National Aeronautics and Space Administration

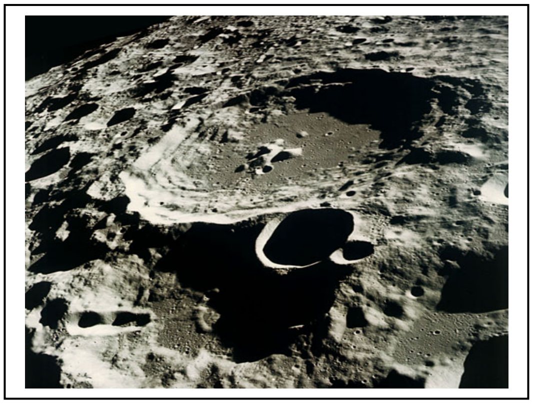
Math Problems Related to

Space Weather

Introduction

This book provides an assortment of mathematics problems that deal with aspects of space weather including estimates for storm frequencies, severities and how coronal mass ejections and solar flares are correlated based upon historical evidence.

Areas and Probabilities



There are many situations in astronomy where probability and area go hand in hand! The problems below can be modeled by using graph paper shaded to represent the cratered areas.

The moon's surface is heavily cratered, as the Apollo 11 photo to the left shows. The total area covered by them is more than 70% of the lunar surface!

1 – A 40km x 40km area of the Moon has 5 non-overlapping craters, each about 5km in radius. A) What fraction of this area is covered by craters? B) What is the percentage of the cratered area to the full area? C) Draw a square representing the surveyed region and shade the fraction covered by craters.

2 - During an 8-day period, 2 days were randomly taken off for vacation. A) What fraction of days are vacation days? B) What is the probability that Day-5 was a vacation day? C) Draw a square whose shaded area represents the fraction of vacation days.

3 – An asteroid capable of making a circular crater 40-km across impacts this same 40km x 40km area dead-center. About what is the probability that it will strike a crater that already exists in this region?

4 – During an 8-day period, 2 days were randomly taken of for vacation, however, during each 8-day period there were 4 consecutive days of rain that also happened randomly during this period of time. What is the probability that at least one of the rain days was a vacation day? (Hint: list all of the possible 8-day outcomes.)

Inquiry – How can you use your strategy in Problem 4 to answer the following question: An asteroid capable of making a circular crater 20-km across impacts this same 40km x 40km area dead-center. What is the probability that it will strike a crater that already exists in this region?

Answer Key

1 – Answer: A) The area of a crater with a circular shape, is A = π (5km)^2 = 78.5 km^2, so 5 non-overlapping craters have a total area of 393 km^2. The lunar area is 40 km x 40km = 1600 km^2, so the fraction of cratered area is 393/1600 **= 0.25.**

B) The percentage cratered is 0.25 x 100% = 25%.

C) Students will shade-in 25% of the squares.

2 - Answer: A) 2 days/8 days = 0.25

B) .25 x 100% = 25%.

C) The square should have 25% of area shaded.

3 –Answer: The area of the impact would be π (20km)^2 = 1240 km^2 . The area of the full region is 1600 km^2. The difference in area is the amount of lunar surface not impacted and equals 360 km^2. Because the cratered area is 393 km^2 and is larger then the unimpacted area, the **probability is 100%** that at least some of the cratered area will be affected by the new crater.

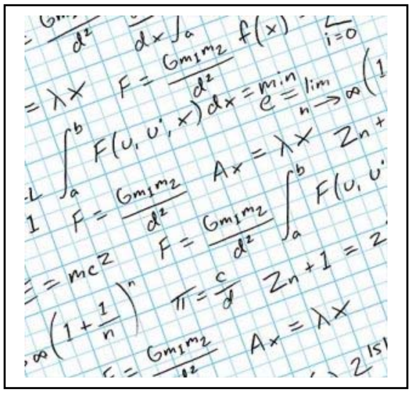
4 – R = Rain days

Vacation: There are 8 x 7/2 = **28 possibilities**

R R R R X X X X
X R R R R X X X
X X R R R R X X
X X X R R R R X
X X X X R R R R

Rain 'area' = 50%. For each of the 5 possibilities for a rain period, there are 28 possibilities for a vacation series, which makes 5 x 28 = 140 combinations of rain and vacation. For a single rain pattern out of the 5, there are 28 vacation patterns, and of these, 50% will include at least one vacation day in the rain, because the other half of the days avoid the rain days entirely. So, out of the 140 combinations, 70 will include rain days and 70 will not, again reflecting the fact that the 'area' of the rain days is 50% of the total days.

Inquiry: The already cratered area is ¼ of the total. The area of the new impact is π (10km)^2 = 314 km^2 which is 314/1600 = 1/5 of the total area. Draw a series of S=20 cells (like the 8-day pattern). The new crater represents 20x1/5 = 4 consecutive cells shaded. Work out all of the possible ways that 20 x 1/4 = 5 previously cratered areas can be distributed over the 20 days so that one of them falls within the 4 consecutive cells of the new crater. Example: X P X P X N N N N X X X X P X P X X P X
X = not cratered area, P = previously cratered area, N = new crater impact.
We are looking for: Probability = (The number of trials where a P is inside the 'N' region) / M , where M = the total number of possible trials. **Because there are 17 possibilities for the Ns, and 20!/ (15! 5!) = (20 x 19) / 2 = 190 possibilities for where the 5 Ps can go, m = 17 x 190 = 3230 . These do not all have to be worked out by hand to find the number of trials where a P is inside an N region in the series. You can also reduce the value of S so long as you keep the relative** areas between N, V and P the same.



Scientific research has a lot in common with solving number puzzles like SODUKO in order to figure out what stars or planets are doing in space. Use your puzzle-solving ability to figure out what event is described by the following number sentences!

1 - Which story matches the sentence 23 – 10 + 6 = 19 ?

A) An astronomer discovers 23 quasars on one photograph, 10 quasars on a second photograph, and 6 additional quasars on a third photograph. How many quasars did she identify?

B) An astronomer spots 23 solar flares on Tuesday and 6 solar flares on Thursday, then decides that 10 of the solar flares were not real. How many real flares did he see?

C) An astronomer counts a total of 19 lunar craters, and classifies 23 of them as asteroid impacts, 6 of them as volcanic calderas and 10 of them as meteor impacts.

2 - Which story matches the sentence 145 + N = 375 ?

A) Two astronomers combined their databases of planets. They observe a total of 375. If one astronomer contributed 145 planets, how many did the second astronomer contribute?

B) The temperature of an asteroid's interior changes by 375 degrees between the center and the surface. If the surface temperature is 145 degrees Centigrade, what is the interior temperature of the asteroid?

C) The width of Saturn's rings is 375 megameters. If the ring system starts at a distance of 145 megameters from Saturn's outer atmosphere, what is the distance to the outer edge of the ring system?

3 - Two astronomers combined their catalogs of cosmic gamma-ray bursts. There were 287 and 598 cataloged by each astronomer with 65 events in common. How many unique events are in the combined catalog?

A) (287 - 65) + (598 - 65) = M
B) 287 + (598 – 65) = M
C) 287 + 598 = M
D) (287 + 65) + (598 + 65) = M

Answer Key

1 - Which story matches the sentence 23 – 10 + 6 = 19 ?

