's surface. Draw a series of semi-circles of different radii similar to the ones in the diagram above, but where the sequence of circles along the circumference have the same ending point as its starting point. This represents a 'standing wave' on the surface. The point on the surface where the arc meets the surface is called the 'node'. How many nodes do you count for circles of the different radii that you create?

Problem 2 - If the location of the shell-burning zone outer edge is half-way to the center of the star, for the arc that just touches the shell-zone, how far apart will the nodes be on the surface of the star, if the radius of the star is 5 million kilometers?

Problem 3 - If the speed of the wave is 1,000 km/sec, how many seconds elapse in the travel between the nodes, and what would you see in the light from this star?

Problem 1 - Draw a circle to represent a star's surface. Draw a series of semi-circles of different radii similar to the ones in the diagram above, but where the sequence of circles along the circumference have the same ending point as its starting point. This represents a 'standing wave' on the surface. The point on the surface where the arc meets the surface is called the 'node'. How many nodes do you count for circles of the different radii that you create?

Answer: For example, divide the circle into four equal quadrants. Place the compass in the upper right quadrant at a position on the circumference that is half-way between the two points where the horizontal and vertical lines intersect the circumference. Set your compass radius so that it touches one of these points, then draw the interior arc. Repeat this for the other three quadrants. Divide the circle into five equal parts and repeat this construction process. The number of nodes for each compass radius length are shown in the table below. The arc radius is given in terms of the length of the radius of the circle taken as R=1.0.

Nodes	Number of arcs	Radius of arc
2	**2**	**1.414**
3	**3**	**1.000**
4	**4**	**0.765**
5	**5**	**0.618**
6	**6**	**0.518**
7	**7**	**0.445**
8	**8**	**0.390**

Students may prove that, if the radius of the circle is defined as R, and the number of nodes on the circumference is N, the general formula for the arc radius is just

$$d = R\sqrt{2 - 2\cos\left(\frac{\pi}{N}\right)}$$

Problem 2 - If the location of the shell-burning zone outer edge is half-way to the center of the star, for the arc that just touches the shell-zone, how far apart will the nodes be on the surface of the star, if the radius of the star is 5 million kilometers?

Answer: If the shells outer edge is half way to the center of the star, its radial location is 0.500 in units of the star's radius. The closest wave arc that come close to this is for the Node=6 arc with R = 0.518. The full circumference of the star's circle is 2π, so the nodes are located along the circumference a distance of $2\pi/N = 2\pi/6$ or $\pi/3$ apart in multiples of the radius of the star. Since the star's radius is 5 million km, the nodes are located (3.141) x (0.333) x (5million km) = **5.22 million km apart**.

Problem 3 - If the speed of the wave is 1,000 km/sec, how many seconds elapse in the travel between the nodes, and what would you see in the light from this star? Answer: The wave travels along the arc between the node points, whose radius is 5.2 million km, and length is 3.141 x (5.22 million km) = 16.4 million km.
T = 16.4 million km/(1000 km/s) = 16,400 seconds or about 4.6 hours.
The light from the star may 'flicker' slightly in brightness with a period of 4.6 hours.

IC-443 supernova remnant located 5,000 light years from Earth. The purple color shows the location of the gamma rays seen by Fermi. The supernova remnant has many filaments of gas in which the cosmic rays are boosted in speed.

Cosmic rays are fast-moving particles that travel through space at nearly the speed of light, though they are not electromagnetic radiation at ll. Instead, they are typically electrons and the nuclei of atoms such as hydrogen and helium. Since their discovery in the 1940s, astronomers have speculated that they are created when stars explode as supernovae. The tremendous energy of the explosion ejects matter from the outer layers of the star into space. As this matter spreads out into individual atoms and nuclei, they become the particles we see in cosmic rays.

NASA's Fermi Gamma Ray Observatory has studied the gamma rays that comes from the remains of two nearby supernovae called IC-443 and W44, and has confirmed that the expanding matter does produce cosmic rays.

In space, the average density of cosmic rays is about 0.005 cosmic rays/meter^3.

Problem 1 – The Milky Way is a disk shaped system of stars with a radius of 50,000 light years and a thickness of about 3,000 light years. One light year is equal to 9.5×10^{15} meters. What is the volume of the Milky Way galaxy in cubic meters?

