



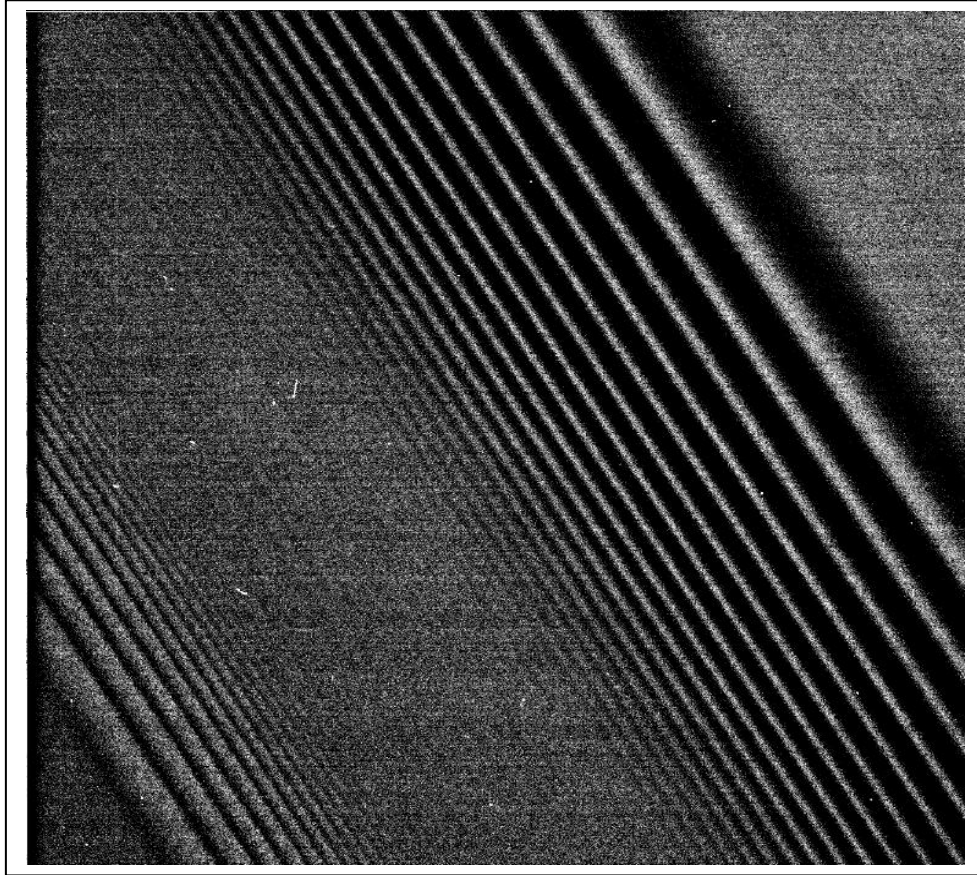
Math Problems Related to **The Solar System I**



Introduction

This book provides an assortment of 20 elementary mathematics problems that cover aspects of the solar system and its planets. Basic concepts such as the scale and size of a planet compared to Earth, exploring the surface of Mars and Saturn's rings are presented.

Page	Level	Math Skill	Topic
1	Beginning	Measurement	Saturn's Rings - A close-up study
2	Beginning	Data Analysis	The Orbit of Comet ISON
3	Beginning	Graphing	Comet ISON and its Close Encounter with Mars
4	Beginning	Data Analysis	Curiosity Heads for Mt Sharp
5	Beginning	Graphing	How to build a planet.
6	Beginning	Percentages	Comparing Comets Up Close with NASA Spacecraft
7	Beginning	Percentages	Searching for Comets
8	Beginning	Data Analysis	Comet Encounters after Discovery
9	Beginning	Arithmetic	Timeline for Planet Formation
10	Beginning	Geometry, Formulas	Volumes of Solids - Packing for a trip to the Moon
11	Beginning	Scale, Ratios	The Relative Sizes of Planets and other Objects.
12	Beginning	Scale	Asteroids and comets and meteors - Oh My! -
13	Beginning	Scale	A Matter of Timing
14	Beginning	Scale	Planets; Fractions and Scales
15	Beginning	Scale, Conversions	Pan's Highway and Saturn's Rings
16	Beginning	Ratios	Fractions in Space
17	Beginning	Arithmetic	Planetary Conjunctions
18	Beginning	Measurement	Exploring Black Holes.
19	Beginning	Scale, Ratios	Big Moons and Small Planets.
20	Beginning	Arithmetic	Telling Time on Mars



This spectacular close-up image of Saturn's A ring was taken in 2004 by the Cassini spacecraft. It shows a 220-km wide snapshot of a magnified portion of the A ring, and how it dissolves into smaller ringlets. Astronomers think that these ringlets are formed by gravitational interactions with Saturn's inner moons, causing ripples and waves to form that 'bunch up' billions of ring particles into separate ringlets. Some of the bright spots you see in the dark bands may be 'shepherding moonlets' only a few kilometers in size, which keep the ring particles orbiting together.

Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers.

Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer?

Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?

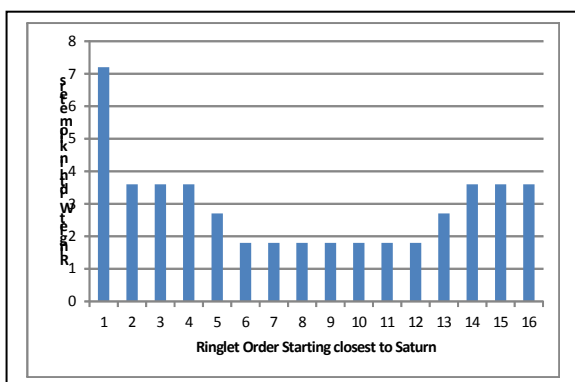
Problem 1 – By using a millimeter ruler, determine the scale of this image in kilometers/millimeter, and estimate the width of a typical ringlet in this image. Answer: When printed on standard 8 ½ x 11 paper, the width of the image is 124 millimeters, so the scale is 220 km/124mm = **1.8 km/mm**.

Problem 2 – Draw a diagonal line from the upper right corner (closest to Saturn) to the lower left corner (farthest from Saturn). Number the 16 ringlets in consecutive order starting from the first complete ringlet in the upper right corner. In a table, state the width of each consecutive ringlet in millimeters and kilometers. Answer: **See below**.

Ringlet	millimeters	kilometers
1	4	7.2
2	2	3.6
3	2	3.6
4	2	3.6
5	1.5	2.7
6	1	1.8
7	1	1.8
8	1	1.8
9	1	1.8
10	1	1.8
11	1	1.8
12	1	1.8
13	1.5	2.7
14	2	3.6
15	2	3.6
16	2	3.6

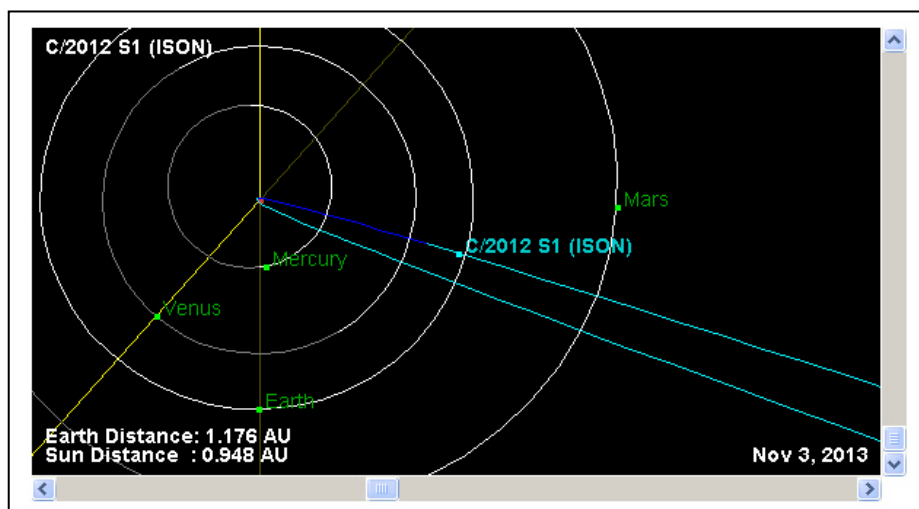
Problem 3 – What is the average width of the 16 ringlets you measured to the nearest kilometer? Answer: $(6 \times 3.6 + 7.2 + 2 \times 2.7 + 7 \times 1.8) / 16 = \mathbf{2.9 \text{ kilometers}}$.

Problem 4 – Plot the ringlet number and the ringlet width in kilometers. What can you say about the ringlet sizes in this portion of the A ring?



Answer: The ringlet widths decrease as you get further from Saturn and are present with distinct dark gaps in between. As the dark gaps become narrower than about 3 km near Ringlet 12, the rings begin to increase slightly in size.

It is 'interesting' that the ringlets in the lower right corner are wider than the upper ringlets, and there are fewer of them.



Comet ISON was discovered on September 21, 2012, and its orbit has been calculated from many observations since then. Instead of the normal circular or elliptical paths that comets, asteroids and planets often take, Comet ISON's orbit is nearly a parabola. This means that it came from far beyond the orbit of Neptune and that this is probably the first time it has entered the inner solar system! The figure shows its calculated path through the orbits of Mercury, Venus, Earth and Mars. The table below gives the distance between Comet ISON and the inner planets for the 15th of each month from July 2013 to March 2014..

Date	Sun	Mercury	Venus	Earth	Mars
July 15	434	497	411	586	283
August 15	365	320	430	502	168
September 15	289	326	397	389	58
October 15	204	267	300	258	48
November 15	91	45	151	135	163
December 15	104	159	76	79	201
January 15	213	255	123	101	233
February 15	299	264	270	224	322
March 15	367	422	410	348	416

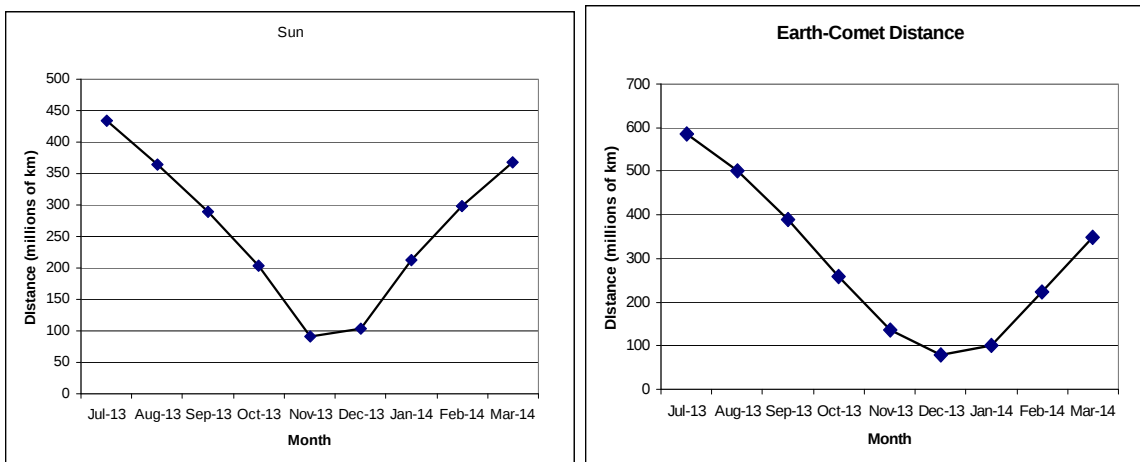
Problem 1 – Create two graphs of the distance data for; A) the Comet-Sun distance and 2) the Comet-Earth distance.

