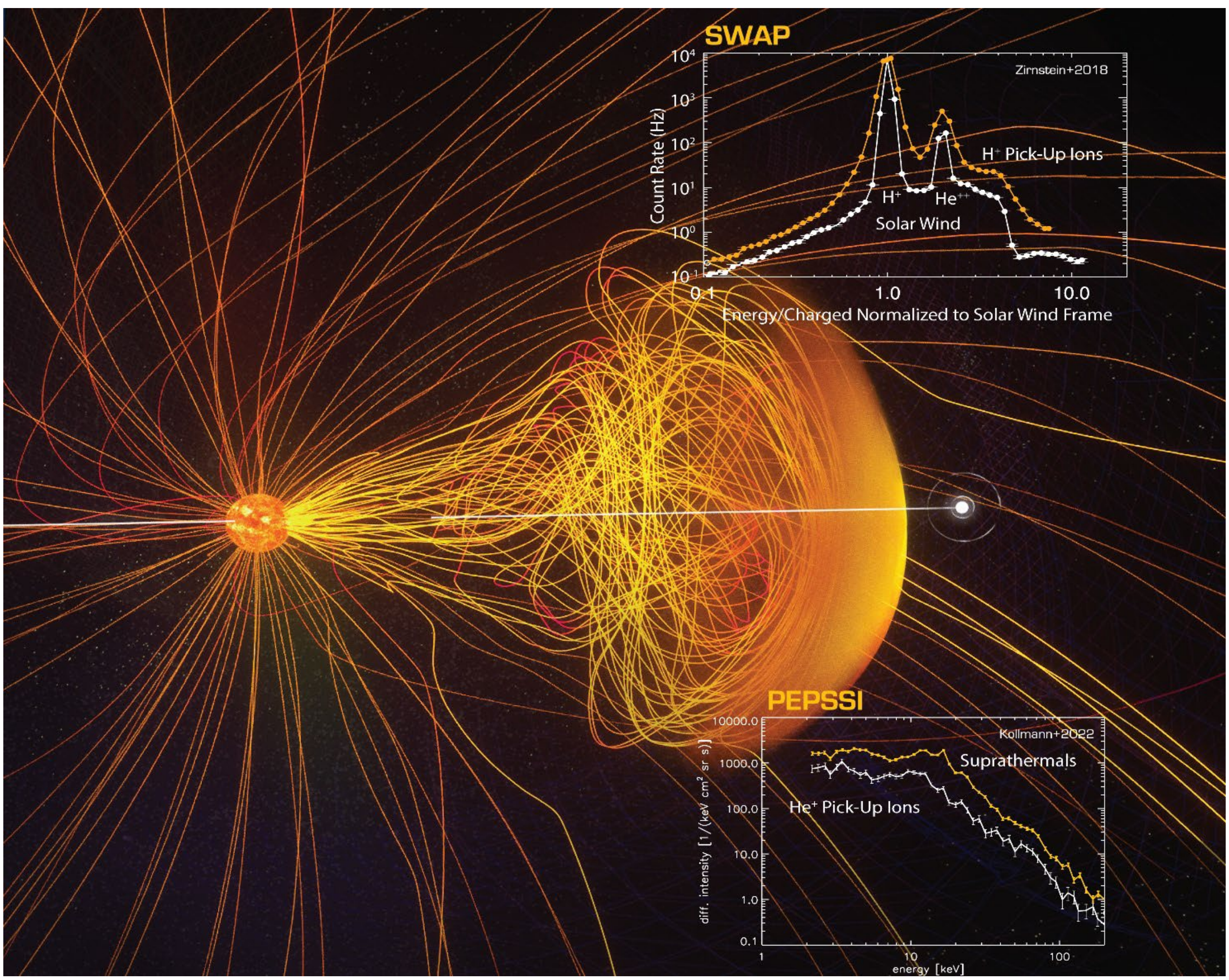
National Aeronautics and Space Administration

Math Problems Related to

Physics



Introduction

This book provides an assortment of mathematics problems that cover aspects of physics in a variety of astronomical settings from gravity and black holes to the temperature of stars, molecules and counting atoms, and time zone math.

37	Advanced	Calculus	Differentiation
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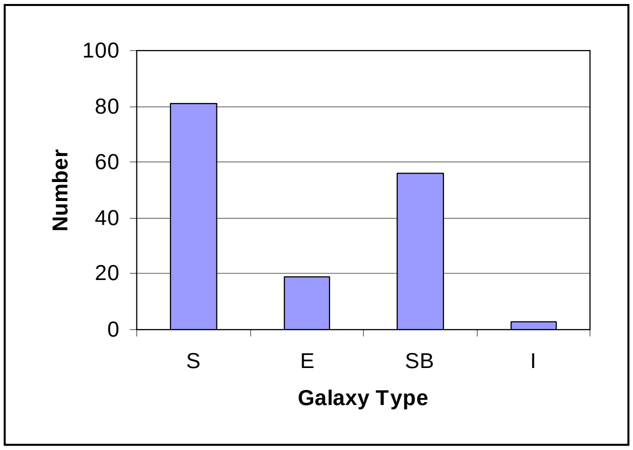
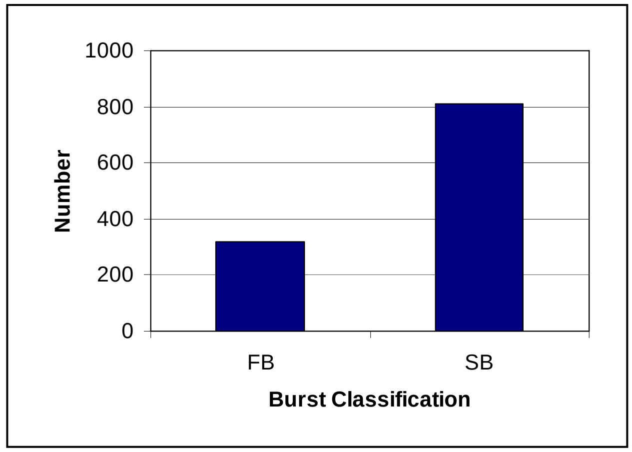
Cosmic Bar Graphs





Problem1 – Astronomers have classified the 160 largest galaxies in the Virgo Cluster according to whether they are spiral-shaped (S and SB), elliptical-shaped (E) or irregular (I). The bar graph to the left shows the number in each category. From the survey, 81 were classed as S, 19 were classed as E, 56 were classed as SB and 3 were classed as I. <u>About what fraction of galaxies in the cluster are spirals?</u>

Problem 2 – Gamma-ray bursts happen about once each day. The bar graph to the right sorts the 1132 bursts detected between 1991-1996 into two categories. There are 320 FBs and 812 SBs indicated in the bar grap. Slow Bursts (SB) are longer than 2 seconds, and may be produced by supernovas in distant galaxies. Fast Bursts (FB) lasting less than 2 seconds may be produced by colliding neutron stars inside our own Milky Way galaxy. <u>What would you predict for 2009 as the number of bursts that might probably come from outside the Milky Way?</u>

Answer Key

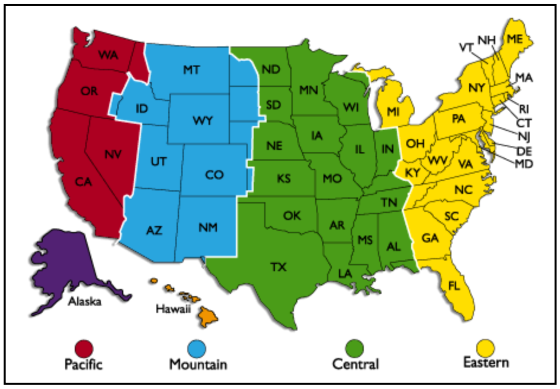
Problem 1 - Astronomers have classified the 160 largest galaxies in the Virgo Cluster according to whether they are spiral-shaped (S and SB), elliptical-shaped (E) or irregular (I). The bar graph to the left shows the number in each category. From the survey, 81 were classed as S, 19 were classed as E, 56 were classed as SB and 3 were classed as I. About what fraction of galaxies in the cluster are spirals?

Answer: There are two categories of spiral galaxies indicated by S and SB for a total of 56 + 81 = 137 galaxies. Since there are 160 total galaxies, the fraction of spirals is **137/160 = 0.86,** or equivalently 86%.

Problem 2 – Gamma-ray bursts happen about once each day. The bar graph sorts the 1132 bursts detected between 1991-1996 into two categories. There are 320 FBs and 812 SBs indicated in the bar grap. Slow Bursts (SB) are longer than 2 seconds, and may be produced by supernovas in distant galaxies. Fast Bursts (FB) lasting less than 2 seconds may be produced by colliding neutron stars inside our own Milky Way galaxy. What would you predict for 2009 as the number of bursts that might probably come from outside the Milky Way?

Answer: There are 320 FBs and 812 SBs indicated in the bar graph. The total number is 1132 over the course of 1996-1991 = 5 years, so that there are 1132/5 = 226 each year. The ones that probably come from outside the Milky Way are the SBs for which the fraction that were seen between 1991-1996 is 812/1132 = 0.72. For a single year, 2009, we would predict about 226 bursts, of which 226 x 0.72 = **163 would be SBs.**

Time Zone Math



Earth is a BIG place! In fact, it is so big that different countries see sunrise and sunset happen at very different times during the day.

If you were living in Germany, you would see sunrise at 6:00 AM, but at that same moment it would be the middle of the night in California!

(Image courtesy http://gis.nwcg.gov/giss_2006/cd_contents.html)

If you have ever gone on a long car or plane ride to the east or west, you will often hear people complain that they have 'gained' or 'lost' hours due to Time Zone change. Here's how it works.

When you travel east, the Sun rises higher and higher in the sky. It is as though you are seeing the Sun as it would be at a later time in the day. When you travel west, the Sun gets lower and lower in the sky. It is as though you are seeing the Sun as it would be at an earlier time in the day.

We can make this more precise by saying that as you travel East you will gain time, and as you travel west you will lose time. The exact amount depends on how many Time Zones you travel through. The figure above shows the Time Zones across North America. During the winter, these Time Zones are called Eastern Standard Time (EST), Central Standard Time (CST), Mountain Standard Time (MST) and Pacific Standard Time (PST).

For example, when you travel westwards, your clock will 'lose' one hour for each Time Zone you pass through. If your watch says 12:00 Noon and you are in New York, which is in the EST Time Zone, you need to set your watch back one hour to 11:00 AM if you are traveling to Chicago in the CST Zone, two hours to 10:00 AM if you are traveling to Denver in the MST Zone, and three hours to 9:00 AM if you are raveling to San Francisco the PST Zone.

1 – A solar astronomer wants to study a flare erupting on the Sun at 12:00 PM (High Noon) at the solar observatory in Denver while taking to his colleague in New York at the same time. At what time should his colleague be ready for the phone call?

2 – A second solar astronomer in Paris, France also wants to participate in this research. If the Paris Time Zone is 4 hours ahead of EST, what time should the Paris astronomer be ready for the same call?

3 – An astronomer sees a solar flare at 2:15 PM EST. A astronomer in Hawaii decides to go out for breakfast between 8:00 and 8:30 AM HST. If Hawaii Standard Time (HST) is 3 hours earlier than the PST Zone, did the Hawaiian astronomer get to see the flare?

Answer Key

1 – A solar astronomer wants to study a flare erupting on the Sun at 12:00 PM (High Noon) at the solar observatory in Denver while taking to his colleague in New York at the same time. At what time should his colleague be ready for the phone call?

Answer: Denver is in the MST Zone, which is 2 hours earlier than the EST. SO, the New York astronomer needs to be ready at 12:00 + 2:00 = 14:00 which is 2:00 PM EST.

2 – A second solar astronomer in Paris, France also wants to participate in this research. If the Paris Time Zone is 4 hours ahead of EST, what time should the Paris astronomer be ready for the same call?

Answer: Paris is 4 hours ahead of EST, so you need to add 4 hours to 12:00 PM Noon EST to get 16:00 hours which is the same as 4:00 PM Paris Time.

3 – An astronomer sees a solar flare at 2:15 PM EST. A astronomer in Hawaii decides to go out for breakfast between 8:00 and 8:30 AM HST. If Hawaii Standard Time (HST) is 3 hours earlier than the PST Zone, did the Hawaiian astronomer get to see the flare?

Answer: The solar flare occurred at 2:15 PM EST. We first convert this to PST by subtracting 3 hours for the time zone change to get 11:15 AM PST. Then, continuing westwards, we subtract another 3 hours to get to the Hawaiian Time Zone, making the time 11:15 AM – 3:00 = 8:15 AM Hawaiian Time. The Hawaiian astronomer missed the flare because he was still having breakfast and not at the observatory.

Atomic Numbers and Multiplying Fractions

5 B	6 C	7 N	8 O	9 F	10 Ne
13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
81 Ti	82 Pb	83 Bi	84 Po	85 At	86 Rn

The Atomic Number, **Z**, of an element is the number of protons within the nucleus of the element's atom. This leads to some interesting arithmetic!

A portion of the Periodic Table of the elements is shown to the left with the symbols and atomic numbers for each element indicated in each square.

Problem 1 - Which element has an atomic number that is 5 1/3 larger than carbon (C)?

Problem 2 - Which element has an atomic number that is 5 2/5 that of neon (Ne)?

Problem 3 - Which element has an atomic number that is 8/9 that of krypton (Kr)?

Problem 4 - Which element has an atomic number that is 2/5 of astatine (At)?

Problem 5 - Which element has an atomic number that is 5 1/8 that of sulfur (S)?

Problem 6 - Which element has an atomic number that is 3 2/3 that of fluorine (F)?

Problem 7 - Which element in the table has an atomic number that is both an even multiple of the atomic number of carbon, an even multiple of the element magnesium (Mg) which has an atomic number of 12, and has an atomic number less than iodine (I)?

Answer Key

Problem 1 - Which element has an atomic number that is 5 1/3 larger than Carbon (C)? Answer: Carbon = 6 so the element is 6 x 5 1/3 = 6 x 16/3 = 96/3 = 32 so **Z=32 and the element symbol is Ge (germanium).**

Problem 2 - Which element has an atomic number that is 5 2/5 that of Neon (Ne)? Answer: Neon = 10, so 10 x 5 2/5 = 10 x 27/5 = 270/5 = 54, so **Z=54 and the element is Xe (xenon).**

Problem 3 - Which element has an atomic number that is 8/9 that of Krypton (Kr)? Answer: Krypton=36 so 36 x 8/9 = 288/9 = 32, so **Z=32 and the element is Ge (germanium).**

Problem 4 - Which element has an atomic number that is 2/5 of Astatine (At)? Answer; Astatine=85 so 85 x 2/5 = 170/5 = 34, **so Z=34 and the element is Se (selenium**).

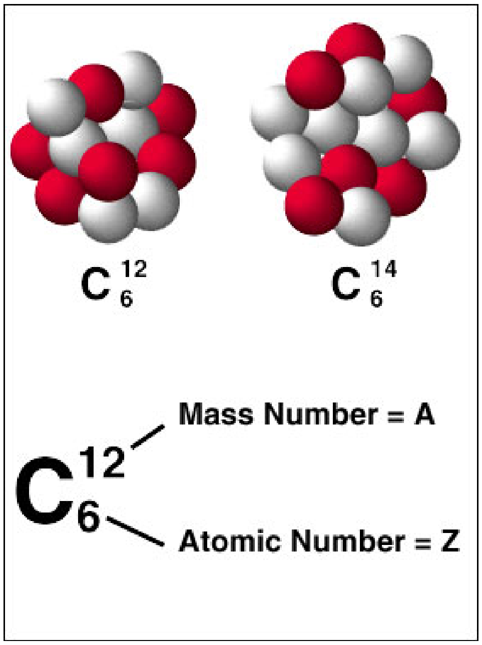
Problem 5 - Which element has an atomic number that is 5 1/8 that of Sulphur (S)? Answer; Sulphur = 16 so 16 x 5 1/8 = 16 x 41/8 = 82, **so Z=82 and the element is lead (Pb).**

Problem 6 - Which element has an atomic number that is 3 2/3 that of Fluorine (F)? Answer: Fluorine = 9 so 9 x 3 2/3 = 9 x 11/3 = 99/3 = 33, **so Z=33 and the element is As (arsenic).**

Problem 7 - Which element in the table has an atomic number that is both an even multiple of the atomic number of carbon, an even multiple of the element magnesium (Mg) which has an atomic number of 12, and has an atomic number less than Iodine (I)?

Answer: The first relationship gives the possibilities: 6, 18, 36, 54. The second clue gives the possibilities 36 and 84. The third clue says Z has to be less than I = 53, **so the element must have Z = 36, which is Kr, (krypton).**

Nuclear Arithmetic



Over 100 elements have been discovered over the last century. The nucleus of each atom contains two kinds of particles: protons and neutrons.

Scientists classify each element by the number of protons (Z) and the mass of the element (A).

Z is called the Atomic Number
A is called the Atomic Mass

The number of neutrons (N) in the nucleus is given by the formula:

$$N = A - Z$$

Problem 1 - In the above example for the element carbon, there are two different forms for carbon. A) How many protons are in the nucleus of carbon-12 and carbon-14? B) How many neutrons are in each nucleus?

Problem 2 - The element praesodymium has an atomic number of 59 and an atomic mass of 141. How many nuclear neutrons does it contain?

Problem 3 - The element nickel (Z=28, A=58) has 30 isotopes that have the same atomic number, but whose atomic masses range from A=48 to A=78. A) How many neutrons does the lightest isotope of nickel have? B) How many neutrons does the heaviest isotope have?

Problem 4 - Solve the formula N = A - Z to determine the missing information:

A) Tin:	A= 125	and Z=50	what is N?
B) Niobium:	N = 54	and Z= 41	what is A?
C) Nobelium:	A = 253	and N = 151	what is Z?
D) Francium:	A=232	and Z= 87	what is N?
E) Oxygen:	Z = 8	and N= 16	what is A?

Answer Key

Problem 1 - In the above example for the element carbon, there are two different forms for carbon. A) How many protons are in the nucleus of carbon-12 and carbon-14? B) How many neutrons are in each nucleus?

Answer: A) Carbon-12 has Z=6 and so does carbon-14 so they both have the same number of protons. B) Answer; The mass of carbon-12 is A=12, while carbon-14 has A=14 so carbon-12 has 12-6 = 6 neutrons while carbon 14 has 14-6 = 8 neutrons. Physicists call carbon-14 an isotope of carbon-14 for this reason.

Problem 2 - The element praesodymium has an atomic number of 59 and an atomic mass of 141. How many nuclear neutrons does it contain?
Answer: Z = 59 and A = 141 so N = 141-59 = **82**.

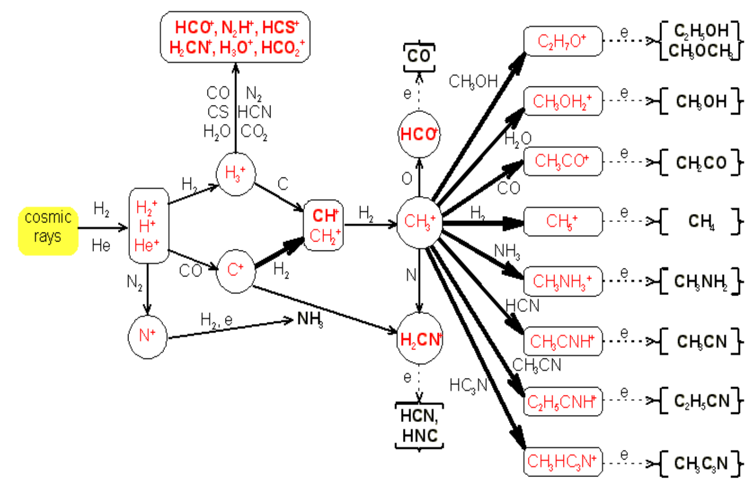
Problem 3 - The element nickel (Z=28, A=58) has 30 isotopes that have the same atomic number, but whose atomic masses range from A=48 to A=78. A) How many neutrons does the lightest isotope of nickel have? B) How many neutrons does the heaviest isotope have?

Answer; A) The lightest isotope is called Nickel-48 and has N = 48 - 28 = **20 neutrons**. B) The heaviest isotope of nickel is called nickel-78 and has N = 78 - 28 = **50 neutrons**.

Problem 4 - Solve the formula N = A - Z to determine the missing information:

A) Tin (A= 125, Z=50) N = ?
Answer: **N = 125-50 = 75**
B) Niobium (N = 54, Z= 41) A= ?
Answer: 54 = A - 41 so **A = 54 + 41 = 95**
C) Nobelium (A = 253, N = 151) Z = ?
Answer; 151 = 253 - Z so Z = 253-151 = **102**.
D) Francium (A=232, Z= 87), N=?
Answer: N = 232 - 87 = **145.**
E) Oxygen (Z = 8 N= 16) A=?
Answer: 16 = A - 8 so **A = 24**.

The initial reactions and radiative association in dense interstellar clouds



Because molecules and atoms come in 'integer' packages, the ratios of various molecules or atoms in a compound are often expressible in simple fractions. Adding compounds together can often lead to interesting mixtures in which the proportions of the various molecules involve mixed numbers.

The figure shows some of the ways in which molecules are synthesized in interstellar clouds. (Courtesy D. Smith and P. Spanel, "Ions in the Terrestrial Atmosphere and in Interstellar Clouds", *Mass Spectrometry Reviews*, v.14, pp. 255-278.)

In the problems below, do not use a calculator and state all answers as simple fractions or integers.

Problem 1 - What makes your car go: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

A) What is the ratio of ethane molecules to water molecules?
B) What is the ratio of oxygen molecules to carbon dioxide molecules?
C) If you wanted to 'burn' 50 molecules of ethane, how many molecules of water result?
D) If you wanted to create 50 molecules of carbon dioxide, how many ethane molecules would you have to burn?

Problem 2 - How plants create glucose from air and water: Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.

A) What is the ratio of glucose molecules to water molecules?
B) What is the ratio of oxygen molecules to the total number of carbon dioxide and water molecules?
C) If you wanted to create 120 glucose molecules, how many water molecules are needed?
D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

Answer Key

Problem 1 - What makes your car go: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

A) In this reaction, 2 molecules of ethane yield 6 molecules of water, so the ratio is 2/6 or 1/3.
B) 7 oxygen molecules and 4 carbon dioxide molecules yield the ratio 7/4

C) The reaction says that 2 molecules of ethane burn to make 6 molecules of water. If you start with 50 molecules of ethane, then you have the proportion:

$$\frac{50\text{ ethane}}{2\text{ ethane}} = \frac{\text{x-water}}{6\text{ -water}} \quad \text{so} \quad X = 25 \times 6 = 150 \text{ water molecules.}$$

D) Use the proportion:

$$\frac{50\text{ Carbon Dioxide}}{4\text{ carbon dioxide}} = \frac{X\text{ ethane}}{2\text{ ethane}} \quad \text{so} \quad X = 2 \times (50/4) = 25 \text{ molecules ethane}$$

Problem 2 - How plants create glucose from air and water: Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.
A) What is the ratio of glucose molecules to water molecules?
B) What is the ratio of oxygen molecules to the total number of carbon dioxide and water molecules?
C) If you wanted to create 120 glucose molecules, how many water molecules are needed?
D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

A) Glucose molecules /water molecules = 1 / 6

B) Oxygen molecules / (carbon dioxide + water) = 6 / (6 + 6) = 6/12 = 1/2

C)

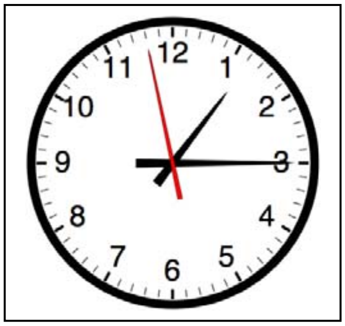
$$\frac{120\text{ glucose}}{1\text{ glucose}} = \frac{X\text{ water}}{6\text{ water}} \quad \text{so} \quad X = 6 \times 120 = 720 \text{ water molecules}$$

D)

$$\frac{100\text{ carbon dioxide}}{6\text{ carbon dioxide}} = \frac{X\text{ glucose}}{1\text{ glucose}} \quad \text{so} \quad X = 100/6 \text{ molecules.}$$

The problem asks for the largest number that can be made, so we cannot include fractions in the answer. We need to find the largest multiple of '6' that does not exceed '100'. This is 96 so that 6 x 16 = 96. That means we can get no more than 16 glucose molecules by starting with 100 carbon dioxide molecules. (Note that 100/6 = 16.666 so '16' is the largest integer).

Time Intervals



Astronomers are often interested in how long a particular event took. This can be used to explore how fast something is moving, or how rapidly it is changing in time. Here are some 'far out' examples!

1 – The gamma-ray burst from the galaxy TXS1510-089 was detected on July 24, 2007 by the AGILE satellite. The burst began at 23:25:07 and ended at 23:25:57 How many seconds did it last?

2 – The black hole orbiting the star A0620-00 in the constellation Monocerous produced a micro-flare on September 29, 2002 visible at radio wavelengths as it swallowed some of the gas falling into it. If the flare began at 01:50:00 and ended at 01:53:20 how many seconds elapsed?

3 – On September 1, 1859 the sun released a cloud of plasma called a Coronal Mass Ejection at 11:18 AM If the cloud reached Earth on September 2 at 04:54 AM how many hours did it take the cloud to travel from the sun to earth?

4 – Full Moon occurred on July 18, 2008 and August 16, 2008. How many days elapsed between the two lunar phases?

5 – A massive star in the Eta Carina cluster erupted in a giant flare in 1843. How many years has it been since this eruption if the current year is 2008?

6 – The Crab Nebula was formed by a supernova in the year 1054 AD. If the next supernova spotted by humans occurred in 1987, how many years elapsed between these events?

7 - The earth was formed 4.5 billion years ago. The asteroid bombardment of its surface ended 3.9 billion years ago. How many million years did the asteroid bombardment era last?

8 - The universe came into existence in the Big Bang 13.7 billion years ago .The most ancient galaxy astronomers have detected so far is A1689-zD1, which was probably formed about 13.0 billion years ago. How many millions of years was the universe in existence before this galaxy began to form?

Answer Key

1 – Answer: 23:25:57 – 23:25:07 = 50 seconds

2 – Answer:
```
     01:53:20
  -  01:50:00
  ----------------
         3:20
```
The burst lasted 3 minutes and 20 seconds.

3 – Answer:
```
    September 2    04:54        add 24h       28:54
  - September 1    11:18                    - 11:18
  -----------------------------             -------------
                                              17:36
```
The CME took 17 hours and 36 minutes to arrive.

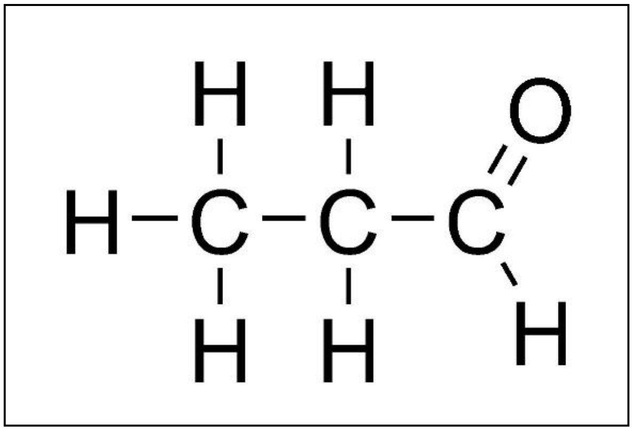
4 – Answer:
```
     August 16, 2008
  -  July   18, 2008
  -------------------------
```
You can use a calendar to count the days, or from 31 days in July, there are 31-18 = 13 days plus the 16 days in August the sum is 13+16 = 29 days.

5 – Answer: 2008 – 1843 = 165 years.

6 – Answer: 1987 – 1054 = 933 years.

7 – Answer: 4.5 billion – 3.9 billion = 0.6 billion years or 600 million years.

8 – Answer: 13.7 billion – 13.0 billion = 0.7 billion years or 700 million years.



The complex molecule Propanal was discovered in a dense interstellar cloud called Sagittarius B2(North) located near the center of the Milky Way galaxy about 26,000 light years from Earth. Astronomers used the giant radio telescope in Greenbank, West Virginia to detect the faint signals from a massive cloud containing this molecule. It is one of the most complex molecules detected in the 35 years that astronomers have searched for molecules in space. Over 140 different chemicals are now known.

Problem 1 - How many atoms of hydrogen (H), carbon (C) and oxygen (O) are contained in this molecule?

Problem 2 - What percentage of all atoms are hydrogen? Carbon? Oxygen?

Problem 3 - What is the ratio of carbon atoms to hydrogen atoms in propanal?

Problem 4 - If the mass of a hydrogen atom is defined as 1 AMU, and carbon and oxygen have masses of 12.0 and 16.0 AMUs, what is the total mass of a propanal molecule in AMUs?

Problem 5 - What is the complete chemical formula for propanal?

C_3 H __ O__

Problem 6 - If this molecule could be broken up, how many water molecules could it make if the formula for water is H_2O?

Answer Key

Problem 1 - How many atoms of hydrogen (H), carbon (C) and oxygen (O) are contained in this molecule?

Answer; There are 6 atoms of hydrogen, 3 atoms of carbon and 1 atom of oxygen.

Problem 2 - What percentage of all atoms are hydrogen? Carbon? Oxygen?

Answer: There are a total of 10 atoms in propanal so it contains 100% x (6 atoms /10 atoms) = 60% hydrogen; 100%x(3/10) = 30% carbon and 100% x (1/10) = 10 % oxygen.

Problem 3 - What is the ratio of carbon atoms to hydrogen atoms in propanal?

Answer: 3 carbon atoms / 6 hydrogen atoms = 1/2.

Problem 4 - If the mass of a hydrogen atom is defined as 1 AMU, and carbon and oxygen have masses of 12.0 and 16.0 AMUs, what is the total mass of a propanal molecule in AMUs?

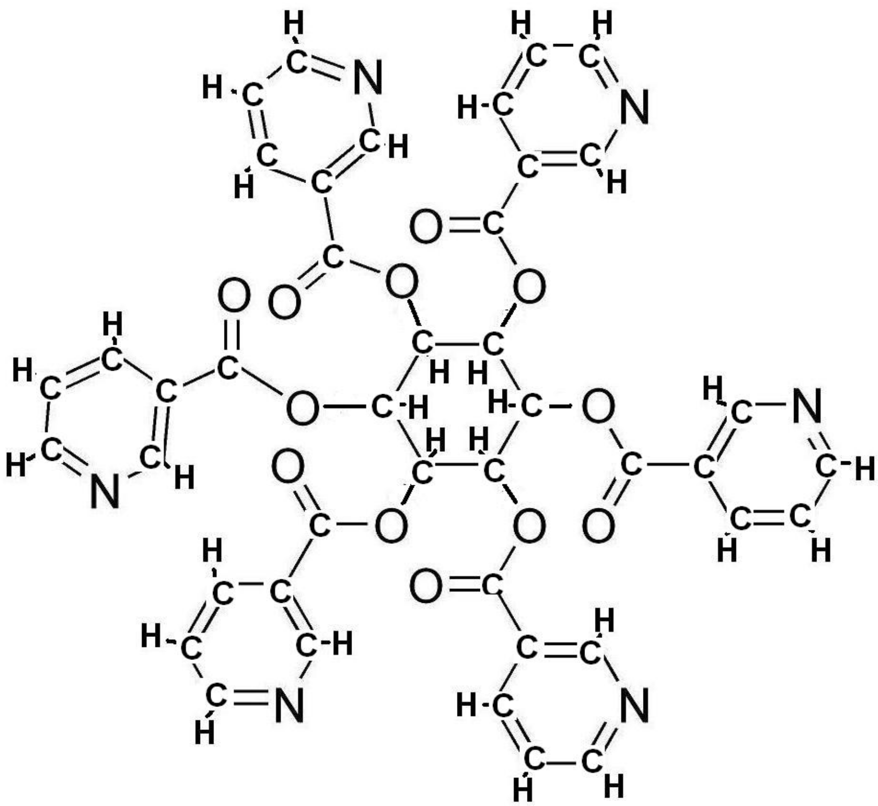
Answer: 1 AMU x 6 hydrogen + 12 AMU x 3 carbon + 16 AMU x 1 oxygen = 6 AMU + 36 AMU + 16 AMU = 58 AMU for the full molecule mass.