A) An astronomer discovers 23 quasars on one photograph, 10 quasars on a second photograph, and 6 additional quasars on a third photograph. How many quasars did she identify? **Answer: No. This would be the sentence 23 + 10 + 6 = N**

B) An astronomer spots 23 solar flares on Tuesday and 6 solar flares on Thursday, then decides that 10 of the solar flares were not real. How many real flares did he see? **Answer: Yes this is correct.**

C) An astronomer counts a total of 19 craters, and classifies 23 of them as asteroid impacts, 6 of them as volcanic calderas and 10 of them as meteor impacts. **Answer: No. This is the sentence 23 + 6 + 10 = 39.**

2 - Which story matches the sentence 145 + N = 375 ?

A) Two astronomers combined their databases of planets. They observe a total of 375. If one astronomer contributed 145 planets, how many did the second astronomer contribute? **Yes. This is correct.**

B) The temperature of an asteroid's interior changes by 375 degrees between the center and the surface. If the surface temperature is 145 degrees Centigrade, what is the core temperature of the asteroid? **Answer: No. This is the sentence 145 + 375 = N**

C) The width of Saturn's rings is 375 megameters. If the ring system starts at a distance of 145 megameters from Saturn's outer atmosphere, what is the distance to the outer edge of the ring system**? Answer: No. This is the sentence 145 + 375 = N**

3 - Two astronomers combined their catalogs of cosmic gamma-ray bursts. There were 287 and 598 cataloged by each astronomer with 65 events in common. How many unique events are in the combined catalog?

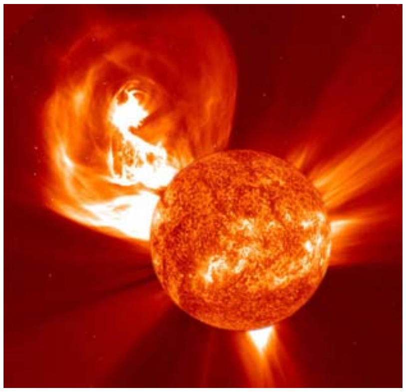
A) (287 - 65) + (598 - 65) = M **No. This eliminates the common bursts from the final catalog.**

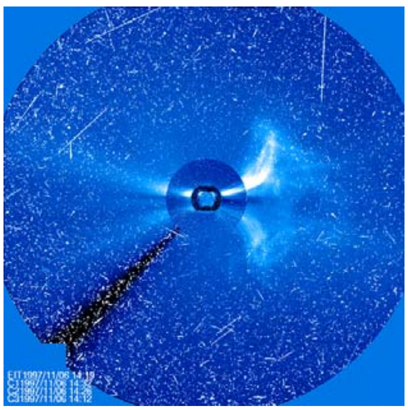
B) 287 + (598 – 65) = M **Yes. This is correct.**

C) 287 + 598 = M - **No. This is just the sum of the two catalogs including duplications.**

D) (287 + 65) + (598 + 65) = **M** **No. The duplications are added to each not subtracted.**

Answer B is correct. Among the 287 + 598 gamma ray bursts in the two catalogs, there are 65 that are in common. Starting with the full catalog provided by one astronomer (287), we add only the non-duplicated bursts in the second astronomers catalog (598-65).





Every once in a while, the sun ejects huge clouds of heated gas, called plasma, which can contains billions of tons of matter and travel at speeds of millions of miles per hour. Occasionally these are directed at earth, and when they arrive they cause brilliant aurora. They can also cause problems for electrical systems on the ground and satellite systems in space.

The top image is a composite that shows the surface of the sun and one of these 'coronal mass ejections' being released. This one is directed away from earth and is harmless to us. When we spot a CME directed towards earth, the cloud seems to form a temporary 'halo' around the edge of the sun. These Halo CMEs are ejected from the sun, and can arrive at earth about 2 to 4 days later.

Soon after a Halo CME is ejected, satellites may detect a rain storm of radiation particles that were ejected from the sun at the same time. These travel so fast that they arrive at earth in only an hour or so. Also called Solar Proton Events (SPEs), these radiation storms are very harmful to astronauts in space and to sensitive satellite electronics. Predicting when SPEs will occur is an important goal of Space Weather Research.

The table below gives the dates for the CMEs detected during the 227 days from January 1, 2013 and August 15, 2013 during the peak of our sun's current storm cycle. Yellow shading indicates that a SPE occurred on the same date.

Date	Type	Date	Type	Date	Type
1-23	Halo	3-15	Halo	5-22	Non-Halo
1-31	Halo	4-11	Halo	6-20	Halo
2-1	Halo	4-20	Non-Halo	7-16	Halo
2-5	Non-Halo	4-21	Non-Halo	7-26	Non-Halo
2-9	Halo	5-17	Halo	8-6	Halo
2-20	Halo	5-19	Halo		

Problem 1 – What is the average number of days between all of the CMEs in this sample?

Problem 2 – What percentage of CMEs are of the Halo-type?

Problem 3 – What percentage of Halo CMEs seem to produce Solar Proton Events?

Problem 4 – If you observed a CME, what is the probability that it may produce a harmful solar proton event?

Answer Key

Problem 1 – What is the average number of days between all of the CMEs in this sample?

Answer: There are a total of 17 CMEs in 227 days so the average interval is about 227/17 = **13 days**.

Problem 2 – What percentage of CMEs are of the Halo-type?

Answer: Of the 17 CMEs, 12 were Halo-type so the percentage is 100%x(12/17) = **71%**

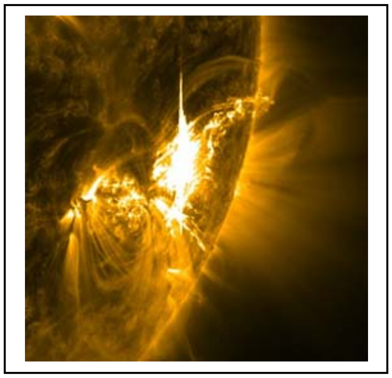
Problem 3 – What percentage of Halo CMEs seem to produce Solar Proton Events?

Answer: Of the 12 Halo-type events, 4 produced SPEs so 100% (4/12) = **33%**

Problem 4 – If you observed a CME, what is the probability that it may produce a harmful solar proton event?

Answer: There were 17 CMEs total of which 6 produced SPEs, so 100% x (6/17) = **35%**.

Note: Of the 6 SPEs, four occurred with Halo CMEs so this means that 4/6 or 67% of all SPEs coincide with Halo-type CMEs, however it is also true that only 4/12=33% of all Halo-type CMEs produce SPEs. Not all Halo events produce solar proton events, so using halo events to predict whether an SPE will occur will lead to a large number of false-positives by about 8 false to 4 positives (8+4 = 12 halo events) for a 2:1 false positive rate.



The following essay describes solar storms. The words that belong in the missing blanks can be found by solving the ten number sentences.

Look in the Word Bank and match the calculated number to the correct answer.

Fill-in the blank with the word you found to complete the essay and answer a question about solar storms!

