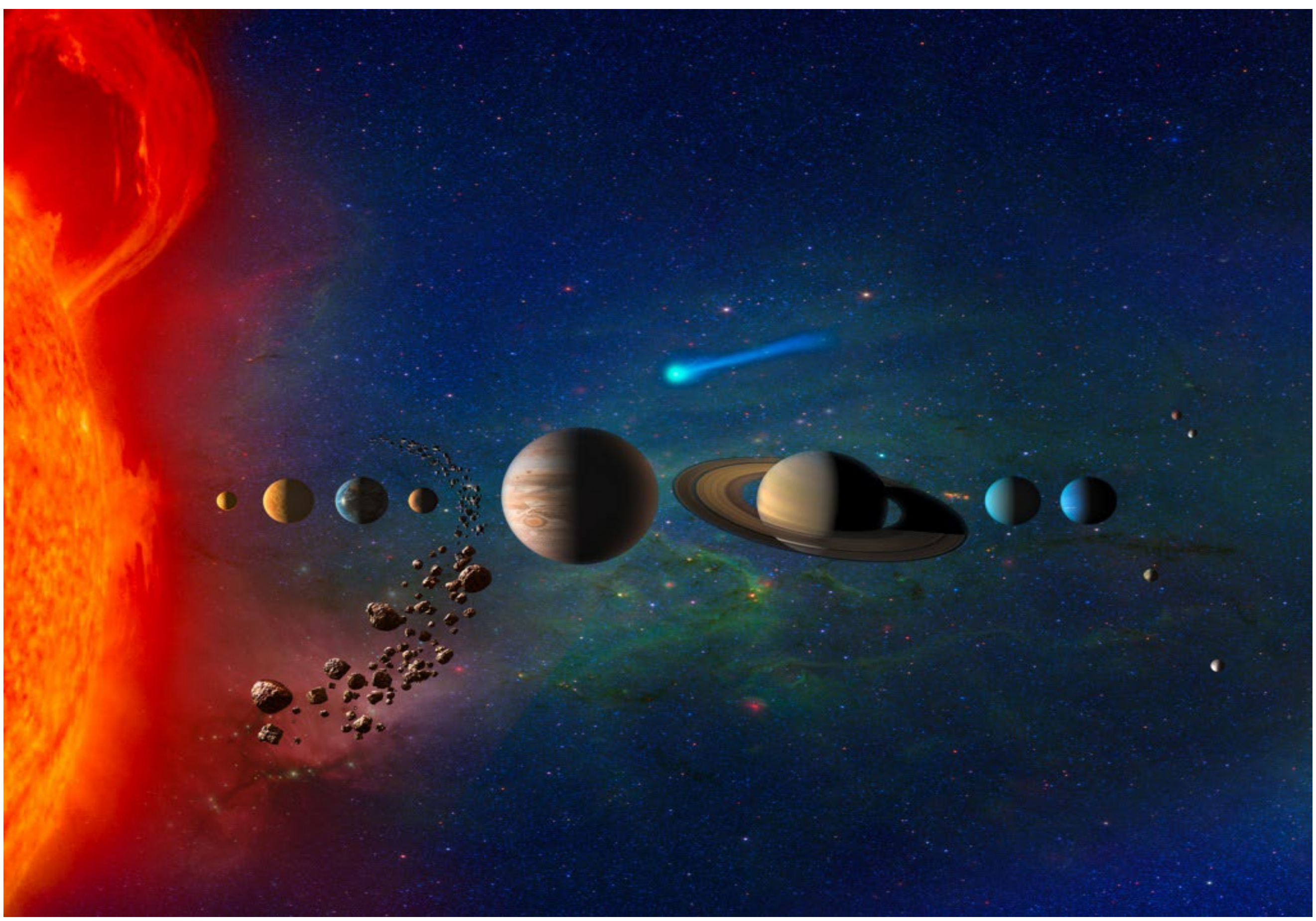
National Aeronautics and Space Administration

Math Problems Related to

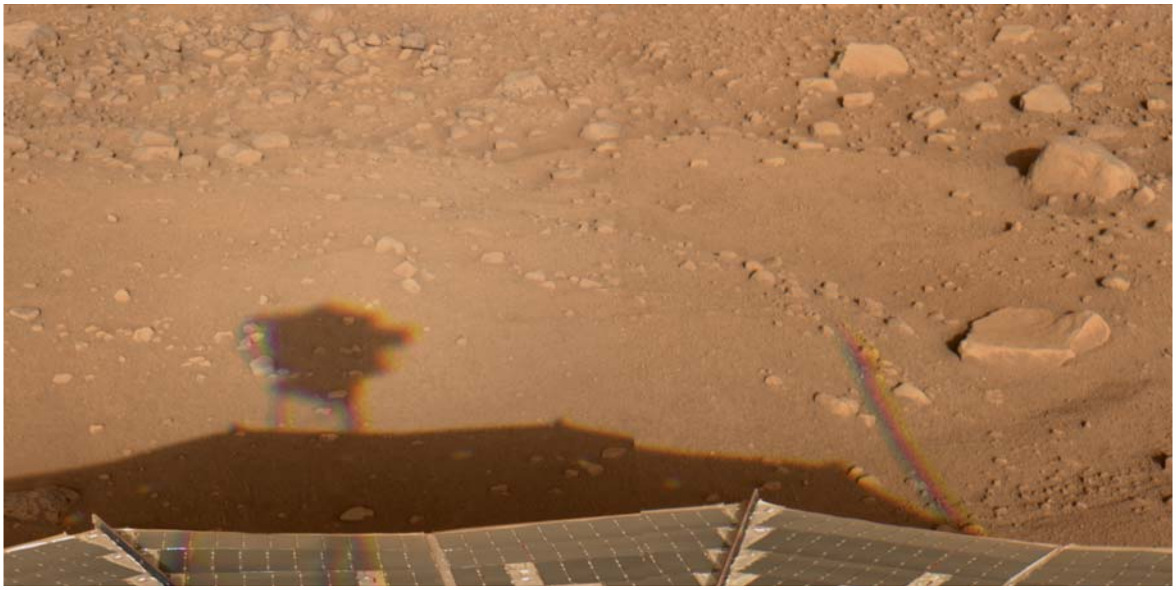
The Solar System II



Introduction

This book provides an assortment of 47 mathematics problems that cover aspects of the solar system and its planets. Basic concepts such as planetary alignments, cratering rates, gravity and calculating the physical sizes of things from images are presented.

39	Intermediate	Graphing	How Quickly are NEOs Being Discovered?
40	Intermediate	Graphing	The 10000th Near Earth Asteroid 2013 MZ5
41	Intermediate	Ratios, Geometry	Visiting the Planets at the Speed of Light
42	Intermediate	Scale, Ratios	Exploring Your Weight Across the Solar System.
43	Intermediate	Functions, Scientific Notation	Exploring Volcanoes and Geysers Across the Solar System.
44	Intermediate	Scale	Comparing the Rings of the Outer Planets
45	Intermediate	Graphing, Conversions, Rates	Exploring the Orbit of the Van Allen Probes.
46	Intermediate	Scale, Geometry	Saturn’s Rings- Shadows from Moons and Ringlets
47	Intermediate	Scale, Ratios, Conversions	How Big is Our Solar System?



The InSight lander will arrive at Mars on September 20, 2016. One of its first jobs will be to photograph a three square meter area in front of the lander where the Seismic Experiment for Interior Structure (SEIS) and the Heat Flow and Physical Properties Package (HP^3) will be deposited with a mechanical arm. The photo above was taken by the successful NASA Phoenix Lander in 2008 using a similar camera system.

The InSight camera system called IDC is a digital camera with a one megapixel square format (1024x1024 pixels). Each pixel can see a square area of the surface that is 0.82 milliradians on each side.

Problem 1 - If one radian is 57.296 degrees, how many arcseconds across is the angular resolution of one pixel?

Problem 2 - What is the width of the camera's field of view in degrees: A) along one side of the image? B) Along the diagonal of the image?

Problem 3 - At a height of one meter from the ground, what is the resolution of the camera in millimeters/pixel?

Problem 4 - If the area to survey is 1.6 meters x 2.4 meters, and each image must overlap 50% of the previous image, how many images have to be taken by the camera to survey the entire instrument area?

Problem 1 - If one radian is 57.296 degrees, how many arcseconds across is the angular resolution of one pixel?

Answer: 0.82 milliradians x (1 radian/1000 milli) x (57.296 degrees/1 radian) x (3600 seconds/1 degree) = **169 arcseconds/pixel.**

Problem 2 - What is the width of the camera's field of view in degrees: A) along one side of the image? B) Along the diagonal of the image?

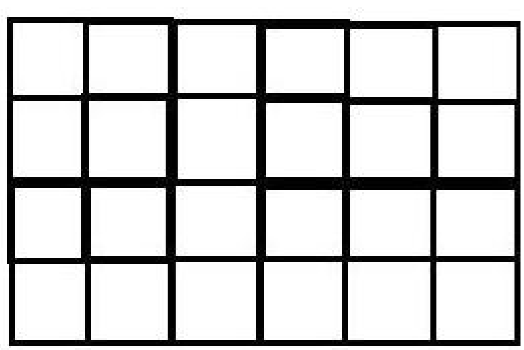
Answer: A) Width = 1024 pixels x 169 arcseconds/pixel x (1 degree/3600 seconds) = **48.1 degrees**. B) The image is a square, so the hypotenuse is 1.414 x side, and so the diagonal length is 1.414 x 48.1 degrees = **68 degrees**.

Problem 3 - At a height of one meter from the ground, what is the resolution of the camera in millimeters/pixel?

Answer: Because the angle is much less than one degree, we use the 'skinny triangle' proportion which says that $x/L = \theta$ / 1radian and so since 169 arcseconds = 0.00082 radians we have x = I meter x 0.00082 so x = **0.82 millimeters per pixel**.

Problem 4 - If the area to survey is 1.6 meters x 2.4 meters, and each image must overlap 50% of the previous image, how many images have to be taken by the camera to survey the entire instrument area?

Answer: Each square image has a side length of 1024 pixels x 0.00082 meters or 0.82 meters. The surface has a width of 1.6 meters and a length of 2.4 meters. If you tiled this field with camera images, you would need a tiling of 2 x 3 images or 6 images with <u>no overlap</u>. For a 50% overlap, the figure below shows how this would work out.



It would take three images vertically and 5 images horizontally for a total of **15 images**.

The InSight Lander will arrive at Mars on September 20, 2016 according to Earth Time, but when will it arrive according to Mars Time?

One Earth Day is exactly 24 hours long, so that the time between two High Noons is exactly 24 hours. But Mars rotates a bit more slowly and by Earth units, one Mars Day is 24 hours and 40 minutes long.

Since the first Viking Lander touched down on Mars in July 20, 1976, NASA scientists use the convention that the first day of operations of a Lander is called Sol 0, and each Mars solar day, called a Sol is 24 hours and 40 minutes long.

This image was taken by the Phoenix lander at sunrise on Sol 86 which corresponded to Earth date August 21, 2008.

Problem 1 - Draw two clock faces for John and Johanna. They live in the same town, but Johanna's clock runs 40 minutes later that John's clock. John says that it is 12:00 Noon. Draw the hour and minute hands for each clock that show when 12:00 Noon happens on Johanna's clock according to John's clock.

Problem 2 - After 10 days, Johanna's clock says that it is 12:00 Noon. What does John's clock say?

Problem 3 - Suppose Johanna is a colonist on Mars and John is back on Earth. On a particular date, time and place on Mars where Johanna was living, it was 12:00 Noon at exactly the same time as it was on Earth where John was living. When John calls Johanna the next Earth day he says that he is having lunch because it is 12:00 Noon by his clock. What time does Johanna's clock read?

Problem 4 - After how many Mars days will both clocks once again show that it is 12:00 Noon?

Problem 1 - Draw two clock faces for John and Johanna. They live in the same town, but Johanna's clock runs 40 minutes behind John's clock. John says that it is 12:00 Noon. Draw the hour and minute hands for each clock that show when 12:00 Noon happens on Johanna's clock according to John's clock.

Answer: John's clock shows the hour and minute hands both on 12, but Johanna's clock has the hands indicating 11:20 am.

Problem 2 - After 10 days, Johanna's clock says that it is 12:00 Noon. What does John's clock say?

Answer: Johanna's clock is 40 minutes behind Johns clock so if John's clock says 12:00 Noon, Johanna's clock says it is **11:20 AM**. It is the same 40 minutes as in Problem 1 because both locations use Earth's 24-hour day.

Problem 3 - Suppose Johanna is a colonist on Mars and John is back on Earth. On a particular date, time and place on Mars where Johanna was living, it was 12:00 Noon at exactly the same time as it was 12:00 Noon on Earth where John was living. When John calls Johanna two Earth days later he says that he is having lunch because it is 12:00 Noon by his clock. What time does Johanna's clock read?

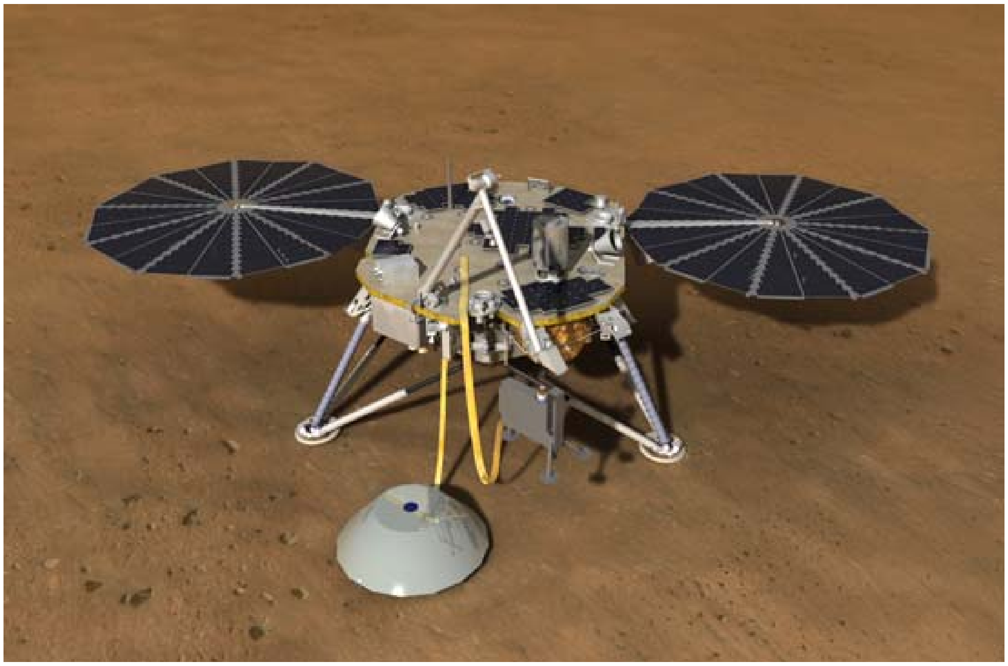
Answer: Because Mars rotates once every 24 hours and 40 minutes, the slower Mars clock will fall behind by 40 minutes after the first Earth day and an additional 40 minutes after the second Earth day, so on Johanna's clock, John will be having Noon at 12:00 - 80 minutes or **10:40 AM**.

Problem 4 - After how many Mars days will both clocks once again show that it is 12:00 Noon?

Answer: For every Earth day, Mars falls behind by 40 minutes. We want N x 40 minutes to add up to one full Mars day of 24h 40 minutes. Solving for N we get:

N = 24h 40m/ 40 m = 24.6666/0.6666 = **37 Mars days**.

Scientists work on Mars Time because Landers use solar panels and so most activity occurs during the Martian daytime. Sunrise and sunset on Mars follow the 24h 40m length of the martian day, not the shorter 24h 00m Earth solar day. This means that by the normal earth clock, martian sunrise occurs 40 minutes earlier each Earth day.



NASA's new mission to Mars called InSight will be launched in March, 2016. It will land in a region of Mars located near the equator and deploy a seismographic station and a heat probe to study the interior of Mars.

Once the InSight lander touches down on September 20, 2016, during the next 60 days it will slowly and carefully deploy the seismometer station called SEIS (the conical package in the image above) and the heat flow experiment called HP^3 (the rectangular instrument package). Both are attached to the lander by yellow cables that carry instrument power and data. A single maneuverable arm will do this work, but it cannot reach closer than 1.5 meter to the Lander, or farther than 2.5 meters from the lander. It can swing left to right, sweeping out an arc of 90 degrees.

Problem 1 – What is the total surface area on Mars near the lander where the maneuverable instrument arm can place its two payloads? (Use $\pi = 3.141$)

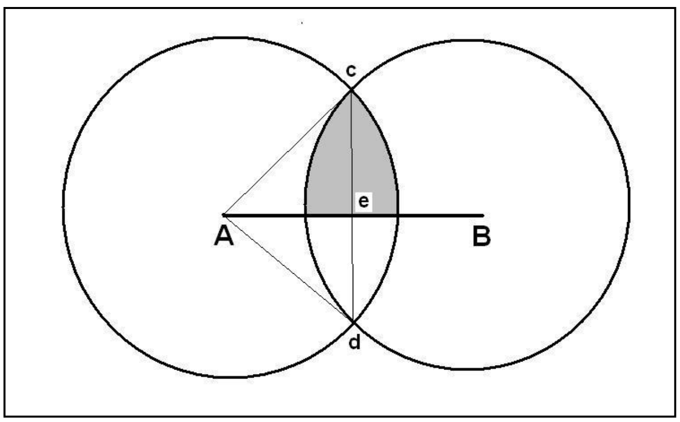
Problem 2 – Suppose the Lander had two identical arms, A and B, whose pivot points were separated by 2.0 meters on the Lander's body. If the arms could reach a maximum of 1.0 meters from their pivot points, what is the area on the martian surface that is common to both arms but above the line joining their pivot points? (Note: For safety reasons, only this common area would be where instruments might be placed so that one or the other arms always have access to them.) Use any method to determine the area.

Problem 1 – What is the total surface area on Mars near the lander where the maneuverable instrument arm can place its two payloads? (Use $\pi = 3.141$)

Answer: The area will be ¼ the area of a circular ring with inner radius of 1.5 meter and outer radius of 2.5 meters. This area is just ¼ the area of the larger circle $A = \pi (2.5)^2 = 19.6\ m^2$ minus the area of the inner circular 'hole' $A = \pi (1.5)^2 = 7.1\ m^2$ which leaves an area of ¼ (12.5) **= 3.1 m^2.**

Problem 2 – Suppose the Lander had two identical arms, A and B, whose pivot points were separated by $2/2^{1/2} = 1.41$ meters on the Lander's body. If the arms could reach a maximum of 1.0 meters from their pivot points, what is the area on the martian surface that is common to both arms but above the line joining their pivot points? (Note: For safety reasons, only this common area would be where instruments might be placed so that one or the other arms always have access to them.) Use any method to determine the area.

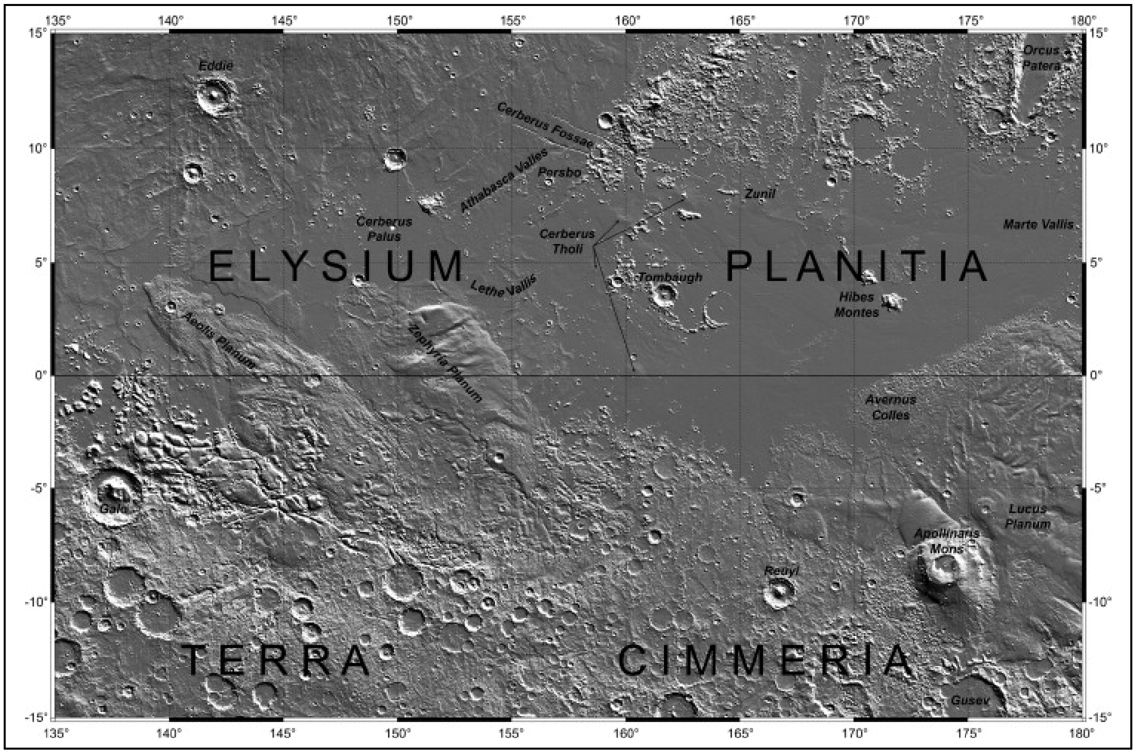
Answer: It always helps to draw a diagram of the area in question given the information in the problem. This visual information helps you formulate a mathematical approach to the problem and can suggest solutions.



Method 1 – Students can reproduce the figure on gridded graph paper and count the number of whole squares in the overlap region.

Method 2 – Students can use Method 1 but add into their sum an estimate for the total area in the fractional squares.

Method 3 – An exact solution to this problem requires a bit of geometry.
The distance $Ae = Be = 1/2^{1/2}$ and the radius of the circle is 1.0, so the triangle **cAe** is a 45-45-90 right triangle with side lengths $R/2^{1/2}$ and an area A=1/2base X height = $1/4\ R^2$ where R=1 for InSight.
The full arc **cAd** is 90^o, so its area is ¼ of the circle whose radius is R=1 meter so Aarc $= \pi/4R^2$.
But we only want the area above the line **AB** so Aarc $= \pi/8R^2$.
The shaded area to the right of the line **ce** has an area A = Aarc – Atriangle = $\pi/8\ R^2 - ¼\ R^2$.
A similar solution by symmetry works for the shaded area to the left of the line **ce**, so the total area of the shaded region is $A = 2 \times (\pi/8\ R^2 - 1/4R^2)$ or $A = R^2/4\ (\pi - 2)$
For the specific case of R=1 meter, $A = (\pi-2)/4 =$ **0.28 meters^2**. This should be close to the answers determined by Method 1 and 2.

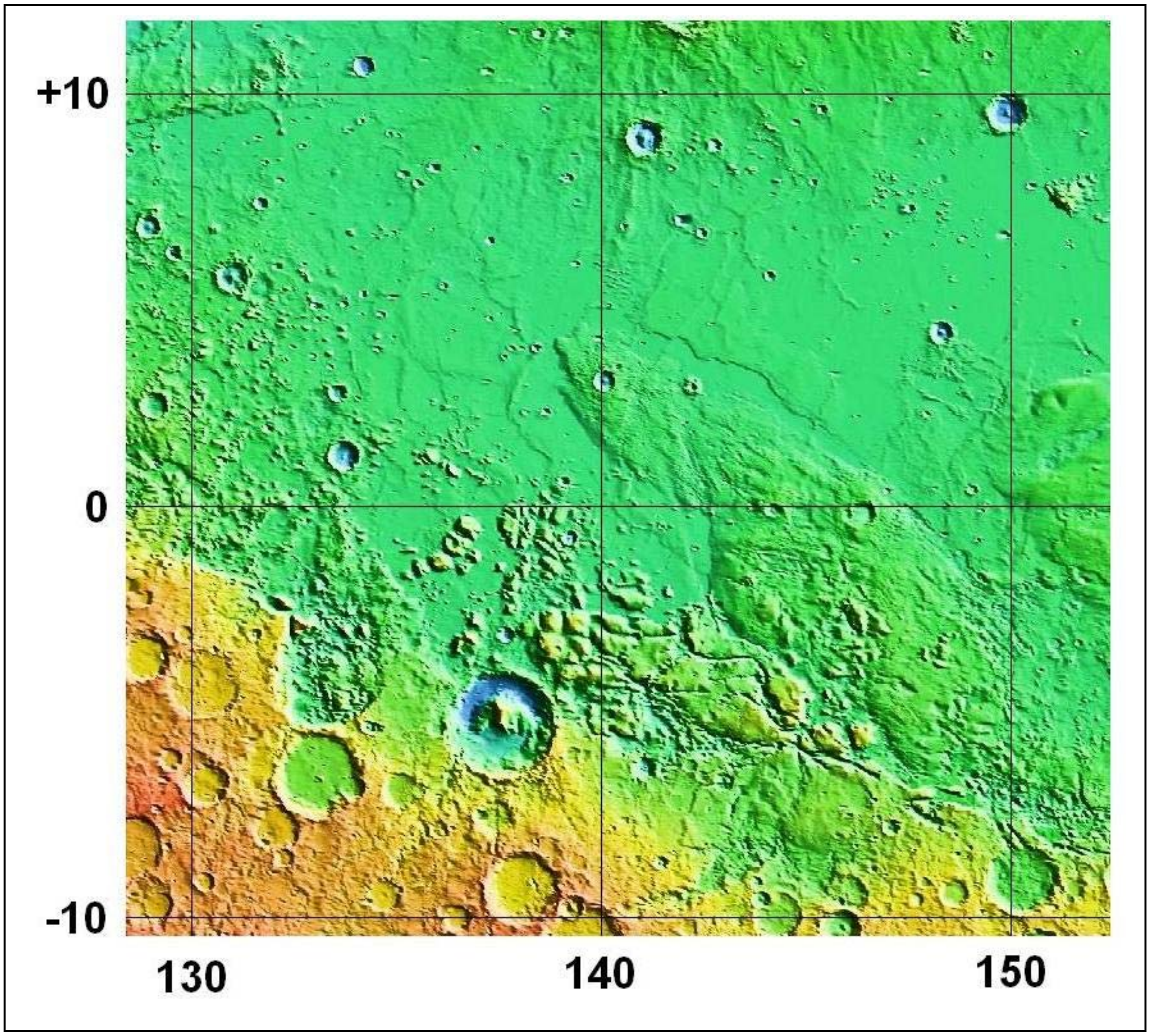


This composite image was obtained by the Mars Reconnaissance Orbiter of the proposed landing area for InSight near Mars latitude +1.5°North and Longitude 139.5°East in the region known as Elysium Planitia.

An equatorial location was selected to make radio communication easier and so that the lander can survive Mars winter conditions, among other reasons.

The lander's temperature probe (HP^3) needs to bore down 5 meters into the martian soil in order to make its measurements of heat flow. The final landing site will be selected based on its having a loose soil-like composition rather than a rocky surface that the drill cannot penetrate.

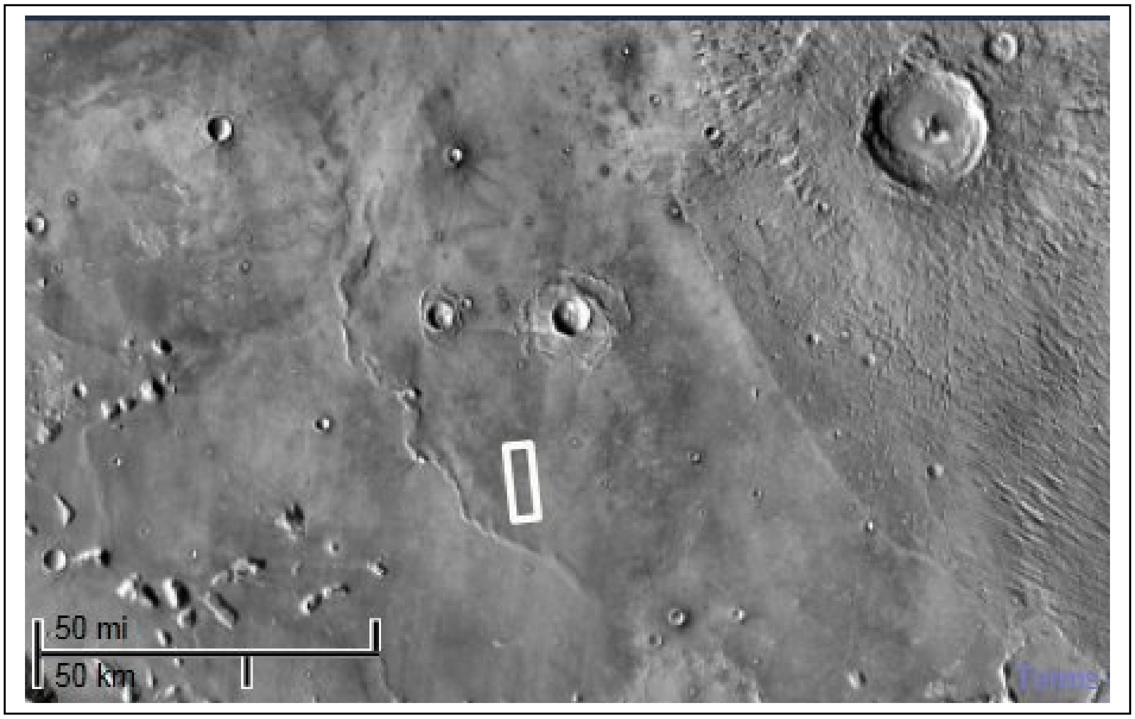
Problem 1 – From the landing coordinates, locate the proposed InSight landing area on the low-resolution map.