Problem 5 - What is the complete chemical formula for propanal?

$$C_3H_6O$$

Problem 6 - If this molecule could be broken up, how many water molecules could it make if the formula for water is H_2O?

Answer: A single water molecule requires exactly 1 oxygen atom ,and since propanal only has 1 atom of oxygen per molecule, you can only get 1 molecule of water by dissolving a single propanal molecule.



This is a figure showing the locations of hydrogen (H), oxygen (O), carbon (C), and nitrogen (N) atoms in one molecule of inositol nicotinate, which is a synthetic drug used to treat coronary heart disease and arteriosclerosis.

Problem 1 - How many atoms of each element are present in one molecule of inositol nicotinate?

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

C__ H__ N__O__

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in inositol nicotinate are H = 1 AMU, C=12 AMU, N= 14 AMU, and O=16 AMU, what is the total mass of one molecule in AMUs?

Problem 1 - How many atoms of each element are present in one molecule of inositol nicotinate?

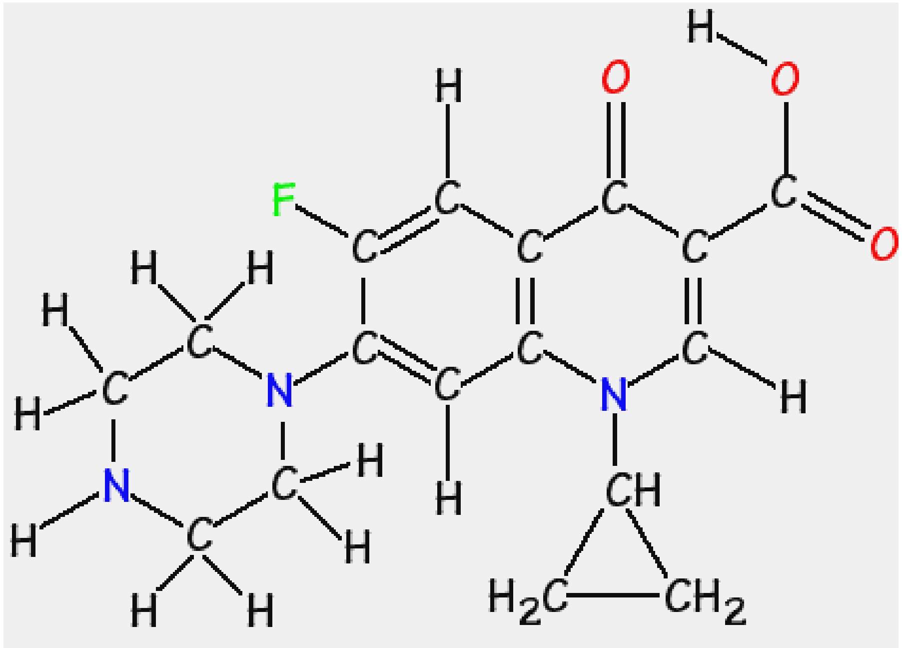
Answer: **Carbon (C) = 42**
Oxygen (O) = 12
Hydrogen (H) = 30
Nitrogen (N) = 6

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

$$C_{42}\ H_{30}\ N_6\ O_{12}$$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in inositol nicotinate are H = 1 AMU, C=12 AMU, N = 14 AMU, and O=16 AMU, what is the total mass of a molecule in AMUs?

Answer: M = 42(12) + 30(1) + 6(14) + 12(16) = 504 + 30 + 84 + 192 = **810 AMU.**



This is a figure showing the locations of hydrogen (H), oxygen (O), carbon (C), nitrogen (N) and fluorine (F) atoms in one molecule of ciprofloxacin. This man-made compound kills bacteria such as anthrax by interfering with the enzymes that cause DNA in the anthrax bacterium to rewind after being copied, which stops DNA and protein synthesis.

Problem 1 - How many atoms of each element are present in one molecule of ciprofloxacin? (Note H_2 means 2 atoms of H)

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

$$C\_\_\ H\_\_\ FN_3O\_\_$$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in ciprofloxacin are H = 1 AMU, C=12 AMU, O=16 AMU, N = 14 AMU and F = 19 AMU, what is the total mass of the ciprofloxacin molecule in units of AMUs?

Problem 1 - How many atoms of each element are present in one molecule of ciprofloxacin?

Answer: **Carbon (C) = 17 atoms**
Hydrogen (H) = 18 atoms
Oxygen (O) = 3 atoms
Fluorine (F) = 1
Nitrogen (N) = 3

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

$\mathbf{C_{17}\,H_{18}\,FN_3O_3}$

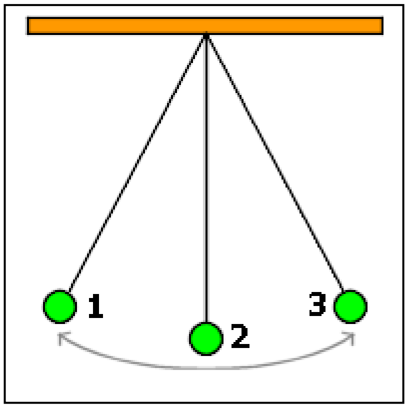
Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in ciprofloxacin are H = 1 AMU, C=12 AMU, O=16 AMU, N = 14 AMU and F = 19 AMU, what is the total mass of the ciprofloxacin molecule in units of AMUs?

Answer: From the count of the numbers of atoms of each kind:

M = 17 (12) + 18(1) + 1(19) + 3(14) + 3 (16)

M **= 331 AMU**.

Measuring Gravity with a Pendulum!



A pendulum is a very simple toy, but you can actually use it to measure gravity! The beat of a pendulum called its period, P, depends on the length of the pendulum, L, and the acceleration of gravity, g ,according to:

$$P = 2\pi\sqrt{\frac{L}{g}}$$

If you measure P in seconds, and know the length of the pendulum, L, in meters, you can figure out how strong the acceleration of gravity is, g, in meters/sec^2. Let's see how this works for explorers working on different planets and moons in the solar system!

Problem 1 – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is 3.8 m/sec^2, how long will the pendulum have to be in meters?

Problem 2 - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

Problem 3 – On Earth, prospectors are looking for a deposit of iron ore beneath the ground. They decide to use the acceleration of gravity to find where the iron is located because the additional iron mass should change the acceleration of gravity. They use a carefully-made pendulum with a length of 2.00000 meters and measure the period of the swing as they walk around the area where they think the deposit is located. To the nearest millionth of a second, how much will the period change if the acceleration of gravity between two spots changes from 9.80000 meters/sec^2 to 9.80010 meters/sec^2? (use p = 3.14159)

Problem 1 – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is 3.8 m/sec^2, how long will the pendulum have to be in meters?

Answer: $4 = 2\pi (L/3.8)^{1/2}$ then $L = (16/4\pi^2)*3.8$ = **1.54 meters**.

Problem 2 - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

Answer: $7.00 = 2\pi (2/g)^{1/2}$ so $g = 8\pi^2/49$ so g = **1.61 meters/sec^2**.

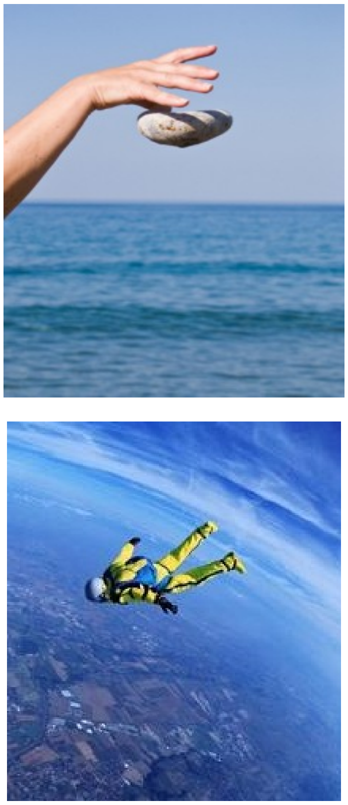
Problem 3 – On Earth, prospectors are looking for a deposit of iron ore beneath the ground. They decide to use the acceleration of gravity to find where the iron is located because the additional mass should change the acceleration of gravity. They use a carefully-made pendulum with a length of 2.00000 meters and measure the period of the swing as they walk around the area where they think the deposit is located. To the nearest millionth of a second, how much will the period change if the acceleration of gravity between two spots changes from 9.80000 meters/sec^2 to 9.80010 meters/sec^2? (use π = 3.14159)

Answer: $P = 2\pi (2.00000/9.80000)^{1/2}$ = **2.838451 seconds**.
And $P2 = 2\pi (2.00000/9.80010)^{1/2}$ = **2.838437 seconds**

So the difference in period will be **0.000014 seconds or 14 microseconds.**

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \cdots\right)$$

Where theta is the start angle from verticle.



One of the simplest kinds of motion that were first studied carefully is that of falling. Unless supported, a body will fall to the ground under the influence of gravity. But the falling does not happen smoothly. Instead, the speed of the body increases in proportion to the elapsed time. This is called acceleration.

Near Earth's surface, the speed of a body increases 32 feet/sec (9.8 meters/sec) for every elapsed second. This is usually written as an acceleration of 32 feet/sec/sec or 32 feet/sec^2. (also 9.8 meters/sec^2) A simple formula gives you the speed of the object after an elapsed time of T seconds:

$$S = 32\ T \quad \text{in feet/sec}$$

If instead of just dropping the object, you threw it downwards at a speed of 12 feet/sec (3 meters/sec) you could write the formula as:

$$S = 12 + 32T \quad \text{feet/sec}$$

In general you could also write this by replacing the selected speed of 12 feet/sec, with a fill-in speed of S_0 to get

$$S = S_0 + 32T \quad \text{feet/sec.}$$

Problem 1 – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

Problem 2 – On Mars, the acceleration of gravity is only 12 feet/sec^2. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

Problem 3 – Two astronauts standing on the surface of two different objects in the solar system want to decide which object is the largest in mass. The first astronaut drops a hammer off a cliff that is exactly 5000 feet tall and measures the impact speed with a radar gun to get 100 feet/sec. It takes 8 seconds for the hammer to hit the bottom. The second astronaut drops an identical hammer off a cliff that is only 500 feet tall and also measures the impact speed at 150 feet/sec, but he accidentally gave the object a release speed of 5 feet/sec. It takes 29 seconds for the hammer to reach the ground. They can't re-do the experiments, but given this information what are the accelerations of gravity on the two bodies and which one has the highest mass?

Problem 1 – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

Answer: 192 = 32T, so T = **6 seconds**.

Problem 2 – On Mars, the acceleration of gravity is only 12 feet/sec^2. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

Answer: 700 = 12T, so T = **58 seconds**.

Problem 3 – Two astronauts standing on the surface of two different objects in the solar system want to decide which object is the largest in mass. The first astronaut drops a hammer off a cliff that is exactly 5000 feet tall and measures the impact speed with a radar gun to get 100 feet/sec. It takes 8 seconds for the hammer to hit the bottom. The second astronaut drops an identical hammer off a cliff that is only 500 feet tall and also measures the impact speed at 150 feet/sec, but he accidentally gave the object a release speed of 5 feet/sec. It takes 29 seconds for the hammer to reach the ground. They can't re-do the experiments, but given this information what are the accelerations of gravity on the two bodies and which one has the highest mass?

Answer: Astronaut 1. 100 = A1 x T1 T1 = 8 seconds, so A1 = 100/8 = 12 feet/sec^2.

Astronaut 2: 150 = 5 + A2 T2 T2 = 29 sec, so A2 = (150-5)/29 = 5 feet/sec^2

The acceleration measured by the second astronaut is much lower than for the first astronaut, so the first astronaut is standing on the more-massive objects. In fact, Astronaut 1 is on Mars and Astronaut 2 is on the moon!

Distance Traveled Under Constant Acceleration

Bungee jumping has become a popular but dangerous sport. It also shows how the acceleration of gravity is connected to the total distance traveled during the fall. The distance traveled is given by the formula

$$D = \frac{1}{2} g T^2$$

Where g is the acceleration of gravity in meters/sec^2, D is the distance in meters, and T is the elapsed time in seconds. For locations near the surface of Earth, g = 9.8 meters/sec^2 (32 feet/sec^2)

Problem 1 - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

Problem 2 – How fast would the Daredevil be traveling at the moment of impact if S = 32T?

Problem 3 – Once he reaches 130 mph (190 feet/sec), called the terminal velocity, his free-fall speed stops increasing. How soon after he jumps does he reach terminal velocity, and how far has he fallen from the plane?

Problem 4 - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

Problem 5 – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec^2, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

Problem 1 - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

Answer: 15,000 = ½ (32) T^2, so T^2 = 937 and so T = **31 seconds**.

Problem 2 – How fast would the Daredevil be traveling at the moment of impact if S = 32T?

Answer: S = 32 x 31 = 992 feet/second or 676 miles/hour!

Problem 3 – Once he reaches 130 mph (190 feet/sec), called the terminal velocity, his free-fall speed stops increasing. How soon after he jumps does he reach terminal velocity, and how far has he fallen from the plane?

Answer: 190 = 32 x T so T = **6 seconds**. He has fallen d = ½ $(32)(6)^2$ = **576 feet.**

Problem 4 - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

Answer: 1240 = 32 T so T = **39 seconds**.
D = ½ (32) $(39)^2$ = 24,336 feet,
so 127,000 – 24333 = **102,700 feet from the ground**.

Problem 5 – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec^2, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

Answer: 23,000 = ½(12)T^2 so T = **62 seconds**.
Speed = 12 x 62 = **744 feet/sec or 507 mph**.

Gravity and Energy

Energy can be changed from one form to another. When you peddle a bike, your body uses up stored food energy (in calories) and converts this into kinetic energy of motion measured in joules. When you connect an electric motor to a battery, electrical energy stored in the battery is converted into rotational kinetic energy causing the motor shaft to turn.

A millstone paddle wheel uses the gravitational energy of falling water to turn the millstone wheel and perform work by grinding wheat, or even running simple machinery to cut wood in a lumber mill.

The energy in Joules of an object falling from a height near the surface of Earth can be calculated from

$$\mathbf{E = mgh}$$

where m is the mass of the falling body in kilograms, g is the acceleration of gravity (9.8 meters/sec^2) and h is the distance of the fall in meters.

Problem 1 – Nevada Falls in Yosemite Valley California has a height of 180 meters. Every second, 500 cubic feet of water goes over the edge of the falls. If 1 cubic foot of water has a mass of 28 kilograms, how much energy does this waterfall generate every day?

Problem 2 – For a science fair project, a student wants to build a water hose powered hydroelectric plant to run a light bulb. Every second, the light bulb needs 60 Joules to operate at full brightness. If the water hose produces a steady flow of 0.2 kilograms every second, how high off the ground does the water hose have to be to turn a paddle wheel to generate the required electrical energy?

Problem 3 – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If g = 0.1 meters/sec^2, and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

Answer Key

Problem 1 – Nevada Falls in Yosemite Valley California has a height of 180 meters. Every second, 500 cubic feet of water goes over the edge of the falls. If 1 cubic foot of water has a mass of 28 kilograms, how much energy does this waterfall generate every day?

Answer: 1 day = 24 x 60 x 60 = 86,400 seconds, then the total mass is 500 x 28 x 86400 = 1.2 billion kilograms. E = 1.2 billion kg x 9.8 x 180 = **2.1 trillion joules** per day. Note: since 1 watt = 1 Joule/second, this waterfall has a wattage of 500 x 28 x 9.8 x 180 = 25 megawatts.

Problem 2 – For a science fair project, a student wants to build a water hose powered hydroelectric plant to run a light bulb. Every second, the light bulb needs 60 Joules to operate at full brightness. If the water hose produces a steady flow of 0.2 kilograms every second, how high off the ground does the water hose have to be to turn a paddle wheel to generate the required electrical energy?

Answer: 60 = 0.2 x 9.8 x h so h = **30.6 meters** (or 90 feet!).

Problem 3 – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If g = 0.1 meters/sec^2, and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

Answer: 1,000,000 = 2000 x 0.1 x h, so h = **5,000 meters or 5 kilometers**.

It would be nice if we could just jump real hard and we would suddenly be in space orbiting Earth. Fortunately it is not that easy as any Olympic High Jumper will tell you!

Because of the pull of gravity, every planet, asteroid or other object in the universe has its own speed limit. If you move slower than this speed you will stay on the body. If you move faster than this speed you will escape into space. Scientists call this the **escape speed** or **escape velocity**.

It's not just a number you guess at. It depends exactly on how much mass the planet or moon has, and how far from its center you are located. That means you can predict what this speed will be as you travel to other planets. That's very handy if you are an astronaut!

For Earth, the escape speed V in kilometers/second (km/s) at a distance R from Earth's center in kilometers, is given by

$$V = \frac{894}{\sqrt{R}}$$

Problem 1 - What is the escape speed for a rocket located on Earth's surface where R = 6378 km?

Problem 2 – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

Problem 3 – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

Problem 4 – To enter a circular orbit at a distance of R from the center of Earth, you only need to reach a speed that is $2^{1/2}$ smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

Problem 1 - What is the escape speed for a rocket located on Earth's surface where R = 6378 km?

Answer: $V = 894/(6378)^{1/2}$ = **11.19 km/s**

Problem 2 – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

Answer: $V = 894/(6378+8.9)^{1/2}$ = **11.18 km/s. This does not change the required escape speed by very much considering the effort to build such a launch facility at this location**.

Problem 3 – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

Answer: $V = 894/(6378+35{,}786)^{1/2}$ = **4.35 km/s**

Problem 4 – To enter a circular orbit at a distance of R from the center of earth, you only need to reach a speed that is $2^{1/2}$ smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

Answer: 4.35 km/s / (1.414) = **3.079 km/sec**.

Note: Satellites at an altitude of 35,800 orbit earth exactly once every day and are called geosynchronous satellites because they remain over the same geographic spot on Earth above the equator.

Circumference of the orbit = $2 \pi R$ = 2 x 3.141 x (42164 km) = 264,924 km.
Speed = 3.079 km/sec so
T = 264924/3.079 = 86,042 seconds or 23.9 hours – Earths rotation period.

Light Travel Times

NASA satellites and space probes are so far away from Earth that serious time delays happen when radio signals are sent to them. This is because radio signals travel at the speed of light, 300,000 kilometers/sec, and the distances from Earth to the spacecraft are huge!!

Spacecraft	Distance (km)	Seconds
Themis P2	30,000	
LRO	382,000	
ACE	1.5 million	
MESSENGER	50 million	
STEREO-A	111 million	
Mars Orbiter	220 million	
Ulysses	800 million	
Cassini	1.8 billion	
Voyager 2	13 billion	

This image was taken by the Hubble Space Telescope and shows the satellite Io The moon is the small disk to the left, and its shadow appears to the right of center. (Courtesy J. Spenser, Lowell Observatory and NASA).

Problem 1 - How long will it take for a radio signal to travel from the satellites to Earth, one-way, in the above table? Complete the last column to find out!

Problem 2 - Suppose a radio message needs to be sent at 01:20 on February 15, 2008. To the nearest minute, what time would it be when the message arrived at each spacecraft, and when the data arrived back at Earth?

Problem 3 - When Jupiter was located farthest from the Sun (aphelion), it was at a distance of 667 million kilometers from Earth. When it was closest to the sun (perihelion) it was 590.5 million kilometers from Earth. Suppose you are calculating a schedule for when the satellite Io will be exactly at dead-center of Jupiter's disk based on when you saw the transit at perihelion. How much of a schedule change will you see when you observe the transit at aphelion, and will the transit occur sooner or later than you predicted?

Answer Key

Problem 1 – Answer:

Spacecraft	Distance (km)	Seconds
Themis P2	30,000	**0.1**
LRO	382,000	**1.3**
ACE	1.5 million	**5.0**
MESSENGER	50 million	**167**
STEREO-A	111 million	**370**
Mars Orbiter	220 million	**733**
Ulysses	800 million	**2,667**
Cassini	1.8 billion	**6,000**
Voyager 2	13 billion	**43,333**

Problem 2 - Answer: see table below:

Spacecraft	Distance (km)	Arrival at satellite	Arrival at Earth	One-way time
Themis P2	30,000	**01:20**	**01:20**	**0.2 sec**
LRO	382,000	**01:20**	**01:20**	**2.6 sec**
ACE	1.5 million	**01:20**	**01:20**	**10 sec**
MESSENGER	50 million	**01:23**	**01:26**	**2.8 minutes**
STEREO-A	111 million	**01:26**	**01:32**	**6.2 minutes**
Mars Orbiter	220 million	**01:32**	**01:44**	**12.2 minutes**
Ulysses	800 million	**02:04**	**02:49**	**44.4 minutes**
Cassini	1.8 billion	**03:00**	**04:44**	**1.7 hours**
Voyager 2	13 billion	**13:20**	**01:20 on Feb 16th**	**12.0 hours**

Problem 3 - When Jupiter was located farthest from the sun (aphelion), it was at a distance of 667 million kilometers from Earth. When it was closest to the sun (perihelion) it was 590.5 million kilometers from Earth. Suppose you are calculating a schedule for when the satellite Io will be exactly at dead-center of Jupiter's disk based on when you saw the transit at perihelion. How much of a schedule change will you see when you observe the transit at aphelion, and will the transit occur sooner or later than you predicted?

Answer: The difference between Jupiter's distance in each case is 667 million - 590.5 million = 76.5 million kilometers. At the speed of light, this equals a time difference of 76500000/300000 = 255 seconds or 4.25 minutes. At the time the transit occurs at aphelion, the event will be seen about 4.25 minutes later than the predicted time.

Kelvin Temperatures and Very Cold Things!

183 K	Vostok, Antarctica
160 K	Phobos
134 K	Superconductors
128 K	Europa summertime
120 K	Moon at night
95 K	Titan Noon
90 K	Liquid oxygen
88 K	Miranda Noon
81 K	Enceladus summertime
77 K	Liquid nitrogen
70 K	Mercury at night
63 K	Solid nitrogen
55 K	Pluto summertime
54 K	Solid oxygen
50 K	Quaoar summertime
45 K	Moon shadowed crater
40 K	Star-forming nebula
33 K	Pluto wintertime
20 K	Hydrogen liquifies
19 K	Bose-Einstein matter
4 K	Helium liquifies
3 K	Cosmic fireball light
1 K	Helium becomes solid
0 K	ABSOLUTE ZERO

To keep track of some of the coldest things in the universe, scientists use the Kelvin temperature scale which begins at 0 Kelvin, which is also called Absolute Zero. Nothing can ever be colder than Absolute Zero because at this temperature, all motion stops. The table to the left shows some typical temperatures of different systems in the universe.

You are probably already familiar with the Centigrade (C) and Fahrenheit (F) temperature scales. The two formulas below show how to switch from degrees-C to degrees-F.

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32$$

Because the Kelvin scale is related to the Centigrade scale, we can also convert from Centigrade to Kelvin (K) using the equation:

$$K = 273 + C$$

Use these three equations to convert between the three temperature scales:

Problem 1: 212 F converted to K

Problem 2: 0 K converted to F

Problem 3: 100 C converted to K

Problem 4: -150 F converted to K

Problem 5: -150 C converted to K

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F. A) What are the equivalent temperatures on the Kelvin scale; B) What is the average daytime temperature on the Kelvin scale?

Answer Key

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32 \qquad K = 273 + C$$

Problem 1: 212 F converted to K :
First convert to C: C = 5/9 (212 - 32) = +100 C. Then convert from C to K:
K = 273 + 100 = 373 Kelvin

Problem 2: 0 K converted to F: First convert to Centigrade:
C = K - 273 so C = -273 degrees. Then convert from C to F:
F = 9/5 (-273) + 32 = -459 Fahrenheit.

Problem 3: 100 C converted to K : K = 273 + C = 373 Kelvin.

Problem 4: -150 F converted to K : Convert to Centigrade
C = 5/9 (-150 - 32) = -101 C. Then convert from Centigrade to Kelvin: K = 273 - 101 = 172 Kelvin.

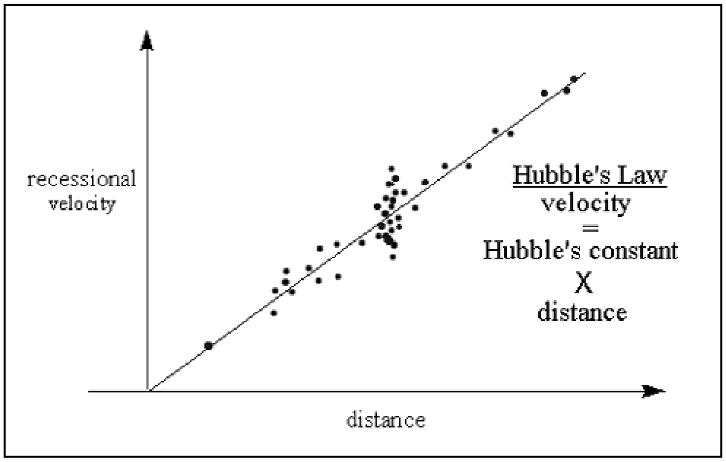
Problem 5: -150 C converted to K : K = 273 + (-150) = 123 Kelvin

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F.

A) What are the equivalent temperatures on the Kelvin scale?;
107 C becomes K = 273 + 107 = 380 Kelvins.
221 F becomes C = 5/9 (221 - 32) = 105 C, and so K = 273 + 105 = 378 Kelvins.

B) What is the average daytime temperature on the Kelvin scale?
Answer: (380 + 378)/2 = 379 Kelvins.

Equations with one variable II



Calculations involving a single variable come up in many different ways in astronomy. One way is through the relationship between a galaxy's speed and its distance, which is known as Hubbel's Law. Here are some more applications for you to solve!

Problem 1 – The blast wave from a solar storm traveled 150 million kilometers in 48 hours. Solve the equation 150,000,000 = 48 V to find the speed of the storm, V, in kilometers per hour.

Problem 2– A parsec equals 3.26 light years. Solve the equation 4.3 = 3.26D to find the distance to the star Alpha Centauri in parsecs, D, if its distance is 4.3 light years.

Problem 3 – Hubble's Law states that distant galaxies move away from the Milky Way, 75 kilometers/sec faster for every 1 million parsecs of distance. Solve the equation, V = 75 D to find the speed of the galaxy NGC 4261 located 41 million parsecs away

Problem 4 – Convert the temperature at the surface of the Sun, 9,900 degrees Fahrenheit to an equivalent temperature in Kelvin units, T, by using T = (F + 459) x 5/9

Problem 5 – The Andromeda Galaxy measures 3 degrees across on the sky as seen from Earth. At a distance of 2 million light years, solve for D, the diameter of this galaxy in light years: 57.3 = 6,000,000/D.

Answer Key

1 – The blast wave from a solar storm traveled 150 million kilometers in 48 hours. Solve the equation 150,000,000 = 48 V to find the speed of the storm, V, in kilometers per hour.

Answer: 150,000,000/48 = V so V = 3,125,000 kilometers/hour.

2 – A parsec equals 3.26 light years. Solve the equation 4.3 = 3.26D to find the distance to the star Alpha Centauri in parsecs, D, if its distance is 4.3 light years.

Answer: D = 4.3/3.26 = 1.3 parsecs.

3 – Hubble's Law states that distant galaxies move away from the Milky Way, 75 kilometers/sec faster for every 1 million parsecs of distance. V = 75 x D. Solve the equation to find the speed of the galaxy NGC 4261 located D = 41 million parsecs away

Answer: V = 75 x 41 so V = 3,075 kilometers/sec.

4 – Convert the temperature at the surface of the sun, 9,900 degrees Fahrenheit (F) to an equivalent temperature in Kelvin units, T, by using T = (F + 459) x 5/9

Answer: T = (F + 459) x 5/9 so T = (9,900 + 459) x 5/9 = 5,755 Kelvins

5 – The Andromeda Galaxy measures 3 degrees across on the sky as seen from Earth. At a distance of 2 million light years, solve for D, the diameter of this galaxy in light years: 57.3 = 6,000,000/D.

Answer: D = 6,000,000/57.3 so D = 104,700 light years in diameter.

Variables and Expressions from Around the Cosmos

There are many simple mathematical formulae that astronomers use to describe different aspects of the universe and the physical world. (The above photo of Saturn was taken by NASA's Cassini spacecraft from behind Saturn looking back towards the Sun.)

Problem 1: Find, P, the length of Earth's day 500 million years in the future if **P = 24 hours + 0.004 Y**, where Y is the number of millions of years that have elapsed.

Problem 2: Find the distance to the Andromeda galaxy in light years, L, if its distance in parsecs, P = 770,000 and **L = 3.26** P.

Problem 3: Find the temperature, T, of a gas cloud emitting X-rays if the energy of the X-rays is E = 12,000 electron Volts and **T = 11,500 E**.

Problem 4: Find the temperature in degrees Centigrade of the air at an altitude of 20 kilometers if H = 20 and T = 25.0 - 6.5 H.

Problem 5: Find the diameter in kilometers, D, of a black hole with a mass of 10 times the sun if **D = 5.6 M** and M = 10.0.

Problem 6: Calculate the speed of sound, S, in meters/second for a temperature of T = 200 Centigrade (that's 392 F), if **S = 331 + 0.6 T**.

Problem 7: Calculate the sunspot number, N, if there are X = 15 individual sunspots and Y = 10 groups of sunspots is **N = X + 11 Y**.

Answer Key

Problem 1: Find, P, the length of Earth's day 500 million years in the future if **P = 24 hours + 0.004 Y**, where Y is the number of millions of years that have elapsed.

Answer; **P = 24 hours + 0.004 (500) = 26 hours.**

Problem 2: Find the distance to the Andromeda galaxy in light years, L, if its distance in parsecs, P = 770,000 and **L = 3.26** P.

Answer: **L = 3.26** (770,000) = **2,200,000 light years.**

Problem 3: Find the temperature, T, of a gas cloud emitting X-rays if the energy of the X-rays is E = 12,000 electron Volts and **T = 11,500 E**.

Answer: **T = 11,500 (12,000) = 138 million K**

Problem 4: Find the temperature in degrees Centigrade of the air at an altitude of 20 kilometers if H = 20 and T = 25.0 - 6.5 H.

Answer: T = 25.0 - 6.5 (20) = **- 105 Centigrade.**

Problem 5: Find the diameter in kilometers, D, of a black hole with a mass of 10 times the sun if **D = 5.6 M** and M = 10.0.