Problem 2 – What is the closest distance to the Earth that Comet ISON gets? If the distance to the moon from Earth is 380,000 km, how many times the Earth-Moon distance is the closest comet distance?

Problem 3 –Which planet with Comet ISON pass the closest too, and during which month?

Problem 4 – On November 28 COMET ISON passes within 1.2 million kilometers of the sun. At the Sun's Tidal Distance, objects are often pulled apart by the sun's intense gravity. For comets, this distance is about 1 million km from the center of the sun. Is it possible that Comet ISON may break apart?

Problem 1 – Create two graphs of the distance data for; A) the Comet-Sun distance and 2) the Comet-Earth distance.



Problem 2 – What is the closest distance to the Earth that Comet ISON gets? If the distance to the moon from Earth is 380,000 km, how many times the Earth-Moon distance is the closest comet distance?

Answer: From the graph, the closest distance is near December 15, 2013 at **79 million km**. This equals $79,000,000 / 380,000 = \mathbf{18 \text{ times the distance to the moon from Earth}}$.

Problem 3 –Which planet with Comet ISON pass the closest too, and during which month?

Answer: It comes closest to Mars around **October 15, 2013 at 48 million km**.

Problem 4 – On November 28 COMET ISON passes within 1.2 million kilometers of the sun. At the Sun's Tidal Distance, objects are often pulled apart by the sun's intense gravity. For comets, this distance is about 1 million km from the center of the sun. Is it possible that Comet ISON may break apart?

Answer: Comet ISON comes very close to the sun's TIDAL Limit on November 28, so it may suffer some fragmentation into smaller comets. Astronomers will closely watch this comet when it once again becomes visible from Earth on November 29 to see if it broke apart!

Note: Because we have given the distance to Comet ISON on the 15th of each month, this hides the fact that the Comet actually passes very close to each of these planets at a different date. SUN: November 28, 1.2 million km; Mercury: November 19, 36.3 million km. Venus: December 21, 73.4 million km; Earth: December 26, 64.2 million km and Mars: October 1, 10.9 million km. These estimates were based on the orbit known on July 29, 2013 but will change as the comet swings around the sun and its orbit changes slightly.



While most people will be admiring Comet ISON as it passes close by Earth and the Sun in the fall of 2013, other robotic observers on Mars will marvel as this same comet lights up the martian sky.

For the first time in human history, we will get to see the same comet from two different outposts of humanity in the solar system!

As a warm up, here is an image of Earth in the martian sky as seen by the Spirit Rover in 2004!

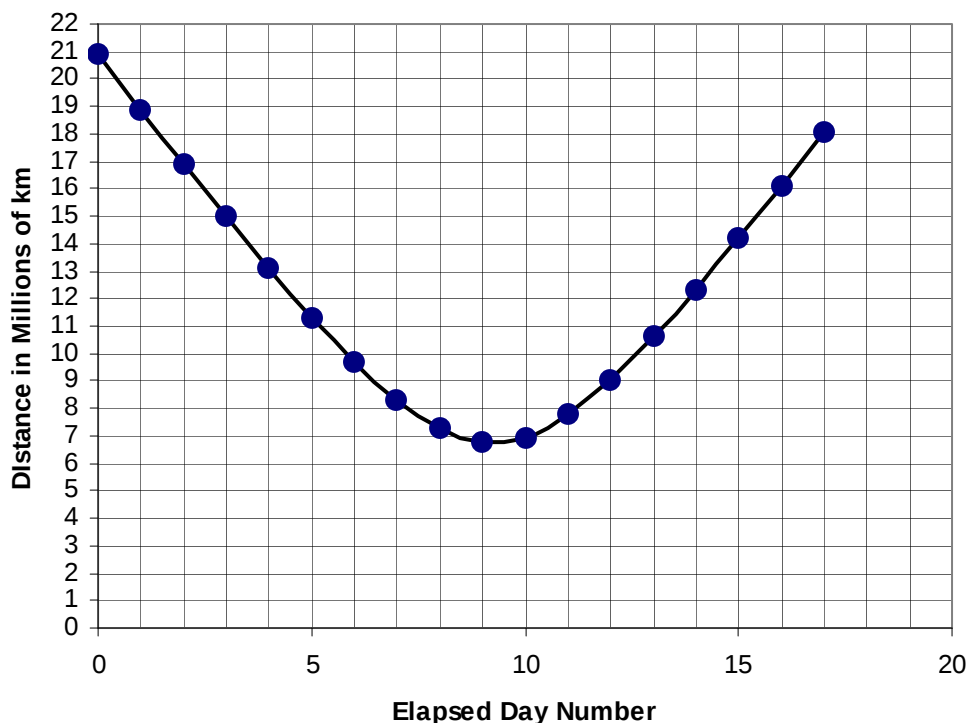
We can use the NASA, *Eyes on the Solar System* simulator to explore the positions of Comet ISON and Mars during the time of the encounter. Click on the following link to set up the program. <http://eyes.nasa.gov/index.html>. You will need to 'Launch' and download the software as instructed. Under 'Visual Controls' make sure the 'Small Bodies' and 'Metric' dots are white by selecting them. Right click on the label for Comet ISON and select 'Measure distance from' then click on the label for Mars to open the measurement window. Now open the 'Date and Time' menu and select the date and time using the sliders. Click on the UT ball to select Universal Time. No hit 'Submit' and the scene will move to the first location at URL: 1.usa.gov/18PtDeF for September 20 at 05:34 PM, UT. As you move forward in time, notice that the distance changes, beginning near 27 million km for this first scene. The table below gives a set of distance measurements (in millions of kilometers) during the close approach period at the same time of the day (13:00 UT). Select these, or create your own set using *Eyes on the Solar System*.

Date	Distance	Date	Distance	Date	Distance
9-22	20.9	9-28	9.7	10-4	9.0
9-23	18.9	9-29	8.3	10-5	10.6
9-24	16.9	9-30	7.3	10-6	12.3
9-25	15.0	10-1	6.8	10-7	14.2
9-26	13.1	10-2	6.9	10-8	16.1
9-27	11.3	10-3	7.8	10-9	18.1

Problem 1 - Graph the data with elapsed day number (9-11 = 0.0) on the horizontal axis, and the distance in millions of kilometers on the vertical axis. Draw a smooth curve through the data points.

Problem 2 – At what day and time will the distance be at its minimum?

Problem 1 - Graph the data with elapsed day on the horizontal axis, and the distance in millions of kilometers on the vertical axis. Draw a smooth curve through the data points. Answer: See below:



Problem 2 – At what day and time will the distance be at its minimum?

Answer: Although the point for October 1 (Day number 9) looks like the closest distance (10.8 million km), students will see that the curve reaches a slightly smaller distance value between Day 9 and Day 10., which correspond to October 1 at 13:00 UT and October 2 at 13:00 UT, 24-hours later. Invite the students to create a better estimate between these two dates and times using their graphs, or by using Eyes on the Solar System. **Answers close to October 1 near 19:00 UT are acceptable. The distance will be close to 6.75 million km.**

Note: The orbit for Comet ISON will be updated as new data is obtained, so the actual numbers used in these problems are only current for July 28, 2013. Students will be using the updated values and will need to use Eyes on the Solar system to get the most up to date values.



In July 2013, Curiosity began its long journey to the base of Mt Sharp, seen in the distance in this image. Because it is operated robotically, it only travels a few meters every hour and communicates with its Earth technicians after each step. Because radio waves take 20 minutes or longer to reach earth, 40 minutes elapse before a transmitted command is received and the results of the action can be verified.

The table below gives the progress made by Curiosity during several days of operation on Mars, called Sols.

Sol	Drive	Duration (minutes)	Odometer (meters)	Azimuth (degrees)	Pitch (degrees)
345	68	83	1490	235	-1
347	69	67	1550	236	-2
349	70	72	1621	190	+1
351	71	94	1706	249	+1
354	72	65	1763	304	+1

Problem 1 – What is the average time that Curiosity drove each day?

Problem 2 – What is the average distance traveled each day?

Problem 3 – ‘Azimuth’ is the direction you are pointing from North so that due North is 0° , East is 90° , South is 180° and West is 270° . What is the average azimuth angle that Curiosity traveled along during the tabulated period?

Problem 4 – ‘Pitch’ is the tilt angle of the land, with straight up being $+90^\circ$, horizontal being 0° and straight down being -90° . What is the average pitch of Curiosity’s travels during this period and what can you tell about the ground over which it traveled?

Problem 5 – The direction to Mt Sharp is at an azimuth of 225° . What does Curiosity’s average azimuth have to be during the next 5 days so that it is back on course to Mt Sharp?

Problem 6 – Mt Sharp is located 7.5 km from Curiosity. About how many more Sols will be required for Curiosity to get there?

Problem 1 – What is the average time that Curiosity drove each day?

Answer: $T = (83+67+72+94+65)/5 = 381 \text{ minutes}/5 = \mathbf{76 \text{ minutes/day}}$

Problem 2 – What is the average distance traveled each day?

Answer: $D = (60 + 71 + 85 + 57)/4 = 273 \text{ meters}/4 = \mathbf{68 \text{ meters/day}}$.

Problem 3 – ‘Azimuth’ is the direction you are pointing from North so that due North is 0° , East is 90° , South is 180° and West is 270° . What is the average azimuth angle that Curiosity traveled along during the tabulated period?

Answer: $A = (235+236+190+249+304)/5 = 1214/5 = \mathbf{243^\circ}$

Problem 4 – ‘Pitch’ is the tilt angle of the land, with straight up being $+90^\circ$, horizontal being 0° and straight down being -90° . What is the average pitch of Curiosity’s travels during this period and what can you tell about the ground over which it traveled?

Answer: $P = (-1 + -2 + +1 + +1 + +1)/5 = 0^\circ$ So the terrain was very level and horizontal.

Problem 5 – The direction to Mt Sharp is at an azimuth of 225° . What does Curiosity’s average azimuth have to be during the next 5 days so that it is back on course to Mt Sharp?

Answer: $225 = [5 (243) + 5(X)] / 10$
 $2250 - 1214 = 5X$ so $\mathbf{X = 207^\circ}$

Problem 6 – Mt Sharp is located 7.5 km from Curiosity. About how many more Sols will be required for Curiosity to get there?

Answer: The average distance traveled was 68 meters/day so to travel 7500 meters will take $T = 7500/68 = 110$ Sols if Curiosity does not stop along the way.



When the solar system was young, planets were built as huge numbers of smaller bodies like asteroids collided with each other. Over millions of years of collisions, planets like Earth grew to their present sizes.

To see how this happened, we will model the process using a ball of clay!

Problem 1 - Take one-half pound of clay and form it into a round ball. Slice this round ball exactly in half and measure its diameter using a millimeter ruler.

Problem 2 – Take the two halves of the clay ball and re-form them into a round ball again. Now divide this ball into ten equal pieces of clay and roll each of these into round balls of about equal sizes. Slice one of these balls in half and measure its diameter.

Problem 3 – On a piece of graph paper, mark the horizontal axis with the number of small balls used from one to ten and the vertical axis with the diameter of the finished ball from 1 centimeter to 20 centimeters. Plot the diameter of one of these small balls on the graph.