Problem 2 - About what is the total number of cosmic rays in the Milky Way galaxy?

Problem 3 – Suppose a single supernova can eject 2.0×10^{30} kilograms of matter into space. If the mass of a single proton is 1.6×10^{-27} kg, how many hydrogen nuclei does this represent?

Problem 4 – About how many supernova would be required to 'fill up' the Milky Way with cosmic rays if all of the ejected mass in hydrogen atoms were converted into cosmic rays?

Problem 1 - The Milky Way is a disk shaped system of stars with a radius of 50,000 light years and a thickness of about 3,000 light years. One light years is equal to 9.5x1015 meters. About what is the volume of the Milky Way in cubic meters?

Answer: The volume of the galaxy is $V = \pi R^2 H = 3.141 \times (50000 \times 9.5 \times 10^{15})^2 (3000 \times 9.5 \times 10^{15})$ = **2.0×10^{61} meters^3**.

Problem 2 - About what is the total number of cosmic rays in the Milky Way galaxy?

Answer: $N = 0.005 \times 2.0 \times 10^{61} = 1.0 \times 10^{59}$ cosmic rays.

Problem 3 – A single supernova can eject 2.0×10^{30} kilograms of matter into space. If the mass of a single proton is 1.6×10^{-27} kg, how many hydrogen nuclei does this represent?

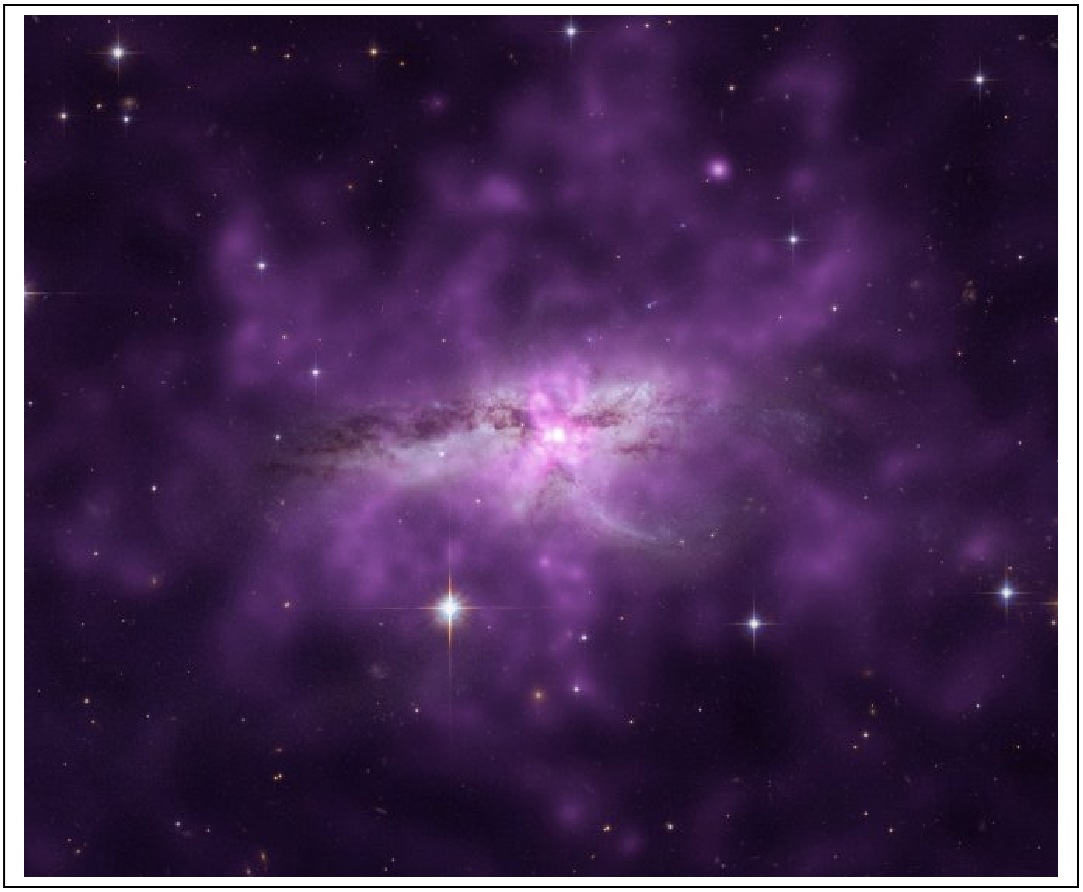
Answer: 2.0×10^{30} kg x (1 hydrogen atom/1.6×10^{-27} kg) = **1.3×10^{57} hydrogen atoms**.

