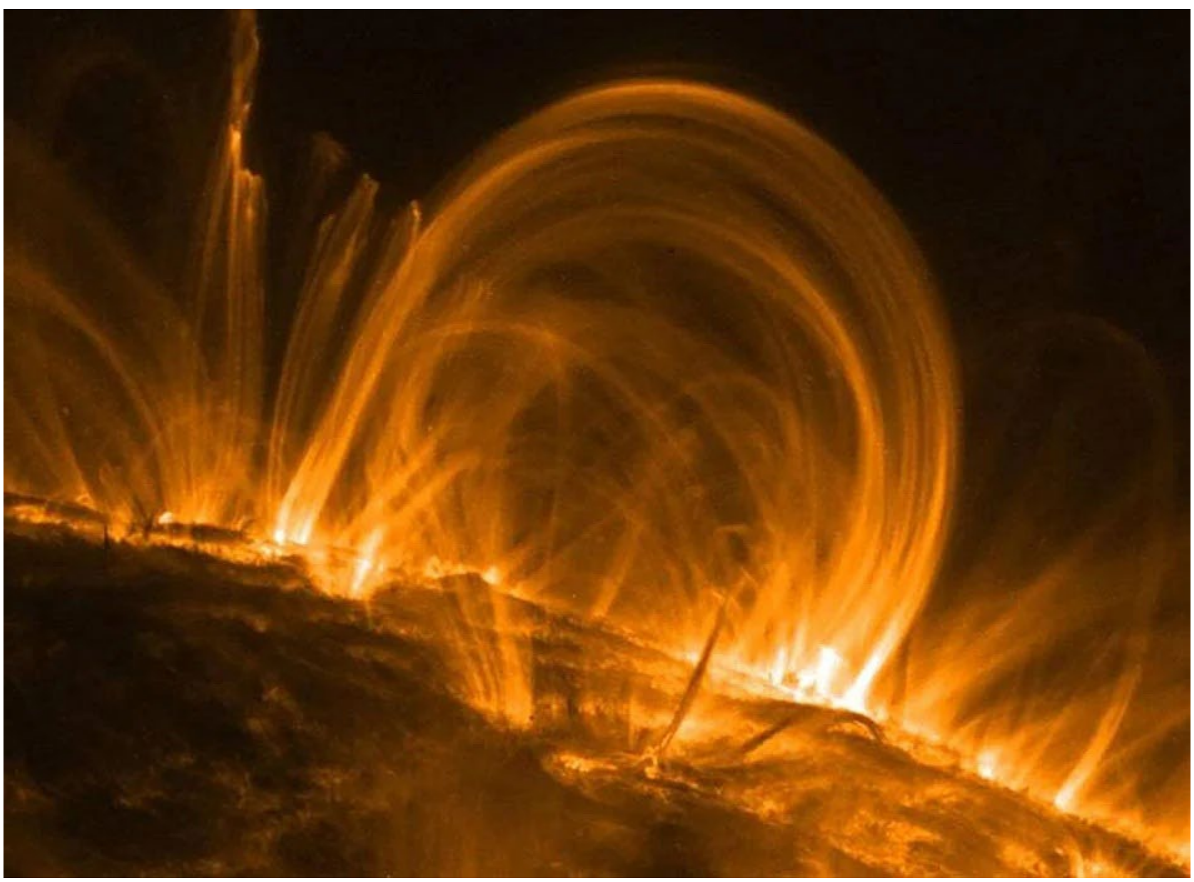
National Aeronautics and Space Administration

Math Problems Related to

Magnetism



Introduction

This book provides an assortment of mathematics problems that cover magnetism and magnetic fields including their shapes, geometry, scale with examples from the sun and Earth. Of particular importance is the idea that magnetic fields can store energy, which is later used to create flares and other energetic phenomena through magnetic reconnection.

Name ______________________________

Scientists have to use mathematics to analyze data, and to build models of the phenomena they are studying. Can you solve this crossword puzzle using the mathematical clues provided? When you're finished, color the blank squares with your favorite aurora colors!

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Word Bank:

+4 proton	- 14 RPI	+6 ionosphere	-1 plasma
-16 FUV	11 photons	+27 spectrum	-27 beams
+3 radiation	0 flares	-10 field	-7 magnetism
-6 earth	-8 solar	-13 envelope	-2 current
-3 aurora	- 23 cone	+2 corona	10 humanity
+1 rays	13 oval	-11 Plasmasphere	-26 echos
-9 polarity	+23 STP	-17 CMEs	+26 UT
+19 TOD	9 ions	-19 sunspot	+25 ENA
-21 FAC	-5 IMAGE	-4 neutron	+8 electrons
+14 watt	17 UV	+7 crown	+5 magnetosphere

The numerical answer to each problem below will tell you which word to use in the word bank to fill in the row or column with the indicated letter.

Down:

l) - 2 (- 5 + 3)

m) - 4 (3 – 1)

e) + 2 (-6 + 3) + 4

n) - 3 (- 5 + 2) + (- 4 + 3)

g) + 5 – (- 8 + 2)

o) - 4 (7 – 3) + (8 – 3)

p) + 1 – 1 + 3 + 4 – 5 – 8

r) - 3 (- 12 + 9)

T) + 2 – 3 + 5 – 10 + 2

u) + 4 (- 5 + 3) – (- 7 + 6)

v) + 3 (- 8 + 6) - 3

x) + 4 (8 – 3) – 3 (+ 6 – 8 + 12)

y) + 2 (+ 8 – 5 + 10) – (+ 10 + 5 – 2)

J) - 2 (- 3 + 2 – 5 – 6) + 2 – 2 (+ 26)

Across:

a) + 2 (- 3 + 8) – 5

b) + 3 (+ 2 + 1) – 2 (- 3 + 9)

c) - 15 (- 2 + 3) + 3 (- 2 + 9)

d) - 1 + 1 – 1 + 1 – 1 + 1

e) - 6 + (+ 11 – 3)

f) - 3 (- 8 + 11) + 4

g) (+ 1 – 1) + (+ 1 – 2)

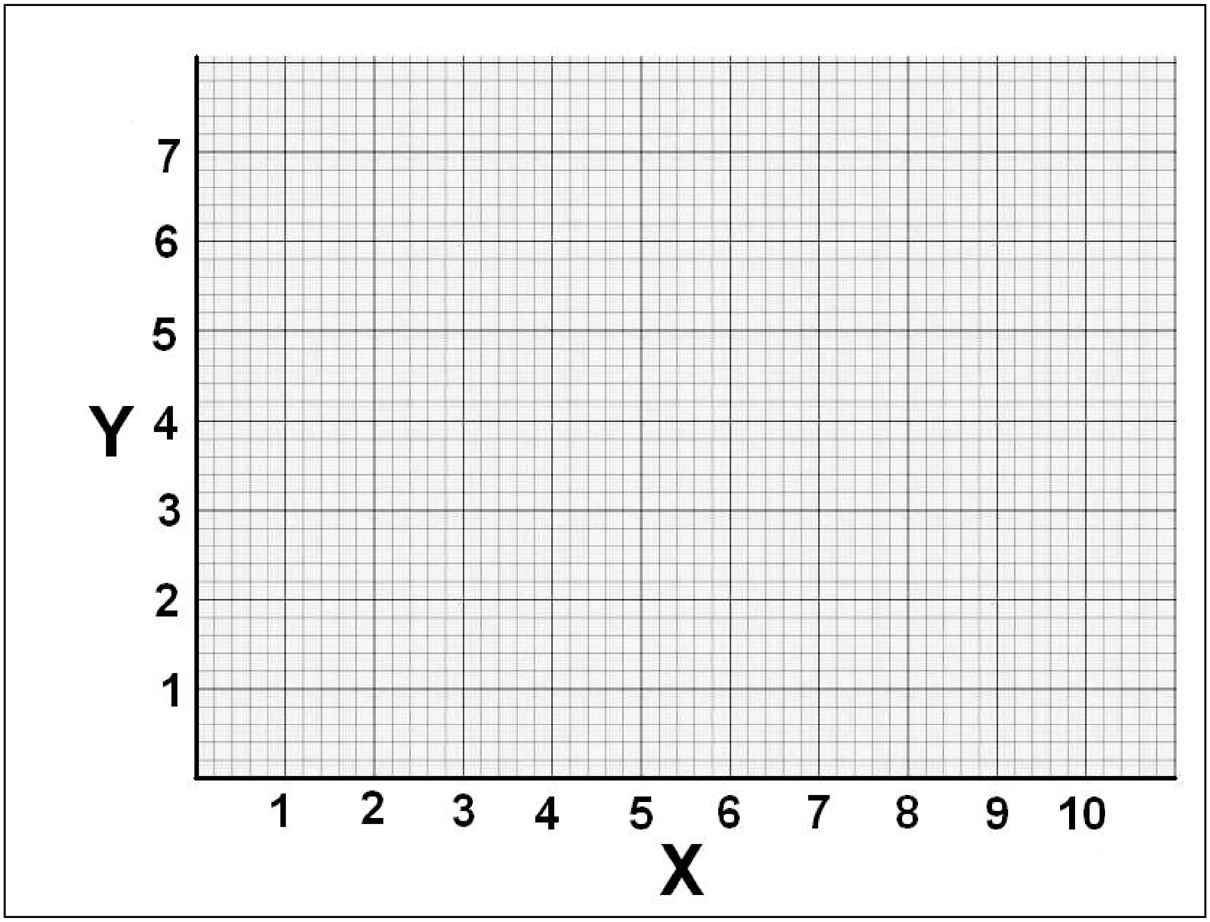
h) + 4 (+ 5 – 2) – 3 (+ 3)

k) + 5 (+ 8 – 5) – 4 (- 6 + 8)

s) - 1 + 1 – 1 + 1 + 1 – 1 + 1

w) + 3 (- 2 + 8) + (- 12 + 4)

z) - 5 + 2 (- 8 + 4)



Problem 1 – The Van Allen Probe A spacecraft travels along its orbit from point A(+3,+4) to point B(+7,+4). Draw an arrowed segment showing the direction that the spacecraft is traveling.

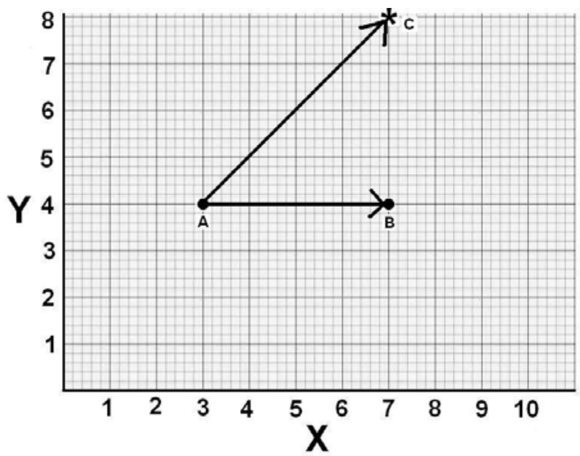
Problem 2 – Earth's magnetic field in this region of space passes through Point A, in the direction of Point C(+7, +8). Draw an arrowed segment showing the magnetic field direction.

Problem 3 – From the geometry of the triangle ABC, what angle does the magnetic field make to the direction of the spacecraft motion?

Problem 4 – If the strength of the magnetic field in the direction AC is 14 nanoTeslas, what would the 'shadows' of the magnetic field strength be along the directions AB and AC? (These are called the components of the magnetic field.)