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1)___________ and incredible explosions of 2)__________ almost every day. Sometimes, its entire surface is speckled by 3)___________ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4)____________ light that can cause problems for radio communication on Earth. They also heat up the 5)___________ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6)__________ called 7)_______________________ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8)_________. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9)_________ in the northern and southern skies. Most CMEs are not directed towards earth and are completely 10)_____________.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!

Word Bank

-5	asteroids	+48	heat	-17	X-rays
-44	aurora	+1	ocean	+3	harmless
-15	ultraviolet	+5	days	+44	rainbows
-27	energy	0	atmosphere	-5	months
-40	rocks	-48	gas	+24	sunspots
-6	harmful	+3	plasma	-3	coronal mass ejections
-2	flares	+20	comets	-7	prominences

Solve these problems to get the Word Bank number key.

1) 1+(1-3)-(5-8)+(-6+2) =
2) 8(3-2)-2(3-8)+5(-6-3) =
3) (1-3)(-5+2)(8-6)(3-1) =
4) 3(2-6)+(-8+4)-(4-3) =
5) 5(-3+2)-2(6-2)-(+7-20) =
6) -2(+2(-3(+2(-3+1)))) =
7) -8/2 +(3+2)/(8-3) =
8) -7+23-6-(-10)+(-3)(4+1) =
9) 12/(-1/4) + (-36)/(-9) =
10) (-4)2 + 21/3 + 4 =

Answer Key

Solve these problems to get the Word Bank number key.

1)	1+(1-3)-(5-8)+(-6+2) = -2		6)	-2(+2(-3(+2(-3+1)))) = -48	
2)	8(3-2)-2(3-8)+5(-6-3) = -27		7)	-8/2 +(3+2)/(8-3) = -3	
3)	(1-3)(-5+2)(8-6)(3-1) = +24		8)	-7+23-6-(-10)+(-3)(4+1) = +5	
4)	3(2-6)+(-8+4)-(4-3) = -17		9)	12/(-1/4) + (-36)/(-9) = -44	
5)	5(-3+2)-2(6-2)-(+7-20) = 0		10)	(-4)2 + 21/3 + 4 = +3	

The words are

Line 1 = -2 = flares
Line 2 = -27 = energy
Line 3 = +24 = sunspots
Line 4 = -17 = X-ray
Line 5 = 0 = atmosphere
Line 6 = -48 = gas
Line 7 = -3 = coronal mass ejections
Line 8 = +5 = days
Line 9 = -44 = aurora
Line 10 = +3 = harmless

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1)____**flares**_____ and incredible explosions of 2)___**energy**_____ almost every day. Sometimes, its entire surface is speckled by 3)__**sunspots**____ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4)___**X-ray**____ light that can cause problems for radio communication on Earth. They also heat up the 5)___**atmosphere**_____ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6)__**gas**____ called 7)___**coronal mass ejections**____ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8)____**days**_____. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9)__**aurora**____ in the northern and southern skies. Most CMEs are not directed towards earth and are completely 10)___**harmless**______.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!

Name ________________________________

In the table below you will find information about the communication satellites that were launched in 2005. Assume that each satellite will survive until the year when its lifespan expires. Most are designed to last 15 years before needing replacement. Solar storms and cosmic rays damage the satellite solar panels and cause a 2% decrease in electrical power. Assume that this means that the satellite loses 2% of its transponders each year. Each satellite transponder can carry 2 channels of regular (analog) TV programs, or 6 channels of digital TV programs.

Name	Lifespan In years	Number of Transponders	Cost (million $)	Retire year	Revenue (million $)	Break even year
Hotbird-7A	15	38	200			
Arabsat 4A	15	40	200			
AMC-12	16	72	280			
StarOne C2	13	44	150			
Insat-4A	12	24	67			
Intelsat IA-8	13	64	320			
Spaceway-2	13	48	250			
DirecTV-8	12	32	260			
AMC-23	15	38	280			
Anik F1R	15	56	250			
Echostar 10	15	32	250			
Chinasat-8	15	52	100			
Telkom-2	15	24	150			
Thaicom-4	12	38	400			
Galaxy-14	15	24	270			
Galaxy-15	15	24	270			
Apstar-6	14	50	225			
Asiasat-6	12	50	200			
Express AM3	12	28	290			
Express AM2	12	28	290			
Measat-3	15	48	132			

Question 1) A) What is the total number of transponders carried by these satellites? B) How many analog satellite TV channels can be supported by these satellites? C) How many digital TV channels can be supported by these satellites?

Question 2) In Column 4, determine the retirement year of the satellite given its launch year and lifespan. A) What is the earliest year when this group of satellites will begin to retire? B) What year will the oldest satellites retire? C) How old will you be when the last satellite is retired?

Question 3) Satellite transponders are rented by the satellite owner to TV companies to carry their programs. A typical transponder costs $1.2 million to lease each year, and this represents income to the satellite owner. In Column 6, calculate the annual revenue from each satellite's transponders in millions of dollars. A) What is the total revenue each year from these satellites? B) Which satellite makes the most money each year? C) Which satellites make the least money each year?

Question 4) For each satellite, by what year will its cumulative revenue equal the cost of the satellite? This is the 'break even' year when the satellite has paid for itself and from this year on is producing a net profit to the owner. Enter the break-even year in Column 7.

For Experts: A) If the Hotbird-7A satellite actually loses 3% of its transponders each year, how much money will the satellite have lost by the break-even year because of space weather? Assume it loses the same number of transponders each year beginning with its first year.

Name	Lifespan In years	Number of Transponders	Cost (million $)	Retire year	Revenue (million $)	Break even year
Hotbird-7A	15	38	200	2020	45.6	2010
Arabsat 4A	15	40	200	2020	48.0	2010
AMC-12	16	72	280	2021	86.4	2009
StarOne C2	13	44	150	2018	52.8	2008
Insat-4A	12	24	67	2017	28.8	2008
Intelsat IA-8	13	64	320	2018	76.8	2010
Spaceway-2	13	48	250	2018	57.6	2010
DirecTV-8	12	32	260	2017	38.4	2012
AMC-23	15	38	280	2020	45.6	2012
Anik F1R	15	56	250	2020	67.2	2009
Echostar 10	15	32	250	2020	38.4	2012
Chinasat-8	15	52	100	2020	62.4	2007
Telkom-2	15	24	150	2020	28.8	2011
Thaicom-4	12	38	400	2017	45.6	2014
Galaxy-14	15	24	270	2020	28.8	2015
Galaxy-15	15	24	270	2020	28.8	2015
Apstar-6	14	50	225	2019	60.0	2009
Asiasat-6	12	50	200	2017	60.0	2009
Express AM3	12	28	290	2017	33.6	2014
Express AM2	12	28	290	2017	33.6	2014
Measat-3	15	48	132	2020	57.6	2008

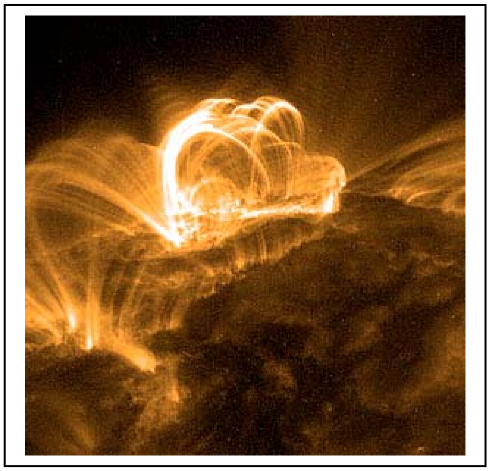
Question 1) A) What is the total number of transponders carried by these satellites? B) How many analog satellite TV channels can be supported by these satellites? C) How many digital TV channels can be supported by these satellites? Answer; A) 854 transponders; B) About 854 x 2 = 1708 analog TV channels. C) About 854 x 6 = 5124 digital TV channels.