Problem 4 – About how many supernova would be required to 'fill up' the Milky Way with cosmic rays if all of the ejected mass in hydrogen atoms were converted into cosmic rays?

Answer: 1.0×10^{59} cosmic rays x (1 Supernova/1.3×10^{57} hydrogen atoms) = **77 supernova**.

Note: These are only estimates that fit the simple origins model we used. In fact, the actual amount of matter converted into cosmic rays per supernova explosion is far less than what was used in Problem 3. The present population of cosmic rays has been built up over billions of years as millions of stars have become supernova. Each supernova adds its share to the Milky Way's cosmic ray 'atmosphere'. Over time, many of these cosmic rays actually leave the Milky Way galaxy entirely. The Milky Way is not a closed vessel that accumulates cosmic rays but a leaky bag.

Giant Gas Cloud in System NGC 6240



Scientists have used Chandra to make a detailed study of an enormous cloud of hot gas enveloping two large, colliding galaxies. This unusually large reservoir of gas contains as much mass as 10 billion Suns, spans about 300,000 light years, and radiates at a temperature of more than 7 million degrees.

This giant gas cloud, which scientists call a "halo," is located in the system called NGC 6240. Astronomers have long known that NGC 6240 is the site of the merger of two large spiral galaxies similar in size to our own Milky Way. Each galaxy contains a supermassive black hole at its center. The black holes are spiraling toward one another, and may eventually merge to form a larger black hole.

Problem 1 - If 1 light year equals 9.5×10^{15} meters, and the cloud is in the shape of a sphere with a diameter of 300,000 light years, what is the volume of this cloud in cubic meters? ($\pi = 3.141$)

Problem 2 - The mass of the sun is 2.0×10^{30} kilograms. What is the density of this cloud in kilograms/m^3?

Problem 3 - If a single hydrogen atom has a mass of 1.7×10^{-27} kilograms, how many hydrogen atoms per cubic meter does the gas density represent?

Giant Gas Cloud in System NGC 6240
April 30, 2013
http://www.nasa.gov/mission_pages/chandra/multimedia/ngc6240.html

Problem 1 - If 1 light year equals 9.5 x 10^{15} meters, and the cloud is in the shape of a sphere with a diameter of 300,000 light years, what is the volume of this cloud in cubic meters? (π = 3.141)

Answer: V = 4/3 π R^3 so

V = 1.333 (3.141) (150,000 x 9.5x1015 meters)^3

= **1.2 x 10^{64} meters^3**

Problem 2 - The mass of the sun is 2.0 x 10^{30} kilograms. What is the density of this cloud in kilograms/m^3?

Answer: Mass of cloud = 10 billion suns = 10^{10} x 2.0x10^{30} kg = 2.0x10^{40} kg.