Problem 1 – The Van Allen Probe A spacecraft travels along its orbit from point A(+3,+4) to point B(+7,+4). Draw an arrowed segment showing the direction that the spacecraft is traveling. Answer: See below.

Problem 2 – Earth's magnetic field in this region of space passes through Point A, in the direction of Point C(+7, +8). Draw an arrowed segment showing the magnetic field direction.



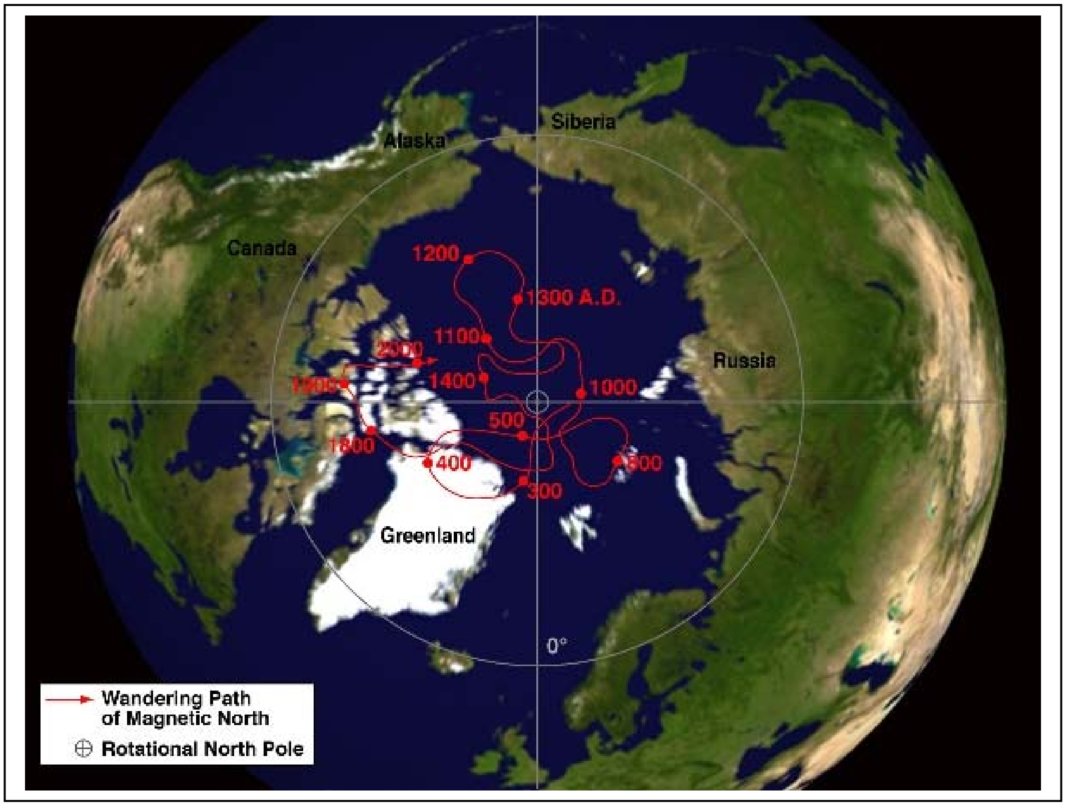
Problem 3 – From the geometry of the triangle ABC, what angle does the magnetic field make to the direction of the spacecraft motion?

Answer: The horizontal segment AB is 4.0 units long and the vertical segment BC is 4.0 units long so this triangle is a 45:45:90 right triangle. The angle that AC makes to segment AB is then 45^o.

Problem 4 – If the strength of the magnetic field in the direction AC is 14 nanoTeslas, what would the 'shadows' of the magnetic field strength be along the directions AB and AC? (These are called the components of the magnetic field.)

Answer: In a 45:45:90 right triangle, the side lengths are 1.0 and the hypotenuse is $\sqrt{2}$ so By using proportions, the sides of the 'magnetic triangle' with a hypotenuse of 14 nanoTeslas are 14 nanoTeslas/$\sqrt{2}$ = 14 nanoTeslas/1.4 = 10 nanoTeslas. So the components to the magnetic field are +10 nanoTeslas along direction AB and +10 nanoTeslas along direction BC. We can also write this as (+10 nT, +10nT)

Name ______________________________



The Earth's magnetic North Pole moves around quite a bit, especially if you plot its position during the last 1700 years! The scale of the above plot is approximately 760 km per centimeter. Use this scale with a piece of string and the dates for each position, to answer the following questions:

Question 1: What is the total distance that the North Magnetic Pole wandered from 300 AD to 2000 AD?

Question 2: What is the shortest distance that the pole wandered in a 100-year period?

Question 3: What is the longest distance that the pole wandered in a 100-year period?

Question 4: What is the average speed of the wander from 300 AD to 2000 AD?

Question 5: What is the fastest speed it traveled in a 100-year period?

Question 6: Is the speed of the wander during the last 100 years unusual compared to the average speed or to the fastest speed?

This exercise introduces students to the idea that Earth's magnetic poles are not fixed in space and time. Since the 1700's, mapmakers have known that the bearings to seaports and other fixed landmarks change in a steady manner from decade to decade so that maps often have to be re-drawn to reflect the new bearings. Geologists call this phenomenon 'polar wander'. (It has nothing to do with Earth's rotation axis!!)

Students will be asked to study a map of the wandering magnetic North Pole and answer some quantitative questions. To measure the distance (along the red track) that the magnetic pole has wandered, have students use a piece of string laid along the track, and then measure the length of the track in centimeters. The scale of their map is about 760 km/cm, so multiplying the string length by this scale factor, they can easily compute the track length and answer the questions. Students will also need to compute the speed of the pole movement between the years indicated on the map, by dividing the relevant track interval they measured by the difference in the years.

1) What is the total distance that the North Magnetic Pole wandered from 300 AD to 2000 AD?

Answer: The string was 24 cm long which equals 24 cm x 760 km/cm = 18,240 km.

2) What is the shortest distance that the pole wandered in a 100-year period?

Answer: Between 500 and 800 AD the string measured 2 cm or 1520 km. The time interval is 3 centuries, so in 1 century the pole traveled 1520/3 = 507 km.

3) What is the longest distance that the pole wandered in a 100-year period?

Answer: The longest distance is between 1300 and 1400 AD for a length of 3.3 cm or about 2500 km

4) What is the average speed of the wander from 300 AD to 2000 AD?

Answer: Average speed is the distance divided by time = 18,240 km/1700 years or 11 km/year.

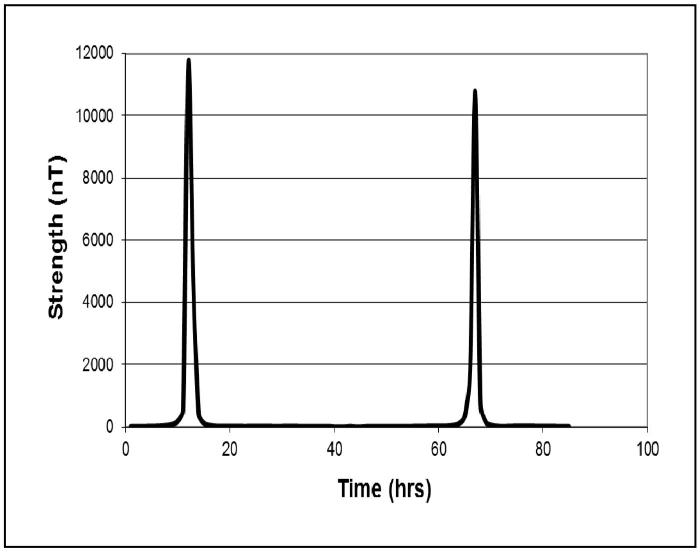
5) What is the average fastest speed it traveled in a 100-year period?

Answer: Between 1300 and 1400 it traveled 2470 km in 100 years or 24.7 km/year!

6) Is the speed of the wander during the last 100 years unusual compared to the average speed or to the fastest speed?

Answer: 1.25 cm = 960 km so 950km/100yrs = 9.5 km/year. It's above average but not a record-holder!

Note to teachers: If you have the students calculate the average speeds for each 100-year period and plot this on a speed vs year graph, the students will see that the polar wander represents accelerated motion because the speed is not constant from century to century!



The Cluster satellite constellation consists of 5 satellites orbiting Earth in a close formation. They were designed to measure Earth's magnetic field, and particles in space such as protons and electrons.

This graph shows the strength of Earth's magnetic field measured by the Cluster C1 satellite as it orbited Earth between January 1 and January 6, 2010.

Problem 1 - About what is the highest magnetic field strength measured along the satellite's orbit?

Problem 2 - The satellite's orbit had a perigee (closest point to Earth) of 10,000 km and an apogee (farthest distance from Earth) of 140,000 km. About what was the strength of the magnetic field at A) Perigee? B) Apogee?

Problem 3 - How many hours did it take the satellite to complete one orbit? Explain how you determined this from the graph.

Problem 4 - Below is a table of data taken at specific distances from Earth during the orbit. The strength of the magnetic field is given in units of the nanoTesla (nT). Graph this data. Does the strength decrease as the inverse-square or inverse-cube of the distance?

Point	Distance (km)	Strength (nT)
1	10,000	10,000
2	30,000	370
3	40,000	160
4	50,000	83
5	60,000	44
6	70,000	30
7	140,000	6

Problem 1 - Answer: The two peaks are at 11,700 nT and 10,800 nT so the maximum value occurs for the first peak with **11,700 nT**.

Problem 2 - Answer: A) At perigee, the satellite is closest to Earth so the strength of the magnetic field should be at its highest point along the orbit or 11,700 nT. B) At apogee the spacecraft is farthest from Earth and the strength is lowest, which from the graph is near-zero.

Problem 3 - Answer: The time between perigee of one orbit and perigee of the next orbit is just the time between the maximum measured magnetic strengths of these two consecutive orbits. From the graph, and using a millimeter ruler to get the correct scale, the time between the peaks is about 54 hours.

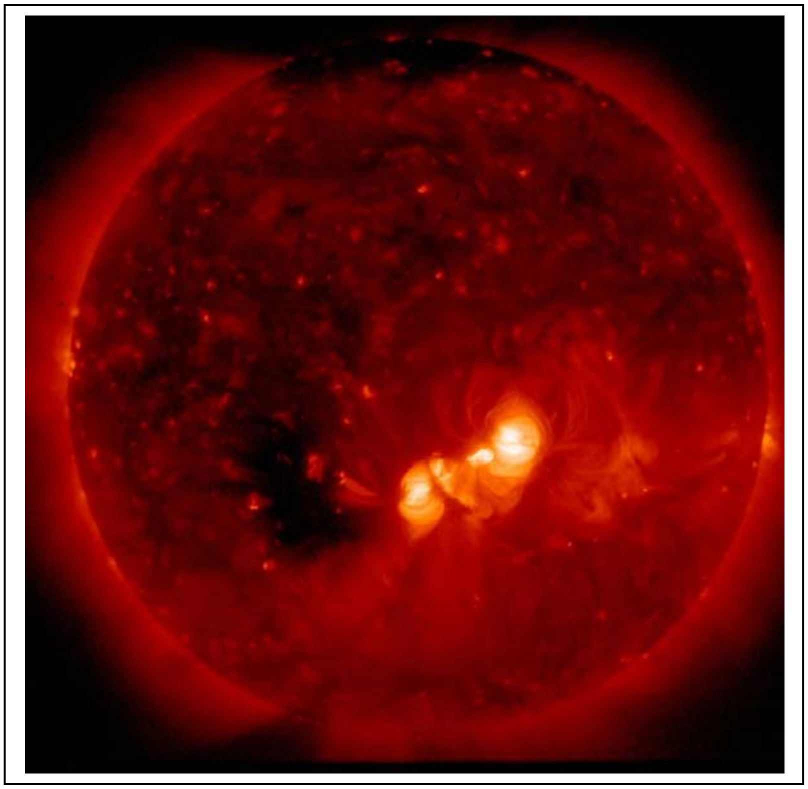
Problem 4 - Below is a table of data taken at specific distances from Earth during the orbit. The strength of the magnetic field is given in units of the nanoTesla (nT). Graph this data. Does the strength of the magnetic field decrease as the inverse-square or inverse-cube of the distance from Earth?

