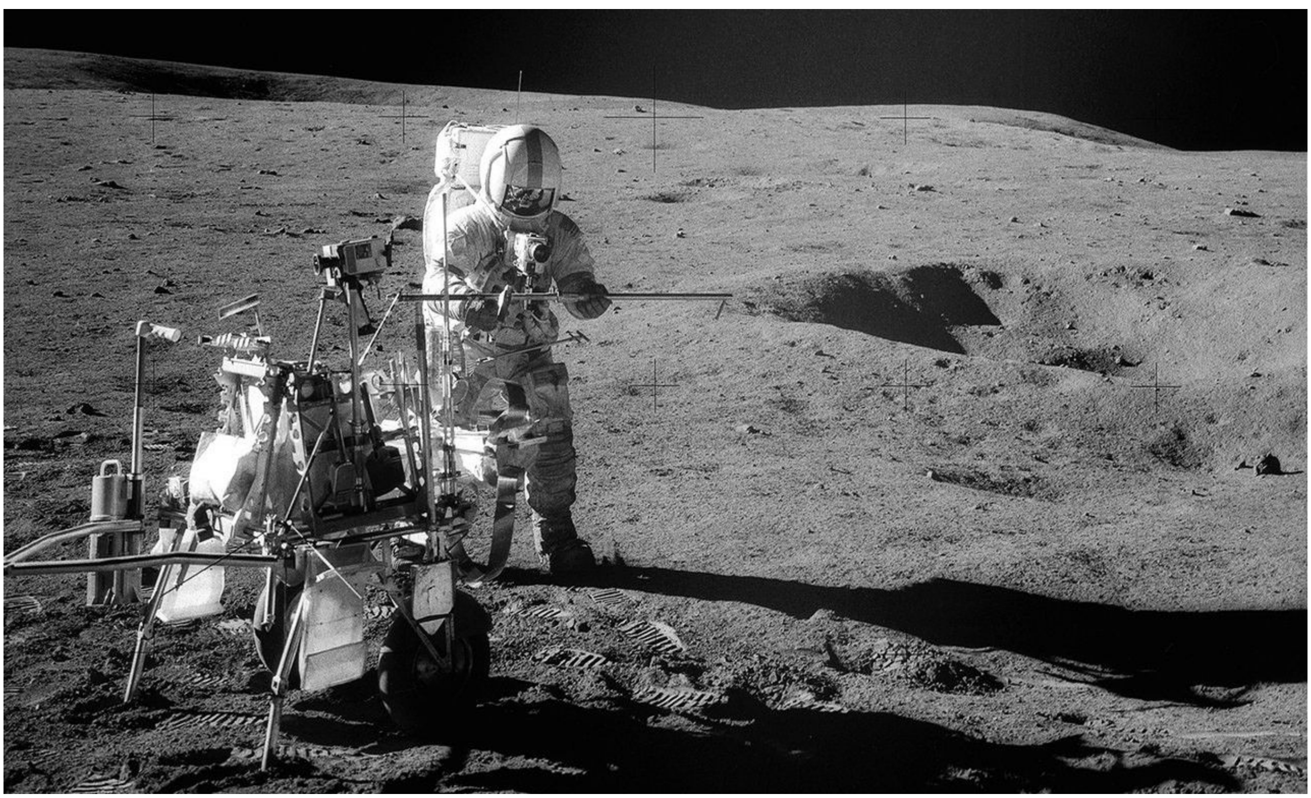
National Aeronautics and Space Administration

Math Problems Related to

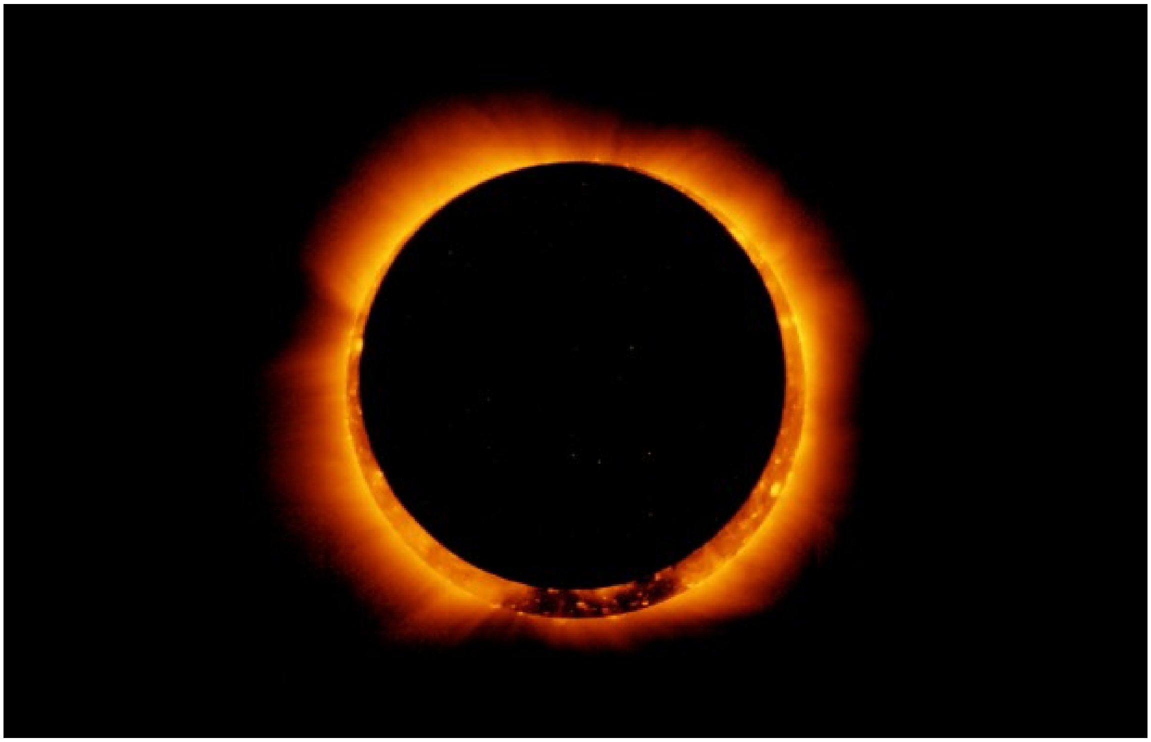
The Moon



Introduction

This book provides an assortment of mathematics problems that deal with our moon. Many of the problems have to do with using simple scaling rules to measure the diameters of craters on imagery of the lunar surface obtained by the Lunar Reconnaissance Orbiter and other spacecraft. Problems are also available for determining the density of the lunar atmosphere and modeling the interior of the moon. Of particular interest to astronomers is gauging the frequency of crater sizes, which can be used to determine the ages of various lunar surfaces. A calculation is also provided for estimating how often an astronaut might expect to be struck by a lunar meteorite since there is no atmosphere to otherwise shield astronauts from this hazard.

X-Ray Satellite Observes a Solar Eclipse from Space



On May 10, 2013, the sun experienced what's called an annular eclipse. This happens when the moon moves directly in front of the sun, but doesn't completely cover the face of the sun. This leaves a thin, fiery ring, called the annulus, visible around the outside. This eclipse was only visible from the South Pacific, Australia, Papua New Guinea, the Solomon Islands and the Gilbert Islands.

The picture shown above was taken by the Hinode X-ray satellite on Jan. 4, 2011 of a previous annular solar eclipse as seen from space. The brilliant corona of the sun can be seen as it glows in X-ray light. The moon appears completely black in front of the sun's surface.

Problem 1 – About two solar eclipses can be seen somewhere on Earth each year. The ones in 2013 occur on May 10 and November 3. In 2014 a pair will occur on April 29 and October 23. The two eclipses in 2015 occur on May 20 and September 13. The one on October 23, 2014 will be visible from North America. On average, about how many days separate the 6 solar eclipses?

Problem 2 – Solar eclipses happen because the size of the Moon from Earth's surface appears the same size as the Sun's disk. If the Sun is located 400 times farther from Earth than the Moon, and the Moon has a diameter of 3,500 kilometers, what is the diameter of the Sun? (Hint: Use a simple proportion)

Annular Eclipse on May 10

May 10, 2013

http://www.nasa.gov/topics/solarsystem/features/2013-annular.html

Problem 1 – About two solar eclipses can be seen somewhere on Earth each year. The ones in 2013 occur on May 10 and November 3. In 2014 a pair will occur on April 29 and October 23. The two eclipses in 2015 occur on May 20 and September 13. The one on October 23, 2014 will be visible from North America. On average, about how many days separate the 6 solar eclipses?

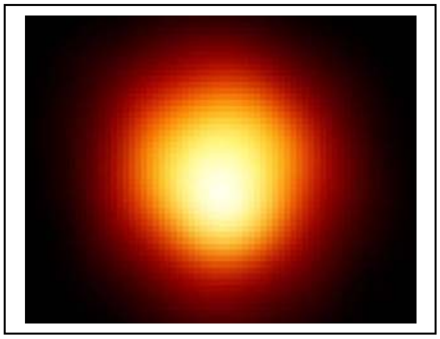
Answer: 178 days, 178 days, 178 days, 210 days, 107 days. **Average = 170 days.**

Problem 2 – Solar eclipses happen because the size of the Moon from Earth's surface appears the same size as the Sun's disk. If the Sun is located 400 times farther from Earth than the Moon, and the Moon has a diameter of 3,500 kilometers, what is the diameter of the Sun? (Hint: Use a simple proportion)

Answer:

$$\frac{\text{Solar Diameter}}{\text{Lunar Diameter}} = \frac{400}{1}$$ so Solar Diameter = 400 x Lunar Diameter = **1,400,000 km**

The relative sizes of the sun and stars



Stars come in many sizes, but their true appearances are impossible to see without special telescopes. The image to the left was taken by the Hubble Space telescope and resolves the red supergiant star Betelgeuse so that its surface can be just barely seen. Follow the number clues below to compare the sizes of some other familiar stars!

Problem 1 - The sun's diameter if 10 times the diameter of Jupiter. If Jupiter is 11 times larger than Earth, how much larger than Earth is the Sun?

Problem 2 - Capella is three times larger than Regulus, and Regulus is twice as large as Sirius. How much larger is Capella than Sirius?

Problem 3 - Vega is 3/2 the size of Sirius, and Sirius is 1/12 the size of Polaris. How much larger is Polaris than Vega?

Problem 4 - Nunki is 1/10 the size of Rigel, and Rigel is 1/5 the size of Deneb. How large is Nunki compared to Deneb?

Problem 5 - Deneb is 1/8 the size of VY Canis Majoris, and VY Canis Majoris is 504 times the size of Regulus. How large is Deneb compared to Regulus?

Problem 6 - Aldebaran is 3 times the size of Capella, and Capella is twice the size of Polaris. How large is Aldebaran compared to Polaris?

Problem 7 - Antares is half the size of Mu Cephi. If Mu Cephi is 28 times as large as Rigel, and Rigel is 50 times as large as Alpha Centauri, how large is Antares compared to Alpha Centauri?

Problem 8 - The Sun is 1/4 the diameter of Regulus. How large is VY Canis Majoris compared to the Sun?

Inquiry: - Can you use the information and answers above to create a scale model drawing of the relative sizes of these stars compared to our Sun.

Answer Key

The relative sizes of some popular stars is given below, with the diameter of the sun = 1 and this corresponds to an actual physical diameter of 1.4 million kilometers.

Betelgeuse	440	Nunki	5	VY CMa	2016	Delta Bootis	11
Regulus	4	Alpha Cen	1	Rigel	50	Schedar	24
Sirius	2	Antares	700	Aldebaran	36	Capella	12
Vega	3	Mu Cephi	1400	Polaris	24	Deneb	252

Problem 1 - Sun/Jupiter = 10, Jupiter/Earth = 11 so Sun/Earth = 10 x 11 = **110 times.**

Problem 2 - Capella/ Regulus = 3.0, Regulus/Sirius = 2.0 so Capella/Sirius = 3 x 2 = **6 times.**

Problem 3 - Vega/Sirius = 3/2 Sirius/Polaris=1/12 so Vega/Polaris = 3/2 x 1/12 = **1/8 times**

Problem 4 - Nunki/Rigel = 1/10 Rigel/Deneb = 1/5 so Nunki/Deneb = 1/10 x 1/5 = **1/50.**

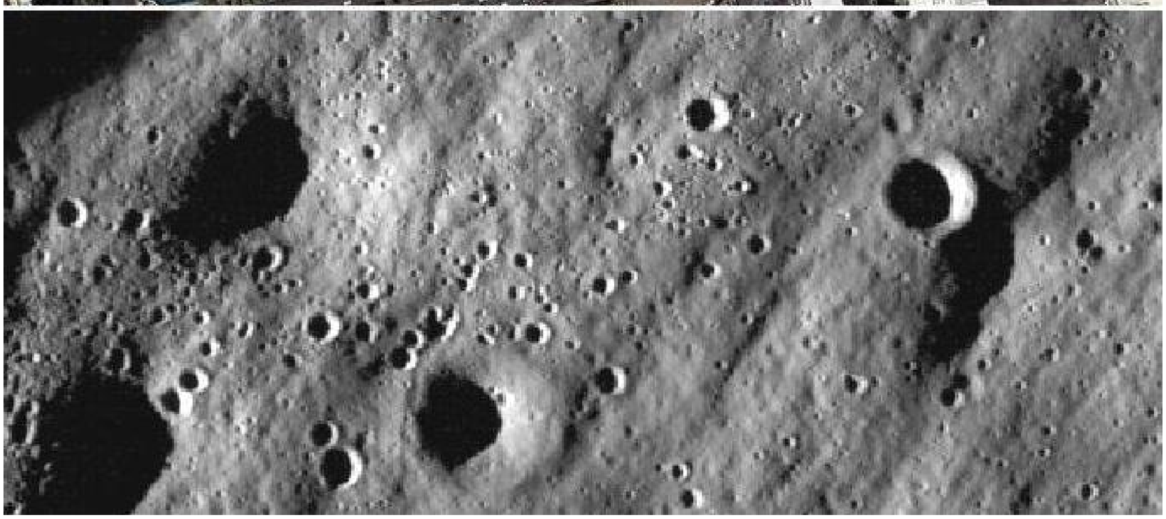
Problem 5 - Deneb/VY = 1/8 and VY/Regulus = 504 so Deneb/Regulus = 1/8 x 504 = **63 times**

Problem 6 - Aldebaran/Capella = 3 Capella/Polaris = 2 so Aldebaran/Polaris = 3 x 2 **= 6 times**.

Problem 7 - Antares/Mu Cep = 1/2 Mu Cep/Rigel = 28 Rigel/Alpha Can = 50, then Antares/Alpha Can = 1/2 x 28 x 50 = **700 times**.

Problem 8 - Regulus/Sun = 4 but VY CMA/Regulus = 504 so VY Canis Majoris/Sun = 504 x 4 = **2016 times the sun's size!**

Inquiry: Students will use a compass and millimeter scale. If the diameter of the Sun is 1 millimeter, the diameter of the largest star VY Canis Majoris will be 2016 millimeters or about 2 meters!



The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above 700-meter wide image shows Downtown Las Vegas, Nevada (Top - Courtesy of Digital Globe, Inc.), and Mare Nubium (bottom - LRO) at this same resolution.

Problem 1 - About how big, in meters, are the large, medium and small-sized craters in the LRO image?

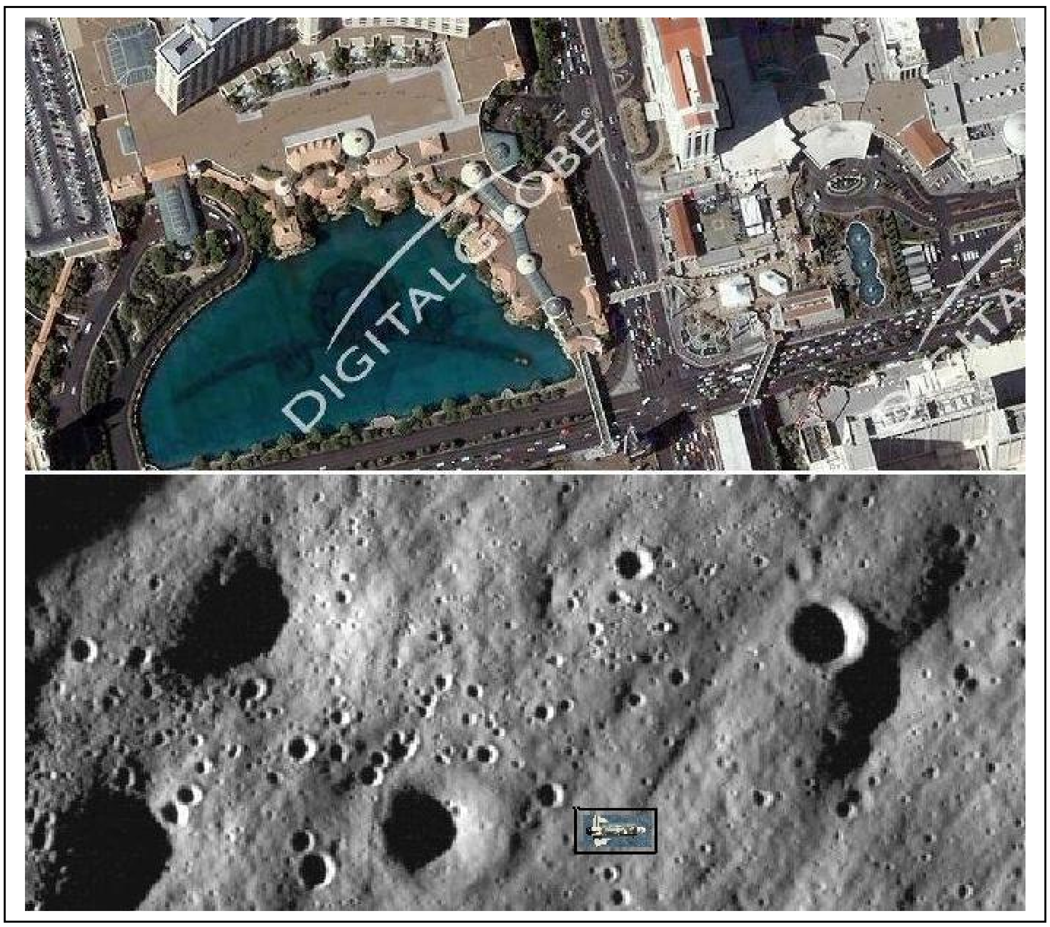
Problem 2 - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood?

Problem 3 - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!).

Problem 1 - About how big, in meters, are the large, medium and small-sized craters in the LRO image? Answer: the image is 153 millimeters wide so the scale is 700 meters/153 mm = 4.6 meters/mm. Small craters are about 4-5 meters across; medium craters are about 10 to15 meters across, and the few large craters are about 30 to 100 meters across.

Problem 2 - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood? Answer: The small craters are about as wide as your car, mini-van or street. The medium craters are about as wide as large as your house. The big craters are as big as your entire yard or a large Boulevard.

Problem 3 - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!). Answer: The shuttle would be 37 meters/4.6 M/mm = 8 millimeters long by 24/4.6 = 5.2 millimeters wide. The figure below shows the Shuttle to the same scale as the LRO image.



The Oldest Lunar Rocks

Apollo astronauts recovered over 840 pounds of lunar rocks, and during the last 30 years, these have been carefully studied to find out which features came first, and the ancient history of the lunar surface including its formation. The table below shows the ages (in millions of years: Myr) of some of the mineral specimens determined by geologist using various radioisotope methods applied to the different rock samples.

Location	Mission	Rock Type	Age (Myr)
Mare Tranquillitatis	Apollo-11	Basalt	3,500
Oceanus Procellarum	Apollo-12	Basalt	3,200
Fra Mauro Formation	Apollo-14	Basalt	4,150
	Apollo-14	Impact melts	3,850
	Apollo-14	KREEP	4,420
Mare Imbrium	Apollo-15	Basalt	3,250
	Apollo-15	Anorthosite	4,090
Descartes Mountains	Apollo-16	Breccia	3,980
	Apollo-16	Basalt	3,790
	Apollo-16	Anorthosite	4,470
	Apollo-16	Plagioclase-old	4,560
	Apollo-16	Plagioclase-young	4,290
Taurus-Littrow Mountains	Apollo-17	Zircon -old	4,418
	Apollo-17	Zircon -Young	4,331
	Apollo-17	Basalt	3,750
	Apollo-17	Impact melts	3,900

Problem 1 - What is the average age of all of the samples in the table in millions of years?

Problem 2 - What is the average age of all of the samples in the table to the nearest 0.1 billion years?

Problem 3 - Order the samples from oldest to youngest age. About how many different groups can you identify in terms of their similar ages, and what is the average age of each group of samples in billions of years?

Problem 4 - To the nearest 100 million years, about how old are the lunar mare (dark regions on moon - basalts) compared to the mountainous highland regions (KREEP, anorthosites, zircons and plagioclase) in billions of years?