Problem 2 – This is a composite image made by the NASA, Mars Orbiter Laser Altimeter (MOLA) instrument of the Elysium Planitia area. The latitude and longitude coordinates are given on a 10^o grid. Can you locate the proposed landing area for InSight in this map?

Problem 3 - The map colors indicate the altitude of the region above the average surface of Mars, whose zero-altitude is coded with yellow. Blue means the area is -4 kilometers below this average. Green is -2 kilometers below the average level. Red means the region is +4 kilometers above this average level. What can you say about the landing area from the altimetry data?

Problem 4 – The scale of this map is 10^o in longitude= 590 kilometers. If the Curiosity Rover is located in Gale Crater at Latitude -4.6^o South and $+137.44^o$ East Longitude, about how far from Curiosity is the proposed InSight landing area?

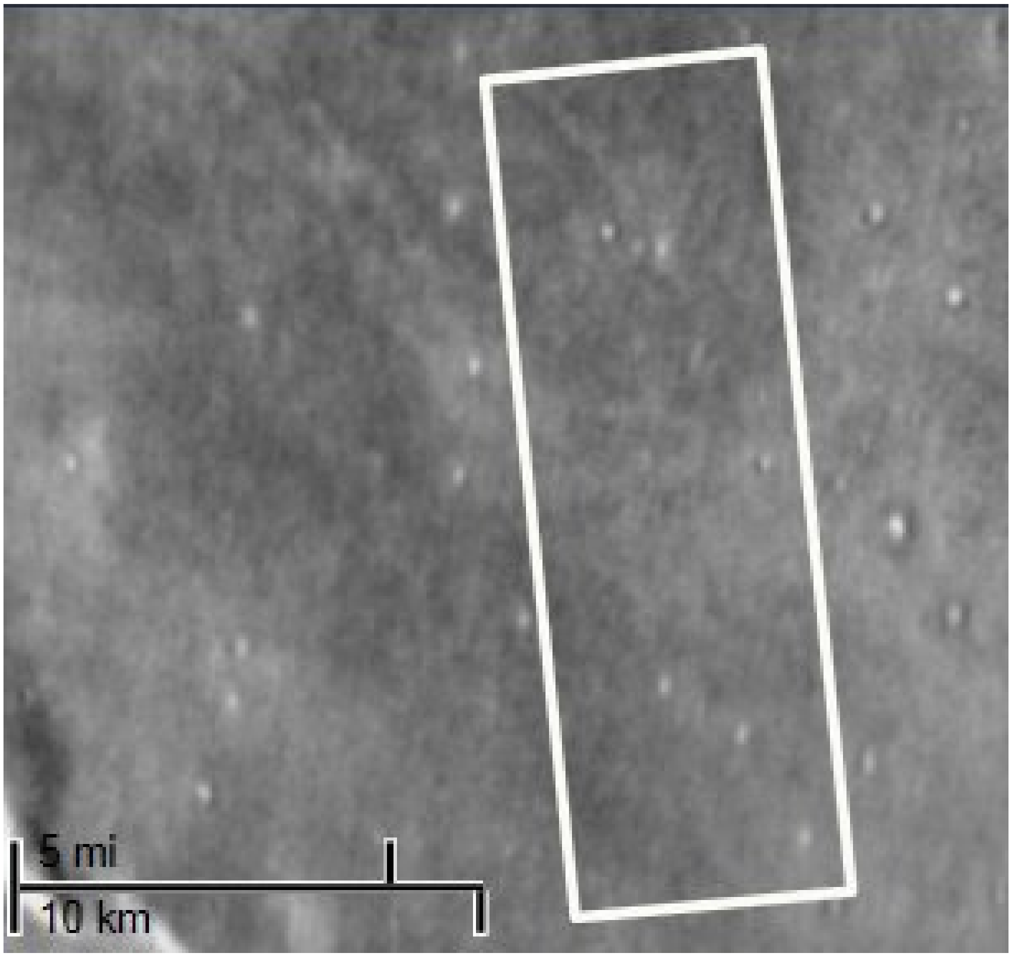


MRO image**: ESP_027702_1815**
Title: Future Landing Site for InSight mission, ellipse E11
Interactive Image: http://www.uahirise.org/hiwish/view/69980
The center of the rectangle is at Latitude +1.617°N and Longitude 138.684° East.

Problem 5 – From this image, and the available information and visual clues, can you identify any features in the MOLA image that appear in this higher-resolution, infrared image?

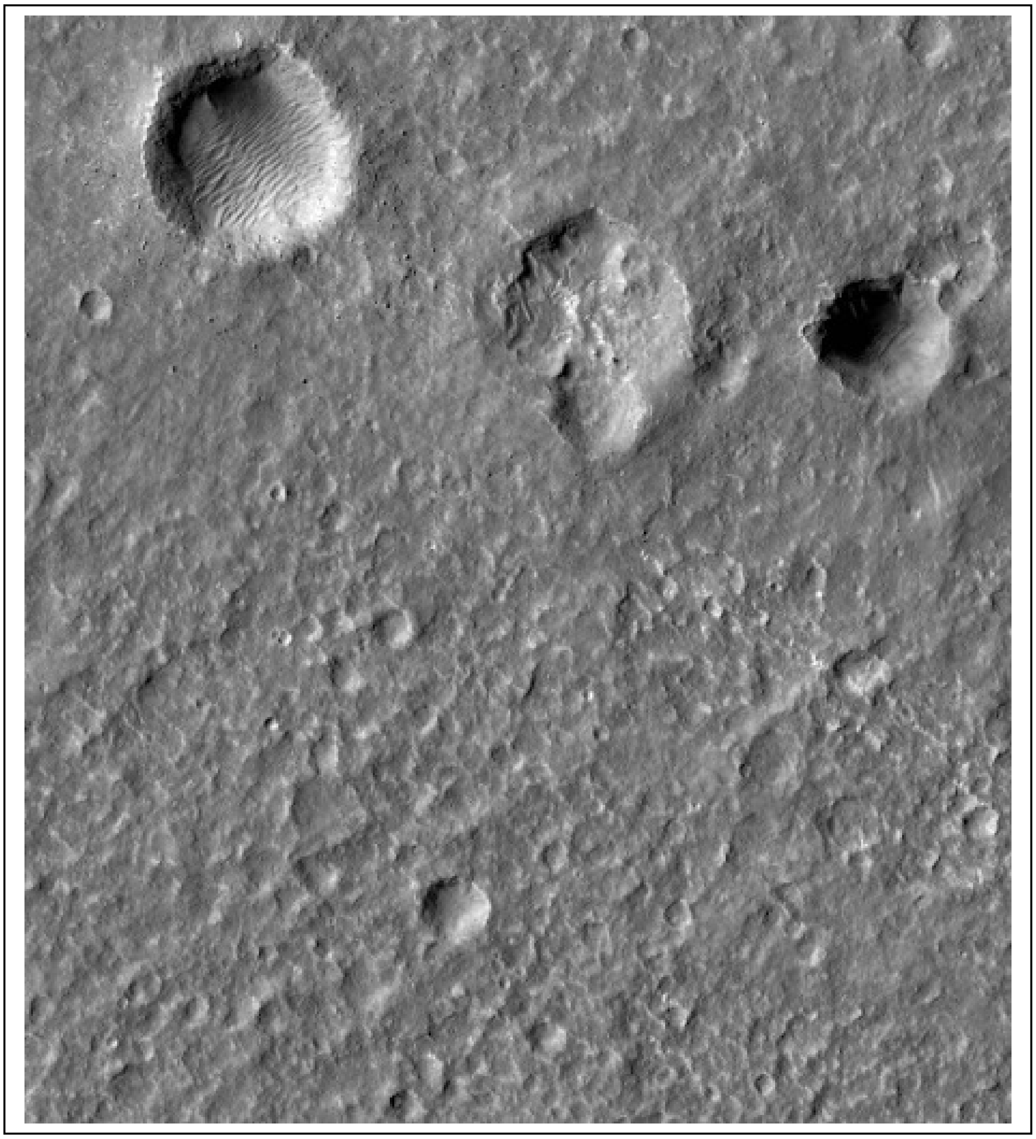
Problem 6 – How big, in meters, are the smallest features you can see, and what are they?

Problem 7 – What clues in this image suggest that, instead of solid rock, the martian ground in this area might actually be much looser?



This is an enlargement of the previous image showing additional detail. The white rectangle is centered on a possible landing area in this region for InSight. Because this is an infrared image sensitive to the temperature of the martian surface, bright features are craters, where the disturbed soil and rock has a higher reflectivity in the infrared spectrum, making the ground appear brighter.

Problem 8 – What is the area of this box in square kilometers?



The Mars Reconnaissance Orbiter High-Resolution (HiRes) imager can take photographs of the martian surface as small as 60-centimeter. This image is an enlargement of a portion of the area inside the landing rectangle. It is 400 meters across.

Problem 9 – Can you find evidence for wind-blown dust?

Problem 10 – Can you find evidence for any large boulders ejected from a meteor impact? If so, how big are they in meters?

Problem 11 – Why do you think that the smaller craters do not show evidence for boulders?

Problem 1 – When printed at normal '81/2x11' page format, the width of the image is 136mm. The scale is 0.33 degrees/mm. The landing area is 13.5 mm from the left-hand edge, and 50 mm from the bottom near the feature called Aeolis Planitia.

Problem 2 – Scale = 55 mm/10 degrees = 5.5 mm/degree.

Problem 3 – The proposed landing area is about 2 kilometers below the average surface level of Mars, and because the color is a constant color, the area is very flat.

Problem 4 – Plot location of Curiosity. Measure distance to InSight with millimeter ruler to get 39.5mm. Scale of the image is 590 km/54.5 mm so distance = 39.5mm x 590km/54.5mm = 427 km. Check your answer at:
http://jmars.mars.asu.edu/maps/?layer=MOC_Atlas_256ppd

Problem 5 – The large crater in the upper right corner, together with the two smaller craters near the center of the image are easy to see in the MOLA map, along with several other features.

Problem 6 – The scale of the image is 5okm/28mm = 1.8 km/mm and the smallest crater you can easily see is about 0.5mm across, so the smallest feature is about 900 meters across.

Problem 7 – The two craters near the center of the image have skirts of ejecta that looks like mud flowing downhill. The wavy ridge to their left also looks like the same kind of flow, so the ground is not solid rock, but a mixture of materials that can flow like mud when struck.

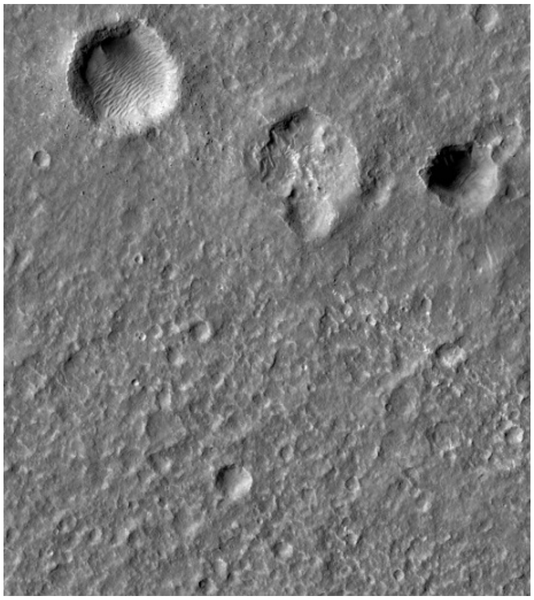
Problem 8 – The scale is 10km/62mm = 0.16 km/mm. The box has dimensions 36mm x 112mm which is 5.8km x 18 km, so the area is 104 km^2.

Problem 9 – In the bottom of the crater in the upper left corner you see trapped 'sand dunes' of martian dust.

Problem 10 – Near the edge of the largest crater you see little dots on the surface that are not craters because the shadow is on the opposite side that for craters, so these dots stick up above the landscape and are boulders. They are less than 1mm across in the image so that means from the scale of the image (400m/130mm) that they are less than 3 meters across.

Problem 11 – One possibility is that the impacts were not powerful enough to launch the boulders out of the crater. Another possibility is that the impacts were shallow and did not penetrate to the ground level where the boulders are located.

Comparing the InSight Landing Area to a City Block!



The image on the left was taken by the Mars Reconnaissance Orbiters High-Resolution camera. It shows a possible landing area for the InSight mission. The image to the right is a satellite view from GOOGLE Earth of a neighborhood somewhere in the United States. Both images have a width of 400 meters.

Problem 1 – How wide are the streets in this neighborhood in meters and feet?

Problem 2 – What is the diameter of the crater towards the bottom edge of the image in meters and feet?

Problem 3 – What is the diameter in meters and feet of the smallest crater you can see in the image?

Problem 4 - Find the tennis court in the neighborhood. Which crater is about the same size as a tennis court?

Problem 1 – How wide are the streets in this neighborhood in meters and feet?

Answer: When reproduced wit a standard printer for '8 1/2x11' paper, the scale of the image is 400 meters = 70 millimeters or 5.7 meters/mm. The street in the center of the image is about 3 mm wide or 17 meters, which is about 51 feet.

Problem 2 – What is the diameter of the crater towards the bottom edge of the image in meters and feet?

Answer: The crater is about 5 mm wide or 28 meters in diameter or 84 feet wide.

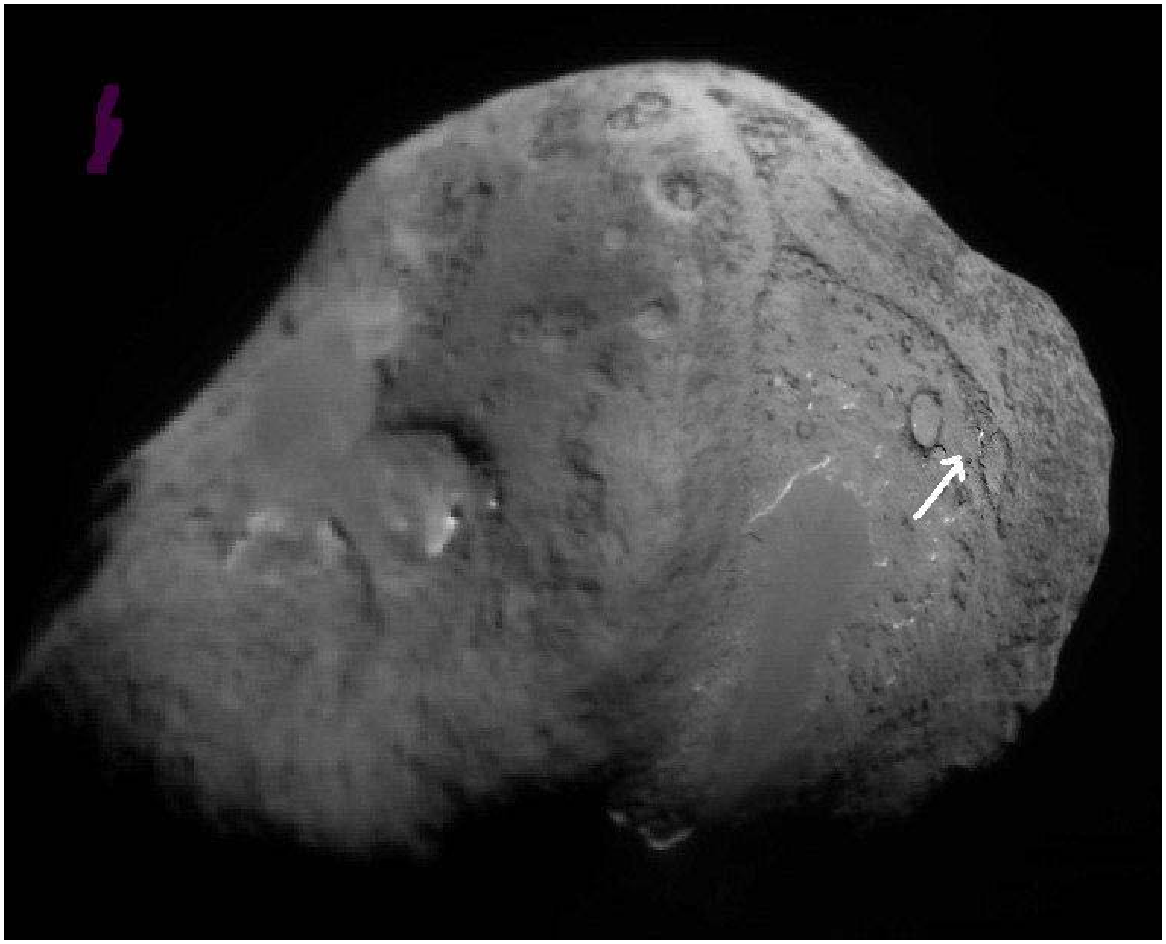
Problem 3 – What is the diameter in meters and feet of the smallest crater you can see in the image?

Answer: Students selections will differ, but 0.3 millimeters equals 1.7 meters or 5 feet is a good estimate. Some ambitious students may see features smaller than this.

Problem 4 - Find the tennis court in the neighborhood. Which crater is about the same size as a tennis court?

Answer: The tennis court is located in the upper left corner of the image as a greenish rectangle. It is about 150 feet long. The large crater near the top right edge is a close match to the size of the tennis court.

Tempel 1 - Close-up of a Comet!



On July 4, 2005, the Deep Impact spacecraft flew within 500 km of comet Tempel 1. This composite image of the surface was put together from images taken by the Impactor probe as it plummeted towards the comet before finally hitting it and excavating a crater. The width of this picture is 8.0 kilometers.

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus by the arrow? D) What is the size of some of the smallest details you can see in the picture?

Problem 2 - The white streak near the center of the picture is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of the arrow. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus? D) What is the size of some of the smallest details you can see in the picture?

Answer: By using a millimeter ruler, what is the scale of this image in meters per millimeter? Answer: A) Width = 153 millimeters, so the scale is 8000 meters/153 mm = **52 meters/mm**. B) Width = 147 mm x 110 mm or **7.6 km x 5.7 km**. C) Although the craters are foreshortened, the maximum size gives a better indication of their 'round' diameters of about 7mm or **360 meters**. D) **Students may find features about 1 millimeter across or 50 meters.**

Note to Teacher: Depending on the quality of your printer, the linear scale of the image in millimeters may differ slightly from the 153 mm stated in the answer to Problem 1. Students may use their measured value as a replacement for the ‘153 mm’ stated in the problem.

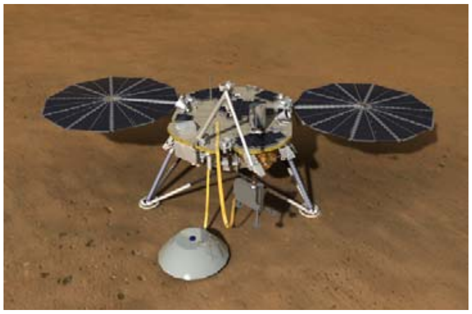
Problem 2 - The white streak near the center of the picture is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

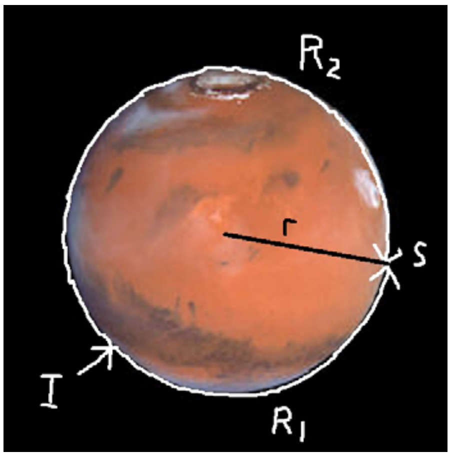
Answer: The width of the irregular white feature is about 0.5 millimeters or 26 meters. The length is about 15 millimeters or 15x52 = **780 meters**.

Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of the arrow. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Answer: The picture shows that the shortest distance to the edge of the nucleus is about 20 millimeters to the right, so this is a distance of about 20 x 52 = **1 kilometer**!

Note to Teacher:
Since the distance to the Earth was about 100 million kilometers, the spacecraft orbit had to be calculated to better than 1 part in 100 million over this distance in order for the probe to hit Tempel-1 as planned.





NASA's new mission to Mars called InSight will be launched in March, 2016. It will land on September 20, 2016 in a region of Mars located near the equator and deploy a seismographic station to study the interior of Mars.

Each time a large meteor strikes the surface of Mars, one seismic wave will travel to the InSight station along a clockwise path around mars, and a second seismic wave will travel in the opposite direction to the station.

InSight will measure the arrival times of the two waves, called R1 and R2. From this timing data and the 5 km/s speed of the seismic wave along the martian surface, InSight will calculate where the impact occurred. The radius of Mars is r=3,397 kilometer.

In the following problems, use $\pi = 3.1416$, and round all answers to the nearest kilometer and second.

Problem 1 – Suppose that the impact occurred at Point I on the above figure, and the time between the arrival of the R1 and R2 waves was exactly 1423 seconds. How far did the R1 and R2 waves travel to get to the InSight station?

Problem 2 – For a large enough impact, InSight scientists expect that after the R1 and R2 waves are detected by the seismometer, that the waves will continue to 'orbit' the surface of Mars and return once again as a second pair of weaker seismic signals called R3 and R4, followed later on by a third pair of even-weaker signals called R5 and R6. For the example in Problem 1, what are the arrival times of all 6 seismic signals if R1 was detected at the clock time of 13:00:00 local time at the lander site?

Problem 3 - Because of a glitch in the recording of the seismic data, InSight scientists were able to detect the R3 and R6 seismic waves, which arrived at 15:25:30 and 17:00:00 Local Mars Time. How far from the Lander did the impact occur, and when would the arrival times for all 6 seismic waves have occurred in the data?

Problem 1 – Suppose that the impact occurred at Point I on the above figure, and the time between the arrival of the R1 and R2 waves was exactly 1423 seconds. How far did the R1 and R2 waves travel to get to the InSight station?

Answer: To travel once around the circumference of Mars, the wave has to travel $2 \pi r = 2 (3.1416) (3397 \text{ km}) = 21344 \text{ km}$, so the round trip time is T = 21344 km / (5 km/s) = 4269 seconds. We know that T2 – T1 = 1423 seconds, so the R2 wave had to travel the same distance as the R1 wave plus an additional 1423 seconds. Since 1423 seconds = 1/3 of the full circumference time of 4269 seconds, that means that R1 traveled 1423 seconds from the impact site, I, and R2 traveled 2 x 1423 = 2846 seconds from the impact site. The distance to the impact site using the R1 wave data is 1423 sec x 5 km/sec = **7,115 kilometers**

Problem 2 – For a large enough impact, InSight scientists expect that after the R1 and R2 waves are detected by the seismometer, that the waves will continue to 'orbit' the surface of Mars and return once again as a second pair of weaker seismic signals called R3 and R4, followed later on by a third pair of even-weaker signals called R5 and R6. For the example in Problem 1, what are the arrival times of all 6 seismic signals if R1 was detected at the clock time of 13:00:00 local time at the lander site?

R1 = **13:00:00**
R2 = 13:00:00 + 1423 seconds = 13:00:00 + 23m 43s = **13:23:43**
R3 = 13:00:00 + 4269 seconds = 13:00:00 + 1h 11m 9s = **14:11:09**
R4 = 13:23:43 + 4269 seconds = 13:23:43 + 1h 11m 9s = **14:34:52**
R5 = 14:11:09 + 4269 seconds = 14:11:09 + 1h 11m 9s = **15:22:18**
R6 = 14:34:52 + 4269 seconds = 14:34:52 + 1h 11m 9s = **15:46:01**

Problem 3 - Because of a glitch in the recording of the seismic data, InSight scientists were able to detect the R3 and R6 seismic waves, which arrived at 15:25:30 and 17:00:00 Local Mars Time. How far from the Lander did the impact occur, and when would the arrival times for all 6 seismic waves have occurred in the data?

Answer: R3 is the R1 wave which has orbited Mars one additional time so that R3-R1 = 4269 seconds. R6 is the R2 wave which has orbited Mars two additional times so that R6-R2 = 2(4269 seconds). The R5 wave would have arrived one full orbit (4269 seconds) after the R3 wave, so the time intervals are as follows:

R1 = 15:25:30 – 4269 seconds = 15:25:30 – 1h 11m 9s = **14:14:21**
R2 = 17:00:00 – 8538 seconds = 17:00:00 – 2h 22m 18s = **14:37:42**
R3 = **15:25:30**
R4 = 17:00:00 – 4269 seconds = 17:00:00 – 1h 11m 9s = **15:48:51**
R5 = 15:25:30 + 4269 seconds = 15:25:30 + 1h 11m 9s = **16:36:39**
R6 = **17:00:00**

Time interval = R2-R1 = 23:21 = 1401 seconds.

R1 + R2 = 4269 seconds
R1 – R2 = 1401 seconds. Then adding the two equations we get 2R1 = 5670 so R1 = 2835 seconds. Traveling at 5 km/sec, the R1 wave originated **14,175 km from the landing site.**

On February 14, 2013 a 10,000 ton meteor about 17-meters in diameter entered Earth's atmosphere over Russia traveling at 40,000 mph (18 km/s). It detonated in the air over the town of Chelyabinsk and the explosion caused major damage to the town injuring 1,000 people. The people were hurt by flying glass when the windows of over 3000 buildings blew out over an area of about 1000 km^2. Unlike the famous Tunguska Event of 1908 which blew down 80 million trees and was nit 'discovered' for many decades afterwards, the Chelyabinsk Meteor was extensively videoed by hundreds of dashcams and cell phones as it happened.

Studies of thousands of meteor sightings by scientists can now tell us just how often asteroids of 4-meters or larger enter Earth's atmosphere. About two of these events happens each year over the entire surface area of Earth, which is 500 million km^2!

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Problem 3 - The kinetic energy in Joules of a large meteor is given by $KE = 1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded if 1 ton = 1000 kg?

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Answer: The inhabited area of Earth is 3% of 28% of the total surface area or just 0.8% of Earth's total surface area. This is 1/125 of the full area. If you had one event per year, you would have to wait 125 years for the next one. If you had 2 events per year, you would have to wait half this time or about **62 years**. So, in a typical 70-year human lifetime, you will hear about one or two of these major impact events in the News!

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Answer: The 50,000 bolides arrive somewhere over Earth each year. The chance that that this area is over an inhabited region of Earth is 0.8% or 1/125. So 1/125 of the bolides arrive over an inhabited area which is 50000/125 = 400 each year. For you to personally see the event, it has to happen within 100 km^2 of where you are standing. The inhabited area of Earth has an area of 1/125 x 500 million km^2 = 4 million km^2. Your 100 km^2 is only 1/40000 of this area. So you will see 400 bolides x 1/40000 = 1/100 bolides each year, or will **have to wait about 100 years to see just one**! If you watched the sky every night for your entire life, you might see one of these events!