Question 2) In Column 4, determine the retirement year of the satellite given its launch year and lifespan. A) What is the earliest year when this group of satellites will begin to retire? B) What year will the oldest satellites retire? C) How old will you be when the last satellite is retired? Answer: A) The year 2017. B) The year 2020 C) For a 14-year old student in 2005, you will be 14+15 = 29 years old when the oldest satellite retires.

Question 3) Satellite transponders are rented by the satellite owner to TV companies to carry their programs. A typical transponder costs $1.2 million to lease each year, and this represents income to the satellite owner. In Column 6, calculate the annual revenue from each satellite's transponders in millions of dollars. A) What is the total revenue each year from these satellites? B) Which satellite makes the most money each year? C) Which satellite makes the least money each year? Answer: A) 1.0248 billion dollars. B) AMC-12, C) Insat-4A, Telkom-2, Galaxy-14 and Galaxy-15.

Question 4) For each satellite, by what year will its cumulative revenue equal the cost of the satellite? This is the 'break even' year when the satellite has paid for itself and from this year on is producing a net profit to the owner. Enter the break-even year in Column 7. Answer Example: Hotbird-7a makes $45.6 million each year. It cost $200 million, so it will take (200/45.6) = 4.4 years. Rounding-up, it was launched in 2005, so by 2005+5 = 2010 it will have paid for itself. Rounding-down, students may also use 2005+4 = 2009.

<u>For Experts.</u> A) If the Hotbird-7A satellite loses 3% of its transponders each year how much money will the satellite have lost by its break-even year because of space weather? Assume it loses the same number of transponders each year beginning with its first year. Answer: Hotbird-7A reaches its break-even year 4 years after launch. It will lose 38 x 0.03 = 1 transponder the first year, and the same number for each of the remaining 3 years. The cumulative transponder loss for each year is 1 ,2, 3 ,4. for a cumulative loss of 1.2 + 2.4 + 3.6 + 4.8 = $12 million. Select some other satellites to do the same calculation. You may also want to do this on a spreadsheet! What will be the total loss of revenue due to space weather for this entire collection of satellites?



The Sun is an active star, which produces solar flares (F) and explosions of gas (C). Astronomers keep watch for these events because they can harm satellites and astronauts in space. Predicting when the next storm will happen is not easy to do. The problems below are solved by writing out all of the possibilities, then calculating the probability of the particular outcome!

Solar flare photo courtesy TRACE/NASA

1 – During a week of observing the sun, astronomers detected 1 solar flare (F). What was the probability (as a fraction) that it happened on Wednesday?

2 – During the same week, two gas clouds were ejected (C), but not on the same days. What is the probability (as a fraction) that a gas cloud was ejected on Wednesday?

3 – Suppose that the flares and the gas clouds had nothing to do with each other, and that they occurred randomly. What is the probability (as a fraction) that both a flare and a gas cloud were spotted on Wednesday? (Astronomers would say that these phenomena are uncorrelated because the occurrence of one does not mean that the other is likely to happen too).

Answer Key

1 – Answer: There are only 7 possibilities:

F X X X X X X	X X X F X X X	X X X X X X F
X F X X X X X	X X X X F X X	
X X **F** X X X X	X X X X X F X	

So the probability for any one day is **1/7.**

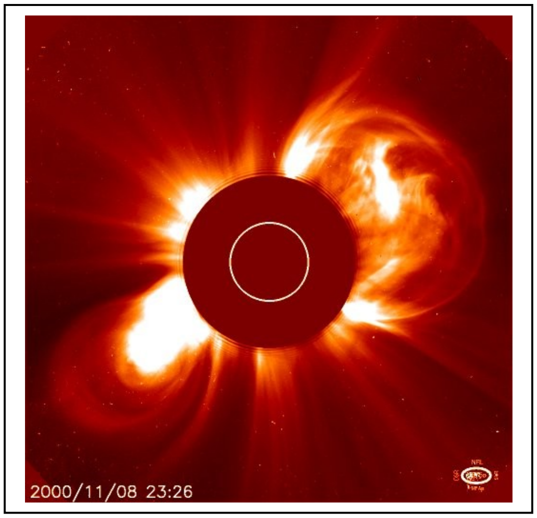
2 – Here we have to distribute 2 storms among 7 days. For advanced students, there are 7! / (2! 5!) = 7 x 6 / 2 = 21 possibilities which the students will work out by hand:

C C X X X X X	X C **C** X X X X	X X **C** C X X X	X X X C C X X
C X **C** X X X X	X C X C X X X	X X **C** X C X X	X X X C X C X
C X X C X X X	X C X X C X X	X X **C** X X C X	X X X C X X C
C X X X C X X	X C X X X C X	X X **C** X X X C	X X X X C C X
C X X X X C X	X C X X X X C		X X X X C X C
C X X X X X C			X X X X X C C

There are 6 possibilities (in red) for a cloud appearing on Wednesday (Day 3), so the probability is **6/21.**

3 – We have already tabulated the possibilities for each flare and gas cloud to appear separately on a given day. Because these events are independent of each other, the probability that on a given day you will spot a flare and a gas cloud is just 1/7 x 6/21 or 6/147. This is because for every possibility for a flare from the answer to Problem 1, there is one possibility for the gas clouds.

There are a total of 7 x 21 = 147 outcomes for both events taken together. Because there are a total of 1 x 6 outcomes where there is a flare and a cloud on a particular day, the fraction becomes (1 x 6)/147 = **6/147.**



The Sun is an active star that produces solar flares (F) and explosions of gas (C). Astronomers keep watch for these events because they can harm satellites and astronauts in space. Predicting when the next storm will happen is not easy to do. The problems below are solved by writing out all of the possibilities, then calculating the probability of the particular outcome!

Photo of a coronal mass ejection courtesy SOHO/NASA.

Problem 1 – During a particularly intense week for solar storms, three flares were spotted along with two massive gas cloud explosions. Work out all of the possible ways that 3 Fs and 2 Cs can be separately distributed among 7 days. Examples include C C X X X X X and F F F X X X X.

What is the probability (as a fraction) that none of these events occurred on Friday?

Inquiry: Does the probability matter if we select any one of the other 6 days?