Answer: Inverse-square: Example of this model would predict between Point 1 and Point 2 that the intensity would drop by $1/(3^2)$ so it would be 1,111 nT
Inverse-cube: it would be $10{,}000/(3^3) =$ 370 nT as shown in the table.

We see that the inverse-cube model fits the data much better than the inverse-square distance law.

Point	Distance (km)	Strength (nT)	**Inverse-square**	**Inverse-cube**
1	10,000	10,000	**10,000**	**10,000**
2	30,000	370	**1,111**	**370**
3	40,000	160	**625**	**156**
4	50,000	83	**400**	**80**
5	60,000	44	**278**	**46**
6	70,000	30	**204**	**29**
7	140,000	6	**51**	**4**

The Hinode satellite views the sun

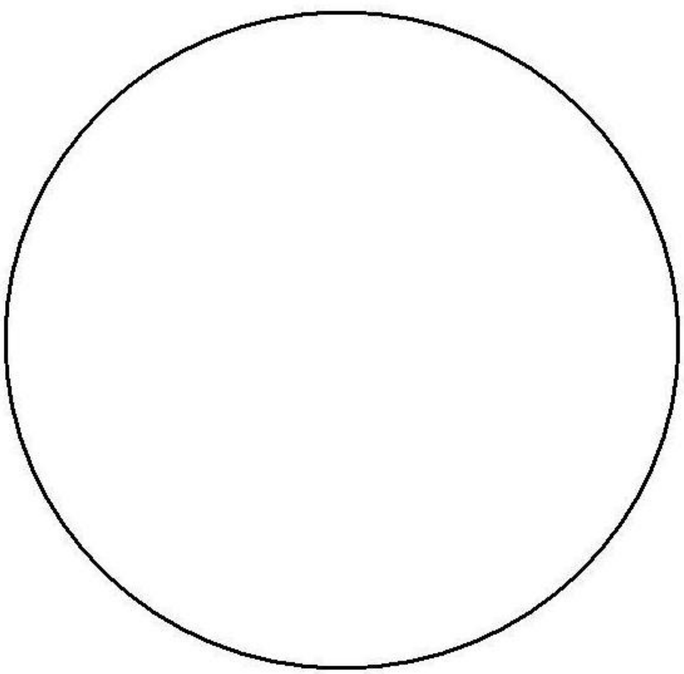


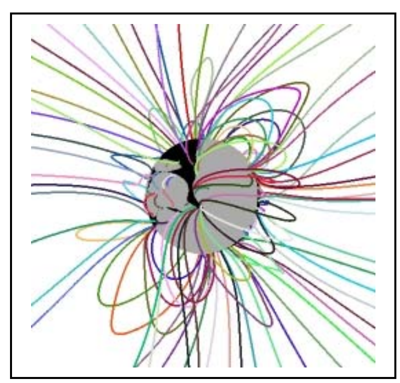
This image was taken by the X-Ray Telescope (XRT) on the Hinode solar observatory in December, 2006. It shows the complex magnetic structure over a large sunspot called Active Region AR930. You can also see large numbers of bright 'freckles' - each representing a small micro-flare.

The large black 'holes' are places in the corona of the sun where high-temperature gas is free to escape from the sun, and so there is little gas to illuminate these regions of the solar corona. This is because in these 'coronal holes' magnetic field lines open out to interplanetary space. Closed field lines near the surface act like magnetic bottles and keep the heated plasma close to the sun, creating the bright areas (red and yellow colors).

Using a black pen or pencil, try your hand at predicting what the magnetic field lines look like using the clues from Hinode picture!

Below is an example of a field line model calculated from an image by the SOHO satellite.





Answer Key:

Students may come up with several different versions. The main thing to look for is that in the regions where the Hinode picture shows orange or yellow, students should draw loops of magnetism…like a bar magnet field….that are close-in to the solar surface. In the black regions (north pole) of sun and the large spot to the left of the sunspot (yellow), they should draw magnetic lines that start in the dark region but end outside the picture because they are continuing on into interplanetary space. Below is a possible drawing! Students may notice that the gases are brightest in the lower-right quadrant so there are more closed magnetic field 'loops' there. There are also more dark areas in the top half of the image so there are probably a mixture of open and closed field lines, and not as many closed ones as in the lower-right quadrant. They should definitely realize that the two large dark areas at the top 'north' pole and to the left of the bright yellow sunspot region contain open field lines. The bright yellow active region should have a number of large loops and a higher density of them than elsewhere.

Hinode Studies Loopy Sunspots!

The solar surface is not only a hot, convecting ocean of gas, but is laced with magnetism. The sun's magnetic field can be concentrated into sunspots, and when solar gases interact with these magnetic fields, their light lets scientists study the complex 'loopy' patterns that the magnetic fields make as they expand into space. The above image was taken by NASA's TRACE satellite and shows one of these magnetic loops rising above the surface near two sunspots. The horseshoe shape of the magnetic field is anchored at its two 'feet' in the dark sunspot regions. The heated gases become trapped by the magnetic forces in sunspot loops, which act like magnetic bottles. The gases are free to flow along the lines of magnetic force, but not across them. The above image only tells scientists where the gases are, and the shape of the magnetic field, which isn't enough information for scientists to fully understand the physical conditions within these magnetic loops. Satellites such as Hinode carry instruments like the EUV Imaging Spectrometer, which lets scientists measure the temperatures of the gases and their densities as well.

Problem 1: The Hinode satellite studied a coronal loop on January 20, 2007 associated with Active Region AR 10938, which was shaped like a semi-circle with a radius of 20,000 kilometers, forming a cylindrical tube with a base radius of 1000 kilometers. What was the total volume of this magnetic loop in cubic centimeters assuming that it is shaped like a cylinder?

Problem 2: The Hinode EUV Imaging Spectrometer was able to determine that the density of the gas within this magnetic loop was about 2 billion hydrogen atoms per cubic centimeter. If a hydrogen atom has a mass of 1.6×10^{-24} grams, what was the total mass of the gas trapped within this cylindrical loop in metric tons?

Answer Key:

Problem 1: The Hinode satellite studied a coronal loop on January 20, 2007 associated with Active Region AR 10938, which was shaped like a semi-circle with a radius of 20,000 kilometers, forming a cylindrical tube with a base radius of 1000 kilometers. What was the total volume of this magnetic loop in cubic centimeters assuming that it is shaped like a cylinder?

Answer: The length (h) of the cylinder is 1/2 the circumference of the circle with a radius of 20,000 km or $h = 1/2\ (2\pi\ R) = 3.14 \times 20{,}000$ km = 62,800 km

The volume of a cylinder is $V = \pi R^2 h$ so that the volume of the loop is

$V = \pi\ (1000\ \text{km})^2 \times 62{,}800$ km

$= 2.0 \times 10^{11}$ cubic kilometers.

1 cubic kilometer = 10^{15} cubic centimeters so

= **2.0×10^{26} cubic centimeters**

Problem 2: The Hinode EUV Imaging Spectrometer was able to determine that the density of the gas within this magnetic loop was about 2 billion hydrogen atoms per cubic centimeter. If a hydrogen atom has a mass of 1.6×10^{-24} grams, what was the total mass of the gas trapped within this cylindrical loop in metric tons?

Answer: The total mass is the product of the density times the volume, so

Density = 2×10^9 particles/cc $\times$ (1.6×10^{-24} grams/particle) = 3.2×10^{-15} grams/cm^3

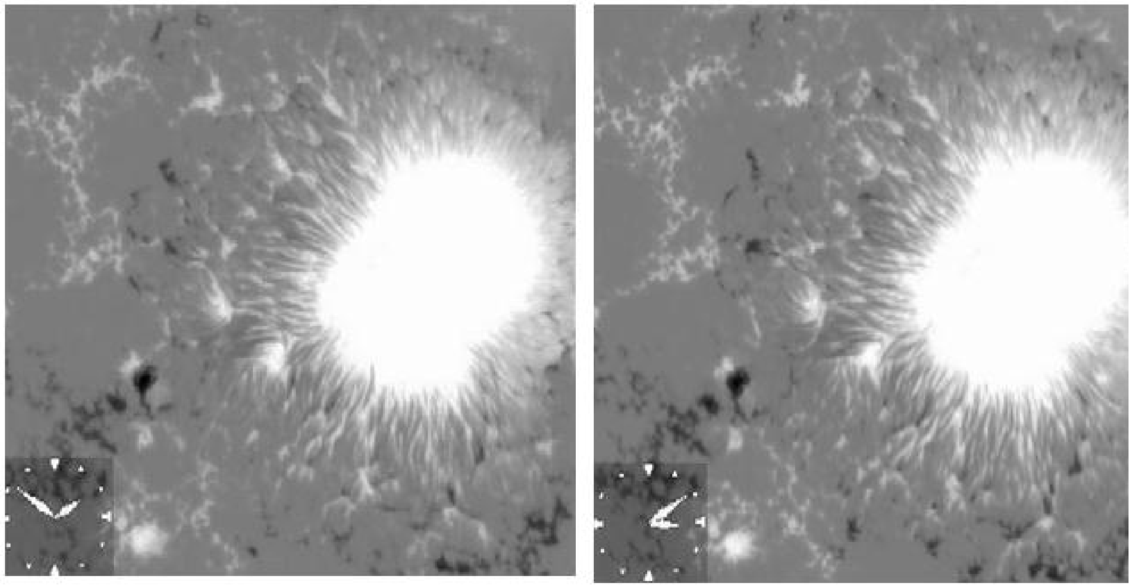
The approximate volume of the magnetic loop in cubic centimeters is

$V = (2.0 \times 10^{11}\ \text{km}^3) \times (1.0 \times 10^{15}\ \text{cm}^3/\text{km}^3)$

$= 2.0 \times 10^{26}\ \text{cm}^3$

Mass = Density x Volume = $(3.2 \times 10^{-15}\ \text{grams/cm}^3) \times (2.0 \times 10^{26}\ \text{cm}^3) = 6.4 \times 10^{26-15}$ $= 6.4 \times 10^{11}$ grams or 6.4×10^{8} kilograms or **640,000 metric tons.**

Moving Magnetic Filaments Near Sunspots



These two images were taken by the Hinode (Solar-B) solar observatory on October 30, 2006. The size of each image is 34,300 km on a side. The clock face shows the time when each image was taken, and represents the face of an ordinary 12-hour clock.

1) What is the scale of each image in kilometers per millimeter?