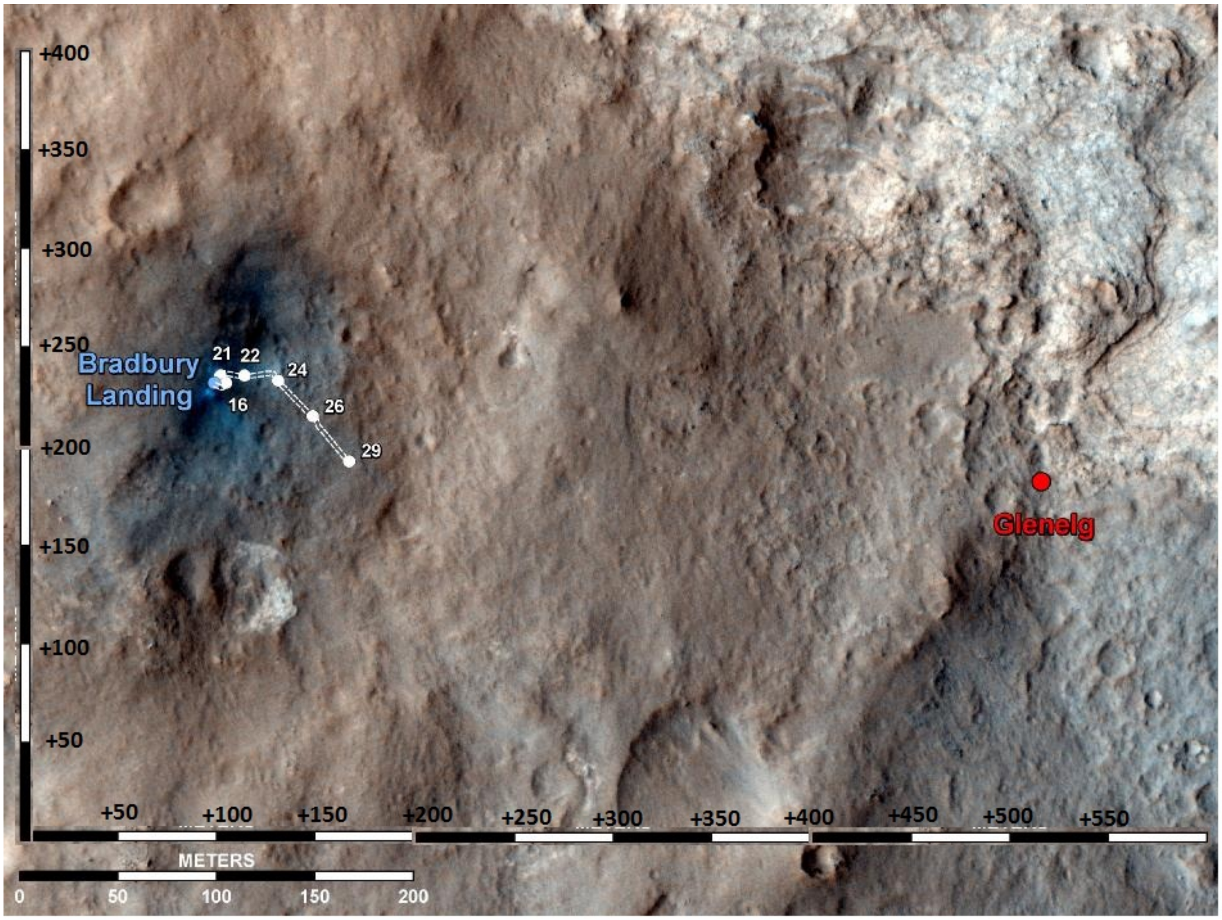
BUT, because of our world-wide news and internet coverage, you could hear about any bolide that flashed over the inhabited area of Earth or 400 bolides each year! We only hear about a few of these because most of them are too unimpressive to get the attention of the news system. After the Chelyabinsk Meteor event on February 14, 2013, there were many announcements of fireballs or bolides over Los Angeles, San Francisco and other cities as this news topic became popular for a few weeks.

Problem 3 - The kinetic energy in Joules of a large meteor is given by $KE = 1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded?

Answer: The mass was 10,000 tons or 10,000 tons x 1000 kg/1 ton = 10 million kg. The speed was 18 km/s x 1000 m/km = 18,000 m/s, so the energy was

$KE = ½ (1.0\times10^7) \times (1.8\times10^4)^2 = 1.62\times10^{15}$ Joules.

This is equal to 1.62×10^{15} Joules x (1 ton/4.2×10^9Joules) = **386,000 tons of TNT** or about 10 times the energy of a small atom bomb. This is similar to the estimates found in the news reports of this event, and explains why it did so much damage!



The Curiosity Rover on Mars landed at Bradbury Station on Day 0 (Called Sol 0) and is headed for an important geological site called Glenelg. This map shows the location of the Rover until Sol 29. Also shown on the map is a coordinate grid marked in intervals of 50-meters. Bradbury Station is located at approximately (+100, +230). The table below gives the location of Curiosity for the period from Sol 29 to Sol 56. Students should use the distance formula to determine interval lengths: $d^2 = (x2-x1)^2 + (y2-y1)^2$ but they may also use millimeter rulers and the image scale to determine the distances between the points.

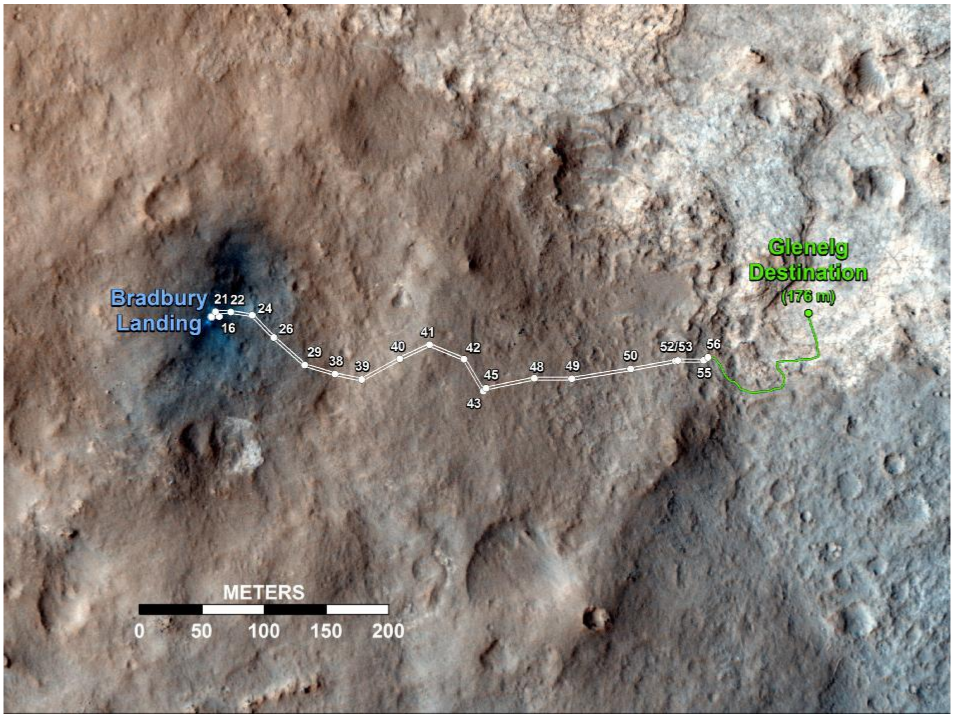
Day	X	Y	Day	X	Y
39	+210	+180	48	+360	+175
41	+270	+210	49	+390	+180
42	+300	+200	52	+470	+200
45	+315	+165	56	+500	+205

Problem 1 – Graph the additional points and connect them with line segments to show Curiosity's path across the martian landscape.

Problem 2 – During which segment was Curiosity traveling the fastest?

Problem 3 – During which segment was Curiosity traveling the slowest?

Problem 4 – What has been the average speed of Curiosity between Sol 39 and Sol 56?



Problem 1 – Graph the additional points and connect them with line segments to show Curiosity's path across the martian landscape. Answer: **See actual course above**.

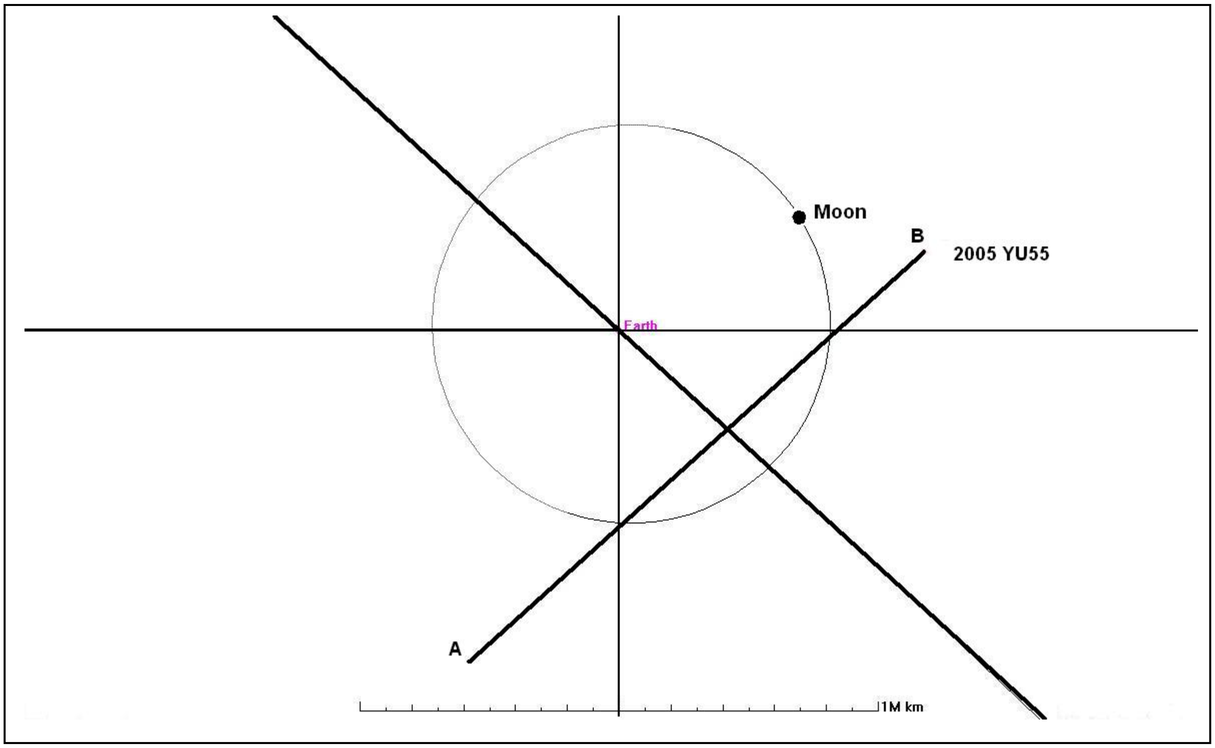
Day	X	Y	Segment Time (days)	Segment Distance (m)	Segment Speed (m/d)
39	+210	+180			
41	+270	+210	2	67	34
42	+300	+200	1	32	32
45	+315	+165	3	38	13
48	+360	+175	3	46	15
49	+390	+180	1	30	30
52	+470	+200	3	82	27
56	+500	+205	4	30	8

Example: Day 45 - Day 42 = 3 days. $D^2 = (315\text{-}300)^2 + (165\text{-}200)^2 = 1450$ so d = 38 meters and speed = 38meters/3days = 13 meters/day.

Problem 2 – During which segment was Curiosity traveling the fastest? **Between Sol 41 and Sol 42 at a speed of 34 meters per day.**

Problem 3 – During which segment was Curiosity traveling the slowest? **Between Sol 52 and Sol 56.**

Problem 4 – What has been the average speed of Curiosity between Sol 39 and Sol 56?
Total segment distance traveled = (67+32+38+46+30+82+30)=325 meters in 17 days
So **average speed = 19 meters/day.**



The Asteroid 2005 YU55 passed inside the orbit of our moon sometime between November 8 and November 9, 2011. The diagram shows the lunar orbit as a circle centered on Earth. The diagonal line is the orbit of Earth around the sun. The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

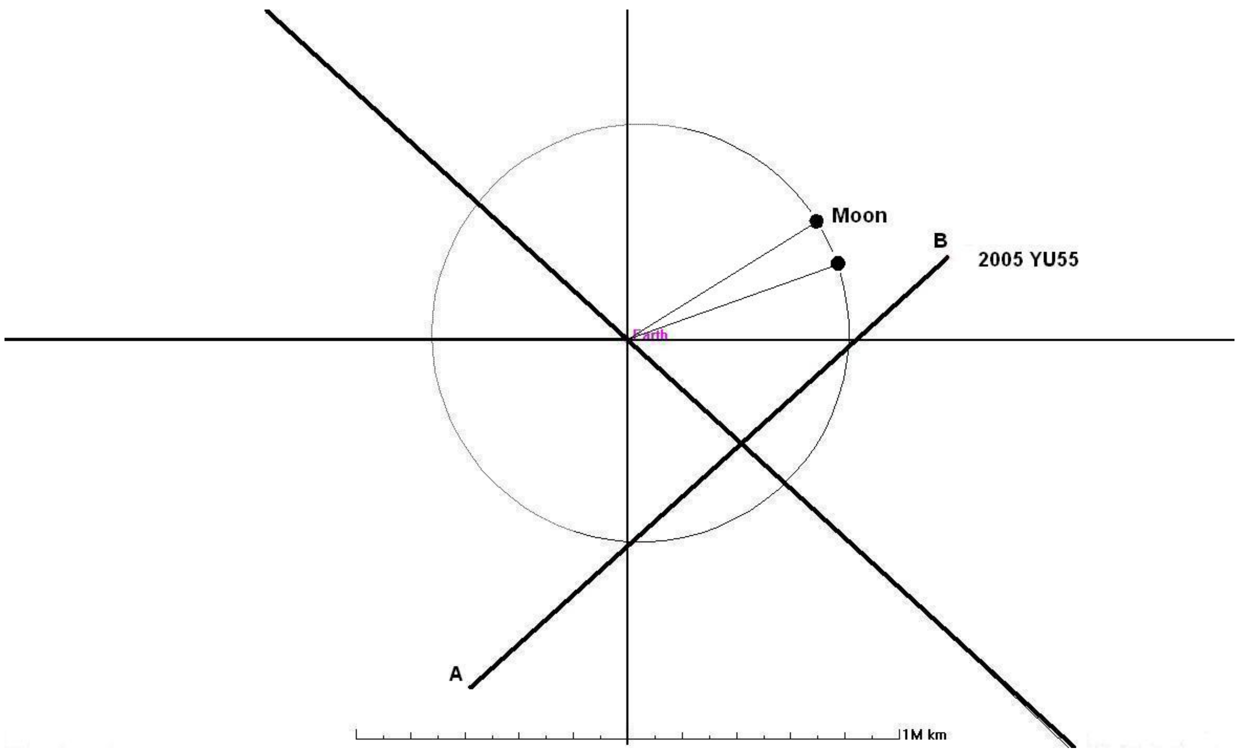
Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time (9.438 days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. Where was the moon located when the asteroid was at Point A?

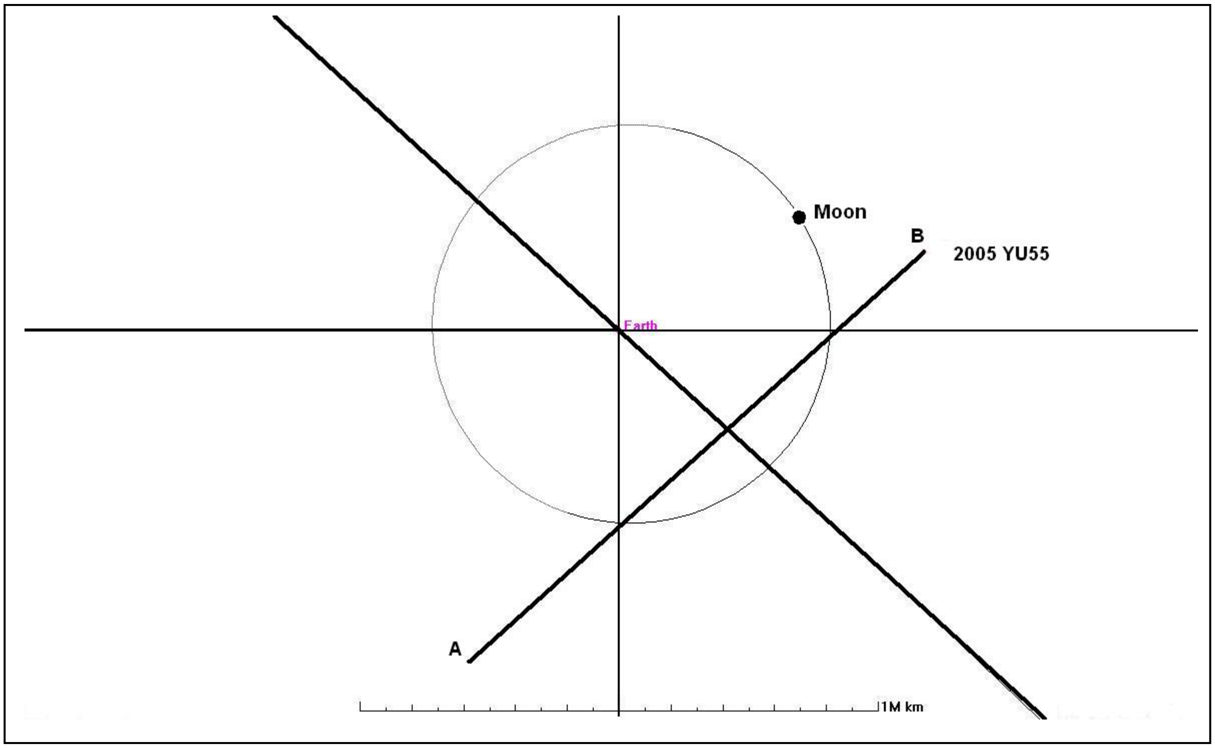
The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time (9.438days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. Where was the moon located when the asteroid was at Point A?

Answer: The time between Point A and Point B is exactly 1.0 days, so we need to determine the position of the moon 1 day earlier that its location in the diagram.

The moon travels 360 degrees in 27.3 days for a speed of 13.2 degrees per day, so we need to find the position of the moon 13.2 degrees clockwise of its position in the diagram. Using a protractor, the figure below shows this position.





The Asteroid 2005 YU55 passed inside the orbit of our moon between November 8 and November 9, 2011. The diagram shows the lunar orbit as a circle center on Earth. The diagonal line is the orbit of Earth around the sun. The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time (UT). For example: 9.500 is 12:00 UT on November 9.

Problem 1 - To the nearest minute, what is the Universal Time hour and minute for each of the endpoint dates?

Problem 2 - Using a millimeter ruler, what is the scale of this diagram in kilometers per millimeter?

Problem 3 - To the nearest 100 km, how far will the asteroid travel between the endpoint times?

Problem 4 - How fast will the asteroid be traveling in kilometers per hour?

Problem 5 - On what date and Universal Time will the asteroid be closest to Earth, and what is this distance?

The line segment AB is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point A is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

Problem 1 - What is the Universal hour and minute for each of the endpoint dates?

Answer - 0.438 days = 0.438 x 24hrs = 10.512h and (10.512-10)x60m = 31 m, so the time is 10:31 UT, then Point A is **November 8 at 10:31 UT** and Point B is **November 9 at 10:31 UT**.

Problem 2 - Using a millimeter ruler, what is the scale of this diagram in thousands of kilometers per millimeter?
Answer - 1 million km / 81 millimeters = **12346 km/mm**.

Problem 3 - How far will the asteroid travel between the endpoint times?
Answer - The distance is 81 mm or **1 million km**.

Problem 4 - How fast will the asteroid be traveling in kilometers per hour?
Answer - 1 million km / (9.438-8.438) = 1 million km/day or **41,667 km/hr**.

Problem 5 - On what date and Universal Time will the asteroid be closest to Earth?

Answer - This happens 46 mm from Point A.
D = 46 x 12346 km = 567,916 km.

The speed is 41,667 km/hr so T = 567,916 / 41667 = 13.63 hours from Point A.
This equals 0.568 days.

Then 8.438 + 0.568 = 9.006 which is **00:09 UT on November 9**.

Dawn Spacecraft sees Asteroid Vesta Up Close!

On July 15, 2011 the NASA spacecraft Dawn completed a 2.8 billion kilometer journey taking four years, and went into orbit around the asteroid Vesta. Vesta is the second largest asteroid in the Asteroid Belt. Its diameter is 530 kilometers. After one year in orbit, Dawn departed in 2012 for an encounter with asteroid Ceres in 2015. Meanwhile, from its orbit around Vesta, it will map the surface and see features less than 1 kilometer across.

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Problem 3 - Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Problem 4 - The Space Shuttle traveled at a speed of 28,000 km/hr in its orbit around Earth. How many times faster than the Shuttle does the Dawn spacecraft travel?

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Answer: When printed using a standard printer, the width of the asteroid is about 123 millimeters. Since the true diameter of the asteroid is 530 km, the scale is then S = 530 km / 123 mm = **4.3 kilometers per millimeter**.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Answer: **Students can find a number of small features in the image that are about 1 mm across, so that is about 4.3 kilometers. Among the largest features are the 9 large craters located along the middle region of Vesta from left to right. Their diameters are about 4 to 7 millimeters or 17 to 30 kilometers across. The large depression located in the upper left quadrant of the image is about 40 mm long and 15 mm wide in projection, which is equivalent to 172 km x 65 km in size.**

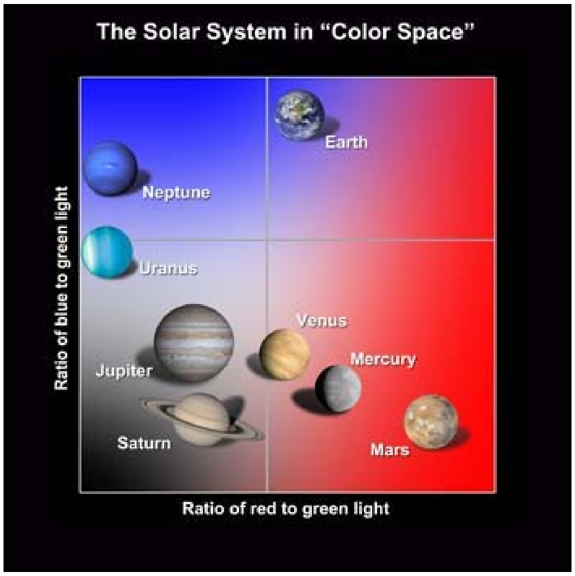
Problem 3 - Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Answer: Time = 4 years, distance = 2.8 billion km, so the speed is A) S = 2.8 billion km / 4 years = **700 million km/year**. B) Converting to an hourly rate, S = 700 million km/yr x (1 year/365 days) x (1 day/24 hours) = **79,900 km/hour**.

Problem 4 - The Space Shuttle traveled at a speed of 28,000 km/hr in its orbit around Earth. How many times faster than the Shuttle does the Dawn spacecraft travel?

Answer: Ratio = (79,900 km/hr) / (28,000 km/hr) = 2.9 . So Dawn is traveling at an average speed that is **2.9 times faster than the Space Shuttle!**

Note: The Space Shuttle is traveling 8 times faster that a bullet (muzzle velocity) from a high-powered M16 rifle! So Dawn is traveling 23 times faster than such a bullet!



Earth is invitingly blue. Mars is angry red. Venus is brilliant white. NASA astronomer Lucy McFadden, and UCLA research assistant in geochemistry Carolyn Crow, have now discovered that a planet's "true colors" can reveal important details.

Mars is red because its soil contains rusty red stuff called iron oxide. Our planet, the "blue marble" has an atmosphere that scatters blue light rays more strongly than red ones. This suggests that astronomers could use color information to identify Earth-like worlds. Their colors will tell us which ones to study in more detail.

As NASA's Deep Impact spacecraft cruised through space, its High Resolution Instrument (HRI) measured the intensity of Earth's light. HRI is a 30-cm telescope that feeds light through seven different color filters. Each filter samples the incoming light at a different portion of the visible-light spectrum, from ultraviolet and blue to near-infrared. A table showing the reflectivity of each body is shown below. The numbers indicate the percentage of light reflected by the planet at 350, 550 and 850 nanometers (nm). For example, compared to the light that it reflects at 550 nm, Venus reflects 116% more light at 850 nm.

Object	350 nm	550 nm	850nm	Object	350nm	550nm	850nm
Mercury	47	100	142	Jupiter	60	100	64
Venus	58	100	109	Saturn	45	100	78
Earth	152	100	110	Titan	34	100	88
Moon	67	100	169	Uranus	98	100	15
Mars	34	100	203	Neptune	125	100	13

Note: Table based upon data published by the astronomers in the Astrophysical Journal (March 10, 2011).

Problem 1 – One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where X = R(850)/R(550) and X = R(350)/R(550). For example, for the Moon, where R(350) = 61% , R(550) = 100%, and R(850) = 155% we have X = 155/100 = 1.55, and Y = 61/100 = 0.61. Using this method, calculate X and Y for each object and then plot the (X,Y) points on a graph.

Problem 2 – The planetary data in the table can be written as an ordered triplet. For example, for Mercury the reflectivities in the table above would be written as (56, 100, 177). Using the definition for X and Y in Problem 1, which of the planets below would you classify as Earth-like, Jupiter-like, or Moon-like, if the planetary reflectivities are: Planet A (61, 82, 156), Planet B (45, 35, 56), Planet C (90, 120, 67).

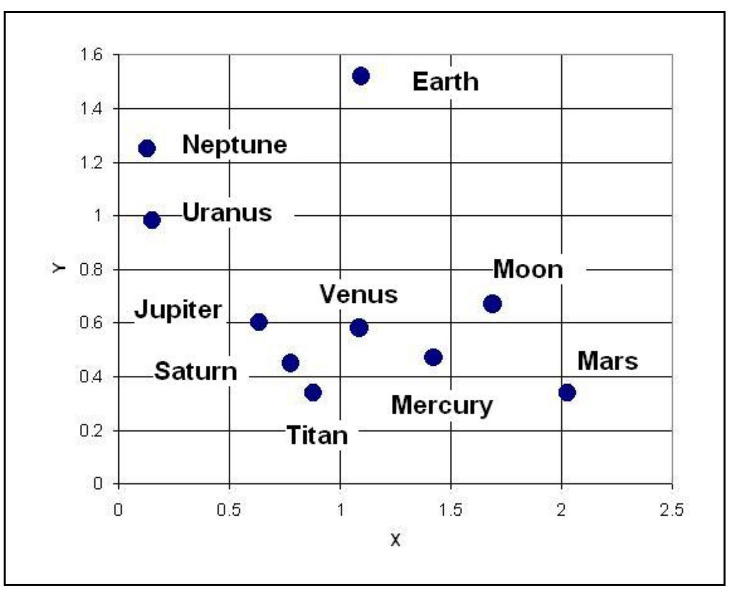
Problem 3 – Can you create a different plot for the planets that makes their differences stand out even more?

Problem 1 – One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where X = R(850)/R(550) and X = R(350)/R(550). For example, for the Moon, where R(350) = 61% , R(550) = 100%, and R(850) = 155% we have X = 155/100 = 1.55, and Y = 61/100 = 0.61. Using this method, calculate X and Y for each object and then plot the (X,Y) points on a graph. See graph below.