Answer Key

Problem 1 - Answer: Add the 16 numbers in the last column together and divide by 16 to get 63,949 million years/16 = 3,997 million years.

Problem 2 -Answer: Add the 16 numbers in the last column together and divide by 16 to get 63,949/16 = 3,997 million years, which equals 3.997 billion years which needs to be rounded to 4.0 billion years.

Problem 3 - The samples ordered from oldest to youngest age:

Descartes Mountains	Apollo-16	Plagioclase-old	4,560
Descartes Mountains	Apollo-16	Anorthosite	4,470
Fra Mauro Formation	Apollo-14	KREEP	4,420
Taurus-Littrow Mountains	Apollo-17	Zircon -old	4,418
Taurus-Littrow Mountains	Apollo-17	Zircon -Young	4,331
Descartes Mountains	Apollo-16	Plagioclase-young	4,290
Fra Mauro Formation	Apollo-14	Basalt	4,150
Mare Imbrium	Apollo-15	Anorthosite	4,090
Descartes Mountains	Apollo-16	Breccia	3,980
Taurus-Littrow Mountains	Apollo-17	Impact melts	3,900
Fra Mauro Formation	Apollo-14	Impact melts	3,850
Descartes Mountains	Apollo-16	Basalt	3,790
Taurus-Littrow Mountains	Apollo-17	Basalt	3,750
Mare Tranquillitatis	Apollo-11	Basalt	3,500
Mare Imbrium	Apollo-15	Basalt	3,250
Oceanus Procellarum	Apollo-12	Basalt	3,200

About how many different groups can you identify in terms of their similar ages, and what is the average age of each group of samples in billions of years? (One possibility is shaded)

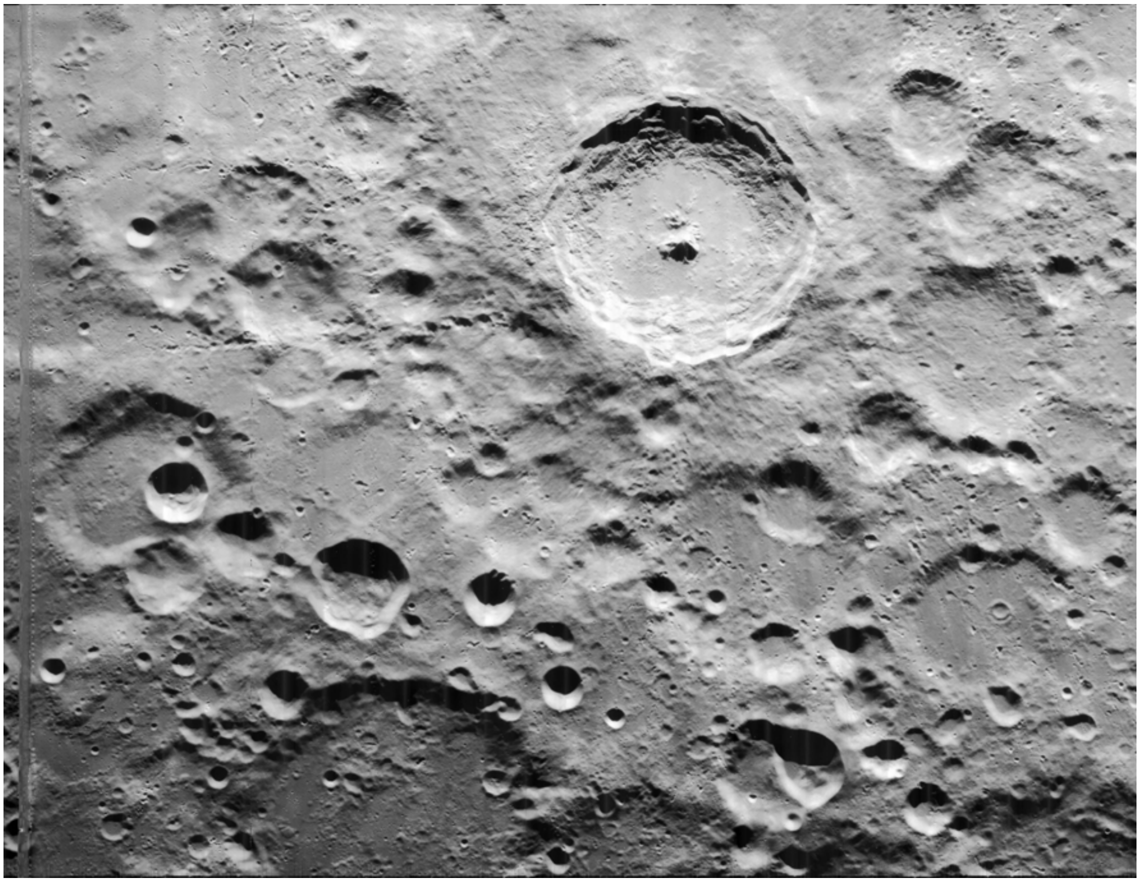
Oldest Group: (4.56+4.47+4.42+4.418+4.331)/5 = 4.4 billion years.
Middle Group: (4.29+4.15+4.09+3.98+3.9+3.85)/6 = 4.0 billion years
Young Group: (3.79+3.75+3.5+3.25+3.2)/5 = 3.5 billion years.

Problem 4 - To the nearest 100 million years, about how old are the lunar mare (dark regions on moon - basalts) compared to the mountainous highland regions (KREEP, anorthosites, zircons and plagioclase) in billions of years?

Answer: The lunar basalts are mostly in the younger group with an age of 3.5 billion years. The highland samples have ages (4.56 + 4.47 + 4.42 + 4.418 + 4.331 + 4.290 + 4.090)/7 = 4.37 billion years, or rounded to the nearest 0.1 you get 4.4 billion years.

Note: The highland materials sampled by the Apollo astronauts are older than the mare samples by nearly 1 billion years, and represent the ancient, and very old, original crust of the moon before volcanism filled the mare with lava during the first billion years after lunar formation.

Craters on the Moon - I



This is a NASA image taken by the Lunar Orbiter IV spacecraft as it captured close-up images of the lunar surface in May, 1967. The large crater at the top-center is Tycho. Other images from the Lunar Orbiter spacecrafts can be found at the Lunar Orbiter Photo Gallery (http://www.lpi.usra.edu/resources/lunarorbiter/) The satellite was at an altitude of 3,000 kilometers when it took this image, which measures 350 km x 270 km.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 350 kilometers x 270 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What is the diameter of the crater Tycho in kilometers?
Question 2: How large is the smallest feature you can see?
Question 3: How large are some of the smaller hills at the floor of the crater, in meters?
Question 4: About how large are the most common craters in the field?
Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?

Answer Key:

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?
Answer: 150 millimeters.

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.
Answer: 350 kilometers.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.
Answer: 350 kilometers / 150 millimeters = 2.3 kilometers / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

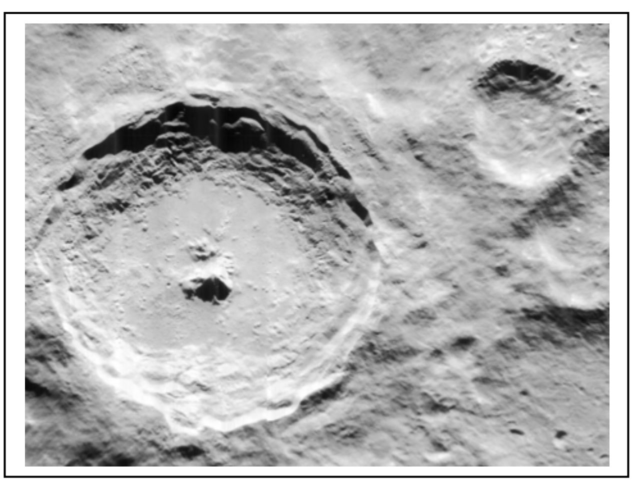
Question 1: What is the diameter of the crater Tycho in kilometers?
Answer: About 35 millimeters x 2.3 km/mm = 80.5 kilometers in diameter which is 80 kilometers to two significant figures.

Question 2: How large is the smallest feature you can see?
Answer: There are many small details in the image, pits, hills, etc, that students can estimate 0.1 to 0.3 millimeters for a physical size of 0.2 to 0.7 kilometers since the measurement is only good to one significant figure.

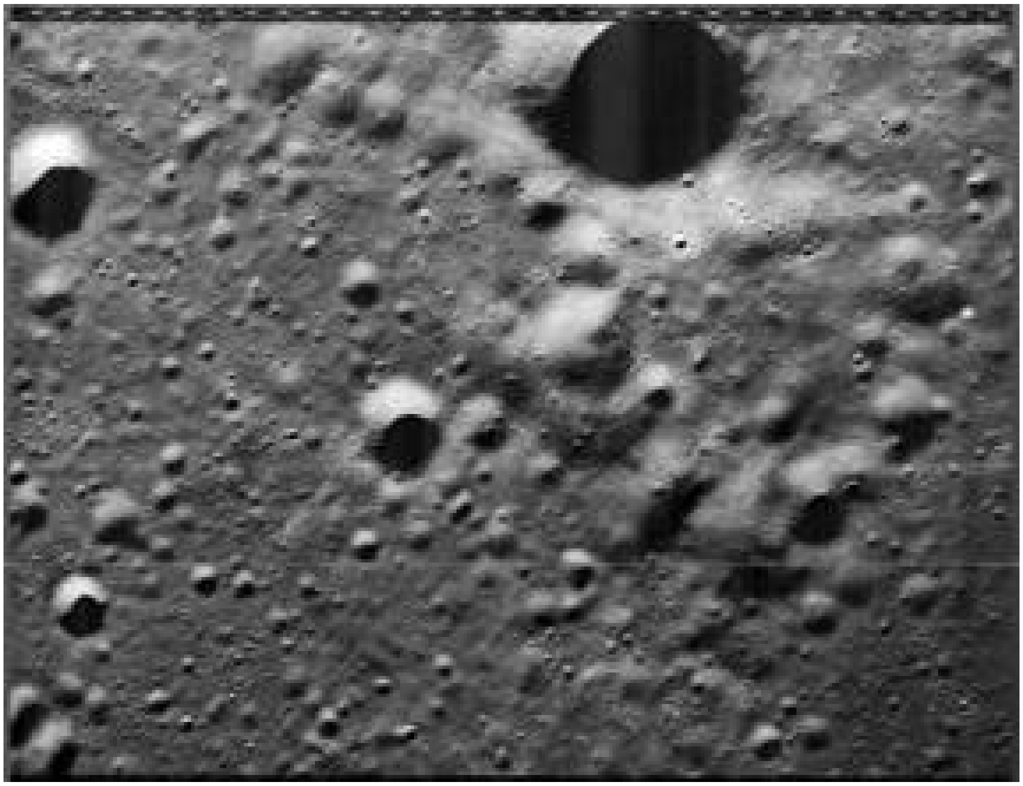
Question 3: How large are some of the smaller hills at the floor of the crater, in meters?
Answer: These small features are about 0.1 millimeters across or 200 meters in size.

Question 4: About how large are the most common craters in the field?
Answer: The answer may vary a bit, but the small craters that are 0.5 millimeters across are the most common. These have a physical size of about 1 kilometer.

Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?
Answer: In order to fit Denver into one of these lunar craters, it will have to appear to be about 25 km x (1.0 millimeter/2.3 km) = 11 millimeters across. There are three caters just to the right of Tycho that are about this big. Students should not get 'lost' trying to exactly match up their estimate with a precise lunar feature. 'Close-enough' estimates are good enough! See below comparison as a guide.



Craters on the Moon



This is a high resolution image of the lunar surface taken by NASA's Lunar Orbiter III spacecraft in February 1967 as it orbited at an altitude of 46 kilometers. It is located near the lunar equator. The field of view is 16.6 kilometers x 4.1 kilometers. Additional Orbiter images can be found at the Lunar Orbiter Gallery (http:// www.lpi.usra.edu/resources/lunarorbiter/). Because of the low sun angle, craters look like circles that are half-black, half-white inside!

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

Question 1: How big is the largest crater in the image?
Question 2: How big is the smallest crater in the image, in meters?
Question 3: About what is the typical distance between craters in the image?
Question 4: How far would you have to walk between the largest, and next-largest craters?

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?
Answer: 134 millimeters.

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.
Answer: 16600 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.
Answer: 16600 meters / 134 millimeters = 124 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

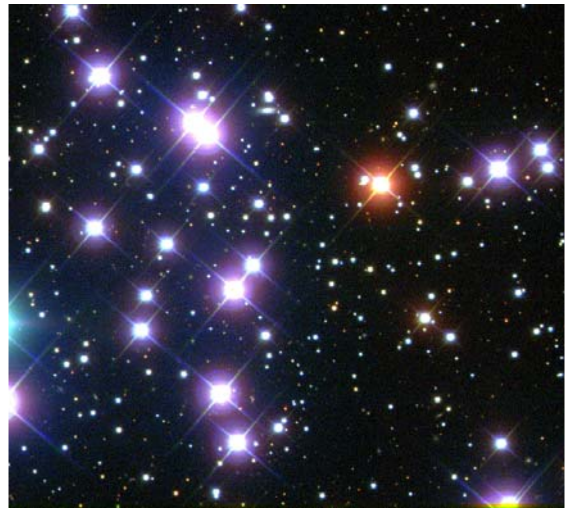
Question 1: How big is the largest crater in the image?
Answer: The one at the top is about 25 millimeters across. 25 mm x 124 meters/mm = 3,100 meters or 3.1 kilometers.

Question 2: How big is the smallest crater in the image, in meters?
Answer: The largest number of features are about 1 millimeter across or 100 meters to one significant figure. Students may also go for the recognizable craters which are about 2 millimeters across or about 200 meters.

Question 3: About what is the typical distance between craters in the image?
Answer: The answer may vary, but the distance between obvious craters (about 2 mm in diameter) is about 5 millimeters or 5 mm x 124 meters/mm = 600 meters to one significant figure.

Question 5: How far would you have to walk between the largest, and next-largest craters?
Answer: The crater rims are about 35 millimeters apart or 35 mm x 124 meters/mm = 4,340 meters or 4.3 kilometers to two significant figures.

Angular Size : The Moon and Stars



Although many astronomical objects may have the same angular size, most are at vastly different distance from Earth, so their actual sizes are very different. If your friends were standing 200 meters away from you, they would appear very small, even though they are as big as you are!

The pictures show the Moon (d = 384,000 km) and the star cluster Messier-34 (d = 1,400 light years). The star cluster photo was taken by the Sloan Digital Sky Survey, and although the cluster appears the same size as the Moon in the sky, its stars are vastly further apart than the diameter of the Moon!

In the problems below, round all answers to one significant figure.

Problem 1 - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter?

Problem 2 - The relationship between angular size, **Θ**, and actual size, **L**, and distance, **D**, is given by the formula:

$$L = \frac{\Theta}{206{,}265} D$$

Where Θ is measured in arcseconds, and L and D are both given in the same units of length or distance (e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years?

Problem 3 - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers?

Problem 4 - What is the smallest star separation you can measure in Messier-34 in among the brightest stars in A) arcseconds? B) Light years?

Answer Key

Problem 1 - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter? Answer: The diameter of the Moon is about 64 millimeters, and since this corresponds to 1,900 arcseconds, the scale is 1,900 asec/64 mm = 29.68 or 30 asec/mm.

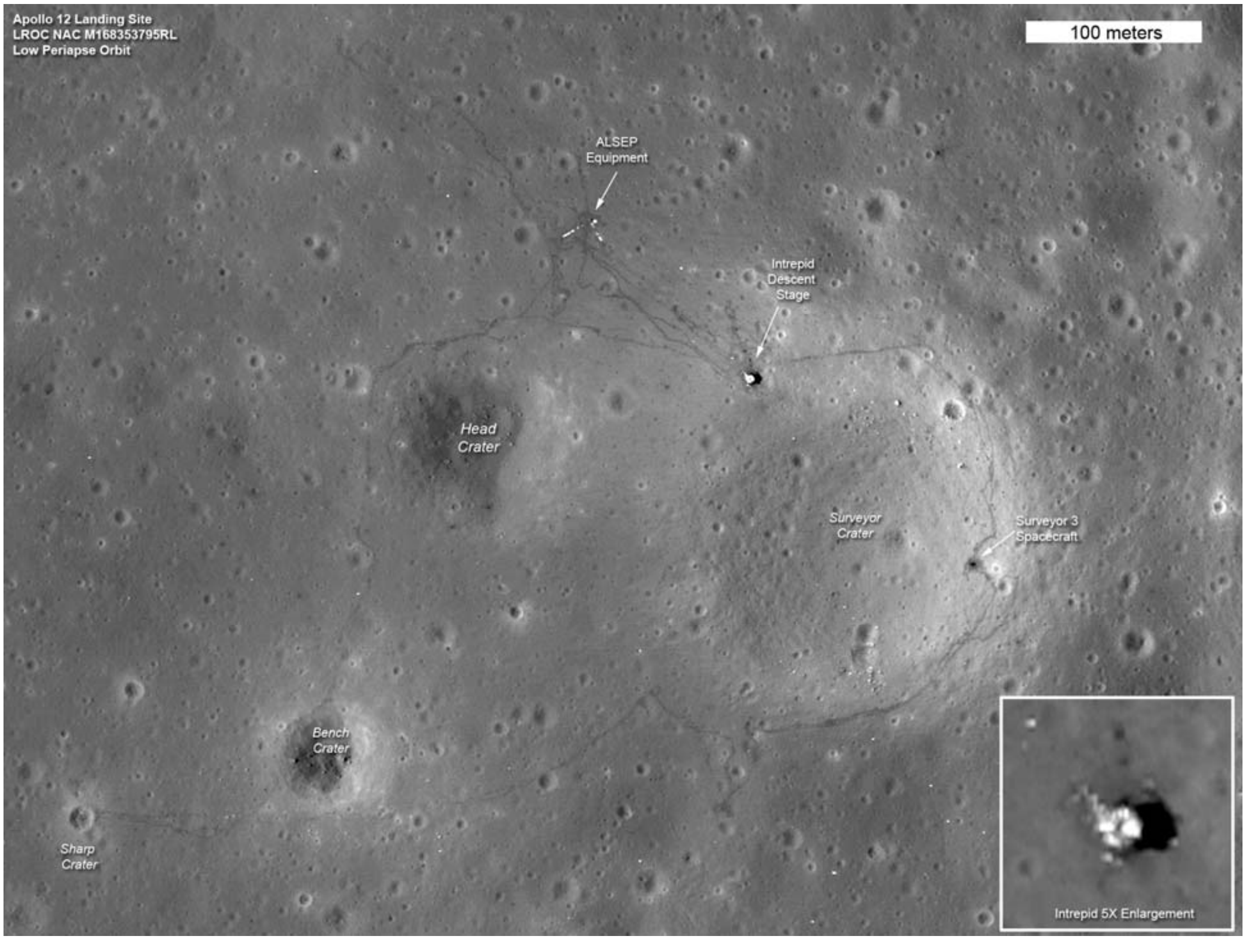
Problem 2 - The relationship between angular size, **Θ**, and actual size, **L**, and distance, **D**, is given by the formula:

$$L = \frac{\Theta}{206,265} D$$

Where Θ is measured in arcseconds, and L and D are both given in the same units of length or distance (e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years? Answer: A) For the Moon: L = 1 arcsec/206265 x (384,000 km) = 1.86 or 2.0 kilometers. B) For the cluster, L = 1 arcsec/206265 x (1,400 light years) = 0.007 light years.

Problem 3 - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers? Answer: A) About 1 millimeter, which corresponds to 1.0 arcsec. B) One arcsec corresponds to 2.0 kilometers.

Problem 4 - What is the smallest star separation you can measure in Messier-34 among the brightest stars in A) arcseconds? B) Light years? Answer: A) Students may find that some of the bright stars are about 3 millimeters apart, which corresponds to 3 mm x 30 asec/mm = 90 arcseconds. B) At the distance of the cluster, 1 asec = 0.007 light years, so 90 asec corresponds to 90 x (0.007 light years/asec) = 0.63 or 0.6 light years to 1 significant figure.



NASA's Lunar Reconnaissance Orbiter (LRO) from a lunar orbit of 21 kilometers (13 miles) captured the sharpest images ever taken from space of the Apollo 12 landing site. Images show the twists and turns of the paths made when the astronauts explored the lunar surface. One of the details that shows up is a bright L-shape in the Apollo 12 image. It marks the locations of cables running from the Apollo Lunar Surface Experiments Package (ALSEP) central station to two of its instruments. Although the cables are much too small for direct viewing, they show up because they reflect light very well.

Problem 1 – Following one of the walking paths, about how many meters did the astronauts have to walk from A) the ALSEP to the Descent Stage, and then around Surveyor Crater to finally reach the Surveyor spacecraft? B) The Surveyor spacecraft to Sharp Crater?

Problem 2 – Using your favorite method, about how many craters can you see across this entire area?

Problem 3 - If the craters were created over a period of about 3 billion years, about what may have been the average time between impacts to form the craters you see?