Answer Key

1 – Here we have to distribute 2 cloud events (C) among 7 days. For advanced students, there are 7! / (2! 5!) = 7 x 6 / 2 = 21 possibilities which the students will work out by hand, in this case starting with Monday as the first place in the sequence:

C C X X **X** X X	X C C X **X** X X	X X C C **X** X X	X X X C C X X
C X C X **X** X X	X C X C **X** X X	X X C X C X X	X X X C **X** C X
C X X C **X** X X	X C X X C X X	X X C X **X** C X	X X X C **X** X C
C X X X C X X	X C X X **X** C X	X X C X **X** X C	X X X X C C X
C X X X **X** C X	X C X X **X** X C		X X X X C X C
C X X X **X** X C			X X X X **X** C C

Next, we have to distribute 3 flares (F) among 7 days. There will be 7! / (3! 4!) = (7 x 6 x 5) /(3 x 2) = 35 possibilities as follows:

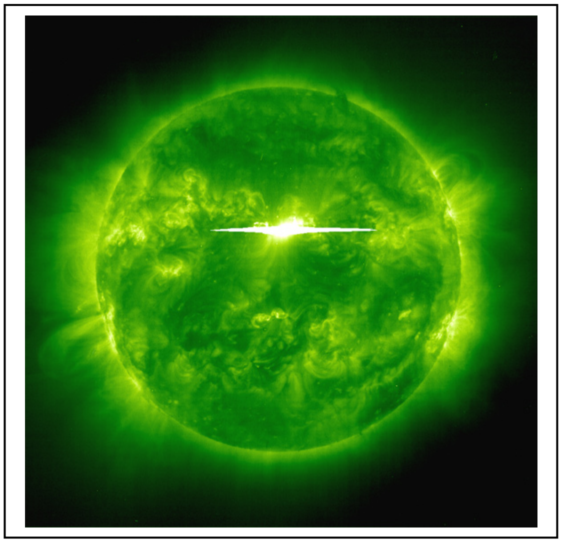
F F F X **X** X X	F X X F F X X	X F F F **X** X X	X X F F F X X
F F X F **X** X X	F X X F **X** F X	X F F X F X X	X X F F **X** F X
F F X X F X X	F X X F **X** X F	X F F X **X** F X	X X F F **X** X F
F F X X **X** F X	F X X X F F X	X F F X **X** X F	X X F X F F X
F F X X **X** X F	F X X X F X F	X F X F F X X	X X F X F X F
F X F F **X** X X	F X X X **X** F F	X F X F **X** F X	X X F X **X** F F
F X F X F X X		X F X F **X** X F	X X X F F F X
F X F X **X** F X		X F X X F F X	X X X F F X F
F X F X **X** X F		X F X X F X F	X X X F **X** F F
		X F X X **X** F F	X X X X F F F

For each cloud event (21 possibilities) there are 35 possibilities for flare events, so the total number of arrangements is 21 x 35 = 735. Friday is the fifth location in each sequence.

The number of sequences of cloud events in which no cloud appears in the fifth slot is highlighted in red. There are 15 possibilities. For the solar flares, there are 20 possibilities, so because these are independent, the total number is 15 x 20 = 300. So, the probability that there will be no solar storms on Friday is 300/735

Inquiry – Students may redo the calculation for any of the other days. The resulting probability should always be 300/735. This is because the events are not correlated with any particular day, so the day choice is random as well.

Solar Storm Timeline



On July 15, 2001 a solar storm was tracked from the Sun to Earth by a number of research satellites and observatories. This activity lets you perform time and day arithmetic to figure out how long various events lasted. This is a very basic process that scientists go through to study an astronomical phenomenon. The image to the left was taken by the TRACE satellite and shows the x-ray flare on the Sun. The 'slinky' shape is caused by magnetic fields.

Photo courtesy SOHO/NASA

The Story: On July 14, 2000, NASA's TRACE satellite spotted a major X5.7-class solar flare erupting at 09:41 from Active Region 9077. The flare continued to release energy until 12:31. At 10:18:27, radio astronomers using the Nancay radio telescope detected the start of a radio-frequency Type-I noise storm. This storm strengthened, and at 10:27:27, four moving radio sources appeared. Meanwhile, the satellite, GOES-10 detected the maximum of the x-ray light from this flare at 10:23. The SOHO satellite, located 92 million miles from the Sun, and 1 million miles from Earth, recorded a radiation storm from fast-moving particles, that caused data corruption at 10:41. The SOHO satellite's LASCO imager also detected the launch of a coronal mass ejection (CME) at 10:54. The CME arrived at the satellite at 14:17 on July 15. Then at 14:37 on July 15, the CME shock wave arrived at Earth and compressed Earth's magnetic field. The IMAGE satellite recorded the brightening of the auroral oval starting at 14:25. Aurora were at their brightest at 14:58. The aurora expanded to the lowest latitude at 17:35. By 20:00, Earth's magnetic field has slightly decreased in strength in the equatorial regions. By 16:47 on July 16, the IMAGE satellite recorded the recovery of Earth's magnetosphere to normal conditions. On January 12, 2001, the CME was detected by the Voyager I satellite located 63 AU from the Sun.

Problem 1 - From this information, create a time line of the events mentioned.

Problem 2 – How long did it take for the CME to reach Earth?

Inquiry: What other questions can you explore using this timing information?

Answer Key

Problem 1

July 14,
09:41 - X5.7-class solar flare
10:19 - Radio astronomers detect Type-I radio storm.
10:23 - GOES-10 detected the maximum of the x-ray light from this flare
10:27 - Four moving radio sources appeared on sun.
10:41 - SOHO satellite radiation storm and data corruption.
10:54 - SOHO sees launch of CME
July 15,
14:17 - CME shock wave arrived at Earth
14:25 – IMAGE satellite sees brightening of the auroral oval
14:58 - Aurora at brightest
17:35 – Aurora expand to lowest latitudes
20:00 - Earth's magnetic field has slightly decreased in strength
July 16
16:47 - IMAGE satellite recorded the recovery of Earth's magnetosphere

January 12, 2001, CME detected by Voyager I satellite 63 AU from the Sun.

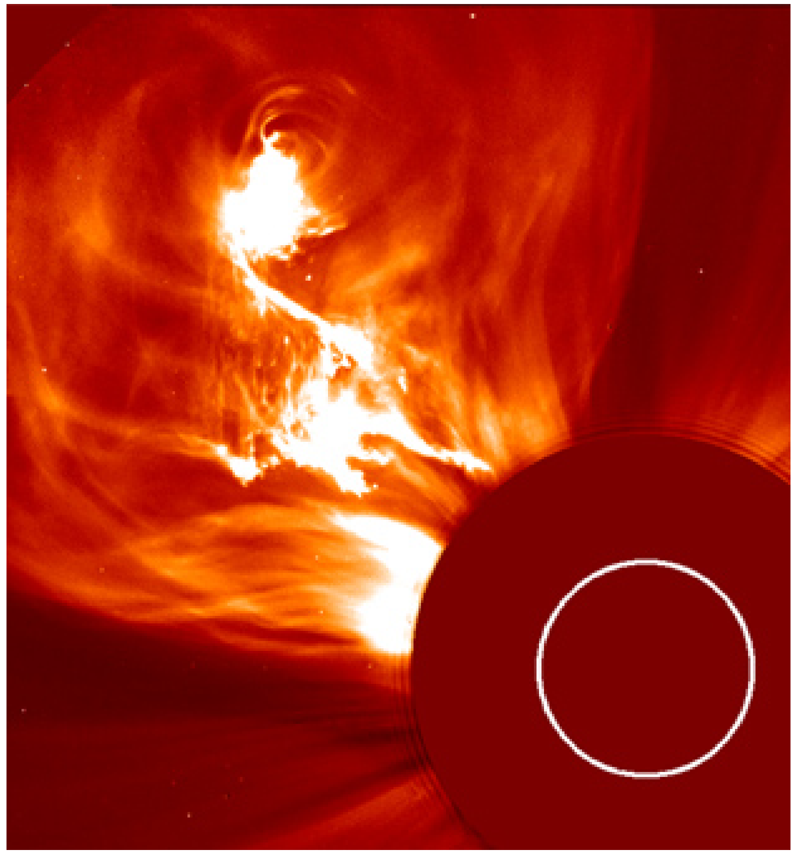
Problem 2 - The CME was launched on July 14 at 10:54 and arrived at Earth on July 15 at 14:17. The elapsed time is 1 full day (24 hours) and the difference between 10:54 and 14:17 which is 14:17 – 10:54 = 13:77 – 10:54 = 3hours and 77-54 = 23 minutes. The total elapsed time is then 24h + 3h 23m = 27hours 23minutes.