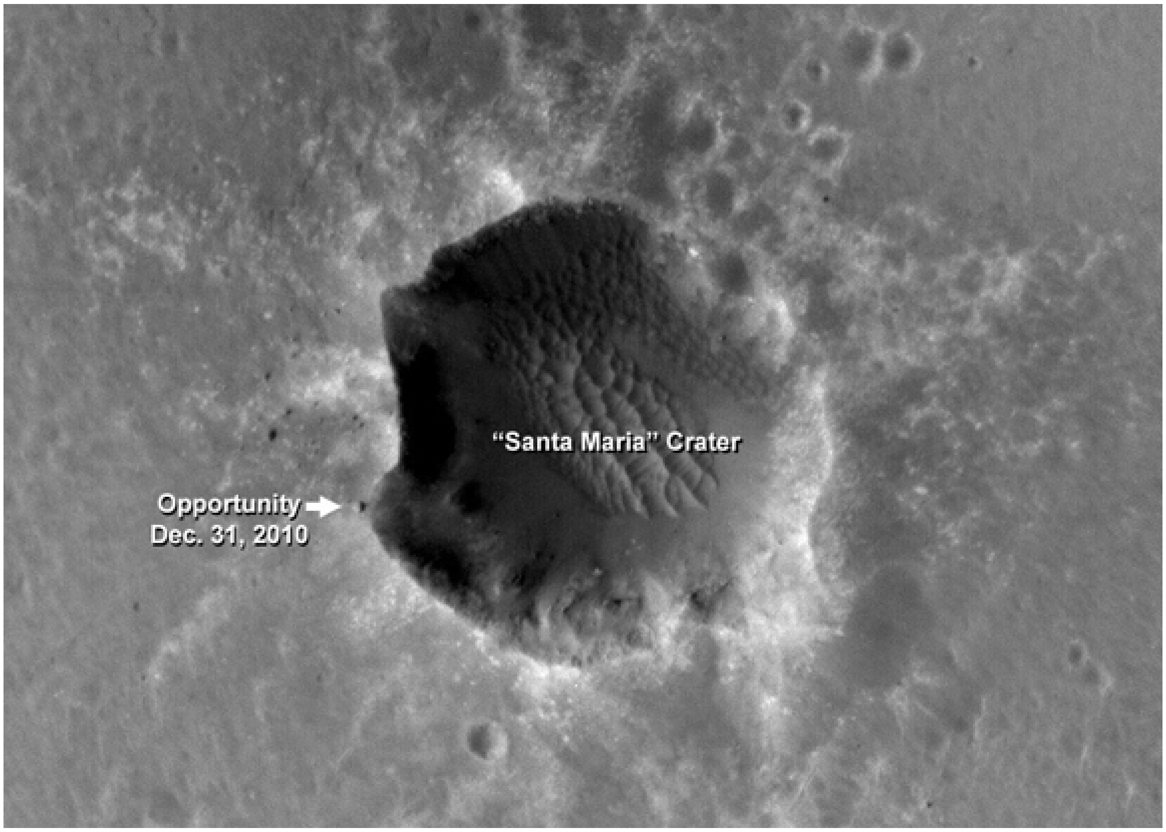
Object	350 nm	550 nm	850nm	X	Y
Mercury	47	100	142	**1.42**	**0.47**
Venus	58	100	109	**1.09**	**0.58**
Earth	152	100	110	**1.10**	**1.52**
Moon	67	100	169	**1.69**	**0.67**
Mars	34	100	203	**2.03**	**0.34**
Jupiter	60	100	64	**0.64**	**0.60**
Saturn	45	100	78	**0.78**	**0.45**
Titan	34	100	88	**0.88**	**0.34**
Uranus	98	100	15	**0.15**	**0.98**
Neptune	125	100	13	**0.13**	**1.25**

Problem 2 – The planetary data in the table can be written as an ordered triplet. For example, for Mercury the reflectivities in the table above would be written as (56, 100, 177). Using the definition for X and Y in Problem 1, which of the planets below would you classify as Earth-like, Jupiter-like, or Moon-like, if the planetary reflectivities are: Planet A (61, 82, 156), Planet B (45, 35, 56), Planet C (90, 120, 67). Answer: **Planet A has X and Y values similar to the moon; Planet B is more like Earth; and Planet C is more like Jupiter.**

Problem 3 – Can you create a different plot for the planets that makes their differences stand out even more? Answer: **There are more than 100 different ways in which students may decide to create new definitions for X and Y such as X = R(350) – R(850); Y = R(850)/R(350) and so on. Some will not visually let you see a big difference between the planet 'colors' while other may. Astronomers try many different combinations, usually with some idea of the underlying physics and how to enhance what they are looking for. There is no right or wrong answer, only ones that make the analysis easier or harder!**



Taking a Stroll around a Martian Crater



The High Resolution Imaging Science Experiment (HiRISE) camera on NASA's Mars Reconnaissance Orbiter acquired this image of the Opportunity rover on the southwest rim of "Santa Maria" crater on New Year's Eve 2010. Opportunity arrived at the western edge of Santa Maria crater in mid-December and will spend about two months investigating rocks there. That investigation will take Opportunity into the beginning of its eighth year on Mars. Opportunity is imaging the crater interior to better understand the geometry of rock layers and the meteor impact process on Mars. Santa Maria is a relatively young, 90 meter-diameter impact crater, but old enough to have collected sand dunes in its interior.

Problem 1 - Using a millimeter ruler, what is the scale of this image in meters per millimeter to one significant figure?

Problem 2 - To one significant figure, about what is the circumference of the rim of this crater in meters?

Problem 3 - The rover can travel about 100 meters in one day. To one significant figure, how long will it take the rover to travel once around this crater?

Problem 4 - A comfortable walking speed is about 100 meters per minute. To one significant figure, how long would it take a human to stroll around the edge of this crater?

Problem 1 - Using a millimeter ruler, what is the scale of this image in meters per millimeter to one significant figure?

Answer: The shape of the crater is irregular, but taking the average of several diameter measurements with a ruler gives a diameter of about 55 millimeters. Since this corresponds to 90 meters according to the text, the scale of this image is about 90 meters/55 mm = 1.8 meters/millimeter, which to one significant figure becomes **2 meters/millimeter.**

Problem 2 - To one significant figure, about what is the circumference of the rim of this crater in meters?

Answer: Students may use a piece of string and obtain an answer of about 200 millimeters. Using the scale of the image in Problem 1, the distance in meters is about 200 mm x 2 meters/mm = **400 meters**.

Problem 3 - The rover can travel about 100 meters in one day. To one significant figure, how long will it take the rover to travel once around this crater?

Answer: 400 meters x (1 day/100 meters) = **4 days**.

Problem 4 - A comfortable walking speed is about 100 meters per minute. To one significant figure, how long would it take a human to stroll around the edge of this crater?

Answer: 400 meters x (1 minute/100 meters) = **4 minutes**.

Yes…It moves!!!

This is an image of a starfield in the constellation Centaurus taken by the Hubble Space Telescope in 1994. In addition to the bright stars, the streak of a single asteroid can also be seen. The Hubble has 'accidentally' detected over 100 asteroids as its cameras have been looking at other targets. Many of the asteroids are new discoveries. The curvature of the asteroid's trail as it moved across the sky was caused by parallax changes as the telescope orbited Earth during the 40-minute exposures. The field is 2.7 arcminutes on a side, and the distance to the asteroid was estimated to be 140 million kilometers from Earth. Based on the faintness of the asteroid at this distance, it was probably only 2 kilometers across!

Problem 1 - At the distance of the asteroid, this field would measure about 110,000 kilometers across. How many kilometers did the asteroid travel during the time of the exposure?

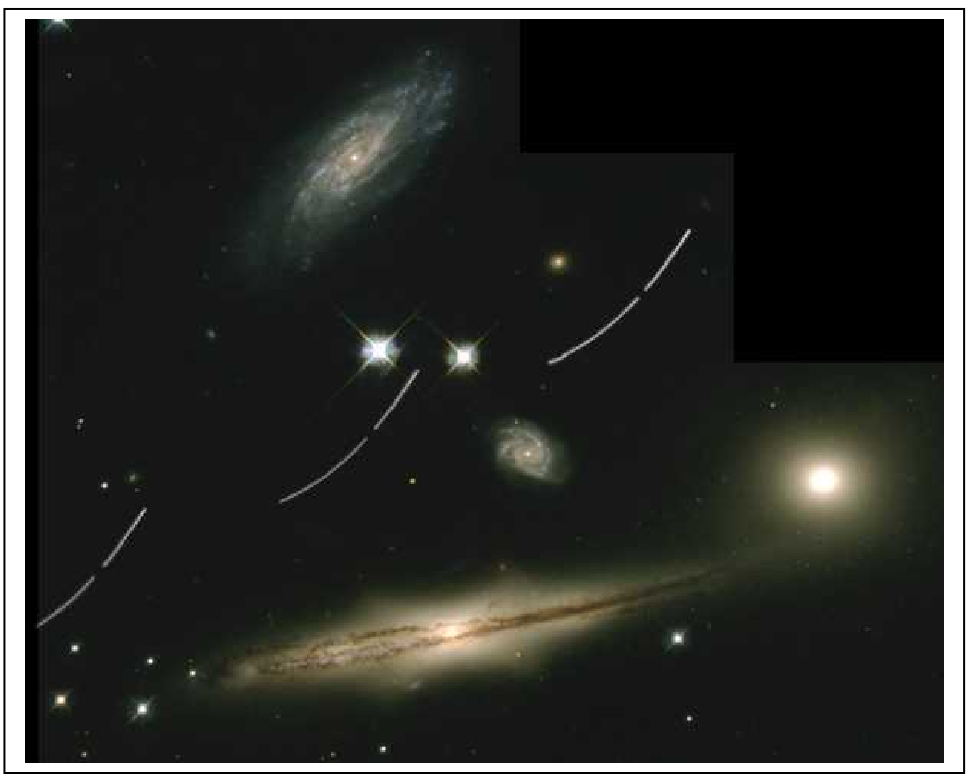
Problem 2 - What was the approximate speed of the asteroid in kilometers/hour from the beginning to the end of the trail?

Answer Key

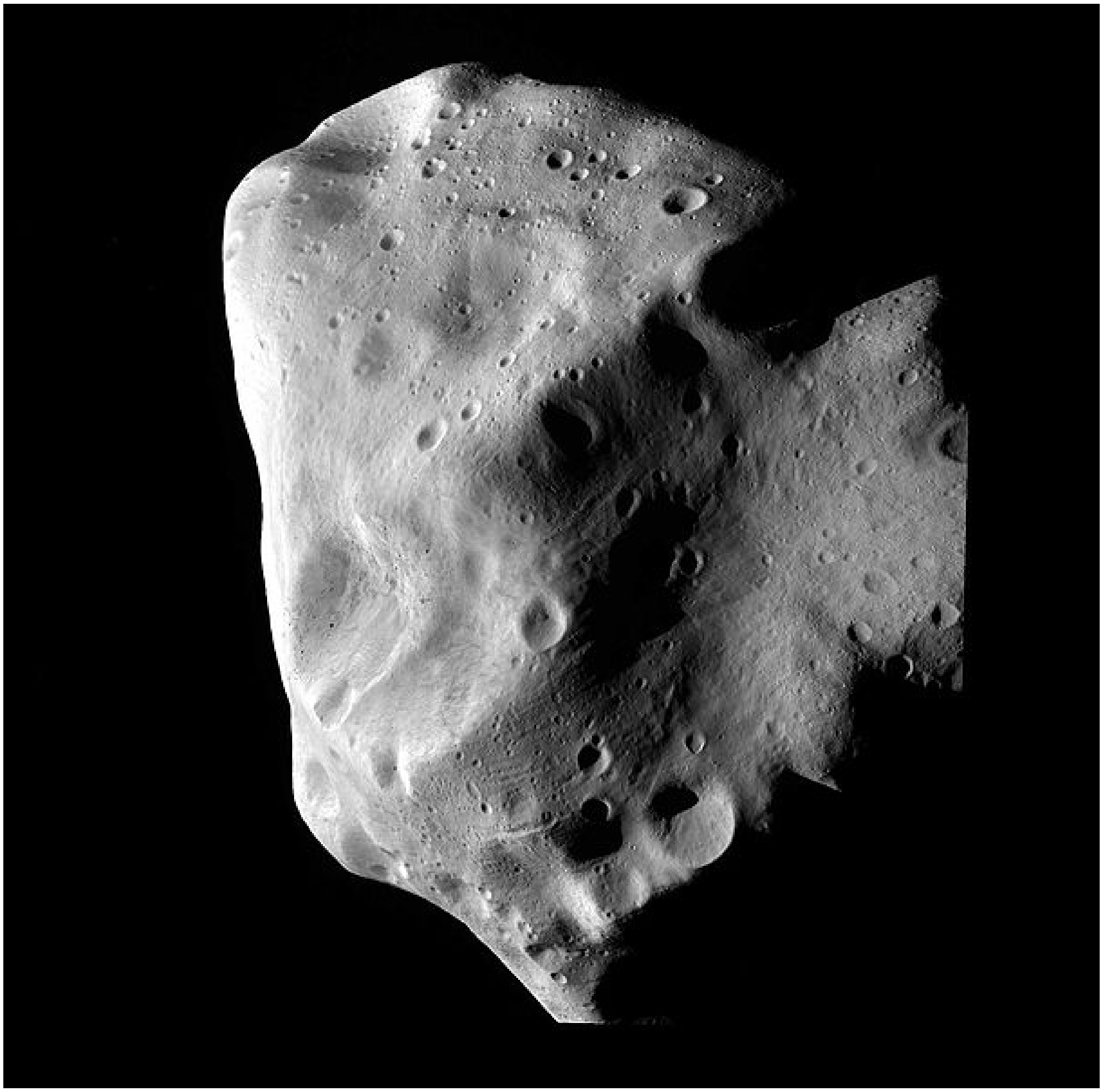
Problem 1 - At the distance of the asteroid, this field would measure about 110,000 kilometers across. How many kilometers did the asteroid travel during the time of the exposure?
Answer: Students will have to convert the length of the streak into kilometers using the scale of the image. Use a millimeter ruler to determine the scale of the image by first measuring the width of the image to get 118 millimeters. This physical length is equal to 110,000 kilometers, so the image scale is just 110,000 km/ 118 millimeters = 932 km/mm. The length of the asteroid streak is 20 millimeters, so its length is 20 x (932 km/mm) = 18,640 kilometers.

Problem 2 - What was the approximate speed of the asteroid in kilometers/hour from the beginning to the end of the trail?
Answer: The paragraph says that the exposure took 40 minutes, so during that time the asteroid moved the distance indicated in Problem 1. The speed is then 18,640 Kilometers/0.66 hours = 28,200 km/hr.

The photograph below shows another asteroid streak across a picture taken of the Hickson Galaxy Group # 87 in the constellation Capricornus. The size of the field is 132,000 km at the distance of the asteroid, and the exposure was about 6.6 hours. During this time the asteroid traveled 166,000 kilometers for a speed of about 25,000 km/hr.



A Flyby of Asteroid Lutetia



Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

Problem 2 - At a distance of 3,162 km, A) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth (0.5 degrees), how much larger was Lutetia?

Problem 3 - If the spacecraft traveled at a speed of 15 km/s on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Image credit: ESA 2010 MPS for OSIRIS Team MPS/UPD/LAM/IAA/RSSD/INTA/UPM/DASP/IDA. European Space Agency's Rosetta spacecraft, with NASA instruments aboard, flew past asteroid Lutetia on Saturday, July 10, 2010. Asteroid diameter about 130 km. This view is from a distance of 3,162 Km. The probe spent several hours shooting images of the irregular shaped space rock, circling more than 450 million km (280 million miles) out from the sun. The space agency says its OSIRIS camera was able to capture detail down to just a few dozen meters.

Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

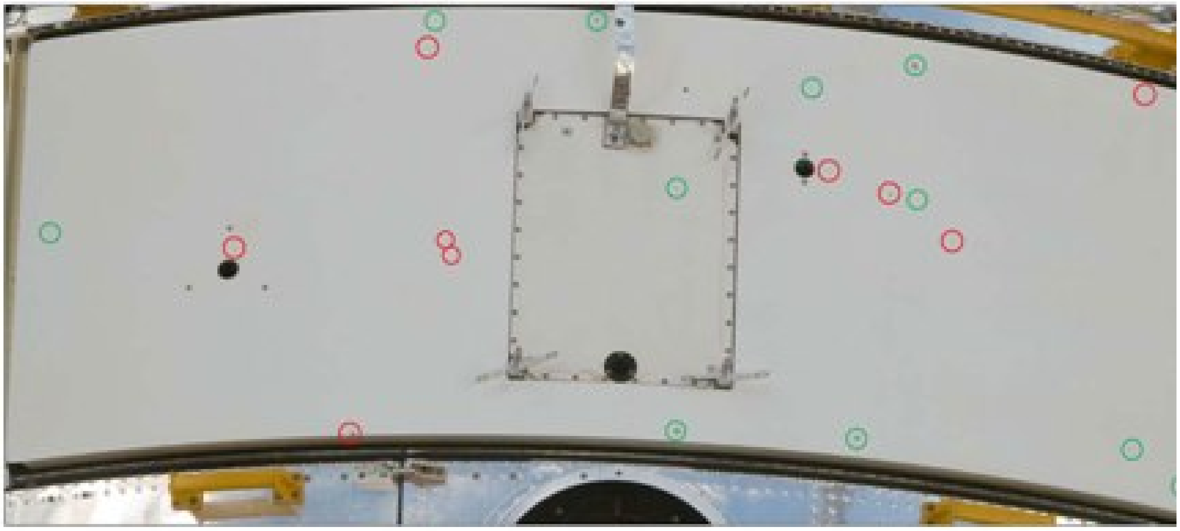
Answer: 130 km / 152 mm = **855 meters/mm**.

Problem 2 - At a distance of 3,162 km, A) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth (0.5 degrees), how much larger was Lutetia?

Answer: A) Tan(θ) = 130/3162 so θ = **2.3 degrees**.
B) Asteroid is 2.3/0.5 = **4.6 times** the diameter of the full moon.

Problem 3 - If the spacecraft traveled at a speed of 15 km/s on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Answer: To have an angular diameter of 0.5 degrees ,the spacecraft has to be 4.6 times farther away than at closest approach, or a distance of 4.6 x 3,162 = 14,545 km. From the Pythagorean theorem, the distance from the closest approach point is just d = $(14545^2 - 3162^2)^{1/2}$ = 14,197 km. To travel this distance takes T = 14197/15 = **946 seconds or 15.8 minutes.**



The STS-125 Atlantis astronauts retrieved the Hubble Space Telescope Wide-field Planetary Camera 2 (WFPC2) during a very successful and final servicing mission in May 2009. The radiator (above photo) attached to WFPC2 has dimensions of 2.2 meters by 0.8 meters. Its outermost layer is a 4-mm-thick aluminum, curved plate coated with white thermal paint. This radiator has been exposed to space since the deployment of WFPC2 in 1993. During this time, it received numerous impacts by micrometeoroids and man-made particles (flecks of paint, etc). The circles drawn on the radiator plate show the locations of these impacts.

Problem 1 - What is the total surface area of the WFPC2 radiator plate in square meters?

Problem 2 - How many impacts were counted over this area?

Problem 3 - What is the surface density of impacts in units of impacts per square meter?

Problem 4 - How many years was this panel in space?

Problem 5 - What is the impact rate in units of impacts per meter^2 per year?

Problem 6 - The solar panels on the International Space Station have a total surface area of 1,632 meters^2. A) How many impacts per year will these solar panels experience? B) What will be the average time, in hours, between impact?

Problem 1 - What is the total surface area of the WFPC2 radiator plate in square meters?
Answer: 2.2 meter x 0.l8 meter = 1.8 meters^2.

Problem 2 - How many impacts were counted over this area?
Answer: There are 20 circular marks, or portions along the edge, on the panel.

Problem 3 - What is the surface density of impacts in units of impacts per square meter?
Answer: 20 impacts / 1.8 meters^2 = 11.1 impacts/meter^2

Problem 4 - How many years was this panel in space?
Answer: 2009 - 1993 = 16 years.

Problem 5 - What is the impact rate in units of impacts per meter^2 per year?
Answer: (11.1 impacts/meter^2) / (16 years) = 0.7 impacts/meter^2/year

Problem 6 - The solar panels on the International Space Station have a total surface area of 1,632 meters^2. A) How many impacts per year will these solar panels experience? B) What will be the average time, in hours, between impact?
Answer: A) 1,632 meters^2 x (0.7 impacts/meter^2/year) = 1,142 impacts/year.
B) There are 1,142 impacts in a year,
so since there are 365 days/year x 24 hours/day = 8760 hours/year,
there will be about 8,760 hours/year x 1 year/ 1,142 impacts
or **7.7 hours between impacts on the average**.

The images below show some close-up images of impact craters on the Hubble Space Telescope as viewed from the Space Shuttle using a telephoto lens.

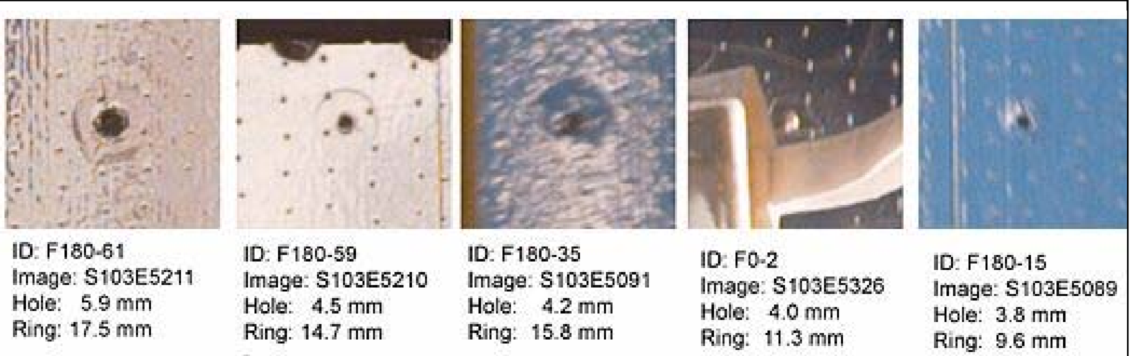
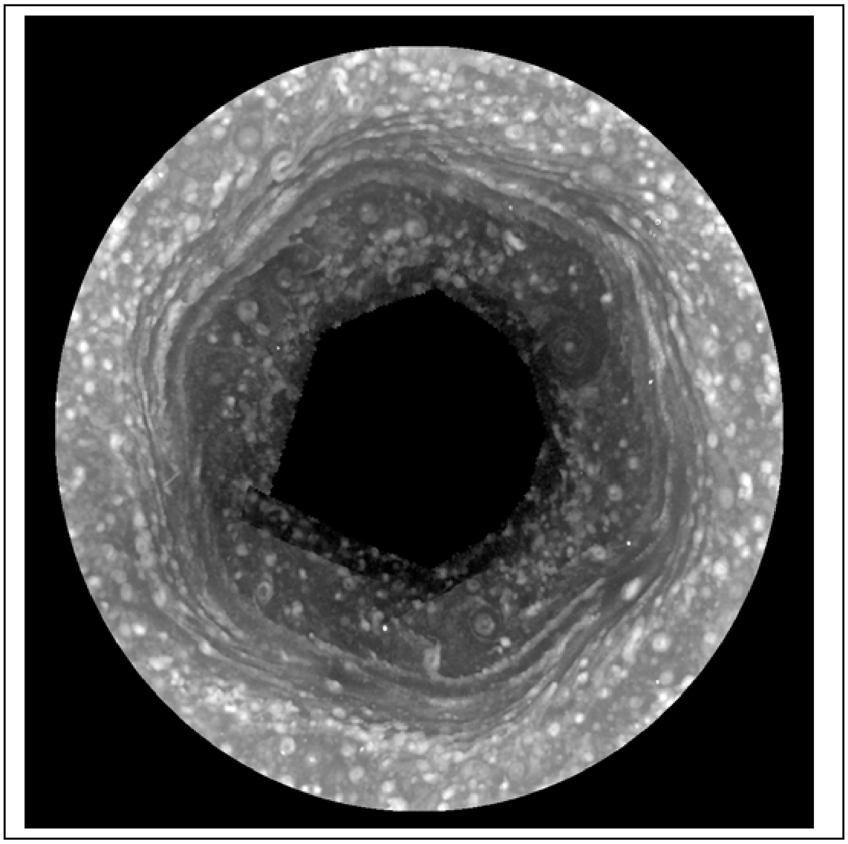


Figure 1. Five HST impact sites photographed with 400 mm lens.

The Mysterious Hexagon on Saturn!



The Cassini spacecraft recently took this high-resolution image of Saturn's north-polar region, which was observed by the Voyager 1 and 2 spacecraft in the early-1980s to have a remarkable hexagonal jet stream! The white spots are individual clouds, much like fair-weather cumulus clouds on Earth. The interior distance between opposite vertices of the hexagon is 25,000 kilometers, and the estimated speed of the winds along the walls of the hexagon is about 100 meters/sec. Use a millimeter ruler, or your knowledge of hexagons, to answer these questions:

Problem 1 - How long does it take the jet stream to make one complete circuit of the hexagon; A) In hours?, B) In days?

Problem 2 - The Earth has a radius of 6,378 kilometers, from a scaled drawing of the hexagon and Earth, how many Earths can fit inside the hexagonal area?

Problem 3 - Acceleration is a measure of the change in the speed and/or direction of motion per unit time interval. A) From the hexagon figure, how much time, T, elapsed in seconds as the velocity of the jet stream completely changed direction as it crossed a vertex region? B) If the total velocity change, V, passing across one vertex is about 173 meters/sec, what was the average acceleration of the jet stream defined as $a = V/T$ in meters/sec^2?

Problem 1 - How long does it take the jet stream to make one complete circuit of the hexagon; A) In hours?, B) In days?

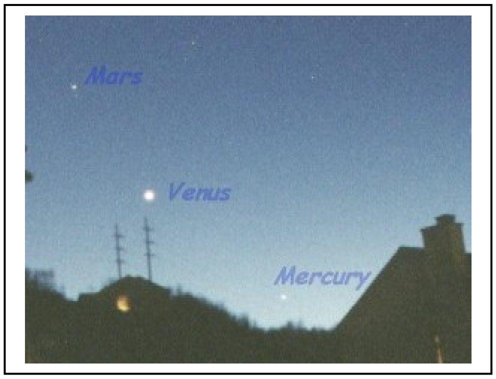
Answer: The interior distance between opposite vertices is 67 mm, which corresponds to 25,000 kilometers, so the scale is 25,000 km/67 mm = 373 km/mm. The length of one side is 35 millimeters (or 13,000 km) so the full perimeter is 6 x 35 mm = 210 mm. Converting this to meters we get 210 mm x (373 km/mm) = 78,300 kilometers or 7.83×10^7 meters. The time taken is T = D/V so for D = 7.83×10^7 meters and V = 100 meters/sec we get T = 783,000 seconds. A) **217 hours** and B) **9 days**.

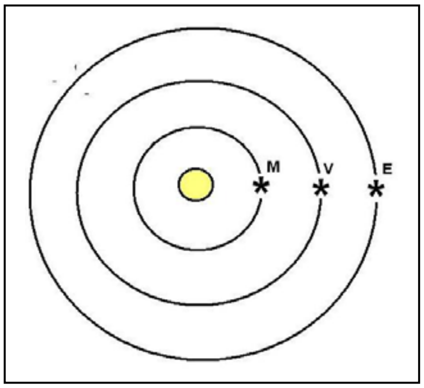
Problem 2 - The Earth has a radius of 6,378 kilometers, from a scaled drawing of the hexagon and Earth, how many Earths can fit inside the hexagonal area?
Answer: The earth circle will have a diameter of 2 x 6,378/373 = 34 millimeters. That will be about **2 Earth disks** spanning the inside of the hexagon. The area of a hexagon is given by $A = 3(3)^{1/2}/2\ L^2$ so the Saturn hexagon has an area of A = 3 (1.732)x(0.5) x $(13{,}000\ \text{km})^2$ = 440,000,000 km^2. The surface area of Earth is $A = 4\ \pi\ R^2$ = 510,000,000 km^2, so the hexagon has nearly the same surface area as all of earth.

Problem 3 - Acceleration is a measure of the change in the speed and/or direction of motion per unit time interval. A) From the hexagon figure, how much time, T, elapsed in seconds as the velocity of the jet stream completely changed direction as it crossed a vertex region? B) If the total velocity change, V, passing across one vertex is about 173 meters/sec, what was the average acceleration of the jet stream defined as a = V/ T in meters/sec^2? Answer: A) The shape of the clouds near a vertex suggest that the turn a distance of about 8 millimeters in going from the direction along one face to the other .The physical distance is 8 mm x (373 km/mm) = 3,000 kilometers or 3 million meters. The speed is 100 meters/sec, so the time taken is 3,000,000 m/(100m/s) = **30,000 seconds**. B) The velocity change during this time was 173 meters/sec, so the acceleration A = V/T = (173 meters/sec) / (30,000 sec) = **0.006 meters/sec^2**.
Note, the acceleration of gravity at Earth's surface is 9.8 meters/sec^2, so the winds feel an acceleration of about 0.0006 Gs.

Note: From a study of vectors, the velocity difference between a flow along one face Vf, and the adjacent face is a vector 'turn' of 60 degrees, so the difference vector, Vd, has a magnitude of $Vd^2 = 2(Vf)^2 + 2(Vf)^2\cos(60)$ so for Vf = 100 m/sec, Vd = 173 meters/sec, which is the value used.

Planetary Alignments





One of the most interesting things to see in the night sky is two or more planets coming close together in the sky. Astronomers call this a conjunction. The picture to the left shows a conjunction involving Mercury, Venus and Mars on June 24, 2005.

As seen from their orbits, another kind of conjunction is called an 'alignment' which is shown in the figure to the lower left and involved Mercury, M, Venus, V, and Earth, E. As viewed from Earth's sky, Venus and Mercury would be very close to the Sun, and may even be seen as black disks 'transiting' the disk of the Sun at the same time, if this alignment were exact. How often do alignments happen?

Earth takes 365 days to travel one complete orbit, while Mercury takes 88 days and Venus takes 224 days, so the time between alignments will require each planet to make a whole number of orbits around the Sun and return to the pattern you see in the figure.