2) What is the elapsed time between each image in; A) hours and minutes? B) decimal hours? C) seconds?

Carefully study each image and look for at least 5 features that have changed their location between the two images. (Hint, use the nearest edge of the image as a reference).

3) What direction are they moving relative to the sunspot?

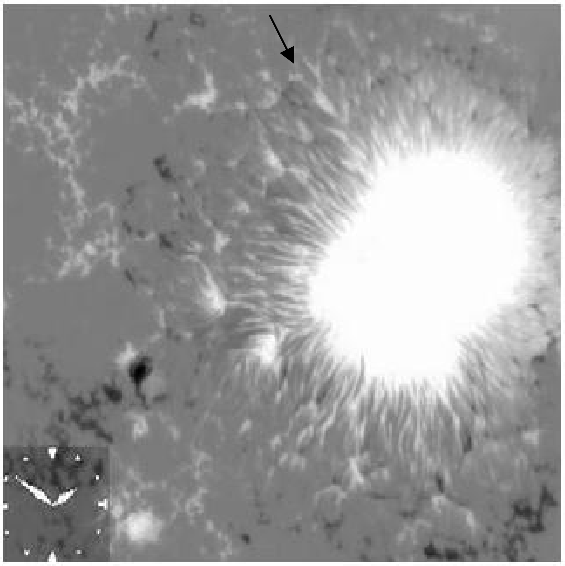
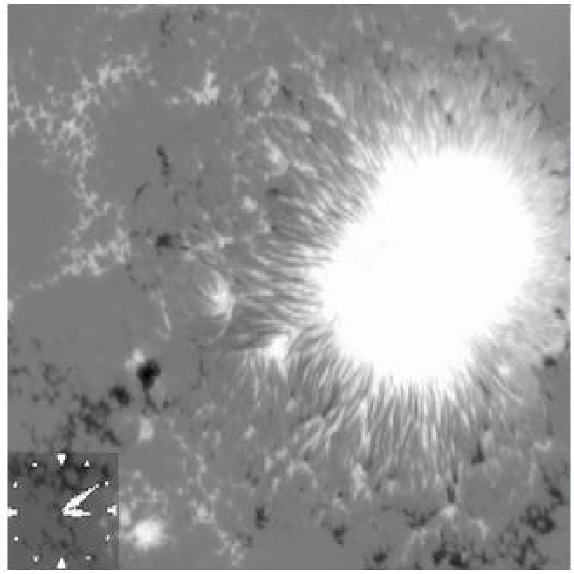
4) How far, in millimeters have they traveled on the image?

5) From your answers to questions 1, 2 and 4, calculate their speed in kilometers per second, and kilometers per hour.

6) A fast passenger jet plane travels at 600 miles per hour. The Space Shuttle travels 28,000 miles per hour. If 1.0 kilometer = 0.64 miles, how fast do these two craft travel in kilometers per second?

7) Can the Space Shuttle out-race any of the features you identified in the sunspot image?

Answer Key:

These two images were taken by the Hinode (Solar-B) solar observatory on October 30, 2006. The size of each image is 34,300 km on a side. The clock face shows the time when each image was taken.

1) What is the scale of each image in kilometers per millimeter? **Answer:** The pictures are 75 mm on a side, so the scale is 34,300 km /75mm = **457 km/mm**

2) What is the elapsed time between each image in;

A) hours and minutes? About 1 hour and 20 minutes.
B) decimal hours? About 1.3 hours
C) seconds? About 1.3 hours x 3600 seconds/hour = 4700 seconds

Carefully study each image and look for at least 5 features that have changed their location between the two images. (Hint, use the nearest edge of the image as a reference). Students may also use transparent paper or film, overlay the paper on each image, and mark the locations carefully.
The above picture shows one feature as an example.

3) What direction are they moving relative to the sunspot?
Answer: Most of the features seem to be moving away from the sunspot.

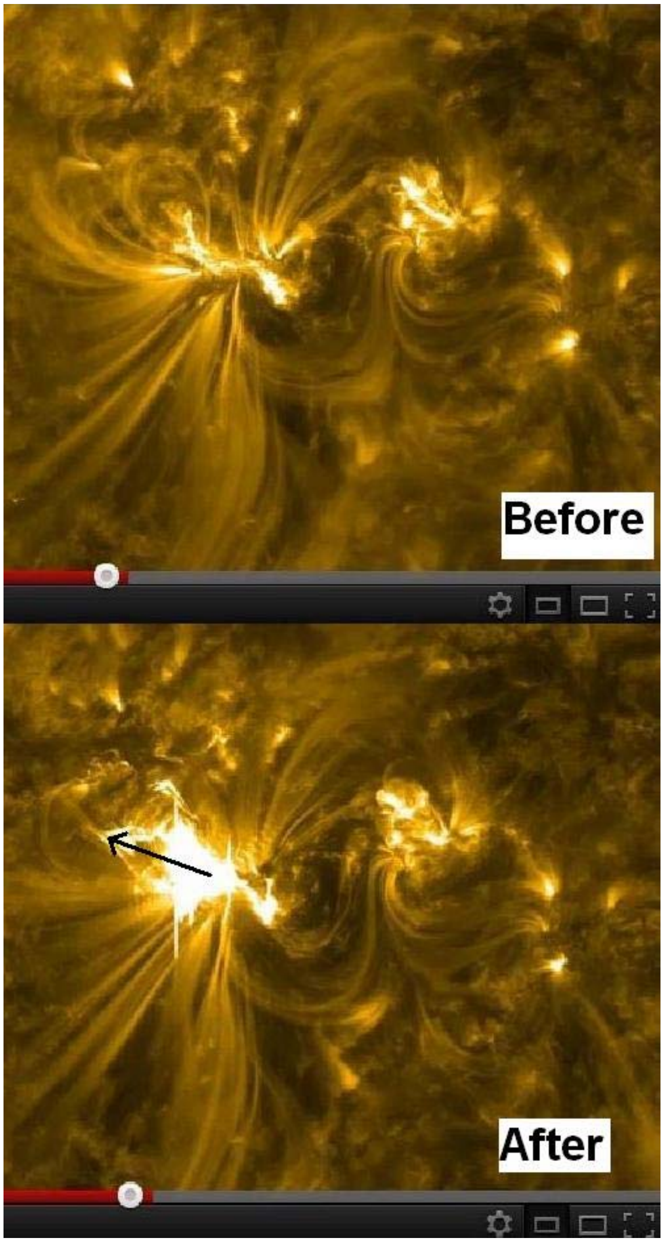
4) How far, in millimeters have they traveled on the image? **Answer:** The feature in the above image has moved about 2 millimeters.

5) From your answers to questions 1, 2 and 4, calculate their speed in kilometers per second, and kilometers per hour. **Answer:** 2 mm x 457 km/mm = 914 kilometers in 4700 seconds = 0.2 kilometers/sec or 703 kilometers/hour.

6) A fast passenger jet plane travels at 600 miles per hour. The Space Shuttle travels 28,000 miles per hour. If 1.0 kilometer = 0.64 miles, how fast do these two craft travel in kilometers per second? Jet speed = 600 miles/hr x (1 / 3600 sec/hr) x (1 km/0.64 miles) = **0.26 km/sec**. Shuttle = 28,000 x (1/3600) x (1/0.64) = **12.2 km/sec.**

7) Can the Space Shuttle out-race any of the features you identified in the sunspot image? **Answer:** Yes, in fact a passenger plane can probably keep up with the feature in the example above!

8) Are the features moving at increasing speed away from the sunspot, or traveling at a constant speed?



On March 6, 2012 Active Region 1429 produced a spectacular X5.4 solar flare shown in the pair of images taken by the NASA Solar Dynamics Observatory. The time between the two images is 2 seconds, and the width of each image is 318,000 kilometers.

Problem 1 - The arrowed line indicates how far the million-degree gas produced by the flare traveled in the time interval between the images. What was the speed of the gas, called a plasma, in kilometers/sec?

Problem 2 - By carefully looking at the two images, what other features can you find that changed their position in the time between the images?

Problem 3 - What would you estimate the speeds to be for the features that you identified in Problem 2?

The explosion can be seen in the movie located on *YouTube* at
http://www.youtube.com/watch?v=4xKRBkBBEP0

The above 'high definition' images of the spectacular March 6 solar flare were taken by the NASA Solar Dynamics Observatory (SDO) located in geosynchronous orbit around Earth. Within a few minutes, this flare produced more energy than a thousand hydrogen bombs going off all at once. This energy caused gasses nearby to be heated to millions of degrees, and produced a blast-wave that traveled across the face of the sun at over 4 million km/hour. In the movie above, watch for magnetic loops and filaments to be disturbed by the explosion as the blast wave passes by them.

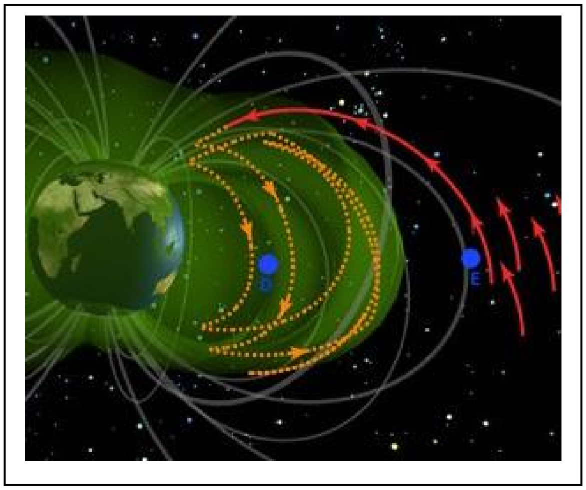
Problem 1 - The arrowed line indicates how far the million-degree gas produced by the flare traveled in the time interval between the images. What was the speed of the gas, called a plasma, in kilometers/sec?

Answer: Students should compare the length of the arrowed line to the width of the image and solve the proportion to get the physical distance that the plasma traveled. Example, for standard printing onto an 81/2 x 11 page, the line width is about 14 millimeters long, and the width of the image is about 84 millimeters so the proportion for the true length of the line in kilometers is just

$$\frac{14}{84} = \frac{X}{318,000 km}$$ so X = 53,000 km

The speed of the flare is simply the distance traveled (53,000 km) divided by the time between the two images (2 seconds) so V = 53,000/2 = **26,500 km/sec**.

The actual speed of the plasma will be much less than this because some of the brightening you see in the second image is because gases trapped in the magnetic field loops were heated in place and brightened rapidly without much movement. As one region dims and another brightens we have the appearance of something moving between the two locations when in fact there was little of no actual movement.



Amateur radio operators have been hearing this sound for decades, especially at 'dawn'. It is an eery sound, like a chorus of birds chirping, so it was called Dawn Chorus.

This sound cannot be heard with ordinary ears even though it is in the right frequency range. Because it is a radio wave, you need a radio receiver to hear it.

Space physicists have tried to understand what produces this electromagnetic 'sound wave', but whatever is producing it is occurring somewhere in the van Allen belts high above Earth.

During its 60-day checkout phase, the twin Van Allen Probes satellites captured chorus waves close to where they are being produced in the Van Allen belts. As the satellites continue to take more data, scientists hope to be able to triangulate the location of these waves to their place of origin. This will provide scientists with a HUGE clue about what is causing them in the first place.

Let's have a look at how they will 'triangulate' the chorus position in space using simple graphing techniques, a protractor and the Pythagorean Theorem!