Problem 4 – Take two of the small balls and roll them together into a single ball. Cut this new ball in half and measure its diameter. Mark the finished ball on the graph by its size in centimeters and the number of small balls '2'. Continue this process until you have collected all ten balls into one large ball and plot the diameter of the large ball and the number of small balls used '10'.

Problem 5 – Connect the ten points on the graph. What do you notice about the curve you plotted?

Problem 6 – If the large ball represented the final size of our Earth with a diameter of 12,000 kilometers, on this scale of the clay balls, how big would each of the ten 'planetessimals' have been that collided to form the final planet?

Problem 7 – Suppose you started with 100 equal-sized small clay balls. Would you get the same kind of plotted curve?

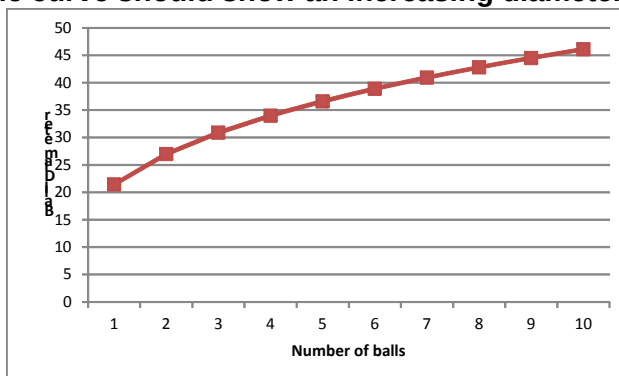
Problem 1 - Take one-half pound of clay and form it into a round ball. Slice this round ball exactly in half and measure its diameter using a millimeter ruler. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 2 – Take the two halves of the clay ball and re-form them into a round ball again. Now divide this ball into ten equal pieces of clay and roll each of these into round balls of about equal sizes. Slice one of these balls in half and measure its diameter. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 3 – On a piece of graph paper, mark the horizontal axis with the number of small balls used from one to ten and the vertical axis with the diameter of the finished ball from 1 centimeter to 20 centimeters. Plot the diameter of one of these small balls on the graph. **Answer will depend on how the students make the ball and how much clay is used.**

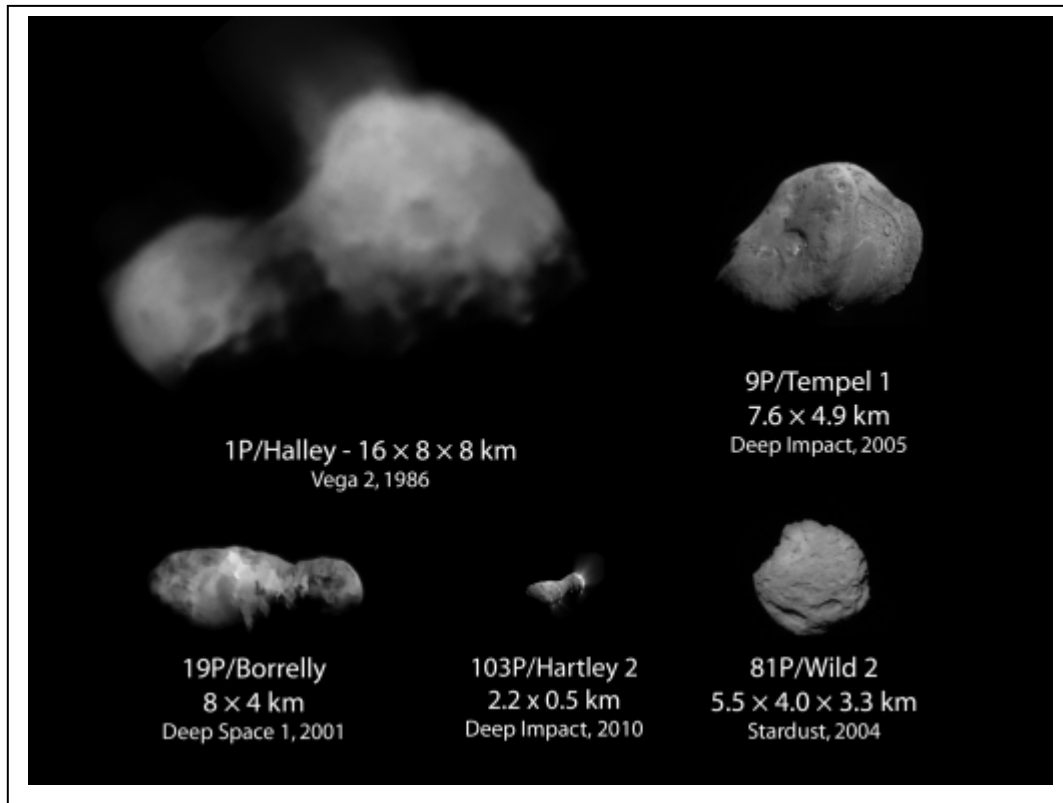
Problem 4 – Take two of the small balls and roll them together into a single ball. Cut this new ball in half and measure its diameter. Mark the finished ball on the graph by its size in centimeters and the number of small balls '2'. Continue this process until you have collected all ten balls into one large ball and plot the diameter of the large ball and the number of small balls used '10'. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 5 – Connect the ten points on the graph. What do you notice about the curve you plotted? **Answer: The curve should show an increasing diameter from left to right.**



Problem 6 – If the large ball represented the final size of our Earth with a diameter of 12,000 kilometers, on this scale of the clay balls, how big would each of the ten 'planetessimals' have been that collided to form the final planet? **Answer:** Suppose the big balls final diameter is 45 millimeters and the small balls diameter is 20 mm. Use a simple proportion to find $12,000/45 = X/20$ and so **$X = 5300 \text{ km}$** . This is a bit larger than the diameter of our moon (3,500 km).

Problem 7 – Suppose you started with 100 equal-sized small clay balls. Would you get the same kind of plotted curve? **Answer: The curve would look the same but the scale on the horizontal axis would be changed. Each of the small balls would have a diameter of about 10 mm and represent a planetesimal with a diameter of about $12000/45 = X/10$ so $X = 2700$ kilometers or slightly smaller than the diameter of our moon!**



Spacecraft have flown-by five comets to study the dense object which produces the dramatic head and tails of these objects as seen from Earth. The figure above shows images of the nuclear objects to the same scale.

Problem 1 – What percentage of comet nuclei are:

- A) round
- B) potatoe-shaped?

Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Problem 1 – What percentage of comet nuclei are:

A) Round - answer $100\% \times (2/5) = 40\%$

B) potatoe-shaped? - answer $100\% \times (3/5) = 60\%$

Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Answer: The smallest dimension is for Comet Hartley 2 at 0.5 km. The largest dimension is for Halleys Comet at 16 km. **So the range is from 0.5 to 16 kilometers.**

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Answer: $S = 0.5$ km so volume $= 0.5 \times 0.5 \times 0.5 = 0.125 \text{ km}^3$
 $S = 16$ km, so volume $= 16 \times 16 \times 16 = 4096 \text{ km}^3$.

The range of volumes is 0.125 to 4096 km³.

Year	Total	Amateurs	Spacecraft	Observatories
2012	60	27	1	32
2011	49	28	2	19
2010	161	29	119	13
2009	227	35	188	4
2008	220	34	182	4
2007	223	35	170	18
2006	206	31	152	23
2005	221	23	169	29
2004	223	8	172	43
2003	193	7	149	37
2002	182	9	131	42
2001	149	4	107	38
2000	135	9	99	27
1999	129	18	87	24

Every year, professional astronomers and dedicated amateur astronomers use everything from simple binoculars to sophisticated computer-driven telescopes to discover new comets. The table to the left gives a count of the number of comets detected between 1999 and 2012.

Comet hunters carefully compare images of the same part of the sky over a period of days or weeks. Although stars remain fixed, comets appear as fuzzy spots of light that change their positions.

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

The tabulated data is based upon the Catalog of Comet Discoveries archive at <http://www.comethunter.doc>

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

Answer: Amateur astronomers: $100\% \times (29/161) = \mathbf{18\%}$
Spacecraft: $100\% \times (119/161) = \mathbf{74\%}$
Observatories : $100\% \times (13/161) = \mathbf{8\%}$

Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Answer: Total for amateurs = 297 over 14 years so the average was **21 comets/year**.
For observatories: 353 over 14 years so the average is **25 comets/year**.

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

Answer: In 2012 the total number of comets was 60, and of these 27 were detected by amateur astronomers, so $60 - 27 = 33$ detected by other means, then $100\% \times (33/60) = \mathbf{55\%}$ of the comets would have remained undetected.

Name	Discovery	Earth Distance	Date
C/2011 UF305	October 2011	375	June 2012
C/2011 L4	June 2011	495	June 2012
C/2011 F1	March 2011	300	August 2012
C/2011 R1	September 2011	330	Sept 2012
C/2010 S1	September 2010	840	Sept 2012
C/2012 J1	May 2012	345	Oct 2012
273P	June 1827	128	Oct 2012
C/2012 J1	May 2012	360	Nov 2012
C/2012 K5	May 2012	50	Dec 2012

Comets are icy bodies that can be many kilometers across. If they struck earth they would do considerable damage and could even lead to extinctions. One of the biggest problems that astronomers face is that, unlike asteroids, comets come from places in our solar system that are far beyond the orbit of Jupiter. They are usually discovered as very faint fuzzy objects once they reach the orbit of Mars. By that time, there is not much warning should the comet have a path that intersects earth's orbit.

The table above gives the month and year when a comet was discovered (column 2), the closest distance that it got to earth in millions of kilometers (column 3) and the month and year when it makes its closest approach to the sun and earth (column 4).

Problem 1 - How many months after the comet was discovered did it make its next closest passage to Earth? Omitting Comet 273P, what is the average number of months between discovery and close passage?

Problem 2 – What percentage of the comets passed further away than 50 million kilometers from Earth?

Problem 3 – During 2012 there were 60 new comets discovered. About how many of these might come within 50 million kilometers of earth during the following year, 2013?

Problem 1 - How many months after the comet was discovered did it make its next closest passage to Earth? Omitting Comet 273P, what is the average number of months between discovery and close passage?

Name	Discovery	Earth Distance	Date	Time
C/2011 UF305	October 2011	375	June 2012	8
C/2011 L4	June 2011	495	June 2012	12
C/2011 F1	March 2011	300	August 2012	17
C/2011 R1	September 2011	330	Sept 2012	12
C/2010 S1	September 2010	840	Sept 2012	24
C/2012 J1	May 2012	345	Oct 2012	6
273P	June 1827	128	Oct 2012	2224
C/2012 J1	May 2012	360	Nov 2012	7
C/2012 K5	May 2012	50	Dec 2012	8

Answer: Omitting 273P , the average is $(8+12+17+12+24+6+7+8)/8 = 12$ months.

Problem 2 – What percentage of the comets passed further away than 50 million kilometers from Earth?

Answer: $100\% \times (8/9) = 89\%$

Problem 3 – During 2012 there were 60 new comets discovered. About how many of these might come within 50 million kilometers of earth during the following year, 2013?

Answer: $60 \times 0.89 = 53$ that pass farther away than 50 million km, and so there are **7 that pass closer than 50 million km.**

Note: What this means is that by the time a comet is discovered, it is only a year from its closest approach to earth, and for a typical year there could be 7 comets that come this close. These are potentially dangerous objects because their sizes are big enough to devastate cities or even entire continents if they were to impact!