Suppose Mercury takes 1/4 earth-year and Venus takes 2/3 of an earth-year to make their complete orbits around the Sun. You can find the next line-up from two methods:

Method 1: Work out the three number series like this:
Earth = 0, 1, **2**, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,…
Mercury = 0, 1/4, 2/4, 3/4 ,4/4, 5/4, 6/4, 7/4, **8/4**, 9/4, 10/4, 11/4, 12/4, 13/4, …
Venus = 0, 2/3, 4/3, **6/3**, 8/3, 10/3, 12/3, 14/3, 16/3, 18/3, 20/3, …
Notice that the first time they all coincide with the same number is at **2 years**. So Mercury has to go around the Sun 8 times, Venus 3 times and Earth 2 times for them to line up again in their orbits.

Method 2: We need to find the Least Common Multiple (LCM) of 1/4, 2/3 and 1. First render the periods in multiples of a common time unit of 1/12, then the sequences are:
Mercury = 0, 3, 6, 9, 12, 15, 18, 21, **24**,
Venus = 0, 8, 16, **24**, 32, 40, …
Earth, 0, 12, **24**, 36, 48, 60, …

The LCM is 24 which can be found from prime factorization:
Mercury: 3 = 3
Venus: 8 = 2 x 2 x 2
Earth: 12 = 2 x 2 x 3
The LCM the product of the highest powers of each prime number or 3 x 2 x 2 x 2 = 24. and so it will take 24/12 = **2 years.**

Problem 1 - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 earth-years and Venus takes 26/42 earth-years. After how many earth-years will the alignment reoccur?

Problem 1 - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 earth-years and Venus takes 26/42 earth-years. After how many earth-years will the alignment reoccur?

Mercury = 7/30 x 365 = 85 days vs actual 88 days
Venus = 26/42 x 365 = 226 days vs actual 224 days
Earth = 1

The common denominator is 42 x 30 = 1,260 so the series periods are
Mercury = 7 x 42 = 294 so 7/30 = 294/1260
Venus = 26 x 30 = 780 so 26/42 = 780/1260
Earth = 1260 so 1 = 1260/1260

The prime factorizations of these three numbers are

294 = 2 x 2 x 3 x 7 x 7
780 = 2 x 2x 5 x 3 x 13
1260 = 2 x 2 x 3 x 3 x 5 x 7

LCM = 2 x 2 x 3 x 3 x 5 x 7 x 7 x 13 = 114,660

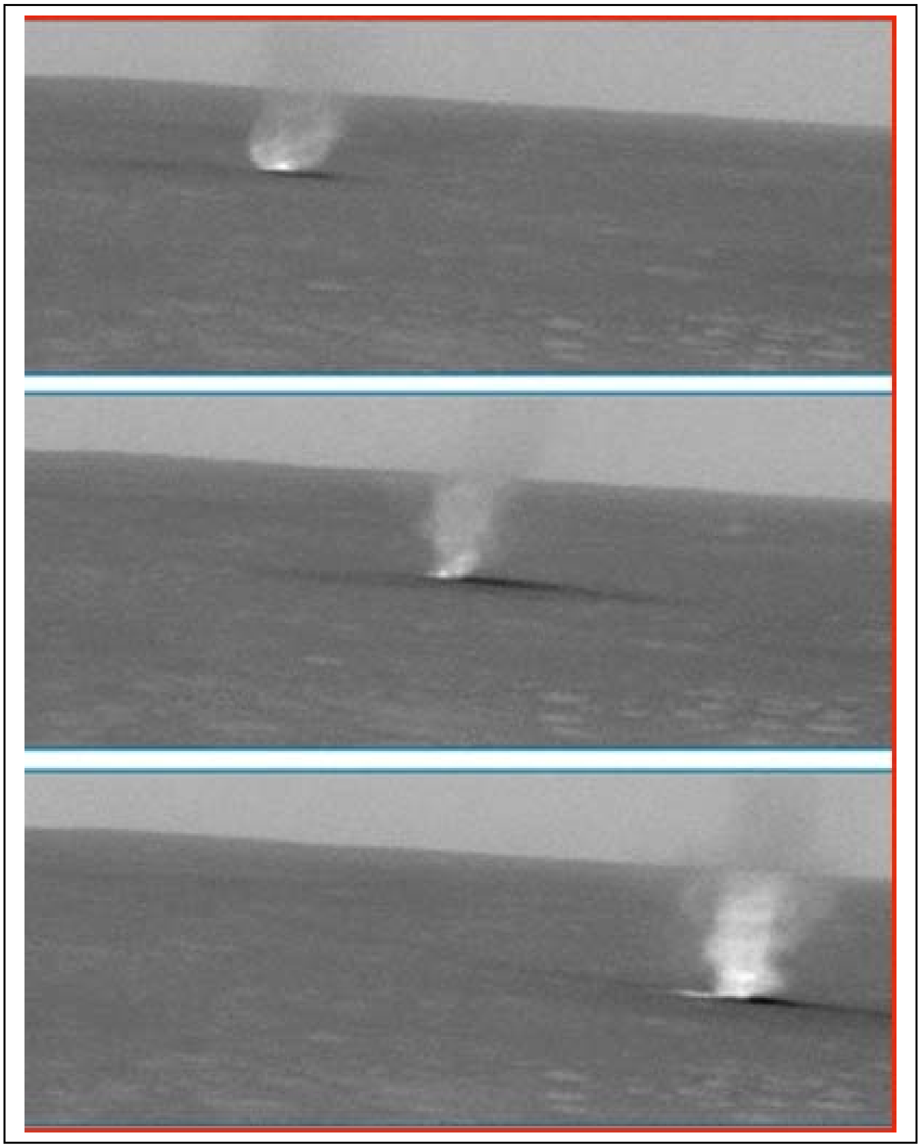
So the time will be 114,660 / 1260 = 91 years! In this time, Mercury will have made exactly 114,660/294 = 390 orbits and Venus will have made 114,660/780 = 147 orbits

Note to Teacher: Why did the example problem give only 2 years while this problem gave 91 years for the 'same' alignment? Because we used a more accurate approximation for the orbit periods of the three planets. Mercury actual period = 88 days but 1/4 earth-year = 91.25 days compared to 7/30 earth year = 85 days. Venus actual period = 224 days but 2/3 earth-year = 243 days and 26/42 earth-year = 226 days.

This means that after 2 years and exactly 8 orbits (8 x 91.25 = 730 days), Mercury will be at 8/4 x 365 = 730 days while the actual 88-day orbit will be at 88 x 8 = 704 days or a timing error of 26 days. Mercury still has to travel another 26 days in its orbit to reach the alignment position. For Venus, its predicted orbit period is 2/3 x 365 = 243.3 days so its 3 orbits in the two years would equal 3 x 243.3 days = 730 days, however its actual period is 224 days so in 3 orbits it accumulates 3 x 224 = 672 days and the difference is 730-672 = 58 days so it has to travel another 58 days to reach the alignment. In other words, the actual positions of Mercury and Venus in their orbits is far from the 'straight line' we were hoping to see after exactly 2 years, using the approximate periods of 1/4 and 2/3 earth-years!

With the more accurate period estimate of 7/30 earth-years (85 days) for Mercury and 26/42 earth-years (226 days) for Venus, after 91 years, Mercury will have orbited exactly 91 x 365 days/88 days = 377.44 times, and Venus will have orbited 91x365/224 = 148.28 times. This means that Mercury will be 0.44 x 88d = 38.7 days ahead of its predicted alignment location, and Venus will be 0.28 x 224 = 62.7 days behind its expected alignment location. Comparing the two predictions, Prediction 1: Mercury= - 26 days, Venus= - 58 days; Prediction 2: Mercury = +26 days and Venus = - 22 days. Our prediction for Venus has significantly improved while for Mercury our error has remained about the same in absolute magnitude. In the sky, the two planets will appear closer together for Prediction 2 in 1911 years than for Prediction 1 in 2 years. If we want an even 'tighter' alignment, we have to make the fractions for the orbit periods much closer to the actual periods of 88 and 224 days.

The Martian Dust Devils



A dust devil spins across the surface of Gusev Crater just before noon on Mars.

NASA's Spirit rover took the series of images with its navigation camera on the rover's martian day, or sol, 486 (March 15, 2005).

The images were taken at:

11:48:00 (T=top)
11:49:00 (M=middle)
11:49:40 (B=bottom)

based upon local Mars time.

The dust devil was about 1.0 kilometer from the rover at the start of the sequence of images on the slopes of the "Columbia Hills."

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top (T) and bottom (B) frames?

Problem 2 - What was the time difference, in seconds, between the images T-M, M-B and T-B?

Problem 3 - What was the distance, in meters, traveled between the images T-M and M-B?

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B?

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B?

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B?

Answer Key

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top and bottom frames? Answer: The location of the dust devil in frame B when placed in image T is a shift of about 65 millimeters, which at a scale of 7.4 meters/mm equals about 480 meters.

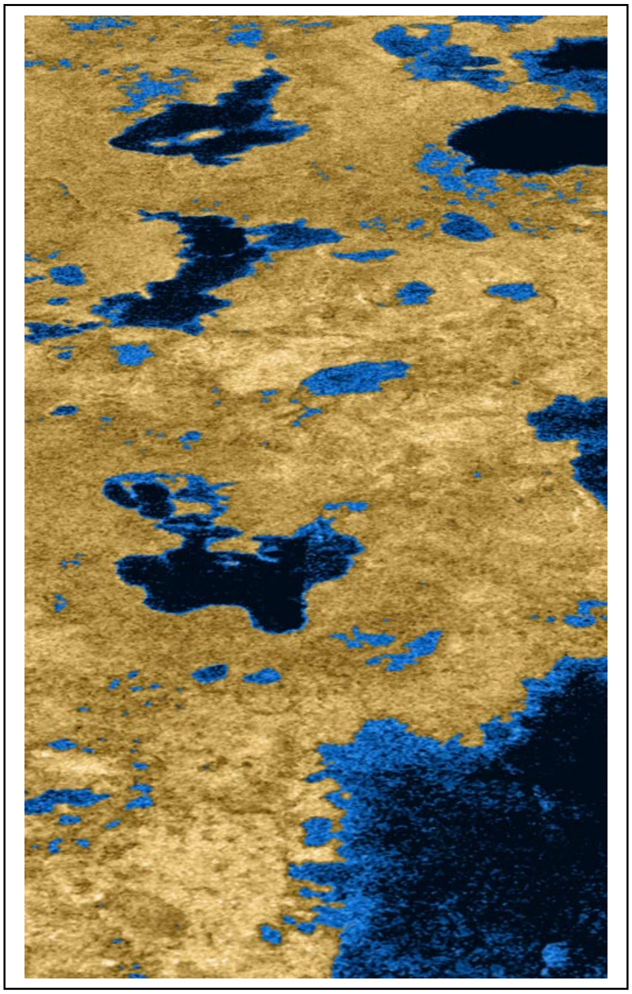
Problem 2 - What was the time difference between the images T-M, M-B and T-B? Answer: T-M = 11:49:00 - 11:48:00 - 1 minute or 60 seconds. For M-B the time interval is 40 seconds. For T-B the time interval is 100 seconds.

Problem 3 - What was the distance traveled between the images T-M and M-B? Answer: T-M = about 30 mm or 222 meters; M-B = about 35 mm or 259 meters.

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B? Answer: Speed = distance/time so 480 meters/100 seconds = 4.8 meters/sec.

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B? Answer: Speed(T-M) = 222 meters/60 seconds = 3.7 meters/sec. Speed(M-B) = 259 meters/40 seconds = 6.5 meters/sec.

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B? Answer: because the speed increased from 4.8 meters/sec to 6.5 meters/sec, the dust devil was accelerating during this time interval from 11:48:00 to 11:49:40.



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Answer: 150 km / 77 millimeters = **1.9 km/mm**.

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Answer: Combining the areas over the rectangular field of view gives about 1/4 of the area covered. The field of view measures 77 mm x 130mm or 150 km x 247 km or an area of 37,000 km^2. The dark areas therefore cover about 1/4 x 37,000 km^2 or **9,300 km^2.**

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Answer: Volume = area x height = 9,300 km^2 x (0.02 km) = **190 km^3**.

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?
.
Answer: 190 km^3 / 150 km^3 = **1.3 Lake Tahoes**.

Avalanche on Mars!

This image was taken by NASA's Mars Reconnaissance Orbiter on February 19, 2008. It shows an avalanche photographed as it happened on a cliff on the edge of the dome of layered deposits centered on Mars' North Pole. From top to bottom this impressive cliff is over 700 meters (2300 feet) tall and reaches slopes over 60 degrees. The top part of the scarp, to the left of the image, is still covered with bright (white) carbon dioxide frost which is disappearing from the polar regions as spring progresses. The upper mid-toned (pinkish-brownish) section is composed of layers that are mostly ice with varying amounts of dust. The dust cloud extends 190 meters from the base of the cliff.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?
Question 2: What is the smallest detail you can see in the ice shelf?
Question 3: What is the average thickness of the red layers on the cliff?
Question 4: What is the total width of the reddish rock cliff?

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?
Answer: 60 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 190 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 190 meters / 60 mm = 3.2 meters / mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

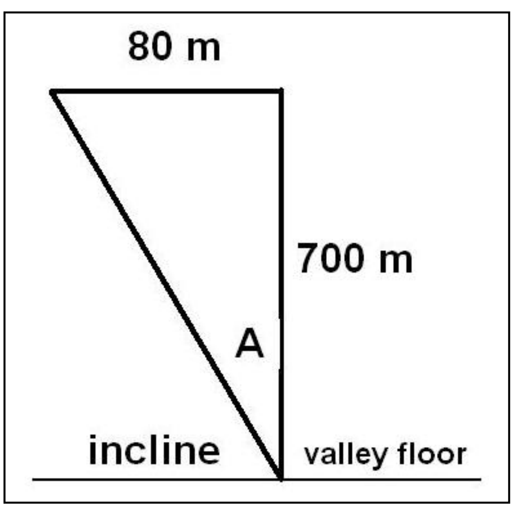
Question 1: What are the dimensions, in meters, of this image?
Answer: 140 mm x 86 mm = 448.0 meters x 275.2 meters, but to two significant figures this becomes 450 meters x 280 meters.

Question 2: What is the smallest detail you can see in the ice shelf?
Answer: 0.2 mm = 0.6 meters

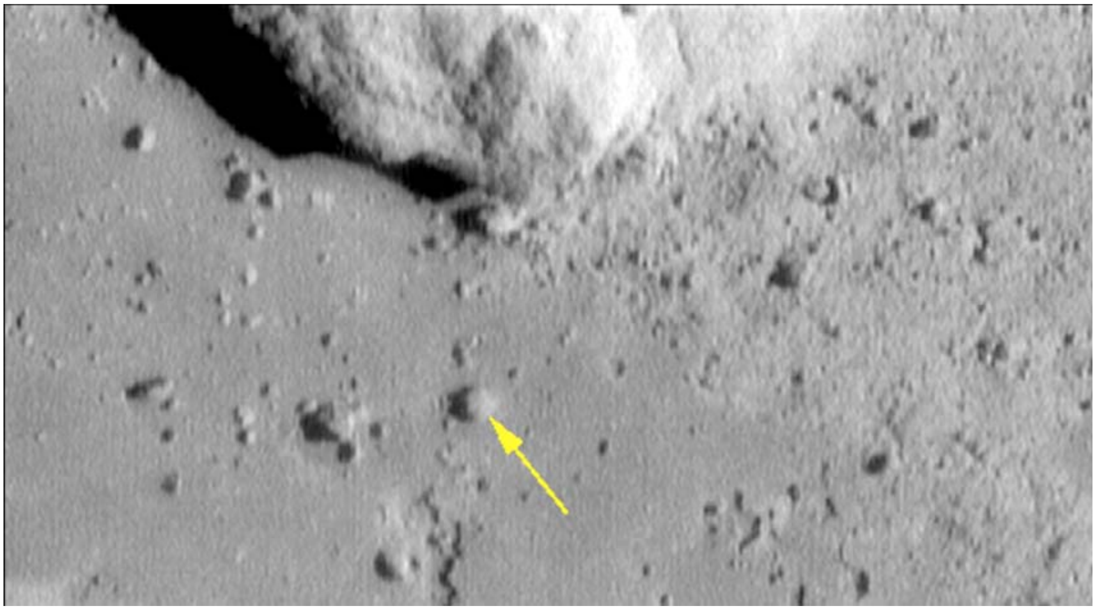
Question 3: What is the average thickness of the red layers on the cliff?
Answer: 1.0 millimeter = 3.2 meters.

Question 4: What is the total width of the reddish rock cliff?
Answer: 25 millimeters = 80 meters.

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?
Answer: Draw the triangle with the 700-meter side being vertical and the 80 meter side being horizontal. The tangent of the angle is (80 / 700) = 0.11 so the angle is 6.5 degrees. This is the angle from the vertical, so the incline angle from the floor of the valley is 90 - 6.5 = 84 degrees. This is a nearly vertical wall!



Asteroid Eros



This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6.0 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?

Question 2: What is the width, in centimeters, of the largest feature?

Question 3: What is the size of the smallest feature you can see?

Question 4: How big is the stone shown by the arrow?

Answer Key:

This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University)). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?
Answer: 144 millimeters

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 6.0 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter.
Answer: 6.0 meters / 144 mm = 600 cm / 144 millimeters = 4.2 cm/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters.

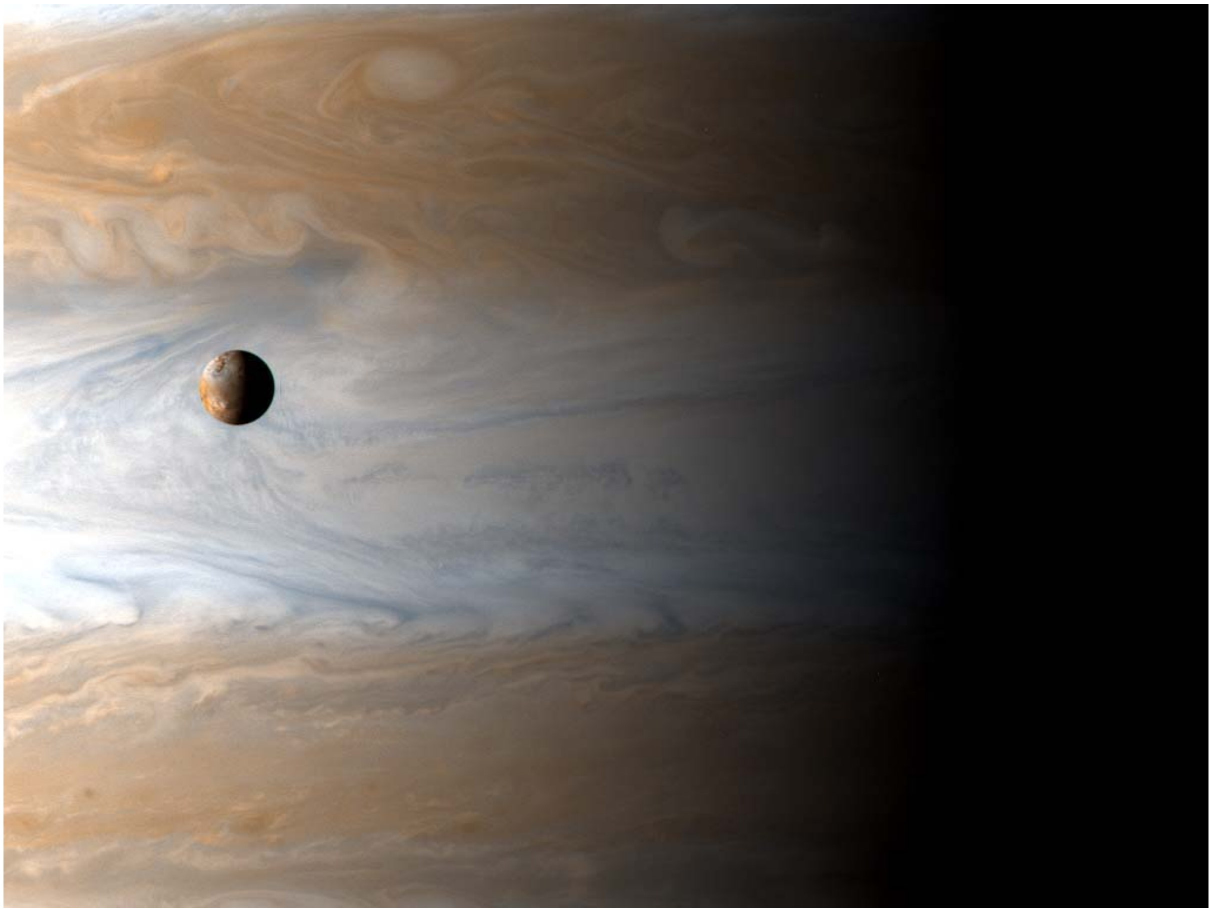
Question 1: What are the dimensions, in meters, of this image?
Answer: Height = 80 mm = 336 cm or 3.4 meters so area is 6.0 m x 3.4 m

Question 2: What is the width, in centimeters, of the largest feature?
Answer: The big rock at the top of the image is about 60 mm across or 2.5 meters.

Question 3: What is the size of the smallest feature you can see?
Answer: The small pebbles are about 0.5 millimeters across or 2.1 centimeters (about 1 inch).

Question 4: How big is the stone shown by the arrow?
Answer: 4 millimeters or 17 centimeters (about 7 inches).

Jupiter and Io



This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?
Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?
Question 4: What is the size of the smallest feature on Io that you can see?

Answer Key

This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?
Answer: 10 mm

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 3,600 km

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.
Answer: 3600 km / 10 mm = 360 km/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Answer: 160 mm x 119 mm = 58,000 km x 19,000 km

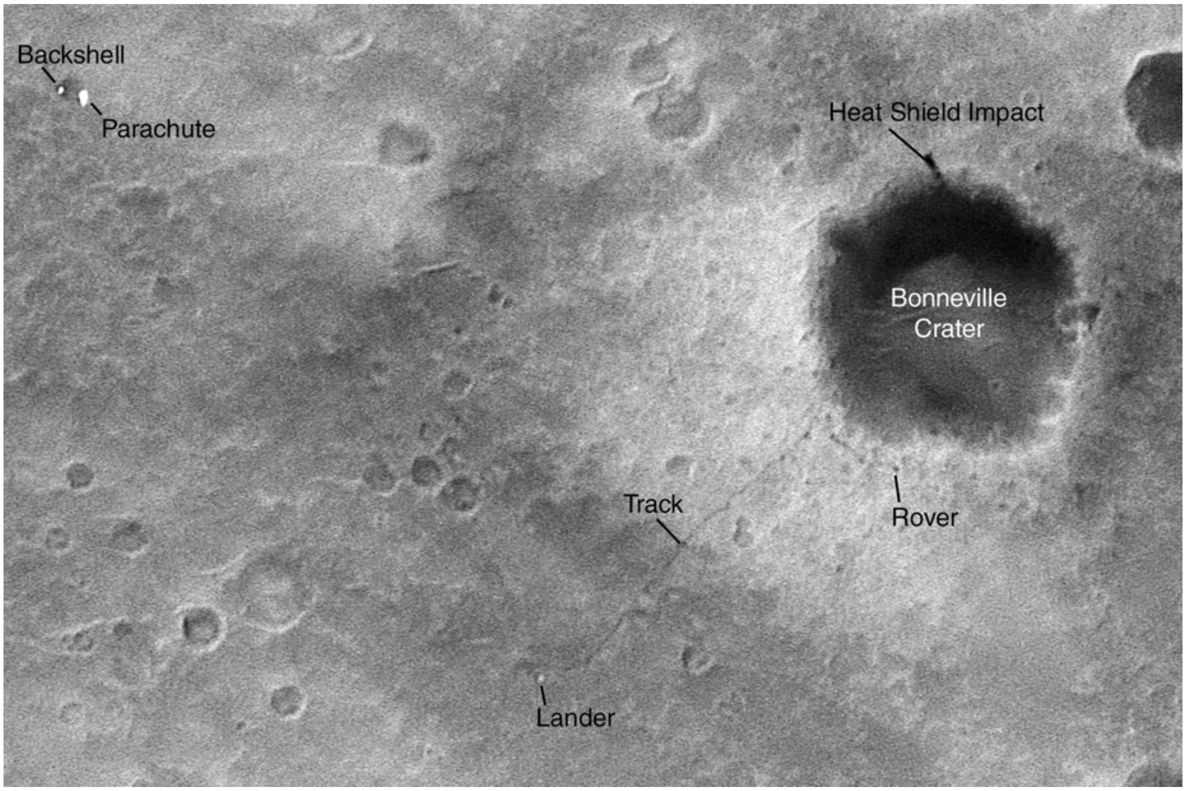
Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?
Answer: The width of the white equatorial band is 45 mm or 16,000 km

Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?
Answer: The faint cloud streaks are 0.5 mm wide or 200 km across to one significant figure.

Question 4: What is the size of the smallest feature on Io that you can see?
Answer: The white spots in the southern hemisphere are 0.5 mm across or 200 km to one significant figure. This is a good time to mention that some details in an image can be artifacts from the printing process or defects in the camera itself. Students may find photocopying artifacts at 0.5 mm or less.

Note to teachers: The correct scale for Io and Jupiter will be slightly different depending on how far away the camera was when taking the picture. If the camera was very close to Io, then the scale you will infer for Io will be very different than for the more distant Jupiter because Io will take up more of the field-of-view in the image. Geometrically, for a fixed angle of separation between features on Io, this angle will subtend a SMALLER number of kilometers than the same angle on the more-distant Jupiter. However, if the distance from the camera to Jupiter/Io is very large, then as seen from the camera, both objects are at essentially the same distance and so there will be little difference between the scales used for the two bodies. Students can check this result with an inquiry assignment.

Mars Rover Landing Site



This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the image is exactly 895 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?
Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.
Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Problem 1: About what is the diameter of Bonneville Crater rounded to the nearest ten meters?

Problem 2: How wide, in meters, is the track of the Rover?

Problem 3: How big is the Rover?

Problem 4: How small is the smallest well-defined crater to the nearest meter in size?

Problem 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

Answer Key:

This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the crater is 200 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?
Answer: 157 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.
Answer: 895 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 895 m/157 mm = 5.7 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

Problem 1: About what is the diameter of Bonneville Crater rounded to the nearest 10 meters?
Answer: Students answers for the diameter of the crater in millimeters may vary, but answers in the range from 30-40 mm are acceptable. Then this equals 30x5.7 = 170 meters to 40x5.7=230 meters. Students may average these two measurements to get (170+230)/2 = 200 meters.

Problem 2: How wide, in meters, is the track of the Rover?
Answer: 0.2 millimeters = 1 meter.

Problem 3: How big is the Rover?
Answer: 0.3 millimeters = 1.7 meters but since the measurement is only 1 significant figure, the answer should be 2 meters.

Problem 4: How small is the smallest well-defined crater in meters?
Answer: 2 millimeters x 5.7 = 11.4 meters, which to the nearest meter is 11 meters.

Problem 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?
Answer: Students answers may vary and lead to interesting discussions about what features are real, and which ones are flaws in the printing of the picture. This is an important discussion because 'image artifacts' are very common in space-related photographs. 5 meters is about 1 millimeter, and there are no obvious rounded objects this large or larger in this image.

Water on Mars!

NASA/JPL/Malin Space Science Systems

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Question 2: How wide, in meters, are the streams half-way down their flow channels?
Question 3: What is the smallest feature you can see in the image?
Question 4: How wide is the top of the crater wall at its sharpest edge?

Answer Key:

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?
Answer: 13 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.
Answer: 300 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 300 meters / 13 mm = 23 meters / millimeter.