Answer: **D = 5.6 (10) = 56 kilometers.**

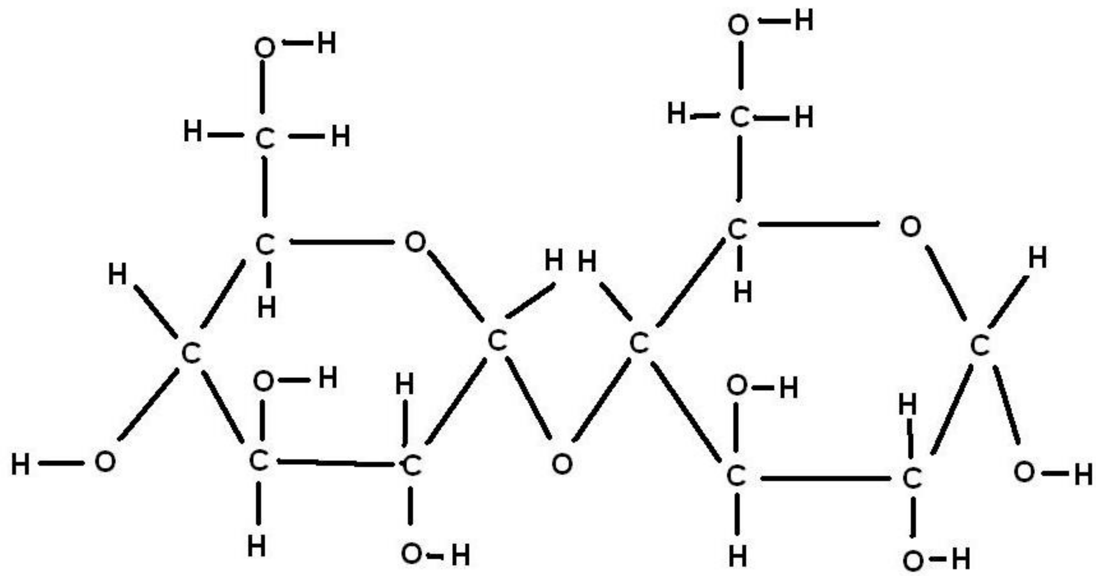
Problem 6: Calculate the speed of sound, S, in meters/second for a temperature of T = 200 Centigrade (that's 392 F), if **S = 331 + 0.6 T**.

Answer: **S = 331 + 0.6 (200)** = 331 + 120 = **451 meters/sec.**

Problem 7: Calculate the sunspot number, N, if there are X = 15 individual sunspots and Y = 10 groups of sunspots is **N = X + 15 Y**.

Answer: **N = 15 + 11 (10) = 15 + 110 = 125 sunspots.**

Atoms - How sweet they are!



Glucose is a very important sugar used by all plants and animals as a source of energy. Maltose is the next most complicated sugar, and is formed from two glucose molecules. The atomic ingredients of the maltose molecule is shown in the diagram above, which is called the structural formula for maltose. As an organic compound, it consists of three types of atoms: hydrogen (H), carbon (C), and oxygen (O).

Problem 1 - How many molecules does maltose contain of A) hydrogen? B) oxygen? C) carbon?

Problem 2 - What is the ratio of the number of hydrogen atoms to oxygen atoms?

Problem 3 - What fraction of all the maltose atoms are carbon?

Problem 4 - If the mass of 1 hydrogen atoms is 1 AMU, and 1 carbon atom is 12 AMU and 1 oxygen atom is 16 AMU, what is the total mass of one maltose molecule in AMUs?

Problem 5 - Write the chemical formula of maltose by filling in the missing blanks:

$C\_\_ \quad H\_\_ \quad O_{11}$

Answer Key

Problem 1 - How many molecules does maltose contain of A) hydrogen? B) oxygen? C) carbon? Answer: A) There are 22 hydrogen atoms; B) There are 11 oxygen atoms; C) there are 12 carbon atoms.

Problem 2 - What is the ratio of the number of hydrogen atoms to oxygen atoms? Answer: 22 hydrogen atoms / 11 oxygen atoms so the ratio is 2/1

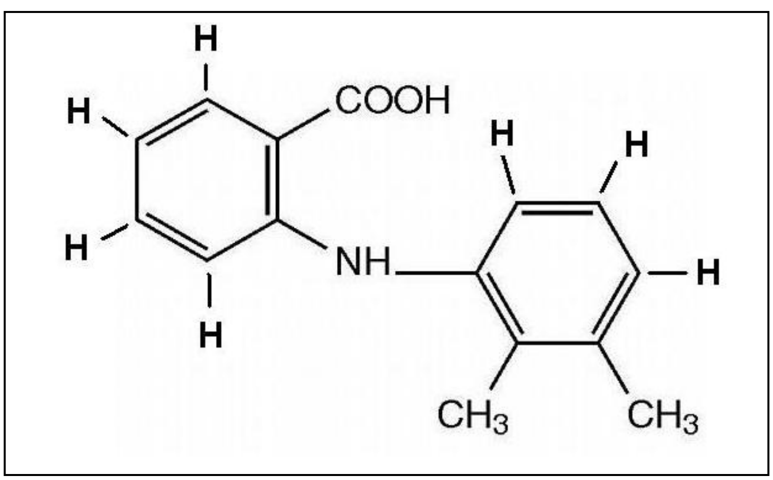
Problem 3 - What fraction of all the maltose atoms are carbon? Answer: The total number of atoms is 12 + 22 + 11 = 45, so carbon atoms are 12/45 of the total.

Problem 4 - If the mass of 1 hydrogen atoms is 1 AMU, and 1 carbon atom is 12 AMU and 1 oxygen atom is 16 AMU, what is the total mass of one maltose molecule in AMUs? Answer: 1 AMU x 22 atoms hydrogen + 12 AMU x 12 atoms carbon + 16 AMU x 11 atoms oxygen = 342 AMU.

Problem 5 - Write the chemical formula of maltose by filling in the missing blanks:

$C_{12}\ H_{22}\ O_{11}$

More Challenging Extra Problem:
Below is the structural formula for Mefenamic Acid. Students can determine its chemical formula as $C_{15}H_{15}\ N\ O_2$ and its mass as 241 AMU. In this kind of diagram, carbon atoms in the hexagonal rings are located at each vertex. Hydrogen atoms at the vertices are not labeled as well. Challenge your students to GOOGLE the term 'structural diagram' in the 'images' area and try to decipher other more complex molecules such as Lorazepam or Vancocin !



Period	Age (years)	Days per year	Hours per day
Current	0	365	
Upper Cretaceous	70 million	370	
Upper Triassic	220 million	372	
Pennsylvanian	290 million	383	
Mississippian	340 million	398	
Upper Devonian	380 million	399	
Middle Devonian	395 million	405	
Lower Devonian	410 million	410	
Upper Silurian	420 million	400	
Middle Silurian	430 million	413	
Lower Silurian	440 million	421	
Upper Ordovician	450 million	414	
Middle Cambrian	510 million	424	
Ediacarin	600 million	417	
Cryogenian	900 million	486	

We learn that an 'Earth Day' is 24 hours long, and that more precisely it is 23 hours 56 minutes and 4 seconds long. But this hasn't always been the case. Detailed studies of fossil shells, and the banded deposits in certain sandstones, reveal a much different length of day in past eras! These bands in sedimentation and shell-growth follow the lunar month and have individual bands representing the number of days in a lunar month. By counting the number of bands, geologists can work out the number of days in a year, and from this the number of hours in a day when the shell was grown, or the deposits put down. The table above shows the results of one of these studies.

Problem 1 - Complete the table by calculating the number of hours in a day during the various geological eras. It is assumed that Earth orbits the sun at a fixed orbital period, based on astronomical models that support this assumption.

Problem 2 - Plot the number of hours lost compared to the modern '24 hours' value, versus the number of years before the current era.

Problem 3 - By finding the slope of a straight line through the points can you estimate by how much the length of the day has increased in seconds per century?

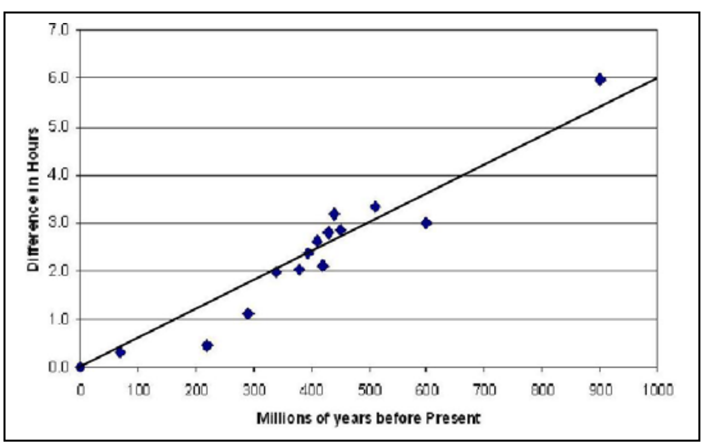
Answer Key

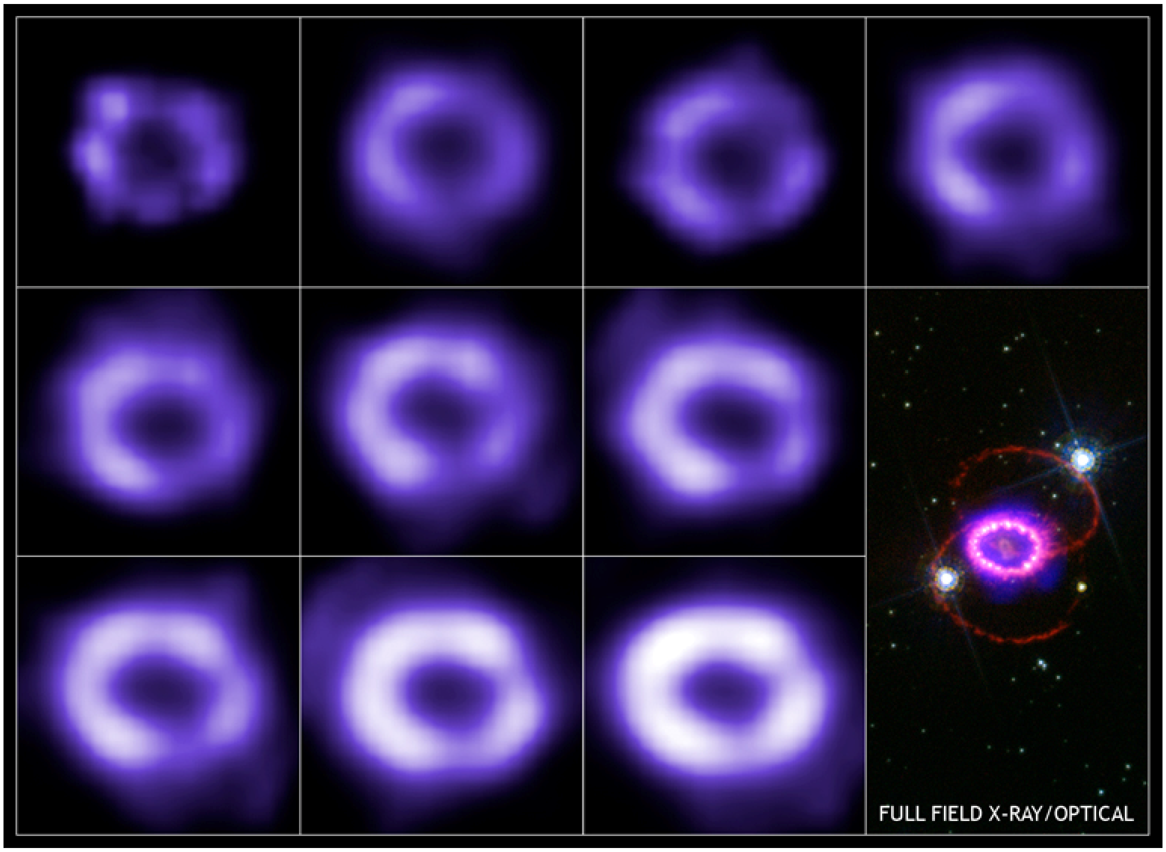
Period	Age (years)	Days per year	Hours per day
Current	0	365	24.0
Upper Cretaceous	70 million	370	23.7
Upper Triassic	220 million	372	23.5
Pennsylvanian	290 million	383	22.9
Mississippian	340 million	398	22.0
Upper Devonian	380 million	399	22.0
Middle Devonian	395 million	405	21.6
Lower Devonian	410 million	410	21.4
Upper Silurian	420 million	400	21.9
Middle Silurian	430 million	413	21.2
Lower Silurian	440 million	421	20.8
Upper Ordovician	450 million	414	21.2
Middle Cambrian	510 million	424	20.7
Ediacarin	600 million	417	21.0
Cryogenian	900 million	486	18.0

Problem 1 - Answer; See table above. Example for last entry: 486 days implies 24 hours x (365/486) = 18.0 hours in a day.

Problem 2 - Answer; See figure below

Problem 3 - Answer: From the line indicated in the figure below, the slope of this line is m = (y2 - y1)/ (x2 - x1) = 6 hours / 900 million years or 0.0067 hours/million years. Since there are 3,600 seconds/ hour and 10,000 centuries in 1 million years (Myr), this unit conversion yields 0.0067 hr/Myr x (3600 sec/hr) x (1 Myr / 10,000 centuries) = 0.0024 seconds/century. This is normally cited as 2.4 milliseconds per century.





In March, 1987 a supernova occurred in the Large Magellanic Cloud; a nearby galaxy to the Milky Way about 160,000 light years away from Earth. The site of the explosion was traced to the location of a blue supergiant star called Sanduleak -69º 202 (SK -69 for short) that had a mass estimated at approximately 20 times our own sun.The series of image above, taken by the Chandra X-ray Observatory, shows the expansion of the million-degree gas ejected by the supernova between January, 2000 (top left image) to January, 2005 (lower right image). The width of each image is 1.9 light years.

Problem 1 - Using a millimeter ruler, what is the scale of each image in light years/millimeter?

Problem 2 - If 1 light year = 9.5×10^{12} kilometers, and 1 year = 3.1×10^{7} seconds, what was the average speed of the supernova gas shell between 2000 and 2005?

Problem 1 - Using a millimeter ruler, what is the scale of each image in light years/millimeter?

Answer: The width of each image is about 38 millimeters, so the scale is 1.9 light years / 38 mm = **0.05 light years/mm**.

Problem 2 - If 1 light year = 9.5×10^{12} kilometers, and 1 year = 3.1×10^{7} seconds, what was the average speed of the supernova gas shell between 2000 and 2005?

Answer: First convert the scale to kilometers/mm to get 0.05 LY/mm x (9.5×10^{12} kilometers / 1 LY) = 4.8×10^{11} kilometers/mm.

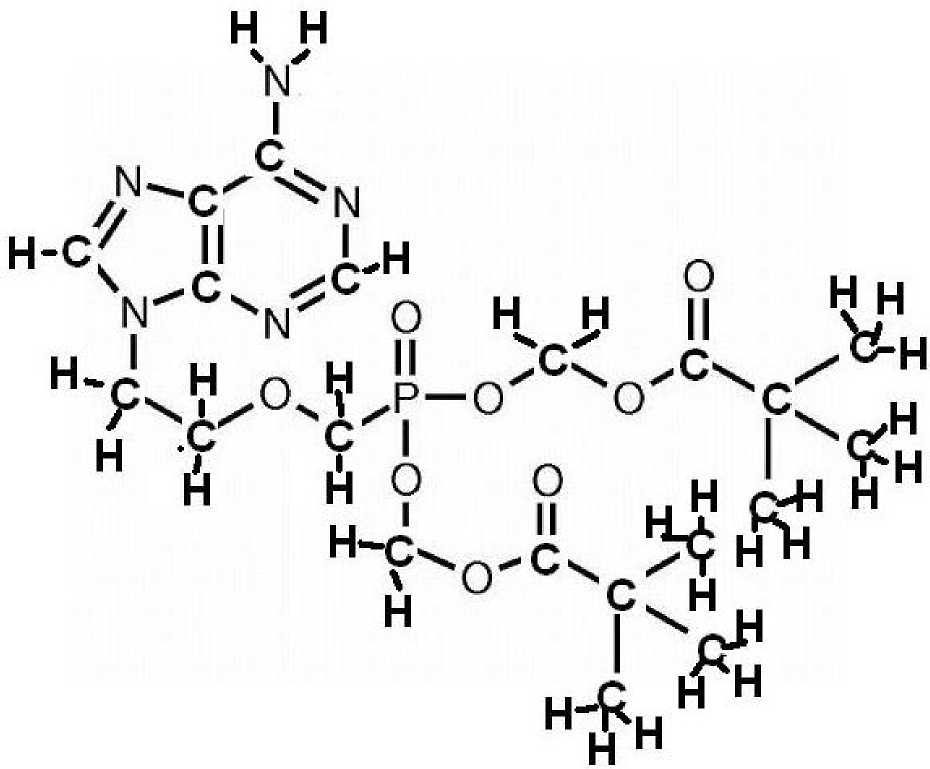
Next, students will have to measure the outer diameter of an irregular shape. They may do this by making several measurements across different chords through the center of the shell, and averaging the numbers. Because the outer edge is not sharp due to brightness gradients, students will also have to decide at what point the edge of the shell occurs and then use this visual definition consistently in the other edge measurements.

The diameter of the top left supernova ring is about 19 millimeters. The bottom-right ring has a diameter of about 27 millimeters, so the change on radius was (27 - 19)/2 = 4 millimeters. This corresponds to a physical distance of 4 mm x (4.8×10^{11} kilometers/mm) = 1.9×10^{12} kilometers. The elapsed time was (January 2005 - January 2000) = 5 years or 5 years x (3.1×10^{7} seconds / 1 year) = 1.6×10^{8} seconds. The average speed is then V = 1.9×10^{12} kilometers / 1.6×10^{8} seconds = **12,000 kilometers/sec**.

Note to Teacher: Although this is the average speed, students can investigate whether the shell has been moving at a constant speed during this 5-year period, or if the gas shell has been accelerating or deceleration by measuring the speed differences between the consecutive images which can be found at:

http://chandra.harvard.edu/photo/2005/sn87a/more.html ………..for specific image dates

http://chandra.harvard.edu/photo/2005/sn87a/index.html ……….. supernova information



This is a figure showing the locations of hydrogen (H), oxygen (O), carbon (C) , nitrogen (N) and phosphorus (P) atoms in one molecule of adefovir dipivoxil, which is a drug designed to treat hepatitis B.

Problem 1 - How many atoms of each element are present in one molecule of tannic acid?

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

C__ H__ N__O__P__

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in adefovir dipivoxil are H = 1 AMU, C=12 AMU, N= 14 AMU, O=16 AMU, and P = 31 AMU, what is the total mass of a single molecule in AMUs?

Problem 4 - If 1 AMU equals 1.7×10^{-27} kilograms, how many molecules are present in a sample with a mass of 1 microgram?

Problem 1 - How many atoms of each element are present in one molecule of tannic acid?

Answer: **Carbon (C) = 20**
Oxygen (O) = 8
Hydrogen (H) = 32
Nitrogen (N) = 5
Phosporus (P) = 1

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

$$C_{20}\, H_{32}\, N_5\, O_8\, P$$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in adefovir dipivoxil are H = 1 AMU, C=12 AMU, N= 14 AMU, O=16 AMU, and P = 31 AMU, what is the total mass of a single molecule in AMUs?

Answer: M = 20(12) + 32(1) + 5(14) + 8(16) + 1(31) = **501 AMU.**

Problem 4 - If 1 AMU equals 1.7×10^{-27} kilograms, how many molecules are present in a sample with a mass of 1 microgram?

Answer: One molecule has a mass of 501 AMU x ($1.7\text{x}10^{-27}$ km/1 AMU) = 8.5×10^{-25} kg. The sample has a total mass of $1.0\text{x}10^{-6}$ grams which equals $1.0\text{x}10^{-9}$ kilograms.

So the number of molecules is N = $1.0\text{x}10^{-9}$ /$8.5\text{x}10^{-25}$ = **1.2×10^{15} molecules**.

Earth's Polar Wander - The Chandler Wobble

Year	Month	X (meters)	Y (meters)
2008	1	-3	+8
2008	2	-4	+10
2008	3	-4	+13
2008	4	-2	+15
2008	5	+1	+16
2008	6	+4	+16
2008	7	+7	+15
2008	8	+9	+13
2008	9	+9	+10
2008	10	+8	+7
2008	11	+6	+5
2008	12	+3	+4
2009	1	0	+4
2009	2	-3	+6
2009	3	-4	+9
2009	4	-4	+12
2009	5	-2	+15
2009	6	+1	+16
2009	7	+4	+16
2009	8	+7	+15

The Earth rotates on its axis once every 24 hours (23 hours 56 minutes and 4 seconds more-accurately!), but like a bobbing, spinning top, the direction doesn't point exactly at an angle of 23 1/5 degrees.

For over 200 years, careful measurements of the rotation axis have shown that it moves slightly. The data in the table to the left gives the axis location of the North Pole as it passes through Earth's surface. The X-axis runs East-West and the Y-axis runs North-South. The units are in meters measured on the ground.

Problem 1 - On a Cartesian graph, plot the location of the North Pole during the time span indicated by the table.

Problem 2 - About how long, in days, does it take the North Pole to return to its starting position based on this data?

Problem 3 - About what is the average speed of this Polar Wander in meters per day?

Problem 4 - Based on your plot, does it look like the motion will exactly repeat itself in space during the next cycle?

Problem 5 - About what is the X and Y location of the center of the movement pattern?

Problem 6 - What would you use as the best location of the North Pole?

Answer Key

Problem 1 - Answer; See below.

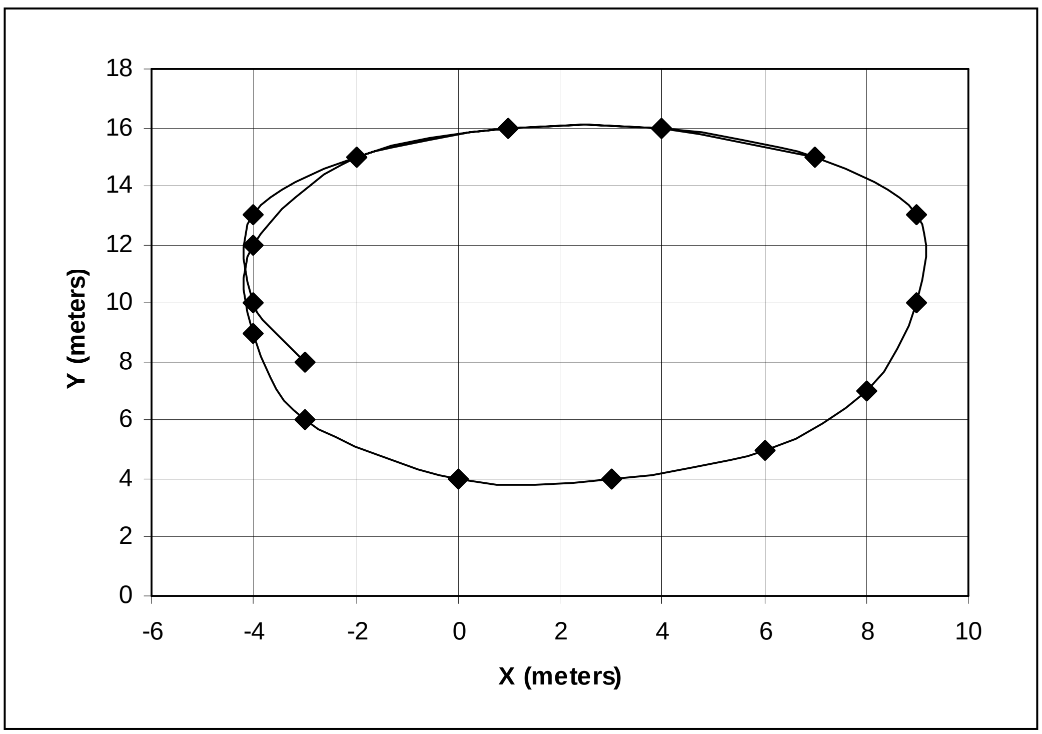
Problem 2 - Answer; About 14 months or 420 days. This is called the Chandler Period.

Problem 3 - Answer: In one month the point moves on the circle about 3 meters, so the speed is about 3 meters/30 days or 0.1 meter/day.

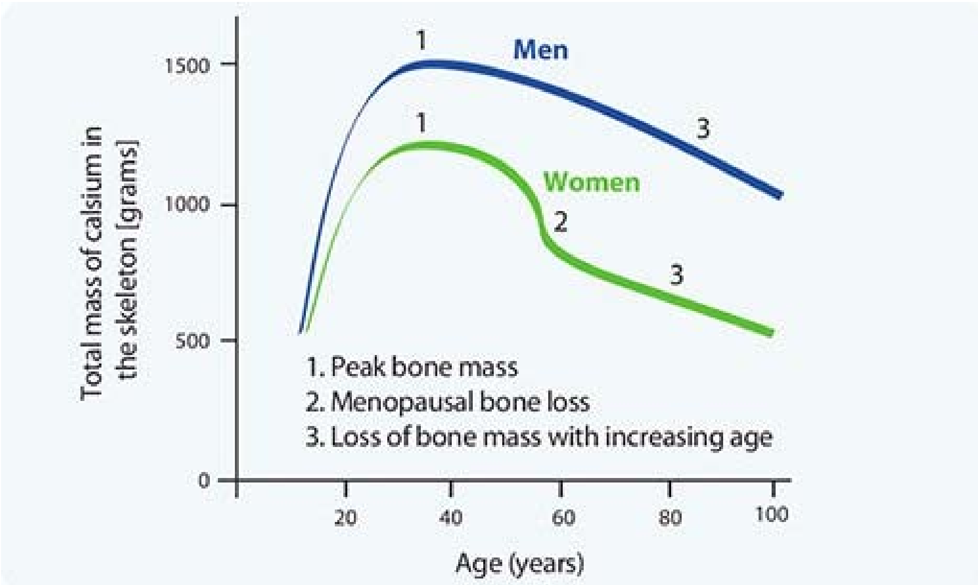
Problem 4 - Answer; Not exactly. The new cycle points do not follow the circle of the older cycle. This motion is called the Chandler Wobble.

Problem 5 - Answer; X = + 2 meters, Y = +10 meters.

Problem 6 - Answer: Probably the point at the center of the circle, because it is about equidistant from all of the other points.



Astronaut Bone Loss in Space



Under low-gravity conditions, human bones respond by losing mass. The longer an astronaut remains in space, the more bone loss occurs. This is considered one of the greatest challenges to humans operating in space for extended periods of time, such as journeys to Mars and living in 'space stations'. The above graph shows normal bone loss with age for humans living on Earth. The average bone loss for some astronauts is 1.9 percent per month.

Problem 1 – From the graph above, what is the average rate of bone loss between age 40 and age 70 for A) Men and B)Women?

Problem 2 – Suppose a 40 year old male astronaut spent 6 months aboard the International Space Station. If he started out with 1500 grams of bone calcium, how much calcium would remain in his bones when he returned to Earth?

Problem 3 - Suppose a 40 year old female astronaut spent 6 months aboard the International Space Station. If she started out with 1200 grams of bone calcium, how much calcium would remain in her bones when she returned to Earth?

Problem 4 – From the graph, how old would both astronauts be in order to have the same amount of calcium as they did after returning to Earth?

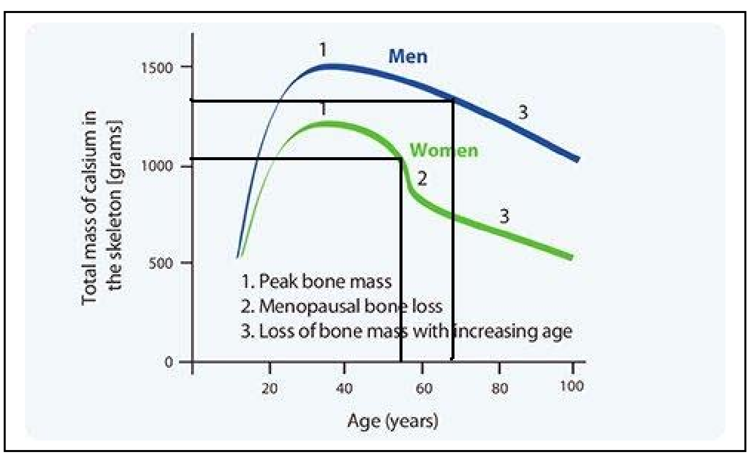
Problem 1 – From the graph above, what is the average rate of bone loss between age 40 and age 70 for A) Men and B)Women? Answer: A) rate = (change in mass) / (change in time) = (1100 – 1500) / (100 – 40) = - 6.7 grams/year. B) rate = (500 – 1200)/(100 – 40) = - 11.7 grams/year.

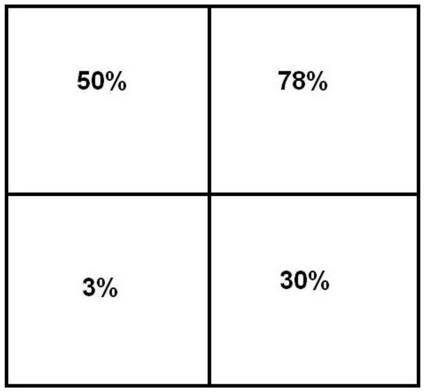
Note: Because the signs are negative, the rate is a mass loss. As a percentage, for men the rate is 100% x (6.7/1500) = 0.4% per year. For women it is 100% x (11.7/1200) = 1.0% per year…more than twice as fast as for men.

Problem 2 – Suppose a 40 year old male astronaut spent 6 months aboard the International Space Station. If he started out with 1500 grams of bone calcium, how much calcium would remain in his bones when he returned to Earth? Answer: The astronaut will lose 0.019 x 6 months x 1500 grams = 171 grams lost

Problem 3 - Suppose a 40 year old female astronaut spent 6 months aboard the International Space Station. If she started out with 1200 grams of bone calcium, how much calcium would remain in her bones when she returned to Earth? Answer: The astronaut will lose 0.019 x 6 months x 1200 grams = 137 grams lost

Problem 4 – From the graph, how old would both astronauts be in order to have the same amount of calcium as they did after returning to Earth? Answer: For the male astronaut, his final bone mass is 1500 grams – 171 grams = 1329 grams. From the graph below, this bone mass is reached when a typical male reaches the age of about 67 years. For the female astronaut, her final bone mass is 1200 grams – 137 grams = 1063 grams. From the graph below, this bone mass is reached when a typical male reaches the age of about 55 years.





Material	Reflectivity
Snow	80%
White Concrete	78%
Bare Aluminum	74%
Vegetation	50%
Bare Soil	30%
Wood Shingle	17%
Water	5%
Black Asphalt	3%

When light falls on a material, some of the light energy is absorbed while the rest is reflected. The absorbed energy usually contributed to heating the body. The reflected energy is what we use to actually see the material! Scientists measure reflectivity and absorption in terms of the percentage of energy that falls on the body. The combination must add up to 100%.

The table above shows the reflectivity of various common materials. For example, snow reflects 80% of the light that falls on it, which means that it absorbs 20% and so 80% + 20% = 100%. This also means that if I have 100 watts of light energy falling on the snow, 80 watts will be reflected and 20 watts will be absorbed.

Problem 1 - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Problem 2 - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Problem 3 - What is the average reflectivity of these four equal-area regions combined?

Problem 4 - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

Answer Key

Problem 1 - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Answer: The reflectivity is 100% x (300 watts/1000 watts) = 30%. From the table, Bare Soil has this same reflectivity and so is a likely composition.

Problem 2 - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Answer: 50% = Vegetation
78% = White Concrete
30% = Bare Soil
3% = Black Asphalt

Problem 3 - What is the average reflectivity of these four equal-area regions combined? Answer: Because each of the four materials cover the same area, we just add up their reflectivities and divide by 4 to get (50% + 78% + 30% + 3%)/4 = 40%.