Era	Time (years)	Description
Pre-solar Nebula Era	0.0	Collapse of cloud to form flattened disk
Asteroid Era	3 million	Formation of large asteroids up to 200 km across ends
Gas Giant Era	10 million	Rapid formation of Jupiter and Saturn ends
Solar Birth Era	50 million	Sun's nuclear reactions start to produce energy in core
Planetesimal Era	51 million	Formation of numerous small planet-sized bodies ends
T-Tauri Era	80 million	Solar winds sweep through inner solar system and strip off primordial atmospheres
Ice Giant Era	90 million	Formation of Uranus and Neptune
Rocky Planet Era	100 million	Formation of rocky planets by mergers of 50-100 smaller bodies
Late Heavy Bombardment Era	600 million	Migration of Jupiter disrupts asteroid belt sending large asteroids to impact planetary surfaces in the inner solar system.
Ocean Era	600 million	LHB transports comets rich in water to Earth to form oceans
Life Era	800 million	First traces of life found in fossils on Earth

For decades, geologists and astronomers have studied the contents of our solar system. They have compared surface features on planets and moons across the solar system, the orbits of asteroids and comets, and the chemical composition and ages for recovered meteorites. From all this effort, and with constant checking of data against mathematical models, scientists have created a timeline for the formation of our solar system.

Our solar system began as a collapsing cloud of gas and dust over 4.6 billion years ago. Over the next 600 million years, called by geologists the Hadean Era, the sun and the planets were formed, and Earth's oceans were probably created by cometary impacts. Comets are very rich in water ice.

The fossil record on Earth shows that the first bacterial life forms emerged about 600 million years after the formation of the solar system. Geologists call this the Archaean Era – The era of ancient life.

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Problem 4 – How many years were there between the Planetesimal Era and the end of the Rocky Planet Era?

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Answer: On the Timeline '0.0' represents a time 4.6 billion years ago, so the Rocky Planet Era ended 100 million years after this or **4.5 billion years ago**.

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Answer: LHB ended 600 million years after Time '0.0' or $4.6 \text{ billion} - 600 \text{ million} = \mathbf{4.0 \text{ billion years ago}}$.

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

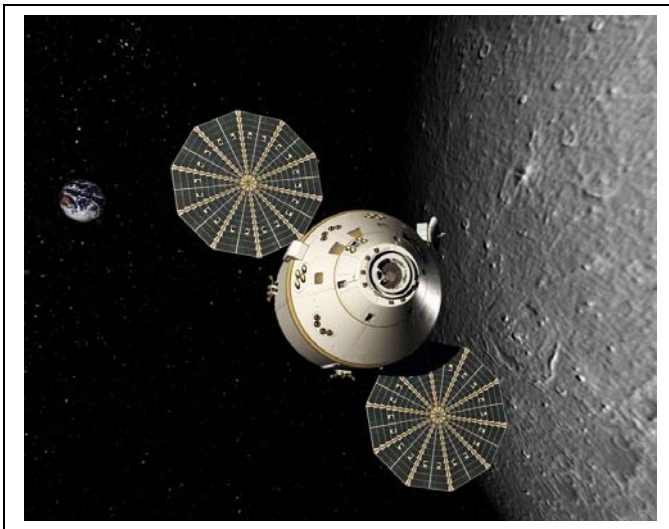
Answer: $4.6 \text{ billion} - 800 \text{ million} = \mathbf{3.8 \text{ billion years ago}}$.

Problem 4 – How many years were there between the Planetesimal Era and the end of the Rocky Planet Era?

Answer: On the timeline the difference is $100 \text{ million} - 51 \text{ million} = \mathbf{49 \text{ million years}}$.

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Answer: The time interval is 49 million years so the average time between impacts would have been $49 \text{ million years} / 80 \text{ impacts} = \mathbf{612,000 \text{ years}}$.



Astronauts traveling to the moon will not be able to bring everything with them. They will have a very strict allowance on the volume and mass of what they can bring.

Although the first moon rocket, Ares-V/Orion, will not be launched for another 10 years, let's suppose the 'Astronaut Preference Kit' volume limit is $2 \frac{1}{2}$ liters per astronaut.

An astronaut wants to bring the following items with him. Assume that the items are all shaped like rectangular solids. Given their dimensions, will he be within the volume limits of the Orion vehicle? (All measurements are in centimeters, and 1,000 cubic centimeters equals 1 liter.)

Item	Length	Width	Height	Total
Toothbrush	17	1	1	
Diary book	25	2	15	
Pen	14	1	1	
Ipod	11	7	1	
Camera	13	9	10	
One paperback book	17	10	2	
Extra glasses	14	5	2	

If the total volume is greater than the PPK limit, what item or items would you leave behind? Is there anything you could do to reduce the volume of these items so that they add up to less than the PPK limit?

If the total volume is less than the PPK limit, what items would you add if the trip was to last 1 week on the Space Shuttle or three months on the International Space Station?

Are there any items you could think of that you would want to substitute instead of the items on this list?

Item	Length	Width	Height	Total
Toothbrush	17	1	1	17
Diary book	25	2	15	750
Pen	14	1	1	14
Ipod	11	7	1	77
Camera	13	9	10	1170
One paperback book	17	10	2	340
Extra glasses	14	5	2	140

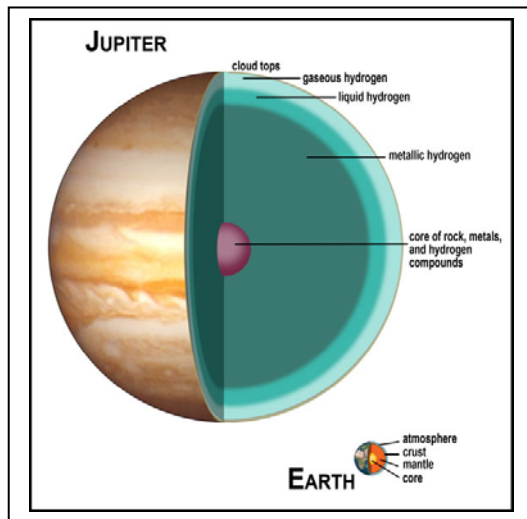
The total volume of all these items is 2,508 cubic centimeters. Since 2 1/2 liters equals 2,500 cubic centimeters, the astronaut has to eliminate at least 8 cubic centimeters of material from this list. The total volume is **greater than the PPK limit**.

Some students may not need glasses, while others may decide that reading a book is the last thing they want to do in space with all those great views out the window!

Here are some of the things that NASA astronauts have brought into space:

- 400 US commemorative stamps (Apollo 15)
- Wedding rings (STS-36)
- 33rd Degree Masonic Ring (Gemini 6)
- Florida Hunting License (Gemini-6)
- Piece of wood from Wright Brothers 1903 airplane (Apollo 11)
- Vial of wine and miniature chalice (Apollo 11)
- Walkman with headphones (Spacelab)
- 1923 US Peace Dollar (Apollo 11)
- Moon Tree seeds (Apollo 14)

Can your students research this topic and find more items?



The actual sizes of the major objects in our solar system range from the massive planet Jupiter, to many small moons and asteroids no more than a few kilometers across.

It is often helpful to create a scaled model of the major objects so that you can better appreciate just how large or small they are compared to our Earth.

This exercise will let you work with simple proportions and fractions to create a scaled-model solar system.

Problem 1 - Jupiter is $\frac{7}{6}$ the diameter of Saturn, and Saturn is $\frac{5}{2}$ the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter?

Problem 2 – Earth is $\frac{13}{50}$ the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn?

Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km), Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

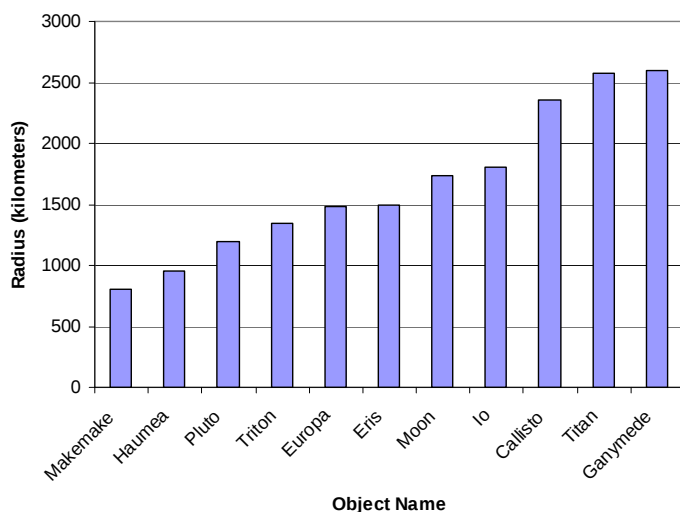
Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters?

Problem 1 - Jupiter is $\frac{7}{6}$ the diameter of Saturn, and Saturn is $\frac{5}{2}$ the diameter of Uranus. Expressed as a simple fraction, how big is Uranus compared to Jupiter? Answer: $\frac{2}{5} \times \frac{6}{7} = \frac{12}{35}$.

Problem 2 – Earth is $\frac{13}{50}$ the diameter of Uranus. Expressed as a simple fraction, how much bigger than Earth is the planet Saturn? Answer: $\frac{5}{2} \times \frac{50}{13} = \frac{250}{26} = \frac{125}{13}$ times.

Problem 3 – The largest non-planet objects in our solar system, are our own Moon (radius=1738 km), Io (1810 km), Eris (1,500 km), Europa (1480 km), Ganymede (2600 km), Callisto (2360 km), Makemake (800 km), Titan (2575 km), Triton (1350 km), Pluto (1,200 km), Haumea (950 km). Create a bar chart that orders these bodies from smallest to largest. For this sample, what is the: A) Average radius? B) Median radius?