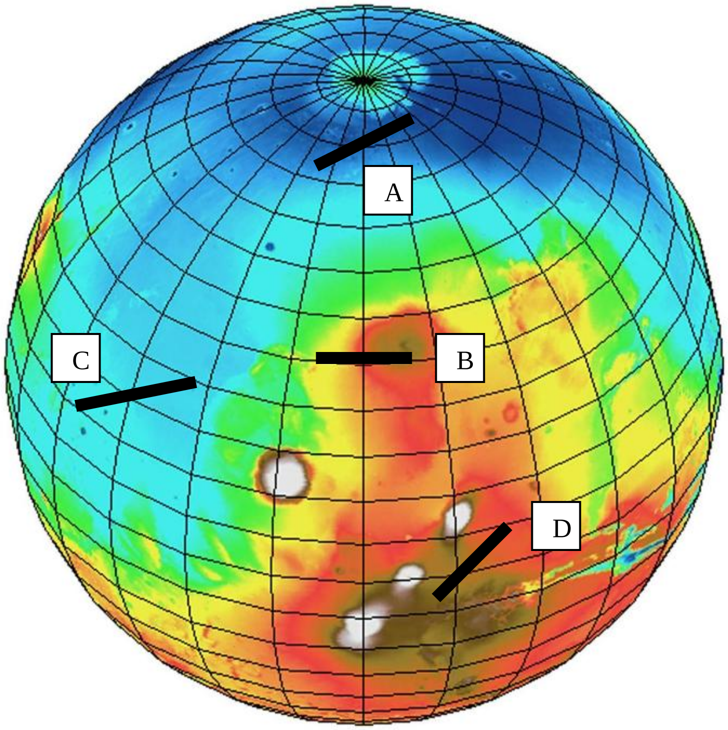
Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Answer: 134 mm x 120 mm = 3.1 km x 2.8 km

Question 2: How wide, in meters, are the streams half-way down their flow channels?
Answer: 0.5 millimeters = 12 meters.

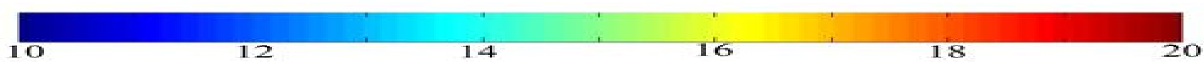
Question 3: What is the smallest feature you can see in the image?
Answer: Sand dunes in upper left of image = 0.3 millimeters or 7 meters wide.

Question 4: How wide is the top of the crater wall at its sharpest edge?
Answer: 0.2 millimeters or 4 meters wide.



Mars has virtually no atmosphere, and this means that, unlike Earth, its surface is not protected from solar and cosmic radiation. On Earth, the annual dosage on the ground is about 0.35 Rem/year, but can vary from 0.10 to 0.80 Rem/year depending on your geographic location, altitude, and lifestyle.

This figure, created with the NASA, MARIE instrument on the Odyssey spacecraft orbiting Mars, shows the unshielded surface radiation dosages, ranging from a maximum of 20 Rem/year (brown) to a minimum of 10 Rem/year (deep blue).



Astronauts landing on Mars will want to minimize their total radiation exposure during the 540 days they will stay on the surface. The Apollo astronauts used spacesuits that provided 0.15 gm/cm^2 of shielding. The Lunar Excursion Module provided 0.2 gm/cm^2 of shielding, and the orbiting Command Module provided 2.4 gm/cm^2. The reduction in radiation exposure for each of these was about 1/4, 1/10 and 1/50 respectively. Assume that the Mars astronauts used improved spacesuit technology providing a reduction of 1/8, and that the Mars Excursion Vehicle provided a 1/20 radiation reduction.

The line segments on the Mars radiation map represent some imaginary, 1,000 km exploration tracks that ambitious astronauts might attempt with fast-moving rovers, and not a lot of food! Imagine a schedule where they would travel 100 kilometers each day. Suppose they spend 20 hours a day within a shielded rover, and they study their surroundings in spacesuits for 4 hours each day.

1) Convert 10 Rem/year into milliRem/day.

2) What is the astronauts radiation dosage per day in a region (brown) where the ambient background produces 20 Rem/year?

3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the approximate total dosage to an astronaut in milliRems (mRems), given the exposure times and shielding information provided.

4) Which track has the highest total dosage in milliRems? The least total dosage? What is the annual dosage that is equivalent to these 20-day trips? How do these compare with the 350 milliRems they would receive if they remained on Earth?

Having a Hot Time on Mars!

1) Convert 10 Rem/year into milliRem/hour.

Answer: (10 Rem/yr) x (1 year/ 365 days) x (1 day/24 hr) = 1.1 milliRem/hour

2) What is the astronauts radiation dosage per day in a region (brown) where the background is 20 Rem/year?

Answer: From Problem 1, 20 Rem/year = 2.2 milliRem/hour.
20 hours x (1/20) x 1.1 milliRem/hr + 4 hours x (1/8) x 1.1 milliRem/hr = 1.1 + 0.55 = 1.65 milliRem/day

3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the total dosage in milliRems to an astronaut, given the exposure times and shielding information provided. The scaling relationship is that for each 20 Rems/year, the daily astronaut dosage is 0.66 milliRem/day (e.g. 0.66/20). The factor of 2 in the answers accounts for the round-trip.

Track A dosage:

2x(12 Rems/yr x 10 days x (1.65 / 20)) = 2x(9.9) = 19.8 mRem.

Track B dosage:

2x(16 Rems/yr x 3.3days + 18 Rems/yr x 3.3days + 20 Rems/yr x 3.3days)(1.65/20) = 2x(14.8) = 29.6 mRem

Track C dosage:

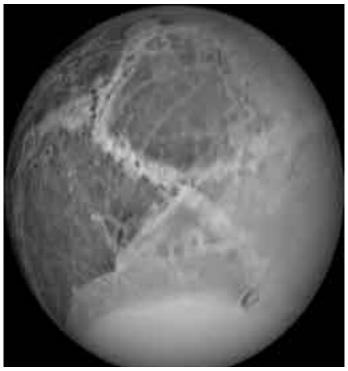
2x(12 Rems/yr x 5 days x (1.65/20) + 14 Rems/yr x 5 days x (1.65/20)) = 2x(5.0 + 5.8) = 21.6 mRem

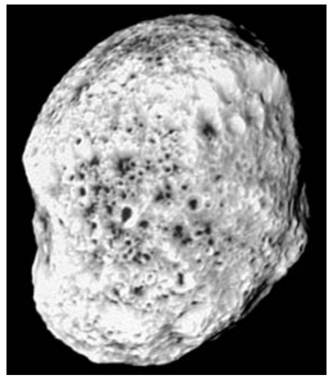
Track D dosage:

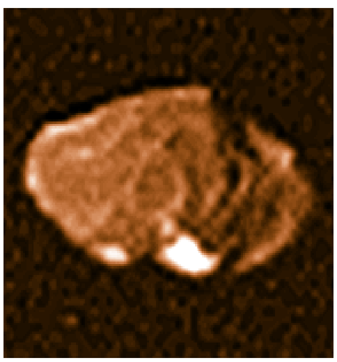
2x(18 Rems/yr x 5 days x (1.65/20) + 20 Rems/yr x 5 days x (1.65/20)) = 2x(7.5 + 8.25) = 31.5 mRem

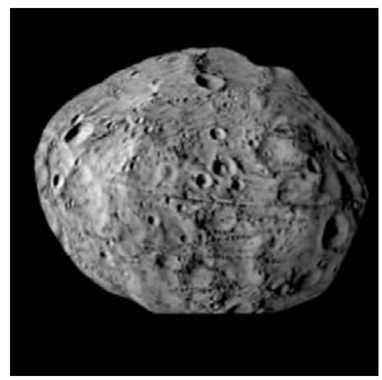
4) For this 20-day excursion, Track D has the highest dosage and Track A has the lowest. The equivalent annual dosage for the lowest-dosage track is 19.8 milliRem x 365 days/10 days = 722 milliRem, which is about twice the annual dosage they would receive if they remained on Earth. For the highest-dosage trip, the annualized dosage is 1,149 milliRems which is about 3 times the dosage on Earth.

Why are many astronomical bodies round? Here is an activity in which you use astronomical photographs of various solar system bodies, and determine how big a body has to be before it starts to look round. Can you figure out what it is that makes a body round?









The images show the shapes of various astronomical bodies, and their sizes: Dione (560 km), Hyperion (205 x 130 km), Tethys (530 km), Amalthea (130 x 85 km), Ida (56 x 24 km), Phobos (14 x 11 km).

Question 1) How would you define the roundness of a body?

Question 2) How would you use your definition of roundness to order these objects from less round to round?

Question 3) Can you create from your definition a number that represents the roundness of the object?

Question 4) On a plot, can you compare the number you defined in Question 3 with the average size of the body?

Question 5) Can you use your plot to estimate the minimum size that a body has to be in order for it to be noticeably round? Does it depend on whether the body is mostly made of ice, or mostly of rock?

The images show the shapes of various astronomical bodies, and their sizes: Dione (560 km), Hyperion (205 x 130 km), Tethys (530 km), Amalthea (130 x 85 km), Ida (56 x 24 km), Phobos (14 x 11 km).

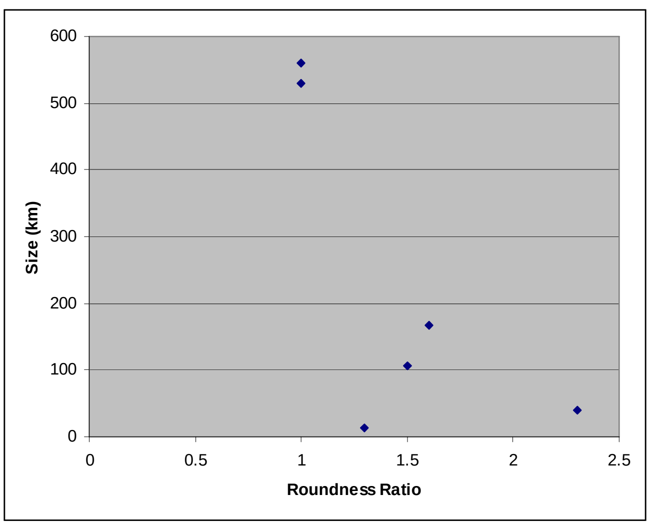
Question 1) How would you define the roundness of a body? **Answer: Students may explore such possibilities as the difference between the longest and shortest dimension of the object; the ratio of the longest to the shortest diameter; or other numerical possibilities.**

Question 2) How would you use your definition of roundness to order these objects from less round to round? **Answer. The order should look something like Ida, Amalthea, Hyperion, Phobos, Tethys and Dione.**

Question 3) Can you create from your definition a number that represents the roundness of the object? **Answer: If we select the ratio of the major to minor axis length, for example, we would get Dione = 1.0; Hyperion = 1.6; Tethys=1.0; Amalthea=1.5; Ida = 2.3 and Phobos = 1.3**

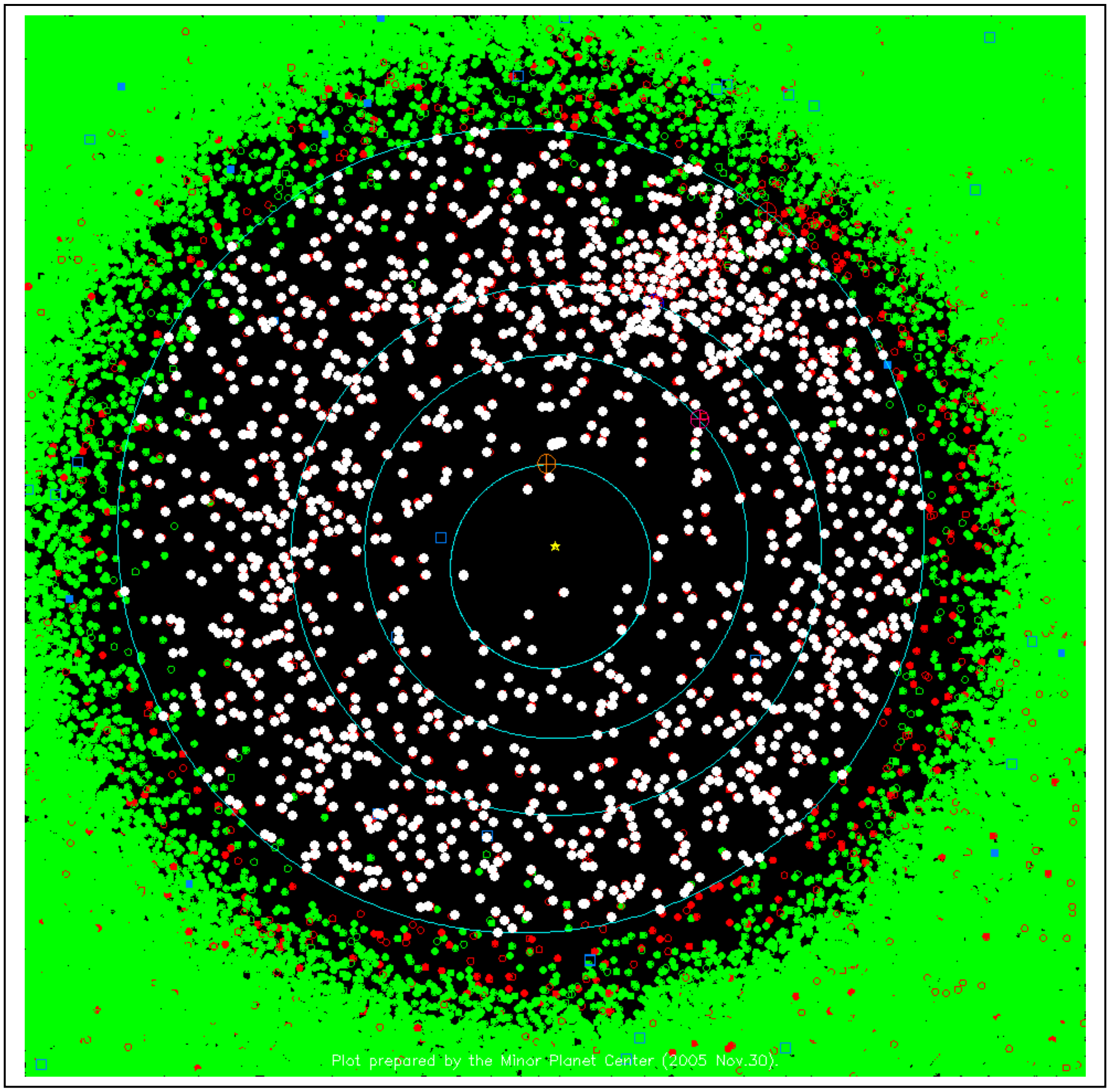
Question 4) On a plot, can you compare the number you defined in Question 3 with the average size of the body? **Answer: See plot below for the numerical definition selected in Question 3. We have used the average size of each irregular body defined as (L + S)/2. so Hyperion = (205+130)/2 = 167 km; Amalthea = (130+85)/2 = 107 km; Ida = (56+24)/2 = 40 km; Phobos = (14+11)/2 = 13 km.**

Question 5) Can you use your plot to estimate the minimum size that a body has to be in order for it to be noticeably round? Does it depend on whether the body is mostly made of ice, or mostly of rock? Answer: By connecting a smooth curve through the points (you can do this by eye), the data suggests that a body becomes noticeably round when it is at a size between 200-400 km. Students can obtain pictures of other bodies in the solar system and see if they can fill-in the plot better with small moons of the outer planets, asteroids (Ceres, etc) or even comet nuclei. Note, inner solar system bodies are mostly rock. Outer solar system bodies are mostly ice, so students might notice that by labeling the points as 'rocky bodies' or 'icy bodies' that they may see two different trends because ice is more pliable than rock. Students should investigate what the size (mass) has to do with roundness, and see that larger bodies have more gravity to deform their substance with.



Name ______________________________

Astronomers have catalogued and determined orbits for 30,000 minor planets in the solar system (asteroids, comets etc). Over 150,000 bodies larger than a few hundred meters across have been spotted and remain to have their orbits exactly determined. Below is a plot made on November 30, 2005 of the locations of all known objects (white dots) within the orbit of Mars whose path skirts the inner edge of the asteroid belt (green dots).



Question 1 – How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars?

Question 2 – If the radius of Earth's orbit is 150 million km, what is the scale of this figure in millions of km per millimeter?

Question 3 – About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel?

Question 4 – How many asteroids crossed Earth's orbit on November 30, 2005?

The plot of the minor planets was obtained from the IAU, Minor Planets Center (http://cfa-www.harvard.edu/iau/lists/InnerPlot2.html). It shows the location of the known asteroids, comets and other 'minor planets' for November 30, 2005. The plot shows the orbits of Mercury, Venus, Earth and Mars. Objects that have parahelias (closest orbit location to the sun) less than 1.3 AU are shown in white circles. More details can be found at http://www.space.com/scienceastronomy/solarsystem/asteroid_toomany_011019-1.html

Question 1 – How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars? **Answer:** Students should count the plotted symbols within (or on) the first inner ring (Mercury's orbit) and get **13** symbols (don't include the sun!). For the space between Venus and Mercury, I count 119 spots which makes the total **132** minor planets inside the orbit of Venus. Between Earth and Venus there are about 280 for a total of 412 minor planets inside Earth's orbit. Between Mars and Earth, a careful student may be able to count about 833 which means there are 833+412 = **1245** minor planets inside the orbit of Mars.

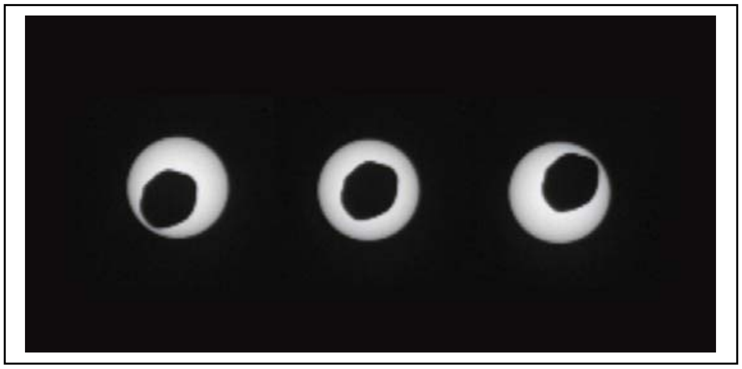
Question 2 – What is the scale of this figure in millions of km per millimeter? **Answer:** The radius of Earth's orbit is 150 million kilometers, which corresponds to 70 millimeters, so the scale is <u>2.1 million km per millimeter</u>.

Question 3 – About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel? **Answer:** Although the asteroids are only plotted as though they are located in the same 2-D plane, we can estimate from the average 'eyeball' separation between asteroids of about 2 millimeters, that they are about 4.2 million kilometers apart. A spacecraft would not collide with a typical asteroid unless it was directed to specifically target an asteroid for investigation…or impact. It is a popular myth about space travel that astronauts have to dodge asteroids when traveling to Mars or the outer solar system. Interplanetary dust grains and micro-meteoroids are, however, a much bigger hazard!!

Question 4 – How many asteroids crossed Earth's orbit on November 30, 2005? **Answer:** Just count the number of white spots that touch the line that represents the orbit of Earth. There are about 70 spots that touch the orbit line.

Could the Earth collide with them? Each dot is about 1 mm in radius, so this represents a distance of 2 million kilometers. Since Earth is only 12,000 km across and a typical asteroid is only 1 km across, collision is extremely unlikely even when the diagram seems to show otherwise. There is another way that this diagram makes the situation look worse than it is. Because the asteroid orbits can be several million miles above or below the orbit of Earth as the asteroids cross this location, there are very few close calls between Earth and any given asteroid in the current catalog. Astronomers call the ones that get close 'Near-Earth Asteroids' and there are about 700 of these known. Only **one** of these known NEAs may get close enough to Earth in the next 30 years to be a potential collision problem, but astronomers are still finding dozens more NEAs every year.

Transit of Phobos Across the Sun Viewed from Mars



This set of three images shows views three seconds apart as the larger of Mars' two moons, Phobos, passed directly in front of the sun as seen by NASA's Mars rover Curiosity.

Curiosity photographed this annular eclipse with the rover's Mast Camera on August 17, 2013 or 'Sol 369' by the Mars calendar.

Curiosity paused during its drive to Mount Sharp to take a set of observations that the camera team carefully calculated to record this celestial event. Because this eclipse occurred near mid-day at Curiosity's location on Mars, Phobos was nearly overhead. This timing made Phobos' silhouette larger against the sun -- as close to a total eclipse of the sun as is possible from Mars.

Angular size is given by

$$\Theta = 57.3 \times \frac{\text{Diameter (km)}}{\text{Distance (km)}} \text{ degrees}$$

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Annular Eclipse of the Sun by Phobos, as Seen by Curiosity
http://www.nasa.gov/mission_pages/msl/news/msl20130828.html
Aug. 28, 2013

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Answer: Phobos: 57.3 x (11/6000) = **0.10 degrees**
or 0.1 degrees x 60 = **6 minutes of arc**

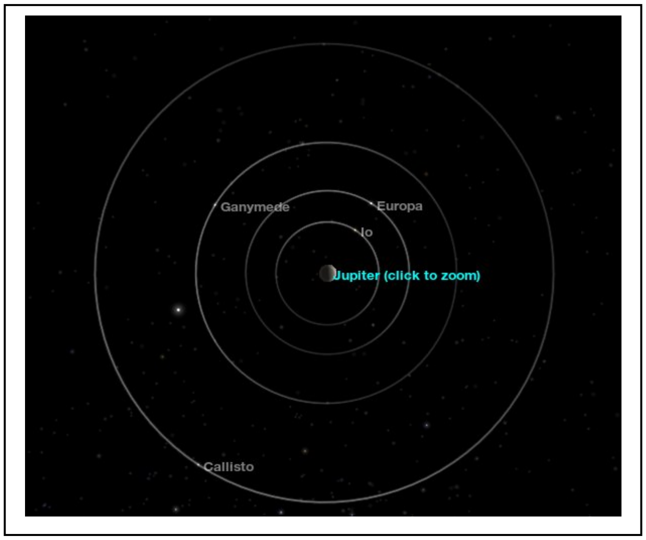
Sun: 57.3 x (1.4/235) = **0.34 degrees** or 0.34 x 60 = **20 minutes of arc**.

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Answer: If the object is located 6000 km from the surface and moves 2.1 km, it will appear to cover an angle of 57.3 x (2.1/6000) = 0.02 degrees. There are 60 arcminutes in 1 degree, so this angle is 1.2 arcminutes. Since this distance is traveled in 1 second, the angular speed is **1.2 arcminutes /second**.

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Answer: The diameter of the sun is 20 arcminutes and the diameter of Phobos is 6 arcminutes. When the center of the disk of Phobos is 3 arcminutes from the eastern edge of the sun, it is just touching the solar disk and about to start its transit. When it is 3 arcminutes from the western edge of the sun, it has just finished its transit, so the total distance it has to travel is 3 arcminutes + 20 arcminutes + 3 arcminutes or 26 arcminutes. It travels at a speed of 1.2 arcminutes per second, to it will cover 26 arcminutes in about 26/1.2 = **22 seconds**.



In the future, rovers will land on the moons of Jupiter just as they have on Mars. Rover cameras will search the skies for the disks of nearby moons. One candidate for landing is Europa with its ocean of water just below its icy crust.

The figure to the left shows the orbits of the four largest moons near Europa. How large will they appear in the European sky compared to the Earth's moon seen in our night time skies? The apparent angular size of an object in arcminutes is found from the proportion:

$$\frac{\text{Apparent size}}{\text{3438 arcminutes}} = \frac{\text{Diameter (km)}}{\text{Distance (km)}}$$

The table below gives the diameters of each 'Galilean Moon' together with its minimum and maximum distance from Europa. Jupiter has a diameter of 142,000 km. Europa has a diameter of 2960 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun hass a diameter of 1.4 million km. Our Moon has a diameter of 3476 km.

	Shortest Distance (km)	Size (arcminutes)	Longest Distance (km)	Size (arcminutes)
Europa to Callisto	1.2 million		2.6 million	
Europa to Io	255,000		1.1 million	
Europa to Ganymede	403,000		1.7 million	
Europa to Jupiter	592,000		604,000	
Europa to Sun	740 million		815 million	
Earth to Moon	356,400		406,700	

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa.

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. . In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa. Jupiter has a diameter of 142,000 km. Io has a diameter of 3620 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun has a diameter of 1.4 million km. Our Moon has a diameter of 3476 km. Answer: see below.

	Shortest Distance (km)	Size (arcminutes)	Longest Distance (km)	Size (arcminutes)
Europa to Callisto	1.2 million	**13.5**	2.6 million	**6.2**
Europa to Io	255,000	**48.8**	1.1 million	**11.3**
Europa to Ganymede	403,000	**44.4**	1.7 million	**10.5**
Europa to Jupiter	592,000	**824.7**	604,000	**808.3**
Europa to Sun	740 million	**6.5**	815 million	**5.9**
Earth to Moon	356,400	**33.5**	406,700	**29.4**

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

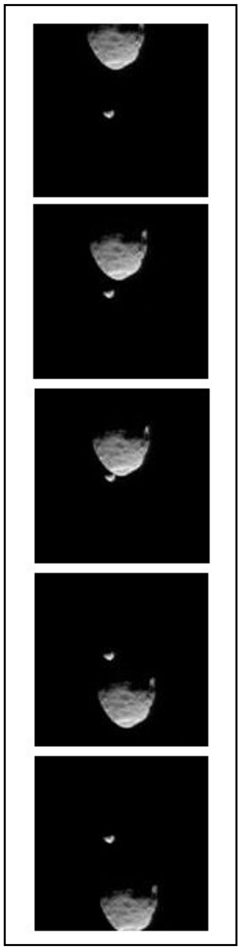
Answer: The solar disk has an angular size between 5.9 and 6.5 arcminutes. Only Callisto at its longest distance has an angular diameter (6.2 arcminutes) close to the solar diameter and so a complete eclipse is possible.

Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Answer: Io and Ganymede have angular diameters between 11 and 49 arcminutes. Our moon has a range of sizes between 29.4 and 33.5 arcminutes, so at some points in the orbits of Io and Ganymede, they will appear about the same size as our moon dies in our night sky.

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Answer: When Jupiter appears at its largest (824 arcminutes) and Io at its smallest (11.3 arcminutes) Jupiter will be 73 times bigger than the disk of Io. When Jupiter is at its smallest (808 arcminutes) and Io is at its largest (48.8 arcminutes) Jupiter will appear to be 16.6 times larger than the disk of Io.



The larger of the two moons of Mars, Phobos, passes directly in front of the other, Deimos, in a new series of sky-watching images from NASA's Mars rover Curiosity. Large craters on Phobos are clearly visible in these images from the surface of Mars. No previous images from missions on the surface caught one moon eclipsing the other.

Deimos (small image), and Phobos (large image), are shown together as they actually were photographed by the Mast Camera (Mastcam) on NASA's Mars rover Curiosity on Aug. 1, 2013.

How do we figure out how big something will look when it's far away? We draw a scale model of the object showing its diameter and its distance as the two sides of a triangle. The angle can then be measured. As the distance to the object increases, the 'angular size' of the object will decrease proportionately. A simple proportion can then be written that relates the angles to the lengths of the sides:

$$\frac{\text{Apparent Angle in degrees}}{57.3 \text{ degrees}} = \frac{\text{True diameter in kilometers}}{\text{Distance in kilometers}}$$

Let's see how this works for estimating the sizes of the moons of Mars as viewed from the Curiosity rover!

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

NASA Rover Gets Movie as a Mars Moon Passes Another
http://www.nasa.gov/mission_pages/msl/news/msl20130815.html
August 15, 2013

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Answer: Apparent size = 57.3 x (3476/370000) = **0.5 degrees**.

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Answer: Diemos: 57.3 x (12/20500) = **0.033 degrees**
Phobos: 57.3 x (22/6240) = **0.2 degrees**.

Note: Diemos appears to be about .033/.2 = 1/6 the diameter of Phobos in the sky.

Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Answer: 57.3 x (1400000/227,000,000) = 57.3 x (1.4/227) = **0.35 degrees**.

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

Answer: Diemos is only 0.033/0.35 = 1/11 the diameter of the sun in the sky as viewed from Mars, so it does not cover the full disk of the sun. As it passes across the sun it would look like a large dark spot. Phobos is 0.2/0.35 = ½ the diameter of the sun in the sky and it would not produce an eclipse like our moon does. It would look like a large black spot ½ the diameter of the sun.

The Late Heavy Bombardment Era

Between 4.1 and 3.8 billion years ago, the surfaces of all the planets were being bombarded by asteroids and other large bodies called impactors that had formed in the solar system by this time. Astronomers call this the Late Heavy Bombardment Era, and it is the era which finalized the formation of the planets at their present sizes.

The surface of our moon shows many large round basins called mare that are all that remains of this era. Similar scars on Earth have long since vanished due to erosion, volcanism and plate tectonic activity.

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Answer: Small: 300 million/20,000 = **15,000 years**. Medium: 300 million/50 = **6 million years**, Large: 300 million/5 = **60 million years**.

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Answer: Small = 20 km/6 = **3 km**. Medium: 1000 km/6 = **166 km**. Large: 5000km/6 = **833 km**.