Density = mass/volume

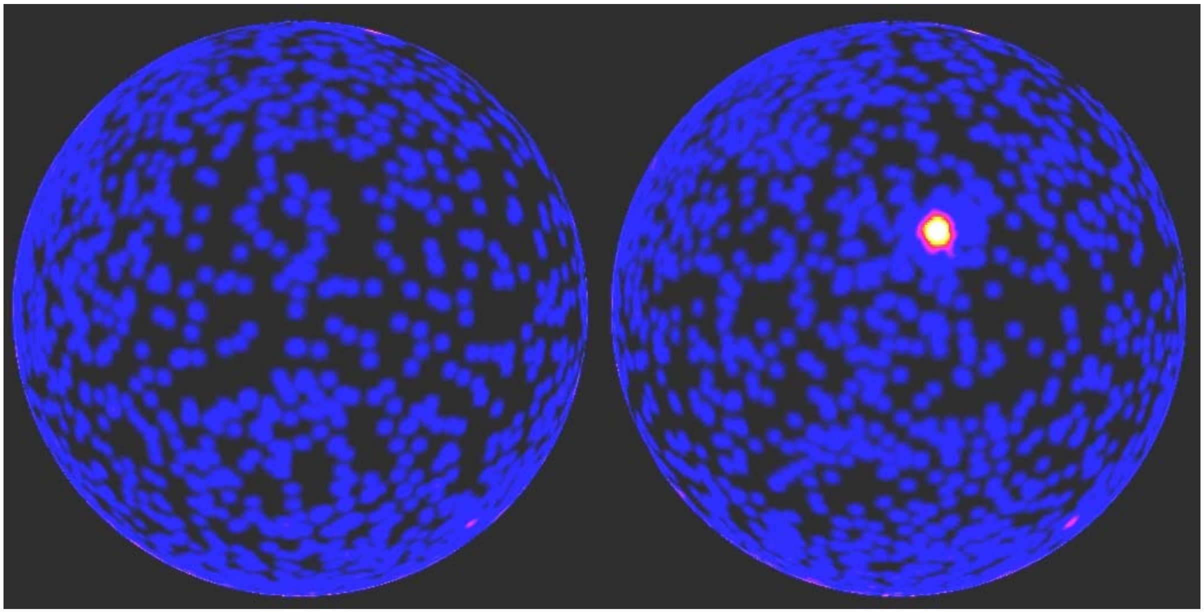
= 2.0x10^{40} kg / 1.2x10^{64} m^3

= **1.7x10^{-24} kg/m^3**

Problem 3 - If a single hydrogen atom has a mass of 1.7x10^{-27} kilograms, how many hydrogen atoms per cubic meter does the gas density represent?

Answer: Density = 1.7x10^{-24} kg/m^3 x (1 atom/1.7x10^{-27} kg)

= **1000 atoms/meter^3**



In April, 2013, a record-setting blast of gamma rays from a dying star in a distant galaxy wowed astronomers around the world. The eruption, which was classified as a gamma-ray burst, or GRB, and designated GRB 130427A, produced the highest-energy light ever detected from such an event. Fermi's Large Area Telescope (LAT) detected the gamma-ray emission from the burst, which lasted for hours, and it remained detectable for the better part of a day, setting a new record for the longest gamma-ray emission from a GRB. The event occurred 3.6 billion light years from Earth in the direction of the constellation Leo. The estimated energy flux from this event at the location of Earth was about $F = 2.0\times10^{-8}$ watts/meter^2.

Problem 1 – Although instruments at Earth can measure how many watts of energy were detected over an area of 1 square meter, called the radiant flux, they would really like to know how much energy in watts, was released by the source of the event. Suppose that the same flux of energy flowed through every square meter of surface, of a sphere whose radius equaled the distance to the source of 3.6 billion light years (1 light year = 9.5×10^{15} meters). What was the total power emitted by the source into space in A) watts, B) solar units where 1 solar unit = 4.0×10^{26} watts)?

Problem 2 - Instead of the energy emitted across the entire sky like our sun does, astronomers think that the energy from a gamma-ray burst is emitted in two narrow beams that come from the core of the star as it becomes a supernova. If the area of each beam on the sky is only 1 square degree, and the full area of the sky is 42,000 square degrees, what will be the estimated total power of the source in watts and in solar units?

NASA's Fermi, Swift See 'Shockingly Bright' Burst
May 3, 2013
http://www.nasa.gov/topics/universe/features/shocking-burst.html

Problem 1 – Although instruments at Earth can measure how many watts of energy were detected over an area of 1 square meter, called the radiant flux, they would really like to know how much energy in watts, was released by the source of the event. Suppose that the same flux of energy flowed through every square meter of surface, of a sphere whose radius equaled the distance to the source of 3.6 billion light years (1 light year = 9.5×10^{15} meters). What was the total power emitted by the source into space in A) watts, B) solar units where 1 solar unit = 4.0×10^{26} watts)?

Answer: The surface area of a sphere with a radius of 3.6 billion light years is
R = 3.6 billion ly x (9.5×10^{15} meters/ly) = 3.4×10^{25} meters.

Then S = 4 (3.141) $(3.4 \times 10^{25})^2 = 1.45 \times 10^{52}$ meters^2.