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0].

Problem 2 – The RBSP-A spacecraft detects the Chorus signal coming from a direction angle of 198^{o}. RBSP-B detects the same signal coming from a direction angle of 135^{o}. Draw two lines at these angles from the locations of VAP-A and B to locate the source of the Chorus signal.

Problem 3 - What is the coordinate of the intersection point of these two lines, and the location of the Chorus signal?

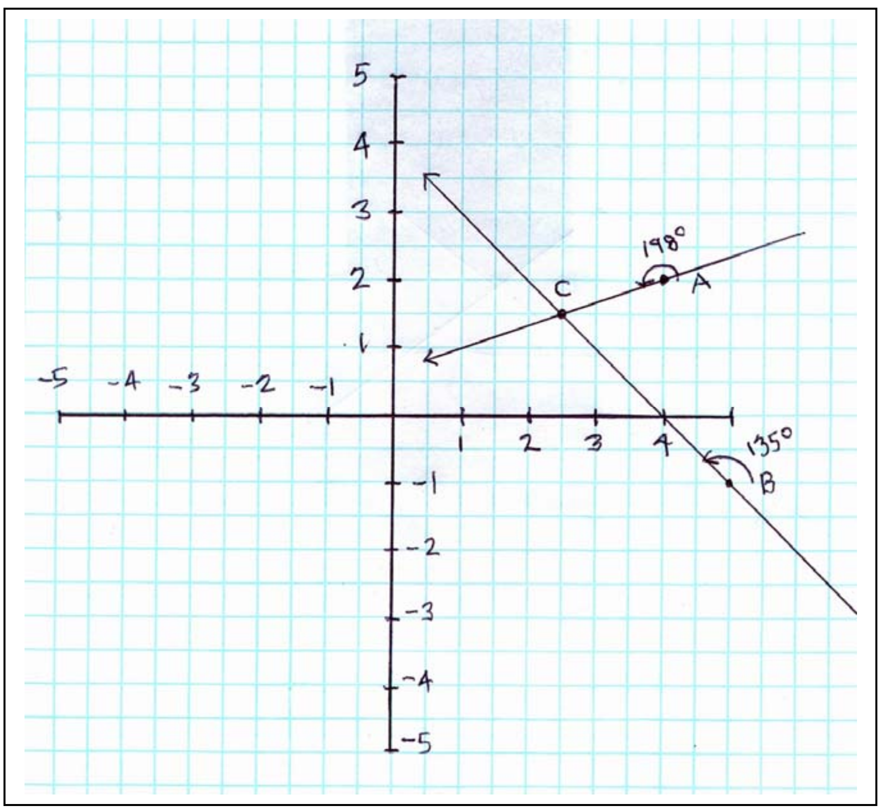
Problem 4 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal?

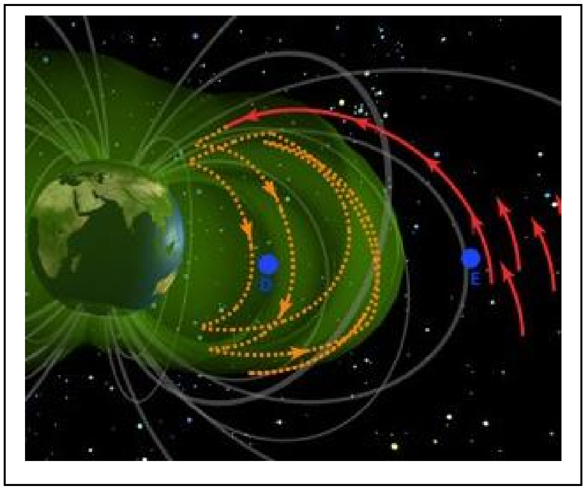
Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0]. Answer: **See figure below**

Problem 2 – The VAP-A spacecraft detects the Chorus signal coming from a direction angle of 198^{o}. RBSP-B detects the same signal coming from a direction angle of 135^{o}. Draw two lines at these angles from the locations of VAP-A and B to locate the source of the Chorus signal. Answer: Students place the protractor centered on each spacecraft point, with the bottom edge parallel to the horizontal X-axis. They measure the two degree angles and draw a line through each point.

Problem 3 - What is the coordinate of the intersection point of these two lines, and the location of the Chorus signal? Answer: The intersection point is at **C:(+2.5, +1.5)**

Problem 4 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal? Answer: Students can use a ruler and from this scaled drawing determine that the length of the chord from VAP-A to Point C is about 1.6 units or 1.6 x 6400 km = **10,200 kilometers.** Students may also use the distance formula $d^2 = (4.0-2.5)^2 + (2.0-1.5)^2$ so d = 1.6 units and so d = 10,200 km.





Amateur radio operators have been hearing this sound for decades, especially at 'dawn'. It is an eery sound, like a chorus of birds chirping, so it was called Dawn Chorus.

This sound cannot be heard with ordinary ears even though it is in the right frequency range. Because it is a radio wave, you need a radio receiver to hear it.

Space physicists have tried to understand what produces this electromagnetic 'sound wave', but whatever is producing it is occurring somewhere in the Van Allen belts high above Earth.

During its 60-day checkout phase, the twin Van Allen Probes satellites captured chorus waves close to where they are being produced in the Van Allen belts. As the satellites continue to take more data, scientists hope to be able to triangulate the location of these waves to their place of origin. This will provide scientists with a HUGE clue about what is causing them in the first place.

Let's have a look at how they will 'triangulate' the chorus position in space using simple linear equation techniques and the 2-point distance formula !

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center, and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0].

Problem 2 – The VAP-A spacecraft detects the Chorus signal coming from a direction along the line given by the formula $3y = x + 2$.

VAP-B detects the same signal coming from a direction somewhere along the line given by the formula $y = -x + 4$.

What are the coordinates of the intersection point of these two direction lines?

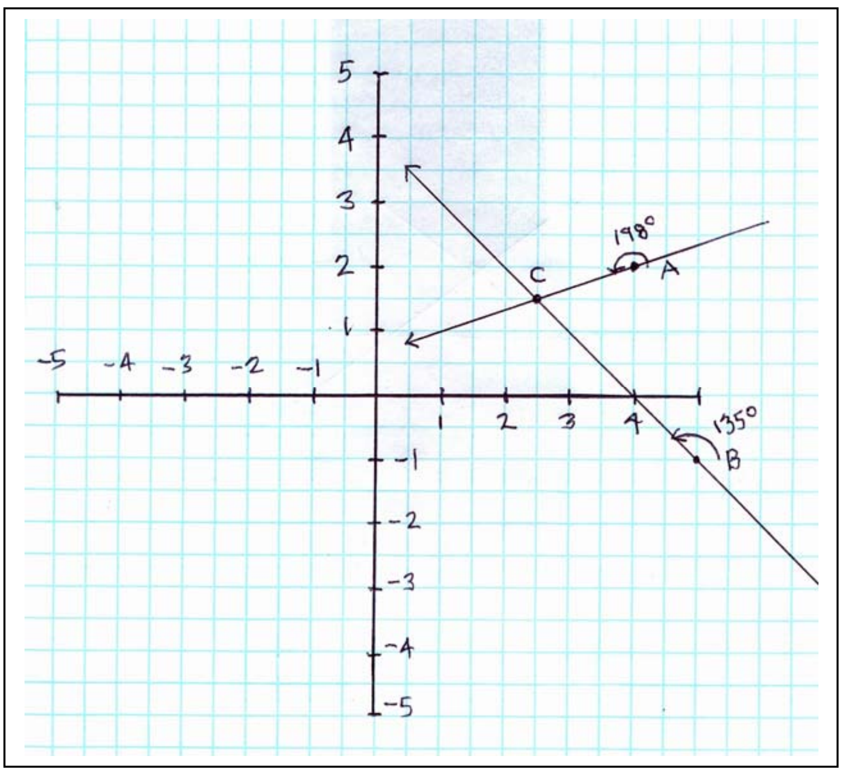
Problem 3 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal?

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center, and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0]. Answer: **See figure below**

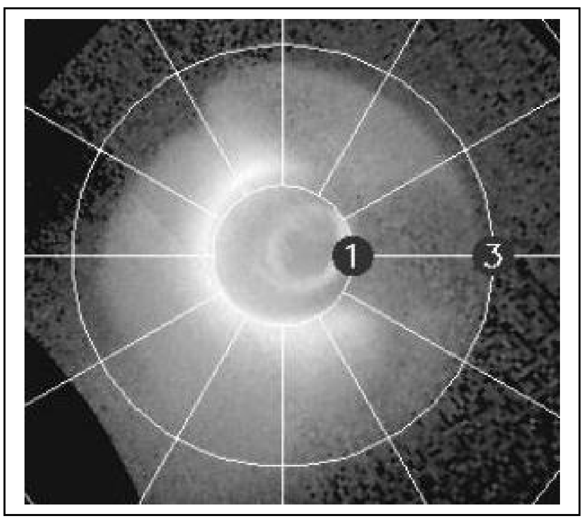
Problem 2 – The VAP-A spacecraft detects the Chorus signal coming from a direction along the line given by the formula 3y = x + 2. VAP-B detects the same signal coming from a direction somewhere along the line given by the formula y= -x + 4. What are the coordinates of the intersection point of these two direction lines?

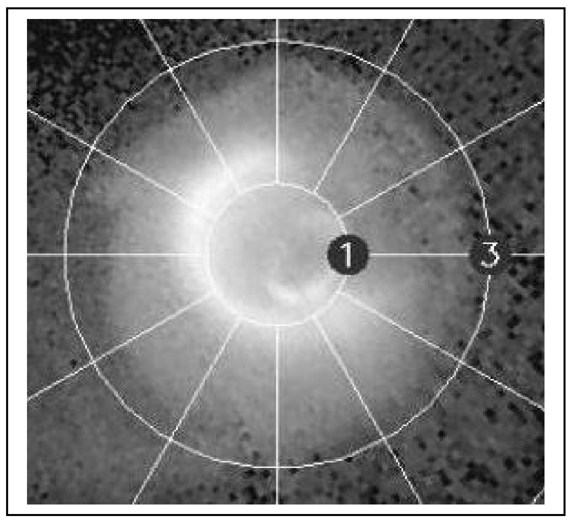
Answer: the intersection point is a solution to both equations simultaneously. Students may use the substitution method, 3(-x+4) = x + 2 so -3x + 12 = x + 2 and so 4x = 10 and x = 2.5, then y = -(2.5)+4 = +1.5 so **C: (+2.5, +1.5)**.

Problem 3 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal? Answer: Students may use the distance formula $d^2 = (4.0-2.5)^2 + (2.0-1.5)^2$ so d = 1.6 units and so **d = 10,200 km**.



The Plasmasphere is a region of Earth's atmosphere above the ionosphere, and extending over 15,000 kilometers into space. The Space Shuttle and the International Space Station 'fly' through this dilute gas which consists of ions of hydrogen and helium. Solar ultraviolet light ionizes these gases causing them to become charged particles. Because the ions are charged particles, they are strongly effected by Earth's magnetic field. Like riders on a carousel, the ions in the plasmasphere are dragged around the Earth by their intimate connection with the magnetic field. They do not orbit earth like spacecraft because gravity is not strong enough to overcome the magnetic forces acting on the plasmasphere gases. The two pictures below were obtained by the IMAGE satellite. They show the plasmasphere at two different times, 3 hours apart. The central round disk is the silhouette of Earth's disk, which has a radius of 6,400 kilometers. In this problem, you will use a protractor to measure the angular change in the plasmasphere during this 3-hour period, and use this to calculate how long it takes the plasmasphere to rotate around Earth. The image on the left looking down at Earth from above the North Pole was taken on May 24, 2004 at 06:52 UT. The image on the right was taken 3 hours later at 09:52 UT. The lines are drawn at 1-hour intervals with 30-degree spacings. (30 x 12 = 360).