Problem 4 - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

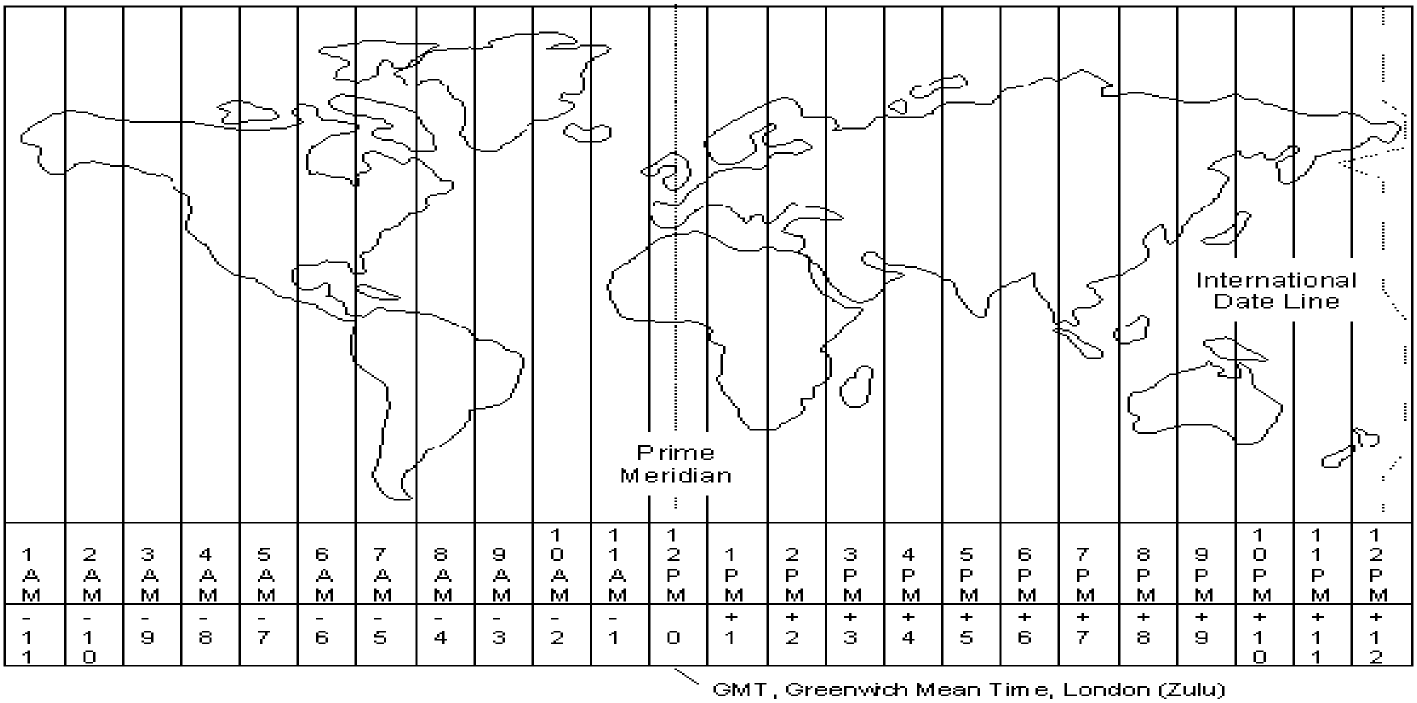
Answer: Each material covers 10 meters x 10 meters = 100 square meters:

A) Vegetation: 0.5 x 1300 watts/m^2 x 100 m^2 = 65,000 watts.
Concrete: 0.78 x 1300 watts/m^2 x 100 m^2 = 101,400 watts.
Bare Soil: 0.30 x 1300 watts/m^2 x 100 m^2 = 39,000 watts.
Black Asphalt: 0.03 x 1300 watts/m^2 x 100 m^2 = 3,900 watts.

B) 65,000 + 100,000 + 39,000 + 3,900 = 209,300 watts.

C) The total wattage entering this area is 1,300 watts/m^2 x 100 m^2 x 4 = 520,000 watts. Since 209,300 watts are reflected, that means that 520,000 watts - 209,300 watts = 310,700 watts are being absorbed.

Name ________________________________



The above diagram shows the 24 time zones around the world. If you live in New York and want to call someone in San Francisco, you have to remember that the local time in San Francisco (Pacific Standard Time) is 3 hours BEHIND New York Time (Eastern Standard Time). By counting the timezones between the locations and keeping track of ‘ahead’ or ‘behind’ you can figure out the correct local times anywhere on the planet. Suppose an astronomer in Texas spots a solar flare at 6:00 AM Central Standard Time near sunrise. Meanwhile, an astronomer in India spots the same solar flare at 5:00 PM India Time near sunset.

Question 1) How can exactly the same event be seen at two different times on Earth?

Question 2) What time would an astronomer in West Africa have seen the same solar flare according to her local time?

Question 3) Would an astronomer living in Central Australia have seen the flare? How about an astronomer living in Alaska?

Question 4) Suppose a solar flare happened at 10:31 AM Greenwich Mean Time. What time would the event have happened in California according to Pacific Standard Time?

Question 5) A satellite registers a major solar explosion that lasts from 3:15 PM to 4:26 PM Greenwich Mean Time. A solar scientist monitoring the satellite data decided to go to Starbucks for a quick coffee between 7:00 AM and 7:45 AM Pacific Standard Time.

e) How long did the explosion last?
f) Did the scientist know about the flare before he left for coffee?
g) How much of the flare event did the scientist get to see in the satellite data as it happened?
h) Should the scientist have gone for coffee?

One of the most basic ideas that astronomers have to work with is the idea of the 'time zone'. When one astronomer observes a phenomenon from one place on Earth, it is often important to be able to communicate when that phenomenon occurred to another astronomer located somewhere else on Earth. To make this as easy as possible, we adopt the time of the phenomenon as it would have been observed in Greenwich England – called Greenwich Mean Time (GMT). This required converting the astronomers local time to GMT.

This activity is an exercise in adding and subtracting time, and how to use a timezone map of the world, to convert from local time to Greenwich Mean Time.

Question 1) How can exactly the same event be seen at two different times on Earth? **Answer** – Because the surface of Earth has different time zones.

Question 2) What time would an astronomer in West Africa have seen the same solar flare according to her local time? **Answer** – From the diagram and the location of West Africa, the local time would be 11:00 AM.

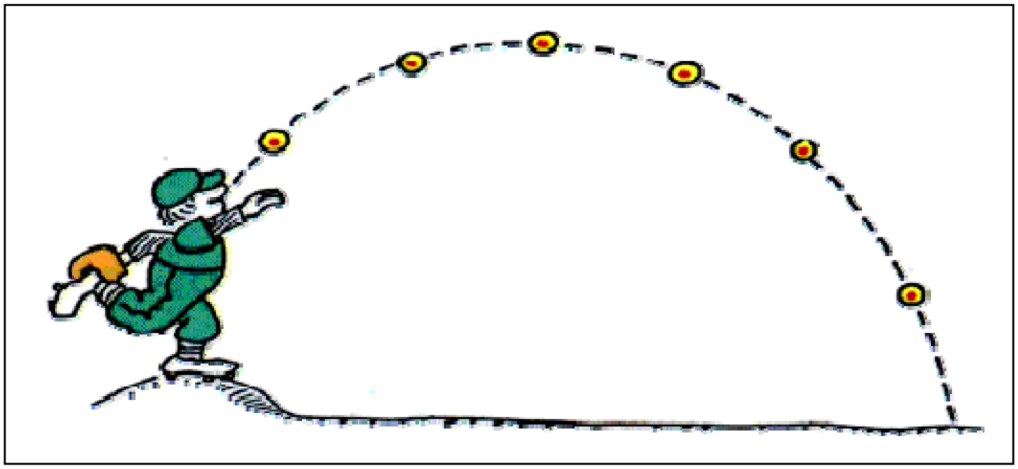
Question 3) Would an astronomer living in Central Australia have seen the flare? How about an astronomer living in Alaska? **Answer** – Neither would have seen it because the sun had already set (Australia) or had not yet risen (Alaska).

Question 4) Suppose a solar flare happened at 10:31 AM Greenwich Mean Time. What time would the event have happened in California according to Pacific Standard Time? **Answer** – By the diagram, PST is 8 hours behind GMT so 10:31 AM – 8hrs = 2:31 AM PST. This is well before sunrise so it wouldn't have been seen.

Question 5) A satellite registers a major solar explosion that lasts from 3:15 PM to 4:26 PM Greenwich Mean Time. A solar scientist monitoring the satellite data decided to go to Starbucks for a quick coffee between 7:00 AM and 7:45 AM Pacific Standard Time.

a) How long did the explosion last? **Answer** 4:26 PM– 3:15 PM = 1hour and 11 minutes.
b) Did the scientist know about the flare before he left for coffee? **Answer:** He left for coffee at 7:00 AM PST which is 7:00 + 8hrs = 3:00 PM GMT so he didn't know about the solar explosion which happened 15 minutes after he left.
c) How much of the flare event did the scientist get to see in the satellite data as it happened? **Answer:** When he returned to the satellite station, the time was 7:45 AM PST + 8hrs = 3:45 PM GMT. The explosion ended at 4:26 PM GMT, so the scientist got to see 4:26 PM – 3:45 PM = 41 minutes of the last part of the explosion.
d) Should the scientist have gone for coffee? **Answer.** Yes, of course! Because if he had been smart, he would have made sure that his data was being recorded for playback, just the way you use a tape in your VCR to record favorite programs you have to miss at the time they broadcast them. Also, no scientist using satellite data ever relies on on-the-spot analysis. We always record the satellite data so we can study it in detail later. Still, I'm sure after a long night at the satellite terminal, he would have loved to have been there to see it happen in 'real-time'!!

The Physics of Rock Throwing



The horizontal motion of a rock (projectile) is given by the formula:

$$X = V_h T$$

Independently, the vertical motion is given by the formula

$$Y = H_0 + V_v T - \frac{1}{2} g T^2$$

The speed of the projectile has been described in terms of its vertical (Vv) and horizontal (Vh) speeds to that the total speed is given by the Pythagorean Theorem $S = (V_h^2 + V_v^2)^{1/2}$.

Problem 1 – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land? ($g = 32$ feet/sec^2)

Problem 2 – On Mars ($g = 12$ feet/sec^2) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

Problem 1 – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land?

Answer: H_0 = 1063 feet, V_h = 40 feet/sec, V_v = 0.0, g = 32 feet/sec^2. The vertical equation give us the time to reach the ground (y=0) : 0 = 1063 – 16 T^2, so 1063/16 = T^2 and T = 8.1 seconds. From the horizontal motion, it travels d = 40 x 8.1 = **324 feet**.

Problem 2 – On Mars (g = 12 feet/sec^2) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

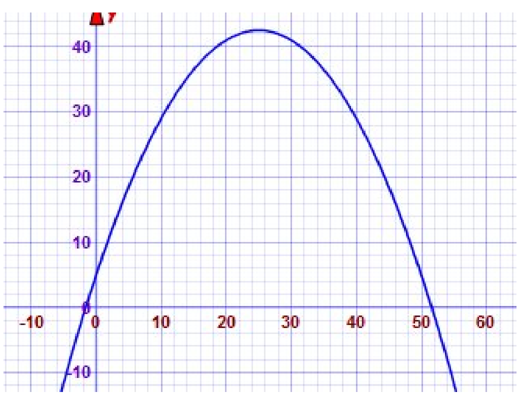
Answer: The two equations are X = 10 T and Y = 5.0 + 30T - 6T^2

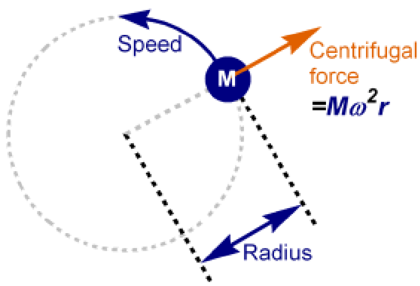
Write Y in terms of X: Y = 5.0 + 30(X/10) - 6 $(X/10)^2$ so **Y = 5.0 + 3.0X - 0.06X^2**

Solve for the roots of Y(X) = -0.06X^2 + 3.0X + 5.0 with coefficients a = -0.06, b=3.0 and c = 5.0, to get the ground intercept points using the Quadratic Formula:

X = [-3.0 +/- $(9-4(-0.06)(5.0))^{1/2}$] / (2x-0.06) so
X = (-3 + 3.19)/-0.12 = -1.6 feet , and the second root is
X = (-3 – 3.19)/-0.12 = +51.6 feet. see graph below.

The peak of the parabola is ½ way between the x-intercepts at x = (51.6-1.6)/2 = +25.0 feet And since X = 10T, we have 25 = 10T so T = 2.5 seconds. From Y(T), the altitude of the peak is Y = 5.0 + 30(2.5) – 6$(2.5)^2$ = **42.5 feet**. From the x-intercept, it reaches a distance of **51.6 feet from the astronaut.**






Gravitron ride at an amusement park

Most science fiction stories require some form of artificial gravity to keep spaceship passengers operating in a normal earth-like environment. As it turns out, weightlessness is a very bad condition for astronauts to work in on long-term flights. It causes bones to lose about 1% of their mass every month. A 30 year old traveler to Mars will come back with the bones of a 60 year old!

The only known way to create artificial gravity it to supply a force on an astronaut that produces the same acceleration as on the surface of earth: 9.8 meters/sec^2 or 32 feet/sec^2. This can be done with bungee chords, body restraints or by spinning the spacecraft fast enough to create enough centrifugal acceleration.

Centrifugal acceleration is what you feel when your car 'takes a curve' and you are shoved sideways into the car door, or what you feel on a roller coaster as it travels a sharp curve in the tracks. Mathematically we can calculate centrifugal acceleration using the formula:

$$A = \frac{V^2}{R}$$

where V is in meters/sec, R is the radius of the turn in meters, and A is the acceleration in meters/sec^2.

Let's see how this works for some common situations!

Problem 1 - The Gravitron is a popular amusement park ride. The radius of the wall from the center is 7 meters, and at its maximum speed, it rotates at 24 rotations per minute. What is the speed of rotation in meters/sec, and what is the acceleration that you feel?

Problem 2 - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity (1 g = 9.8 meters/sec^2)?

Problem 3 – Another way to create a 1-G force is for the rocket to continuously fire during its journey so that its speed increases by 9.8 meters/sec every second. Suppose that NASA could design such a rocket system for a 60 million km trip to Mars. The idea is that it will accelerate during the first half of its trip, then make a 180-degree flip and decelerate for the remainder of the trip. If

$$\text{distance} = 1/2aT^2$$
$$\text{speed} = aT,$$

where **T** is in seconds, **a** is in meters/sec^2, and distance is in meters, how long will the trip take in hours, and what will be the maximum speed of the spacecraft at the turn-around point?

Problem 1 - The Gravitron is a popular amusement park ride. The radius of the wall from the center is 7 meters, and at its maximum speed, it rotates at 24 rotations per minute. What is the speed of rotation in meters/sec, and what is the acceleration that you feel?

Answer: For circular motion, the distance traveled is the circumference of the circle C = 2 π (7 meters) = 44 meters. At 24 rpm, it makes one revolution every 60 seconds/24 = 2.5 seconds, so the rotation speed is 44 meters/2.5 sec = 17.6 meters/sec.

The acceleration is then A = $(17.6)^2/7$ = **44.3 meters/sec^2**. Since 1 earth gravity = 9.8 meters/sec^2, the 'G-Force' you feel is 44.3/9.8 = 4.5 Gs. That means that you feel 4.5 times heavier than you would be just standing in line outside!

Problem 2 - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity (1 g = 9.8 meters/sec^2)?

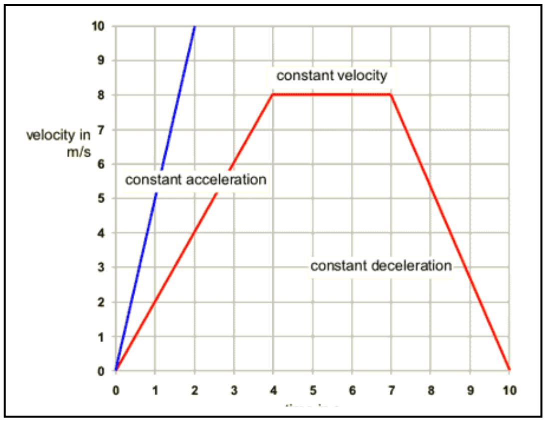
Answer: The circumference is C = 2 π (30) = 188 meters. 1 RPM is equal to rotating one full circumference every minute, for a speed of 188/60 sec = 3.1 meters/second. So V = 3.1 meters/sec x RPM. Then A = $(3.1\text{RPM})^2$ / 188 = 0.05 x RPM^2. We need 9.8 = 0.05RPM^2 so **RPM = 14**.

Problem 3 – Another way to create a 1-G force is for the rocket to continuously fire during its journey so that its speed increases by 9.8 meters/sec every second. Suppose that NASA could design such a rocket system for a 60 million km trip to Mars. The idea is that it will accelerate during the first half of its trip, then make a 180-degree flip and decelerate for the remainder of the trip. If distance = 1/2aT^2 and V = aT, how long will the trip take in hours, and what will be the maximum speed of the spacecraft at the turn-around point?

Answer: The turn-around point happens at the midway point 30 million km from Earth, so d = $3.0\text{x}10^{10}$ meters, a = 9.8 meters/sec^2, and so solving for T,

$3.0\text{x}10^{10}$ = ½ (9.8) T^2 so
T = 78,246 seconds or
for a full trip = 2x78246 seconds = **43 hours**!

Speed = 9.8 x 78246 = 767,000 meters/sec = **767 kilometers/sec**.



In the same way that speed = distance divided by time, we can also look at acceleration as the change in speed over the time that the change occurred. Both of these quantities can be thought of as rates of change or 'slopes' on a graph like the one to the left.

When the final speed is, larger than the initial speed, the slope of the line is positive (upward) and we say that the object is accelerating. When the final speed is less than the initial speed, the slope is negative (downward) and we say that the object is decelerating.

Problem 1 – A car leaves its parking spot and accelerates to 30 mph (13 m/s) in 10 seconds. It travels on a road at a constant speed of 30 mph for another 30 seconds and enters the onramp of a highway where it accelerates from 30 mph to a speed of 60 mph (26 m/s) after 6 seconds. It stays at this speed for another 2 minutes, then the car exits an off ramp, slowing to a speed of zero after 2 seconds. It then accelerates to 30 mph after 3 seconds as it merges into the local street traffic. After 1 minute at this speed the car approaches a gas station and decelerates to zero after 4 seconds. Draw a speed versus time graph in <u>metric units</u> that represents the car's journey.

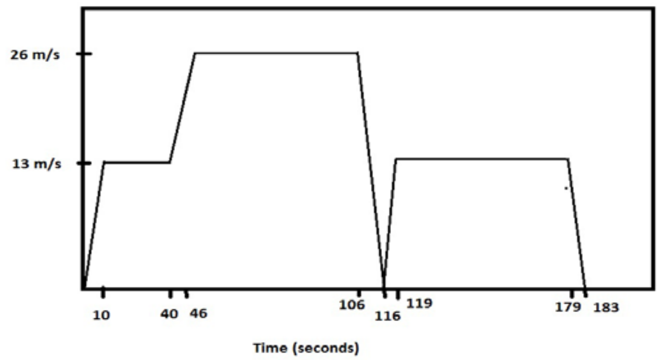
Time (Sec)	Speed (m/s)
0	0
1	3
2	4
3	7
4	10
5	12
6	15
7	16
8	20
9	23
10	25
11	27
12	31
13	34
14	36
15	39
16	43
17	46
18	49
19	53
20	56

Problem 2 – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

Saturn V Rocket Launch Speed vs Time

Problem 3 – The table to the left shows the speed of the Saturn V rocket during a launch from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT). What was the average acceleration of the Saturn V rocket during its first 20 seconds of constant thrust? How far did it travel during this time?

Problem 1 – A car leaves its parking spot and accelerates to 30 mph (13 m/s) in 10 seconds. It travels on a road at a constant speed of 30 mph for another 30 seconds and enters the onramp of a highway where it accelerates from 30 mph to a speed of 60 mph (26 m/s) after 6 seconds. It stays at this speed for another 1 minute, then exits an off ramp, slowing to a speed of zero after 10 seconds. It then accelerates to 30 mph after 3 seconds as it merges into the local street traffic. After 1 minute at this speed the car approaches a gas station and decelerates to zero after 4 seconds. Draw a speed versus time graph that represents the car's journey.

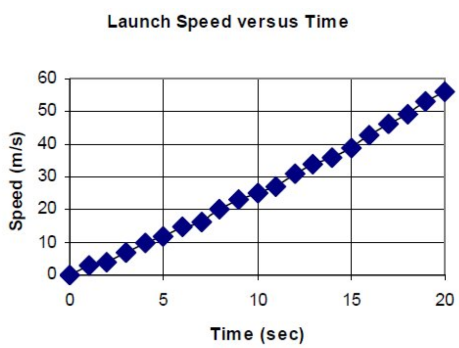


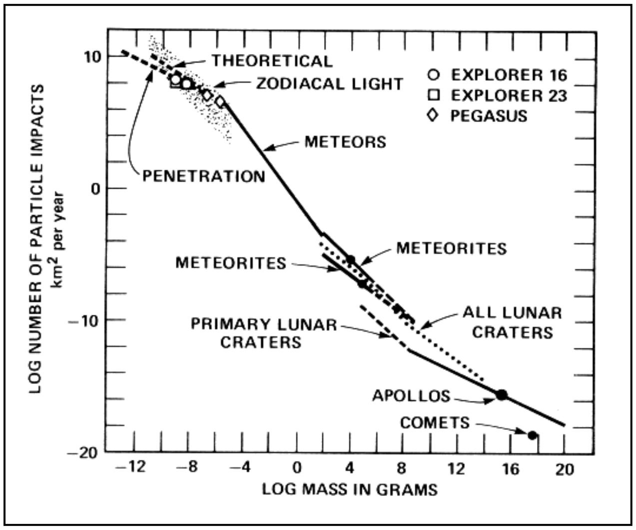
Problem 2 – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

Answer: **Area = speed x time = meters/sec x seconds = meters. So the area under the graph has the units of distance in meters**. This figure has five triangular areas as the car is accelerating and decelerating, and three rectangular areas as the car is traveling at constant speed. The sum of the triangular areas is A = ½ time interval x speed = ½ [10 x 13 + 6 x (26-13) + 10x26 + 3x13 + 4x13) = 279 meters. The sum of the rectangular areas is A = time x speed = (30x13 + (106-46)x26 + (179-119)x13) = 2730 meters, so the total sum is 2730+279 = **3009 meters or 3 kilometers of total travel**.

Problem 3 – The table to the left shows the speed of the Saturn V rocket during a launch from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT). What was the average acceleration of the Saturn V rocket during its first 20 seconds of constant thrust? How far did it travel during this time?

Answer: Acceleration = (56 m/s – 0 m/s)/20 sec = **28 m/s^2**. (Note this is about 2.8 times the acceleration of gravity at earth's surface so the astronauts would have felt about 2.8 times heavier). The triangular area is ½(20sec)(56 m/s) = **560 meters**.





The space between the planets is filled with fragments of asteroids, comets and material left over from the formation of the planets. These rocks and debris rain down upon exposed surfaces at speeds up to 30 km/sec.

The figure on the left summarizes the impact frequency of various sizes of particles in space. Note the graph is plotted in the Log-Log format due to the enormous range of masses and rates being described.

Replacing a data graph with an empirical model:

Problem 1 - What linear function of the form y = mx + b best describes the data in the LogLog graph?

Problem 2 - What function N(m) is obtained from your answer to Problem 1?

Problem 1 - What linear function of the form y = mx + b best describes the data in the LogLog graph?

Answer: Use the two-point formula:

$$(y - y1) = \frac{y2 - y1}{x2 - x1}(x - x1)$$ with p1 = (x1,y1) and p2 = (x2,y2)

Select: p1 = (-4, +2) p2 = (16, -16)

and get **y = -0.9x - 1.6**

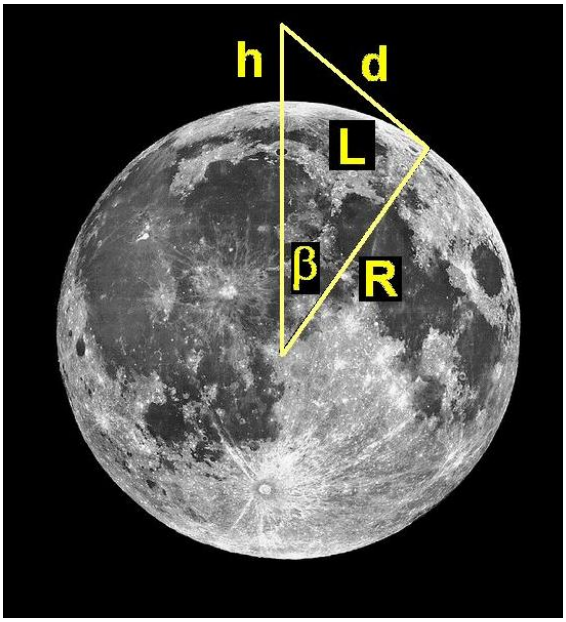
Problem 2 - What function N(m) is obtained from your answer to Problem 2?

Answer: Since y = Log(N(m)) and x = Log(m) we have

Log(N(m)) = -0.9 Log(m) - 1.6 then

$10^{\text{Log(Nm))}} = 10^{\text{Log(m)}}$

And $N(m) = 10^{-1.6}\, m^{-0.9}$ so **$N(m) = 0.025\, m^{-0.9}$**



An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the diameter of the planet (or asteroid!) and the height of the observer above the ground.

Another application of this geometry is in determining the height of a transmission antenna on the Moon in order to insure proper reception out to a specified distance.

Teachers: *Problems 1-4 can be successfully accomplished by algebra students. Problems 5 and 6 require a knowledge of derivatives and can be assigned to calculus students after they have completed Problem 1 and 2.*

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure above to derive the formula for the line-of-sight (LOS) horizon distance, d, to the horizon tangent point.

Problem 2: Derive the distance along the planet, L, to the tangent point.

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6,378 km); B) The Moon (1,738 km)

Problem 4: A radio station has an antenna tower 50 meters tall. What is the maximum line-of-sight (LOS) reception distance in the Moon?

Problem 5: What is the rate of change of the lunar LOS radius, d, for each additional meter of antenna height in Problem 4?

Problem 6: What is the rate-of-change of the distance to the lunar radio tower, L, at the LOS position in Problem 4?

Answer Key:

Problem 1: By the Pythagorean Theorem $d^2 = (R+h)^2 - R^2$

so $d = (R^2 + 2Rh + h^2 - R^2)^{1/2}$

and so $d = (h^2 + 2Rh)^{1/2}$

Problem 2: Derive the distance along the planet, l, to the tangent point. From the diagram,

$\mathrm{Cos}(\beta) = R/(R+h)$ and so $L = R\ \mathrm{Arccos}(R/(R+h))$

Problem 3: Use the equation from Problem 1. A) For Earth, R=6378 km and h=2 meters so d = ((2 meters)^2 + 2 x 2 meters x 6.378 x 10^6 meters))$^{1/2}$ = 5051 meters or 5.1 kilometers.
B) For the Moon, R=1,738 km so d = 2.6 kilometers

Problem 4: h = 50 meters, R=1,738 km so d = 13,183 meters or 13.2 kilometers.

Problem 5: Use the chain rule to take the derivative with respect to h of the equation for d in Problem 1. Evaluate dd/dh at h=50 meters for R=1,738 km.

Let $U = h^2 + 2Rh$ then $d = U^{1/2}$ so dU/dh = (dd/dU) (dU/dh)

Then $dd/dh = +1/2\ U^{-1/2}$

$dU/dh = +1/2\ (2h + 2R)\ (h^2 + 2Rh)^{-1/2}$

For h=50 meters and R = 1,738 km,

$dd/dh = +0.5 \times (100 + 3476000)(2500 + 2 \times 50 \times 1738000)^{-1/2}$

= +131.8 meters in LOS distance per meter of height.

Problem 6: Let U = R/(R+h), then $L = R \cos^{-1}(U)$.

By the chain rule dL/dh = (dL/dU) x (dU/dh).

Since $dL/dU = R \times (-1)(1 - u2)^{1/2}$ and $dU/dh = R \times (-1) \times (R + h)^{-2}$ then

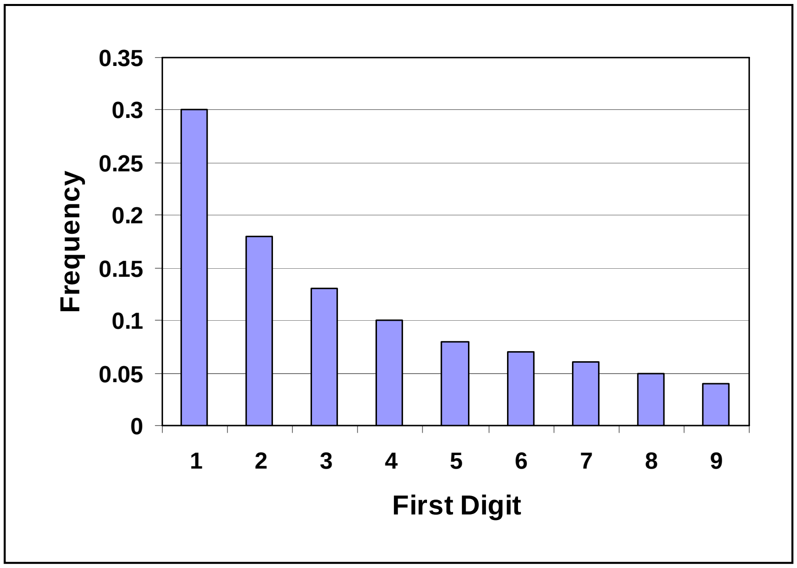
$dL/dh = R^2\ (R + h)^{-2}\ (R + h)^{1/2}\ /(\ (R+ h)^2 - R^2\ )^{1/2}$

$dL/dh = R^2\ (R+h)^{-1}\ (h^2 + 2Rh)^{-1/2}$

Since R >> h, $dL/dh = R/(2Rh)^{1/2}$

Evaluating this for R = 1,738 km and h = 50 meters gives dL/dh = +131.8 meters per kilometer.

Benford's Law



One remarkable feature of numbers used in the real world (measured in physical units like dollars, meters, grams etc) is that the first leading digit of a collection of these numbers is not random, although the numbers, taken together, may seem so. For example, in the sequence $123.43, $327.30, $457.67, $1327.03 the first digits are 1, 3, 4 and 1. We could plot this in a histogram so that '1' has a frequency of 2, '3' has a frequency of 1 and '4' has a frequency of 1. When tallied for an entire tax return with several hundred numbers, the frequency table follows Benford's Law.

The Internal Revenue Service uses Benford's Law to identify tax cheats! For example, if we take all the numbers (in dollar units) you enter in a tax form and count up the frequency of 1's ,2's ,3's …9's, we will get a histogram that shows that 1's appear 30% of the time, 2s appear 17% of the time, 3s appear 12% of the time, and so on.

American astronomer Simon Newcomb noticed that in logarithm books (used at that time to perform calculations), the earlier pages (which contained numbers that started with 1) were much more worn than the other pages.

What is interesting about this law is that, for example, if you made one measurement in inches and computed the first-digit frequency, you would get the same first-digit histogram if you then changed the units to meters or any other unit of measurement!

Does the universe obey this law too?

Below are some archives of data online. Tally the first digit frequencies and test if Benford's Law holds.

Problem 1: **Sunspot Numbers** (Pure numbers: no units) from the National Geophysical Data Center:
http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html#american

Problem 2: **Solar Wind Magnetism**: NASA ACE satellite measurements of the solar wind.
http://www.swpc.noaa.gov/ftpmenu/lists/ace2.html
Open a file with 'mag' as part of the file name such as 200712_ace_mag_1h.txt
Look at Column 11 labeled 'Bt' which are the measurements of the total solar wind magnetic field strength in units of nanoTeslas.