Inquiry – There are many possibilities, for example, how long did it take for the CME to reach Voyager in days? Hours? What was the speed of the CME as it traveled to Earth? How long after the flare did SOHO experience a radiation storm?

The sun is an active star that produces explosions of matter and energy. The space between the planets is filled with invisible clouds of gas that sometimes collide with Earth. Scientists call them Coronal Mass Ejections. They can travel at millions of miles per hour and carry several billion tons of gas called a plasma. When 'CMEs' collide with Earth, they produce the Northern Lights and magnetic storms.

In this exercise, you will examine one of these 'solar storm' events by examining a timeline of events that it caused.

The picture to the right was taken by the SOHO spacecraft showing a spectacular CME. The white circle is the size of the sun.



Solar Storm Timeline

Day	*Time*	*What Happened*
Tuesday	4:50 PM	Gas eruption on Sun
Thursday	3:36 AM	Plasma storm reaches Earth.
Thursday	5:20 AM	Storm at maximum intensity.
Thursday	5:35 AM	Auroral power at maximum.
Thursday	11:29 AM	Aurora power at minimum.
Thursday	2:45 PM	Space conditions normal

1) How much time passed between the solar gas eruption and its detection near Earth?

2) How long after the plasma storm reached Earth did the aurora reach their maximum power?

3) How long did the storm last near Earth from the time the plasma was detected, to the time when space conditions returned to normal?

Extra for Experts!
If the Earth is 150 million kilometers from the sun, how fast did the storm travel from the Sun in kilometers per hour? How long will the trip to Pluto take is Pluto is 40 times farther away from the sun than Earth?

Goal: Students will interpret a timeline table to extract information about a solar storm using time addition and subtraction skills.

Day	***Time***	***What Happened***
Tuesday	**4:50 PM**	**Gas eruption on Sun**
Thursday	**3:36 AM**	**Plasma storm reaches Earth.**
Thursday	**5:20 AM**	**Storm at maximum intensity.**
Thursday	**5:35 AM**	**Auroral power at maximum.**
Thursday	**11:29 AM**	**Aurora power at minimum.**
Thursday	**2:45 PM**	**Space conditions normal**

1) How much time passed between the solar gas eruption and its detection near Earth?

Answer: There are various ways to do this problem. You want to subtract the final time from the initial time so: (Tuesday 4:50 PM) – (Thursday, 3:36 AM) = (Thursday – Tuesday) + (3:36 AM – 4:50 PM) = 48 hrs – (4:50PM – 3:36AM) = 48h – 13h 14m = 34hours and 46minutes.

2) How long after the plasma storm reached Earth did the aurora reach their maximum power?

Answer: Storm arrived at 3:36 AM. Aurora at maximum at 5:35AM. Difference in time is 1 hour and 59 minutes.

3) How long did the storm last near Earth from the time the plasma was detected, to the time when space conditions returned to normal?

Answer: On Thursday, the storm started at 3:36 AM and ended at 2:45 PM, so the storm effects at Earth lasted from 03:36 to 14:45 so the difference is 14:45 – 03:36 = 11 hours and (45-36 =) 9 minutes.

Extra for Experts!

If the Earth is 150 million kilometers from the sun, how fast did the storm travel from the Sun in kilometers per hour? How long will the trip to Pluto take is Pluto is 40 times farther away from the sun than Earth?

Answer: The answer to Problem 1 is 34hours and 46minutes, which in decimal form is 34 + (46/60) = 34.8 hours with rounding. The speed is therefore 150 million km/34.8 hours or 4.3 million km/h. The trip to Pluto would take 40 x 34.8 hours = 1,392 hours or about 58 days. Note, the Space Shuttle is our fastest manned spacecraft and travels at 44,000 km/h so it would take about 58 x (4.3 million/44,000) = 5668 days to make this trip, which equals 15.5 years!!!! Of course, the Space Shuttle will be out of fuel and supplies within a week.

Solar Storms: Odds, Fractions and Percentages

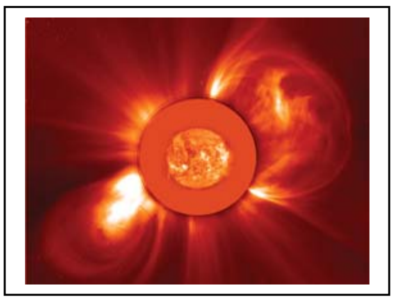
One of the most basic activities that scientists perform with their data is to look for correlations between different kinds of events or measurements in order to see if a pattern exists that could suggest that some new 'law' of nature might be operating. Many different observations of the Sun and Earth provide information on some basic phenomena that are frequently observed. The question is whether these phenomena are related to each other in some way. Can we use the sighting of one phenomenon as a prediction of whether another kind of phenomenon will happen?

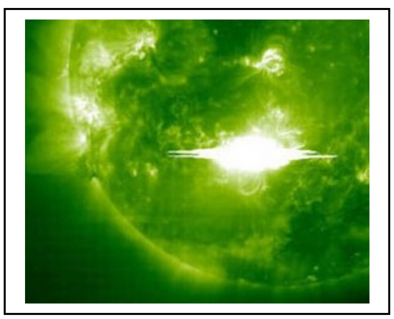
During most of the previous sunspot cycle (January-1996 to June-2006), astronomers detected 11,031 coronal mass ejections, (CME: Top image) of these 1186 were 'halo' events. Half of these were directed towards Earth.

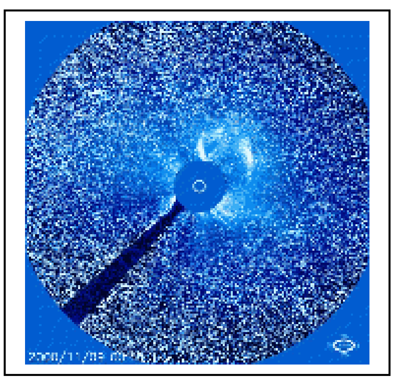
During the same period of time, 95 solar proton events (streaks in the bottom image were caused by a single event) were recorded by the GOES satellite network orbiting Earth. Of these SPEs, 61 coincided with Halo CME events.