Step 1 – Look carefully at each image and find a feature that has moved between the two images.

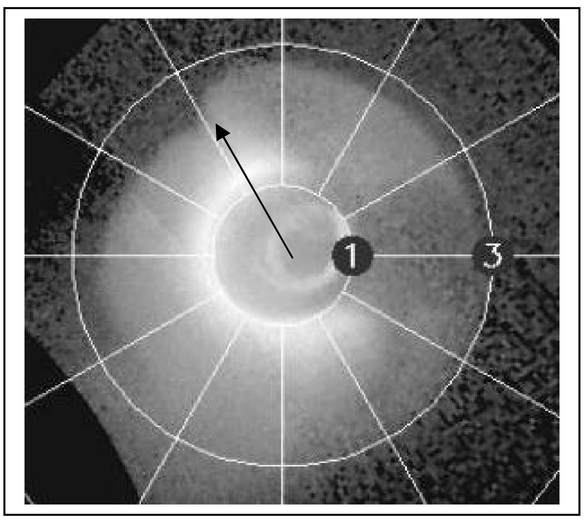
Step 2: From the 30-degree markings, or with a protractor, determine how many degrees the feature has moved between the two images.

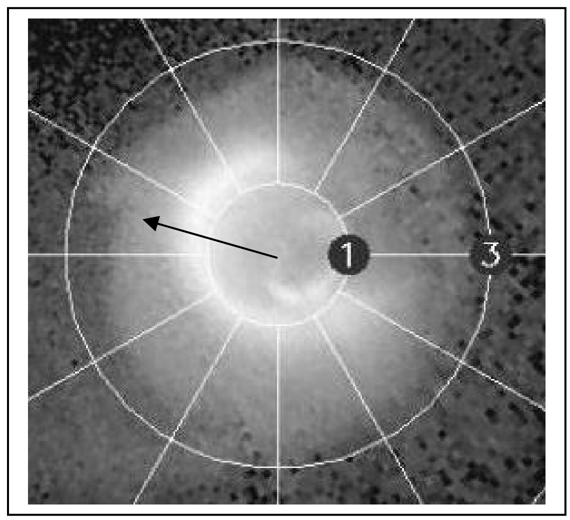
Question 1: How long would it take your feature to make one complete rotation around Earth?

Question 2: A satellite takes about 100 minutes to orbit Earth. Does the plasmasphere behave like a satellite?

The Plasmasphere is a region of Earth's atmosphere above the ionosphere, and extending over 15,000 kilometers into space. The Space Shuttle and the International Space Station 'fly' through this dilute gas which consists of ions of hydrogen and helium. Solar ultraviolet light ionizes these gases causing them to become charged particles. Because the ions are charged particles, they are strongly effected by Earth's magnetic field. Like riders on a carousel, the ions in the plasmasphere are dragged around the Earth by their intimate connection with the magnetic field. They do not orbit earth like spacecraft because gravity is not strong enough to overcome the magnetic forces acting on the plasmasphere gases.

In this activity, students will examine two images of the plasmasphere from space obtained by the IMAGE satellite. By using a protractor, they will measure the motion of the plasmasphere and calculate its rotation period.





Question 1: How long would it take your feature to make one complete rotation around Earth?
Answer: In the left image, the 'notch' at the 11:00 o'clock position has moved in the right image to between the 9 and 10:00 position. The angular change for the feature in the example marked with arrows above, is about 45 degrees. This change took 3 hours, so by a simple proportion (360/45)= (T/3hrs) the time, T, it takes to move a full 360 degrees is about 24 hours.

Question 2: A satellite takes about 100 minutes to orbit Earth. Does the plasmasphere behave like a satellite?
Answer: No. The plasmasphere moves much more slowly than a satellite does. This is because the plasmasphere is not in motion because of gravitational forces, as the reading material discusses. It is anchored to Earth's magnetic field, which rotates with the Earth. That means that it will rotate about once each say even at distances of 15,000 kilometers from the ground.

Name ________________________________

Most science courses discuss the various familiar forms of energy such as kinetic, potential, chemical, thermal and electrical. Most science courses also discuss magnetism, and electromagnetism. Combining these two ideas, we can imagine that magnetic fields also store and release energy – **magnetic energy**. This energy plays an important role in creating many of the most dramatic events scientists study, from solar flares to the Northern lights.

The amount of magnetic energy, **Em**, for a field strength, **B**, given in Gauss units and a volume, **V**, given in cubic centimeters can be calculated from this formula:

$$E_m = \frac{B^2}{8\pi} V$$

Most objects that astronomers study have shapes that can be approximated as spheres, or cylinders. Here are the formulas for the volumes, **V**, of these shapes. In these formulae, the variables are as follows:

R = radius of sphere or cylinder
h = height of cylinder

Sphere $\frac{4}{3}\pi R^3$

Cylinder $h \pi R^2$

Using the formulas above, calculate the total magnetic energy, Em, of each system in the table below. You will need to use scientific notation and a calculator!

Object	Shape	B (Gauss)	R (cm)	H (cm)	Em (ergs)
Earth	Sphere	0.5	6.4×10^8		
Geotail	Cylinder	0.002	5×10^9	1.5×10^{10}	
Sun	Sphere	5.0	6.9×10^{10}		
Solar Prominence	Cylinder	100	5×10^9	2.0×10^9	

Answer - Extra Credit Problem 12

Worked Answers:

Earth: Volume of sphere = 1.333 x 3.141 x (6.4 x 10^8 cm)^3 = 1094.8 x 10^{24} cm^3 = 1.1 x 10^{24} cm^3

Em = 0.0398 x (.5)^2 x 1.1 x 10^{24} = 1.1 x 10^{22} ergs.

Geotail: Volume of cylinder = 3.141 x (5 x 10^9 cm)^2 x 1.5 x 10^{10} cm = 117.8 x 10^{28} cm^3 = 1.2 x 10^{30} cm^3

Em = 0.0398 x (0.002)^2 x 1.2 x 10^{30} = 1.9 x 10^{23} ergs

Sun: Volume of sun = 1.38 x 10^{33} cm^3

Em = 1.4 x 10^{33} ergs

Solar Prominence: Volume of cylinder = 1.6 x 10^{29} cm^3

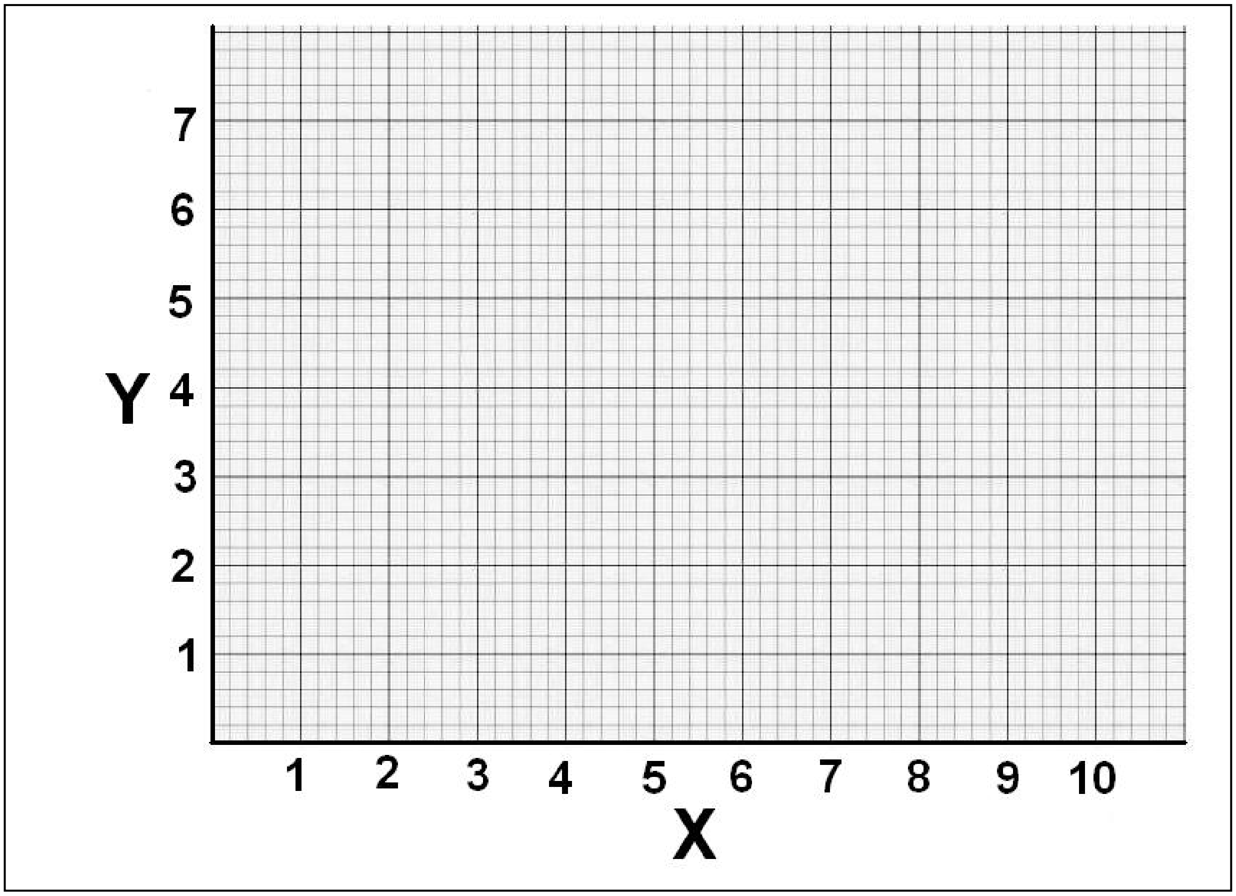
Em = 6.4 x 10^{31} ergs

Definitions:

The **Geotail** is the extension of Earth's magnetosphere into a comet-like tail in the opposite direction of the Sun. The magnetic energy released in the geotail causes charged particles to flow along the magnetic lines of force into Earth's Polar Regions, causing the Aurora Borealis and Aurora Australis.

Solar prominences are loops of magnetic force above large sunspots. The magnetic energy released by prominences can lead to solar flares and expulsions of gas called Coronal Mass Ejections.

Object	Shape	B (Gauss)	R (cm)	H (cm)	Em (ergs)
Earth	Sphere	0.5	6.4×10^{8}		1.1×10^{22}
Geotail	Cylinder	0.002	5×10^{9}	1.5×10^{10}	1.9×10^{23}
Sun	Sphere	5.0	6.9×10^{10}		1.4×10^{33}
Solar Prominence	Cylinder	100	5×10^{9}	2.0×10^{9}	6.4×10^{31}



Problem 1 – During part of its orbit around Earth, the Van Allen Probes travel along the line given by the equation $y = -1/2\, x + 2$. Graph this line on the grid above.

Problem 2 – Earth's magnetic field is oriented along lines that are parallel to $y = \frac{3}{4} X$. Draw three of these lines across the grid above.