Problem 3: **Depth of latest earthquakes** in kilometers from the USGS:
http://earthquake.usgs.gov/eqcenter/recenteqsww/Quakes/quakes_all.html

Inquiry Questions: Can you find any patterns in the frequency histograms for the above problems? Why don't the digits appear with about the same frequency? Can you think of other astronomical data that you can use to further test your hunches, or refine any patterns you may have found?

Answer Key:

Problem 1: **Sunspot Numbers** (Pure numbers: no units) from the National Geophysical Data Center:
http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html#american

Answer:
Here are some sample numbers for the month of April 1998

53, 55, 48, 47, 57, 61, 91, 97, 103, 102, 93, 75, 61, 60, 63, 56, 34, 26, 30, 31, 31, 24, 17, 19, 14, 13, 13, 27, 30, 39

The leading digits are 5,5,4,4,5,6,9,9,1,1,9,7,6,6,6,5,3,2,3,3,3,2,1,1,1,1,1,2,3,3
Ordered from 1 to 9 by frequency: 7,3,6,2,4,4,1,0,3
Note: There are significantly more 1s.

Problem 2: **Solar Wind Magnetism**: NASA ACE satellite measurements of the solar wind.

http://www.swpc.noaa.gov/ftpmenu/lists/ace2.html
Open a file with 'mag' as part of the file name such as 200712_ace_mag_1h.txt
Look at Column 11 labeled 'Bt' which are the measurements of the total solar wind magnetic field strength in units of nanoTeslas.

Answer:
Sample numbers from the above file are: 4.7, 5.4, 4.7, 1.6, 1.6, 2.2, 2.7, 1.9, 2.5, 3.9, 3.7, 3.6, 3.7, 3.0, 3.1, 2.4, 2.7, 2.6, 1.2, 3.1, 4.2, 3.6, 3.6, 3.6, 1.1, 2.8, 2.1, 3.4, 3.4, 2.4, 2.2, 1.7, 0.4, 2.7, 2.8

The leading digits are: 4,5,4,1,1,2,2,1,2,3,3,3,3,3,3,2,2,2,1,3,1,2,3,3,3,1,2,2,3,3,2,2,1,4,2,2
Ordered from 1 to 9 by frequency: 7,13,12,3,1,0,0,0,0
Note: this data is bounded by a maximum value near 6, so it does not show Benford's Law.

Problem 3: **Depth of latest earthquakes** in kilometers from the USGS:

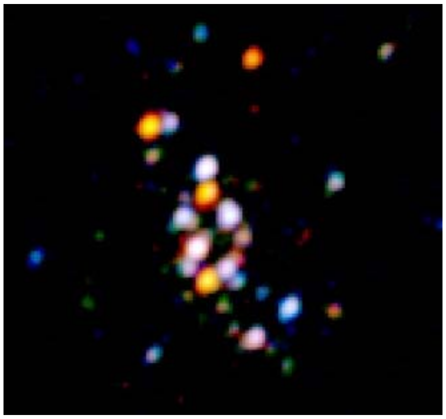
http://earthquake.usgs.gov/eqcenter/recenteqsww/Quakes/quakes_all.html

Answer:
Sample depths: 10.0, 36.5, 12.5, 64.6, 3.9, 10.2, 10.0, 10.0, 98.4, 3.5, 500.7, 92.7, 81.6, 22.4, 2.0, 46.0, 53.3, 100.0, 10.0, 49.1, 57.0

The leading digits are: 1,3,1,6,3,1,1,1,9,3,5,9,8,2,2,4,5,1,1,4,5
Ordered from 1 to 9 by frequency: 7,2,3,2,3,1,0,1,2
Note: this data does show Benford's Law because it is not apparently bounded.

Inquiry Questions: Students should discover that numbers that are limited in some way, such as the height of your students (all about the same height) will not show Benford's law. Only numbers that are not in some way bounded above or below in value (such as solar wind magnetism, which has a rather narrow range of values near 5.0 nTeslas). Also, street address and other pure numbers do not follow Benford's Law, only numbers related to unbounded, measured parameters with physical units (temperature, density, mass, energy, length, time, etc) will work.

Chandra Explores Angular Size



The above image of the globular cluster NGC-6266 was taken by the Chandra Satellite and shows only the stars in the cluster that are producing X-rays. Each spot is a binary star in which one of the members is a dense neutron star which is consuming gas from its normal star companion to produce x-rays. The field is 66.8 arcseconds wide.

Problem 1 - Measure the width of the image in millimeters, and calculate the image scale in arcseconds per millimeter.

Problem 2 - About what is the smallest separation between stars in the picture in: A) millimeters? B) arcseconds?

Problem 3 - What is the diameter of the densest part of the x-ray cluster in: A) millimeters? B) arcseconds?

Problem 4 - The distance to the cluster is 22,500 light years. What is: A) the smallest star separation and B) the diameter of the x-ray cluster?

Astronomers measure the sizes of objects in the sky in terms of their angular size. For instance, the moon appears to be about 1/2 degree in diameter. The planet Venus, when it is closest to Earth, appears to be even smaller - only about 1/60 of a degree. This small angle is called the arc-minute. There are 60 arcminutes in one degree.

What we really would like to know is, physically, how big something is in kilometers, instead of how big it appears to be in angular measure. To get this information, all we need to know is how far away the object is from us.

The moon is 324,000 kilometers away, and Venus is about 40 million kilometers away from Earth at its closest distance. The following formula lets us convert angles into kilometers:

Size = Distance x Diameter in degrees / 57.3 degrees

With the data for the Moon and Venus we get

Moon = 324,000 km x (0.5) / 57.296 = 2,800 km
Venus = 40 million km x (1/60)/ 57.296 = 11,600 km.

For more distant objects, we use a slightly different formula. Instead of an angular size measured in degrees, we use angles measured in arcseconds. There are 60 arcseconds in one arc minute, so one arcsecond is 1/60 x 1/60 = 1/3600 degree, so we get:

Diameter = distance x Size / (3600x57.296)

which becomes

Diameter = distance x Size/(206,265)

The binary star system P Eridani is 26.6 light years from Earth. Its two stars have a maximum separation of 11.8 arcseconds, which will occur in the year 2048. How far apart will they be if 1 light year (LY) equals 6 trillion kilometers?

Separation = 26.6 x11.8/206265=0.0015 LY.

This equals 0.0015 LY x 6 trillion km/LY = 9 billion kilometers, or twice the distance to Pluto!

For more on angular size, and an interactive calculator, visit the NASA, Chandra website:
http://chandra.harvard.edu/photo/scale_distance.html

Answer Key:

Problem 1 - Measure the width of the image in millimeters, and calculate the image scale in arcseconds per millimeter.
Answer: 58 millimeters corresponds to 66.8 arcseconds, so the scale is 66.8/58 = **1.2 arcseconds / millimeter.**

Problem 2 - About what is the smallest separation between stars in the picture in A) millimeters? B) arcseconds?
Answer: Students answers may vary depending on which stars they choose: A) **About 2 mm** B) 2 mm x 1.2 arcsec/mm = **2.4 arcseconds.**

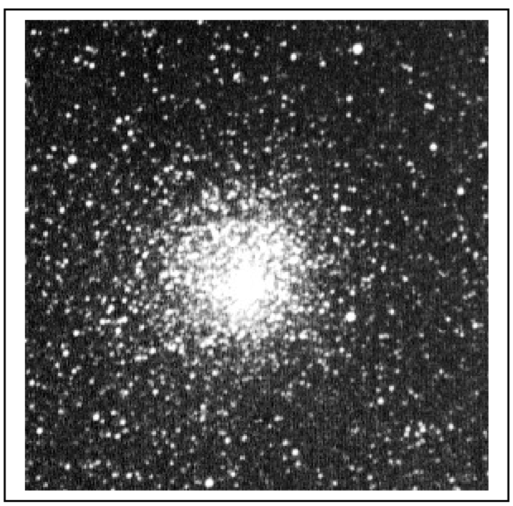
Problem 3 - What is the diameter of the densest part of the x-ray cluster in A) millimeters? B) arcseconds?
Answer: Students answers may vary: A) **10 millimeters**; B) 10 mm x 1.2 arcsec/mm = **12 arcseconds.**

Problem 4 - The distance to the cluster is 22,500 light years. What is A) the smallest star separation and B) the diameter of the x-ray cluster?

Answer:
A) Separation = 22,500 light years x (2.4 arcseconds) / 206265 = **0.26 light years.**

B) Diameter = 22,500 light years x (12 arcseconds) / 206265 = **1.3 light years.**

Students estimates will differ depending on which star pairs or diameters they measure.



NGC-6266 is also called Messier-62 and is located in the constellation Ophiuchus. Here is a picture of the entire cluster. The diameter of the star cluster in this image is 15 arcminutes. At a distance of 22,500 light years, its diameter is

22,500 LY x (15/60)/57.296 = 98 light years!

The Chandra image is only a very small part of the center of this cluster!

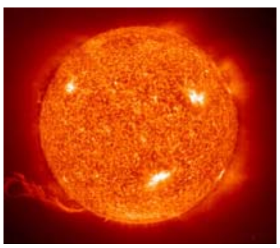
The Oscillation Period of Gaseous Spheres

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Any collection of matter that is governed by the force of gravity has a natural period of oscillation. For example, a simple pendulum such as a playground swing, will move back and forth with a time period in seconds, T, given by the formula to the right, where L is the length of the swing in centimeters, and g is the acceleration of gravity at Earth's surface given by 980 cm/sec^2. The period of a swing that is 3 meters long is then T = 3.5 seconds.

$$T^2 = \frac{3\pi}{G\ D}$$

This behavior also applies to any body held together by gravity whether it is a star or a planet. The natural oscillation period of such bodies is given by the formula to the right, where D is the density of the body in grams/cm^3 and G is the Newtonian constant of Gravity G = 6.6 x 10^{-8} dynes cm^2 gm^2 and T is in seconds. For example, the planet Jupiter has a density of about D = 1.3 gm/cm^3 so its period, T, will be about 10,500 seconds or T = 3 hours. From the information below, calculate the natural periods for the various astronomical bodies.



Courtesy SOHO-EIT

The sun is about 1.5 million kilometers across, and has an average density of about 1.5 grams/cm^3

T = ______________ hours.

Courtesy NASA-Apollo

The Earth has a diameter of about 12,500 kilometers, and has an average density of about 5.5 grams/cm^3

T = ______________ minutes.

Courtesy NASA-Dana Berry

A neutron star is about 50 kilometers in diameter, and has an average density of about 2 x 10^{14} grams/cm^3

T = ______________ seconds.

Answer Key:

The sun is about 1.5 million kilometers across, and has an average density near its surface of about 1.5 grams/cm^3

$$T^2 = 3 \times (3.141)/(1.5 \times 6.6 \times 10^{-8})$$
$$= 9.5 \times 10^7$$

So T = 9,800 seconds
= 2.7 hours

The Earth has a diameter of about 12,500 kilometers, and has an average density of about 5.5 grams/cm^3

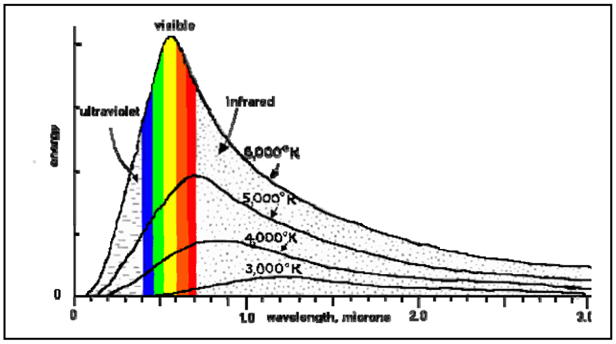
$$T^2 = 3 \times (3.141)/(5.5 \times 6.6 \times 10^{-8})$$
$$= 2.5 \times 10^7$$

So T = 5,100 seconds
= 85 minutes

A neutron star is about 50 kilometers in diameter, and has an average density of about 2 x 10^{14} grams/cm^3

$$T^2 = 3 \times (3.141)/(2.0 \times 10^{14} \times 6.6 \times 10^{-8})$$
$$= 7.1 \times 10^{-7}$$

So T = 0.00085 seconds



When radiation is produced by a heated body, the intensity of electromagnetic radiation depends on frequency (wavelength) in a manner defined by the Planck Function. There is a simple law, called the Wein Displacement Law, that relates the temperature of a body to the frequency where the Planck curve has its maximum value. In this exercise, we will use two different methods to derive this law.

$$I(\lambda, T) = \frac{A}{\lambda^5 \left(e^{\left[\frac{14394}{T\lambda}\right]} - 1 \right)}$$

where: $A = 3.747 \times 10^{14}$ watts microns^4/m^2/str

Temperature (K)	Peak Wavelength (microns)
10,000	**0.2898**
9,000	**0.322**
8,000	**0.362**
7,000	**0.414**
6,000	**0.483**
5,000	**0.579**
4,000	**0.724**
3,000	**0.966**
2,000	**1.449**
1,000	**2.828**
500	**5.796**
300	**9.660**

Algebra Problem:

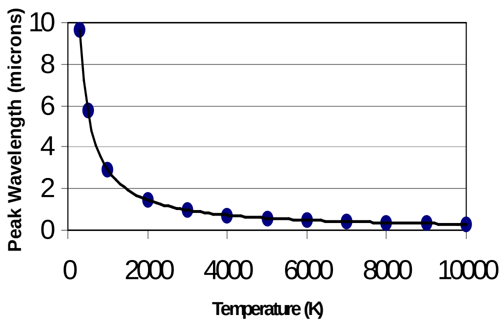
A) From the data in the table, use a calculator to find a formula that fits the data. Some possibilities might include a linear equation, $\lambda = a\,T$, power laws such as $\lambda = b\,T^{-1}$, $\lambda = c\,T^{-2}$ or $\lambda = d\,T^{2}$, $\lambda = e\,T^{3}$ or exponential functions such as $\lambda = f\,e^{(gT)}$ where a,b,c,d,e,f,g are constants determined by the fitting process. B) Which function fits the tabulated data the best, and what is the value of the constant?

Calculus Problem:

A) Find Equation 2 for the maximum of the function $I(\lambda,T)$ by differentiating with respect to the wavelength, λ, and setting the derivative equal to zero.

B) Find the solution to Equation 2, which you found in Part A, using the technique of 'successive approximation', or 'trial and error'. (Note, ignore trivial solutions involving zero! From this iterated solution, find the form of the function for the maximum wavelength as a function of temperature.)

Answer Key:



Algebra Problem: The figure above shows the data plotted in the table, and the best fit curve. This was done by copying the table into an Excel spread sheet, plotting the data as an XY scatter plot, and using 'Add Trend line'. Students may experiment with various choices for the fitting function using an HP-83 graphing calculator, or the Excel spread sheet trend line options, but should find that the best fit has the form: $\lambda = b\ T^{-1}$ where b = 2898.0 micronsKelvins. Students may puzzle over the units 'micronsKelvins' but it is often the case in physics that units have complex forms that are not immediately intuitive.

Calculus Problem: A) Below is a recommended strategy:

Let $U = A\ \lambda^{-5}$ then $dU/d\lambda = -5\ A\ \lambda^{-6}$

Let $V = e^{(14329/(\lambda\ T))} - 1$ then $dV/d\lambda = -14329 / (\lambda^2\ T)\ e^{14329/(\lambda t)}$

Then use the quotient rule: $d/d\lambda\ (U/V) = 1/V\ dU/d\lambda - U/V^2\ dV/d\lambda$

To get $dU/d\lambda - U/V\ dV/d\lambda = 0$

Then by substitution and a little algebra

$5\ l\ T\ (e^{14329/(\lambda T)} - 1) - 14329\ e^{14329/\lambda T} = 0$

Let $X = 14329/ (\lambda\ T)$ then we get a simpler equation to solve:

$5(e^x - 1) - x\ e^x = 0$ (Equation 2)

Equation 3, when solved, will give the location of the extrema for the Planck Function, however, it cannot be solved exactly. You will need to program a graphing calculator or use an Excel spreadsheet to find the value for X that gives, in this case, the maximum value of the Planck Function.

Calculus Problem: B) The table shows some trial-and-error results for Equation 3, and a convergence to approximately X = 4.965 The formula for the peak wavelength is then

$4.965 = 14394 / (\lambda\ T)$ or $\mathbf{\lambda = 2899 / T}$

This is similar to the function derived by fitting the tabulated data.

X	Eq. 3
1	5.873127
2	17.16717
3	35.17107
4	49.59815
4.5	40.00857
4.9	8.428978
4.95	2.058748
4.96	0.703752
4.965	0.015799
4.966	-0.12263

Gas clouds in interstellar space are acted upon by external pressure and their own gravity, and would otherwise collapse, but if they are hot enough, they can remain stable for a long time. That seems to be the case for objects called Bok Globules.

This photo of Thackeray's Globule (IC-2944) taken by the Hubble Space Telescope may be a stable dark cloud containing 10 times the mass of our sun at a temperature of less than 100 K.

A gas sphere with a radius, R, a mass, M, and a temperature, T, is subject to an external pressure, P so that

$$P = \frac{3\,M\,k\,T}{4\,\pi\,R^3\,\mu\,m} - \frac{3\,G\,M^2}{20\,\pi\,R^4}$$

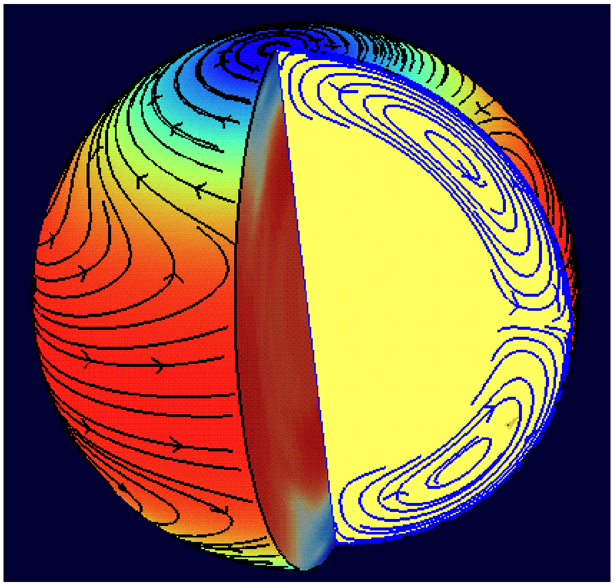
where k, G and μ are constants.

Problem 1 - At a given external temperature, P, the cloud will be in equilibrium and not collapse if the sum of the two internal pressure terms on the right-side of the equation add up to the same value as the external pressure.

At what minimum radius will the cloud start to collapse for a given mass and temperature because the internal pressure no longer equals the external pressure? (Hint: determine dP/dR and find minimum.

Problem 1 - The problem states that the mass and temperature are held constant, so the only free variable is R. For complicated equations, it is always a good idea to group all constants together and define new constants. You can later replace the new constants by the old ones. Let's define A = (3MkT/4$\pi\mu$m) and B=(3GM^2/20π), then the equation becomes P = AR^{-3} - BR^{-4}. To find the extremum, we calculate dP/dR and set this equal to zero. This gives us dP/dR = A (-3)R^{-4} - B(-4)R^{-5} = 0. This leads to R = 4 B/3 A which upon substituting back for the definitions of A and B gives us

$$R_c = \frac{12G\mu m}{45kT}$$



There are many situations in which differentiation has to be performed on formulae in astrophysics. Many objects such as stars and galaxies display 'differential rotation' which leads to many interesting and unusual phenomena. The formula describing these phenomena are usually 'differential equations' that relate changes in one quantity to changes in another.

Here are some popular equations used in astrophysics whose differentiation will test your basic skills!

Image: Model of solar differential rotation (Courtesy: Stanford Solar Center / NASA SOHO)

$$m = \frac{M}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L = 4\pi R^2 \sigma T^4$$

$$R = \left(\frac{3\,m\,V}{\pi\ \rho}\,t\right)^{1/4}$$

$$V = \frac{(Z+1)^2 + 1}{(Z+1)^2 - 1}$$

$$D = \left(\frac{k\ T}{4\,\pi\ e^2 N}\right)^{1/2}$$

$$\Lambda = \frac{D^2(D-1)^2(D+2)}{8\,\pi\,(D+4)^2}\,m^2$$

Problem 1 - Find dm/dv - the rate of change of mass with velocity near the speed of light.

Problem 2 - Find dL/dT - The rate of change of a star's luminosity with its temperature.

Problem 3 - Find dR/dt - The rate of change of the size of an expanding supernova remnant with time (in other words, its expansion speed!).

Problem 4 - Find dV/dz - The rate of change of the apparent speed of a body with its gravitational redshift.

Problem 5 - Find dD/dN - the rate of change of the Debye shielding radius in a plasma with a change in the density of the plasma.

Problem 6 - Find $(1/m^2)\ d\Lambda/dD$ - the rate of change of the energy of empty space as you change the number of dimensions to space.

Answer Key

Problem 1 - Find dm/dv - the rate of change of mass with velocity near the speed of light.

$$\frac{dm}{dv} = \frac{M\,v}{c^2\,(1-[v^2/c^2])^{3/2}}$$

Problem 2 - Find dL/dT - The rate of change of a star's luminosity with its temperature.

$$\frac{dL}{dT} = 16\,\pi\,R^2\,\sigma\,T^3$$

Problem 3 - Find dR/dt - The rate of change of the size of an expanding supernova remnant with time (in other words, its expansion speed!).

$$\frac{dR}{dt} = 1/4\,(3\,mV/(\pi\,\rho))^{1/4}\,t^{-3/4}$$

Problem 4 - Find dV/dz - The rate of change of the apparent speed of a body with its gravitational redshift. This requires using the quotient rule for differentiation $d(U/V) = (1/V)\,dU - (U/v^2)\,dV$

$$\frac{dV}{dz} = \frac{1}{(z+1)^2+1}\,2(z+1) - \frac{1}{[(z+1)^2+1]^2}\,2(z+1) = \frac{2(z+1)^3}{[(z+1)^2+1]^2}$$

Problem 5 - Find dD/dN - the rate of change of the Debye shielding radius in a plasma with a change in the density of the plasma.

$$\frac{dD}{dN} = -1/2\,(kT/4\pi\,e^2)^{1/2}\,N^{-3/2}$$

Problem 6 - Find $d\Lambda/dD$ - the rate of change of the energy of empty space as you change the number of dimensions to space. Again the quotient rule is needed where $U = D^5 - 3D^3 + 2D^2$ $V = D^2 + 8D + 16$

$$\frac{1}{m^2}\frac{d\Lambda}{dD} = \frac{5D^4 - 9D^2 + 4D}{8\,\pi\,(D^2+8D+16)} - \frac{[D^5 - 3D^3 + 2D^2]\,(2D+8)}{[8\,\pi\,(D^2+8D+16)]^2}$$

Whether you are talking about atoms in a gas, stars in a star cluster, or galaxies in intergalactic space, eventually some members will collide with each other. Predicting the collision times for various systems is an important way to estimate important events in their history.

Image: Seyfert's Sextet galaxy cluster showing collisions (Courtesy Hubble Space Telescope)

Problem 1 - **Cross Sectional Area:** Draw two circles on a paper that overlap. These represent the geometrical cross sectional areas of two bodies. For example, the cross sectional area of a marble with a radius of 5 millimeters is $A = \pi\ (5\text{mm})^2 = 78.5\ \text{mm}^2$. Find the cross sectional areas of: A) an oxygen atom with a radius of 50 picometers (in square meters); B) a star with a radius of 698,000 km (in square kilometers); C) a galaxy with a radius of 50,000 light years (in square light years). (1 ly = 9.4 trillion km)

Problem 2 - **Swept out volume:** A moving body sweeps out a cylindrical volume whose base area equals the bodies cross sectional area, and whose height equals the speed of the body times the elapsed time. Calculate the cylindrical volumes, in cubic meters, for; A) an atom of oxygen traveling at 500 meters/sec for 10 seconds; B) a star in the Omega Centauri cluster traveling at 22 km/sec for 1 million years (in cubic light years); C) the Milky Way galaxy traveling at 200 km/sec for 100 million years (in cubic light years). (1 year = 31 million seconds)

Problem 3 - **Average particle volume:** The average volume occupied by a body in a system depends on the volume of the system and the number of bodies present. The 'number density' measures the number of particles divided by the volume of the system. The inverse of this number is the average volume per particle. Calculate the average volume for the following bodies: A) Sea-level atmosphere with 3×10^{25} atoms/cubic meter; B) Omega Centauri cluster with a diameter of 160 light years and containing 10 million stars (in stars per cubic light year) ; C) The Local Group of galaxies containing 35 galaxies including the Milky Way, with a diameter of 10 million light years (in galaxies per cubic mega light year).

Problem 4 - **Collision time:** The collision time is the time it takes a body to sweep out the same volume as is occupied by an average particle in the system. It is given by the formula $T = 1/(N\ A\ V)$ where N is the density of particles, V is their average speed, and A is their average cross-sectional area. (All units should be in terms of meters and seconds.) From the area calculated in Problem 1, the speeds given in Problem 2, and the average particle densities from Problem 3, compute the particle collision times for A) an oxygen atom at sea level (in nanoseconds); B) a star in the Omega Centauri cluster (in years) and C) a galaxy in the Local Group (in years).

Inquiry Question: There are many more stars than galaxies, so why is it that galaxies collide nearly a million times more often?

Answer Key

Problem 1 - **A)** an oxygen atom with a radius of 50 picometers; $A = \pi (50 \times 10^{-12}\ m)^2 =$ **7.9×10^{-21} meters^2** **B)** a star with a radius of 698,000 km; $A = \pi (698{,}000\ km)^2 =$ **1.5×10^{12} km^2** **C)** a galaxy with a radius of 50,000 light years. $A = \pi(50{,}000\ ly)^2 =$ **7.9×10^9 ly^2**

Problem 2 - Use the cross sectional areas calculated in Problem 1 and Volume = area x distance traveled; **A)** an atom of oxygen traveling at 500 meters/sec for 10 seconds; L = 500 m/s x 10 s = 5000 meters. Volume = 7.9×10^{-21} meters^2 x 5000 meters = **4.0×10^{-17} meters^3** **B)** a star in the Omega Centauri cluster traveling at 22 km/sec for 1 million years; L = 22 km/s x 1 million yrs x 31 million sec/yr = 6.8×10^{14} kilometers. Volume = 1.5×10^{12} km^2 x 6.8×10^{14} km = **1.0×10^{27} km^3**. **C)** the Milky Way galaxy traveling at 200 km/sec for 100 million years. L = 200 km/s x 100 million years x 31 million sec/yr = 6.2×10^{17} kilometers = 66,000 light years. Volume = 7.9×10^9 ly^2 x 6.6×10^4 ly = **5.2×10^{14} ly^3**.

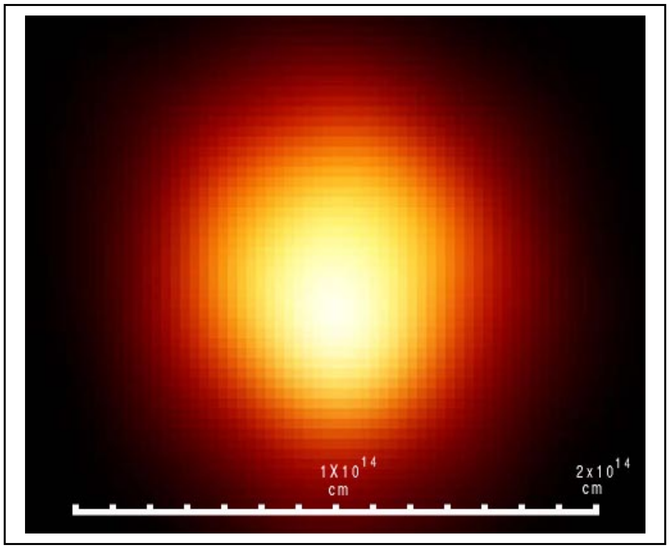
Problem 3 - Calculate the average volume for the following bodies: **A)** Sea-level atmosphere with 3×10^{25} atoms/cubic meter; The number density, N = 3×10^{25} atoms/meter^3 so 1/N = V = **3.3×10^{-26} meters^3 per atom**. **B)** Omega Centauri cluster with a diameter of 160 light years and containing 10 million stars; Assume the cluster is a sphere with a volume $V = 4/3\ \pi\ (80\ ly)^3 = 2.1 \times 10^6\ ly^3$. Then N = 10 million / 2.1×10^6 ly^3 = 4.8 stars/ly^3. The individual star volume = 1/N = **0.2 ly^3 per star**. **C)** The Local Group of galaxies containing 35 galaxies including the Milky Way, with a diameter of 10 million light years (i.e. 10 Mly). Volume = $4/3\ \pi\ (5\ Mly)^3 = 523$ Mly^3 . N = 35 galaxies/ 523 Mly^3 = 0.07 galaxies/Mly^3 . Then 1/N = **14 Mly^3 per galaxy**.

Problem 4 - A) Atom collision time: A = 7.9×10^{-21} meters^2 V = 500 m/sec and N = 3×10^{25} atoms/m^3. so T = 1/(7.9×10^{-21} meters^2 x 500 m/sec x 3×10^{25} atoms/m^3) = **8.4×10^{-9} seconds or about 8 nanoseconds**.

B) Star collision time: A = 1.5×10^{12} km^2 V = 22 km/sec and N = 4.8 stars/Ly^3 . converting them to the same units, in this case kilometers, N = $4.8/(9.4\times10^{12})^3$ = 5.8×10^{-39}, so T = 1/(1.5×10^{12} x 22 km/sec x 5.8×10^{-39}) = 5.2×10^{24} seconds or **1.7×10^{17} years**.

C) Galaxy collision time: A = 7.9×10^9 ly^2 ; V = 200 km/sec, which is equal to 200 km/sec x (1 Ly/9.4×10^{12} km) x (31 million seconds/1 year) = 0.0066 Ly/year; and N = 0.07 galaxies/Mly^3 which is 0.07×10^{-18} galaxies/ly^3; so T = 1/(7.9×10^9 x 0.0066 x 7.0×10^{-20}) = **2.7×10^{11} years.**

Inquiry Question: **Students should recognize that, although stars and galaxies travel about the same speed, galaxies have much larger cross sections compared to the volume of space they occupy, so there will be more collisions between them.**



The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun!

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4 \pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzman radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in degrees Kelvin. The formula then becomes,

$$L = 4 \pi R^2 \sigma T^4$$

Problem 1 - The Sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

Problem 2 - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

Problem 3 - The nearby star, Sirius, has a temperature of 9,200 K and a radius of 1.76 times our Sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our Sun?

Calculus:

Problem 4 - Compute the total derivative of L(R,T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Problem 1 - We use $L = 4\,(3.141)\ R^2\,(5.67 \times 10^{-5})\ T^4$ to get L (ergs/sec) = 0.00071 $R(cm)^2$ $T(degreesK)^4$ then,

A) L(ergs/sec) = 0.00071 x (696,000 km x 10^5 cm/km)^2 $(5700)^4$ = **3.6 x 10^{33} ergs/sec**

B) L(watts) = 3.6 x 10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6 x 10^{25} watts**.