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Answer: $V = 4/3\ \pi\ R^3$ and there were 20,000 of these so
Small = $20{,}000 \times 4/3 \times 3.141 \times (3\text{ km}/2)^3$ = **282,000 km^3**
Medium: There were 50 of these so $V = 50 \times 4/3 \times 3.141 \times (166\text{ km}/2)^3$ = **120 million km^3**
Large: There were 5 of these so $V = 5 \times 4/3 \times 3.141 \times (833\text{ km}/2)^3$ = **1.5 billion km^3**

Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

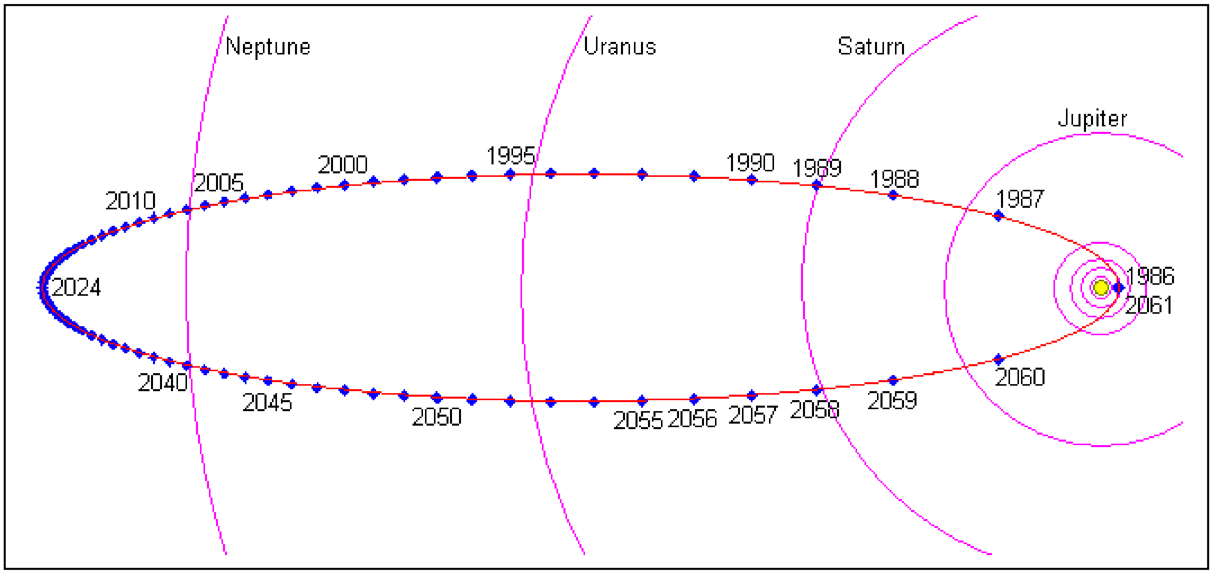
Answer: The total volume of a spherical Earth is $V = 4/3 \times 3.141 \times (6378)^3$ = 1.1 trillion km^3

So the three kinds of impactors contributed:

Small = 100% x (282000/1.1 trillion) = **0.00003 % of the final volume**.
Medium = 100% x (120 million/1.1 trillion) = **0.01 % of the final volume**.
Large = 100% x (1.5 billion/1.1 trillion) = **0.13% of the final volume**.

So the infrequent (every 60 million years) but largest impactors changed Earth's size the most rapidly during the Late Heavy Bombardment Era!

Exploring Comet Orbits



Comets are giant icebergs in space, sometimes over 50 miles across, that shed water vapor as they are heated while approaching the sun. Some comets only come around once and are never seen again. Others travel on elliptical paths called orbits, that take them far beyond the orbit of Jupiter before they, once again, loop back towards the sun.

Halleys Comet, which made a pass near the sun in 1986 is one of the most famous Periodic Comets, and will return to Earth's skies in the year 2061. This figure shows the orbit of Halleys Comet to the same scale as the orbits of the planets. Each dot is the position after one Earth year has elapsed.

Problem 1 – What is the period of Halleys Comet in Years?

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

Problem 3 – The distance between Saturn's orbit and the orbit if Venus is 1.3 billion km. About how fast is Halleys Comet traveling in km/year as it travels the Venus-Saturn distance?

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halleys Comet during this time in km/year?

Answer Key

Problem 1 – What is the period of Halleys Comet in Years?

Answer: 2061 – 1986 = **75 years.**

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

Answer: First determine the scale of this figure using a millimeter ruler and the actual Jupiter-Saturn distance. When printed on standard 8 ½ x 11 paper, the separation should be about 19 millimeters, so the scale is 650 million km/19mm = 34 million km/mm. The length of the ellipse is 143 mm, so the actual distance is 143 x 34 million = **4.86 billion kilometers**.

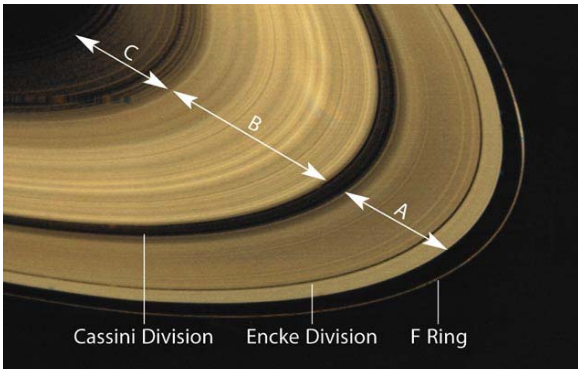
Problem 3 – The distance between Saturn's orbit and the orbit if Venus is 1.3 billion km. About how fast is Halleys Comet traveling in km/year as it travels the Venus-Saturn distance?

Answer: Counting the number of years, it takes 3 years to travel this distance, so the speed is 1.3 billion km/3 years = **433 million km/year**.

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halleys Comet during this time in km/year?

Answer: Counting the year marks, it takes 12.5 years, so the speed is about 1.6 billion km/12.5 years = **128 million km/year.**

Orbit Speeds and Times for Saturn's Rings



The trillions of particles in Saturn's rings orbit the planet like individual satellites. Although the rings look like they are frozen in time, in fact, the rings orbit the planet at thousands of kilometers per hour! The speed of each ring particle is given by the formula:

$$V = \frac{29.4}{\sqrt{R}} \text{ km/s}$$

where R is the distance from the center of Saturn to the ring in multiples of the radius of Saturn (R = 1 corresponds to a distance of 60,300 km).

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Answer: R = (60300 km + 7,000 km) / 60300 km = 1.12, so V = 23.6 km/sec
R = 140300 km/60300 km = 2.33, so V = 16.3 km/sec.

Note: The International Space Station orbits Earth at a speed of 7.7 km/s.

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent 'gap' in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Answer: R = 117580/60300 = 1.95 V = 17.83 km/s
R = 122170/60300 = 2.02 V = 17.52 km/s
The outer edge particles travel about 17.83 – 17.52 = 0.31 km/sec slower than the inner edge particles.

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Answer: Orbit circumference = 2 π r km, but r = 60300 R so C = 2 (3.141) x 60300 R ,
C = 379,000 R km, wnhere R is in Saturn radius units. Since the orbit speed is V = $24.9/R^{1/2}$, then Time = C/V = 15220 $R^{3/2}$ seconds. Since 1 hour = 3600 seconds, we have
T = 4.22 $R^{3/2}$ hours.

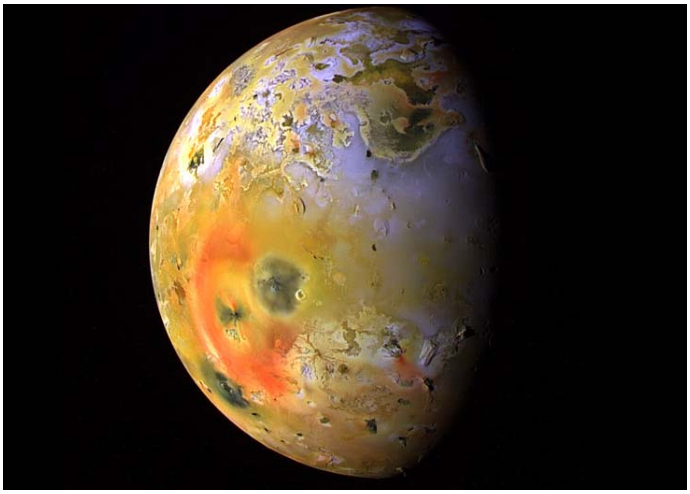
Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Answer: R = 1.95 so T = 11.49 hours.
R = 2.02 so T = 12.11 hours.

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Answer: 22.5/11.49 = 1.94 which is nearly 2.0. This means that every time Mimas orbits once, the particles in the Cassini Division orbit about twice around Saturn. This is an example of an orbit resonance. Because the ring particles encounter a push from Mimas's gravitational field at the same location every two orbits, they will be ejected. This is an explanation for why the Cassini Division has so few ring particles.

Io's Volcanoes and Resurfacing



This 1997 image taken by NASA's Galileo spacecraft shows the complex surface of Io. Sulfur dioxide frost appears in white and grey hues while yellowish and brownish hues are from other sulfurous materials. The new dark spot 400 km in diameter, surrounds a volcanic center named Pillan Patera. The spot did not exist 5 months earlier, and is the source of a 120 km high plume that has been seen erupting from this location.

Although no impact craters have been found, over 420 calderas and active vents have been mapped. About 15 are actively spewing fresh material within 175 km of each vent. This means that Io quickly resurfaces itself, covering over all of the impact craters within a million years or less.

Problem 1 – Assume Io is a sphere with a radius of 1820 km, and is covered to a depth of 1 kilometers to cover any new craters. What volume of fresh material must be produced in cubic meters?

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Problem 5 – What is the total resurfacing rate each year in centimeters/year?

Problem 1 – Assume Io is a sphere with a radius of 1820 km, and is covered to a depth of 1 kilometers to cover any new craters. What volume of fresh material must be produced in cubic meters?

Answer: Area = $4 \pi R^2$, and R = 1820,000 meters, so
Area = $4 \times 3.14 \times (1820000)^2 = 4.2\times10^{13}$ m^2.
The volume of the surface shell 1km thick is that V = A x 1km = **4.2×10^{16} m^3.**

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Answer: 4.2×10^{16} m^3 / 420 = **1.0×10^{14} m^3 per caldera**.

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

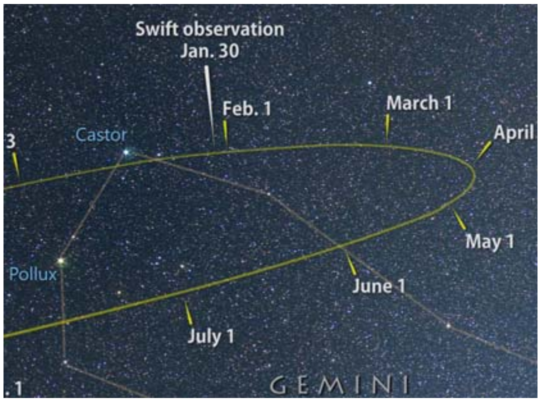
Answer: 1.0×10^{14} m^3/500000yrs = **2.0×10^{8} m^3/year.**

Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Answer: 2.0×10^{8} m^3/year would be the rate if each caldera continuously operated for 500,000 years. If they only are active for 1% of this time, then the average rate has to be 100x higher or **2.0×10^{10} m^3/yr.**

Problem 5 – What is the total resurfacing rate each year in centimeters/year?

Answer: If the 1km depth is generated over 500,000 years, then each year the depth added is 100000 centimeters/500000 yr = **0.2 centimeters/year.**



Comet ISON will be carefully watched as it makes its closest approach to the sun in November, 2013. Some astronomers predict that it may break up into smaller comets because of the Sun's enormous gravity. The comet will travel to within 1,800,000 km of the center of the sun, or about 1,100,000 km from the hot solar surface! As it travels, it will also get very close to Mars and the asteroid 3362 Khufu, though no impacts are predicted!

A portion of its track across the sky is shown in the figure for January-July, 2013.

The table below gives the location of Comet ISON as it approaches the sun. The sun is located at the point (-0.4, +14.7) where all of the coordinate units are in millions of kilometers.

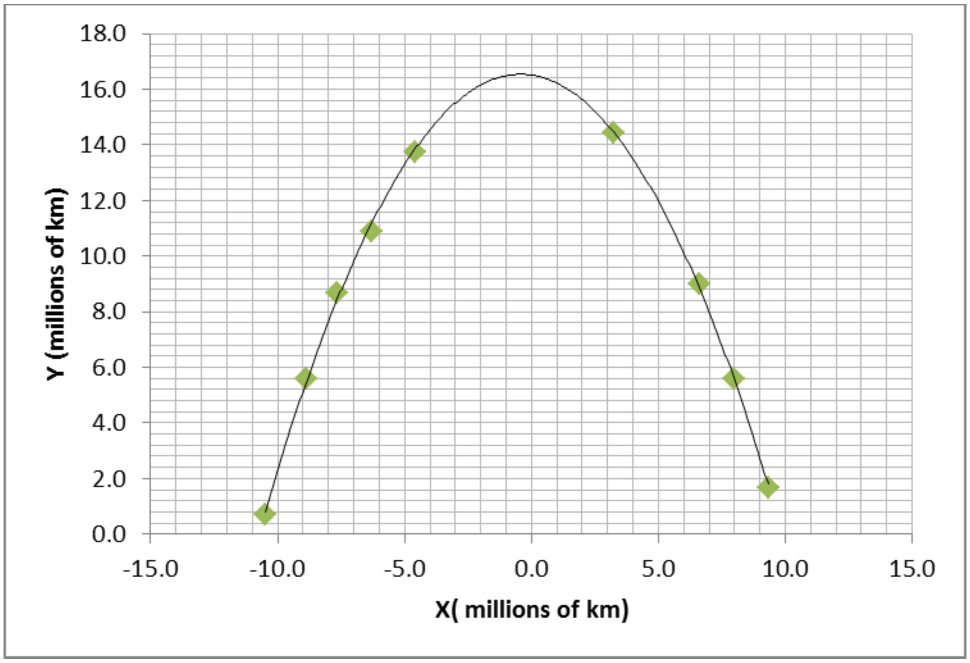
Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-10.5	+0.7	
November 27, 13:00 UT	-8.8	+5.6	
November 28, 01:00 UT	-7.7	+8.7	
November 28, 08:00 UT	-6.3	+10.9	
November 28, 14:00 UT	-4.6	+13.7	
November 28, 23:00 UT	+3.2	+14.5	
November 29, 10:00 UT	+6.6	+9.0	
November 29, 19:00 UT	+8.0	+5.6	
November 30, 10:00 UT	+9.3	+1.7	

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve.

Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table.

Problem 3 – What is your prediction for the time when Comet ISON is at its closest point in its orbit to the sun?

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve. **Note: These coordinates are valid for the orbit as known on November 24, 2013 but may change as a more precise orbit is eventually determined.**



Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table. Answer: For advanced students using the distance formula: $D = ((x2-x1)^2 + (y2-y1)^2)^{1/2}$. For the x2 at (-10.5, +0.7) and x1 (sun) at (-0.4,+14.7) so D = 17.3 million km.

Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-10.5	+0.7	**17.3**
November 27, 13:00 UT	-8.8	+5.6	**12.4**
November 28, 01:00 UT	-7.7	+8.7	**9.4**
November 28, 08:00 UT	-6.3	+10.9	**7.0**
November 28, 14:00 UT	-4.6	+13.7	**4.3**
November 28, 23:00 UT	+3.2	+14.5	**3.6**
November 29, 10:00 UT	+6.6	+9.0	**9.1**
November 29, 19:00 UT	+8.0	+5.6	**12.4**
November 30, 10:00 UT	+9.3	+1.7	**16.3**

Problem 3 – What is your prediction for the exact time when Comet ISON is at its closest point in its orbit to the sun? Answer: Students may interpolate between Points 5, 6 and 7 using any convenient method. The answers should be close to **November 28 at 18:00 UT and a distance to the sun of about 1,840,000 km.**

Using Calculus: The best fit parabola is $y = -0.155X^2 - 0.132X + 16.51$. Setting the first derivative equal to zero we get $0 = -0.31X - 0.132$ so X= -0.43 and Y = +16.54. The distance to the sun is then 1.84 million km.

This image of Comet ISON was taken by the Hubble Space Telescope on April 10, 2013 when the comet was 386 million miles from the sun, and 394 million miles from Earth. The icy core of the comet may only be about 5 km in diameter, but the bright head or 'Coma' is about 3,000 miles across.

Although it is too far from the sun to have a dramatic tail, from just beyond the orbit of Mars, its tail has already grown to 57,000 miles.

Measurements by NASA's Swift satellite in January, 2013 show that at a distance of 460 million miles from the sun (375 million miles from Earth), and traveling at about 21 km/sec (13 miles/sec), Comet ISON was ejecting 51,000 kg of dust and 60 kg of water every minute as its surface has been steadily heated from 40 K to about 150 K. This pace of ejection will increase enormously as the comet gets closer to the sun. By some estimates the comet will lose 10% of its total mass as it swings by the sun and heads back out into deep space in early - 2014.

Problem 1 – Suppose that the average density of the comet nucleus is 1000 km/m^3, and it is a sphere with a diameter of about 6 km. What is the total mass of the nucleus of the comet?

Problem 2 – Suppose that Comet ISON continues to lose dust mass at the rate of 51,000 kg/minute as it travels to the orbit of Earth (93 million miles from the sun) at a speed of 13 miles/sec. How much mass will it lose, and what percentage of its total mass is this?

Problem 1 – Suppose that the average density of the comet nucleus is 1000 km/m3, and it is a sphere with a diameter of about 6 km. What is the total mass of the nucleus of the comet?

Answer: $V = 4/3 \pi R^3$ so
$V = 1.33 \times 3.14 \times (3000m)^3$
$= 1.1 \times 10^{11} m^3$.

Since Mass = density x volume, the comet's mass is
$M = 1000 \times 1.1 \times 10^{13}$
= $\mathbf{1.1 \times 10^{16}}$ **kg**.

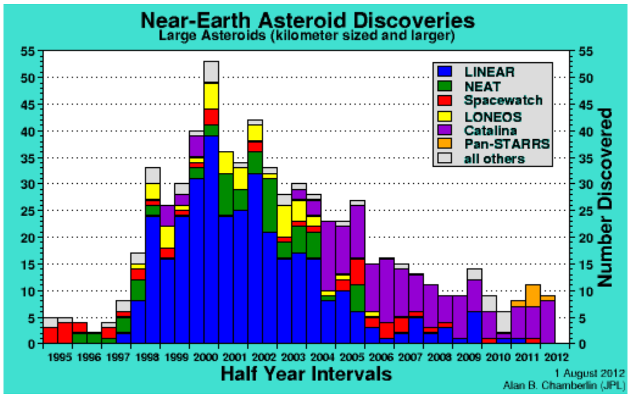
Problem 2 – Suppose that Comet ISON continues to lose dust mass at the rate of 51,000 kg/minute as it travels to the orbit of Earth (93 million miles from the sun) at a speed of 13 miles/sec. How much mass will it lose, and what percentage of its total mass is this?

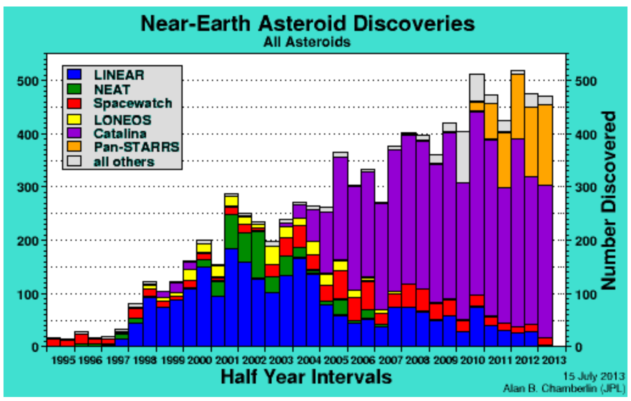
Answer: It travels from 460 million miles to 93 million miles covering a total of 460-93 = 367 million miles.

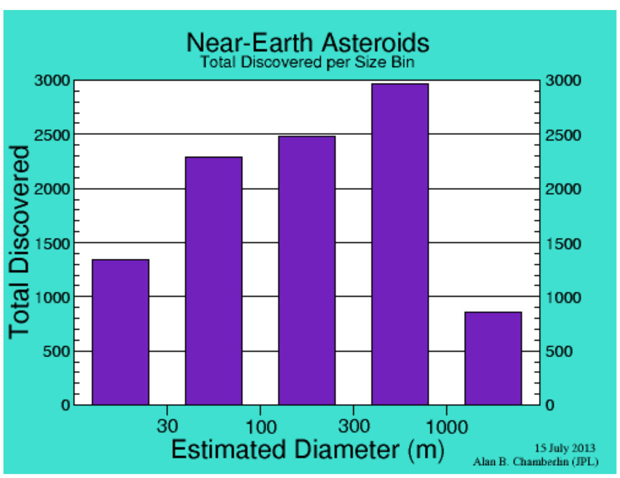
At a speed of 13 miles/sec, this takes 367,000,000/13 = 28 million seconds or 470,000 minutes.

At a rate of 51,000 kg/minute, it will loose M = 51,000 x 470,000 = $\mathbf{2.4 \times 10^{10}}$ **kg**.

This represents P = 100% x $(2.4 \times 10^{10}/1.1 \times 10^{16})$ = **0.0004% of the total mass**!







Near Earth Objects (NEOs) are asteroids that have orbits very close to Earth's orbit in the solar system. That means that, over time, they might collide with Earth. The most devastating asteroids are larger than 1 km and can cause world-wide extinction events. Smaller bodies from 10 to 300-meters can damage cities but are too small to affect continent-sized areas.

Astronomers have, for decades detected and tracked these smaller bodies as they come near Earth, and from this determine their orbits. The graphs to the left show the progress of these searches since 1995.

Problem 1 – The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Problem 2 – What is the total number of asteroids discovered in 2012?

Problem 3 – What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B)Smaller than 1 kilometer?

Problem 4 – From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Problem 5 – Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Problem 1 – The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Answer: The last bar in the graph shows that **9** were discovered in 2012.

Problem 2 – What is the total number of asteroids discovered in 2012?

Answer: The second graph shows that for 2012 the column indicates about **470** were discovered that year.

Problem 3 – What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B) Smaller than 1 kilometer?

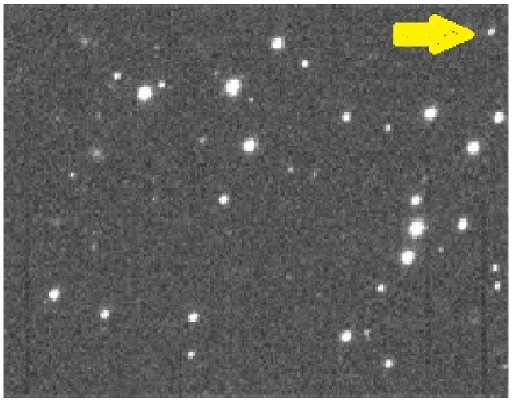
Answer: A) P = 100% x (9/470) = **1.9%.**
B) P = 100% x (461/470) = **98.1%**

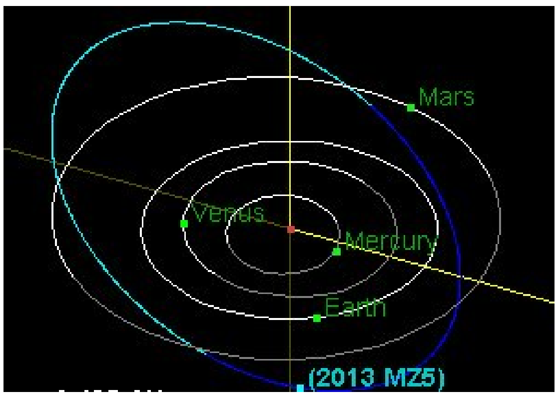
Problem 4 – From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Answer: A) Adding up all 5 columns gives a total of 1350+2300 + 2500 + 3000 + 850 = 10,000
Then Small asteroids P = 100% x (10,000-850)/10000 = **91.5%**
B) Large asteroids P = 100% - 91.5% = **8.5%**

Problem 5 – Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Answer: The top graph shows that the detection rate for these large asteroids has declined steadily since 2000. This means that each year the surveys are finding fewer and fewer large asteroids that they did not previously know about. This means that the surveys have almost completely detected all of the NEO asteroids that are this large. Because these large NEOS are only 2% of the detected asteroids, the majority of NEO asteroids are much smaller than the large ones, and more numerous. The middle graph shows that the numbers we are detecting continues to grow each year with no sign of decreasing. This means that there are many more of these to be discovered than we have found already. By some estimates, we have only discovered about 5% of all that there are near Earth, which is why we have to keep searching. Once the number of new discoveries begins to follow the profile of the top figure, we will know that we have discovered the vast majority of the small asteroids that remain.





It was only 300 meters in diameter, but on June 18, 2013 it was discovered by the Pan-STARRS-1 survey telescope in Hawaii (top image), making it the 10,000th Near Earth Object (NEO).

NASA estimates that there are 15,000 asteroids that are 140-meters or larger with orbits close to Earth. We have now discovered 30% of them according to statistical estimates based on the rate of discovery of these NEOs. Objects of this size or larger are considered severe hazards should they impact near cities. The February 2013 ‘Russian’ meteor was 17 meters in diameter, yet it injured over 3000 people, though it left no crater. A 140-meter asteroid impact would produce a 25-meter crater and the atmospheric shock wave would damage thousands of buildings.

The orbit is shown in the left. The figure to the left. The orbit is tilted nearly 45^o to Earth’s orbit plane.

The distance from Earth to 2013 MZ5 is given in the table below, every 10 days, and in units of millions of kilometers.

Date	Distance	Date	Distance	Date	Distance
May 22	115	July 1	71	August 9	82
June 1	100	July 10	69	August 19	89
June 11	88	July 20	71	August 29	97
June 21	77	July 30	76	September 8	104

Problem 1 – Graph the tabulated distance function and connect the points with a smooth curve.

Problem 2 – On what date is the asteroid closest to Earth?

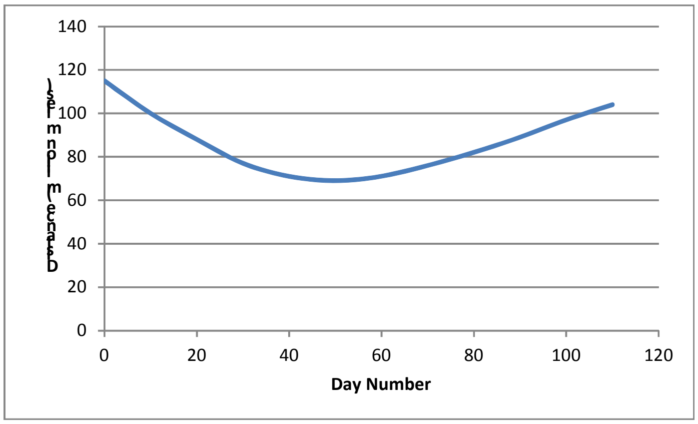
Problem 3 – The approximate formula for the approach speed of the asteroid to Earth in km/hr is given by S = 1100 T – 62000, where T is the elapsed days from May 22. How fast was it approaching Earth on June 18, the day of its discovery?

Orbit - http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2013%20MZ5;orb=1;cov=0;log=0;cad=0#orb

Data - http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2013%20MZ5

http://www.nasa.gov/centers/jpl/news/neo20130624.html
NASA - Ten Thousandth Near-Earth Object Unearthed in Space
June 24, 2013.

Problem 1 – Graph the tabulated distance function and connect the points with a smooth curve. Answer: **See graph below**.



Problem 2 – On what date is the asteroid closest to Earth?

Answer: The table shows that on **July 10 (Elapsed day 50)** it reached its minimum distance of 69 million kilometers.

Problem 3 – The approximate formula for the approach speed of the asteroid to Earth in km/hr is given by S = 1100 T – 62000, where T is the elapsed days from May 22. How fast was it approaching Earth on June 18, the day of its discovery?

Answer: June 18 is elapsed day 27, so T = 27 and so S = 1100(27)-6200 = **-23,500 km/hr**. The negative sign means that the distance to Earth was decreasing.

The fastest way to get from place to place in our solar system is to travel at the speed of light, which is 300,000 km/sec (670 million miles per hour!). Unfortunately, only radio waves and other forms of electromagnetic radiation can travel exactly this fast.

When NASA sends spacecraft to visit the planets, scientists and engineers have to keep in radio contact with the spacecraft to gather scientific data. But the solar system is so vast that it takes quite a bit of time for the radio signals to travel out from Earth and back.

Problem 1 – Earth has a radius of 6378 kilometers. What is the circumference of Earth to the nearest kilometer?