Problem 3 – What is the equation of the line that is perpendicular to the spacecraft trajectory? Plot this line on the graph above.

Problem 4 – What angle does the magnetic field make with respect to the direction along the spacecraft trajectory?

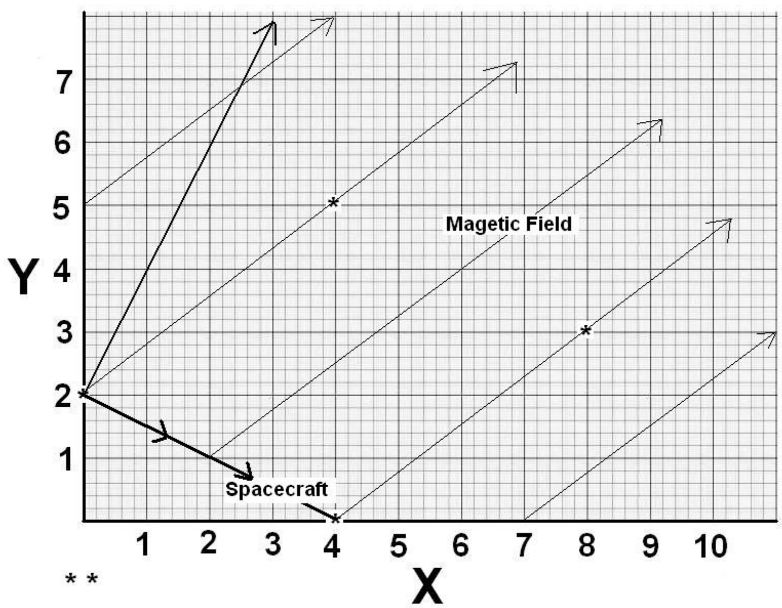
Problem 5 - What angle does the magnetic field make with respect to the direction perpendicular to the spacecraft trajectory?

Problem 1 – During part of its orbit around Earth, the Van Allen Probes travel along the line given by the equation y = -1/2 x + 2. Graph this line on the grid above. Answer: See below, labeled 'spacecraft'

Problem 2 – Earth's magnetic field is oriented along lines that are parallel to y = ¾ X. Draw three of these lines across the grid above. Answer: See below: Labeled 'magnetic field'

Problem 3 – What is the equation of the line that is perpendicular to the spacecraft trajectory? Plot this line on the graph above.

Answer: The perpendicular line to y = mx+b is y = -1/m x + b. The slopes are the negative reciprocals of each other. If the spacecraft direction is y = -1/2 X + 2, then the perpendicular is y = 2x+2 at point (0,+2), as shown in the figure.



* *

Problem 4 – What angle does the magnetic field make with respect to the direction along the spacecraft trajectory?

Answer: Use a protractor to measure the angle. It is **63 degrees**.

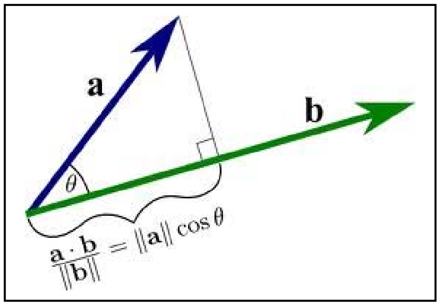
Problem 5 - What angle does the magnetic field make with respect to the direction perpendicular to the spacecraft trajectory?

Answer: It will be the compliment angle, 90-63 = **27 degrees**.

Point A: (4.0, 3.0)
Point B: (6.0, 10.0)

Vector Notation:

A = +4.0**x** +3.0**y**
B = +6.0**x** + 10.0**y**



The Van Allen Belt Probes spacecraft determine their orientation in space by using the direction of the local magnetic field of Earth. This is like using a compass to figure out where you should go next!

The VABP spacecraft measure the intensity of the magnetic field along, and perpendicular to their direction of motion at a given time. They can then use these measurements to find the angle between their motion (vector **A**) and the magnetic field (vector **B**). They compare their measurements against a mathematical model of Earth's magnetic field to find the field's true direction, then they rotate the spacecraft until the angles match up.

To do these calculations we have to determine the projection of the magnetic field onto the direction of the spacecraft motion, and to the axis perpendicular to this motion. This involves using the Vector Dot Product:

$$\mathbf{A}\ \mathbf{B} = ||A||\ ||B|| \cos(\theta)$$

which gives the <u>component</u> of vector **A** along the <u>direction</u> of the unit vector **B** where $||B||=1$.

Problem 1 – The spacecraft moves from point A (0,2) to point B (2,1). Write these points in vector notation and calculate the spacecraft motion vector **C** = **B** – **A**.

Problem 2 – Direction vectors are like motion vectors except they have unit length. What do you have to do to the motion vector, **C**, to give it a magnitude of exactly 1.0?

Problem 3 – What is the slope of the vector, **C**?

Problem 4 – The predicted magnetic field is given by the vector **B** = +4**x** + 3**y** in units of nanoTeslas. What is the projection of the magnetic field along the direction of motion of the spacecraft?

Problem 5 – The spacecraft measures a magnetic field strength of 1 nanoTesla along the spacecraft motion. What is the angle, θ, between the spacecraft motion and the local magnetic field?

Problem 1 – The spacecraft moves from point A (0,2) to point B (2,1). Write these points in vector notation and calculate the spacecraft motion vector **C** = **B** – **A**.

Answer: **A** = +2**y**, **B**= +2**x** + 1y, therefore **C = +2x – 1y**

Problem 2 – Direction vectors are like motion vectors except they have unit length. What do you have to do to the motion vector, **C**, to give it a magnitude of exactly 1.0?

Answer: You have to rescale the vector **C** so that its magnitude is 1.0. Since |C| = $(5)^{1/2}$, the direction vector is **D** = +2/$(5)^{1/2}$**x** – 1/$(5)^{1/2}$**y,** therefore |D| = 1.0

Problem 3 – What is the slope of the vector, **C**? Answer: **-1/2**

Problem 4 – The predicted magnetic field is given by the vector **M** = +4**x** + 3**y** in units of nanoTeslas. What is the projection of the magnetic field along the direction of motion of the spacecraft?

Answer: Use the dot product with **M** = 4**x**+3**y**, and **B** = (2**x**-1**y**) where ||B||=$(5)^{1/2}$.

M dot B/||B|| = (4x + 3y) dot (2x – 1y) / $(5)^{1/2}$

The projection is (8-3)/$(5)^{1/2}$ or $5^{1/2}$ **nanoTeslas.**

Problem 5 – The spacecraft measures a magnetic field strength of 1 nanoTesla along the spacecraft motion. What is the angle, θ, between the spacecraft motion and the local magnetic field?

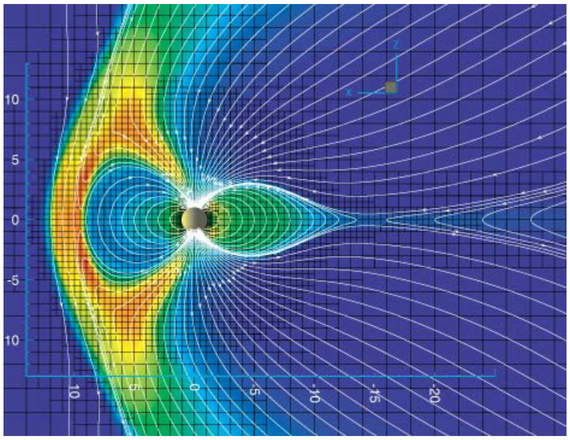
Answer: Use A B = ||A|| ||B|| cos(θ)

1 nano Tesla = M dot B/||B|| = ||M|| cos θ

Therefore 1 nanoTesla / ||B|| = cos θ and ||B|| = $5^{1/2}$

cos(θ) = 1/$5^{1/2}$
= 0.447

Therefore θ = **63 degrees**.



$$R = \left(\frac{1.8x10^{12}}{NV^2} \right)^{\frac{1}{6}}$$

$$Pr = 1.6x10^{-8} NV^2$$

$$Pm = 4.0x10^{-6} B^2$$

The ACE satellite measures the density and speed of the solar wind as it approaches Earth, and also measures the strength of its magnetic field. Both the magnetic field, and the kinetic energy of the particles, cause a build-up of pressure acting upon Earth's magnetic field. This forces Earth's magnetic field closer to the planet's surface, and can expose satellites orbiting Earth to the potentially harmful effects of cosmic rays and other high-energy particles. Based on actual data from the ACE satellite, in this problem you will calculate the particle and magnetic pressure and determine the distance from Earth of the pressure equilibrium region of the magnetic field, called the magnetopause.

The equations to the left give the magnetopause distance, R, in multiples of Earth's radius (6,378 km), and the magnetic (Pm) and ram (Pr) pressures in units of microErgs/cm^3, once the sped of the cloud (V in km/sec), density of the cloud (N in particles/cm^3) and the strength of the cloud's magnetic field (B in nanoTeslas) are specified.

Date	Flare	N (particle/cc)	V (km/s)	B (nT)	Pr	Pm	Distance (Re)
9-7-2005	X-17	50	2500	50	5.0	0.01	4.2
7-13-2005	X-14	30	2000	20			
1-16-2005	X-2.8	70	3700	70			
10-28-2003	X-17	100	2700	70			
11-4-2003	X-28	80	2300	49	6.8	0.01	4.0
4-21-2002	X-1.5	20	2421	10			
7-23-2002	X-4.8	40	1200	15			
4-6-2001	X-5.6	20	1184	20			
7-14-2000	X-5.7	30	2300	60			
11-24-2000	X-1.8	50	2000	10	3.2	0.0004	4.6
8-24-1998	X-1	15	1500	10			

Problem 1: Use the formulae and the values cited in the table to complete the last three columns. A few cases have been computed as examples.

Problem 2: A geosynchronous communications satellite is orbiting at a distance of 6.6 Re (1 Re = 1 Earth radius= 6,378 km). For which storms will the satellite be directly affected by the solar storm particles?

Date	Flare	N (particle/cc)	V (km/s)	B (nT)	Pr	Pm	Distance (Re)
9-7-2005	X-17	50	2500	50	5.0	0.01	4.2
7-13-2005	X-14	30	2000	20	**1.9**	**0.002**	**5.0**
1-16-2005	X-2.8	70	3700	70	**15.3**	**0.02**	**3.5**
10-28-2003	X-17	100	2700	70	**11.7**	**0.02**	**3.7**
11-4-2003	X-28	80	2300	49	6.8	0.01	4.0
4-21-2002	X-1.5	20	2421	10	**1.9**	**0.0004**	**5.0**
7-23-2002	X-4.8	40	1200	15	**0.9**	**0.0009**	**5.6**
4-6-2001	X-5.6	20	1184	20	**0.4**	**0.002**	**6.3**
7-14-2000	X-5.7	30	2300	60	**2.5**	**0.01**	**4.7**
11-24-2000	X-1.8	50	2000	10	3.2	0.0004	4.6
8-24-1998	X-1	15	1500	10	0.5	0.0004	6.1

Note: Density and magnetic field strength are estimates for purposes of this calculation only.

Problem 1: Use the formulae and the values cited in the table to complete the last three columns.

Answer: See above shaded table entries. This is a good opportunity to use an Excel spreadsheet to set up the calculations. This also lets students change the entries to see how the relationships change, as an aid to answering the remaining questions.