Problem 1 – Following one of the walking paths, about how many meters did the astronauts have to walk from A) the ALSEP to the Descent Stage, and then around Surveyor Crater to finally reach the Surveyor spacecraft? B) The Surveyor spacecraft to Sharp Crater?

Answer: Print out the problem on a typical laser printer and measure the '100 meter' bar with a millimeter ruler. An answer of about 23 millimeters yields an image scale of abouy 100 meters/23 mm = 4.3 meters/mm.

A) Using a piece of string or a millimeter ruler, measure the segments of the thin black 'track' that astronauts took. An answer of about 90 millimeters will be adequate. Using the image scale of 4.3 meters/mm you will get a distance of 90 mm x (4.3 m/mm) = 387 meters. This can be rounded to **390 meters**.

B) Measuring the track segments, a string length of about 200 millimeters is adequate. From the scale factor, this equals a physical distance of 200 x 4.3 = **860 meters** traveled.

Problem 2 – Using your favorite method, about how many craters can you see across this entire area?

Answer: Divide the area into a grid of squares. Count the number of craters you can see in one square, and multiply by the total number of squares. For example, if you make the squares 40mm x 40mm, you can fit 4 columns and 3 rows. Selecting the one in the second column, first row, you can count about 80 craters (from 0.5 to 2 millimeters across on the image) so the total number of craters is about 80 x 12 = 960 craters. **Answers between 800 and 1100 are also reasonable estimates.**

Note: From the image scale, the most common craters range in size from 0.2 millimeters to 2 millimeters, which corresponds to an actual size between 0.9 meters and 8.6 meters.

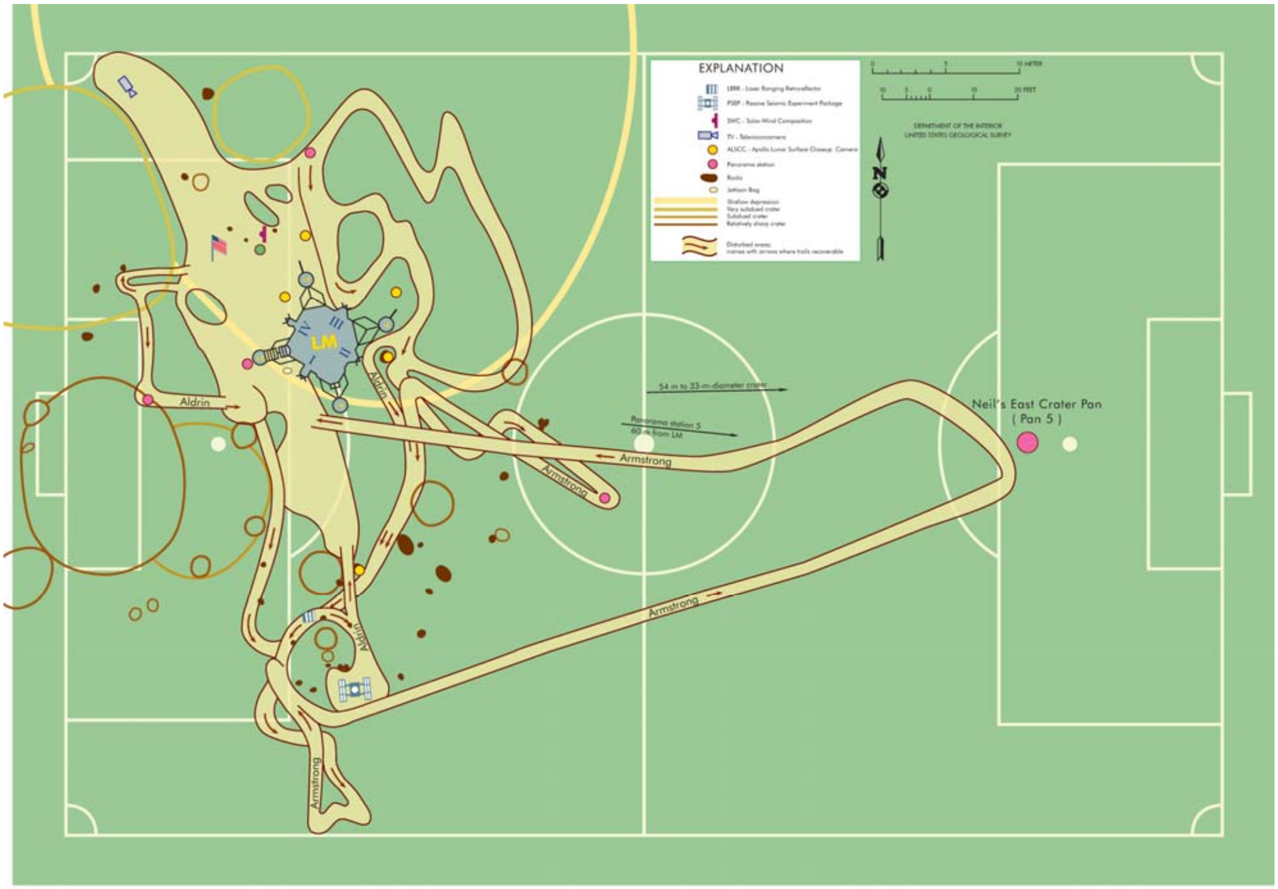
Problem 3 - If the craters were created over a period of about 3 billion years, about what may have been the average time between impacts to form the craters you see?

Answer: If we select 1000 craters as the average estimate, then the rate of cratering is about 1000 craters / 3 billion years or **1 crater every 3 million years**.

For more information about these images, see the NASA press release at:

NASA Spacecraft Images Offer Sharper Views of Apollo Landing Sites
Sep 6, 2011

http://www.nasa.gov/mission_pages/LRO/news/apollo-sites.html



The NASA Lunar Reconnaissance Orbiter (LRO), launched in 2009, will be able to see details on the lunar surface at much higher resolution than any previous lunar mapping mission. The images will have a resolution of about 0.7 meters per pixel. The map shows the area surrounding the Apollo 11 landing site where astronauts deployed experiments and walked around the landing site to gather rock and soil samples. The grid lines show a standard soccer field in comparison, with a distance between the white goal boundaries of 110 meters.

Problem 1 - Use a millimeter ruler and the information provided to determine the scale of the figure in meters per millimeter.

Problem 2 - Draw an overlay of 10 rows and 10 columns on the above figure at a location near the Apollo-11 landing area, with individual cells representing the individual LRO pixels.

Problem 3 - Will LRO be able to see: A) the Lunar Module 'LM' marked on the map? B) The discolorations (shown in yellow) of the lunar soil caused by the paths taken by the astronauts? C) Details on the LM?

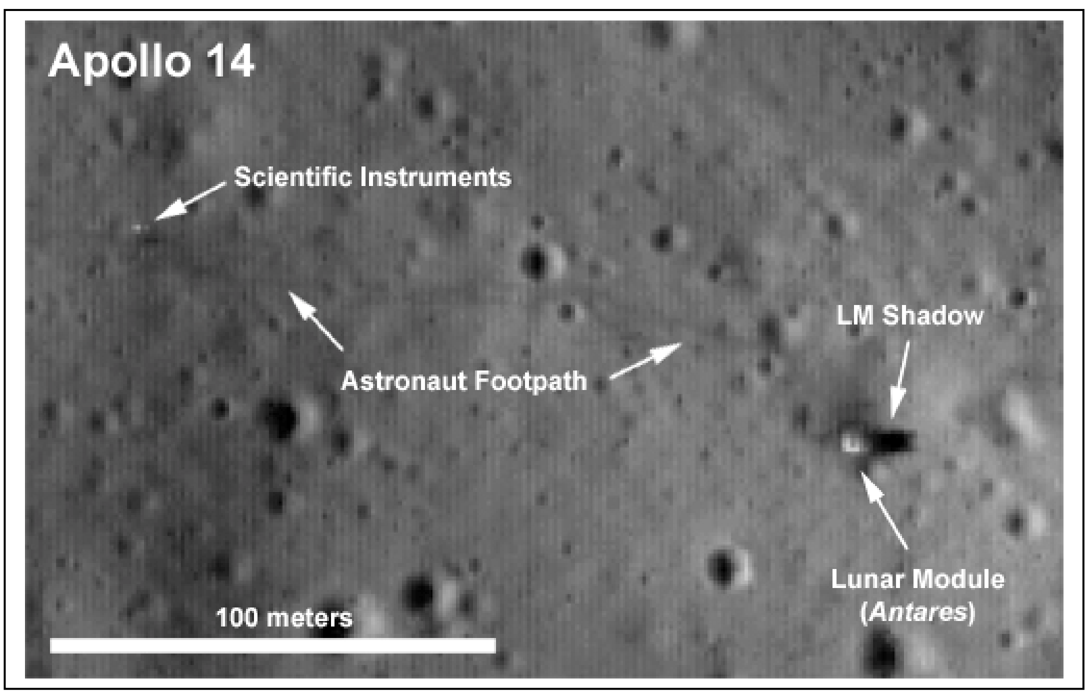
Problem 1 - Use a millimeter ruler and the information provided to determine the scale of the figure in meters per millimeter.

Answer: A millimeter ruler placed along the long-edge of the figure would indicate that the distance between the vertical white goal lines is 153 millimeters .Since this corresponds to 110 meters on the actual field, the scale of this figure is 110 meters/153 mm = 0.7 meters per millimeter. Alternately, students may measure the width of the green area which is 167 millimeters and corresponds to 120 meters.

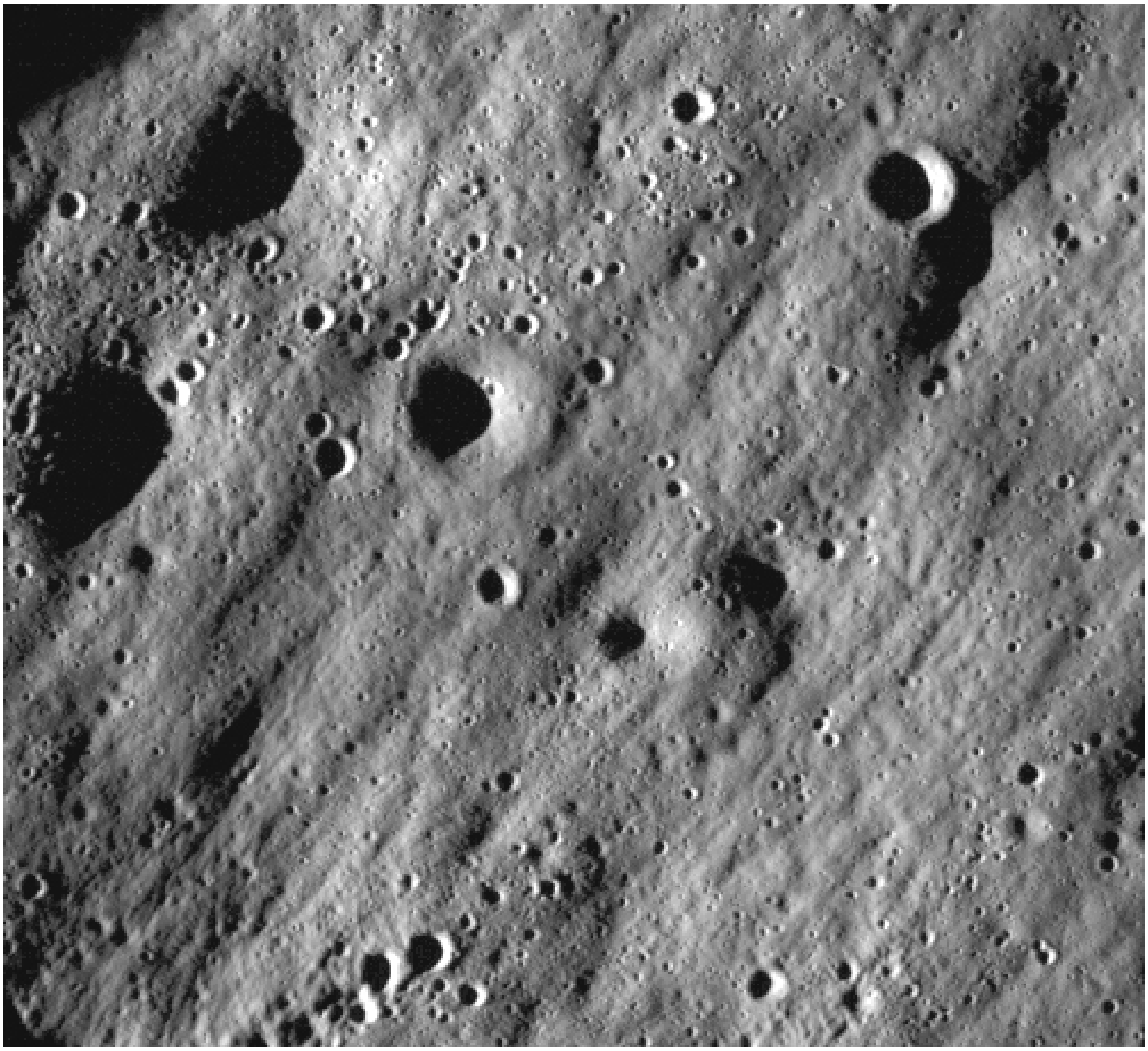
Problem 2 - Draw an overlay of 10 rows and 10 columns on the above figure at a location near the Apollo-11 landing area, with individual cells representing the individual LRO pixels. Answer: The LRO pixels are 0.7 meters wide, so the students would have to draw a grid with rows and columns 1 millimeter wide.

Problem 3 - Will LRO be able to see: A) The Lunar Module 'LM' marked on the map? B) The discolorations (shown in yellow) of the lunar soil caused by the paths taken by the astronauts? C) Details on the LM?

Answer: Below is an example of the details seen by the LRO cameras at the Apollo 14 landing site!



LRO's First Image of Mare Nubium



This is one of the first images taken by LRO showing details in Mare Nubium. The width of the image is 700 meters (500 pixels).

Problem 1 - Use a millimeter ruler to determine the scale of the image in meters per millimeter, and meters per pixel.

Problem 2 – What is the diameter, in meters, of the smallest recognizable crater you can find?

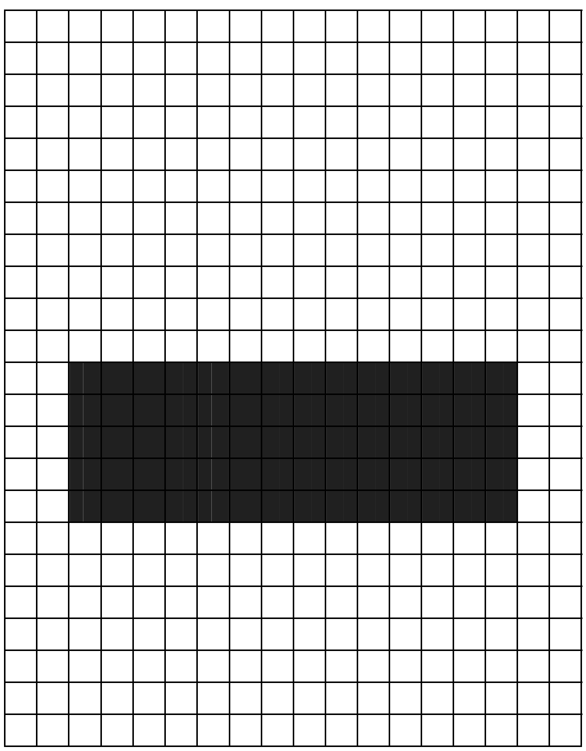
Problem 3 – Suppose your house is 42 feet wide and 60 feet long, and its sits on a property that is 75 feet wide and 96 feet long. Draw two squares at the same pixel scale as the LRO image. (Assume 1 meter = 3 feet)

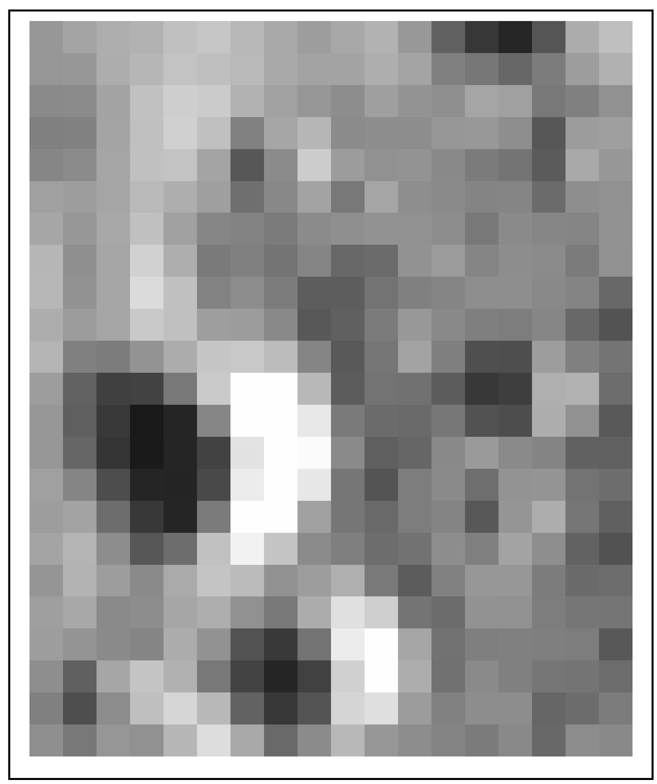
Answer Key

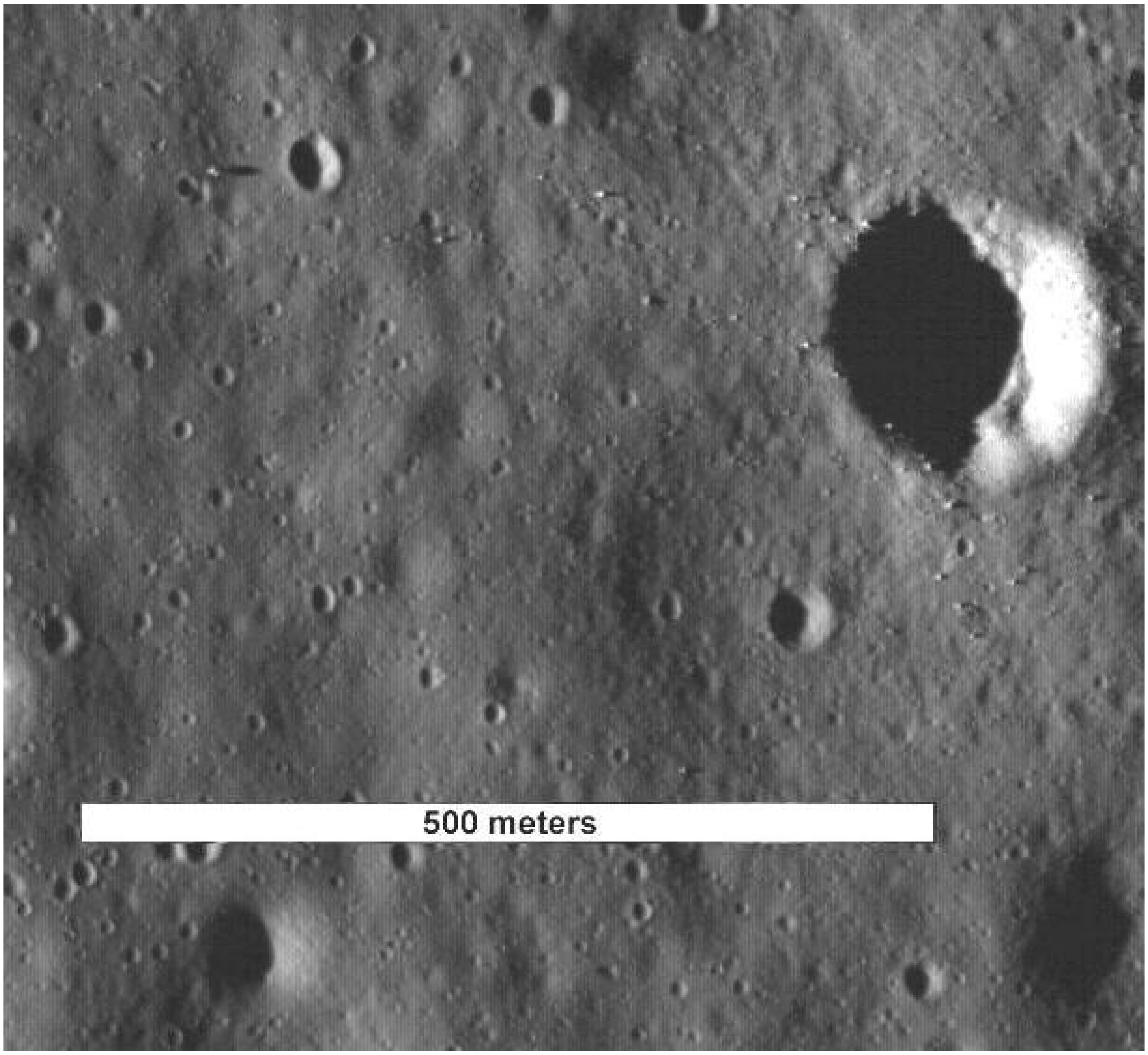
Problem 1 - Use a millimeter ruler to determine the scale of the image in meters per millimeter, and meters per pixel. Answer: Width = 153 millimeters so the scale is 700 meters/153 mm = 4.6 meters/mm, and 700 meters/500 pixels = 1.4 meters/pixel.