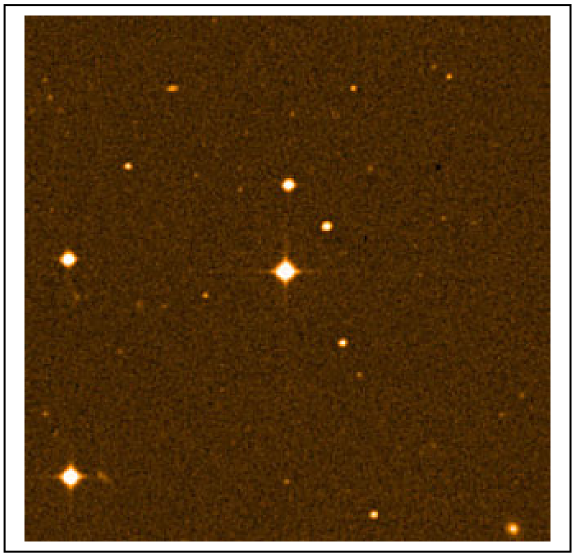
A) The total power = 2.0×10^{-8} watts/m^2 x 1.45×10^{52} m^2 = **2.9×10^{44} watts**.

B) 2.9×10^{44} watts x (1 solar unit / 4.0×10^{26} watts) = **7.3×10^{17} solar units**.

Note: If the source radiation was emitted over the entire surface of the sphere, it would equal the energy produced each second by 730 thousand trillion stars like our sun!

Problem 2 - Instead of the energy emitted across the entire sky like our sun does, astronomers think that the energy from a gamma-ray burst is emitted in two narrow beams that come from the core of the star as it becomes a supernova. If the area of each beam on the sky is only 1 square degree, and the full area of the sky is 42,000 square degrees, what will be the estimated total power of the source in watts and in solar units?

Answer: Each beam only emits energy into 1/42,000 of the full sky, so the area on the spherical surface for each beam is only 1/42000 x 1.45×10^{52} meters^2 or 3.5×10^{47} meters^2. For both beams the total area is 2 x 3.5×10^{47} meters^2 or 7.0×10^{47} meters^2. The total power from the gamma-ray source is then 2.0×10^{-8} watts/meter^2 x 7.0×10^{47} meters^2 = **1.4×10^{40} watts**. This equals the power from **35 trillion suns**!



Cayrel's Star (See ESO photo) is a faint star located 13,000 light years away in the constellation Cetus the Whale. In 2001 a group of astronomers led by Roger Cayrel made the first direct measurement of the age of this star by using the radioactive decay of Uranium-238, which decays to lead with a half-life of 4.47 billion years.

A detailed spectroscopic study revealed numerous, clear lines of uranium in the star's spectrum. From this they could determine the abundances of the elements, and use their ratios to figure out an age for this star.

The measurements showed that the uranium abundances are about 1/7 that of our own sun. The age our sun is known to be 4.6 billion years.

Problem 1 – The half-life formula states that the initial abundance of a radioactive element, N_o, with a half-life of T is related to its current abundance N(t) by

$$N(T) = N_0 e^{-0.69(t/T)}$$

In the case of our sun, what fraction of the Uranium-238 isotope remains in the sun today given an age of 4.6 billion years?

Problem 2 – If the abundance of Uranium-238 in Cayrel's Star is 1/7 of our sun, what is the age of Cayrel's Star?

Problem 3 – Astronomers were able to detect the element Thorium-232 in the atmosphere of Cayrel's Star. It has a half-life of 14.1 billion years. If the amount of thorium-232 in our sun today is 4.4×10^{19} kg, how much was there in the sun when it was first formed?

Problem 1 – The half-life formula states that the initial abundance of a radioactive element, No, with a half-life of T is related to its current abundance N(t) by

In the case of our sun, what fraction of the Uranium-238 isotope remains in the sun today given an age of 4.6 billion years?

Answer:
$N(4.6)=n_o\, e^{(-0.69 \times 4.6/4.7)}$ so $N/n_0 = 0.49$ and so there is **half as many** atoms of uranium-238 today as there was when the sun was formed.

Problem 2 – If the abundance of Uranium-238 in Cayrel's Star is 1/7 of our sun, what is the age of Cayrel's Star?

Answer:

$1/7 = e^{(-0.69\, t/4.47)}$ solving for t we get $\mathrm{Ln}(1/7)= -0.69(T/4.47)$ then **t= 12.6 billion years**.