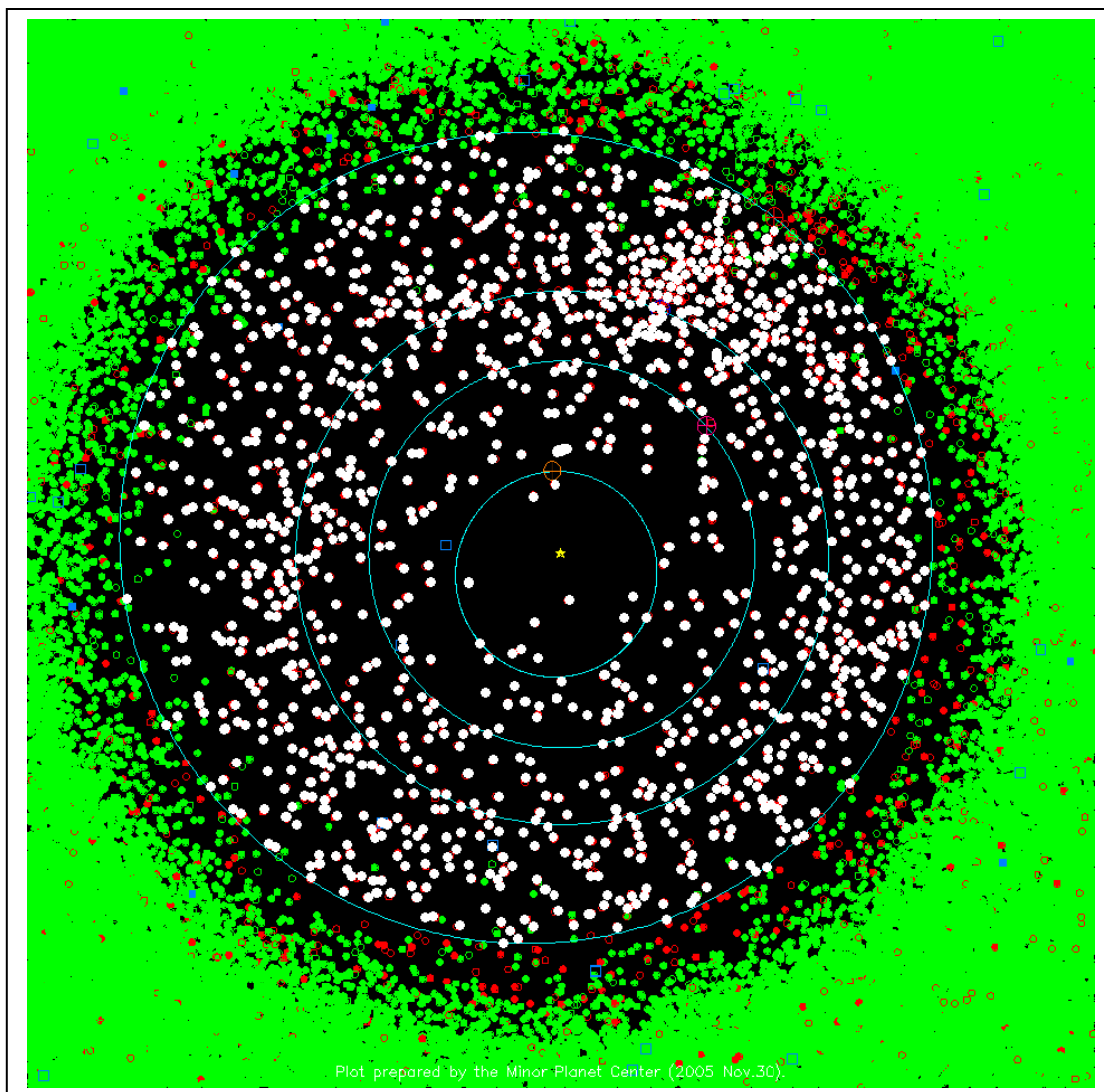
Answer: Average = $(1738+1810+1500+1480+2600+2360+800+2575+1350+1200+950)/11$ so
Average radius = 1669 km. Median radius = 1500 km.



Problem 4 – Mercury is the smallest of the eight planets in our solar system. If the radius of Mercury is 2425 km, how would you create a scaled model of the non-planets if you selected a diameter for the disk of Mercury as 50 millimeters? Answer: The scaled disk diameters are shown in Column 3 in millimeters.

Object	Radius (km)	Diameter (mm)
Makemake	800	16
Haumea	950	20
Pluto	1200	25
Triton	1350	28
Europa	1480	30
Eris	1500	30
Moon	1738	36
Io	1810	38
Callisto	2360	48
Titan	2575	54
Ganymede	2600	54

Astronomers have catalogued and determined orbits for 30,000 minor planets in the solar system (asteroids, comets etc). Over 150,000 bodies larger than a few hundred meters across have been spotted and remain to have their orbits exactly determined. Below is a plot made on November 30, 2005 of the locations of all known objects (white dots) within the orbit of Mars whose path skirts the inner edge of the asteroid belt (green dots).



Question 1 – How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars?

Question 2 – If the radius of Earth's orbit is 150 million km, what is the scale of this figure in millions of km per millimeter?

Question 3 – About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel?

Question 4 – How many asteroids crossed Earth's orbit on November 30, 2005?

The plot of the minor planets was obtained from the IAU, Minor Planets Center (<http://cfa-www.harvard.edu/iau/lists/InnerPlot2.html>). It shows the location of the known asteroids, comets and other 'minor planets' for November 30, 2005. The plot shows the orbits of Mercury, Venus, Earth and Mars. Objects that have perihelion (closest orbit location to the sun) less than 1.3 AU are shown in white circles. More details can be found at http://www.space.com/scienceastronomy/solarsystem/asteroid_toomany_011019-1.html

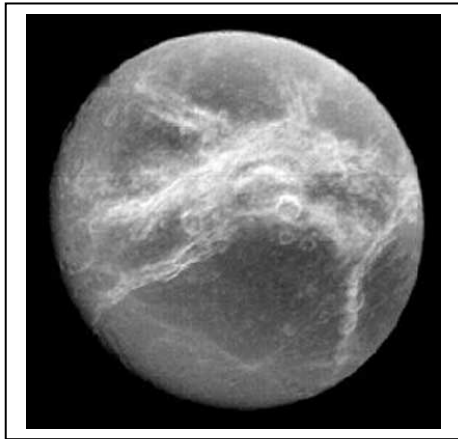
Question 1 – How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars? **Answer:** Students should count the plotted symbols within (or on) the first inner ring (Mercury's orbit) and get **13** symbols (don't include the sun!). For the space between Venus and Mercury, I count 119 spots which makes the total **132** minor planets inside the orbit of Venus. Between Earth and Venus there are about 280 for a total of 412 minor planets inside Earth's orbit. Between Mars and Earth, a careful student may be able to count about 833 which means there are $833+412 = 1245$ minor planets inside the orbit of Mars.

Question 2 – What is the scale of this figure in millions of km per millimeter? **Answer:** The radius of Earth's orbit is 150 million kilometers, which corresponds to 70 millimeters, so the scale is 2.1 million km per millimeter.

Question 3 – About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel? **Answer:** Although the asteroids are only plotted as though they are located in the same 2-D plane, we can estimate from the average 'eyeball' separation between asteroids of about 2 millimeters, that they are about 4.2 million kilometers apart. A spacecraft would not collide with a typical asteroid unless it was directed to specifically target an asteroid for investigation...or impact. It is a popular myth about space travel that astronauts have to dodge asteroids when traveling to Mars or the outer solar system. Interplanetary dust grains and micro-meteoroids are, however, a much bigger hazard!!

Question 4 – How many asteroids crossed Earth's orbit on November 30, 2005? **Answer:** Just count the number of white spots that touch the line that represents the orbit of Earth. There are about 70 spots that touch the orbit line.

Could the Earth collide with them? Each dot is about 1 mm in radius, so this represents a distance of 2 million kilometers. Since Earth is only 12,000 km across and a typical asteroid is only 1 km across, collision is extremely unlikely even when the diagram seems to show otherwise. There is another way that this diagram makes the situation look worse than it is. Because the asteroid orbits can be several million miles above or below the orbit of Earth as the asteroids cross this location, there are very few close calls between Earth and any given asteroid in the current catalog. Astronomers call the ones that get close 'Near-Earth Asteroids' and there are about 700 of these known. Only **one** of these known NEAs may get close enough to Earth in the next 30 years to be a potential collision problem, but astronomers are still finding dozens more NEAs every year.



Astronomers who study planets and their satellites often have to work out how often satellites or planets 'line up' in various ways, especially when they are closest together in space.

Figure shows the satellite Dione (Courtesy: NASA/Cassini)

Problem 1 – The two satellites of Tethys and Dione follow circular orbits around Jupiter. Tethys takes about 2 days for one complete orbit while Dione takes about 3 days. If the two satellites started out closest together on July 1, 2008, how many days later will they once again be at 'opposition' with one another?

- A) Find the Least Common Multiple between the orbit periods.
- B) Draw two concentric circles and work the solution out graphically.
- C) What is the relationship between your answer to A and B?

Problem 2 - Two planets have orbit periods of 3 years and 5 years. How long will it take them to return to the same locations that they started at?

Problem 1 - A) The Least Common Multiple between 2 and 3 is 6, so it will take 6 days for the two moons to return to their original positions. B) The figure below shows the progression in elapsed days, with the moons moving counterclockwise. C) The LCM between the orbit periods tells you how long it will take for the two bodies to return to their same locations when they started.

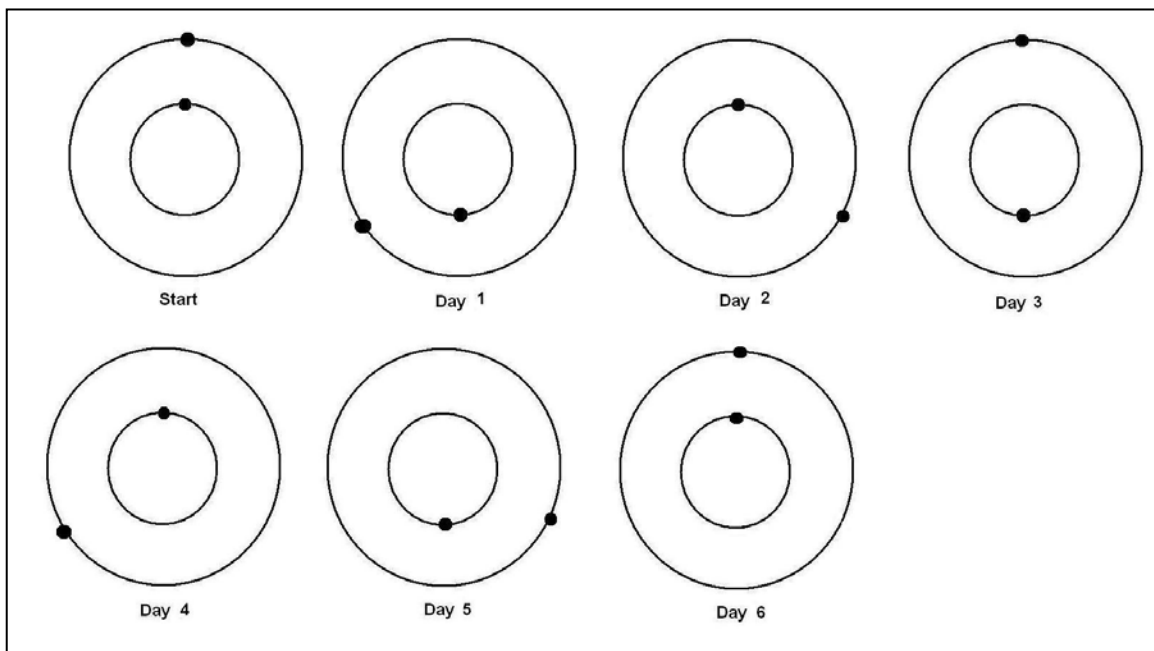
Problem 2 - Two planets have orbit periods of 3 years and 5 years. How long will it take them to return to the same locations that they started at?

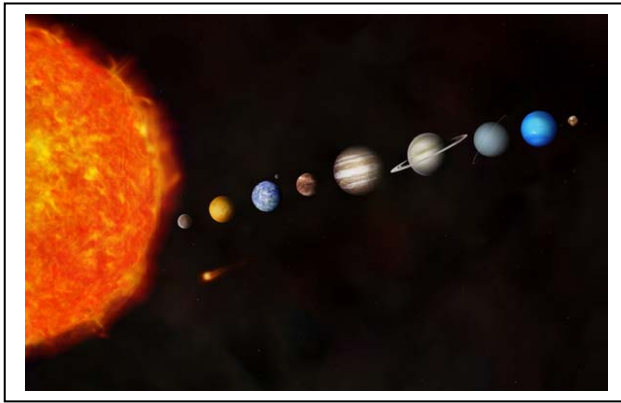
Answer; The LCM for 3 and 5 is found by forming the multiples of 3 and 5 and finding the first number they share in common.

For 3: 3, 6, 9, 12, **15**, 18, 21, 24, 27, 30, 33, 36,

For 5: 5, 10, **15**, 20, 25, 30, ...

The smallest common multiple is '15', so it will take the two planets 15 years to return to the positions they started with.