Problem 2 – What is the diameter, in meters, of the smallest recognizable crater you can find? Answer: Students should see craters as small as 0.5 millimeters or 0.5 mm x 4.6 m/mm = 2.3 meters.

Problem 3 – Suppose your house is 42 feet wide and 60 feet long, and its sits on a property that is 75 feet wide and 96 feet long. Draw two squares at the same pixel scale as the LRO image. Answer: First convert the feet into metric units. Three feet equals about 1 meter, so the yard measures 75 feet x 96 feet = 25 meters x 32 meters, and the house measures 7 meters x 20 meters. At the scale of the LRO image of 1.4 meters/pixel, the property is 18 pixels x 23 pixels, and the house measures 5 pixels x 14 pixels. See sketch below, and the comparison lunar image enlargement.







The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows the region near the Apollo-11 landing area. The Lunar Module (LM) and its shadow are shown to the left of the large crater in the upper right corner. Sunlight comes from the left, and so craters will have their shadow zones on the left-hand side of their depressions. Objects above the surface, like the Apollo LM, will be bright on the left side, and have their right-side in shadow.

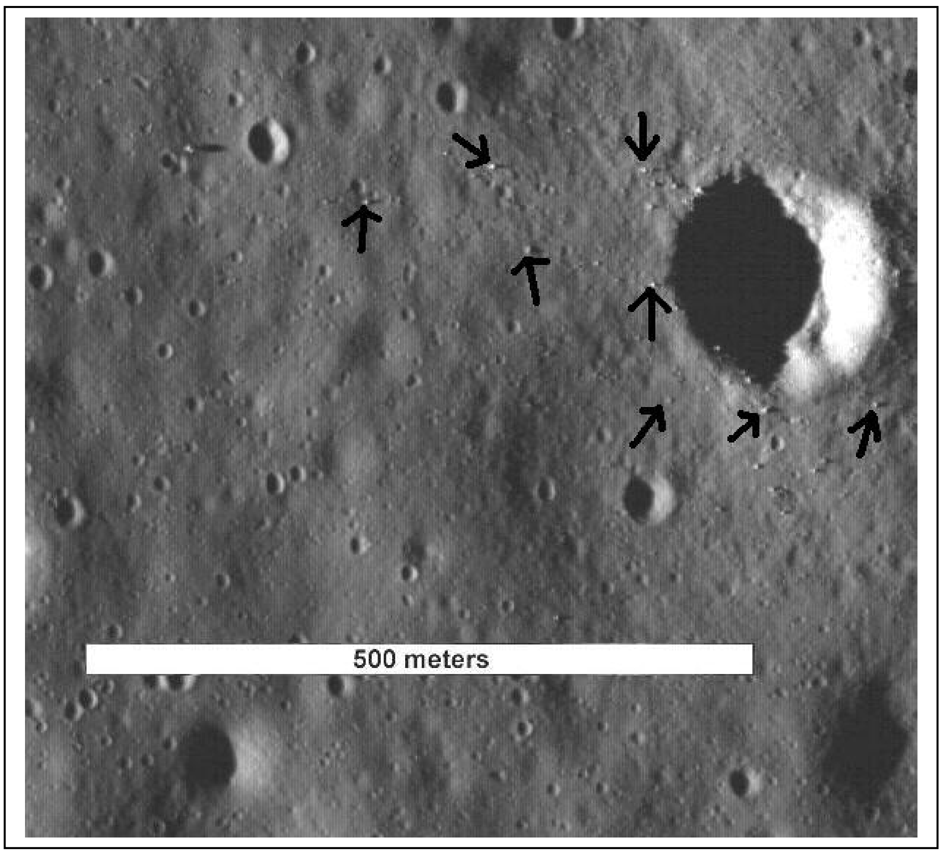
Problem 1 - From the information given, and using a millimeter ruler: A) determine the scale of the image (meters per millimeter); and B)the length of the Apollo LM shadow.

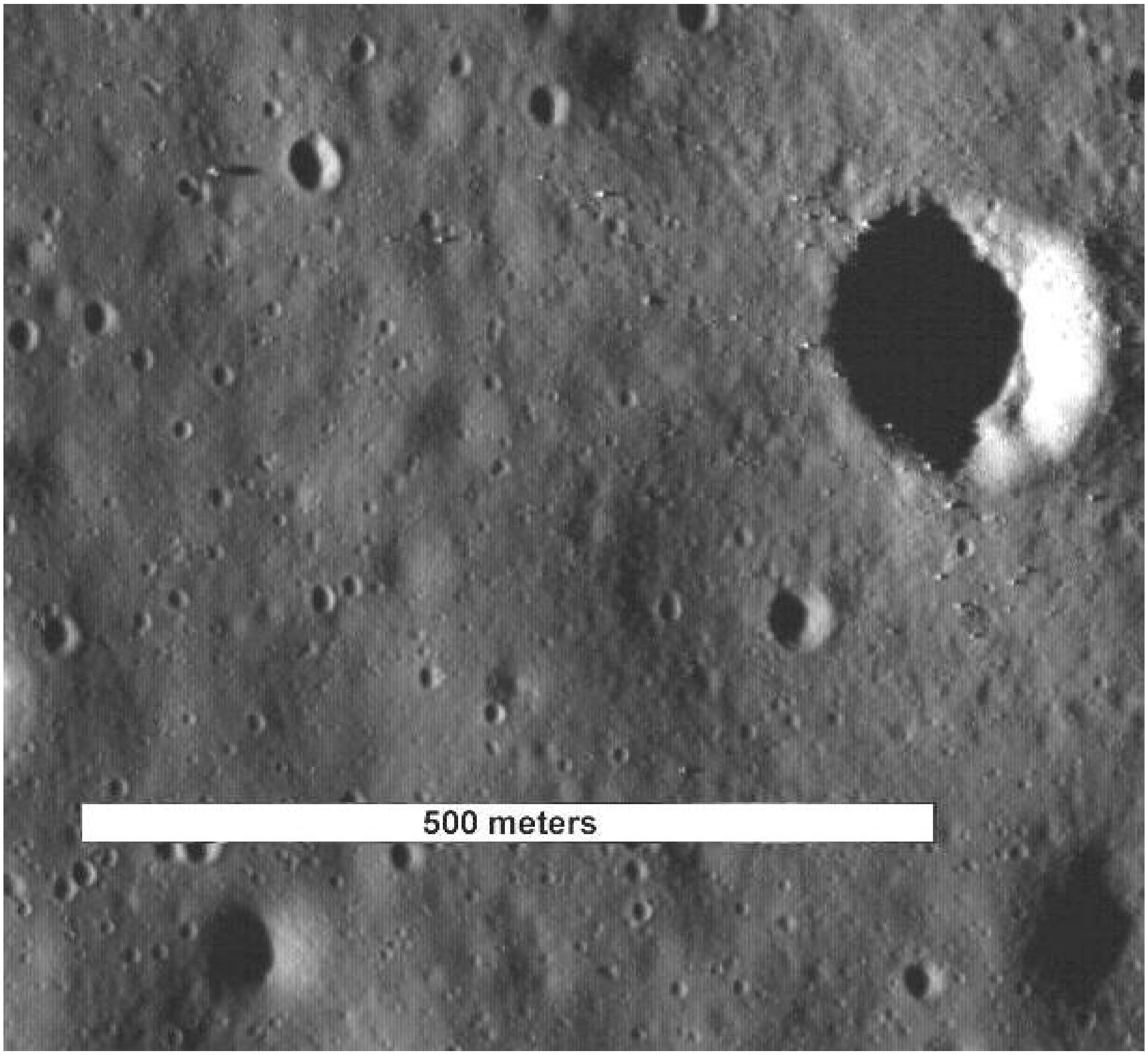
Problem 2 - Find as many boulders as you can, and determine their approximate size using the height of the LM (3.5 meters) and the length of the LM shadow to establish their sizes. Do you think there are smaller boulders that the ones you can easily spot?

Problem 1 - From the information given, and using a millimeter ruler: A) determine the scale of the image (meters per millimeter); and B) the length of the Apollo LM shadow.

Answer: Measure the length of the white bar on the image, which corresponds to a physical length of 500 meters on the lunar surface. Students should get answers near 111 millimeters. A) so the scale of the image is 4.5 meters/millimeter. The length of the LM shadow is 6 mm, so its physical length is 6 x 4.5 = 27 meters.

Problem 2 - Find as many boulders as you can, and determine their approximate size using the height of the LM (3.5 meters) and the length of the LM shadow to establish their sizes. Do you think there are smaller boulders that the ones you can easily spot?
Answer: The image below shows some examples. The LM is 3.5 meters tall and casts a 27-meter shadow, so by using similar triangles and proportions, that means that a 1-meter boulder will cast a shadow that is 1/3.5 x (27 meters) = 7.7 meters long. At the image scale of 4.5 meters/mm, that corresponds to a length on the image that is just under 2-mm long. Students may create tables for actual, numbered, boulders in the image and determine more accurate boulder sizes. Students should realize that, although they cannot directly see boulders smaller than about 1-pixel (1.4 meters) they can easily see the shadows of boulders much smaller than this. For example, a shadow that is 1 millimeter long is from an unobservable boulder about 0.5 meters across!





The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows a region near the Apollo-11 landing site. The Lunar Module (LM) can be seen from its very long shadow near the large crater in the upper left corner of the image.

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Problem 2 - Measure all of the craters larger than or equal to 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_c}$: the Areal Crater Density (craters/km^2).

Problem 3 - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_c}$. About what is the average distance between craters with a diameter close to 5 meters?

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Answer: The 500-meter bar is 111 millimeters long so the scale is 500 M/111mm = 4.5 meters/mm. The image has the dimensions of 149 mm x 136 mm or 670m x 612 m for an area of 0.41 $kilometers^2$.

Problem 2 - Measure all of the craters larger than 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_c}$: the Areal Crater Density ($craters/km^2$). Answer: The following table shows an example. Students bin intervals may differ.

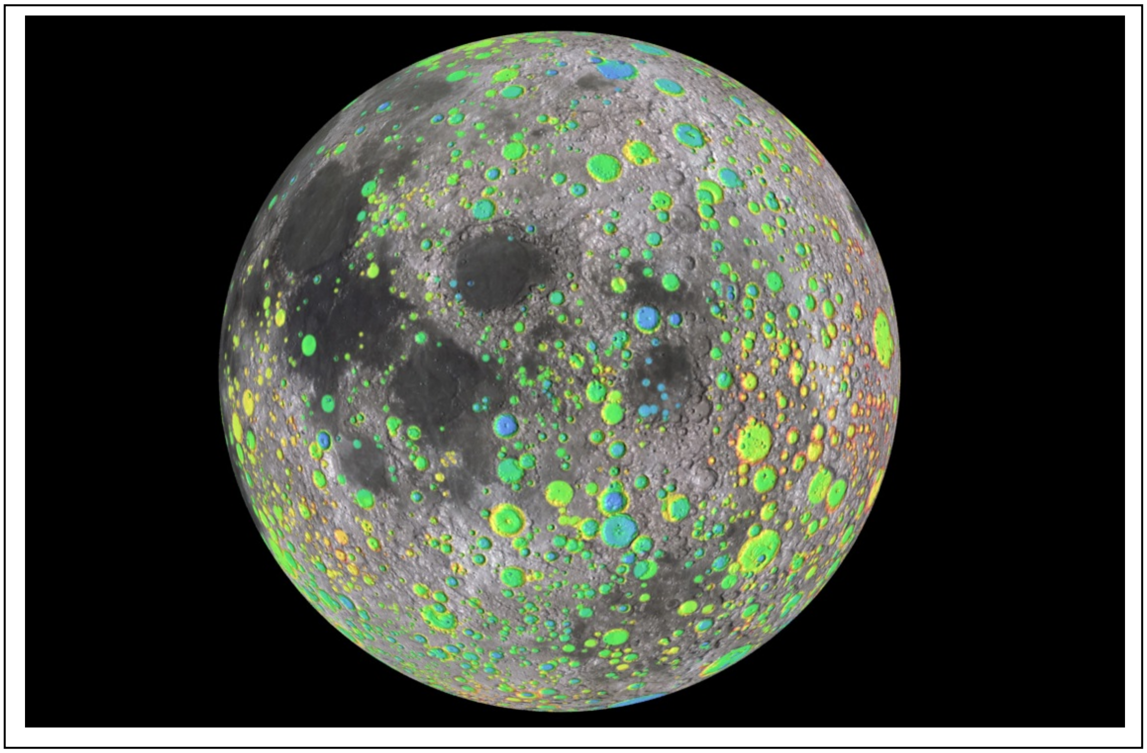
Crater diameter (mm)	Crater diameter (meters)	Number of craters close to this size	Areal Density	**Problem 3** Average distance in kilometers
2 mm	9	70	70/0.41 = 171	0.08
4 mm	18	6	15	0.25
6 mm	27	3	7	0.38
8 mm	36	2	5	0.44
10 mm	45	1	2	0.71

Students may extend this table to include craters of 1-mm diameter and also the single, very large crater that is 35mm in diameter. The number of counted craters, especially in the smallest bins, will vary. Student data may be averaged together to improve accuracy in each bin.

Problem 3 - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_c}$. About what is the average distance between craters with a diameter close to 5 meters?

Answer: See above table for tabulated values. Students may also convert the answers to meters. For example, '0.08 km' = 80 meters. Students will need to estimate the Areal Crater Density for craters just below the tabulated threshold of 9 meters. This can be done by estimating the shape of the plotted curve through the points, and extrapolating it to 5 meters. It is also possible to use Excel Spreadsheets by entering the data and plotting the 'scatter plot' with a trendline added. Reasonable values for the Areal Creater Density would range from 171 $craters/km^2$ to 1000 $craters/km^2$,which lead to distances between 80 meters and 30 meters, but probably closer to 30 meters given the rapidly decreasing trend of the curve based on the data in the bins for 18-meter and 9-meter crater diameters

LRO Determines Lunar Cratering History



The Lunar Reconnaissance Orbiter used millions of measurements of the lunar surface to establish the history of cratering on the surface.

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.
Answer: The image diameter is 90 millimeters so the scale is 3400 km/90 mm = **38 km/mm.**

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?
Answer: 70 km equals 2 millimeters at this image scale. There are about **56 craters larger than 2 mm on the image. Students answers may vary from 40 to 60.**

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?
Answer: You would expect to find about 0.2 x 56 = **11 craters larger than 70 km.**

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Answer: **The lunar highlands were present first and were impacted by asteroids until just before the maria formed. There are few/no craters in the maria regions larger than 70 km, so the maria formed after the episode of large impactors ended.**

For more information, see the LRO press release at:

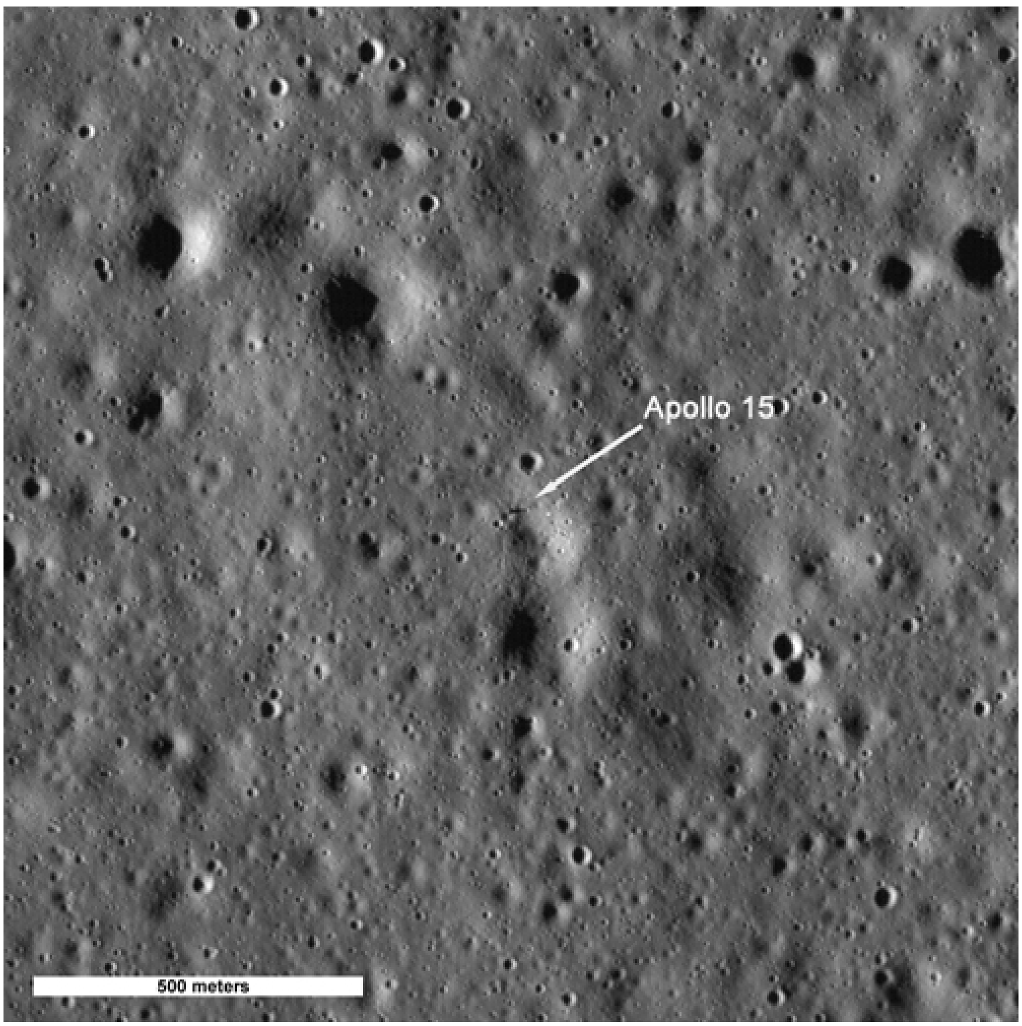
"LRO Exposes Moon's Complex, Turbulent Youth"

http://www.nasa.gov/mission_pages/LRO/news/turbulent-youth.html

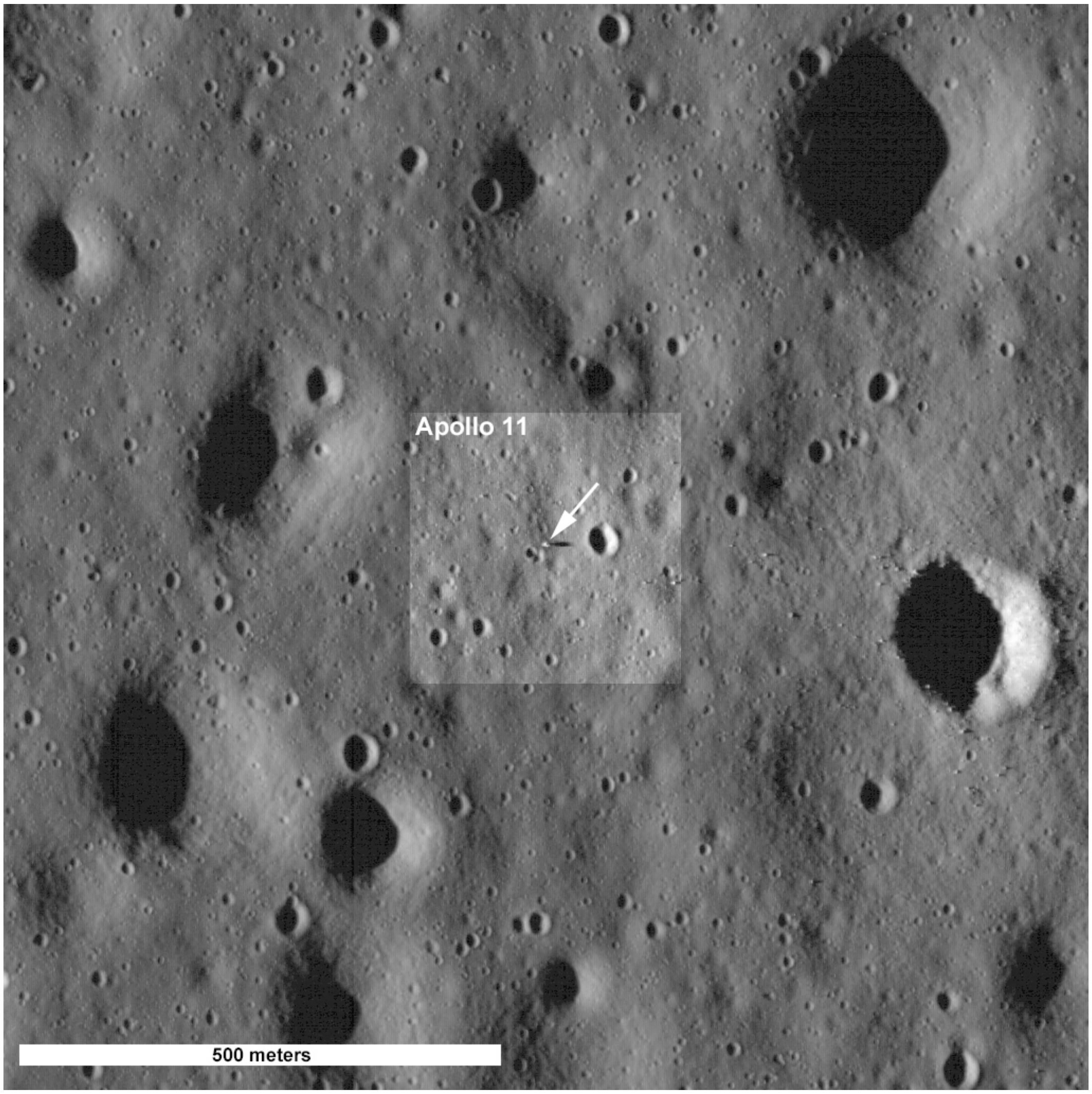
We have all sees pictures of craters on the moon. The images on the next two pages show close-up views of the cratered lunar surface near the Apollo 15 and Apollo 11 landing areas. They were taken by NASA's Lunar Reconnaissance Orbiter (LRO) from an orbit of only 25 kilometers!