Solar flares (middle image) were also recorded by the GOES satellites. During this time period, 21,886 flares were detected, of which 122 were X-class flares. Of the X-class flares, 96 coincided with Halo CMEs, and 22 X-class flares also coincided with 22 combined SPE+Halo CME events. There were 6 X-flares associated with SPEs but not associated with Halo CMEs. A total of 28 SPEs were not associated with either Halo CMEs or with X-class solar flares.

From this statistical information, construct a Venn Diagram to interrelate the numbers in the above findings based on resent NASA satellite observations, then answer the questions below.







1 - What are the odds that a CME is directed towards Earth?

2 - What fraction of the time does the sun produce X-class flares?

3 - How many X-class flares are not involved with CMEs or SPEs?

4 - If a satellite spotted both a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur?

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?

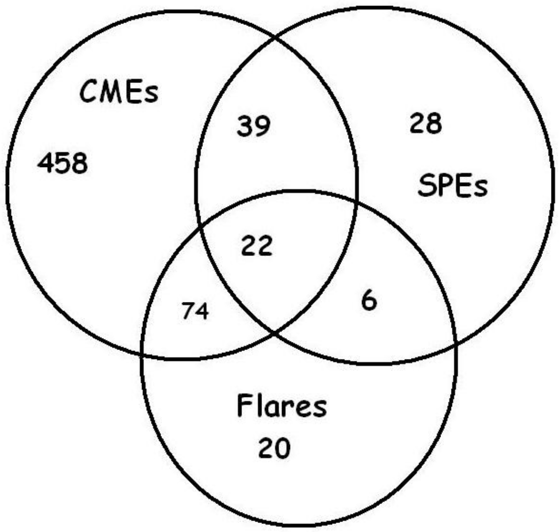
6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

Answer Key:



Venn Diagram Construction.

1. There are 593 Halo CMEs directed to Earth so 593 = 74 with flares + 39 with SPEs + 22 both SPEs and Flares + 458 with no SPEs or Flares..

2. There are 95 SPEs. 95 = 39 with CMEs + 6 with flares + 22 with both flares and CMEs + 28 with no flares or CMEs

3. There are 122 X-class flares. 122 = 74 With CMEs only + 6 with SPEs only + 22 both CMEs and SPEs + 20 with no CMEs or SPEs.

1 - What are the odds that a CME is directed towards Earth? 593/11031 = 0.054 **odds = 1 in 19**

2 - What fraction of the time does the sun produce X-class flares? 122/21886 = **0.006**

3 - How many X-class flares are not involved with CMEs or SPEs? 122 - 74 - 22 - 6 = **20.**

4 - If a satellite spotted BOTH a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur? 22/(74+22) = **0.23**

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?
100% x (39+22+6 / 95) = **70.1 %**

6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?
39+22+74 / 593 = 0.227 so the odds are 1/0.227 or about **1 in 4.**

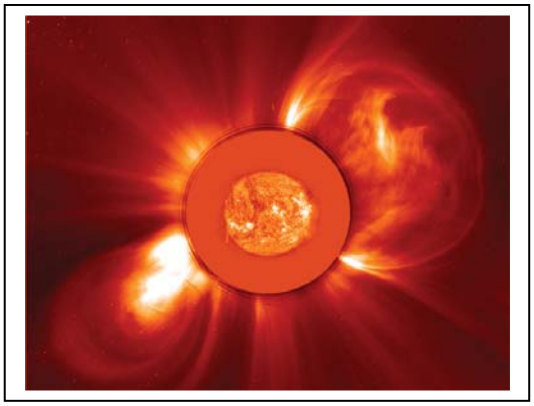
7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?
(6+22)/95 = 0.295 or 1 out of 3 times for X-flares
(39+22)/95 = 0.642 or 2 out of 3 for Halo CMEs
It is more likely to detect an SPE if a Halo CME occurs by 2 to 1.

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?
39+6 / 95 = 0.50 so the odds are 1/0.50 or **2 to 1 .**

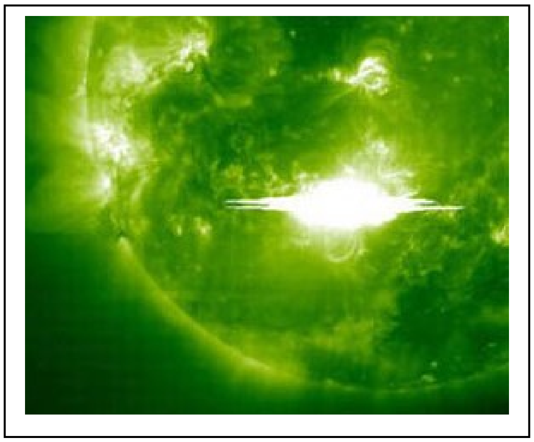
9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

100 x 22/(95+122+593) = **3 times**

Solar storms come in two varieties:

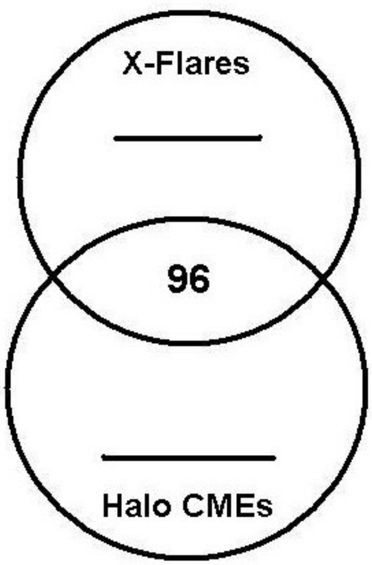


Coronal Mass Ejections (CMEs) are clouds of gas ejected from the sun that can reach Earth and cause the Aurora Borealis (Northern Lights). These clouds can travel at over 2 million kilometers/hr, and carry billions of tons of matter in the form of charged particles (called a plasma). The picture to the left shows one of many CMEs witnessed by the SOHO satellite.



Solar Flares are intense bursts of X-ray energy that can cause short-wave radio interference on Earth. The picture to the left shows a powerful X-ray flare seen by the SOHO satellite on October 28, 2003.

Between 1996 and 2006, astronomers detected 11,031 coronal mass ejections (CMEs), and of these, 593 were directed towards Earth. These are called 'Halo CMEs' because the ejected gas surrounds the sun's disk on all sides and looks, like a halo around the sun. During these same years, astronomers also witnessed 122 solar flares that were extremely intense X-flares. Of these X-flares, 96 happened at the same time as the Halo CMEs.