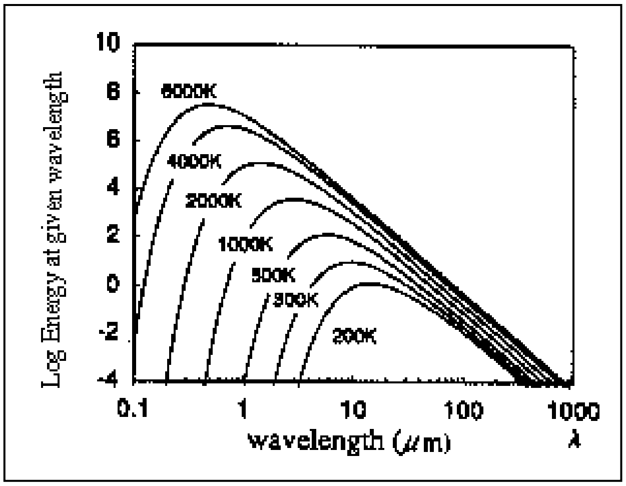
Problem 3 – Astronomers were able to detect the element Thorium-232 in the atmosphere of Cayrel's Star. It has a half-life of 14.1 billion years. If the amount of thorium-232 in our sun today is 4.4×10^{19} kg, how much was there in the sun when it was first formed?

Answer: The decay fraction is just $e^{(-0.69 \times 4.6/14.1)} = 0.80$. So the initial amount was N(4.6)/0.8 = no and so n_o = **5.5×10^{19} kg.**

Note: The abundance of Thorium-232 in our sun today is estimated to be 0.0335 parts per million of silicon atoms, which are 650 ppm of the mass of the sun, which in turn has a total mass of 2.0×10^{30} kg. So,

Amount of thorium = 2.0×10^{30} kg x 650 x 10^{-6} (silicon/Msun) x 0.0335 $\times 10^{-6}$ (thorium/silicon) = 4.4×10^{19} kg.

Star Light…Star Bright



Astronomers can 'take the temperature' of a star by measuring the star's brightness through two filters that pass radiation in the 'blue' and 'visual' regions of the visible spectrum.

From the ratio of these brightnesses, a simple cubic relationship yields the temperature of the star, in Kelvin degrees as follows:

$$T(x) = 9391 - 8350\,x + 5300\,x^2 - 1541\,x^3$$

This formula works for star temperatures between 9,000 and 3,500 Kelvin.

Problem 1 - From the indicated temperature range, what is the domain of this function?

Problem 2 – The sun has a temperature of 5770 K. What is the corresponding value for x?

Problem 3 – To save computation time, an astronomer uses the approximation for T(x) based on a quadratic formula given by $F(x) = 1844x^2 - 6410x + 9175$. What is the formula that gives the percentage error, P(x), between F(x) and T(x)?

Problem 4 – A what temperature, T(x), does P(x) defined in Problem 3 have its maximum absolute percentage error in the domain of x: [0,1.4], and what is this value?

Problem 1 - From the indicated temperature range, what is the domain of this function?
Answer: We have to solve T(x) = 9,000 and T(x) = 3,500. This can be done using a graphing calculator by programming T(X) into an Excel spreadsheet. Acceptable answers for [9000,3500] should be near **[0.06, 1.50]**

Problem 2 – The sun has a temperature of 5770 K. What is the corresponding value for x? Answer: T(x) = 5770, so **x = 0.67**

Problem 3 – To save computation time, an astronomer uses the approximation for T(x) based on a quadratic formula given by $F(x) = 1844x^2 - 6410x + 9175$. What is the formula that gives the percentage error, P(x) between F(x) and T(x)?

Answer: P(x) = 100% x (estimate – actual) / actual

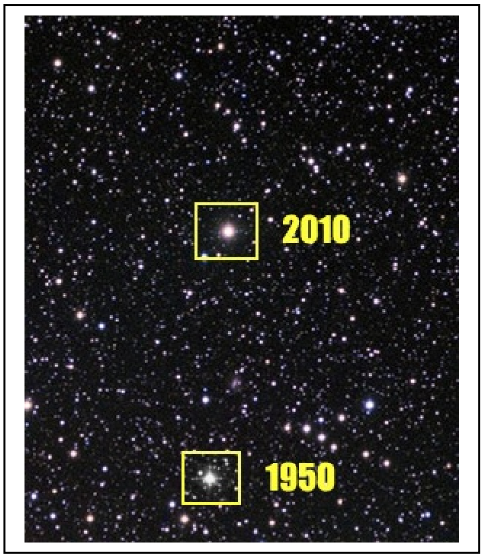
$$= 100\,(F(x)-T(x)) / T(x)$$

$$= \frac{100(1844x^2 - 6410x + 9175 - (9391 - 8350x + 5300x^2 - 1541x^3))}{9391 - 8350x + 5300x^2 - 1541x^3}$$

$$= \frac{100\,[1541x^3 - 3456x^2 + 1940x - 216]}{9391 - 8350x + 5300x^2 - 1541x^3}$$