Problem 2: A geosynchronous communications satellite is orbiting at a distance of 6.6 Re. For which storms will the satellite be directly affected by the solar storm particles?

Answer: If the equilibrium radius is less than 6.6 Re, the satellite will be outside Earth's protective magnetosphere and within the region of space directly affected by the storm particles and fields. This condition is satisfied for all of the storms except for the ones on April 6, 2001 and August 24, 1998

Note to Teacher: Ram pressure is the pressure produced by a cloud of particles traveling at a particular speed with a particular density. We call this a ‘ram’ pressure because it is also the pressure that you feel as you ‘ram’ your way through the air when you are in motion. Because only the relative speed is important, you will feel the same pressure if you are ‘stationary’ and a gas is moving past you at a particular speed, or if the gas is ‘stationary’ and you are trying to move through it at the same speed. Technically, ram pressure is the product of the gas density and the square of this relative speed.

The values for the ram pressure (Pr) are all substantially larger than the values for the magnetic pressure (Pm), so we conclude that ram pressure is stronger than the cloud’s magnetic pressure. This means that when the cloud impacts another object such as Earth, it is mostly the ram pressure of the cloud that determines the outcome of the interaction.

The image used in this problem shows a computer-calculated model of Earth's magnetic field during compression. The yellow coloration indicates regions of maximum pressure. Image courtesy the University of Michigan
http://www.tecplot.com/showcase/studies/2001/michigan.htm

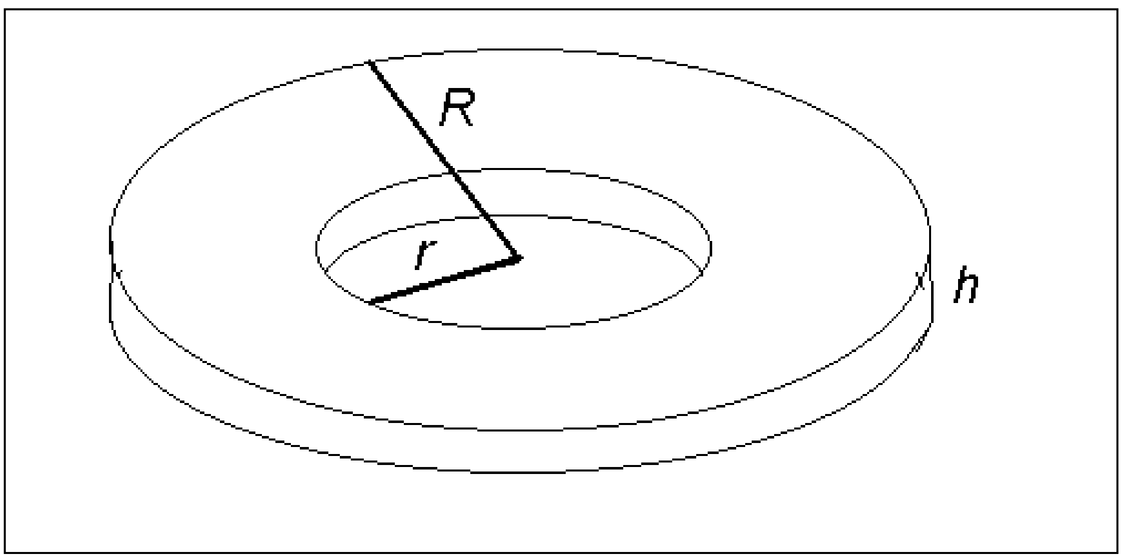
During a severe magnetic storm, Earth is surrounded by a disk of plasma called the Ring Current. The outer diameter of this disk is about 4 times Earth's radius (1 Re = 6,378 kilometers) while the inside radius is about 1.5 times Earth's radius. The thickness of this disk is about 2,000 kilometers. The volume of a ring-shaped disk is given by the formula

$V = \pi \times (R^2 - r^2) \times h$

where **R** is the outer radius, **r** is the inner radius, and **h** is the thickness of the disk. Use this formula to answer the questions below.

Question 1 - If the density of the Ring Current particles is about 10,000 atoms per cubic centimeter, how many atoms are present in this disk of plasma?

Question 2 – If the atoms are mostly oxygen atoms, and an oxygen atom has a mass of about 2.0×10^{-20} kilograms, what is the total mass of the Ring Current?



This problem is suitable for students who have taken Algebra 1.

In this activity, students will use the formula for the volume of a ring.
They will substitute numerical values into the formula.
They will use scientific notation throughout.
They will work with positive and negative exponents.

Question 1 - If the density of the Ring Current particles is about 10,000 atoms per cubic centimeter, how many atoms are present in this disk of plasma?

Answer: We have to multiply the density of the gas by the volume of the disk to find the number of atoms. The volume of a ring-shaped disk is given by $V = \pi \times (R^2 - r^2) \times h$, where R is the outer radius, r is the inner radius, and h is the thickness of the disk.

The outer radius $R = 4.0 \times 6378 \times 100000$ cm $= 2.55 \times 10^9$ cm.
The inner radius $r = 1.5 \times 6378 \times 100{,}000$ cm $= 9.57 \times 10^8$ cm.
The height $h = 2000 \times 100000$ cm $= 2.0 \times 10^8$ cm.

So from the formula:
$V = (3.14) \times [(2.55 \times 10^9)^2 - (9.57 \times 10^8)^2] \times 2.0 \times 10^8$ cubic centimeters

$V = 3.14 \times [6.50 \times 10^{18} - 9.16 \times 10^{17}) \times 2.0 \times 10^8$

$V = 3.51 \times 10^{27}$ cubic centimeters.

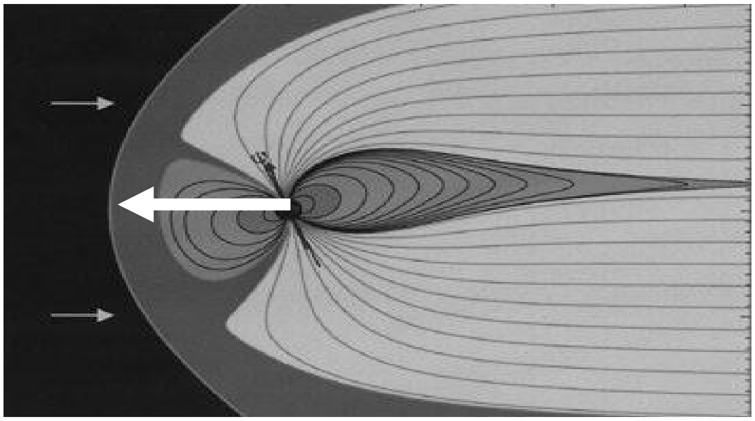
The total number of oxygen atoms is then Density x Volume or

$N = 10{,}000 \times 3.51 \times 10^{27}$ atoms.
$N = 3.51 \times 10^{31}$ atoms.

Question 2 – If the atoms are mostly oxygen atoms, and an oxygen atom has a mass of about 2.0 x 10-20 kilograms. If one metric ton equals 1000 kilograms, what is the total mass of the Ring Current?

Answer: Multiply the answer from Question 1 by the mass of an oxygen atom, and convert from kilograms to metric tons.

Mass $= (3.51 \times 10^{31}) \times (2.0 \times 10^{-20}) = 7.02 \times 10^{11}$ kilograms
$= 7.02 \times 10^8$ metric tons.



When the solar wind flows past Earth, it pushes on Earth's magnetic field and compresses it. There is a point in space called the magnetopause where the pressure of the solar wind balances the outward pressure of Earth's magnetic field. The distance from the Earth, R, (white arrow in drawing) where these two pressures are balanced is given by the equation:

$$R^6 = \frac{0.72}{8\pi D V^2}$$

In this equation, D is the density in grams per cubic centimeter (cc) of the gas (solar wind, etc) that collides with Earth's magnetic field, and V is the speed of this gas in centimeters per second. Let's do an example to see how this equation works!

The solar wind has a typical speed of 450 km/s or equivalently V = 4.5 x 10^7 cm/s. To find the density of the solar wind in grams/cc we have to do a two-step calculation. The wind usually has a particle density of about 5 particles/cc, and since these particles are typically protons (each with a mass of 1.6 x 10^{-24} gm) the density is then 5 x (1.6 x 10^{-24} gm)/cc so that D = 1.28 x 10^{-23} gm/cc.

We substitute D and V into the equation and get R^6 = 1105242.6. so that R = $(1105242.6)^{1/6}$. To solve this, we use a calculator with a key labeled **Y^x** First type '1105242.6' and hit the '**Enter**' key. Then type '0.1666' (which equals 1/6) and press the **Y^X** key. In this case the answer will be '10.16' and it represents the value of R in multiples of the radius of Earth (6378 kilometers). Scientists simplify the mathematical calculation by using the radius of Earth as their unit of distance, but if you want to convert 10.16 Earth radii to kilometers, just multiply it by '6378 km' which is the radius of Earth to get 64,800 kilometers. That is the distance from the center of Earth to the magnetopause where the magnetic pressure is equal to the solar wind pressure for the selected speed and density. These will change significantly during a 'solar storm'.

Now lets apply this example to finding the magnetopause distance for some of the storms that have encountered Earth in the last five years. Complete the table below, rounding the answer to three significant figures:

Storm	Date	Day Of Year	Density (particle/cc)	Speed (km/s)	R (km)
1	11/20/2003	324	49.1	630	
2	10/29/2003	302	10.6	2125	
3	11/06/2001	310	15.5	670	
4	3/31/2001	90	70.6	783	
5	7/15/2000	197	4.5	958	

Question: The fastest speed for a solar storm 'cloud' is 1500 km/s. What must the density be in order that the magnetopause is pushed into the orbits of the geosynchronous communication satellites at 6.6 Re?

The information about these storms and other events can be obtained from the NASA ACE satellite by selecting data for H* density and V_x (GSE)

http://www.srl.caltech.edu/ACE/ASC/level2/lvl2DATA_MAG-SWEPAM.html

Storm	Date	Day Of Year	Density (particle/cc)	Speed (km/s)	R (km)
1	11/20/2003	324	49.1	630	42,700
2	10/29/2003	302	10.6	2125	37,000
3	11/06/2001	310	15.5	670	51,000
4	3/31/2001	90	70.6	783	37,600
5	7/15/2000	197	4.5	958	54,800

Question: The fastest speed for a solar storm 'cloud' is 3000 km/s. What must the density be in order that the magnetopause is pushed into the orbits of the geosynchronous communication satellites at 6.6 Re (42,000 km)?

Answer: Solve the equation for D to get:

$$\boxed{D = \frac{0.72}{8\pi R^6 V^2}}$$

For 1500 km/s V = 1.5 x 10^8 cm/s, and for R = 6.6, we have

$$D = 0.72/(8 \times 3.14 \times 6.6^6 \times (1.5 \times 10^8)^2) = 1.52 \times 10^{-23} \text{ gm/cc}$$

Since a proton has a mass of 1.6 x 10^{-24} grams, this value for the density, D, is equal to (1.52 x 10^{-23}/ 1.6 x 10^{-24}) = 9.5 protons/cc.

For Extra Credit, have students compute the density if the solar storm pushed the magnetopause to the orbit of the Space Station (about R = 1.01 RE).
Answer: D = 3 x 10^{-19} gm/cc or 187,000 protons/cc. A storm with this density has never been detected, and would be catastrophic!