Some of the planets in our solar system are much bigger than Earth while others are smaller. By using simple fractions, you will explore how their sizes compare to each other.

Image courtesy NASA/Chandra Observatory/SAO

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is $\frac{1}{4}$ the size of Neptune. How much larger is Saturn than Neptune?

Problem 2 - Earth is twice as big as Mars, but only $\frac{1}{11}$ the size of Jupiter. How large is Jupiter compared to Mars?

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?

Problem 4 - Mercury is $\frac{3}{4}$ the size of Mars. How large is Earth compared to Mercury?

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?

Problem 6 - The satellite of Saturn, called Titan, is $\frac{1}{10}$ the size of Uranus. How large is Titan compared to Earth?

Problem 7 - The satellite of Jupiter, called Ganymede, is $\frac{2}{5}$ the size of Earth. How large is it compared to Jupiter?

Problem 8 - The Dwarf Planet Pluto is $\frac{1}{3}$ the diameter of Mars. How large is the diameter of Jupiter compared to Pluto?

Problem 9 - If the diameter of Earth is 13,000 km ,what are the diameters of all the other bodies?

Note to teachers: The actual diameters of the planets, in kilometers, are as follows

Mercury	4,900 km	Jupiter	143,000 km
Venus	12,000 km	Saturn	120,000 km
Earth	13,000 km	Uranus	51,000 km
Mars	6,800 km	Neptune	50,000 km

Also: Titan = 5,100 km, Ganymede = 5,300 km Ceres = 950 km, and Pluto 2,300 km

Advanced students (Grades 4 and above) may use actual planetary size ratios as decimal numbers, but for this simplified version (Grades 2 and 3), we approximate the size ratios to the nearest simple fractions. **Students may also use the information in these problems to make a scale model of the solar system in terms of the relative planetary sizes.**

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is $\frac{1}{4}$ the size of Neptune. How much larger is Saturn than Neptune?

Answer: Neptune is 4x Venus and Saturn is 10x Venus, so Saturn is $\frac{10}{4} = \frac{5}{2}$ times as big as Neptune.

Problem 2 - Earth is twice as big as Mars, but only $\frac{1}{11}$ the size of Jupiter. How large is Jupiter compared to Mars?

Answer: Jupiter is 11 x Earth, and Mars is $\frac{1}{2}$ Earth, so Jupiter is 22x Mars.

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?

Answer: If Saturn is 10 x Venus, and Jupiter is 11 x Earth, Jupiter is $\frac{11}{10}$ times Saturn.

Problem 4 - Mercury is $\frac{3}{4}$ the size of Mars. How large is Earth compared to Mercury?

Answer: Mars is $\frac{1}{2}$ x Earth, so Mercury is $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ x Earth

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?

Answer: Neptune was 4x Venus, but since Venus = earth and Neptune=Uranus, we have Uranus = 4x Earth.

Problem 6 - The satellite of Saturn, called Titan, is $\frac{1}{10}$ the size of Uranus. How large is Titan compared to Earth?

Answer: Titan / Uranus = $\frac{1}{10}$, but Uranus/Earth = 4, so Titan/Earth = $\frac{3}{10} \times 4 = \frac{2}{5}$.

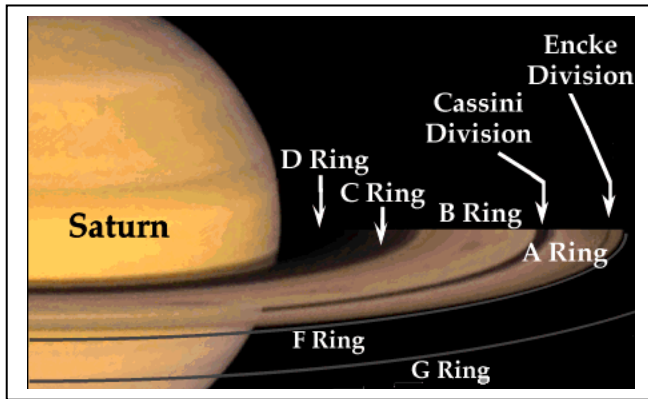
Problem 7 - The satellite of Jupiter, called Ganymede, is $\frac{2}{5}$ the size of Earth. How large is it compared to Jupiter?

Answer: Earth = $\frac{1}{11}$ Jupiter so Ganymede is $\frac{1}{11} \times \frac{2}{5} = \frac{2}{55}$ x Jupiter.

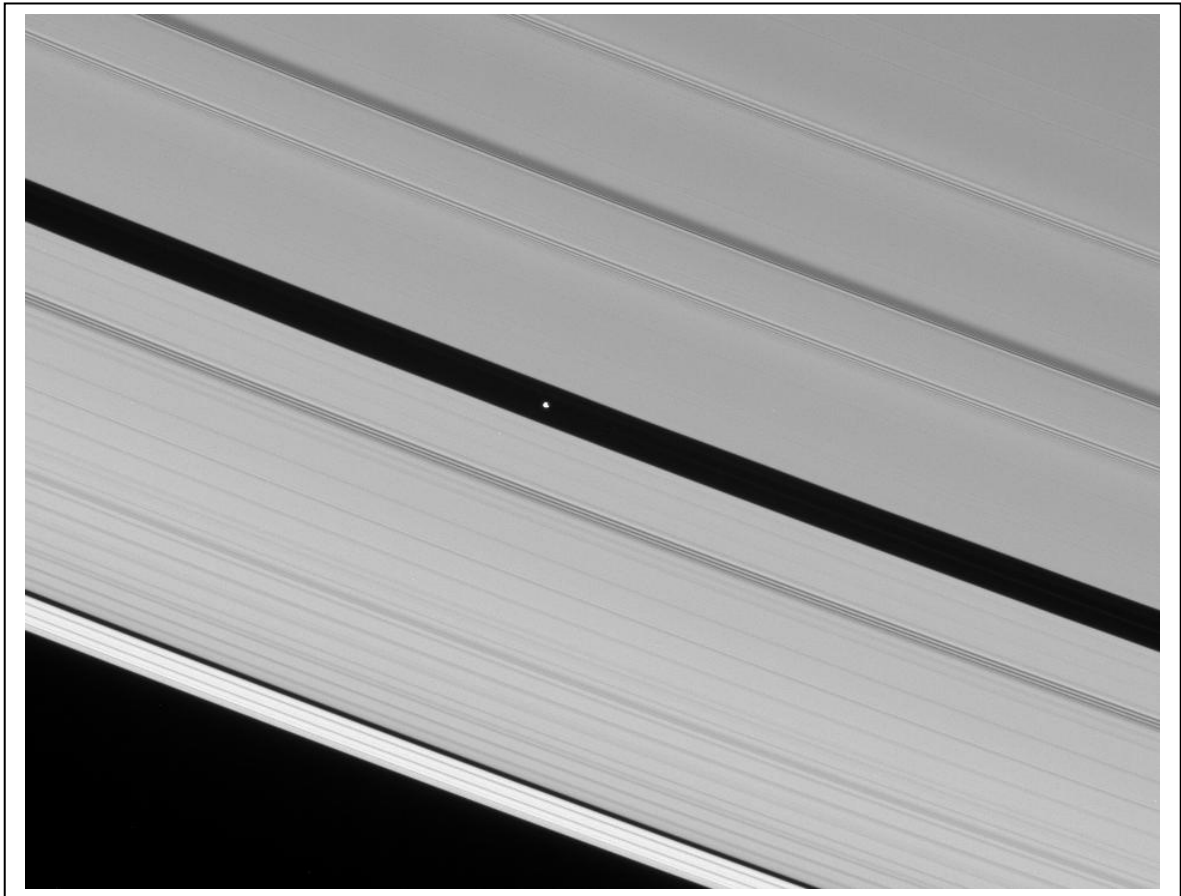
Problem 8 - The Dwarf Planet Pluto is $\frac{1}{3}$ the size of Mars. How large is Jupiter compared to Pluto?

Answer: Jupiter = $\frac{1}{11}$ Earth, Mars= $\frac{1}{2}$ Earth, so Pluto= $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ Earth, and $\frac{1}{66}$ Jupiter.

Problem 9 – **Answer: Students should, very nearly, reproduce the numbers in the table at the top of the page.**



The Encke Gap is a prominent feature of Saturn's outer A-ring system that has been observed since the 1830's. The arrival of the Cassini spacecraft in July 2004 revealed the cause for this gap. A small moonlet called Pan clears out the ring debris in this region every 12 hours as it orbits Saturn!



Problem 1 - This image was taken by Cassini in 2007 and at the satellite's distance of 1 million kilometers, spans a field of view of 5,700 km x 4,400 km. With the help of a millimeter ruler, what is the scale of the image in kilometers per millimeter?

Problem 2 - Pan is that bright spot within the black zone of the Encke Gap. About how many kilometers in diameter is Pan?

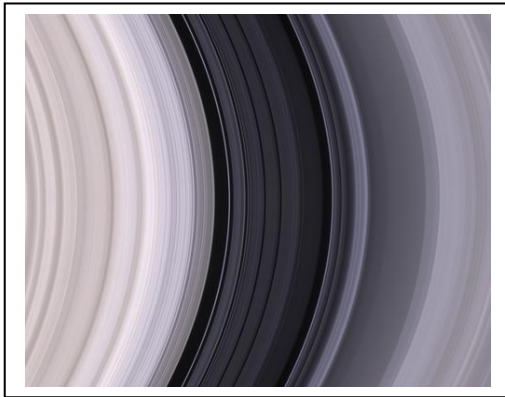
Problem 3 - About how wide is the Encke Gap?

Problem 4 - About what is the smallest feature you can discern in the photo?

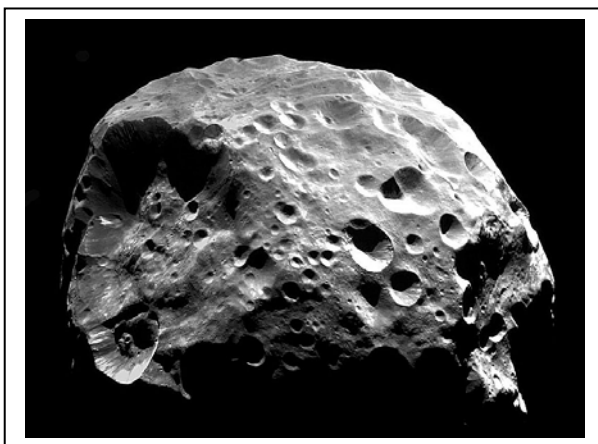


Problem 1 - The width of the picture is 150 millimeters, so the scale is $5,700 \text{ km}/150 \text{ mm} = \mathbf{38 \text{ km/mm}}$.

Problem 2 - Pan is about 1.0 millimeters in diameter which is $38 \text{ km/mm} \times 1 \text{ mm} = \mathbf{38 \text{ kilometers in diameter}}$.



Problem 3 - Students should measure a width of about 5.0 millimeters which is $38 \text{ km/mm} \times 5.0 \text{ mm} = \mathbf{190 \text{ kilometers}}$. The actual width of the Encke Gap is 325 km, but projection effects will foreshorten the gap as it appears in the photo. With the actual gap width (325 km) as the hypotenuse, and 190 km as the short side, the angle opposite the short side is the viewing angle of the camera relative to the ring plane. This angle can be found by constructing a scaled triangle and using a protractor to measure the angle, which will be about 36 degrees.