Problem 2 - A) The radius of Antares is 700 x 696,000 km = 4.9 x 10^8 km.
L(ergs/sec) = 0.00071 x (4.9 x 10^8 km x 10^5 cm/km)^2 $(3500)^4$ = **2.5 x 10^{38} ergs/sec**
B) L(Antares) = (2.5 x 10^{38} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **71,000 L(sun).**

Problem 3 - Sirius-A radius = 1.76 x 696,000 km = 1.2 x 10^6 km
L(Sirius-A) = 0.00071 x (1.2 x 10^6 km x 10^5 cm/km)^2 $(9200)^4$ = **7.3 x 10^{34} ergs/sec**
L = (7.3 x 10^{34} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = 0.00071 x (4900 km x 10^5 cm/km)^2 $(27,400)^4$ = **9.5 x 10^{31} ergs/sec**
L(Sirius-B) = 9.5 x 10^{31} ergs/sec / 3.6 x 10^{33} ergs/sec = **0.026 L(sun).**

Advanced Math:
Problem 4 *(*Note: In the discussion below, the symbol d represents a partial derivative)

$$dL(R,T) = \frac{d L(R,T)}{d R}\, dR + \frac{d L(R,T)}{d T}\, dT$$

$$dL = [\,4\pi\ (2)\ R\,\sigma\, T^4\,]\ dR + [\,4\pi\ (4)\ R2\,\sigma\, T^3\,]\ dT$$

$$dL = 8\pi\ R\,\sigma\, T^4\ dR + 16\pi\ R^2\,\sigma\, T^3\ dT$$

To get percentage changes, divide both sides by $L = 4\pi\ R^2\,\sigma\, T^4$

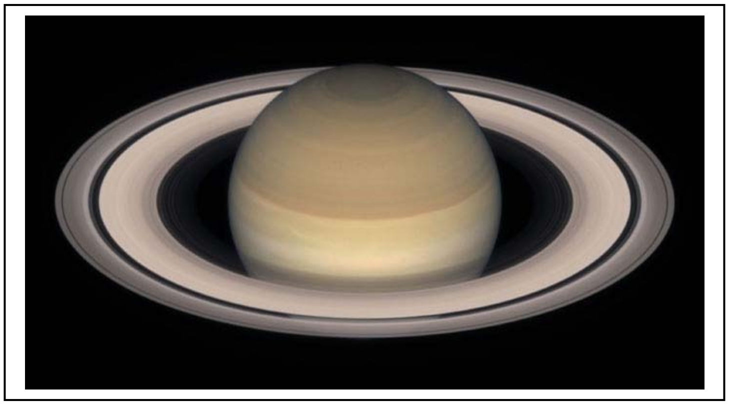
$$\frac{dL}{L} = \frac{8\pi\ R\,\sigma\, T^4}{4\pi\ R^2\,\sigma\, T^4}\, dR + \frac{16\pi\ R^2\,\sigma\, T^3}{4\pi\ R^2\,\sigma\, T^4}\, dT$$

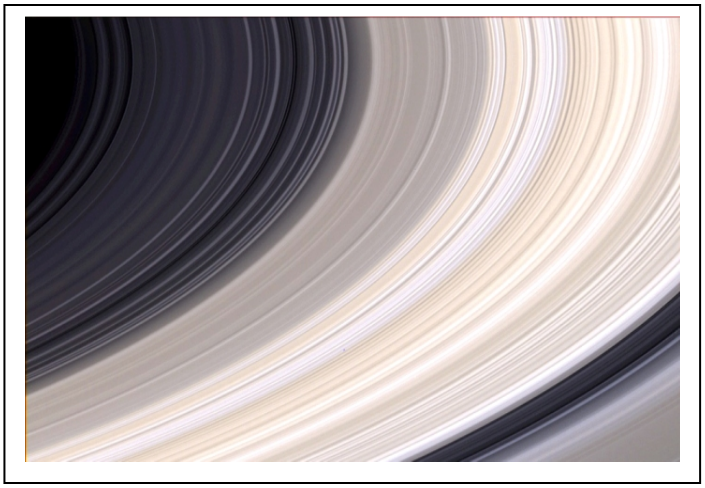
Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40**
The star's luminosity will increase by 40%.

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0. This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 71,000 L(sun) or 2.5 x 10^{38} ergs/sec. If I increase its radius by 10% from 4.9 x 10^8 km to 5.4 x 10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = 0.00071 x (5.4 x 10^8 km x 10^5 cm/km)^2 $(3325)^4$ = **2.5 x 10^{38} ergs/sec**

Tidal Forces - Let 'er Rip!





As the Moon orbits Earth, its gravitational pull raises the familiar tides in the ocean water, but did you know that it also raises 'earth tides' in the crust of earth? These tides are up to 50 centimeters in height and span continent-sized areas. The Earth also raises 'body tides' on the moon with a height of 5 meters!

Now imagine that the moon were so close that it could no longer hold itself together against these tidal deformations. The distance were Earth's gravity will 'tidally disrupt' a solid satellite like the moon is called the tidal radius. One of the most dramatic examples of this is the rings of Saturn, where a nearby moon was disrupted, or prevented from forming in the first place!

Images courtesy NASA/Hubble and Cassini.

Problem 1 - The location of the tidal radius (also called the Roche Limit) for two bodies is given by the formula $d = 2.4x\ R\ (\rho_M/\rho_m)^{1/3}$ where ρ_M is the density of the primary body, ρ_m is the density of the satellite, and R is the radius of the main body. For the Earth-Moon system, what is the Roche Limit if R = 6,378 km, ρ_M = 5.5 gm/cm^3 and ρ_m = 2.5 gm/cm^3? (Note, the Roche Limit, d, will be in kilometers if R is also in kilometers, and so long as the densities are in the same units.)

Problem 2 - Saturn's moons are made of ice with a density of about 1.2 gm/cm^3 .If Saturn's density is 0.7 gm/cm^3 and its radius is R = 58,000 km, how does its Roche Limit compare to the span of the ring system which extends from 66,000 km to 480,000 km from the planet's center?

Problem 3 - In searching for planets orbiting other stars, many bodies similar to Jupiter in mass have been found orbiting sun-like stars at distances of only 3 million km. What is the Roche Limit for a star like our Sun if its radius is R = 600,000 km, and the densities are ρ(planet) = 1.3 gm/cm^3 and ρ(star) = 1.5 gm/cm^3?

Answer Key

Problem 1 -
$d = 2.4xR\ (\rho_M/\rho_m)^{1/3}$
with R = 6,378 km, ρ_M = 5.5 gm/cm^3 and ρ_m = 2.5 gm/cm^3
$d = 2.4 \times 6{,}378\ (5.5/2.5)^{1/3}$
d = 19,900 km.

This distance is inside the orbits of geosynchronous communications satellites (42,164 kilometers). They are not destroyed because the tensile strength of aluminum is far higher than the gravitational tidal forces they feel.

Fortunately our moon is at a distance of 363,000 (perigee) and is steadily moving farther away by 3 centimeters per year!

Problem 2 -
$d = 2.4 \times 58{,}000\ (0.7/1.2)^{1/3}$
d = 116,900 kilometers.

The span of the ring system which extends from 66,000 km to 480,000 km from the planet's center, so the Roche Limit is inside the ring system.

Problem 3 - In searching for planets orbiting other stars, many bodies similar to Jupiter in mass have been found orbiting sun-like stars at distances of only 3 million km. What is the Roche Limit for a star like our Sun if its radius is R = 600,000 km, and the densities are ρ(planet) = 1.3 gm/cm^3 and ρ(star) = 1.5 gm/cm^3?

$d = 2.4 \times 600{,}000\ (1.5/1.3)^{1/3}$
d = 1.5 million kilometers.

So the large planets being detected are very close to the Roche Limits for their stars!

The Many Faces of Energy

1 Joule = 10 million ergs
1 electron Volt (eV) = 1.6 x 10^{-19} Joules
1 degree (K) = 8.62 x 10^{-5} eV
1 calorie = 4.2 Joules
1 kiloWatt hour = 3.6 x 10^{6} Joules
1 eV = 1.78 x 10^{-33} grams
1 AMU = 931.5 million eV (MeV)

Energy comes in many forms, and each one can be measured in terms of its own convenient units. For example, if you were interested in creating a balanced diet, you would measure food energy by its calorie content, not by its number of Joules!

The table to the left shows a few of the equivalent units that scientists use to keep track of energy in different kinds of systems.

Problem 1 - In a chemical reaction, an energy of about 2.5 eV is required to activate the reaction to create a new compound. To form a single molecule of the compound: A) How many Joules of energy is this? B) How many calories is this?

Problem 2: A star has a surface temperature of 20,000 K. About what is the average energy per atom in electron Volts?

Problem 3: The mass of an electron is 9.11 x 10^{-28} grams. What is its equivalent mass in kiloelectron Volts (keV)?

Problem 4: A proton and a neutron are combined to form a deuterium nucleus. Their total individual masses equal 2.016490 AMU, but the mass of a deuterium nucleus is only 2.014102 AMU. If the mass difference to form the deuterium is 0.002388 AMU, how much energy does this energy difference represent in: A) million electron volts (MeV)? B) grams? (Note: this is called the binding energy of the nucleus.)

Problem 5: An astronomer detects X-ray light from a pulsar with an energy of 15 keV. About what is the temperature of the gas emitting this light?

Answer Key

Problem 1 - In a chemical reaction, an energy of about 2.5 eV is required to activate the reaction to create a new compound. To form a single molecule of the compound: A) How many Joules of energy is this? B) How many calories is this?

A) Answer: 2.5 eV x (1.6 x 10^{-19} Joules / 1 eV) = **4.0 x 10^{-19} Joules per molecule.**
B) Answer: 4.0 x 10^{-19} Joules x (1 calorie / 4.2 Joules) = **9.5 x 10^{-20} Joules per molecule.**

Problem 2: A star has a surface temperature of 20,000 K. About what is the average energy per atom in electron Volts? Answer: 20,000 K x (8.62 x 10^{-5} eV / K) **= 1.7 eV**.

Problem 3: The mass of an electron is 9.11 x 10^{-28} grams. What is its equivalent mass in kiloelectron Volts (keV)? Answer: 9.11 x 10^{-28} grams x (1 eV/1.78 x 10^{-33} grams) = 512,000 eV = **512 keV**.

Problem 4: A proton and a neutron are combined to form a deuterium nucleus. Their total individual masses equal 2.016490 AMU, but the mass of a deuterium nucleus is only 2.014102 AMU. If the mass difference to form the deuterium is 0.002388 AMU, how much energy does this energy difference represent in: A) million electron volts (MeV)? B) grams?

Answer: A) 0.002388 AMU x (931.5 MeV/1 AMU) = **2.2 MeV**.

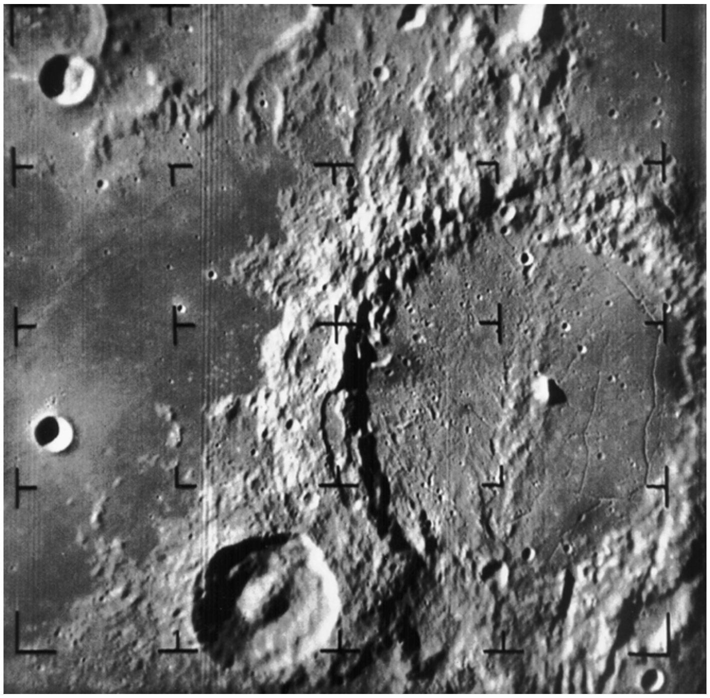
Answer: B) 2.2 MeV x (1,000,000 eV/1 MeV) x (1.78 x 10^{-33} grams/ eV) = **3.9 x 10^{-27} grams.**

Problem 5: An astronomer detects X-ray light from a pulsar with an energy of 15 keV. About what is the temperature of the gas emitting this light? Answer; 15 keV x (1,000 eV/ 1 keV) x (1 K/8.62 x 10^{-5} eV) = **174 million degrees Kelvin**.

Problem 6: A physicist wants to create a proton with a mass of 938 MeV in his accelerator. What is the minimum energy, in Joules, that he will need to provide? Answer: 938 MeV x (1,000,000 eV/1 MeV) x (1.6 x 10^{-19} Joules/ 1 eV) = **1.5 x 10^{-10} Joules.**

Note: $E = mc^2$ is the basis for stating the mass of a particle 'm' in terms of its equivalent electron-Volt energy, E. Physicists understand that $m = E/c^2$ but drop the speed of light constant, c, as a shorthand way of stating a particle's mass..

Craters are a Blast!



Have you ever wondered how much energy it takes to create a crater on the Moon. Physicists have worked on this problem for many years using simulations, and even measuring craters created during early hydrogen bomb tests in the 1950's and 1960's. One approximate result is a formula that looks like this:

$$\mathbf{E = 4.0 \times 10^{15}\ D^3\ Joules.}$$

where D is the crater diameter in kilometers.

As a reference point, nuclear bomb with a yield of one-megaton of TNT produces 4.0×10^{15} Joules of energy!

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified.

Note: To get a better sense of scale, the table below gives some equivalent energies for famous historical events:

Event	Equivalent Energy (TNT)
Cretaceous Impactor	100,000,000,000 megatons
Valdiva Volcano, Chile 1960	178,000 megatons
San Francisco Earthquake 1909	600 megatons
Hurricane Katrina 2005	300 megatons
Krakatoa Volcano 1883	200 megatons
Tsunami 2004	100 megatons
Mount St. Helens Volcano 1980	25 megatons

Answer Key

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

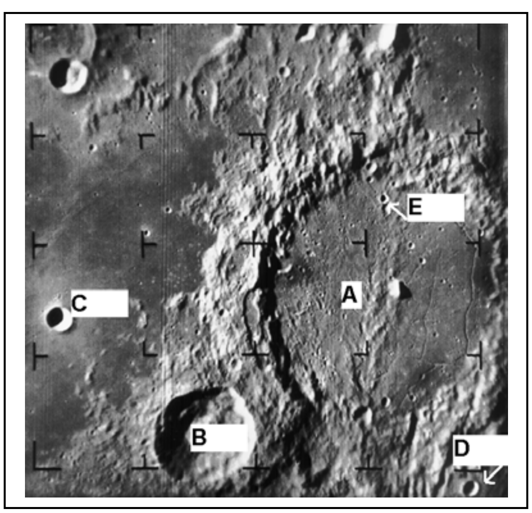
Answer: $E = 4.0 \times 10^{15} D^3$ Joules x (1 megaton TNT/4.0×10^{15} Joules)

$E = 1.0\ D^3$ Megatons of TNT

Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture.
Answer: The width of the image is 92 mm, so the scale is 183/92 = 2.0 km/mm. See figure below for some typical examples: See column 3 in the table below for actual crater diameters.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified. Answer: See the table below, column 4. Crater A is called Alphonsis. Note: No single formula works for all possible scales and conditions. The impact energy formula only provides an estimate for lunar impact energy because it was originally designed to work for terrestrial impact craters created under Earth's gravity and bedrock conditions. Lunar gravity and bedrock conditions are somewhat different and lead to different energy estimates. The formula will not work for laboratory experiments such as dropping pebbles onto sand or flour. The formula is also likely to be inaccurate for very small craters less than 10 meters, or very large craters greatly exceeding the sizes created by nuclear weapons. (e.g. 1 kilometer).

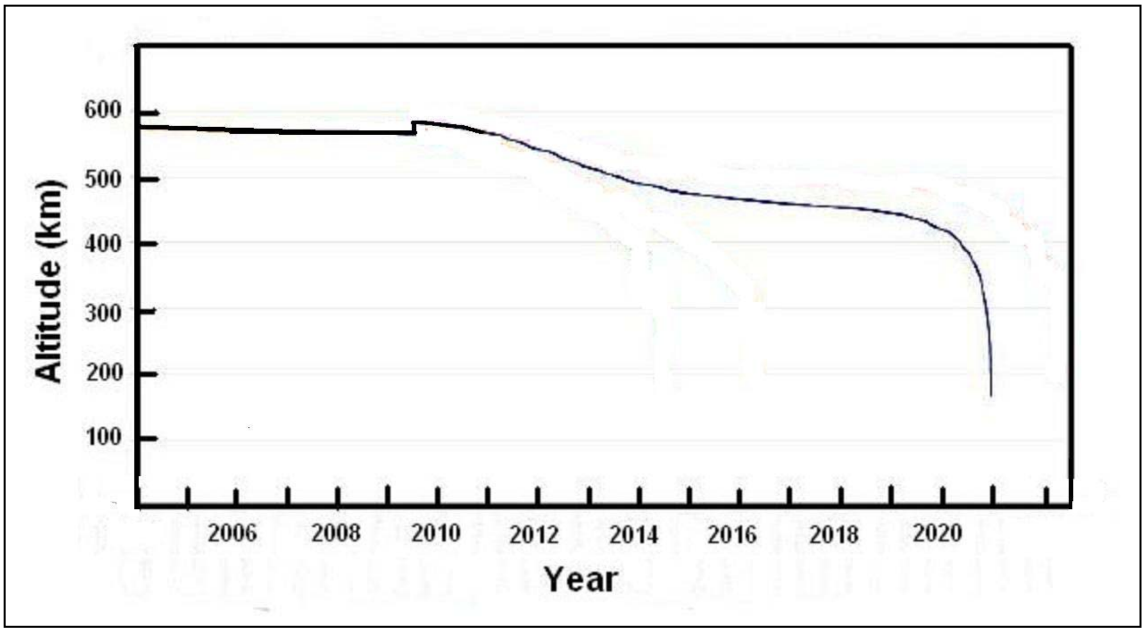
Crater	Size (mm)	Diameter (km)	Energy (Megatons)
A	50	**100**	**1,000,000**
B	20	**40**	**64,000**
C	5	**10**	**1,000**
D	3	**6**	**216**
E	1	**2**	**8**



Satellite Drag and the Hubble Space Telescope

The Hubble Space Telescope was never designed to operate forever. What to do with the observatory remains a challenge for NASA once its scientific mission is completed in 2012. Originally, a Space Shuttle was proposed to safely return it to Earth, where it would be given to the National Air and Space Museum in Washington DC. Unfortunately, after the last Servicing Mission, STS-125, in May, 2009, no further Shuttle visits are planned. As solar activity increases, the upper atmosphere heats up and expands, causing greater friction for low-orbiting satellites like HST, and a more rapid re-entry.

The curve below shows the predicted altitude for that last planned re-boost in 2009 of 18-km. NASA plans to use a robotic spacecraft after ca 2015 to allow a controlled re-entry for HST, but if that were not the case, it would re-enter the atmosphere sometime after 2020.



Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Problem 1 – The last Servicing Mission in 2009 only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Answer: The Servicing Mission occurred in 2009. The upgrades and gyro repairs extend the satellite's operations by 5 more years, so if it re-enters after 2020 it will have about 6 years to go before uncontrolled re-entry.

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

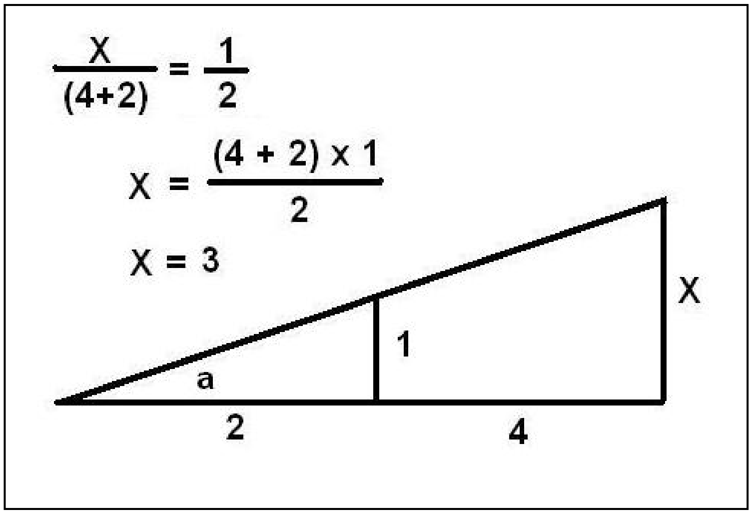
Answer: Use a millimeter ruler to determine the scale of the horizontal axis in weeks per millimeter. Mark the point on the curve that corresponds to a vertical value of 400 km. Draw a line to the horizontal axis and measure its distance from 2013 in millimeters. Convert this to weeks using the scale factor you calculated. The answer should be about 50 weeks.

"NASA's 23-year-old Hubble Space Telescope is still going strong, and agency officials said Tuesday (Jan. 8, 2013) they plan to operate it until its instruments finally give out, potentially for another six years at least.

After its final overhaul in 2009, the Hubble telescope was expected to last until at least 2015. Now, NASA officials say they are committed to keeping the iconic space observatory going as long as possible.

"Hubble will continue to operate as long as its systems are running well," Paul Hertz, director of the Astrophysics Division in NASA's Science Mission Directorate, said here at the 221st meeting of the American Astronomical Society. Hubble, like other long-running NASA missions such as the Spitzer Space Telescope, will be reviewed every two years to ensure that the mission is continuing to provide science worth the cost of operating it, Hertz added."

(Space.com, January 9, 2013.)

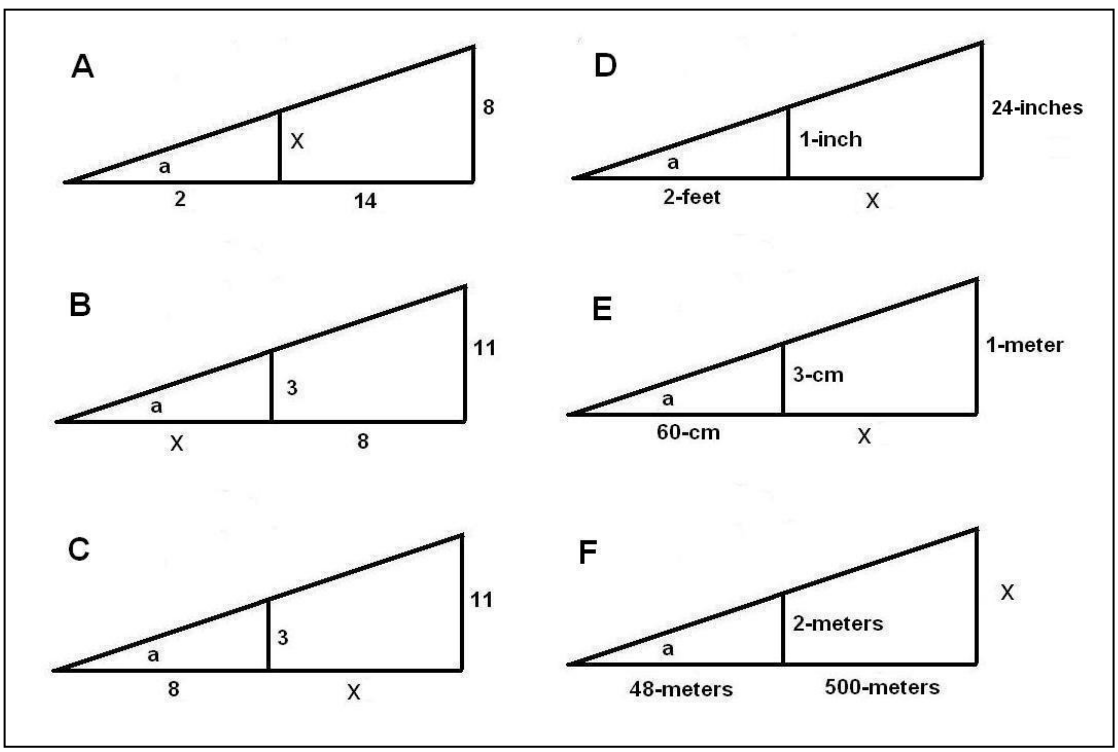


The corresponding sides of similar triangles are proportional to one another as the illustration to the left shows. Because the vertex angle of the triangles are identical in measure, two objects at different distances from the vertex will subtend the same angle, **a** . The corresponding side to 'X' is '1' and the corresponding side to '2' is the combined length of '2+4'.

Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

Problem 2: Which triangles must have the same measure for the indicated angle **a** ?

Problem 3: The sun is 400 times the diameter of the moon. Explain why they appear to have about the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?



Answer Key

Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

A) X / 2 = 8 / 16 so **X = 1**

B) 3 / X = 11 / (X+8) so 3 (X + 8) = 11 X ; 3X + 24 = 11 X ; 24 = 8X and so **X = 3.**

C) 3 / 8 = 11 / (x + 8) so 3 (x + 8) = 88 ; 3X + 24 = 88 ; 3X = 64 and so **X = 21 1/3**

D) 1-inch / 2-feet = 24 inches / (D + 2 feet); First convert all units to inches;
1 / 24 = 24 / (D + 24); then solve (D + 24) = 24x24 so D = 576 - 24;
D = 552 inches or 46 feet.

E) 3 cm / 60 cm = 1 meter / (X + 60 cm). 3/60 = 1 meter / (X + 0.6 m) then
3(X + 0.60) = 60 ; 3X + 1.8 = 60 ; 3X = 58.2 meters so **X = 19.4 meters.**

F) 2 meters / 48 meters = X / 548 meters ; 1/24 = X/548 ; X = 548 / 24 ; so **X = 22.8.**

Problem 2: Which triangles must have the same measure for the indicated angle **a** ?

Answer: Because the triangle (D) has the side proportion 1-inch /24-inches = 1/24 and triangle (F) has the side proportion 2 meters / 48 meters = 1/24 these two triangles, **D and F, have the same angle measurement for angle a**

Problem 3: The Sun is 400 times the diameter of the Moon. Explain why they appear to have the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?

Answer: From one of our similar triangles, the long vertical side would represent the diameter of the sun; the short vertical side would represent the diameter of the moon; the angle **a** is the same for both the sun and moon if the distance to the sun from Earth were 400x farther than the distance of the moon from Earth. Since the lunar distance is 384,000 kilometers, the sun must be at a distance of 154 million kilometers, which is close to the number given.

Evaluating Secondary Physical Constants

Symbol	Name	Value
c	Speed of light	2.9979×10^{10} cm/sec
h	Planck's constant	6.6262×10^{-27} erg sec
m	Electron mass	9.1095×10^{-28} gms
e	Electron charge	4.80325×10^{-10} esu
G	Gravitation constant	6.6732×10^{-8} dyn cm^{2} gm^{-2}
M	Proton mass	1.6726×10^{-24} gms

Also use $\pi = 3.1415926$

Although there are only a dozen fundamental physical constants of Nature, they can be combined to define many additional basic constants in physics, chemistry and astronomy.

In this exercise, you will evaluate a few of these 'secondary' constants to three significant figure accuracy using a calculator and the defined values in the table.

Problem 1 - Bremstrahlung Radiation Constant: $\dfrac{32\pi^2 e^6}{3(2\pi)^{1/2} m^3 c}$

Problem 2 - Photoionization Constant: $\dfrac{32\pi^2 e^6 (2\pi^2 e^4 m)}{3^{3/2} h^3}$

Problem 3 - Stark Line Limit: $\dfrac{16\pi^4 m^2 e^4}{h^4 M^5}$

Problem 4 - Thompson Scattering Cross-section: $\dfrac{8\pi}{3}\left(\dfrac{e^2}{mc^2}\right)^2$

Problem 5 - Gravitational Radiation Constant: $\dfrac{32}{5}\dfrac{G^5}{c^{10}}$

Problem 6 - Thomas-Fermi Constant: $\dfrac{324}{175}\left(\dfrac{4}{9\pi}\right)^{2/3}$

Problem 7 - Black Hole Entropy Constant: $\dfrac{c^3}{2hG}$

Answer Key

Method 1: Key-in to a calculator all the constants with their values as given to all indicated significant figures, write down final calculator answer, and round to three significant figures.

Method 2: Round all physical constants to 4 significant figures, key-in these values on the calculator, then round final calculator answer to 3 significant figures.

Note: When you work with numbers in scientific notation, Ex 1.23×10^5, the leading number '1.23' has 3 significant figures, but 1.23000 has 6 significant figures if the '000' are actually measured to be '000', otherwise they are just non-significant placeholders.

Also, you cannot have a final answer in a calculation that has more significant figures than the smallest significant figure number in the set. For example, 6.25*5.1 which a calculator would render as 31.875 is 'only good' to 2 significant figures (determined from the number 5.1) so the correct, rounded, answer is 32.

Problem	Method 1	Method 2
1	2.28×10^{16}	2.27×10^{16}
2	2.46×10^{-39}	2.46×10^{-39}
3	2.73×10^{135}	2.73×10^{135}
4	6.65×10^{-25}	6.64×10^{-25}
5	1.44×10^{-140}	1.44×10^{-140}
6	5.03×10^{-1}	5.03×10^{-1}
7	3.05×10^{64}	3.05×10^{64}

Note Problem 1 and 4 give slightly different results.

Problem 1: Method 1 answer 3.8784/1.7042 = 2.27578 or 2.28
Method 2 answer 3.8782/1.7052 = 2.2743 = 2.27

Problem 4: Method 1 answer 1.3378/2.0108 = 0.6653 = 0.665
Method 2 answer 1.3376/2.0140 = 0.6642 = 0.664

How Hot is that Planet?

Planets have been spotted orbiting hundreds of nearby stars, but this makes for a variety of temperatures depending on how far the planet is from its star and the stars luminosity.

The temperature of the planet will be about

$$T=273\left(\frac{(1-A)L}{D^2}\right)^{1/4}$$

where A is the reflectivity (albedo) of the planet, L is the luminosity of its star in multiples of the sun's power, and D is the distance between the planet and the star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). The resulting temperature will be in units of Kelvins. (i.e. 0^o Celsius = +273 K, and Absolute Zero is defined as 0 K).

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4?

Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of our sun were increased 1000 times and all other quantities remained the same?

Problem 3 - The recently discovered planet CoRoT-7b (see artist's impression above, from ESA press release), orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	
Water+Land	Earth	0.40	
Gas	Jupiter	0.70	

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4? **Answer: $T = 273 (0.6)^{1/4} = 240$ K**

Note: The equilibrium temperature of Earth is much lower than the freezing point of water. Were it not for the trace gases of carbon dioxide and to a lesser extent water vapor and methane providing 'greenhouse heating' our planet would be unlivable even with an atmosphere!

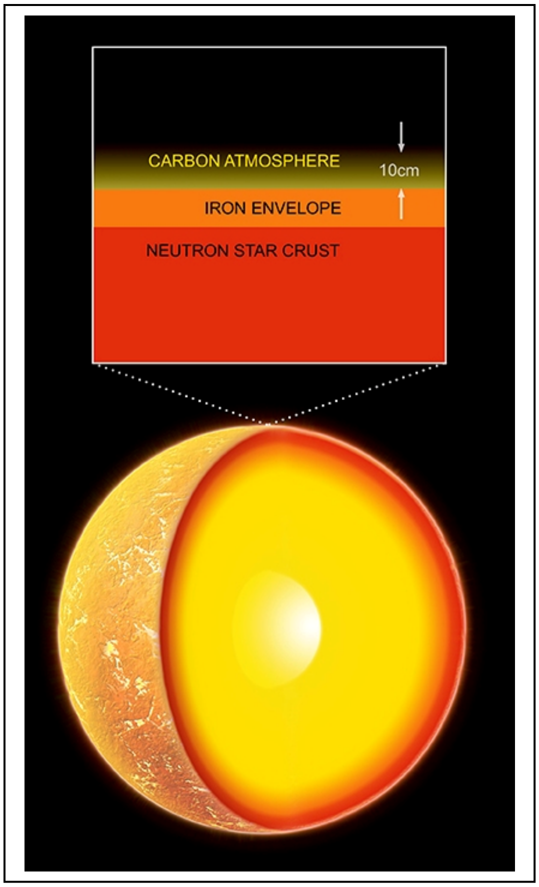
Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of our sun were increased 1000 times and all other quantities remained the same? Answer: From the formula, T = 240 and L = 1000 so
$240 = 273(0.6 \times 1000/D^2)^{1/4}$ and so **D = 5.6 AU**. This is about near the orbit of Jupiter.