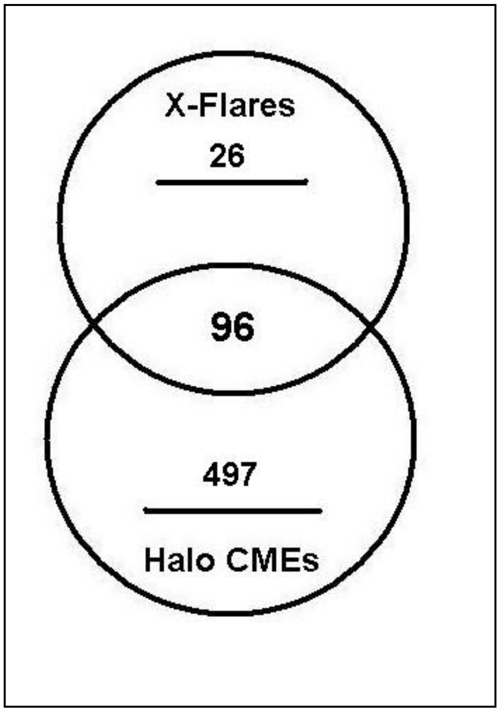
Problem 1 - From this statistical information, fill-in the mission numbers in circular Venn Diagram to the left.

Problem 2 – What percentage of X-Flares also happened at the same time as a Halo CME?

Problem 3 – What percentage of Halo CMEs happened at the same time as an X-Flare?

Problem 4 – What percentage of all CMEs detected between 1996 and 2006 produced X-Flares?

Answer Key:



Answer 1 - The total number of Halo CMEs is 593 and the total number of X-Flares is 122. The intersecting area of the two circles in the Venn Diagram shows the 96 events in which a Halo CME and X-Flare are BOTH seen together. The areas of the circles not in the intersection represent all of the X-flares that are not spotted with Halo CMEs (top ring) and all of the Halo CMEs that are not spotted with X-Flares (bottom ring).

The missing number in the X-Flare ring is just 122 – 96 = 26, and for the Halo CMes we have 593 – 96 = 497.

Answer 2 – The total number of X-Flares is 122 and of these only 96 occurred with a Halo CME, so the fraction of X-Flares is just 96/122 = 0.79. In terms of percentage, this represents 79 %.

Answer 3 – The total number of Halo CMEs is 593 and of these only 96 occurred with an X-Flare, so the fraction of Halo CMEs is just 96/593 = 0.16. In terms of percentage, this represents 16 %.

Answer 4 – There were 11,031 CMEs detected, and of these only 96 coincided with X-Flares, so the fraction is 96/11031 = 0.0087. In terms of percentage, this represents 0.87 % or less than 1 % of all CMEs.

Parametric Functions and Substitution

Image courtesy Dick Hutchinson

Our sun is an active star that ejects a constant stream of particles into space called the 'solar wind'. From time to time, magnetic activity on its surface also launches fast-moving clouds of plasma into space called 'coronal mass ejections' or CMEs.

When some of these clouds directed at Earth arrive after traveling 93 million miles (150 million km), they cause intense disturbances in Earth's magnetic field. Since the 1800's, these disturbances have been called 'magnetic storms', because instruments on Earth can measure the strength of these disturbances, and they resemble storms in an otherwise very calm magnetic field.

Scientists measure the strength of these magnetic storms in terms of the size of the change they make in the Earth's magnetic field. The strength of Earth's field at the ground is about 0.7 Gauss or 70,000 nanoTeslas. The most intense magnetic storms can change the ground-level field by several percent.

According to research by V. Yurchyshyn, H. Wang and V. Abramenko, which was published in 2004 in the journal Space Weather (vol. 2) the relationship between the magnetic field disturbance, Dst and the Z-component of the interplanetary magnetic field, B_z, is given by:

$$(1) \qquad Dst = -2.846 + 6.54\, B_z - 0.118 B_z^2 - 0.002 B_z^3$$

where Dst and Bz are measured in nanoTeslas (nT).

In 2004, W. D. Gonzales and his colleagues published a paper in the Journal of Atmospheric and Solar Terrestrial Physics, in which they determined a relation between the speed of a solar coronal mass ejection V, in km/sec, and the strength of Dst in nT according to

$$(2) \qquad Dst = 0.00052 \times (0.22\, V + 340)^2$$

The relationship between the travel time to Earth from the sun and the speed of the CME was determined from catalogs of CME events by M. J. Owens and P. J. Cargill in research published in 2002 in the Journal of Geophysical Research (vol. 107, p. 1050) in terms of the transit time in days, T, for these coronal mass ejections and their speed, V, in km./sec by

$$(3) \qquad T = -0.0042 \times V + 5.14$$

They also found that the maximum interplanetary magnetic field strength of the CME was given by

$$(4) \qquad B_T = 0.047\, V + 0.644$$

1) From the equation 2 and 3 above, find a function that gives Dst in terms of the transit time of the CME. Write the result in expanded form as a quadratic equation.

2) Assuming that $B_Z = B_T / (2)^{1/2}$ use equations 1 and 4 to find a function that gives Dst in terms of V.

3) From equations 2 and 4, find a function that gives Dst in terms of B_T.

Answer Key:

(1) $Dst = -2.846 + 6.54\ B_z - 0.118B_z^2 - 0.002B_z^3$

(2) $Dst = 0.00052 \times (0.22\ V + 340)^2$

(3) $T = -0.0042 \times V + 5.14$

(4) $B_T = 0.047\ V + 0.644$

Problem 1: From the equation 2 and 3: Dst in terms of the transit time of the CME.

Eqn 3: solve for V. $V = (T - 5.14)/(-0.0042) = -238.1\ T + 1223.8$
Eqn 2: Substitute for V in terms of T:

$$Dst = 0.00052 \times (0.22\ (-238.1\ T + 1223.8) + 340)^2$$
$$= 0.00052 \times (609.2 - 52.4\ T)^2$$

In expanded form: $Dst = 1.4\ T^2 - 33.2\ T + 193.0$ in nT units

Problem 2: Assuming that $B_Z = B_T / (2)^{1/2}$ use equations 1 and 4 and find Dst in terms of V.

Eqn 4: $Bz = B_T / (2)^{1/2} = (0.047\ V + 0.644)/(2)^{1/2}$
$= 0.033\ V + 0.46$

Substituting into Eq 1:

$$Dst = -2.846 + 6.45\ (0.033\ V + 0.46) - 0.118\ (0.033\ V + 0.46)^2 - 0.002\ (0.033\ V + 0.46)^3$$

$$= (-2.846 + 0.46*6.45 - 0.118*0.46^2 - 0.002*0.46^3) +$$
$$(6.45*0.033 -0.118*2*0.46*0.033 -0.002*3.0*0.46^2*0.033)\ V +$$
$$(-0.002*3.0*0.46*0.033^2)\ V^2 -0.002 * 0.033^3\ V^3$$

$$Dst = 0.096 + 0.21\ V - 3.0 \times 10^{-6}\ V^2 - 7.2 \times 10^{-8}\ V^3$$

Problem 3: From equations 2 and 4, find a function that gives Dst in terms of B_T.
Eq 4: Solve for V

$$V = (B_T - 0.644)/0.047 = 21.3\ B_T - 13.7$$

Substitute into Eqn 1 : $Dst = 0.00052 \times (0.22\ (21.3\ B_T - 13.7) + 340)^2$
$= 0.00052\ (4.7\ B_T + 337)^2$

Expanded: $Dst = 0.011\ B_T^2 + 1.65\ B_T + 59.1$ in nT units