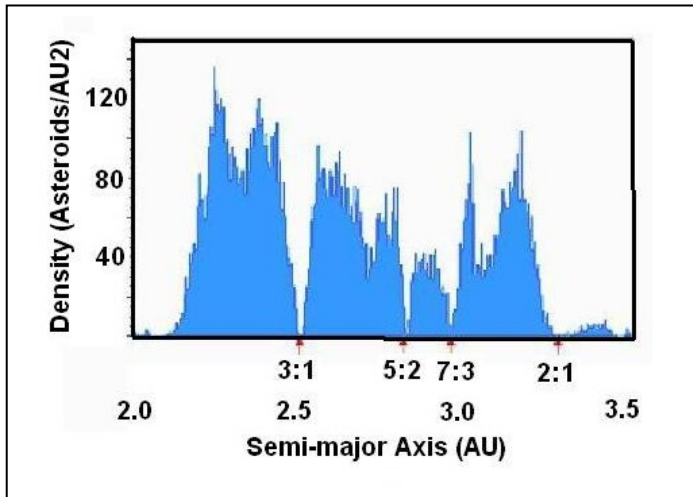


Problem 4 -

It is difficult to estimate lengths smaller than a millimeter. Students may consider using a photocopying machine to make a more convenient enlargement of the image, then measure the features more accurately. Small dark ring bands are about 0.1 mm wide, which is **about 4 km**.

*NASA/Cassini mages, top to bottom:
Saturn Rings closeup showing
Cassini Division and Encke Gap;
Rings closeup showing detail;
One of Saturn's outer
satellites, Phoebe, is about 200 km
across, and may have been a captured
comet.*

Fractions in Space



Simple fractions come up in astronomy in many ways. One common way is shown to the left. The asteroids in the Asteroid Belt have gaps where their orbit period is a simple fraction ($3:1$, $5/2$, or $2:1$) of Jupiter's orbit period.

Here are a few other 'far out' examples!

1 – The satellites of Jupiter, Ganymede and Europa, orbit the planet in 7.1 day and 3.5 days. About what is the ratio of the orbit period of Europa to Ganymede expressed as a simple fraction involving the numbers 1, 2, 3 or 4 in the numerator or denominator?

2 – The planet Pluto orbits the Sun in 248 years while Neptune takes 164 years. What is the simplest fraction involving the numbers 1, 2, 3 or 4 in the numerator and denominator that approximates the ratio of Neptune's period to Pluto's?

3 – Draw two concentric circles labeling the outer circle Earth and the inner circle Venus. Place a point on each circle at the 12 o'clock position, representing the two planets when they are closest to each other. Venus and Earth are opposite each other in their orbits every $3/2$ of an Earth year. During this time, Venus travels $6/5$ of its orbit around the Sun. Where will the planets be after 5 opposition periods, and rounded to the nearest integer year, how many Earth years will this take?

Answer Key

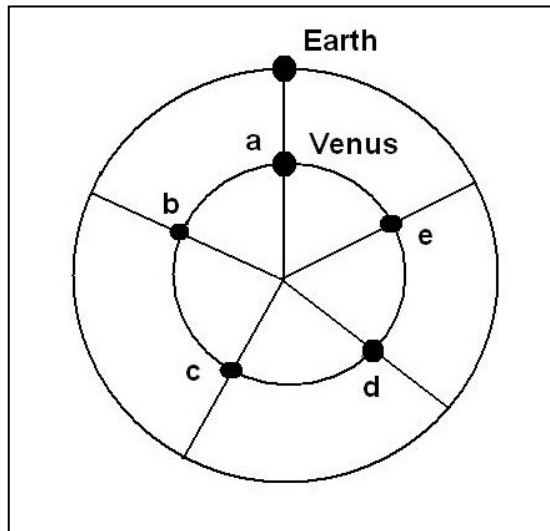
1 – The satellites of Jupiter, Ganymede and Europa, orbit the planet in 7.1 day and 3.5 days. About what is the ratio of the orbit period of Europa to Ganymede expressed as a simple fraction involving the numbers 1, 2, 3 or 4 in the numerator or denominator?

Answer: $3.5 / 7.1 = 0.49$ which is close to 0.5, so the nearest simple fraction is **$1/2$** .

2 – The planet Pluto orbits the sun in 248 years while Neptune takes 60 years. What is the simplest fraction involving the numbers 1, 2, 3 or 4 in the numerator and denominator that approximates the ratio of Neptune's period to Pluto's?

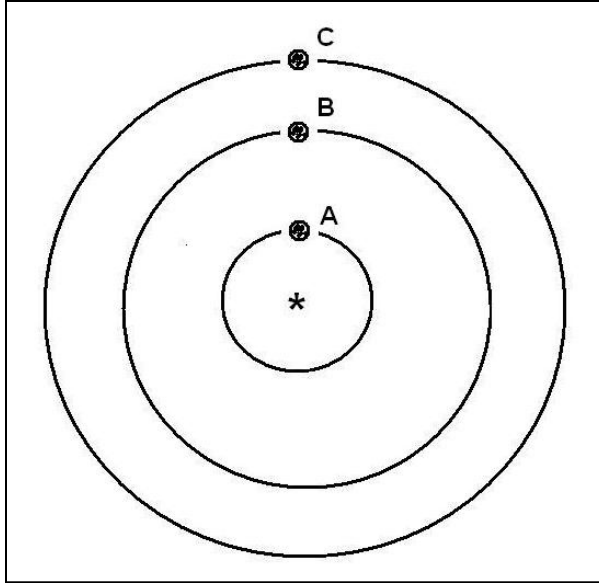
Answer: $164 \text{ years} / 248 \text{ years} = 0.66$. **The closest ratio is $2/3$** . This means that in the time it takes Pluto to orbit the sun twice, Neptune orbits almost exactly three times.

3 – Draw two concentric circles labeling the outer circle Earth and the inner circle Venus. Place a point on each circle at the 12 o'clock position, representing the two planets closest to each other. Venus and Earth are opposite each other in their orbits every $3/2$ of an earth year. During this time, Venus travels $6/5$ of its orbit around the Sun. Where will the planets be after 5 opposition periods, and rounded to the nearest integer year, how many Earth years will this take?



Answer: The 5 opposition periods labeled a, b, c, d, and e in the diagram, form the sequence in earth years of $3/2$, $6/2$, $9/2$, $12/2$, $15/2$. The decimal values are 1.5, 3.0, 4.5, 6.0 and 7.5 years. During this time, Venus has moved $6/5$, $12/5$, $18/5$, $24/5$ and $30/5$ of its orbit. Eliminating full orbits, Venus has moved an extra $b = 1/5$, $c = 2/5$, $d = 3/5$, $e = 4/5$ and $a = 5/5$ and returned to its original position, a, after the 5th opposition. This happened 7.5 years after the opposition sequence started, which rounds up to 8 years later. Every 8 years, Earth and Venus will return to their start positions at the 12 o'clock position in the diagram.

Planetary Conjunctions



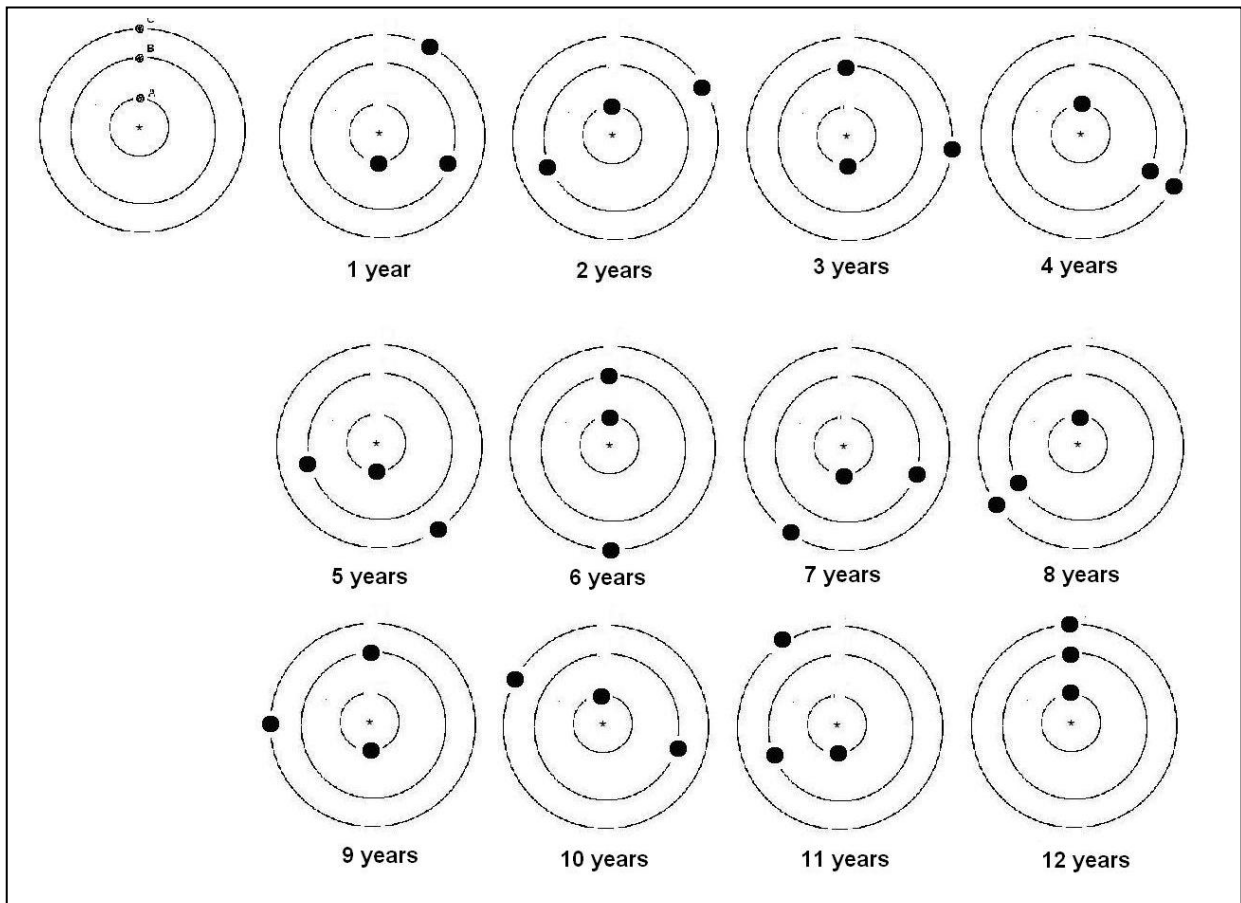
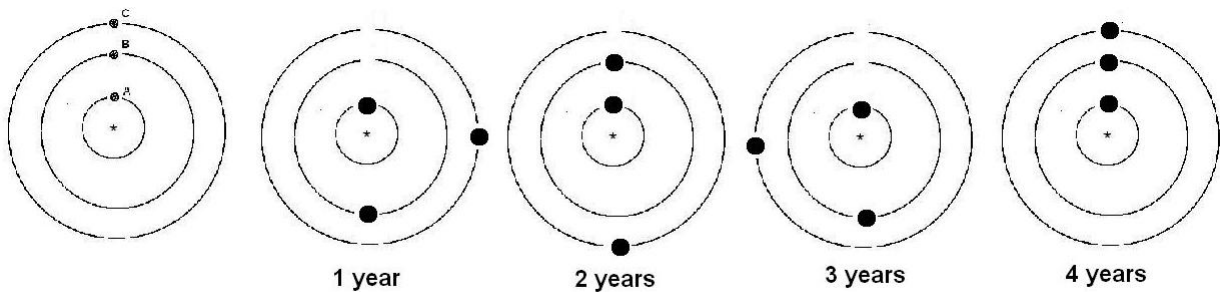
Since 1995, astronomers have detected over 350 planets orbiting distant stars. Our solar system has 8 planets, and for thousands of years astronomers have studied their motions. The most interesting events happen when planets are seen close together in the sky. These are called conjunctions, or less-accurately, alignments. The figure shows a simple 3-planet solar system with the planets starting out 'lined up' with their star. Each planet revolves around the star at a different pace, so it is a challenge to predict when they will all line up again.