Problem 4 – A what temperature, T(x), does P(x) defined in Problem 3 have its maximum absolute percentage error in the domain of x: [0,1.4], and what is this value?

Answer: Students can use a graphing calculator to display this function, or use Excel to program and plot it, to manually determine the maximum. The graph shows a local minimum of -4.6% at x=1.1, and a local maximum of +1.55% at x=0.4. The absolute maximum error is then 4.6% at x=1.1. The function T(X) evaluated at x=1.1 yields **T = 4,600 K**.



Stars travel along different orbits through the Milky Way. Near our sun, stars are going mostly in the same direction, but from time to time they pass close together. Barnard's Star is currently in the constellation Ophiuchus, but travels across the sky so quickly that it traverses the diameter of the full moon every 180 years.

With the sun at the center of a Cartesian coordinate grid, Barnard's Star can be represented as a point located at (2.0, 5.6) where the units are in light years. But because it is moving through space at a speed of 143 km/sec, its future position relative to the sun changes quickly in time.

The parametric equations for the X and Y location of Barnard's Star can be approximated as follows:

$$X(T) = 2.0 + 0.09\,T \qquad Y(T) = 5.67 - 0.25\,T$$

where T is in thousands of years from the present time, and all units are in light years.

Problem 1 – What is the distance to Barnard's Star at the present time?

Problem 2 – By using the differential calculus, what is the equation of the line y = Mx + B that is represented by the parametric functions?

Problem 3 – By using the Pythagorean distance formula and using the differential calculus to find the minimum distance, at what time, T will Barnard's Star be closest to the sun along this trajectory?

Problem 1 – What is the distance to Barnard's Star at the present time?

Answer: d = ($2.0^2 + 5.67^2)^{1/2}$ = 6.0 light years.

Problem 2 – What is the equation of the line Y = Mx + B that is represented by the parametric functions?

$$M = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$ so M = -0.25/0.09 and so M = -2.8

Since a point on this line is at (2.0, 5.6) we have y – 5.6 = M(x-2.0)
Then y = 5.6 -2.8x +5.6
Therefore **y = 11.2 – 2.8x**

Problem 3 – At what time, T, will Barnard's Star be closest to the sun along this trajectory?

$D(T)^2 = (2.0 + 0.09T)^2 + (5.67 – 0.25T)^2$

Take the derivative to get $2 D \frac{dD}{dt}$ = 2 (2.0 + 0.09T)(0.09) + 2(5.67-0.25T)(-0.25)

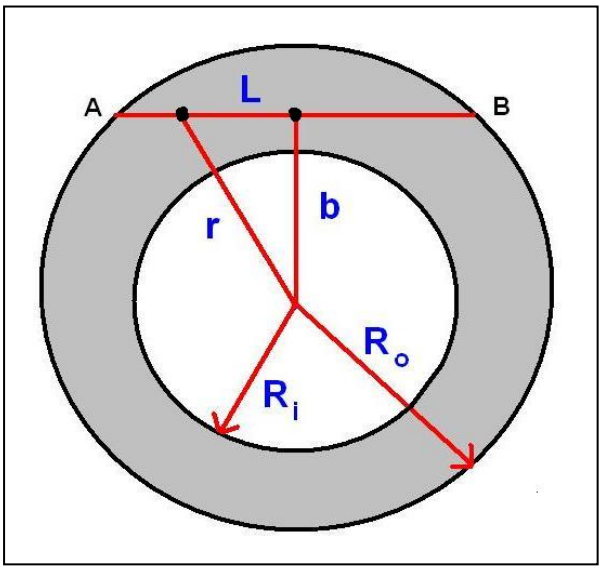
This simplifies to $D \frac{dD}{dt}$ = 0.18 + 0.0081T -1.42 + 0.0625T = -1.24 + 0.0706T

Set this equal to zero to get 0 = -1.24 + 0.0706T then **T = 17.6**.