Meteors do not arrive on the moon at the same rates. Very large meteors that produce the largest craters are much less common than the smaller bodies producing the smallest craters. That's because there are far more small bodies in space than large ones. Astronomers can use this fact to estimate the ages of various surfaces in the solar system by just comparing the number of large craters and small craters that they find in a given area.

Let's have a look the images below, and figure out whether Apollo-11 landed in a relatively younger or older region than Apollo 15!



This is the Apollo-15 landing area near the foot of the Apennine Mountain range. Note the bar indicating the 'scale' of the image. The arrow points to the location of the Lunar Descent Module.



This image taken by LRO is of the Apollo-11 landing area in Mare Tranquilitatus, with the arrow pointing to the Lunar Descent Module. The LDM was the launching platform for the Apollo-11 Lunar Excursion Module, which carried astronauts Neil Armstrong and Buzz Aldrin back to the orbiting Command Module for the trip back to Earth. Note the length of the '500 meter' bar, which gives an indication of the physical scale of the image. How long would it take you to walk 500 meters?

Astronomers assume that during the last 3 billion years following the so-called 'Late Heavy Bombardment Era' the average time between impacts that created craters has been constant. That means that the more time that passes, the more craters you will find, and that they are produced at a more or less steady number for each million years that passes. Also, by the Law of Superposition, younger craters lie on top of older craters.

Dating a cratered surface.

For each of the above images, perform these steps.

Step 1 - With the help of a millimeter ruler, and the '500 meter' line in the image, calculate the scale factor for the image in terms of meters per millimeter.

Step 2 - Calculate the total area of the image in square kilometers.

Step 3 - Identify and count all craters that are bigger than 20 meters in diameter.

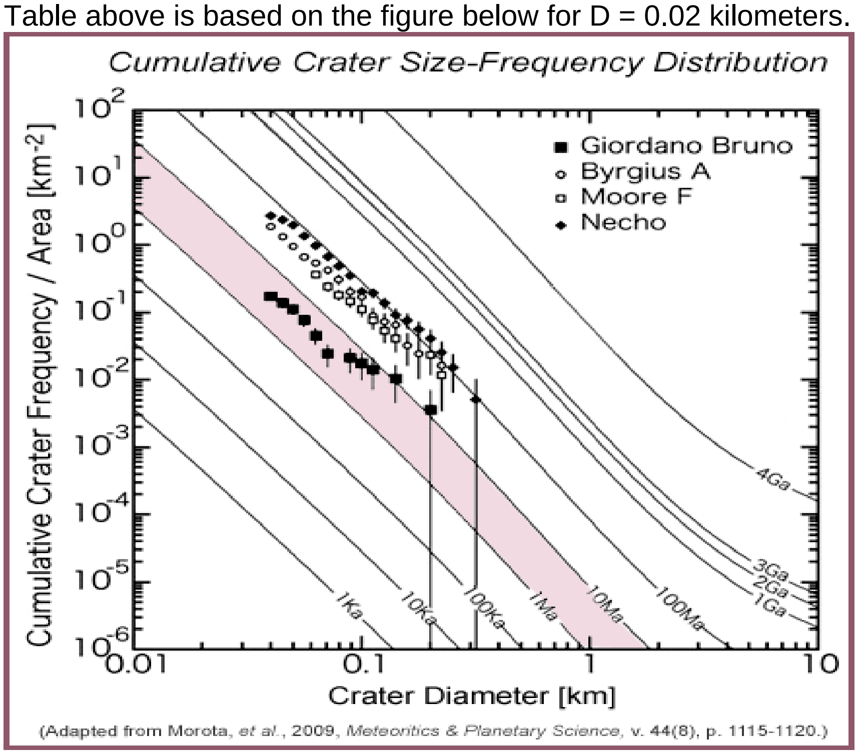
Step 4 - Divide your answer in Step 3 by the area in square kilometers in Step 2.

Step 5 - Look at the table below and estimate the average age of the surface.

Problem 1 - From your answer for each Apollo landing area in Step 5, which region of the moon is probably the youngest?

Estimated Age	Total number of craters per square kilometer
1000 years	0.0008
10000 years	0.008
100,000 years	0.08
1 million years	0.8
10 million years	8.0
100 million years	80.0
1 billion years	800.0

Table above is based on the figure below for D = 0.02 kilometers.



Another thing you might consider doing is to measure the diameters of as many craters as you can, and then plot a histogram (bar graph) of the number of craters you counted in a range of size intervals such as 5 - 10 meters, 11 to 20, 21 to 30 and so on. Because erosion (even on the moon!) tends to eliminate the smallest craters first, you can compare two regions on the moon in terms of how much erosion has occurred.

Problem 2 - Select the same-sized area on each of the Apollo images and count all the craters you can find within the size intervals you selected. How do the two landing areas compare to one another in terms of their crater frequency histograms?

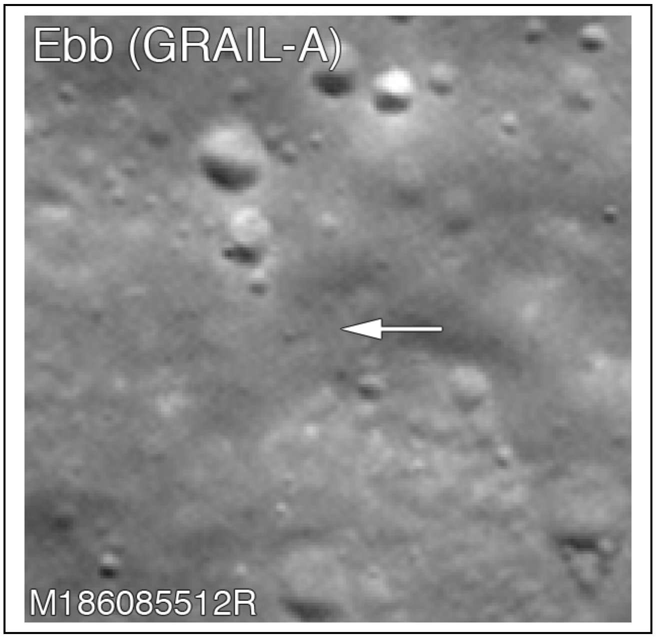
Problem 3 - Which surface do you think has experienced the most re-surfacing or erosion?

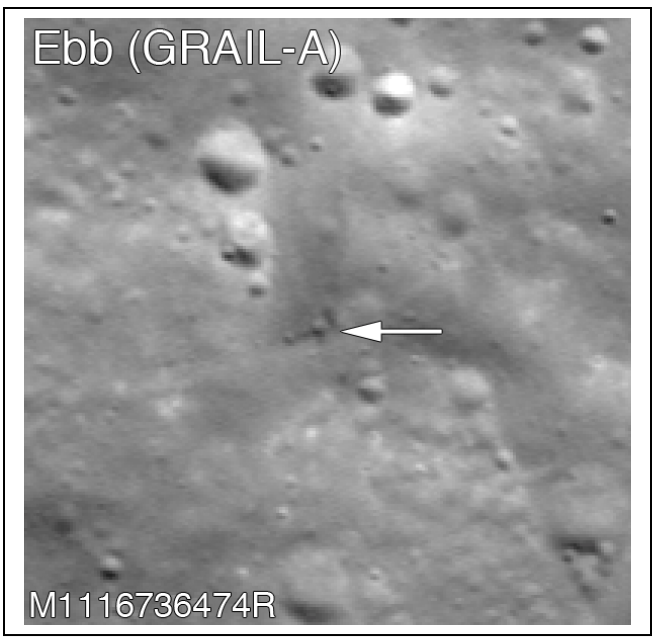
Problem 4 - Without an atmosphere, winds or running water, what do you think could have caused changes in the lunar surface after the craters were formed?

Note to Teachers: More technical information on crater dating can be found at

"How young is the Crater Giordano Bruno"

http://www.psrd.hawaii.edu/Feb10/GiordanoBrunoCrater.html





On December 17, 2012 each of the twin Grail spacecraft ended their lunar mapping missions by crashing into the lunar surface. The two images to the left were taken by the Lunar Reconnaissance Orbiter as it flew over the impact site for the Grail-A spacecraft.

The width of each image is 213 meters.

At the moment of impact, each Grail spacecraft was traveling at a speed of 6,070 km/h and carried a mass of 200 kg.

Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater assuming it has the same density as Grail?

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

NASA's LRO Sees GRAIL's Explosive Farewell
http://www.nasa.gov/mission_pages/LRO/news/grail-results.html

Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Answer: Use a millimeter ruler to measure the width of the image. You should get about 80 mm .The scale is then 213 meters/80 mm = 2.7 meters/mm. The crater is about 1mm across so this is just **2.7 meters**. Since 3 feet = 1 meter, this is about **8 feet across.**

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Answer: About **2.7 times larger**.

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater?

Answer: The largest crater is about 9 mm across or 24 meters. Using Grail, we get 24 meters/2.7 = **8.9 meters across**.

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

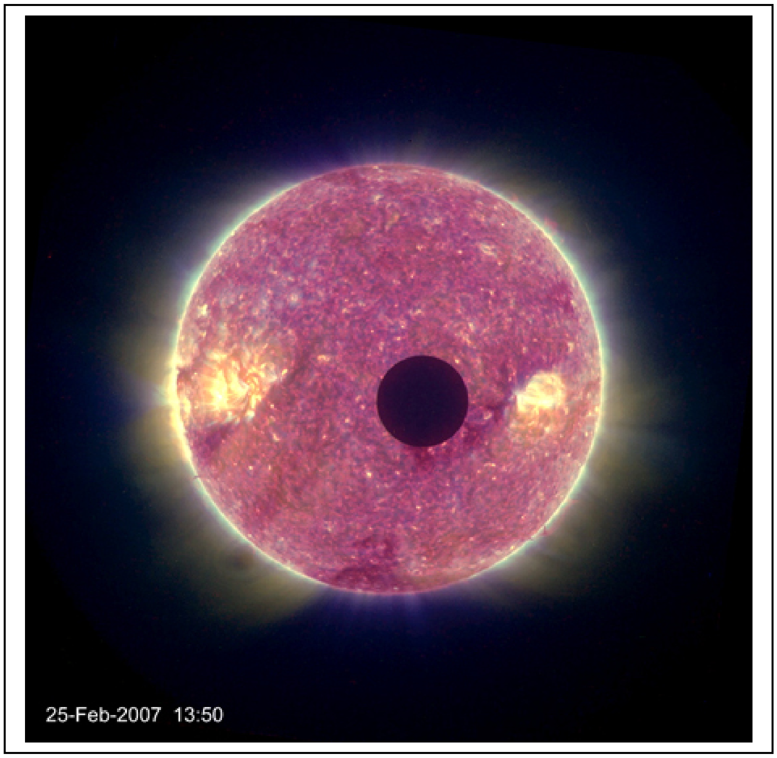
Answer: Volume = $\pi R^2 h$ so V = 3.14 x $(2.7/2)^2$ x (2.7/4) = 3.8 meters^3.
Mass = density x volume so Mass = 2700 x 3.8 = **10,000 kilograms**.

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

Answer: 10,000 kilograms/200 kg = **50**.

So the amount of excavated mass is 50 times the mass of the impacting spacecraft.

A Lunar Transit of the Sun from Space



The twin STEREO satellites captured this picture of our Moon passing across the sun's disk on February 25, 2007. The two satellites are located approximately in the orbit of Earth, but are moving away from Earth in opposite directions. From this image, you can figure out how far away from the Moon the STEREO-B satellite was when it took this picture! To do this, all you need to know is the following:

1) The diameter of the Moon is 3,476 km

2) The distance to the Sun is 150 million km.

3) The diameter of the Sun is 0.54 degrees

Can you figure out how to do this using geometry?

Problem 1: Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the Sun was 0.54 degrees across on February 25. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

Problem 2: As seen from the distance of Earth, the Moon has an Apparent Size of 0.54 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?

Problem 3: From your answer to Problem 1, and Problem 2, what is the distance to the Moon from where the above photo was taken by the STEREO-B satellite?

Problem 4: On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters to the same scale!) with a compass, ruler and protractor?

Answer Key:

Problem 1: Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the sun is 0.5 degrees across. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

Answer: The diameter of the sun is 57 millimeters. This represents 0.54 degrees, so the image scale is 0.54 degrees / 57 millimeters = 0.0095 degrees/mm
The diameter of the Moon is 12 millimeters, so the angular size of the Moon is

12 mm x 0.0095 degrees/mm = 0.11 degrees.

Problem 2: As seen from the distance of Earth, the Moon has an Apparent Size of 0.54 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?

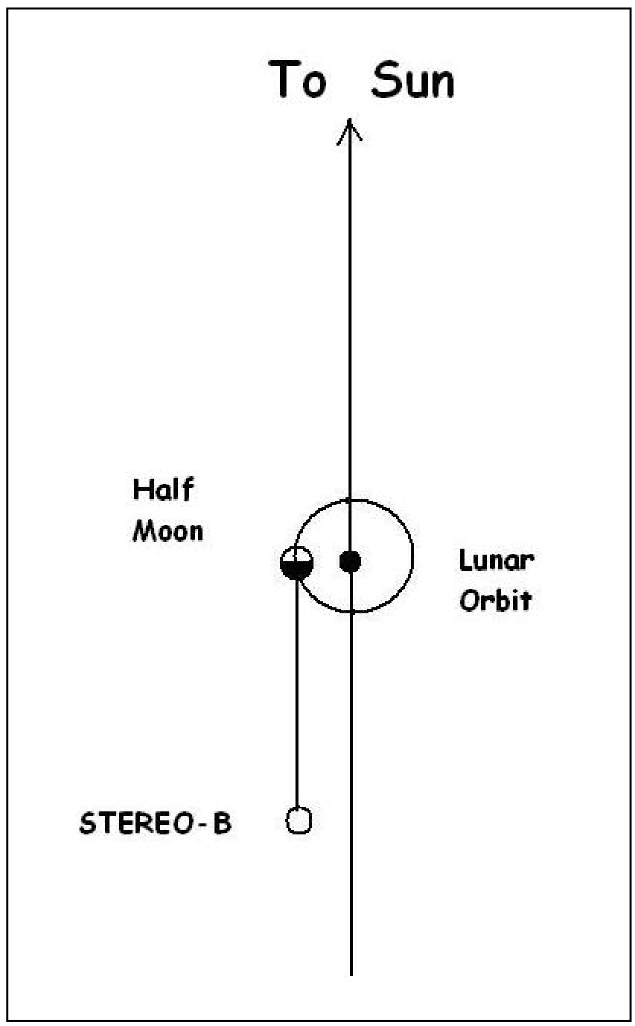
Answer: It would have an Apparent Size half as large, or 0.27 degrees.

Problem 3: From your answer to Problem 1, and Problem 2, what is the distance to the moon from where the above photo was taken by the STEREO-B satellite?

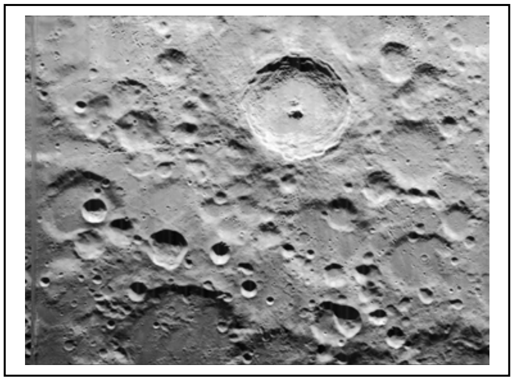
Answer: The ratio of the solar diameter to the lunar diameter is 0.54 degrees/0.11 degrees = 4.9. This means that from the vantage point of STEREO, it is 4.9 times farther away than it would be at the Earth-Moon distance. This means it is 4.9 times farther away than 384,000 km, or 1.9 million kilometers.

Problem 4: On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters!) using a compass, ruler and protractor?

Answer: The figure to the right shows the locations of the Earth, Moon and STEREO satellite. The line connecting the Moon and the Satellite is 4.9 times the Earth-Moon distance.



Lunar Cratering - Probability and Odds



The moon has lots of craters! If you look carefully at them, you will discover that many overlap each other. Suppose that over a period of 100,000 years, four asteroids struck the lunar surface. What would be the probability that they would strike an already-cratered area, or the lunar mare, where there are few craters?

Problem 1 - Suppose you had a coin where one face was labeled 'C' for cratered and the other labeled U for uncratered. What are all of the possibilities for flipping C and U with four coin flips?

Problem 2 - How many ways can you flip the coin and get only Us?

Problem 3 - How many ways can you flip the coin and get only Cs?

Problem 4 - How many ways can you flip the coin and get 2 Cs and 2 Us?

Problem 5 - Out of all the possible outcomes, what fraction includes only one 'U' as a possibility?

Problem 6 - If the fraction of desired outcomes is 2/16, which reduces to 1/8, we say that the 'odds' for that outcome are 1 chance in 8. What are the odds for the outcome in Problem 4?

A fair coin is defined as a coin whose two sides have equal probability of occurring so that the probability for 'heads' = 1/2 and the probability for tails = 1/2 as well. This means that P(heads) + P(tails) = 1/2 + 1/2 = 1. Suppose a tampered coin had P(heads) = 2/3 and P(tails) = 1/3. We would still have P(heads) + P(tails) = 1, but the probability of the outcomes would be different…and in the cheater's favor. For example, in two coin flips, the outcomes would be HH, HT ,TH and TT but the probabilities for each of these would be HH = (2/3)x(2/3)=4/9; HT and TH = 2 x (2/3)(1/3) = 4/9, and TT = (1/3)x1/3) = 1/9. The probability of getting more heads would be 4/9 + 4/9 = 8/9 which is much higher than for a fair coin.

Problem 7: From your answers to Problem 2, what would be the probability of getting only Us in 4 coin tosses if A) P(U) = 1/2? B) P(U) = 1/3?

Problem 8 - The fraction of the lunar surface that is cratered is 3/4, while the mare (dark areas) have few craters and occupy 1/4 of the surface area. If four asteroids were to strike the moon in 100,000 years, what is the probability that all four would strike the cratered areas?

Answer Key

Problem 1 - The 16 possibilities are as follows:

C U U U	C C U U	U C U C	C U C C
U C U U	C U C U	U U C C	U C C C
U U C U	C U U C	C C C U	C C C C
U U U C	U C C U	C C U C	U U U U

Note if there are two outcomes for each coin flip, there are 2 x 2 x 2 x 2 = 16 independent possibilities.

Problem 2 - There is only one outcome that has 'U U U U'

Problem 3 - There is only one outcome that has 'C C C C'

Problem 4 - From the tabulation, there are 6 ways to get this outcome in any order.

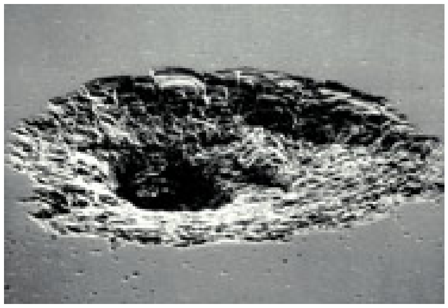
Problem 5 - There are 4 outcomes that have only one U out of the 16 possible outcomes, so the fraction is 4/16 or 1/4.

Problem 6 - The fraction is 6 / 16 reduces to 3/8 so the odds are 3 chances in 8.

Problem 7: A) There is only one outcome that has all Us, and if each U has a probability of 1/2, then the probability is 1 x (1/2) x (1/2) x (1/2) x (1/2) = 1/16.
B) If each U has a probability of 1/3, then the probability is (1/3) x (1/3) x (1/3) x (1/3) = 1/81.