Problem 1 - An astronomer detects three planets, A, B, C, that orbit their star once every 1, 2 and 4 years in a clockwise direction. Using the diagram above, draw a series of new diagrams that show where the planets will be in their orbits after: A) 1 year? B) 2 years? C) 3 years? D) 4 years?

Problem 2 - Suppose the three planets, A, B and C, orbited their star once every 2 years, 3 years and 12 years. A) How long would it take for all three planets to line up again? B) Where would the planets be after 6 years?

Problem 1 - Students will draw dots located as follows in the top diagram series:

Problem 2 - The bottom series of 12 possibilities indicates it will take 12 years to return to the original line-up. The pattern after 6 years is also shown.





All bodies produce gravity. The more mass a body has, the more gravity it creates.

It is also true that the smaller you make a body by compressing it, the more intense its gravity is at its surface.

Suppose you made a body that had such an intense gravity that even light could not escape from it.

That body would be called a **black hole**, because anything falling into it, even light, could never escape from it again.

Black holes can come in all imaginable sizes. Suppose that some aliens could turn the planets and moons in our solar system into black holes. How big would they be?

On a black piece of paper, use a ruler and a compass to make circles that are as large as the black holes mentioned in each of the following problems.

Cut these circles out, and make a black hole mobile of the smaller bodies in the solar system!

Problem 1 - Mercury is a black hole with a radius of 0.5 millimeters.

Problem 2 - Venus is a black hole with a radius of 7 millimeters

Problem 3 - Earth is a black hole with a radius of 9 millimeters

Problem 4 - The moon is a black hole with a radius of 0.1 millimeters

Problem 5 - Mars is a black hole with a radius of 1.0 millimeter

Problem 6 - Pluto is a black hole with a radius of 0.02 millimeters

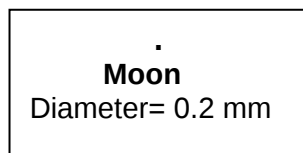
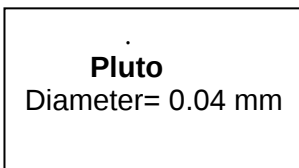
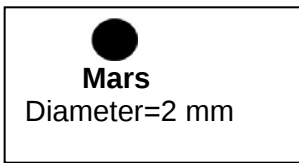
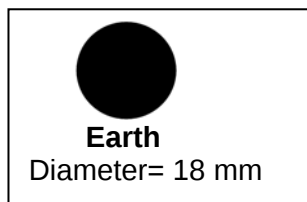
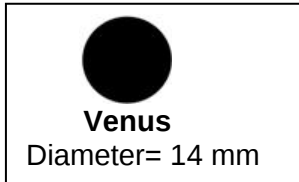
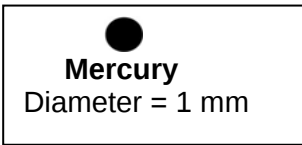
The giant planets will need black circles that are much bigger!

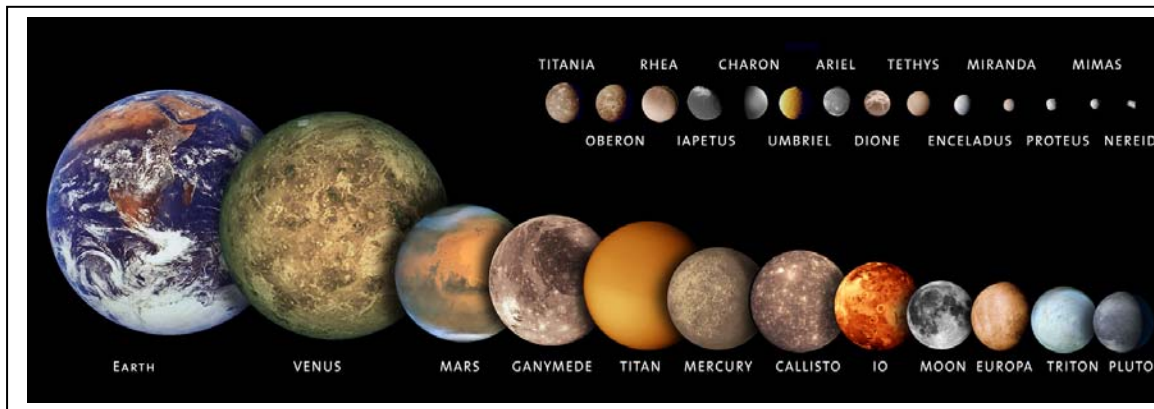
Problem 7 - Jupiter is a black hole with a radius of 280 centimeters

Problem 8 - Saturn is a black hole with a radius of 83 centimeters

Problem 9 - Uranus is a black hole with a radius of 13 centimeter

Problem 10 - Neptune is a black hole with a radius of 15 centimeter





This diagram shows the Top-26 moons and small planets in our solar system, and drawn to the same scale.

Problem 1 – What fraction of the objects are smaller than our moon?

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Problem 4 – If Saturn's moon Titan is $\frac{1}{2}$ the diameter of Earth, and Saturn's moon Dione is $\frac{1}{6}$ the diameter of Titan, how large is the diameter of Dione compared to Earth?

Problem 5 – Oberon is $\frac{1}{7}$ the diameter of Earth, Io is $\frac{1}{3}$ the diameter of Earth, and Titania is $\frac{4}{9}$ the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Problem 1 – What fraction of the objects are smaller than our moon?

Answer: **17/26**

Problem 2 – What fraction of the objects are larger than our moon but are not planets?

Answer: Io, Callisto, Titan and Ganymede : $4/26$ or **2/13**

Problem 3 – What fraction of the objects, including the moon, are about the same size as our moon?

Answer: Moon, Europa, Triton and Pluto so $4/26 = \mathbf{2/13}$.

Problem 4 – Saturn's moon Titan is $\frac{1}{2}$ the diameter of Earth, and Saturn's moon Dione is $\frac{1}{6}$ the diameter of Titan, how large is the diameter of Dione compared to Earth?

Answer: $\frac{1}{2} \times \frac{1}{6} = \mathbf{1/12}$ the size of Earth.

Problem 5 – Oberon is $\frac{1}{7}$ the diameter of Earth, Io is $\frac{1}{3}$ the diameter of Earth, and Titania is $\frac{4}{9}$ the diameter of Io. Which moon is bigger in diameter: Oberon or Titania?

Answer: Oberon is $\frac{1}{7}$ the diameter of Earth.

Titania is $\frac{4}{9}$ the diameter of Io, and Io is $\frac{1}{3}$ the diameter of Earth

So Titania is $(\frac{4}{9}) \times (\frac{1}{3})$ the diameter of Earth

So Titania is $\frac{4}{27}$ the diameter of Earth.

Comparing Oberon, which is $\frac{1}{7}$ the diameter of Earth with Titania, which is $\frac{4}{27}$ the diameter of Earth, which fraction is larger: $\frac{1}{7}$ or $\frac{4}{27}$?

Find the common denominator $7 \times 27 = 189$, then cross-multiply the fractions:

Oberon: $\frac{1}{7} = \frac{27}{189}$ and Titania: $\frac{4}{27} = \frac{(4 \times 7)}{189} = \frac{28}{189}$ so

Titania is $\frac{28}{189}$ Earth's diameter and Oberon is $\frac{27}{189}$ Earth's diameter, and so **Titania is slightly larger!**



The InSight Lander will arrive at Mars on September 20, 2016 according to Earth Time, but when will it arrive according to Mars Time?

One Earth Day is exactly 24 hours long, so that the time between two High Noons is exactly 24 hours. But Mars rotates a bit more slowly and by Earth units, one Mars Day (called a Sol) is 24 hours and 40 minutes long.

This photo was taken by the NASA Phoenix Lander on Sol 90, which is Earth date August 25, 2008

The Curiosity Rover can only safely move during the Martian daytime. This is when scientists on Earth can use TV cameras to watch where the rover is traveling. The Martian day is 40 minutes longer than the Earth day. That means scientists have to move their work schedule forward by 40 minutes each Earth day to keep up with sunrise and sunset on Mars. The rover landed on August 5, 2012 at about 10:30 pm Pacific Daylight Time (PDT), which was defined as Sol 0 for this mission. All of the navigation is performed by Navigation Engineers working at the Jet Propulsion Laboratory in Pasadena, California.

Problem 1 - What time and date will it be on Earth on Sol 5?

Problem 2 - Suppose that sunrise at the Curiosity lander happened on February 16, 2013 at 5:20 pm Pacific Standard Time (Sol 190 at 6:00 am Local Mars Time). A Navigation Engineer begins his work shift exactly at that moment. When will his shift have to start after 5 Sols have passed on Mars?

Problem 3 – How many Sols have to elapse before his work shift once again starts at the same Earth time of 5:20 pm PST?

Mars Sunrise and Sunset Tables
<http://www.curiosityrover.com/sundata/>

Problem 1 - What time and date will it be on Earth on Sol 5?

Answer: This will be 5 x (1 day and 40 minutes) added to August 5 at 10:30 pm PDT.
 5 days and 200 minutes = 5 days 3 hours and 20 minutes after August 5 at 10:30 pm, or
August 11 at 1:50 am PDT.

Problem 2 - Suppose that sunrise at the Curiosity lander happened on February 16, 2013 at 5:20 pm Pacific Standard Time (Sol 190 at 6:00 am Local Mars Time). A Navigation Engineer begins his work shift exactly at that moment. When will his shift have to start after 5 Sols have passed on Mars?

Answer: By his clock, each sunrise will happen 40 minutes later in Earth time, so after 5 Sols have passed, sunrise will happen 5 x 40 minutes = 200 minutes later or 3 hours and 20 minutes added to 5:20 pm PST, which becomes **8:40 pm PST on February 21.**

Problem 3 – How many Sols have to elapse before his work shift once again starts at the same Earth time of 5:20 pm PST?

Answer: One day on Earth is 24 hours long and 1 Sol on Mars is 24 hours 40 minutes long, so we have to wait until the extra 40 minutes after each Sol adds up to exactly 24 hours. Every 3 Sols equals 120 minutes or 2 hours. The progression look like this

Sols	3	6	9	12	15	18...etc
Hours	2	4	6	8	10	12 ... etc

So if 18 Sols equals 12 hours, then 36 Sols equals 24 hours.

The answer is **36 Sols.**

Note to Teacher: 1 Earth day equals 24x60= 1440 minutes so 1 Sol = 1 day plus 40m/
 1440m = 1.028 Earth Days.

We can then convert **36 Sols** to 36 x 1.028 = **37 Earth Days.**

So In the problem above, if the Navigation Engineer starts on February 16 at 5:20 pm for his first Sunrise shift, after 37 days he will start his next Sunrise shift at the same time on March 25, 2013.