Problem 3 - The recently discovered planet CoRoT-7b orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	1840
Water+Land	Earth	0.40	1699
Gas	Jupiter	0.70	1422

Example: For an albedo similar to that of our Moon:
$T = 273 * ((1-0.06)*0.71/(0.017)^2)^{.25}$
= **1,892 Kelvin**

Note: To demonstrate the concept of Significant Figures, the values for L, D and A are given to 2 significant figures, so the answers should be rounded to 1900, 1800, 1700 and 1400 respectively.



The atmosphere of Earth has a thickness of over 100 kilometers, however the Chandra X-Ray Observatory recently detected an atmosphere on the neutron star Cassiopeia-A that measured only 10 centimeters thick. How can a small planet have a deeper atmosphere than an entire stars-worth of matter?

The size of an atmosphere depends on a balance between the force of gravity pulling it towards the center of the planet, and the pressure of the atmosphere due to its temperature and density, pushing in the opposite direction. A pair of simple equations then defines how the density of the atmosphere has to rearrange itself with height above the surface so that gravity and pressure are always in balance. The equations look like this:

$$n(z) = n_0 e^{-\frac{z}{H}} \quad \text{where} \quad H = \frac{kT}{mg}$$

The exponential equation says that as you get farther from the surface, the density of the gas, N, drops very fast. The quantity, H in meters, is called the 'scale height' and its value is defined by the atmosphere's temperature, T, in Kelvins, and the acceleration of gravity at the surface, g, in multiples of Earth's acceleration (9.8 meters/sec^2). It also depends on the average mass, m, of the particles in the atmosphere. A light atmosphere made from hydrogen (m=1) will produce a value for H that is much larger than one made from pure oxygen (m=16). In this equation, k is Boltzman's Constant and equals 1.38 x 10^{-23} Joules/degree.

Problem 1 – Graph the density of Earth's atmosphere using H = 8 kilometers and n = 0.0013 grams/cc.

Problem 2 – The surface acceleration of the neutron star is 100 billion times that of Earth, the temperature of the gas is 3 million Kelvins compared to Earth's of 300 Kelvins, and the neutron star atmosphere is composed of carbon (A=12) rather than Earth's mixture of nitrogen and oxygen (A=28). From the formula for H, and the way in which it scales with m, T and g, what would you predict as the scale height for the neutron star atmosphere if for Earth, H = 8 kilometers?

Problem 3 – How far from the surface would you have to travel in order for the density of the atmosphere to fall by 1 million times for: A) Earth and B) the neutron star?

Problem 1 – Graph the density of Earth's atmosphere using H = 8 kilometers and n = 0.0013 grams/cc.

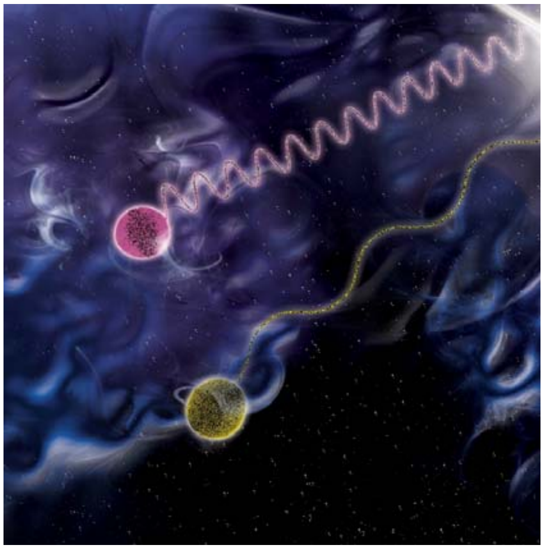
Problem 2 – The surface acceleration of the neutron star is 100 billion times that of Earth, the temperature of the gas is 3 million Kelvins compared to Earth's of 300 Kelvins, and the neutron star atmosphere is composed of carbon (A=12) rather than Earth's mixture of nitrogen and oxygen (A=28). From the formula for H, and the way in which it scales with m, T and g, what would you predict as the scale height for the neutron star atmosphere if for Earth, H = 8 kilometers?
Answer: This is an exercise in scaling and proportionality using equations and their variables. H is proportional to T and inversely proportional to the product of m and g, so that means that if we double only T, H increases by a factor of 2; if we double only g, H decreases by a factor of 2; and if we double only the mass of the atmosphere particles, H decreases by a factor of 2. Combining these proportionalities and starting with the value of H=8 kilometers for Earth, we have that T increases by 3 million/300 = 10,000 times; m decreases by 12/28 = 0.4 and g increases by 100 billion g/1g = 100 billion times. The new scale height is then H = 8 kilometers x (10,000)/(0.4 x 100 billion) **= 0.2 centimeters**.

Problem 3 – How far from the surface would you have to travel in order for the density of the atmosphere to fall by 1 million times for: A) the Earth and B) the neutron star?

Answer: We use the equation for the atmosphere density N = n0 e-x/H and determine for what value of the height, x, that $e^{-x/H}$ = 1/1000000. We can solve the equation as follows:

Take reciprocals of both sides:	1,000,000 = e(x/h)
Take the log, base-e, of both sides	13.8 = x/H
Solve for x	13.8 H = x
A) For Earth, H = 8 kilometers, so	13.8 (8 kilometers) = x
	x = **110 kilometers!**
B) For the neutron star, H = 0.13 centimeters so	13.8 (0.2 cm) = x
	x = **2.8 centimeters.**

Note to teacher: At the densities of the neutron star atmosphere (3.5 grams/cc) the carbon is not a gas but acts more like a crystalline solid so that the thickness, H, will not be completely determined by the equilibrium equations we have been using for Earth's atmosphere.


An artistic impression of two gamma-ray photons traveling through a lumpy space (Courtesy: NASA /Sonoma State University/Aurore Simonnet.)

The most advanced theories of how gravity works have proposed that empty space is not smooth, but may be filled by invisible limps and bumps that distort space into a froth of bubbles that are billions of times smaller than atomic nuclei. That's why we have not detected them in laboratory experiments thus far.

By studying how intense gamma-rays travel through the vast spaces between galaxies in the universe, NASA's Fermi Gamma-ray Observatory may have placed limits on this frothiness that eliminate many of the theories being explored to date.

As the gamma-rays travel through space, the shortest-wavelength gamma-rays take a slightly different path through space than the longer-wavelength gamma-rays. Although all gamma-rays travel at the speed of light, the invisible lumps in space scatter the short-wavelength (high-energy) gamma rays more than the long-wavelength (low-energy) ones so that there is a difference in the travel times of the long and short-wavelength gamma rays. This means that in traveling a distance, L, to Earth, the time should be about $t = L/c$ where c is the speed of light. The theories predict that the lumps in space cause the arrival times for the long and short-wavelength gamma-rays to differ by an amount $\mathbf{T = (L/c) \times (d/\lambda)}$. In this equation, λ is the wavelength difference between the two gamma-rays (related to their energy-difference), **d** is the length scale corresponding to the lumpiness in space, and **c** is the speed of light, 3×10^{10} cm/sec.

Problem 1 – Suppose two rays of visible light differed in wavelength by 100 nanometers ($\lambda = 10^{-9}$ cm) and traveled from the sun to Earth (L=150 million kilometers) through space with a lumpiness of about the scale of an atomic nucleus ($d=10^{-14}$ cm). What would be the arrival time delay in seconds at Earth between the two visible light photons?

Problem 2 – Suppose there is no indication that arriving visible light photons (10^{-9} cm) experience any delays in arrival time longer than 1 second, from quasars as far away as 5 billion light years. What is the maximum size of the lumps in space that are consistent with this limit?

Problem 3 – The Fermi Telescope measured a gamma-ray pulse from a distant object located 10 billion light years from Earth. The time delay was no more than 0.7 seconds. The wavelength difference in the gamma-rays was 4.0×10^{-12} centimeters (33 billion electron volts of energy). What is the largest size that can be involved in the lumpiness of empty space given this Fermi measurement.

Problem 1 – Suppose two rays of visible light differed by 100 nanometers (10^{-9} cm) in wavelength, and traveled from the sun to earth (150 million kilometers) through space with a lumpiness of about the scale of an atomic nucleus (10^{-14} cm). What would be the arrival time delay in seconds at Earth between the two visible light photons?

Answer: $T = (1.5 \times 10^{13}\text{ cm}) \times (1.0 \times 10^{-14}) / (3 \times 10^{10} \times 1.0 \times 10^{-9})$
$= \mathbf{0.005}$ **seconds**.

Problem 2 – Suppose there is no indication that arriving visible light photons (10^{-9} cm) experience any delays in arrival longer than one second, from quasars as far away as 5 billion light years. What is the maximum size of the lumps in space that are consistent with this limit?

Answer: L = 5 billion light years; $\lambda = 10^{-9}$ cm ; T = 1 second
Also L/c = 5 billion light years/c
= 5 billion years.
$= 5 \times 10^{9}$ years $\times 3.1 \times 10^{7}$ sec/year
$= 1.6 \times 10^{17}$ seconds

So $d = T \lambda\ c / L$
$= (1\text{ sec}) \times (1.0 \times 10^{-9}\text{ cm}) / (1.6 \times 10^{17}\text{ sec})$
$= \mathbf{6.3 \times 10^{-27}}$ **centimeters.**
Note: This is 10 trillion times smaller than the size of an atomic nucleus!

Problem 3 – The Fermi Telescope measured a gamma-ray pulse from a distant object located 10 billion light years from Earth. The time delay was no more than 0.7 seconds. The wavelength difference in the gamma-rays was 4.0×10^{-12} centimeters. What is the largest size that can be involved in the lumpiness of empty space given this Fermi measurement.

Answer: L = 10 billion light years; $\lambda = 4.0 \times 10^{-12}$ cm ; T = 0.7 second
Also L/c = 10 billion light years/c
= 10 billion years.
$= 1.0 \times 10^{10}$ years $\times 3.1 \times 10^{7}$ sec/year
$= 3.1 \times 10^{17}$ seconds

So $d = T \lambda\ c / L$
$= (0.7\text{ sec}) \times (1.0 \times 10^{-12}\text{ cm}) / (3.1 \times 10^{17}\text{ sec})$
$= \mathbf{2.2 \times 10^{-30}}$ **centimeters.**

Note: This is 22,000 trillion times smaller than the size of an atomic nucleus! According to some theories of the structure of space, the fundamental limit is about 1.6×10^{-33} centimeters and is called the Planck Length. The Fermi limit corresponds to about 1,400 Planck Lengths or smaller.

The Elementary Particle Masses

Particle	Mass	Status
Higgs Boson	>110 GeV	Predicted
Top Quark	170 GeV	Found
Z Boson	91 GeV	Found
W Boson	80 GeV	Found
Bottom Quark	4.2 GeV	Found
Tauon	1.8 GeV	Found
Charm Quark	1.2 GeV	Found
Strange Quark	120 MeV	Found
Muon	105 MeV	Found
Down Quark	4.0 MeV	Found
Up Quark	2.0 MeV	Found
Tau Neutrino	< 16 MeV	Found
Electron	0.5 MeV	Found
Muon Neutrino	< 0.2 MeV	Found
Electron Neutrino	< 2 eV	Found
Axion	< 1 eV	Predicted
Graviton	0	Predicted
Gluon	0	Found
Photon	0	Found

Most people are familiar with the Periodic Table of the Elements, which summarizes the properties of the 110 known elements starting from Hydrogen. These elements are composed of elementary particles called electrons, protons and neutrons, which combine in various numbers to build up all of the elements.

Since the 1950's, physicists have discovered an even more fundamental collection of particles that seem to be truly elementary, and which combine to produce not only all of the elements, but the very forces that hold them together. The table to the left shows the names of these particles and their mass. Some particles, such as the Higgs Boson are being proposed to exist because some theories require them in order to complete our understanding of how forces and particles interact.

A basic property of a particle is its mass. Because the masses of the fundamental particles are vastly smaller than a gram or a kilogram, physicists use another unit called the electron Volt. This is actually a unit of energy, but because Albert Einstein demonstrated that energy, E, and mass, m, are basically the same things ($E = mc^2$), physicists conveniently state mass as $M = E/c^2$, and then drop the speed-of-light factor c^2 because it is understood to be part of the definition of mass when energy units are used as in the table above. For example, 1 billion electron Volts (1 GeV) are equal to 1.8×10^{-27} kilograms, which is about equal to the mass of a single proton, which is 1.7×10^{-27} kilograms (938 MeV). (Note also that 1 MeV = 1 million electron Volts.)

Problem 1 - What is the mass, in kilograms, of A) An electron? B) A Strange Quark? C) A Top Quark? and D) A Muon?

Problem 2 - A proton consists of two Up Quarks and one Down Quark held tightly together by the strong nuclear force. What is the total quark mass of a proton in A) MeV? B) In kilograms?

Problem 3 - From your answer to Problem 2, what is the mass difference between 1 proton and the combination of the two Up Quarks and one Down Quark in units of A) MeV? B) kilograms?

Problem 4 - An oxygen atom contains a total of 16 protons and neutrons and has a mass of 2.7×10^{-26} kilograms. What is the mass of a Top Quark in terms of the number of oxygen atoms?

Problem 1 - What is the mass, in kilograms, of A) An electron? B) A Strange Quark? C) A Top Quark? and D) A Muon?
Answer:
A) 0.5 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **9.0 x 10^{-31} kilograms.**
B) 120 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **2.1 x 10^{-28} kilograms.**
C) 170 GeV x (1.8 x 10^{-27} kilograms / 1 GeV) = **3.1 x 10^{-25} kilograms.**
D) 105 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **1.9 x 10^{-28} kilograms.**

Problem 2 - A proton consists of two Up Quarks and one Down Quark held tightly together by the strong nuclear force. What is the total quark mass of a proton in A) MeV? B) In kilograms?

Answer: A) 2 Up + 1 Down = 2 x (2 MeV) + 1 x (4 MeV) = **8 MeV**. B) Mass = 8 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **1.4 x 10^{-29} kilograms.**

Problem 3 - From your answer to Problem 2, what is the mass difference between 1 proton and the combination of the two Up Quarks and one Down Quark in units of A) MeV? B) kilograms?

Answer:
A) 938 MeV - 8 MeV = **930 MeV.**
B) Mass = 930 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **1.7 x 10^{-27} kilograms.**

Problem 4 - An oxygen atom contains a total of 16 protons and neutrons and has a mass of 2.7x10^{-26} kilograms. What is the mass of a Top Quark in terms of the number of oxygen atoms?

Answer: From Problem 1C, the Top Quark mass is 3.1 x 10^{-25} kilograms, and so the number of oxygen atoms needed to equal the mass of one Top Quark is 2.7x10^{-26} kilograms / 3.1 x 10^{-25} kilograms is 8.7 or about **9 oxygen atoms**.

The Higgs Boson and the Mystery of Mass

Why is it that some elementary particles such as the photon and gluon do not carry any mass, while other elementary particles such as the electron, the quarks and the W and Z bosons carry different amounts of mass? In 1979, physicists Steven Weinberg, Sheldon Glashow and Abdus Salam received the Nobel Prize in Physics for explaining this mystery. To do so, they needed to add a new particle called the Higgs Boson to the known collection of fundamental particles. Depending on how strongly a particle such as electrons or neutrinos interacted with the Higgs Boson, it would 'pick up' varying amounts of mass. Photons didn't interact at all so they gained no mass, while the Top Quark interacted very strongly and gained a lot of mass.

The search is on for the Higgs Boson at the Large Hadron Collider, which began operation on November 23, 2009. The mass of the Higgs Boson is actually not constant, but depends on the amount of energy that is used to create it. This remarkable behavior can be described by the properties of the following equation:

$$\boxed{V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}}$$

This equation describes the potential energy, V, that is stored in the field that creates the Higgs Boson. The variable x is the mass of the Higgs Boson, and T is the collision energy being used to create this particle. The Higgs field represents a new 'hyper-weak' force in Nature that is stronger than gravity, but weaker than the electromagnetic force. This new field permeates space everywhere in the universe. The Higgs Boson is the particle that transmits the Higgs field just as the photon is the particle that transmits the electromagnetic field.

Problem 1 - What is the shape of the function V(x) over the domain [0,+1] for a collision energy of; A) T=0? B) T=0.5? C) T = 0.8 and D) T=1.0?

Problem 2 - The mass of the Higgs Boson is defined by the location of the minimum of V(x) over the domain [0, +1]. If the mass, M in GeV, of the Higgs Boson is defined by **M = 300x**, how does the predicted mass of the Higgs Boson change as the value of T increases from 0 to 1?

Problem 3 - The measured mass of the Top Quark is 170 GeV, and the Up Quark is 2 MeV and the electron is 0.5 MeV. According to the current models of the Higgs field, the masses are determined by the equations $Me = aM_H$, $Mt = bM_H$ and $Mu = cM_H$ where a, b and c are adjustable constants that have to be selected once the actual mass of the Higgs Boson, M_H, is established experimentally. If the current quark and electron masses are determined for T=0, what would be the predicted quark and electron masses for T = 0.5?

Problem 4 - The LHC will achieve collision energies of about E = 5,000 GeV. If T = E/300 GeV ,what will the function V(x) look like, and what will be the predicted mass of the Top Quark at these energies?

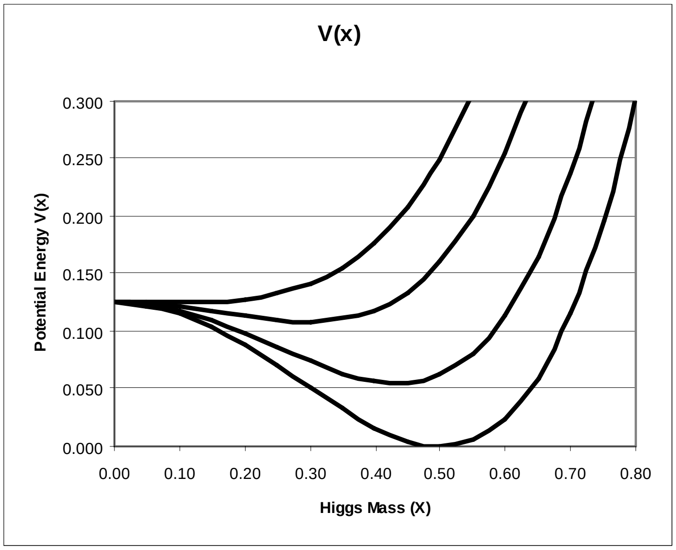
Problem 1 - Answer: The function can be programmed on an Excel spreadsheet or a graphing calculator. Select x intervals of 0.05 and a graphing window of x: [0,1] y:[0,0.3] to obtain the plot to the left below. The curves from top to bottom are for T = 1, 0.8, 0.5 and 0 respectively.

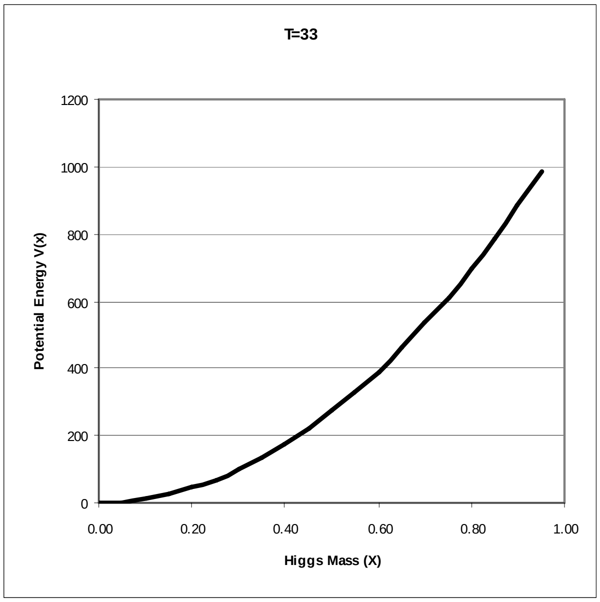
Problem 2 - Answer: The minima of the curves can be found using a graphing calculator display or by interpolating from the spreadsheet calculations. The x values for T = 1 ,0.8, 0.5 and 0 are approximately 0, 0.3, 0.45 and 0.5 so the predicted Higgs Boson masses from the formula M = 300x will be 0 Gev, 90 GeV, 135 GeV and 150 GeV respectively. Note that the observed mass will increase as the collision energy decreases, and consequently as the collision energy increases, the mass will decrease to zero!

Problem 3 - Answer: First we have to determine what the constants are for the condition T=0 where M_H = 150 GeV. For the electron, Me = 0.5 MeV = a x (150 GeV) and since 1 GeV = 1,000 MeV we get a = (0.5/150,000) = 3.3 x 10^{-6} then similarly, b = (170 GeV/150 GeV) = 1.13, and c = (2 MeV/150 GeV) = 1.3 x 10^{-5}. At T = 0.5, Problem 2 says that the Higgs mass is lowered to 135 GeV, so we have the mass of the electron Me = 3.3 x 10^{-6} x 135 GeV = **0.45 MeV**, the Top Quark is M = 1.13 x 135 GeV = **152 GeV** and the Up Quark is M = 1.3 x 10^{-5} x 135 GeV = **1.8 MeV**.

Problem 4 - The LHC will achieve collision energies of about E = 10,000 GeV. If T = E/300 GeV, what will the function V(x) look like, and what will be the predicted mass of the Top Quark at these energies? Answer: T = 10,000/300 = 33. The graph is shown to the lower right. The mass will be **near zero** because the curve is nearly a parabola with its vertex at x=0.

Note to Teacher: The function V(x) in this problem is meant to illustrate an important concept related to the way in which the Higgs Boson allows particles such as electrons and quarks to gain mass, rather than to remain massless particles. The answers to Problems 2-4 are not meant to be exact, but only to illustrate the basic mathematics. In actuality, V(X) and its changes with actual collision energies at the Large Hadron Collider are more complex than presented in these problems, and the masses of known particles are not expected to change my more than a few percent over the energy range for T being explored.





The Energy of Empty Space

An important concept in cosmology is that the 'empty space' between stars and galaxies is not really empty at all! Today, the amount of invisible energy hidden in space is just enough to be detected as Dark Energy, as astronomers measure the expansion speed of the universe. Soon after the Big Bang, this Dark Energy caused the universe to expand by huge amounts in less than a second. Cosmologists call this early period of the Big Bang Era, Cosmic Inflation.

Physicists have developed a number of theories to quantify how Cosmic Inflation occurred. All the theories link this phenomenon to changes in the way that the forces of nature operated at the very high temperatures and energies that existed soon after the Big Bang. The basis for these explanations is a mathematical equation of the form shown below, which connects the energy of empty space to the existence of a new force in nature. This force, called the Hyper-weak Force, is transmitted between a new field hidden in empty space, and the known particles such as electrons, quarks and the particles that transmit the other three forces: electromagnetism and the strong and weak nuclear forces.

An interesting property of this new field, whose energy is represented by the function V(x), is that the shape of this function changes as the temperature of the universe changes. The result is that the way that this field, represented by the variable x, interacts with the other elementary particles in nature, changes. As this change from very high temperatures (T=1) to very low temperatures (T=0) occurs, the universe undergoes Cosmic Inflation!

$$\boxed{V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}}$$

Problem 1 - What are the domain and range of the function V(x)?

Problem 2 - What is the axis of symmetry of V(x)?

Problem 3 - Is V(x) an even or an odd function?

Problem 4 - For T=0, what are the critical points of the function in the domain [-2, +2]?

Problem 5 - Over the domain [0,+2] where are the local minima and maxima located for T=0?

Problem 6 - Using a graphing calculator or an Excel spreadsheet, graph V(x) for the values T=0, 0.5, 0.8 and 1.0 over the domain [0,+1]. Tabulate the x-value of the local minimum as a function of T. In terms of its x location, what do you think happens to the end behavior of the minimum of V(x) in this domain as T increases?

Problem 7 - What is the vacuum energy difference V = V(0) - V(1/2) during the Cosmic Inflation Era?

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?

Problem 1 - **Answer: Domain [- infinity, + infinity], Range [0,+infinity]**

Problem 2 - **Answer: The y-axis: x=0**

Problem 3 - **Answer: It is an even function.**

Problem 4 - **Answer: X = 0, X = -1/2 and x = +1/2**

Problem 5 - **Answer: The local maximum is at x=0; the local minimum is at x = +1/2**

Problem 6 - **Answer: See the graph below where the curves represent from top to bottom, T = 1.0, 0.8, 0.5 and 0.0. The tabulated minima are as follows:**

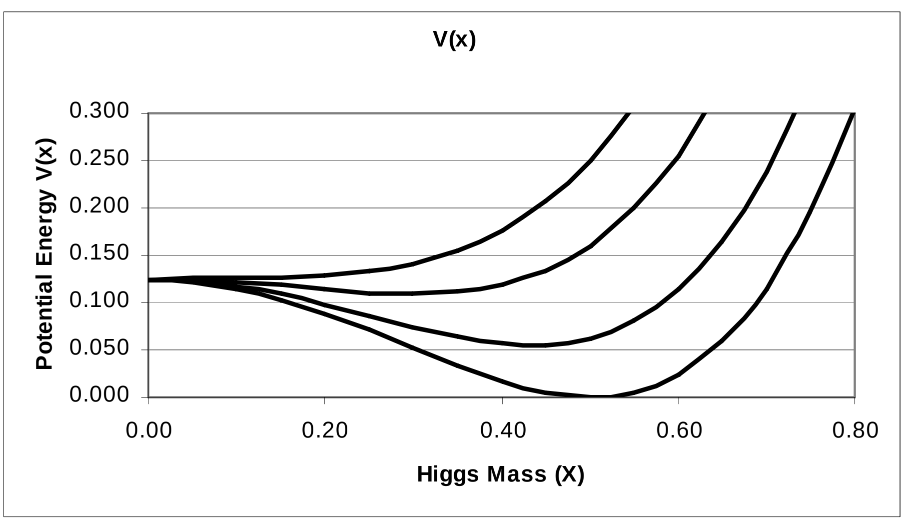
T	X
0.0	0.5
0.5	0.45
0.8	0.30
1.0	0.0

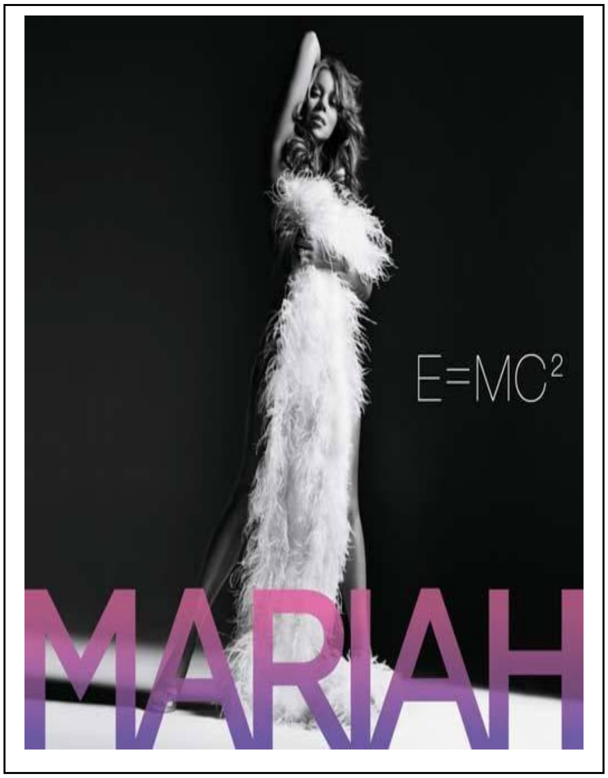
The end behavior, in the limit where T becomes very large, is that V(x) becomes a parabola with a vertex at (0, +1/8)

Problem 7 - What is the vacuum energy difference V = V(0) - V(1/2) during the Cosmic Inflation Era? **Answer: V(0) = 1/8 V(1/2) = 0 so V = 1/8.**

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?
Answer: V = 1/8 x 10^{35} Joules/meter^3 = **1.2 x 10^{34} Joules/meter^3**.

Note to Teacher: This enormous energy was available in every cubic meter of space that existed soon after the Big Bang, and the time it took the universe to change from the V(0) to V(1/2) state lasted only about 10^{-35} seconds. This was enough time for the universe to grow by a factor of 10^{35} times in its size during the Cosmic Inflation Era.





Einstein's formula $E = mc^2$ is almost legendary, and sometimes appears even on T-shirts or even Mariah Carey's pop music album cover! But what does the formula really mean?

It means that, what we consider to be energy in its many forms (like light and heat), which we measure in Joules, can also be considered equivalent to mass in its many forms (like grains of sand or mountains), which we measure in units of kilograms!

The problem is that it takes a LOT of energy to make a kilogram of mass! That's why we never see matter suddenly just appearing out of nowhere in our daily lives. That little formula says that 1 kilogram of mass, m, is exactly equal to 9.0×10^{16} Joules of energy, E. That's the energy released by just one, 20 megaton hydrogen bomb!

The formula depends on the speed of light-squared, c^2, to do the conversion from energy units (E in Joules) to matter units (m in kilograms). If c = 300 million meters/sec, we have the simpler formula: E(Joules) = 9×10^{16} x (mass in kilograms). Use this formula to perform the following conversions using a calculator, and providing answers to 2 significant figures in scientific notation:

Problem 1 1-proton mass = 1.6726×10^{-27} kg equals --------------- Joules

Problem 2 1 Joule of Energy = ------------------- kilograms

Problem 3 1 electron Volt = 1.6×10^{-19} Joules = ------------------ kilograms

Problem 4 1 neutron = 1.6749×10^{-27} kg equals --------------- Joules

Problem 5 1 deuterium nucleus (1 proton+1 neutron) = 3.3444×10^{-27} kg equals --------------- Joules

Problem 6 What is: A) The difference in mass between a single proton plus a single neutron, and the deuterium nucleus? B) The difference in energy? C) The difference in Volts?

Answer Key

Problem 1 1-proton mass = 1.6726×10^{-27} kg

$9 \times 10^{16} \times 1.6726 \times 10^{-27}$ kg = 1.505×10^{-10} Joules

Problem 2 1 Joule of Energy = $1 / (9 \times 10^{16})$ = 1.1×10^{-17} kilograms

Problem 3 1 electron Volt = 1.6×10^{-19} Joules = 1.8×10^{-36} kilograms

Problem 4 1 neutron = 1.6749×10^{-27} kg equals 1.507×10^{-10} Joules

Problem 5 1 deuterium nucleus (1 proton+1 neutron) = 3.3444×10^{-27} kg = 3.010×10^{-10} Joules

Problem 6 What is:

A) The difference in mass between a single proton plus a single neutron, and the deuterium nucleus?

$(1.6726 \times 10^{-27}$ kg + 1.6749×10^{-27} kg) - 3.3444×10^{-27} kg = 3.1×10^{-30} kg

B) The difference in energy?