Problem 8 - P(U) = 1/4 while P(C)= 3/4, so the probability that all of the new impacts are also in the cratered regions is the outcome C C C C which is (3/4) x (3/4) x (3/4) x (3/4) = 81 / 256.

Is there a lunar meteorite hazard?



Damage to Space Shuttle Endeavor in 2000 from a micrometeoroid or debris impact . The crater is about 1mm across. (Courtesy - JPL/NASA)

Without an atmosphere, there is nothing to prevent millions of pounds a year of rock and ice fragments from raining down upon the lunar surface.

Traveling at 10,000 miles per hour (19 km/s), they are faster than a speeding bullet and are utterly silent and invisible until they strike.

Is this something that lunar explorers need to worry about?

Problem 1 - Between 1972 and 1992, military infra-sound sensors on Earth detected 136 atmospheric detonations caused by meteors releasing blasts carrying an equivalent energy of nearly 1,000 tons of TNT - similar to small atomic bombs, but without the radiation. If the radius of Earth is 6,378 km, A) what is the rate of these deadly impacts on Earth in terms of impacts per km^2 per year? B) Assuming that the impact rates are the same for Earth and the Moon, suppose a lunar colony has an area of 10 km^2. How many years would they have to wait between meteor impacts?

Problem 2 - Between 2005-2007, NASA astronomers counted 100 flashes of light from meteorites striking the lunar surface - each equivalent to as much as 100 pounds of TNT. If the surveyed area equaled 1/4 of the surface area of the Moon, and the lunar radius is 1,737 km, A) What is the arrival rate of these meteorites in meteorites per km^2 per year? B) If a lunar colony has an area of 10 km^2, how long on average would it be between impacts?

Problem 3 - According to H.J. Melosh (1981) meteoroids as small as 1-millimeter impact a body with a 100-km radius about once every 2 seconds. A) What is the impact rate in units of impacts per m^2 per hour? B) If an astronaut spent a cumulative 1000 hours moon-walking and had a spacesuit surface area of 10 m^2, how many of these deadly impacts would he receive? C) How would you interpret your answer to B)?

Answer Key

Problem 1 - A) The surface area of Earth is $4 \pi (6378)^2 = 5.1 \times 10^8$ km^2. The rate is R = 136 x 10 impacts / 20 years / 5.1×10^8 km^2 = **1.3×10^{-7} impacts/km^2/year.**

B) The number of impacts/year would be 1.3×10^{-7} impacts/km^2/year x 10 km^2 = 1.3×10^{-6} impacts/year . The time between impacts would be
$1/1.3 \times 10^{-6}$ = **769,000 years!**

Problem 2 - A) The total surface area of the Moon is $4 \pi (1737)^2 = 3.8 \times 10^7$ km^2. Only 1/4 of this is surveyed so the area is 9.5×10^5 km^2. Since 100 were spotted in 2 years, the arrival rate is R = 100 impacts/2 years/ 9.5×10^5 km^2 = **5.3×10^{-5} impacts/km^2/year.**

B) The rate for this area is 10 km^2 x 5.3×10^{-5} impacts/km^2/year = 5.3×10^{-4} impacts/year, so the time between impacts is about 1/ 5.3×10^{-4} = **1,900 years**

Problem 3 - A) A sphere 100-km in radius has a surface area of $4 \pi (100{,}000)^2 = 1.3 \times 10^{11}$ m^2. The impacts arrive every 2 seconds on average, which is 2/3600 = 5.6×10^{-4} hours. The rate is, therefore, R = 1 impacts / (1.3×10^{11} m^2 X 5.6×10^{-4} hours) = **1.4×10^{-8} impacts/m^2/hour.**

B) The number of impacts would be 1.4×10^{-8} impacts/m^2/hour x 10 m^2 x 1000 hours = **1.4×10^{-5} impacts.**

C) Because the number of impacts is vastly less than 1 (a certainty), he should not worry about such deadly impacts unless he had reason to suspect that the scientists miscalculated the impact rates for meteorites this small. Another way to look at this low number is to turn it around and say that the astronaut would have to take 1/ 1.4×10^{-5} about 71,000 such 1000-hour moon walks in order for one impact to occur. Alternately, the time between such events is 71,000 x 1000 hours = 71 million years!

The Launch of LADEE to the Moon

NASA launched its Lunar Atmosphere and Dust Environment Explorer (LADEE) at 11:27 p.m. EDT Friday, September 6, 2013 from the agency's Wallops Flight Facility in Virginia. LADEE is scheduled to arrive at the moon in 30 days, then enter lunar orbit.

NASA's LADEE mission caused a sensation in the Eastern United States. Friday's launch was visible from Virginia to Massachusetts. The fireball was captured by photographers and videographers in many locations. Crowds gathered in Times Square in New York and on the steps of the Lincoln Memorial in Washington to watch.

The table below gives the flight data during the Stage 3 ignition period beginning 190 seconds after launch and just before Stage 3 burnout at 210 seconds.

Time (seconds)	Altitude (kilometers)	Range (kilometers)	Speed (meters/sec)
190	150	446	5339
192	151	456	5344
194	153	467	5535
196	154	478	5629
198	156	489	5744
200	158	500	5853
202	159	511	5962
204	161	522	5991
206	163	533	5993
208	164	546	5990

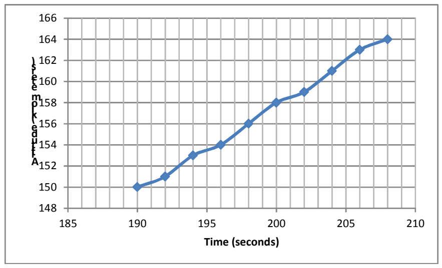
Problem 1 – Create a graph of the altitude in kilometers versus the time after launch in seconds. Over the plotted interval, what is the average slope of the line, and what does it represent? (Use proper units for the slope based on the graph)

Problem 2 – The distance between the launch gantry and the point directly under the current position of the rocket is called the range. Create a graph of the range of the rocket over this time interval. What is the average slope of the plotted line, and what does it represent? (Use proper units for the slope based on the graph)

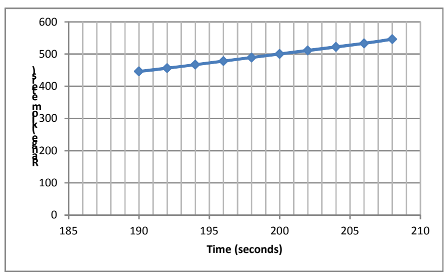
Problem 3 – Graph the speed of the rocket over the interval between 192 and 202 seconds. What is the average slope of the line, and what does this represent?
(Use proper units for the slope based on the graph)

LADEE Lights Up the East Coast
http://www.nasa.gov/content/ladee-lights-up-the-east-coast/index.html#.Uiq1e3eO6So
September 7, 2013

Problem 1 – Create a graph of the altitude in kilometers versus the time after launch in seconds. Over the plotted interval, what is the average slope of the line, and what does it represent? (Use proper units for the slope based on the graph). Answer: Average slope = (164-150)/(208-190) = +14 km/18 sec = **+0.78 km/sec. This represents the vertical speed of the rocket**.

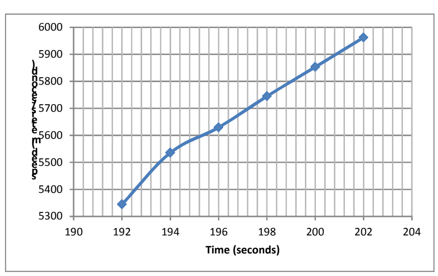


Problem 2 – The distance between the launch gantry and the point directly under the current position of the rocket is called the range. Create a graph of the range of the rocket over this time interval. What is the average slope of the plotted line, and what does it represent? (Use proper units for the slope based on the graph). Answer: Slope = (546-446)/(208-190) = + 100km/18sec = **+5.56 km/sec. This represents the horizontal speed of the rocket.**



Note to Teacher: The vertical speed and horizontal speed are the legs of a right-triangle. The hypothenuse is the total speed of the rocket. Using the Pythagorean Theorem we can find $S^2 = (0.78^2 + 5.56^2)$ so S = 5600 meters/sec, which is close to the average rocket speed in the time interval, which is graphed in Problem 3.

Problem 3 – Graph the speed of the rocket over the interval between 192 and 202 seconds. What is the average slope of the line, and what does this represent? (Use proper units for the slope based on the graph). Answer: slope = (5962-5344)/(202-192) = (+618 meters/sec)/10sec = **+61.8 meters/sec^2. This represents the average acceleration of the rocket.** Note 1 Earth gravity = +9.8 meters/sec^2, so the average acceleration of the rocket is about ‘6 Gs’ which would be very unpleasant for humans if this were a manned flight!



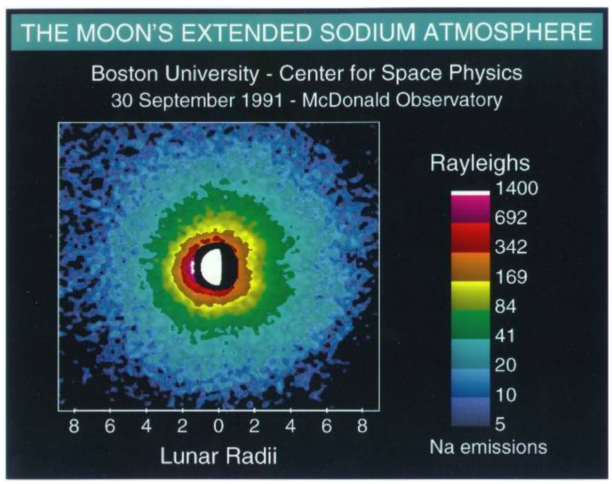
The Moon's Atmosphere!

Courtesy: T.A.Rector, I.P.Dell'Antonio (NOAO/AURA/NSF)

Experiments performed by Apollo astronauts were able to confirm that the moon does have a very thin atmosphere.

The Moon has an atmosphere, but it is very tenuous. Gases in the lunar atmosphere are easily lost to space. Because of the Moon's low gravity, light atoms such as helium receive sufficient energy from solar heating that they escape in just a few hours. Heavier atoms take longer to escape, but are ultimately ionized by the Sun's ultraviolet radiation, after which they are carried away from the Moon by solar wind.

Because of the rate at which atoms escape from the lunar atmosphere, there must be a continuous source of particles to maintain even a tenuous atmosphere. Sources for the lunar atmosphere include the capture of particles from solar wind and the material released from the impact of comets and meteorites. For some atoms, particularly helium and argon, outgassing from the Moon's interior may also be a source.



Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6×10^{-24} grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms? C) metric tons?

Answer Key:

Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Answer: Each element contributes 1/4 of the total particles so hydrogen = 40,000 particles/cc; helium = 40,000 particles/cc, argon=40,000 particles/cc and argon=40,000 particles/cc

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6 x 10-24 grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Answer: A) Hydrogen = 1.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 6.4 x $10^{\mathbf{-20}}$ grams
B) Helium = 4.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.6 x $10^{\mathbf{-19}}$ grams
C) Neon = 20.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 1.3 x $10^{\mathbf{-18}}$ grams
D) Argon = 36.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.3 x $10^{\mathbf{-18}}$ grams
E) Total = (0.064 + 0.26 + 1.3 + 2.3) x 10^{-18} grams = <u>3.9 x $10^{\mathbf{-18}}$ grams per cc.</u>

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Answer: Compute the difference in volume between A sphere with a radius of Ri = 1,738 km and Ro = 1,738+170 = 1,908 km. V = 4/3 π $(1908)^{\mathbf{3}}$ - 4/3π $(1738)^{\mathbf{3}}$ = 2.909 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ - 2.198 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$

Volume = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ **x** (10^5 cm/km) **x** (10^5 cm/km) **x** (10^5 cm/km)
= 7.1 x 10^{24} cm^3

Note: If you use the 'calculus technique' of approximating the volume as the surface area of the shell with a radius of Ri, multiplied by the shell thickness of h = 170 km, you will get a slightly different answer of 6.5 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ or 6.5 x 10^{24} cm^3

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms?

A) Mass = density x volume = (3.9 x $10^{\mathbf{-18}}$ gm/cc) x 7.1 x $10^{\mathbf{24}}$ $cm^{\mathbf{3}}$ = 2.8 x $10^{\mathbf{7}}$ grams

B) Mass = 2.8 x $10^{\mathbf{7}}$ grams x (1 kg/1000 gms) = 28,000 kilograms.

C) Mass = 28,000 kg x (1 ton / 1000 kg) = 28 tons.

Teacher note: You may want to compare this mass to some other familiar objects. Also, the Apollo 11 landing and take-off rockets ejected about 1 ton of exhaust gases. Have the students discuss the human impact (air pollution!) on the lunar atmosphere from landings and launches.

Lunar Meteorite Impact Risks

A December 4, 2006 CNN.Com news story, based on the research by Bill Cooke, head of NASA's Meteoroid Environment Office suggests that one of the largest dangers to lunar explorers will be meteorite impacts. Between November 2005 and November 2006, Dr. Cooke's observations of lunar flashes (see image) found 12 of these events in a single year. The flashes were caused primarily by Leonid Meteors about 3-inches across, impacting with the equivalent energy of 150-300 pounds of TNT.

The diameter of the moon is 3,476 kilometers.

Problem 1: From the formula for the surface of a sphere, what is the area, in square kilometers, of the side of the moon facing Earth?

Problem 2: Although an actual impact only affects the few square meters within its immediate vicinity, we can define an impact zone area as the total area of the surface being struck, by the number of objects striking it. What was the average impact zone area for a single event?

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts?

Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Problem 6: The lunar image shows that the impacts are not really random, but seem clustered into three groups. Each group covers an area about 700 kilometers on a side. What is the average impact zone area for four strikes per zone?

Problem 7: If you were an colony located in one of these three zones, what would be your answer to Problem 5?

Answer Key:

Problem 1: From the formula for the surface of a sphere, A) what is the area, in square kilometers, of the side of the moon facing Earth?

Answer: $2 \times 3.141 \times (1738 \text{ km})^2 = 1.89 \times 10^7 \text{ km}^2$

Problem 2: Answer: $1.89 \times 10^7 \text{ km}^2 /12 = 1.58 \times 10^6 \text{ km}^2$

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Answer: A) $S = (1.58 \times 10^6 \text{ km}^2)^{1/2}$ about 1,257 kilometers B) 1,257 kilometers.

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts?

Answer: 1 year / 12 impacts = One month.

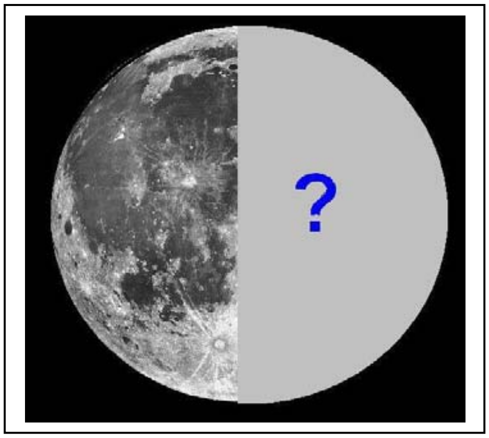
Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Answer: 1 impact per 1.58×10^6 square kilometers per month. The area of the horizon region around the colony is about $\pi \times (3\text{km})^2 =$ 27 square kilometers. This area is $1.58 \times 10^6 \text{ km}^2 /27 \text{ km}^2$ or about 60,000 times smaller that the average, monthly impact area. That suggests you will have to wait about 60,000 times longer than the time it takes for one impact or 60,000 months, which equals 5,000 years, assuming that the distribution of impacts is completely random, unbiased and has a uniform geographic distribution across the Moon's surface.

Problem 6: Answer: (700 km) x (700 km) / 4 = 1 impact per 122,500 km^2 zone area.
Horizon area = 27 km^2, so the impact zone area is 122,500 / 27 = 4,500 times larger. You would need to wait about 4,500 x 1 month or 375 years for an impact to happen within your horizon.

Note to Teacher: This calculation assumes that the clustering of impacts is a real effect that persists over a long time. In fact, this is very unlikely, and it is more statistically probable that when thousands of impacts are plotted, a more uniform strike distribution will result. This is similar to the result of flipping a coin 12 times and getting a different outcome than half-Heads and half-Tails.

The Moon's Density - What's inside?



The Moon has a mass of 7.4×10^{22} kg and a radius of 1,737 km. Seismic data from the Apollo seismometers also shows that there is a boundary inside the Moon at a radius of about 400 km where the rock density or composition changes. Astronomers can use this information to create a model of the Moon's interior.

Problem 1 - What is the average density of the Moon in grams per cubic centimeter (g/cm^3) ? (Assume the Moon is a perfect sphere.)

Problem 2 - What is the volume, in cubic centimeters, of A) the Moon's interior out to a radius of 400 km? and B) The remaining volume out to the surface?

You can make a simple model of a planet's interior by thinking of it as an inner sphere (the core) with a radius of R(core), surrounded by a spherical shell (the mantle) that extends from R(core) to the planet's surface, R(surface). We know the total mass of the planet, and its radius, R(surface). The challenge is to come up with densities for the core and mantle and R(core) that give the total mass that is observed.

Problem 3 - From this information, what is the total mass of the planet model in terms of the densities of the two rock types (D1 and D2) and the radius of the core and mantle regions R(core) and R(surface)?

Problem 4 - The densities of various rock types are given in the table below.

Type	Density
I - Iron+Nickle mixture (Earth's core)	15.0 gm/cc
E - Earth's mantle rock (compressed)	4.5 gm/cc
B - Basalts	2.9 gm/cc
G - Granite	2.7 gm/cc
S - Sandstone	2.5 gm/cc

A) How many possible lunar models are there? B) List them using the code letters in the above table, C) If denser rocks are typically found deep inside a planet, which possibilities survive? D) Find combinations of the above rock types for the core and mantle regions of the lunar interior model, that give approximately the correct lunar mass of 7.4×10^{25} grams. [Hint: use an *Excel* spread sheet to make the calculations faster as you change the parameters.] E) If Apollo rock samples give an average surface density of 3.0 gm/cc, which models give the best estimates for the Moon's interior structure?

Answer Key

Problem 1 - Mass = 7.4×10^{22} kg x 1000 gm/kg = 7.4×10^{25} grams. Radius = 1,737 km x 100,000 cm/km = 1.737×10^{8} cm. Volume of a sphere = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (1.737 \times 10^8)^3 = 2.2 \times 10^{25}$ cm^3, so the density = 7.4×10^{25} grams / 2.2×10^{25} cm^3 = **3.4 gm / cm^3.**

Problem 2 - A) V(core) = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (4.0 \times 10^7)^3$ **= 2.7 x 10^{23} cm^3**
B) V(shell) = V(Rsurface) - V(Rcore) = 2.2×10^{25} cm^3 - 2.7×10^{23} cm^3 = **2.2 x 10^{25} cm^3**

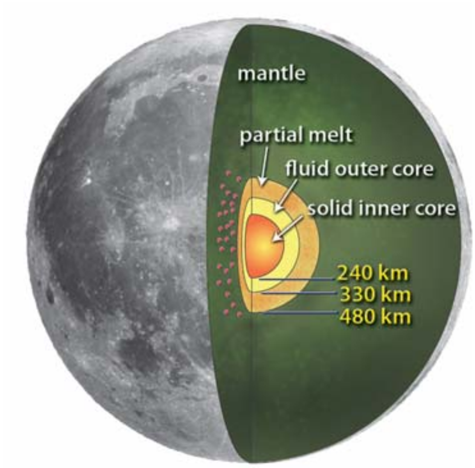
Problem 3 - The total core mass is given by M(core) = $4/3\ \pi\ (Rcore)^3 \times D1$. The volume of the mantle shell is given by multiplying the shell volume V(shell) calculated in Problem 2B by the density : Mshell = V(shell) x D2. Then, the formula for the total mass of the model is given by: MT = $4/3\ \pi\ (Rc)^3 \times D1 + (4/3\ \pi\ (Rs)^3 - 4/3\ \pi\ (Rc)^3) \times D2$, which can be simplified to:

$$\mathbf{MT = 4/3\ \pi\ (D1 \times Rc^3 + D2 \times Rs^3 - D2 \times Rc^3)}$$

Problem 4 - A) There are 5 types of rock for 2 lunar regions so the number of unique models is 5 x 5 = 25 possible models. B) The possibilities are: II, IE, IB, IG, IS, EE, EI, EB, EG, ES, BI, BE, BB, BG, BS, GI ,GE, GB, GG, GS, SI, SE, SB, SG, SS. C) The ones that are physically reasonable are: IE, IB, IG, IS, EB, EG, ES, BG, BS, GS. The models, II, EE, BB, GG and SS are eliminated because the core must be denser than the mantle. D) Each possibility in your answer to Part C has to be evaluated by using the equation you derived in Problem 3. This can be done very efficiently by using an Excel spreadsheet. The possible answers are as follows:

Model Code	Mass (in units of 10^{25} grams)
I E	10.2
I B	6.7
E B	6.4
I G	6.3
E G	6.0
B G	6.0
I S	5.8
E S	5.5
B S	5.5
G S	5.5

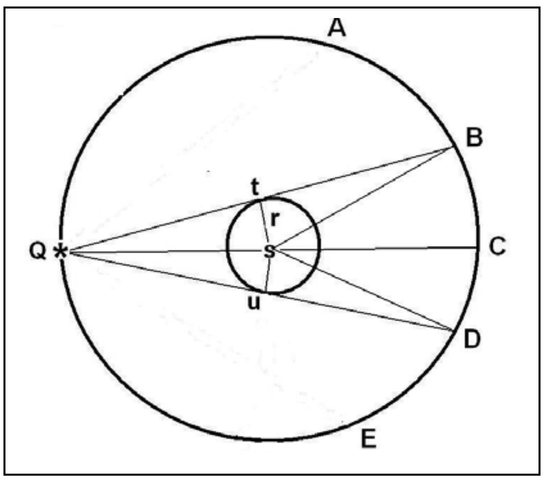
E) The models that have rocks with a density near 3.0 gm/cc as the mantle top layer are the more consistent with the density of surface rocks, so these would be IB and EB which have mass estimates of 6.7×10^{25} and 6.4×10^{25} grams respectively. These are both very close to the actual moon mass of 7.4×10^{25} grams (e.g. 7.4×10^{25} grams) so it is likely that the moon has an outer mantle consisting of basaltic rock, similar to Earth's mantle rock (4.5 gm/cc) and a core consisting of a denser iron/nickel mixture (15 gm/cc).