Problem 2 – At the speed of light, how long would it take for a radio signal to travel once around Earth?

Problem 3 – The Moon is located 380,000 kilometers from Earth. During the Apollo-11 mission in 1969, engineers on Earth would communicate with the astronauts walking on the lunar surface. From the time they asked a question, how long did they have to wait to get a reply from the astronauts?

Problem 4 – In the table below, fill in the one-way travel time from the sun to each of the planets. Use that fact that the travel time from the Sun to Earth is 8 ½ minutes. Give your answer to the nearest tenth, in units of minutes or hours, whichever is the most convenient unit.

Planet	Distance from Sun in Astronomical Units	Light Travel Time
Mercury	0.38	
Venus	0.72	
Earth	1.00	8.5 minutes
Mars	1.52	
Jupiter	5.20	
Saturn	9.58	
Uranus	19.14	
Neptune	30.20	

Problem 1 – Earth has a radius of 6378 kilometers. What is the circumference of Earth to the nearest kilometer?

Answer: $C = 2 \pi R$ so $C = 2 \times 3.141 \times (6378 \text{ km})$ = **40,067 km.**

Problem 2 – At the speed of light, how long would it take for a radio signal to travel once around Earth?

Answer: Time = distance/speed so
Time = 40,067/300,000 = **0.13 seconds. This is about 1/7 of a second**.

Problem 3 – The Moon is located 380,000 kilometers from Earth. During the Apollo-11 mission in 1969, engineers on Earth would communicate with the astronauts walking on the lunar surface. From the time they asked a question, how long did they have to wait to get a reply from the astronauts?

Answer: From the proportion:

$$\frac{0.13 \text{ seconds}}{40067 \text{ km}} = \frac{X}{380000 \text{ km}}$$ we have $X = (380000/40067) \times 0.13 = 1.23$ seconds.

This is the one-way time for the signal to get to the moon from Earth, so the round-trip time is twice this or **2.46 seconds**.

Problem 4 – In the table below, fill in the one-way travel time from the sun to each of the planets. Use that fact that the travel time from the Sun to Earth is 8 ½ minutes. Give your answer to the nearest tenth, in units of minutes or hours, whichever is the most convenient unit.

Answer: Use simple proportions based on 8.5 minutes of time = 1.00 AU of distance.

Planet	Distance from Sun in Astronomical Units	Light Travel Time
Mercury	0.38	3.2 minutes
Venus	0.72	6.1 minutes
Earth	1.00	8.5 minutes
Mars	1.52	12.9 minutes
Jupiter	5.20	44.2 minutes
Saturn	9.58	1.4 hours
Uranus	19.14	2.7 hours
Neptune	30.20	4.3 hours

One of the best ways to explore the effects of gravity on different bodies in the solar system is to calculate what your weight wound be if you were standing on their surfaces!

Scientists use kilograms to indicate the mass of an object, and it is common for Americans to use pounds as a measure of weight. On Earth, the force that one kilogram of mass has on the bathroom scale is equal to 9.8 Newtons or a weight of 2.2 pounds.

The surface gravity of a planet or other body is what determines your weight by the simple formula W = Mg where W is the weight in Newtons, M is the mass in kilograms, and g is the acceleration of gravity at the surface in meters/sec2. For example, on Earth, g = 9.8 m/sec, and for a person with a mass of 64 kg, the weight will be W = 64 x 9.8 = 627 Newtons. Since 9.8 Newtons equals 2.2 pounds, this person weighs 627 x (2.2/9.8) = 140 pounds.

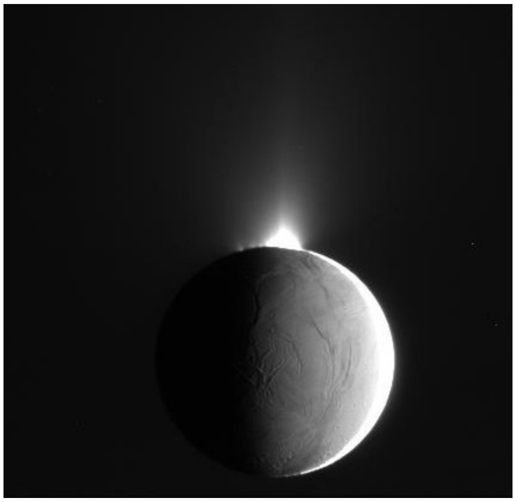
Problem 1 - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G (m/sec^2)	Weight (pounds)
Earth	Planet	9.8	110
Mercury	Planet	3.7	
Mars	Planet	3.7	
Io	Jupiter moon	1.8	
Moon	Earth moon	1.6	
Titan	Saturn moon	1.4	
Europa	Jupiter moon	1.3	
Pluto	Planet	0.58	
Charon	Pluto moon	0.28	
Vesta	Asteroid	0.22	
Enceladus	Saturn moon	0.11	
Miranda	Uranus moon	0.08	
Deimos	Mars moon	0.003	

Problem 1 - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G (m/sec^2)	Weight (pounds)
Earth	Planet	9.8	**110**
Mercury	Planet	3.7	**41.5**
Mars	Planet	3.7	**41.5**
Io	Jupiter moon	1.8	**20.2**
Moon	Earth moon	1.6	**18.0**
Titan	Saturn moon	1.4	**15.7**
Europa	Jupiter moon	1.3	**14.6**
Pluto	Planet	0.58	**6.5**
Charon	Pluto moon	0.28	**3.1**
Vesta	Asteroid	0.22	**2.5**
Enceladus	Saturn moon	0.11	**1.2**
Miranda	Uranus moon	0.08	**0.9**
Deimos	Mars moon	0.003	**0.03**

Note: For the Mars moon Deimos, which is a rocky body only 12 km (7.5 miles) in diameter, your weight would be 0.03 pounds or just ½ ounce! Astronauts that visit this moon would not 'land' on its surface but 'dock' with the moon the way that they do with visits to the International Space Station!



For decades, astronomers thought that the only geologically active object in our entire solar system was Earth. During the last 50 years of spacecraft studies, we now know that there are many locations where recent and even current volcanism can be found.

This image, taken by the Cassini spacecraft, shows plumes of gas and dust ejected from cracks in the surface of Saturn's moon Enceladus. Similar plumes have been found in Jupiter's satellite Io, and Neptune's moon Triton. They are called cryovolcanos because the temperatures are so low, only 100 kelvins (173 Celsius), and instead of rocky lava they eject water, methane or other frozen gases.

A simple 'square-root' formula relates the height of a plum, h, and its ejection speed, V, to the surface gravity of the body, g :

$$V = (2gh)^{1/2}$$

where h is in meters, V is in meters/sec and g is the acceleration of gravity at the surface in meters/sec^2.

Problem 1 – Complete the following table to estimate the ejection speeds and heights of volcanic plumes on the indicated bodies.

Object	Type	G (m/s^2)	V (m/s)	H (m)
Venus	Volcano	8.8	100	
Earth	Volcano	9.8		8,000
Earth	Geyser	9.8	35	
Mars	Volcano	3.8	100	
Io	Volcano	1.8		200,000
Enceladus	Geyser	0.1	60	
Titan	Volcano	1.4	100	
Triton	Geyser	0.8		5,000

Problem 2 – If 1 meter/sec = 2.2 miles/hr, which objects have the fastest and slowest ejection speeds in mph?

Problem 3 - Calculate the average ejection speeds for Volcanos and geysers. What do you notice about the kind of event and its ejection speed?

Problem 1 – Complete the following table to estimate the ejection speeds and heights of volcanic plumes on the indicated bodies.

Object	Type	G (m/s^2)	V (m/s)	H (m)
Venus	Volcano	8.8	100	**570**
Earth	Volcano	9.8	**400**	8,000
Earth	Geyser	9.8	35	**63**
Mars	Volcano	3.8	100	**1,300**
Io	Volcano	1.8	**850**	200,000
Enceladus	Geyser	0.1	60	**18,000**
Titan	Volcano	1.4	100	**3600**
Triton	Geyser	0.8	**89**	5,000

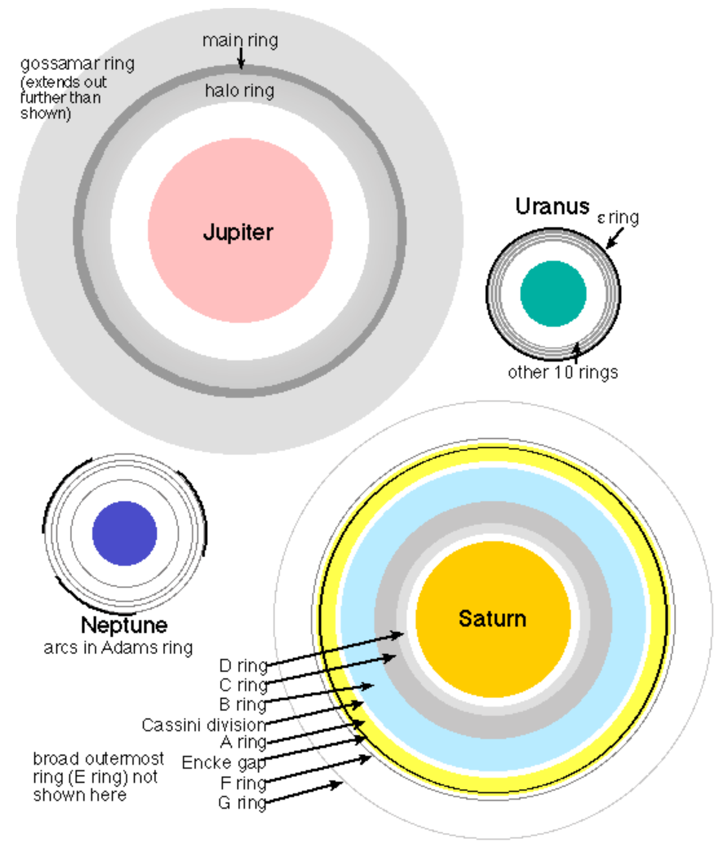
Problem 2 – If 1 meter/sec = 2.2 miles/hr, which objects have the fastest and slowest ejection speeds in mph?

Answer: Fastest is Io at 850 m/s or 1,870 mph. Slowest is Earth geyser at 35 m/s or 77 mph.

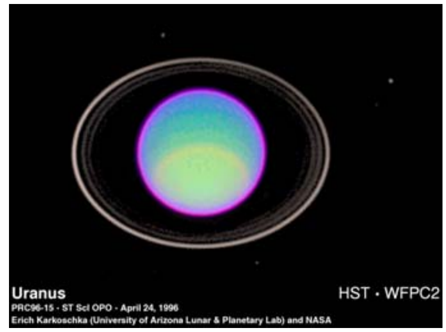
Problem 3 - Calculate the average ejection speeds for Volcanos and geysers. What do you notice about the kind of event and its ejection speed?

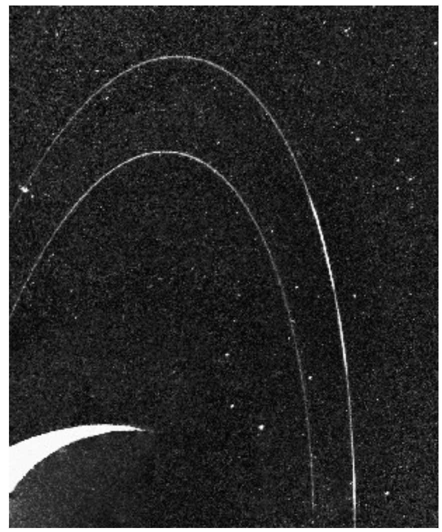
Answer: Volcanos: (100+400+100+850+100)/5 = 310 m/sec (682 mph).
Geysers: (35+60+89)/3 = 61 m/sec (134 mph.)

Volcanos have the highest speeds. This is because they are produced by intense pressure of magma trapped in rocky 'pipes' that reach hundreds of kilometers below the surface, while geysers are created by more modest pressures in the surface crust.



Rings of the jovian planets shown to the same scale
Courtesy of Nick Strobel at www.astronomynotes.com





The rings of Uranus (top right) and Neptune (bottom right) are similar to those of Jupiter and consist millions of small rocky and icy objects in separate orbits. The figure shows a comparison of the scales of the planets and their ring systems.

Problem 1 – Compare the extent of the ring systems for each planet in terms of their size in units of the radius of the corresponding planet. For example, ‘The rings of Uranus extend from 1.7 to 2.0 times the radius of Uranus’.

Problem 2 – An icy body will be destroyed by a planet if it comes within the Tidal Limit of the planet. At this distance, the difference in gravity between the near and the far side of the body exceeds the body’s ability to hold together by its own gravity, and so it is shredded into smaller pieces. For Jupiter (2.7), Saturn (2.2), Uranus (2.7) and Neptune (2.9), the Tidal Limits are located between 2.2 and 2.9 times the radius of each planet from the planet’s center. Describe where the ring systems are located around each planet compared to the planets Tidal Limit. Could the rings be explained by a moon or moon’s getting too close to the planet?

Problem 1 – Compare the extent of the ring systems for each planet in terms of their size in units of the radius of the corresponding planet. For example, 'The rings of Uranus extend from 1.7 to 2.0 times the radius of Uranus'.

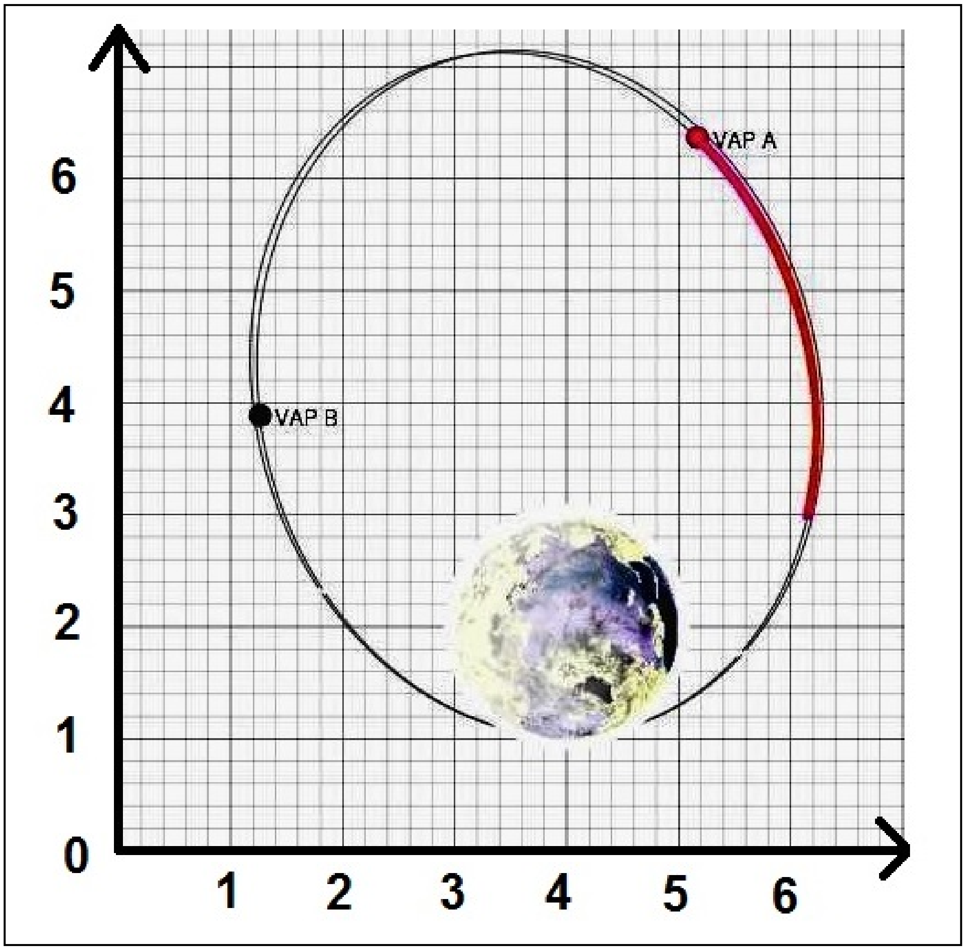
Answer: Jupiter from 1.4 to 2.3
Saturn from 1.1 to 3.6
Uranus from 1.7 to 2.0
Neptune from 1.7 to 2.7

Problem 2 – An icy body will be destroyed by a planet if it comes within the Tidal Limit of the planet. At this distance, the difference in gravity between the near and the far side of the body exceeds the body's ability to hold together by its own gravity, and so it is shredded into smaller pieces. For Jupiter (2.7), Saturn (2.2), Uranus (2.7) and Neptune (2.9), the Tidal Limits are located between 2.2 and 2.9 times the radius of each planet from the planet's center. Describe where the ring systems are located around each planet compared to the planets Tidal Limit. Could the rings be explained by a moon or moon's getting too close to the planet?

Answer: Jupiter 1.4 2.3 (2.7)
Saturn 1.1 (2.2) 3.6
Uranus 1.7 2.0 (2.7)
Neptune 1.7 2.7 (2.9)

The location of the tidal radius for each planet is given in parenthesis on this scaled model. We see that for all of the planets, most of the ring material is located inside the Tidal Limit. In the case of Saturn, which seems to be the exception, there is also some ring material outside R = 2.2 Rs and includes the very sparse F and G rings. But even for Saturn, the majority of the visible rings (seen through a telescope) are inside the Tidal Limit.

The rings can be explained by moons that were tidally destroyed as they passed inside the Tidal Limit for each planet. These moons could not have been formed this close because the same tidal forces that destroy these moons would have prevented them from assembling, so the moons must have been formed outside the Tidal Limit and over millions of years their orbits carried them closer and closer to the Tidal Limit until they were finally destroyed.



Problem 1 – The graph above shows the orbit of the Van Allen Probes spacecraft. Each division, 1.0, 2.0, etc) is given in terms of Earth's radius, which is 6,378 kilometers. For example, '2.0' = twice Earth's radius or 12,756 kilometers. Suppose that the spacecraft move along their orbit from point (+5.6, +1.8) to point (+6.2, +3.0). Plot these two points on the graph, and convert their coordinates into kilometers.

Problem 2 – Using which ever method you like, determine the distance in kilometers that the spacecraft traveled between the two points in their orbit.

Problem 3 – Suppose the spacecraft reached the point (+5.6, +1.8) at a time of 11:22:30 and point (+6.2, +3.0) at a time of 11:50:00 what was the elapsed time to travel this distance in seconds?

Problem 4 – What was the speed of the spacecraft in kilometers/hour?

Problem 1 – The graph above shows the orbit of the Van Allen Probes spacecraft. Each division, 1.0, 2.0, etc) is given in terms of Earth's radius, which is 6,378 kilometers. For example, '2.0' = twice Earth's radius or 12,756 kilometers. Suppose that the spacecraft move along their orbit from point (+5.6, +1.8) to point (+6.2, +3.0). Plot these two points on the graph, and convert their coordinates into kilometers.

Answer: 5.6 x 6378 = 35,717 km, 1.8 x 6378 = 11,480; 6.2 x 6378 = 39,544, 3.0 x 6378 = 19,134 so (35717 , 11480) and (39544 , 19134).

Problem 2 – Using which ever method you like, determine the distance in kilometers that the spacecraft traveled between the two points in their orbit.

Answer: Determine the scale of the graph using a millimeter ruler in km/mm, then measure the distance between the two points in millimeters and convert to kilometers. For advanced students use the 2-point distance formula. $D^2 = (39544-35717)^2 + (19134-11480)^2 = 3827^2 + 7654^2$, so **D = 8,557 kilometers**.

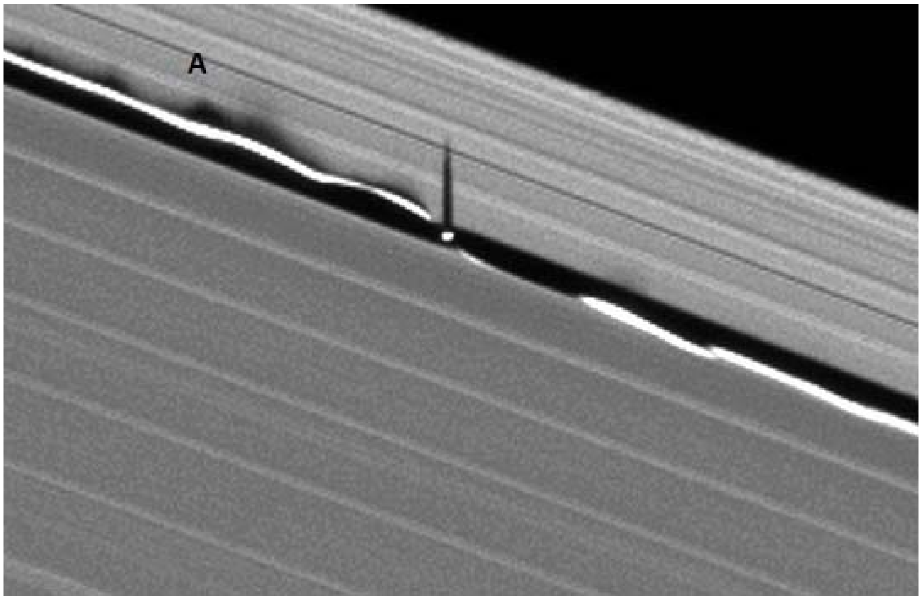
Problem 3 – Suppose the spacecraft reached the point (+5.6, +1.8) at a time of 11:22:30 and point (+6.2, +3.0) at a time of 11:50:00 what was the elapsed time to travel this distance in seconds?

Answer: Elapsed time = 11:50:00 – 11:22:30 = 27m 30 s or 27x60 + 30 = **1650 seconds**.

Problem 4 – What was the speed of the spacecraft in kilometers/hour?

Answer: Time, in hours, = 1650 seconds x (1 minute/60 sec) x (1 hr/60 min) = 0.46 hours.

Speed = distance/time = 8557 km / 0.46 hours = **18,602 km/hr**.



Never-before-seen looming vertical structures, created by the tiny moon Daphnis, cast long shadows across Saturn's A Ring in this startling image taken by the Cassin spacecraft. The 8-kilometre-wide moon Daphnis orbits within the 42-kilometre-wide Keeler Gap in Saturn's outer A Ring, and its gravitational pull perturbs the orbits of the particles forming the gap's edges. The Keeler Gap is foreshortened and appears only about 30 km wide because the image was taken at an angle of about 45 degrees to the ring plane.

Problem 1 – If the apparent perpendicular width of the Keeler Gap is 30 km, what is the length of the shadow of Daphnis in this image?

Problem 2 – Rounded to the nearest kilometer, what is the length of Shadow A to the left from Daphnis?

Problem 3 – Create a scaled model of this ring area and its 45 degree inclination, and using right triangles, estimate the elevation angle of the sun above the ring plane.

Problem 4 – From your scaled model, what is the height of the feature that is casting Shadow A on the ring plane?

Problem 1 – If the apparent (foreshortened) perpendicular width of the Keeler Gap is 30 km, what is the corresponding foreshortened length of the shadow of Daphnis in this image?

Answer: After printing this page on standard 8 ½ x 11 paper, the perpendicular width of the Keeler Gap is about 4 millimeters, which corresponds to 30 km, so the scale is 30 km/4mm = 7.5 km/mm. The length of the moonlets shadow is 12 mm, so its projected length is 12 x 7.5 = **90 km**.

Problem 2 – Rounded to the nearest kilometer, what is the length of Shadow A to the left from Daphnis?

Answer: Students may obtain 5 mm if they measure from the inner edge of the white band, or 3 mm if they measure from the outer edge of the white band. These measurements correspond to **23 km or 38 km**.

Problem 3 – Create a scaled model of this ring area and its 45 degree inclination, and using right triangles, estimate the elevation angle of the sun above the ring plane.

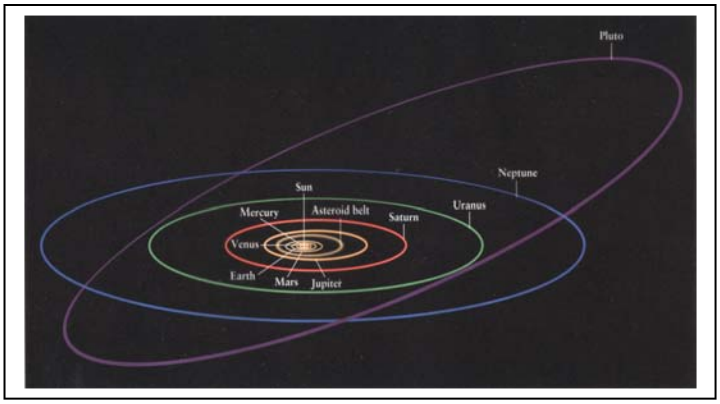
Answer: For the shadow of Daphnis, the true shadow length is the hypotenuse of a 45-45-90 right triangle ABD, and its shadow length in the image is the horizontal side along segment BC of this triangle, so the hyponenuse (segment AB) length is 90 km x $(2)^{1/2}$ = 127 km. The diameter of the moon is 8 km, so the sun angle for the shadow on the ring plane is a triangle whose sides are AC=8 km and BC = 127 km. The angle to the sun ABC can be measured with a protractor and is Tan(Theta) = 8km/127km so to the nearest degree, **Theta = 4 degrees**.

Problem 4 – From your scaled model, what is the height of the feature that is casting Shadow A on the ring plane?

Answer: The projected height of the shadow is 23 or 38 km. The true shadow length is then
23 x $(2)^{1/2}$ = 33 km or
38 x $(2)^{1/2}$ = 54 km.

The sun angle above the ring plane is 4 degrees, so if the right triangle ABC has side BC = 33 km then the height of segment AC can be measured with a protractor as about 33xTan(4) = **2.3 km**, or using the second shadow measurement 54xtan(4) = **3.8 km**.

Note: Astronomers find it amazing that even though the A ring has a thickness of about 10 meters, the wave produced by this small moon rises over 200 times higher above the ring plane than the thickness of the rings themselves!



Our solar system is so big it is almost impossible to imagine its size if you use ordinary units like feet or miles. The distance from Earth to the Sun is 93 million miles (149 million kilometers), but the distance to the farthest planet Neptune is nearly 3 billion miles (4.5 billion kilometers). Compare this to the farthest distance you can walk in one full day (70 miles) or that the International Space Station travels in 24 hours (400,000 miles).

The best way to appreciate the size of our solar system is by creating a scaled model of it that shows how far from the sun the eight planets are located. Astronomers use the distance between Earth and sun, which is 93 million miles, as a new unit of measure called the Astronomical Unit. It is defined to be exactly 1.00 for the Earth-Sun orbit distance, and we call this distance 1.00 AUs.

Problem 1 - The table below gives the distance from the Sun of the eight planets in our solar system. By setting up a simple proportion, convert the stated distances, which are given in millions of kilometers, into their equivalent AUs, and fill-in the last column of the table.

Planet	Distance to the Sun in millions of kilometers	Distance to the Sun in Astronomical Units
Mercury	57	
Venus	108	
Earth	149	
Mars	228	
Jupiter	780	
Saturn	1437	
Uranus	2871	
Neptune	4530	

Problem 2 – Suppose you wanted to build a scale model of our solar system so that the orbit of Neptune was located 10 feet from the yellow ball that represents the sun. How far from the yellow ball, in inches, would you place the orbit of Jupiter?

Problem 1 - The table below gives the distance from the Sun of the eight planets in our solar system. By setting up a simple proportion, convert the stated distances, which are given in millions of kilometers, into their equivalent AUs, and fill-in the last column of the table.

Answer: In the case of Mercury, the proportion you would write would be

$$\frac{149 \text{ million km}}{1 \text{ AU}} = \frac{57 \text{ million km}}{X} \quad \text{then } X = 1 \text{ AU} \times (57/149) = 0.38$$

Planet	Distance to the Sun in millions of kilometers	Distance to the Sun in Astronomical Units
Mercury	57	0.38
Venus	108	0.72
Earth	149	1.00
Mars	228	1.52
Jupiter	780	5.20
Saturn	1437	9.58
Uranus	2871	19.14
Neptune	4530	30.20

Problem 2 – Suppose you wanted to build a scale model of our solar system so that the orbit of Neptune was located 10 feet from the yellow ball that represents the sun. How far from the yellow ball, in inches, would you place the orbit of Jupiter?

Answer: The proportion would be written as:

$$\frac{30.20 \text{ AU}}{10 \text{ feet}} = \frac{5.2 \text{ AU}}{X} \quad \text{then } X = 10 \text{ feet} \times (5.2/30.2) \quad \text{so } X = \mathbf{1.72 \text{ feet}}$$

Since 1 foot = 12 inches, the unit conversion is written as

$$1.72 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = \mathbf{20.64 \text{ inches}}.$$