$(1.505 \times 10^{-10}$ Joules + 1.507×10^{-10} Joules) - 3.010×10^{-10} Joules = 2.0×10^{-13} Joules

C) The difference in Volts?

Answer: Since 1 volt = 1.8×10^{-36} kilograms

3.1×10^{-30} kg x (1 Volt / 1.8×10^{-36} kg) = 1.7 million Volts or 1.7 MeV.

A more accurate calculation yields 2.2 MeV

The Volume of a Hypersphere

This spherical propellant tank is an important component of testing for the Altair lunar lander, an integral part of NASA's Constellation Program. It will be filled with liquid methane and extensively tested in a simulated lunar thermal environment to determine how liquid methane would react to being stored on the moon.

The volume of a sphere is a mathematical quantity that can be extended to spaces with different numbers of dimensions with some very interesting, and surprising, consequences!

The mathematical formula for the volume of a sphere in a space of N dimensions is given by the recursion relation

$$V(N) = \frac{2\pi R^2}{N} V(N-2)$$

For example, for 3-dimensional space, N = 3 and since from the table to the left, V(N-2) = V(1) = 2R, we have the usual formula

$$V(3) = \frac{4}{3}\pi R^3$$

Dimension	Formula	Volume
0	1	1.00
1	2R	2.00
2	πR^2	3.14
3	$\frac{4}{3}\pi R^3$	4.19
4		
5		
6		
7		
8		
9		
10		

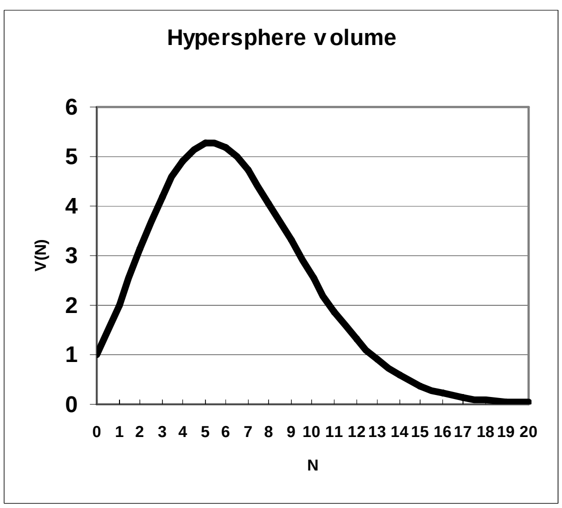
Problem 1 - Calculate the volume formula for 'hyper-spheres' of dimension 4 through 10 and fill-in the second column in the table.

Problem 2 - Evaluate each formula for the volume of a sphere with a radius of 1.00 and enter the answer in column 3.

Problem 3 - Create a graph that shows V(N) versus N. For what dimension of space, N, is the volume of a hypersphere its maximum possible value?

Problem 4 - As N increases without limit, what is the end behavior of the volume of an N-dimensional hypersphere?

Dimension	Formula	Volume
0	1	1.00
1	2R	2.00
2	πR^2	3.14
3	$\frac{4}{3}\pi R^3$	4.19
4	$\frac{\pi^2 R^4}{2}$	4.93
5	$\frac{8\pi^2 R^5}{15}$	5.26
6	$\frac{\pi^3 R^6}{6}$	5.16
7	$\frac{16\pi^3 R^7}{105}$	4.72
8	$\frac{\pi^4 R^8}{24}$	4.06
9	$\frac{32\pi^4 R^9}{945}$	3.30
10	$\frac{\pi^5 R^{10}}{120}$	2.55



Problem 1 - Answer for N=4:

$$V(4) = \frac{2\pi R^2}{4} V(4-2)$$

$$V(4) = \frac{2\pi R^2}{4} V(2)$$

$$V(4) = \frac{2\pi R^2}{4} (\pi R^2)$$

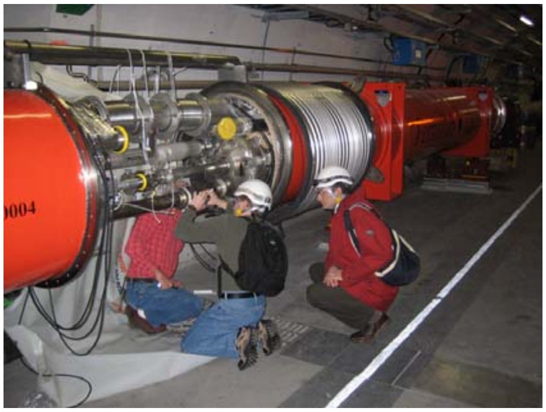
$$V(4) = \frac{\pi^2 R^4}{2}$$

Problem 2 - Answer for N=4:
$V(4) = (0.5)(3.141)^2 = 4.93$.

Problem 3 - The graph to the left shows that the maximum hypersphere volume occurs for spheres in the fifth dimension (N=5). Additional points have been calculated for N=11-20 to better illustrate the trend.

Problem 4 - In the limit for spaces with very large dimensions, the hypersphere volume approaches zero!

Exploring the Large Hadron Collider



A small section of the LHC (Photo: Peter Limon)

During November, 2009 the Large Hadron Collider experiment at CERN began a slow, step-by-step process of being turned on. Its goal is to accelerate two beams of protons and anti-protons to very high energies, and collide them to form new sub-atomic particles for physicists to discover and study.

Although it is not a NASA research program, it will directly impact the findings of many NASA research satellites such as Chandra and WMAP in searching for Dark Matter and Dark Energy. To understand the operation of LHC, we must first learn how to measure and discuss very small amounts of energy!

The 27-kilometer diameter LHC ring, buried deep underground, uses thousands of magnets to steer two beams of protons so that they collide at specific points along the ring. The beams are no bigger than a human hair in diameter, but contain millions of protons and anti-protons traveling at nearly the speed of light and circulating in the 27-kilometer ring in opposite directions. When these particles collide, their energies combine to create a momentary explosion out of which is created hundreds of particles including electrons, quarks and even more massive particles. Thanks to Albert Einstein's $E=mc^2$, the energy of the two protons can create a burst of new particles, m, literally created out of the raw energy, E, of the collisions.

Physicists use the 'electron volt' as a convenient unit of energy. One 'eV' is the amount of energy that an electron gains as it falls through a voltage difference of exactly 1 volt. This energy is equal to 1.6×10^{-19} Joules. Physicists find it far easier to remember and write energy measured in units of eV than in Joules! The following problems exercise your ability to translate between eV units and Joules.

Problem 1 - The mass of an electron is equivalent to 511,000 eV of energy (also written as 511 keV). How many Joules is this?

Problem 2 - The mass of a Top Quark is 175 billion eV (or 175 Giga-electron Volts abbreviated as 175 GeV). How many Joules is this?

Problem 3 - A proton has an energy equivalent to 1.5×10^{-10} Joules. How many electron volts is this?

Problem 4 - A small gnat in flight carries about 1.6×10^{-7} Joules of energy. About how many electron Volts is this equivalent to?

Problem 5 - When the LHC began operation, on November 30, 2009 it achieved an energy of 1.2 trillion electron Volts (1.2 Terra eV or 1.2 TeV). How many Joules of energy is this?

Problem 6 - When fully operational, the LHC will carry 100 trillion protons along each beam, with each proton carrying an energy of 15 TeV. What is the total energy of each proton beam in the LHC compared to: A) A baseball in flight (120 Joules)? B) A small car traveling at 60 mph (300,000 Joules)? C) A Boeing 767 in flight (4 billion Joules)?

Problem 1 - The mass of an electron is equivalent to 511,000 eV of energy (also written as 511 keV). How many Joules is this?
Answer: 511,000 eV x (1.6 x 10^{-19} Joules/1 eV) = **8.2 x 10^{-14} Joules.**

Problem 2 - The mass of a Top Quark is 175 billion eV (or 175 Giga-electron Volts abbreviated as 175 GeV). How many Joules is this?
Answer: 175,000,000,000 eV x (1.6 x 10^{-19} Joules/1 eV) = **2.8 x 10^{-6} Joules.**

Problem 3 - A proton has an energy equivalent to 1.5 x 10^{-10} Joules. How many electron volts is this?
Answer; 1.5 x 10^{-10} Joules x (1 eV/ 1.6 x 10^{-19} Joules) = **938,000,000 eV**
Also written as 938 MeV.

Problem 4 - A small gnat in flight carries about 1.6 x 10^{-7} Joules of energy. About how many electron Volts is this equivalent to?
Answer; 1.6 x 10^{-7} Joules x (1 eV/ 1.6 x 10^{-19} Joules) = **1,000,000,000,000 eV**
Also written as 1 terra eV or 1 TeV.

Problem 5 - When the LHC began operation, on November 30, 2009 it achieved an energy of 1.2 trillion electron Volts (1.2 Terra eV or 1.2 TeV). How many Joules of energy is this?
Answer: 1,200,000,000,000 eV x (1.6 x 10^{-19} Joules/1 eV) = **1.9 x 10^{-7} Joules.**

Problem 6 - When fully operational, the LHC will carry 100 trillion protons along each beam, with each proton carrying an energy of 15 TeV. What is the total energy of each proton beam in the LHC compared to: A) A baseball in flight (120 Joules)? B) A small car traveling at 60 mph (300,000 Joules)? C) A Boeing 767 in flight (4 billion Joules)?

Answer: Each proton will carry 15,000,000,000,000 eV x (1.6 x 10^{-19} Joules/1 eV) = 2.4 x 10^{-6} Joules. So the total energy of each beam will be 100 trillion times this or **2.4 x 10^{8} Joules!**

A) 2.4 x 10^{8} Joules / 120 Joules = **2 million times a baseball in flight**.

B) 2.4 x 10^{8} Joules / 300,000 Joules = **800 times a car in motion.**

C) 2.4 x 10^{8} Joules / 4,000,000,000 Joules = **1/17 of a passenger jet.**

Significant Figures: Oh my!

Example 1:
<u>1</u>.<u>234</u> grams has 4 significant figures,

Example 2:
<u>30</u>.<u>07</u> Liters has 4 significant figures.

Example 3:
0.0**<u>12</u>** grams has 2 significant figures.
0.**<u>20</u>** grams has 2 significant figures.

Example 4:
0.0**<u>230</u>** Liters has 3 significant figures,

For problems involving pure mathematics, one seldom takes the point of view that some digits in a stated number are more important than others, however in science where numbers represent measurements, not all digits are created equally!

The examples to the left show the four basic rules of how to count 'significant figures'. The basic rule is that you never state more digits in a number than the precision of your measurement. For example, if you can only measure to an accuracy of one meter, you should never state a measurement as 102.56 meters!

Problem 1 - For the numbers below, indicate how many SFs are involved:

A) 450.12 B) 450.120 C) 1.234×10^{-11} D) 0.00234500 E) 0.002345

Problem 2 - When multiplying or dividing numbers, the answer must have the same number of significant figures. Pure numbers such as '5' can be written with as many decimal places as needed. Evaluate the following expressions to the correct number of SF.

A) M = 3.275 x 1.3 grams B) T = 5 (356.19)/8 + 24.0347 degrees

C) S = π $(1.0850)^2$ meters^2 D) Y = [1.03 x 3.98720] / 1.1087 kilograms

Problem 3 - The Rydberg Constant is an import number in atomic physics and is given by the formula

$$R_\infty = \frac{2\pi^2 m e^4}{c h^3}$$

Evaluate the formula to the correct number of SF where:

Electron mass: $m = 9.10956 \times 10^{-28}$ grams
Speed of light: $c = 2.997925 \times 10^{10}$ cm/sec
Electron charge: $e = 4.80325 \times 10^{-10}$ ESU
Planck's Constant: $h = 6.62620 \times 10^{-27}$ erg sec

Problem 1 - For the numbers below, indicate how many SFs are involved:

A) 5 SF B) 6 SF C) 4 SF D) 6 SF E) 4 SF

Problem 2 - When multiplying or dividing numbers, the answer must have the same number of significant figures. Pure numbers such as '5' can be written with as many decimal places as needed. Evaluate the following expressions to the correct number of SF.

A) M = 3.3 grams B) T = 246.65 degrees

C) S = 3.6984 meters^2 D) Y = 3.70 kilograms

Problem 3 - The Rydberg Constant is an import number in atomic physics and is given by the formula

$$R_\infty = \frac{2\pi^2 m e^4}{ch^3}$$

Evaluate the formula to the correct number of SF where:

Electron mass: m = 9.10956 x10^{-28} grams

Speed of light: c = 2.997925 x 10^{10} cm/sec

Electron charge: e = 4.80325x10^{-10} ESU

Planck's Constant: h = 6.62620 x 10^{-27} erg sec

Answer: First evaluate the function using only the exponents:

R = $(10^{-28})(10^{-10})^4 / (10^{10})(10^{-27})^3 = 10^3$. Now evaluate all the measured constants:

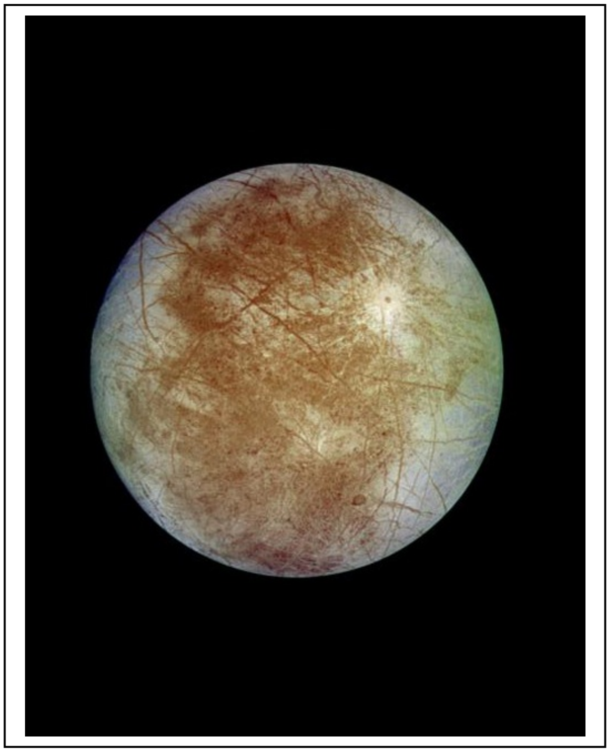
$$R = \frac{2\,(3.141592654)^2\,(9.10956)(4.80325)^4}{(2.997925)(6.62620)^3}$$

Using a calculator, one obtains the 'answer' R = 109.7371112 which has to be combined with the previous calculation for the exponents of 10^3 to get

R = 109.7371112 x 10^3 or in scientific notation R = 1.097371112 x 10^5

The smallest number of SFs occurs for the constants m, e and h which have 6 SF, so we round the answer to **R = 1.09737 x 10^5**.

Ice or Water?



There are no known extremophiles that can exist at a temperature lower than the freezing temperature of water. It is believed that liquid water is a crucial ingredient to the chemistry that leads to the origin of life. To change water-ice to liquid water requires energy.

First, you need energy to raise the ice from wherever temperature it is, to 0 Celsius. This is called the Specific Heat and is 2.04 kiloJoules/kilogram C

Then you need enough energy added to the ice near 0 C to actually melt the ice by increasing the kinetic energy of the water molecules so that their hydrogen bonds weaken, and the water stops acting like a solid. This is called the Latent Heat of Fusion and is 333 kiloJoules/kilogram.

Let's see how this works!

Example 1: You have a 3 kilogram block of ice at a temperature of -20 C. The energy needed to raise it by 20 C to a new temperature of 0 C is Eh = 2.04 kiloJoules/kg C) x 3 kilograms x (20 C) = 2.04 x 3 x 20 = 122 kiloJoules.

Example 2: You have a 3 kilogram block of ice at 0 C and you want to melt it completely into liquid water. This requires Em = 333 kiloJoules/kg x 3 kilograms = 999 kiloJoules.

Example 3: The total energy needed to melt a 3 kilogram block of ice from -20 C to 0C is E = Eh + Em = 122 kiloJoules + 999 kiloJoules = 1,121 kiloJoules.

Problem 1 - On the surface of the satellite Europa (see NASA's Galileo photo above), the temperature of ice is -220 C. What total energy in kiloJoules is required to melt a 100 kilogram block of water ice on its surface? (Note: Calculate Eh and Em separately then combine them to get the total energy.)

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8 x 10^{16} kilograms. How many Joules would be required to melt the entire surface of Europa to this depth? (Note: Calculate Eh and Em separately then combine them to get the total energy. Then convert kiloJoules to Joules)

Problem 3 - The sun produces 4.0 x 10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the Sun's energy could be used? (Note: The numbers are BIG, but don't panic!)

Problem 1 - On the surface of the satellite Europa, the temperature of ice is -220 C. What total energy is required to melt a 100 kilogram block of water ice on its surface?

Answer: You have to raise the temperature by 220 C, then

E = 2.04 x 220 x 100 + 333 x 100
= 44,880 + 33,300
= 78,180 kiloJoules.

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8×10^{16} kilograms. How many joules would be required to melt the entire surface of Europa to this depth?

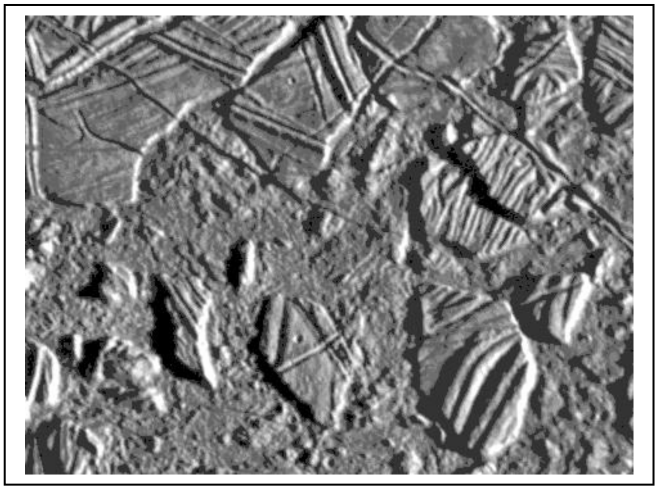
Note: The radius of Europa is 1,565 km. The surface area is $4 \times \pi \times (1{,}565{,}000\ \mathrm{m})^2 = 3.1 \times 10^{13}$ meters^2. A 1 meter thick shell at this radius has a volume of 3.1×10^{13} meters^2 x 1 meter = 3.1×10^{13} meters^3. The density of water ice is 917 kilograms/m^3 ,so this ice layer on Europa has a mass of $3.1 \times 10^{13} \times 917 = 2.8 \times 10^{16}$ kilograms.

Energy = $(2.04 \times 220 + 333) \times 2.8 \times 10^{16}$ kg
= 2.2×10^{19} kiloJoules
= 2.2×10^{22} Joules.

Problem 3 - The sun produces 4.0×10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the sun's energy could be used?

Answer: Time = Amount / Rate
= 2.2×10^{22} Joules / 4.0×10^{26} Joules
= 0.000055 seconds.

Water on Planetary Surfaces



Space is very cold! Without a source of energy, like a nearby star, water will exist at a temperature at nearly -270 C below zero and frozen solid. To create a permanent body of liquid water in which pre-biotic chemistry can occur, a steady source of energy must flow into the ice to keep it melted and in liquid form. Common sources of energy on Earth are volcanic activity, oceanic vents and fumaroles, and sunlight.

The picture above was taken by NASA's Galileo spacecraft of the surface of Jupiter's moon Europa. Its icy crust is believed to hide a liquid-water ocean beneath. The energy for keeping the water in a liquid state is probably generated by the gravity of Jupiter, which distorts Europa's shape through tidal action. The tidal energy may be enough to keep the oceans liquid for billions of years.

A common measure of energy flow or usage is the Watt. One Watt equals one Joule of energy emitted or consumed in one second.

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Water ice at 0 C requires 330,000 Joules of energy to become liquid for each kilogram of ice. Suppose the ice acted like a battery and absorbed all the energy that fell on it. Ice doesn't really work that way, but let's suppose that it does just to make a simple mathematical model!

Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Answer: 100 watts is the same as 100 Joules/sec, so if 1 hour = 3600 seconds, the energy consumed is 100 Joules/sec x 3600 seconds = 360,000 Joules.

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Answer: 3,000 kilowatts x (1,000 watts/1 kilowatt) = 3,000,000 watts. Since this equals 3,000,000 Joules/sec, in 1 hour (3600 seconds) the consumption is 3,000,000 watts x 3,600 seconds = 10,800,000,000 Joules or 10.8 billion Joules.

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Answer: 3,600,000 Joules in 1 hour is an average rate of 3,600,000 Joules/3,600 seconds = 1,000 Joules/sec or 1,000 Watts. This is the maximum rate at which the appliances can consume before exceeding the capacity of the solar system.

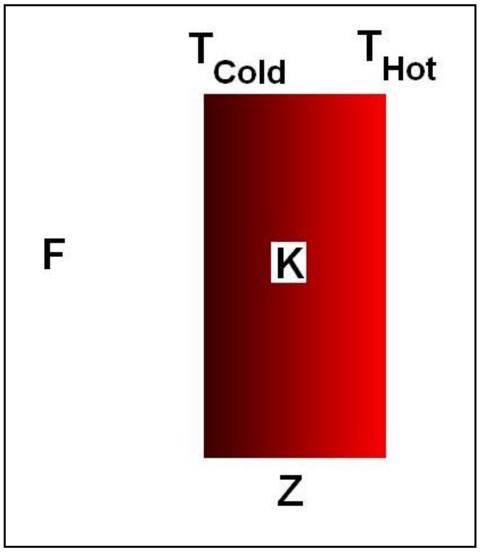
Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Answer: From the information provided, it takes 330,000 Joules to melt 1 kilogram of ice. So since the mass of the ice block is 10 kilograms, it will take 330,000 x 10 = 3,300,000 Joules

If the ice stores the energy falling on it from the hair dryer, all we need to do is to calculate how long a 2,000-watt hair dryer needs to be run in order to equal 3,300,000 Joules. This will be Time = 3,300,000 Joules/2000 watts = 1650 seconds or about 27.5 minutes.

Proving that the unit will automatically be in seconds is a good exercise in unit conversions and using the associative law and reciprocals:

1 Joule / (1 watt)	= 1 Joule / (1 Joule/sec)	
	= 1 Joule x (1 sec/1 joule)	: Multiply be the reciprocal
	= (1 Joule x 1 sec)/ 1 joule	: Re-write
	= 1 sec x (1 joule / 1 joule)	: Re-group common terms
	= 1 sec.	:after cancling the 'joules' unit.



Every time you put on clothing to go outside in the winter you are experimenting with heat flow and insulation. Your body is at a temperature of 98.6°F (37°C) and emits about 100 watts of heat. By putting on a layer of clothing, you are preventing this 100 watts from quickly leaking out into the cold air, and this keeps you nice and warm.

The type (K value) of the insulation you wear (cotton versus down) will determine how rapidly this 100 watts leaks out, and how cold you will feel.

Another factor involved in keeping warm is the temperature difference given by DT= (T_{hot} - T_{cold}) between your body and the surrounding air. If there is a big temperature difference, the heat will flow more rapidly and you will cool off more quickly. Also, a thin insulation (Z small) will make you feel cooler than a thick insulation (Z large) and allow more heat to escape more quickly. Scientists can create a formula that follows all of these relationships.

Problem 1 - Convert the following statement into a mathematical formula using P = heat flux in watts/m^2, K = heat diffusion coefficient of the insulation, z = thickness of insulator in meters and DT = temperature difference in Celsius:

The amount of heat flux in watts per square meter is proportional to the temperature difference in degrees Celsius and inversely proportional to the thickness of the insulator in meters. The constant of proportionality is given by K.

Problem 2 - A cook is using an aluminum pot (K=250) and a stainless steal pot (K=16) to boil water. If the pots have a thickness of 2 millimeters and the temperature difference between the hot plats and the inside of the pot is 200^{o}C, which pot will boil the water the fastest?

Problem 3 - On the surface of Mars, the heat flux is measured to be 20 milliWatts/m^2. The temperature gradient DT/Z = 5^{o}C/20 meters. What is the heat diffusion coefficient K for the martian soil?

Problem 1 - Convert this statement into a mathematical formula using P = heat flux in watts/m^2, K = heat diffusion coefficient of insulation, z = thickness of insulator in meters and DT = temperature difference in Celsius: *The amount of heat power per square meter is proportional to the temperature difference and inversely proportional to the thickness of the insulator. The constant of proportionality is given by K*.

Answer: $$F = K\frac{DT}{Z}$$

Problem 2 - A cook is using an aluminum pot (K=250) and a stainless steal pot (K=16) to boil water. If the pots have a thickness of 2 millimeters and the temperature difference between the hot plate and the inside of the pot is 200^{o}C, which pot will boil water the fastest?

Answer: Aluminum: F = 250 x (200/0.002) = 25,000,000 watts/m^2
Stainless Steal: F = 16 x (200/.002) = 1,600,000 watts m^2

There is 16 times more heat entering the aluminum pot so water will boil faster in an aluminum pot that a stainless steel pot.

Problem 3 - On the surface of Mars, the heat flux is measured to be 20 milliWatts/m^2. The temperature gradient DT/Z = 5^{o}C/20 meters. What is the heat diffusion coefficient K for the martian soil?

Answer: 20 miliWatts/m^2 = 0.020 Watts/m^2. From the formula we have

0.020 watts/m^2 = K x (5^{o}C/20meters) so

K = 0.020 x 20 meters/5^{o}C and so

K = 0.08 watts/meter ^{o}C

Note: Solid granite has a value of K = 3.0
Volcanic pumice rock has K = 0.5
Lunar soil has K between 0.1 and 0.01

So the martian surface is similar as an insulator to lunar soil which has been produced by numerous asteroid bombardments and has lots of space between the soil particles.

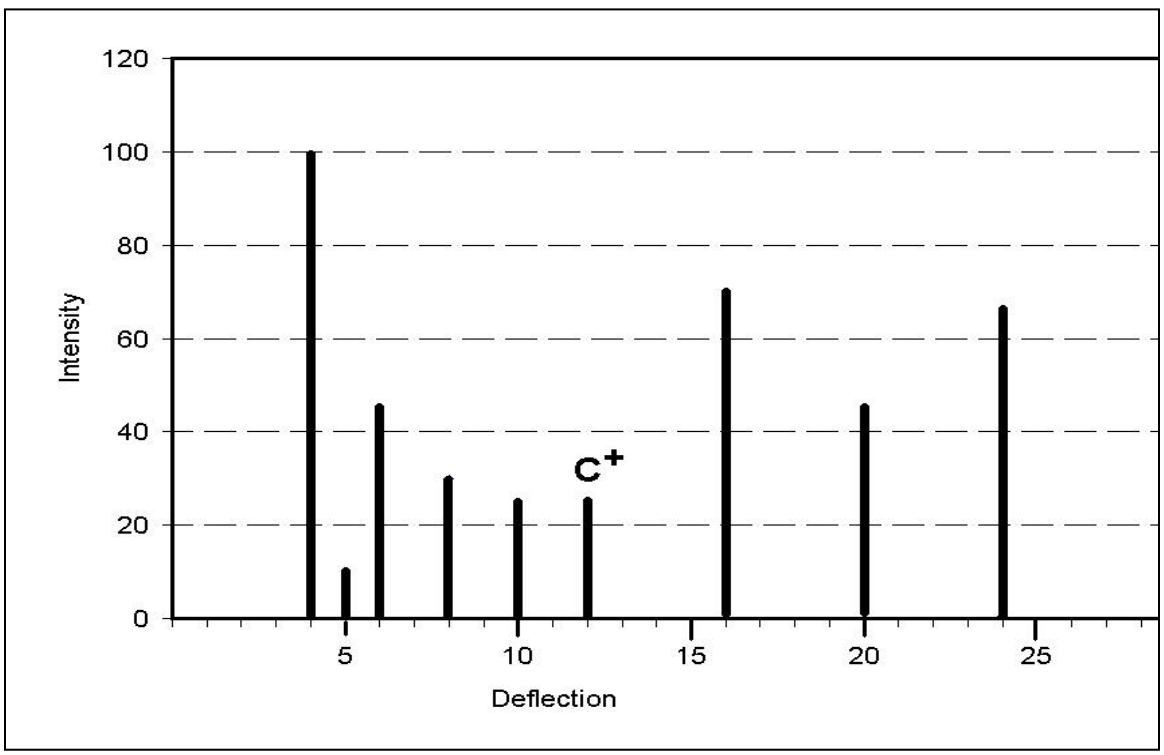
The IMAGE spacecraft uses an instrument called a mass spectrometer to identify the elements in the gases surrounding Earth, and trapped within Earth's magnetic field. The Detectives on the TV program CSI also use instruments like this to identify traces of substances found at the crime scene, or ingested by the victim. A mass spectrometer sorts out the atoms of matter by their mass. Scientists can then identify the kind of atom in the gas by their masses. Some instruments can detect a single atom from among trillions. Here's how the instrument works:

A mass spectrograph lets charged atoms (ions) pass through a magnetic field. As they travel, the ions are turned through a radius that depends on their mass and charge. Atoms with high masses are turned through a larger radius. Atoms with low mass are turned through a smaller radius. For example, in a hypothetical mass spectrometer, a helium ion, He+, with only one electron (a charge of 1 unit) might be turned through a radius of 4 units, and a helium ion, H++, with both electrons lost (a charge of 2 units) would be turned through a radius of only 2 units. An ion of carbon, C+, with 3 times the mass of a helium atom, and with one missing electron (a charge of 1 unit) would be turned through a radius of 4 units x 3 = 12 units.

Question 1: From the above example, what radius would a carbon atom with 2 missing electrons be turned through in this instrument? Symbol = C^{++}

Question 2: From the above example, what radius would an ion of magnesium, Mg+, with one missing electron be turned through?

With a copy of the periodic table of the elements, the clues above, and the mass spectrum below with its one identified ion, C^{+}, identify the other elements found by the mass spectrometer. Are you ready to join the CSI Team?



The IMAGE spacecraft uses an instrument called a mass spectrometer to identify the elements in the gases surrounding Earth, and trapped within Earth's magnetic field. An ordinary prism or 'diffraction grating' takes light and sorts it into a spectrum from red to blue. A mass spectrometer, such as the HENA, MENA and LENA instruments on IMAGE, sorts out the atoms of matter passing through its window by their mass and charge. The kind of data you get from this instrument looks very much like a spectrum plot in which the spectral 'lines' you would see in a light spectrum from individual atoms is replaced by 'lines' that represent the masses of the atoms themselves. Scientists can then identify the kind of atom in the gas directly by its mass, rather than the light it emits. Note: The formula relating the mass (M) and charge (Q) of the ion to its turning radius (R) looks like this:

$$R^2 = \text{constant} \times M / Q$$

For example, in a hypothetical mass spectrometer, a helium ion with only one electron (Q= 1) might be turned through a radius of 4 units, and a helium atom with both electrons lost (a charge of Q=2) and written as He++ would be turned through a radius of 2 units. An atom of carbon with 3 times the mass of a helium atom, and with one missing electron (a charge of Q = 1) would be turned through a radius of 12 units. **Note: The 'units' are in terms of centimeters^2, but students don't have to know this to answer the questions.**

Question 1: From the above example, what radius would a carbon atom with 2 missing electrons (C++) be turned through in this instrument?
Answer: With one missing charge the radius is 12 units, with 2 missing electrons it will be 12 / 2 = 6 units.

Question 2: From the above example, what radius would an atom of magnesium with one missing electron be turned through?
Answer: Students will look up the atomic mass of magnesium and determine that it is twice the mass of a carbon atoms. Since, as for carbon, there is only one missing electron, Q=1, and so the radius is just 12 x 2 = **24 units.**