Sometimes old data can uncover new secrets! Four seismometers were deployed by Apollo astronauts between 1969 and 1972. They were able to record continuous lunar seismic activity until late-1977.

A detailed mathematical analysis of this data reveals that the seismic data are consistent with a model in which the moon has a solid, iron-rich inner core and a fluid, primarily liquid-iron outer core. The core contains a small amount of elements such as sulfur, which is a composition similar to the core of our own Earth.

The analysis they used can be shown with a simple geometry problem:



Suppose a 'moonquake' occurs at Point Q in the figure. Its shock waves travel along the chords from Q to the various seismometers located at points A, B, C, D and E. In addition:

1 - The radius of the moon, R has a length of 1,738 km.

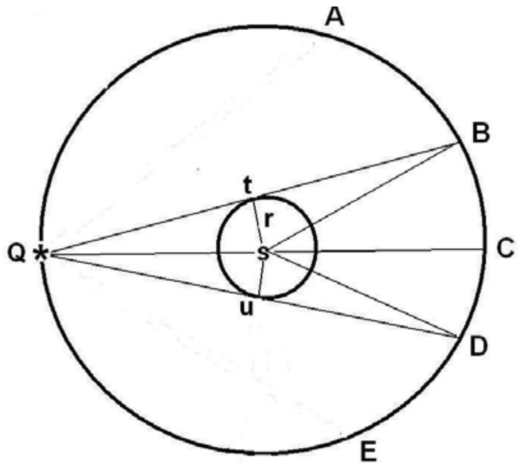
2 - The radius of the core, r, is defined by the segment **ts**.

3 - Angles **Qts** and **Qus** are right-angles.

Basic Seismology: An earthquake generates two kinds of shock wave signals called P-waves and S-waves. When rock is compressed like a sound wave, it produces a pressure wave called the P-wave along its direction of travel. When it moves from side-to-side perpendicular to its direction of travel, it is called a shear wave or S-wave. Although P-waves can travel through a liquid, S-waves are strongly reduced in strength, or sometimes absent all-together.

Suppose on the moon, Stations A and E record normal seismic S and P-wave signals, however, Stations B and D record signals in which the S-wave is slightly reduced in strength compared to Station's A and E. Station C records P-waves but no S-waves. Assume that Station C is in the shadow zone of the liquid lunar core, and that Stations B and D define seismic signals grazing the outer edge of a hypothetical lunar liquid core. Stations B and D are separated by 900 km along the lunar surface.

Problem 1 - From the figure above, and your knowledge of the properties of inscribed arcs what is the radius in kilometers of the core, r, based on this seismic date?



The arc, BCD = 900 km.
Qs = R = 1,738 km

Angle BsD = 360 x (900 km/ $2\pi R$)
= 360 x (900 km / 10918 km)
= 30 degrees

Angle BsC = 15 degrees

Angle BQC = 1/2 Angle BsC = 7.5 degrees

Angle BQC = Angle tQs

Segment st = r

Then:

Sin (Angle tQs) = r/R

r = R sin(7.5 degrees)

r = 1,738 (0.13)

r = 226 km.

The Mass of the Moon

On July 19, 1969 the Apollo-11 Command Service Module and LEM entered lunar orbit. The orbit period was 2.0 hours, at a distance of 1,737 km from the lunar center.

Believe it or not, you can use these two pieces of information to determine the mass of the moon. Here's how it's done!

Problem 1 - Assume that Apollo-11 went into a circular orbit, and that the inward gravitational acceleration by the Moon on the capsule, Fg, exactly balances the outward centrifugal acceleration, Fc. Solve Fc = Fg for the mass of the Moon, M, in terms of V, R and the constant of gravity, G, given that:

$$F_g = \frac{G M m}{R^2} \qquad F_c = \frac{m V^2}{R}$$

Problem 2 - By using the fact that for circular motion, $V = 2 \pi R / T$, re-express your answer to Problem 1 in terms of R, T and M.

Problem 3 - Given that $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-2}$, R = 1,737 km and T = 2 hours, calculate the mass of the Moon in kilograms!

Problem 4 - The mass of Earth is 5.97×10^{24} kg. What is the ratio of the Moon's mass, derived in Problem 3, to Earth's mass?

Problem 1 - From Fg = Fc, and a little algebra to simplify and cancel terms, you get

$$\boxed{M = \frac{R V^2}{G}}$$

Problem 2 – Substitute 2 π R/T for V and with a little algebra you get:

$$\boxed{M = \frac{4\pi^2 R^3}{G T^2}}$$

Problem 3 - First convert all units to meters and seconds: R = 1.737×10^6 meters and T = 7,200 seconds. Then substitute values into the above equation:

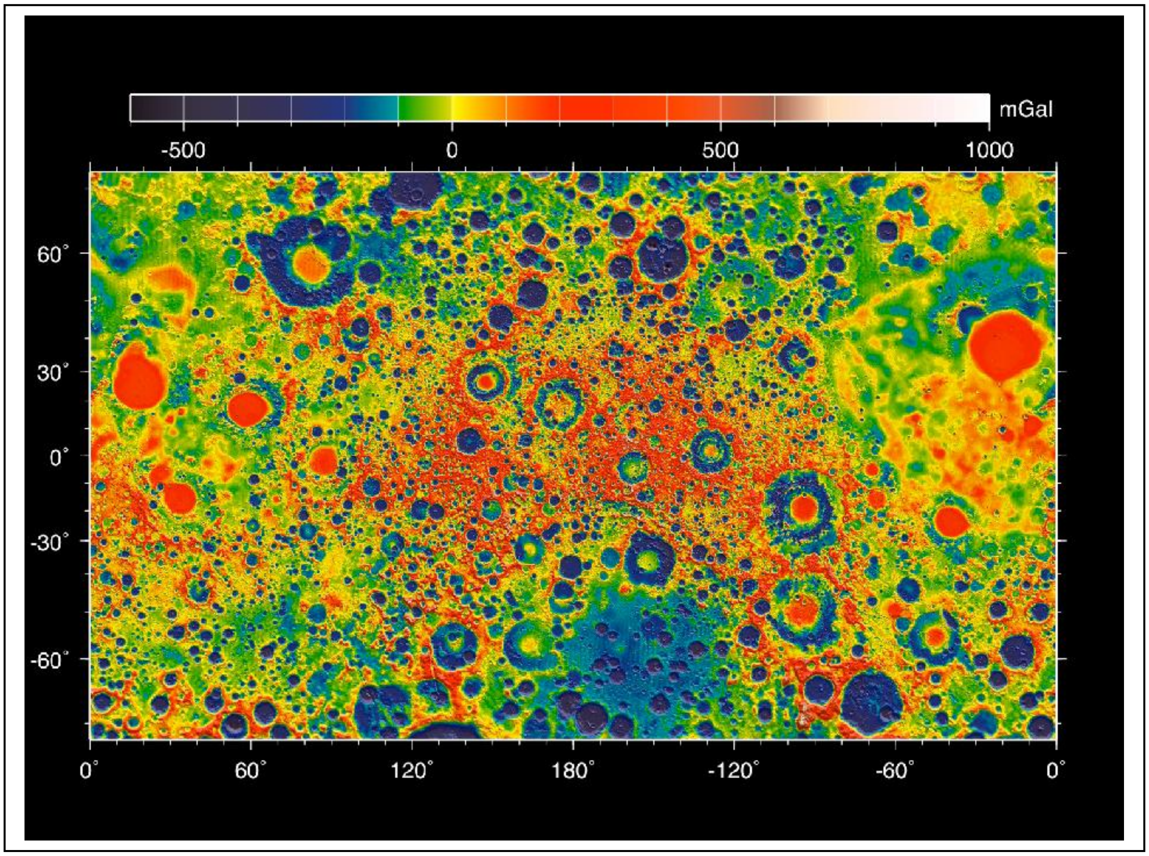
$M = 4 \times (3.14)^2 \times (1.737 \times 10^6)^3 / (6.67 \times 10^{-11} \times (7200)^2))$

$M = (39.44 \times 5.24 \times 10^{18}) / (3.46 \times 10^{-3})$

M = $\mathbf{5.97 \times 10^{22}}$ **kg**

More accurate measurements, allowing for the influence of Earth's gravity and careful timing of orbital periods, actually yield 7.4×10^{22} kg.

Problem 4 - The ratio of the masses is 5.97×10^{22} kg / 5.97×10^{24} kg which equals **1/100.** The actual mass ratio is 1 / 80.



During 2012, NASA's twin Grail satellites orbited the moon at altitudes of only 30 km. As they traveled, minute changes in their speeds tracked from Earth revealed changes in the gravitational field of the moon. These changes could be mapped, and revealed density changes in the lunar surface below them. In this way, scientists could look hundreds of kilometers beneath the lunar surface and explore how the surface was formed billions of years ago! On Earth, the acceleration of gravity is 9,807 cm/sec^2. The normal acceleration of gravity on the average lunar surface is 1620 cm/sec^2, but in the blue regions of the map this is as low as 1520 cm/sec^2, and in the red regions it is as high as 1920 cm/sec^2. A pendulum clock has a swinging period, T in seconds, given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in centimeters, and g is the acceleration of gravity in cm/sec^2.

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length L = 100 cm. What is the swing period? (use π = 3.141)

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period? (use π = 3.141)

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendumum length L = 100 cm. What is the swing period?

Answer: T = 2 (3.141) $(100/1520)^{1/2}$ = **1.61 seconds**.

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period?

Answer; T = 2 (3.141) $(100/1920)^{1/2}$ = **1.43 seconds**.

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Answer: Each swing on the slower Blue Clock pendulum is behind the faster Red Clock by 1.61 - 1.43 = 0.18 seconds. We want this difference to be 3600 seconds in 1 hour, which will take N = 3600/0.18 = **20,000 swings** on the Blue Clock.

Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

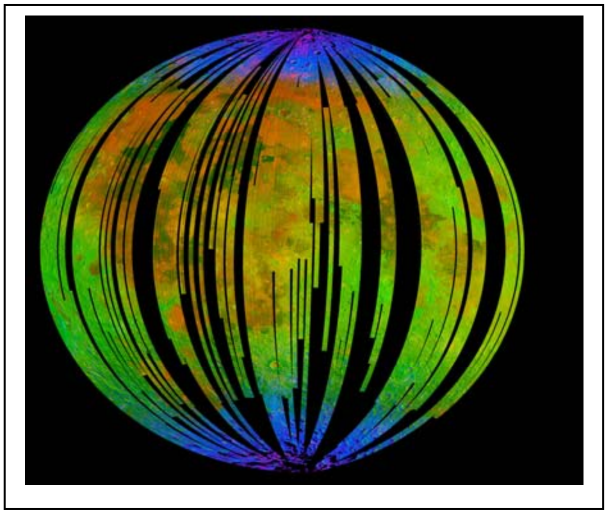
Answer: On the Blue Clock, 20,000 swings have to pass, each taking 1.61 seconds for a total time of 32,200 seconds or 8 hours, 56 minutes, 40 seconds. So the time on the Blue Clock will read **09:56:40 am local time**.

On the Red Clock, because after 20,000 swings it is exactly 1 hour behind the Blue Clock, its time will read 08:56:40 am local time. Another 'long way' to see this is that we still need 20,000 swings to add up to a 1 hour time difference, but on the Red Clock each swing is only 1.43 seconds long and so this takes 28,600 seconds or 7 hours, 56 minutes, 40 seconds. The time on the Red Clock will be **08:56:40 am local time**.

This is why colonists will NOT be using pendulum clocks on the moon!!

Note: Devices that act like pendulum clocks were once used by prospectors on Earth to search for oil and other valuable materials below ground before the advent of more accurate magnetometer-based technology. Minute changes in the pendulum period indicate changes in the density of rock below ground and these can be used to identify high-gravity, density regions (like iron ore) or low-gravity regions (like caverns). Another way to measure minute gravity changes is by the shape of a satellite orbit, or by the subtle changes in speed between two satellites on the same orbit. Lunar scientists used this orbit method with the two Grail spacecraft only 200 kilometers apart.

Water on the Moon !



The debate has gone back and forth over the last 10 years as new data are found, but measurements by Deep Impact/EPOXI, Cassini and most recently the Lunar Reconnaissance Orbiter and Chandrayaan-1 are now considered conclusive. Beneath the shadows of polar craters, and clinging to the lunar regolith, billions of gallons of water are available for harvesting by future astronauts.

The image to the left created by the Moon Minerology Mapper (M3) instrument onboard Chandrayaan-1, shows deposits and sources of hydroxyl molecules. The data has been colored blue and superimposed on a lunar photo.

Complimentary data from the Deep Impact/EPOXI and Cassini missions of the rest of the lunar surface also detected hydroxyl molecules covering about 25% of the surveyed lunar surface. The hydroxyl molecule consists of one atom of oxygen and one of hydrogen, and because water is basically a hydroxyl molecule with a second hydrogen atom added, detecting hydroxyl on the moon is an indication that water molecules are also present.

How much water might be present? The M3 instrument can only detect hydroxyl molecules if they are in the top 1-millimeter of the lunar surface. The measurements also suggest that about 1 metric ton of lunar surface has to be processed to extract 1 liter (0.26 gallons) of water.

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1?

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the lunar surface contains water?

Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters).

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Answer: The surface area of a sphere is given by $S = 4\pi r^2$ and so the volume of a layer with a thickness of L is $V = 4\pi r^2 L$ provided that L is much smaller than r.
$V = 4 \times (3.141) \times (1731000)^2 \times 0.001 = 3.76 \times 10^{10}\ m^3$

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1? Answer: $3.76 \times 10^{10}\ m^3 \times (3000\ kg/m^3) \times (1\ ton/1000\ kg) = 1.13 \times 10^{11}$ metric tons.

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the surface contains water? Answer: 1.13×10^{11} tons x (1 liter water/1 ton regolith) x 1/4 = 2.8×10^{10} liters of water.

Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters). Answer: 2.8×10^{10} liters x (1 gallon / 3.78 liters) = 7.5×10^9 gallons of water or about 8 billion gallons of water.

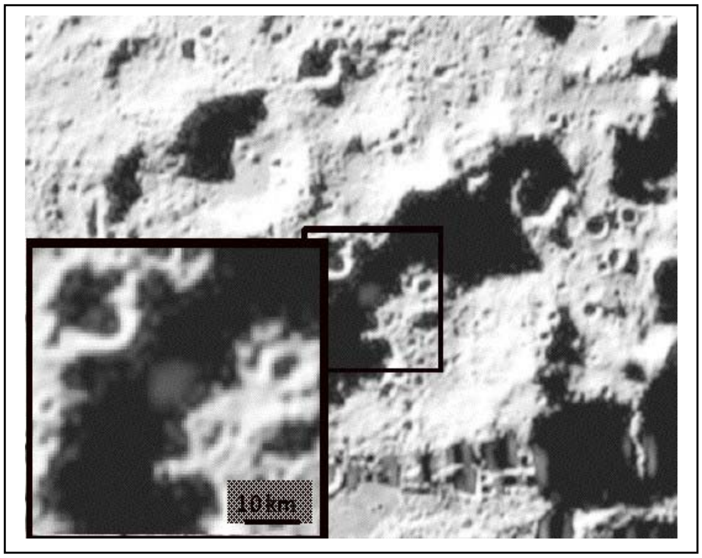
Note: This is similar to the roughly '7 billion gallon' estimate made by the M3 scientists as described in the NASA Press Release for this discovery in September 2009.

For more information, visit:

Moon Minerology Mapper News - http://moonmineralogymapper.jpl.nasa.gov/

The front picture of the moon is from NASA's Moon Mineralogy Mapper on the Indian Space Research Organization's Chandrayaan-1 mission. It is a three-color composite of reflected near-infrared radiation from the sun, and illustrates the extent to which different materials are mapped across the side of the moon that faces Earth. Small amounts of water and hydroxyl (blue) were detected on the surface of the moon at various locations. This image illustrates their distribution at high latitudes toward the poles. **Blue** shows the signature of water and hydroxyl molecules as seen by a highly diagnostic absorption of infrared light with a wavelength of three micrometers. **Green** shows the brightness of the surface as measured by reflected infrared radiation from the sun with a wavelength of 2.4 micrometers. **Red** shows an iron-bearing mineral called pyroxene, detected by absorption of 2.0-micrometer infrared light.

LCROSS 'Sees' Water on the Moon!



On October 9, 2009 the LCROSS spacecraft and its companion Centaur upper stage, impacted the lunar surface within the shadowed crater Cabeus located near the moon's South Pole. The Centaur impact speed was 9,000 km/hr with a mass of 2.2 tons.

The impact created a crater about 20 meters across and about 3 meters deep. Some of the excavated material formed a plume of debris visible to the LCROSS satellite as it flew by. Instruments on LCROSS detected about 25 gallons of water.

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Answer: V = (3.14) x (10 meters)^2 x 3 = **942 cubic meters**.

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Answer: 3000 kg/m^3 x (942 meters^3) = 2,800,000 kilograms. Since 1000 kg = 1 ton, there were **2,800 tons of regolith excavated**.

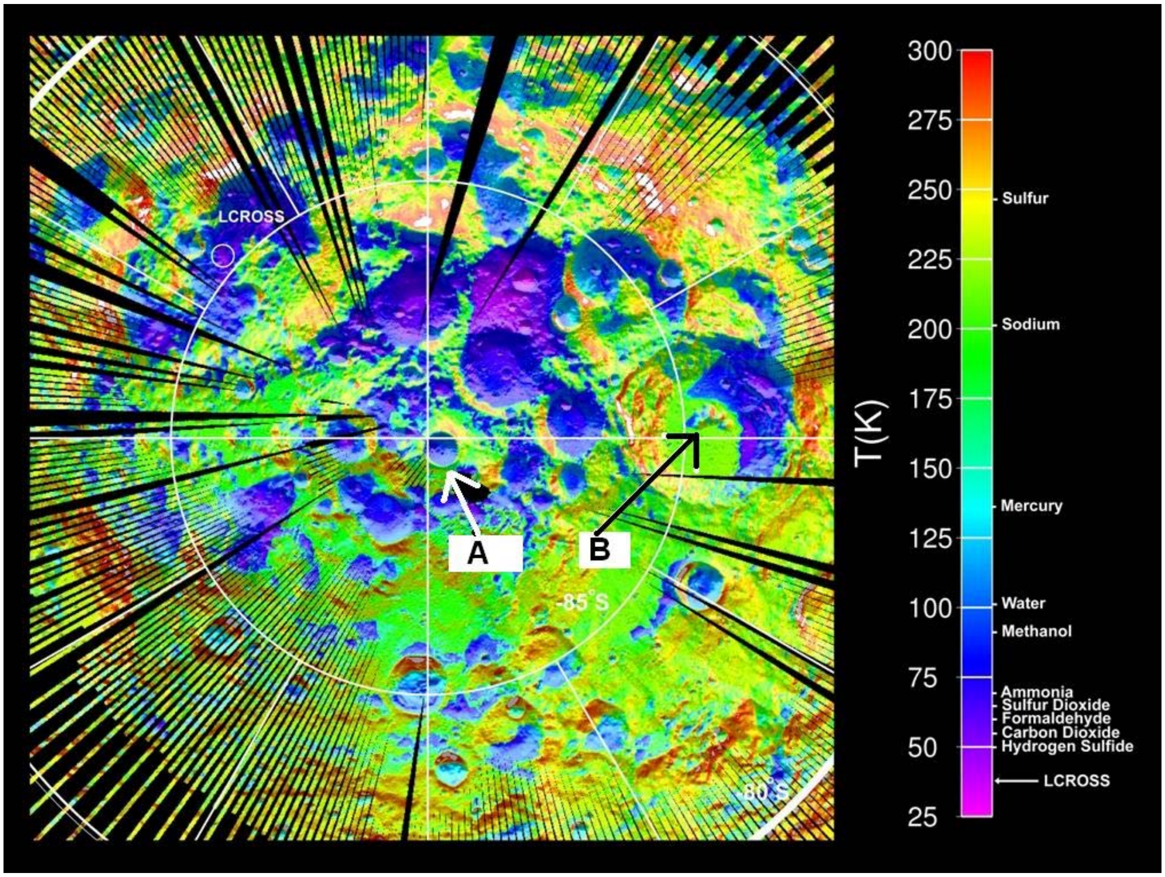
Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Answer: The area of the ejecta blanket is given by A = π(35 meters)^2 - π(10 meters)^2 = 3,846 - 314 = 3500 meters^2. The volume is A x h = (3500 meters^2) x 0.2 meters = 700 meters^3 . Then the mass is just M = (700 meters^3) x (3,000 kilograms/meter^3) = 2,100,000 kilograms or **2,100 tons in the ejecta blanket**.