So in about 18,000 years from now, Barnard's Star will be at its closest to the sun. Its distance at this time will be $d^2 = (2.0+0.09*17.6)^2 + (5.67 – 0.25*17.6)^2$ = 14.5 so d= **3.8 light years**.

Modeling a Planetary Nebula

Planetary nebula are the outer atmospheres of dying stars ejected into space. Astronomers model these nebulae to learn about the total mass they contain, and the details of how they were ejected. The image is of a rare, spherical-shell planetary nebula, Abell 38, photographed by astronomer George Jacoby (WIYN Observatory) and his colleagues using the giant, 4-meter Mayall Telescope at Kitt Peak, Arizona. Abell-38 is located 7,000 light years away in the constellation Hercules. The nebula is 5 light years in diameter and 1/3 light year thick. For other spectacular nebula images, visit the Hubble Space Telescope archive at
http://hubblesite.org/newscenter/archive/releases/nebula



Statement of the Problem:

We want to calculate the intensity of the nebula (shaded shell) at different radii from its center (b) along a series of chords through the nebula (AB). The intensity, I(b) will be proportional to the density of gas within the nebula, which we define as D(r).

The shell is spherically-symmetric, as is D(r), so there are obvious symmetries in the geometry of the problem.

Because D(r) varies along the chord AB, we have to sum-up the contribution to I(b) from each spot along AB.

Problem 1 - Using the Pythagorean Theorem, define the distance L between the two points on segment AB in terms of b and r.

Problem 2 - Calculate the differential, dL in terms of r and b, assuming b is a constant.

Problem 3 - Construct the differential dI(b,r) = D(r)dL explicitly in terms of r and b.

Problem 4 - Integrate dI(b,r) to get I(b) with the assumption that D(r) = 0 from r = 0 to r = R_i, and is constant throughout the shell from r=R_i to r = R_0, and that D(r) = D_0.

Problem 5 - Assuming that all linear units are in light years, plot the 1-d function I(b) for R_i = 2.2 and R_0 = 2.5, and from b=0 to b=5.0, and compare it with Abell-38. Does Abell-38 seem to follow a constant-density shell model?

Problem 1 - $L^2 = r^2 - b^2$

Problem 2 - $2L\,dL = 2r dr$ so $dL = r\,dr / L$, and by substitution for $L = (r^2 - b^2)^{1/2}$, you get

$$dL = r\,dr / (r^2 - b^2)^{1/2}$$

Problem 3 - Construct the differential $dI(b,r) = D(r)dL$ explicitly in terms of r and b.

$$dI = D(r)\, r / (r^2 - b^2)^{1/2}\, dr$$

Problem 4 - Integrate $dI(b,r)$ to get $I(b)$ with the assumption that $D(r) = 0$ from $r = 0$ to $r = R_i$, and is constant throughout the shell from $r=R_i$ to $r = R_0$, and that $D(r) = D_0$.

$$I(b) = \int_{R_i}^{R_o} \frac{D_o\, r}{\sqrt{r^2 - b^2}}\, dr$$

D_0 is a constant, so it can be factored out from the integrand. Use the substitution $U = r^2 - b^2$, so that $dU = 2rdr$, and the integrand becomes $(1/2)\, D_0\, dU/U^{1/2}$. The integral can easily be solved to get $D_0\, (r^2 - b^2)^{1/2} + C$. Evaluating this 'front half integral' for the stated limits, and remembering that there are two halves to the shell, leads to the function:

$$I(b) = 2\, D_0\, (R_0^2 - b^2)^{1/2} - 2D_0\, (R_i^2 - b^2)^{1/2}.$$

Problem 5 - There are actually two parts to evaluating this function, making it a 'piecewise' function. The first part is for b <= 2.2 ly, for which you get:

$$I(b) = 2D_0\, (6.25 - b^2)^{1/2} - 2D_0\, (4.84 - b^2)^{1/2}$$

The second part is the solution for b > 2.2 ly for which you will get

$$I(b) = 2D_0\, (6.25 - b^2)^{1/2}$$

This function, plotted below, does seem to show an increase in brightness in the nebula between R=2.0 and 2.5 light years which matches the picture of the nebula Abell-38, so the nebula may be a hollow, thin spherical shell with a uniform density of gas!