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Answer: The amount of ejected regolith was 2,800 tons - 2,100 tons or 700 tons. The detected water amounted to 25 gallons or 25 gallons x (3.78 liters/ 1 gallon) = 95 liters. So the concentration was about C = 700 tons/95 liters = **7 tons/liter**.

Note to teacher: The estimated concentration, C, in Problem 4 is based on an approximated geometry for the crater (cylinder), an average thickness for the ejecta blanket (about 0.2 meters) and whether all of the remaining material (700 tons) was actually involved in the plume measured by LCROSS. Students may select, by scaled drawing, other geometries for the crater, and thickness for the ejecta blanket to obtain other estimates for the concentration, C. The scientific analysis of the LCROSS data may eventually lead to better estimates for C.



The Lunar Reconnaissance Orbiter (LRO) has recently created the first surface temperature map of the south polar region of the moon using date taken between September and October, 2009 when south polar temperatures were close to their annual maximum values. The colorized map shows the locations of several intensely cold impact craters that are potential cold traps for water ice as well as a range of other icy compounds commonly observed in comets. The approximate maximum temperatures at which these compounds would be frozen in place for more than a billion years is shown on the scale to the right. The LCROSS spacecraft was targeted to impact one of the coldest of these craters, and many of these compounds were observed in the ejecta plume. *(Courtesy: UCLA/NASA/JPL)*

Problem 1 - The width of this map is 500 km. What are the diameters of Crater A (Shackleton) and Crater B (Amundsen) in kilometers?

Problem 2 - In which colored areas might an astronaut expect to find conditions cold enough to recover all of the elements and molecules indicated in the vertical temperature scale to the right?

Problem 3 - The Shackleton Crater (Crater A) is cold enough to trap water and methanol. From Problem 1, and assuming that the thickness of the water deposit is 100 meters, and occupies 10% of the volume of the circular crater, how many cubic meters of water-ice might be present?

Problem 1 - The width of this map is 500 km. What are the diameters of Crater A (Shackleton) and Crater B (Amundsen) in kilometers?

Answer: If the page is printed as 8.5 x 11-inches, the width of the colorized image is 106 mm wide, which corresponds to 500 km, so the image scale is 500 km/106mm = 4.7 km/mm. Crater A has a diameter of 4.4 mm so its actual diameter is 4.4 mm x (4.7 km/mm) = **20 kilometers**. Crater B has a diameter of 34 mm, so its actual diameter is about 34 x 4.7 = **160 kilometers**.

Problem 2 - In which colored areas might an astronaut expect to find conditions cold enough to recover all of the elements and molecules indicated in the vertical temperature scale to the right?

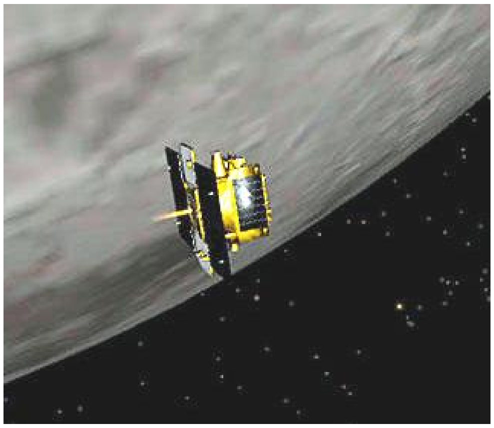
Answer: Students should note that all compounds above a given temperature will be present in conditions cold enough to 'trap' these compounds. **Areas where the temperature is below 50 K will be cold enough to trap all of the identified compounds. These are areas colored lavender at the bottom of the temperature scale, and in areas similar to where LCROSS impacted.**

Problem 3 - The Shackleton Crater (Crater A) is cold enough to trap water and methanol. From Problem 1, and assuming that the thickness of the water deposit is 100 meters, and occupies 10% of the volume of the circular crater, how many cubic meters of water-ice might be present?

Answer: The diameter of the circular crater is 20 km or 20,000 meters, so its radius is 10,000 meters. The area of the crater is then $A = \pi (10,000)^2 = 314,000,000$ meters^2. Since the thickness of the deposit is perhaps 100 meters, the volume of this disk of material is V = 100 meters x 314,000,000 meters^2 = 3.1×10^{10} meters^3. Since only 10% of this is hypothetically water-ice, the ice volume is just 0.1V or **3.1×10^9 meters^3.**

Note: This ice volume is similar to a small glacier!

The Grail and LRO Spacecraft Encounters in Lunar Orbit



Time	Distance	Time	Distance
4:00:38	190	4:01:23	110
4:00:42	180	4:01:35	105
4:00:46	170	4:01:46	110
4:00:51	160	4:01:56	120
4:00:56	150	4:02:03	130
4:01:01	140	4:02:09	140
4:01:07	130	4:02:14	150
4:01:14	120	4:02:20	160

On May 31, 2012 the Grail 'Ebb' spacecraft and the Lunar Reconnissance Orbiter (LRO) will come very close to each other in their orbits around the moon. LRO is in a polar orbit, while Grail Ebb is in an equatorial orbit. Although there is no scientific value in the encounter, it does represent one of the first times that two NASA spacecraft orbiting the same astronomical body have passed so close to each other, and with the capability of actually seeing each other.

The table to the left gives the encounter times in the afternoon (Eastern Standard Time in hours, minutes and seconds) and distances (in kilometers) between the spacecraft.

Problem 1 – At what time were the spacecraft at their closest distances from one another?

Problem 2 – About how fast, in kilometers/hour was the distance between them changing just before closest approach?

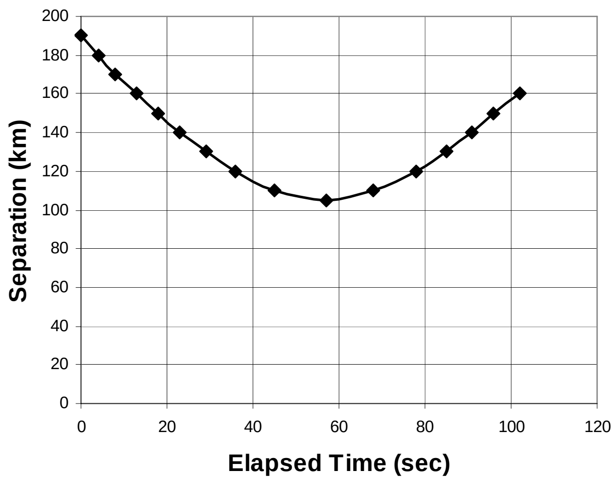
Problem 3 - Calculate the elapsed time of the encounter since 4:00:38 in seconds. Graph the tabular data in terms of elapsed time in seconds and distance in kilometers. What shape does the plotted curve resemble?

Problem 4 - The Grail 'Ebb' spacecraft will attempt to take a picture of LRO. At a distance of 50 kilometers, Grail/Ebb can just resolve an object if it is 8-meters across. That means that the angle corresponding to 8 meters at a distance of 50 kilometers is just large enough to be discerned by Grail. The LRO spacecraft is about 4 meters across. Using simple proportions, and the fact that the angular size of an object is inversely proportional to its distance, will Grail be able to see any details on the LRO spacecraft at the time of closest approach?

Problem 1 – At what time were the spacecraft at their closest distances from one another? Answer: This would be at **4:01:35** at a distance of 105 kilometers.

Problem 2 – About how fast, in kilometers/hour was the distance between them changing just before closest approach? Answer: Speed = distance/time. Distance = 110-105 = 5 km; time = 4:01:35 – 4:01:23 = 12 seconds so speed = 5km/12sec = 0.417 km/sec. In terms of km/hr, speed = 0.41 km/s x (3600 sec/1 hr) = **1500 km/hr**.

Problem 3 - Calculate the elapsed time of the encounter since 4:00:38 in seconds. Graph the tabular data in terms of elapsed time in seconds and distance in kilometers. What shape does the plotted curve resemble? Answer: **A parabola!**

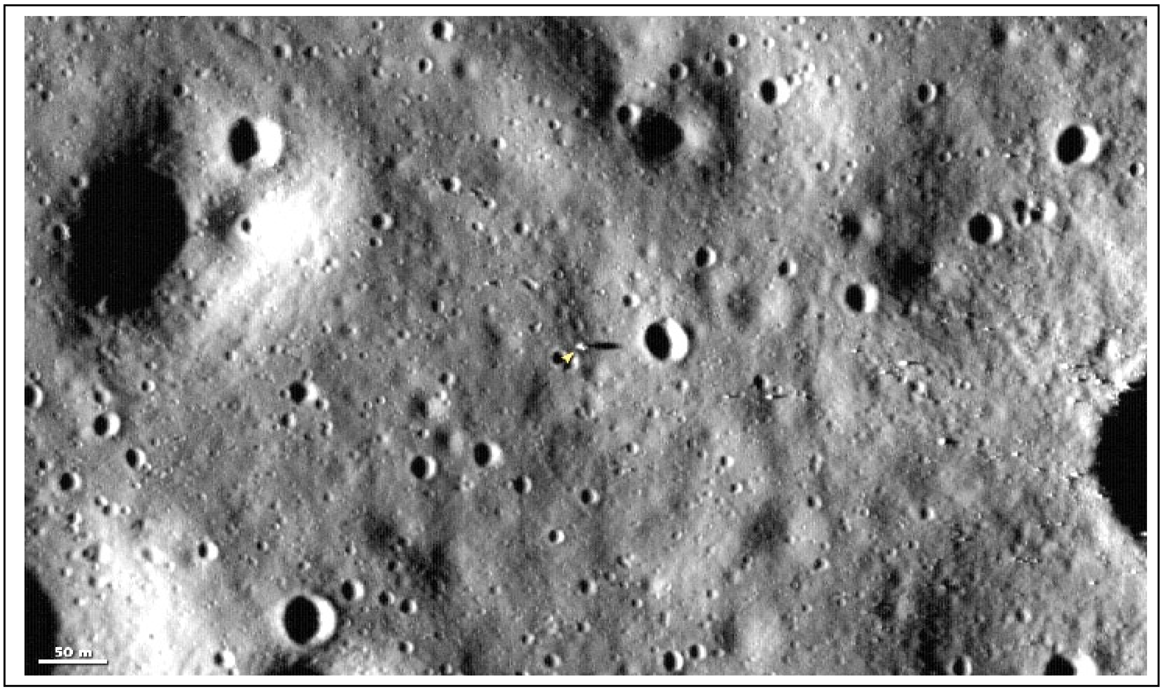


Problem 4 - The Grail 'Ebb' spacecraft will attempt to take a picture of LRO. At a distance of 50 kilometers, Grail/Ebb can just resolve an object if it is 8-meters across. That means that the angle corresponding to 8 meters at a distance of 50 kilometers is just large enough to be discerned by Grail. The LRO spacecraft is about 4 meters across. Using simple proportions, and the fact that the angular size of an object is inversely proportional to its distance, will Grail be able to see any details on the LRO spacecraft at the time of closest approach?

Answer: At their closest separation, 105 kilometers is about twice as large as 50 km, and so for the same angle as seen by an 8 meter object at 50 km, the object would have to be about 8 x 105 km/50 km = 16.8 meters long in order to be seen by Grail. But LRO has a maximum size of only 4 meters, so that means Grail will only see LRO as an unresolved 'dot' of light as it passes by.

Note: Explore the encounter by using NASA's Eyes on the Solar System orbit simulator. Select the Moon, and the date and time of the encounter, then manipulate the scene until the two spacecraft orbits are highlighted. Step the time forward until you come to the encounter scene. To jump to this scene from here click on this link after first setting up EOSS to run on your computer: **http://1.usa.gov/LoD2YE**

Lunar Crater Frequency Distributions



This image of the 800-meter x 480-meter region near the Apollo-11 landing pad (arrow) was taken by the Lunar Reconnaissance Orbiter (LRO). It reveals hundreds of craters covering the landing area with sizes as small as 5 meters. The Apollo-11 landing pad is near the center of the image, and is casting a long horizontal shadow to the right of the pad.

Astronomers use counts of the number of craters per kilometer^2 as a function of crater diameter to determine the age of a given lunar landscape, and the distribution of the sizes of the impactors. Crater counts are also used to determine which areas are safe to land. The power-law function below is based upon the above image from LRO and gives the surface density of craters near the Apollo-11 landing site in terms of craters per kilometer^2 of a given diameter, x, in meters. The range of validity is from 2 meters to 40 meters for this particular lunar area. Apollo-11 astronauts did not find any craters smaller than 2-meters near the landing area.

$$S(x) = 22000\, x^{-2.4} \quad \text{craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters.

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters.

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? (Assume that the craters do not overlap, which is a good approximation to what the image shows.)

$$S(x) = 22000\, x^{-2.4} \text{ craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters. Answer: The limits to the definite integral extend from m to infinity:

$$\int_m^\infty 22000x^{-2.4}dx \quad = \quad N(x>m) = \frac{22000}{1.4m^{1.4}}$$

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters. Answer: The limits to the definite integral extend from 2 to m, because Apollo-11 astronauts did not see any craters smaller than 2 meters (x=2) in this area:

$$\int_2^m 22000x^{-2.4}dx \qquad N = \frac{22000}{1.4(2)^{1.4}} - \frac{22000}{1.4m^{1.4}} \quad \text{so} \quad N = 5955 - \frac{22000}{1.4m^{1.4}}$$

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? Answer: S(x) gives the number of craters with a diameter of x per km^2. The maximum area occupied by these craters assuming that they are non-overlapping is given by $\pi (x/2)^2$ S(x). The total area covered by non-overlapping craters larger than 2 meters is given by the integral:

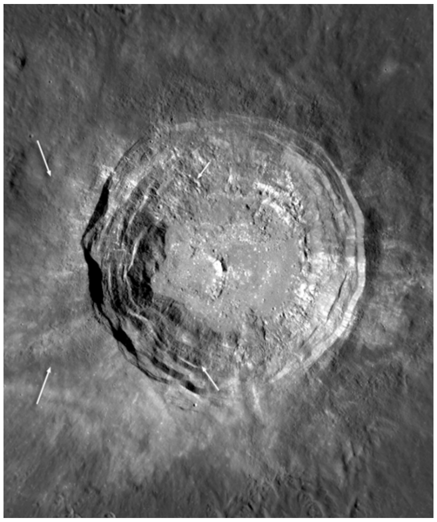
$$A = \int_2^{40} \pi \left(\frac{x}{2}\right)^2 22000x^{-2.4}dx$$

so $$A = \frac{22000\pi}{4}\int_2^{40} x^{-0.4}dx$$ then $$A = \frac{22000\pi}{4}\left(\frac{1}{0.6}\right)\left[x^{0.6}\right]_2^{40}$$

$$A = \frac{22000\pi}{4(0.6)}\left[40^{0.6} - 2^{0.6}\right]$$

so that the cratered area is A = 28783 (9.14 - 1.52) = 220,000 square meters. The area in the image is 800 meters x 480 meters = 384,000 square meters, so the cratered area represents 100% x (220,000/384,000) = 57% of the surface area. So, the **maximum area that is covered by craters is 57%**. Note: That means that 43% of the area was safe to land on.

The volume of a lunar impact crater



Lunar craters have been excavated by asteroid impacts for billions of years. This has caused major remodeling of the lunar surface as the material that once filled the crater is ejected. Some of this returns to the lunar surface hundreds of kilometers from the impact site.

An example of a lunar crater is shown in the image to the left taken of the crater Aristarchus by the NASA Lunar Reconnaissance Orbiter (LRO).

Astronomers create models of the rock that was displaced that try to match the overall shape of the crater. The shape of a crater can reveal information about the density of the rock, and even the way that it was layered below the impact area.

One such mathematical model was created for a 17-kilometer crater located at lunar coordinates 38.4 °North, and 194.9 °West. For this particular crater, its depth, D, at a distance of x from its center can be approximated by the following 4th-order polynomial:

$$D = 0.0001x^4 - 0.0055x^3 + 0.0729x^2 - 0.2252x - 0.493$$

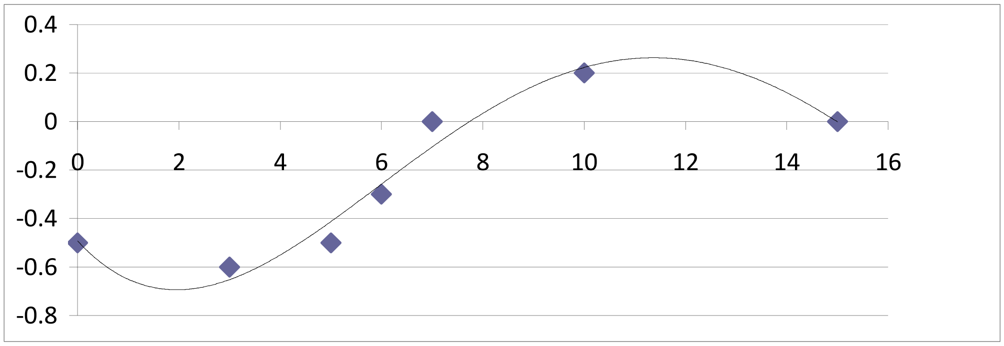
where D and x are in kilometers. The crater is symmetric around the axis x=0 and the surface of the moon is located at a height D=0.2.

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.

Problem 2 - To 2 significant figures, what is the approximate excavated volume below the lunar surface of this symmetric crater using the method of inscribed and circumscribed disks?

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the lunar surface defined by the line y = +0.2 km, and rotated about the y-axis? (Use $\pi = 3.141$)

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.



Problem 2 - To 2 significant figures, what is the approximate excavated volume of the symmetric crater using the method of inscribed and circumscribed disks? Answer; The volume of a disk is V = π R$^{\mathbf{2}}$ h. For the crater, the inscribed disk has a radius of 4 kilometers and a height of (0.2 + 0.5) = 0.7 km, so its volume is Vi = 3.141 (5)$^{\mathbf{2}}$ (0.7) = 55 km$^{\mathbf{3}}$. The circumscribed disk has a radius of 10 km and a height of 0.8 km, so Vc = 3.141 (10)$^{\mathbf{2}}$ (0.8) = 251 km$^{\mathbf{3}}$. The estimated volume is then the average of these two or V = (Vc + Vi)/2 = 153 km$^{\mathbf{3}}$. To 2 Significant Figures, the correct answer would be V = 150 km$^{\mathbf{3}}$.

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141) Answer: Using the method of shells, the volume differential for this problem is dV = 2π x [0.2 - h(x)] dx

The definite integral to evaluate is then $V = \int_0^{10} 2\pi x[0.2 - h(x)]dx$ Then :

$$V = 2\pi(0.2)\int_0^{10} x dx - 2\pi\int_0^{10} 0.0001x^5 - 0.0055x^4 + 0.0729x^3 - 0.2252x^2 - 0.493x\, dx$$

$$V = 2\pi(0.2)\frac{x^2}{2}\Big]_0^{10} - 2\pi\left(0.0001\frac{x^6}{6} - 0.0055\frac{x^5}{5} + 0.0729\frac{x^4}{4} - 0.2252\frac{x^3}{3} - 0.493\frac{x^2}{2}\right)_0^{10}$$

$$V = 2\pi(0.2)\frac{10^2}{2} - 2\pi\left(0.0001\frac{10^6}{6} - 0.0055\frac{10^5}{5} + 0.0729\frac{10^4}{4} - 0.2252\frac{10^3}{3} - 0.493\frac{10^2}{2}\right)$$

V = 2(3.141) [10 - 16.7 + 110 - 182.3 + 75.1 + 24.7] V = 6.24 [20.8]

V = 129.8 km^3 To 2 significant figures this becomes **V = 130 km$^{\mathbf{3}}$**. So to check that Vi < V < Vo we have 55 km^3 < 130 km^3 < 251 km^3.