

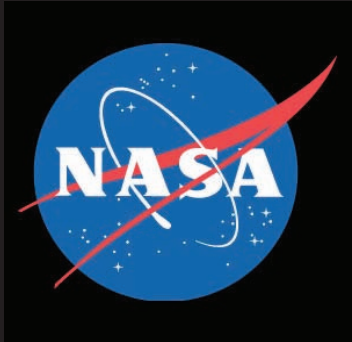


Image Scale Math

How to Use this Book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Mathematics offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that “Similarity also can be related to such real-world contexts as photographs, models, projections of pictures” which can be an excellent application for students to practice using the scale of an image..

Image Scale Math is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery. This set of math problems can be used as an application for students to gain mastery of scale of an image using a variety of images.

The following scenario demonstrates how the series of math problems are used in one classroom:

Ms. Jones teaches 5th grade math and science. She decided to use the Image Scale Math Book to make the concept more interesting and demonstrate the application of what they were learning. The class was divided into 5 groups; each group was given 2 math problems to solve. At the same time they were to write 2 facts they learned about space from their math problem. Each group would share their results and facts (teaching the class). The “For Experts” section was offered as extra credit for any student who wanted the challenge.

Image Scale Math can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2004-2008 school years. They were intended as supplementary problems for students looking for additional challenges in the math and physical science curriculum in grades 6 through 8. The problems are designed to be 'one-pagers' consisting of a Student Page, and Teacher's Answer Key. This compact form was deemed very popular by participating teachers.

The topic for this collection is **Image Scaling**, which is an important First Step that all astronomers perform in understanding image-type data produced by satellites in space, and telescopes on the ground.

General Procedure:

- 1) Each activity includes an image of some solar system feature, and an indication of the physical field of view in meters or kilometers.
- 2) Students will use a metric ruler to measure the size of the image in millimeters.
- 3) They will divide the physical size by the image size to calculate the image scale in meters per millimeter, or kilometers per millimeter.
- 4) They will then investigate the image further by identifying the smallest object they can discern, and determine its physical size.

This booklet was created by the NASA Space Math program

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For more weekly classroom activities about astronomy and space science, visit
<http://spacemath.gsfc.nasa.gov>

Add your email address to our mailing list by contacting Dr. Sten Odenwald at
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Cover credits: Tycho Crater (NASA/Orbiter); Saturn Ring (NASA/Cassini); Sombrero Galaxy (NASA/Spitzer); Helix Nebula (NASA/Spitzer)

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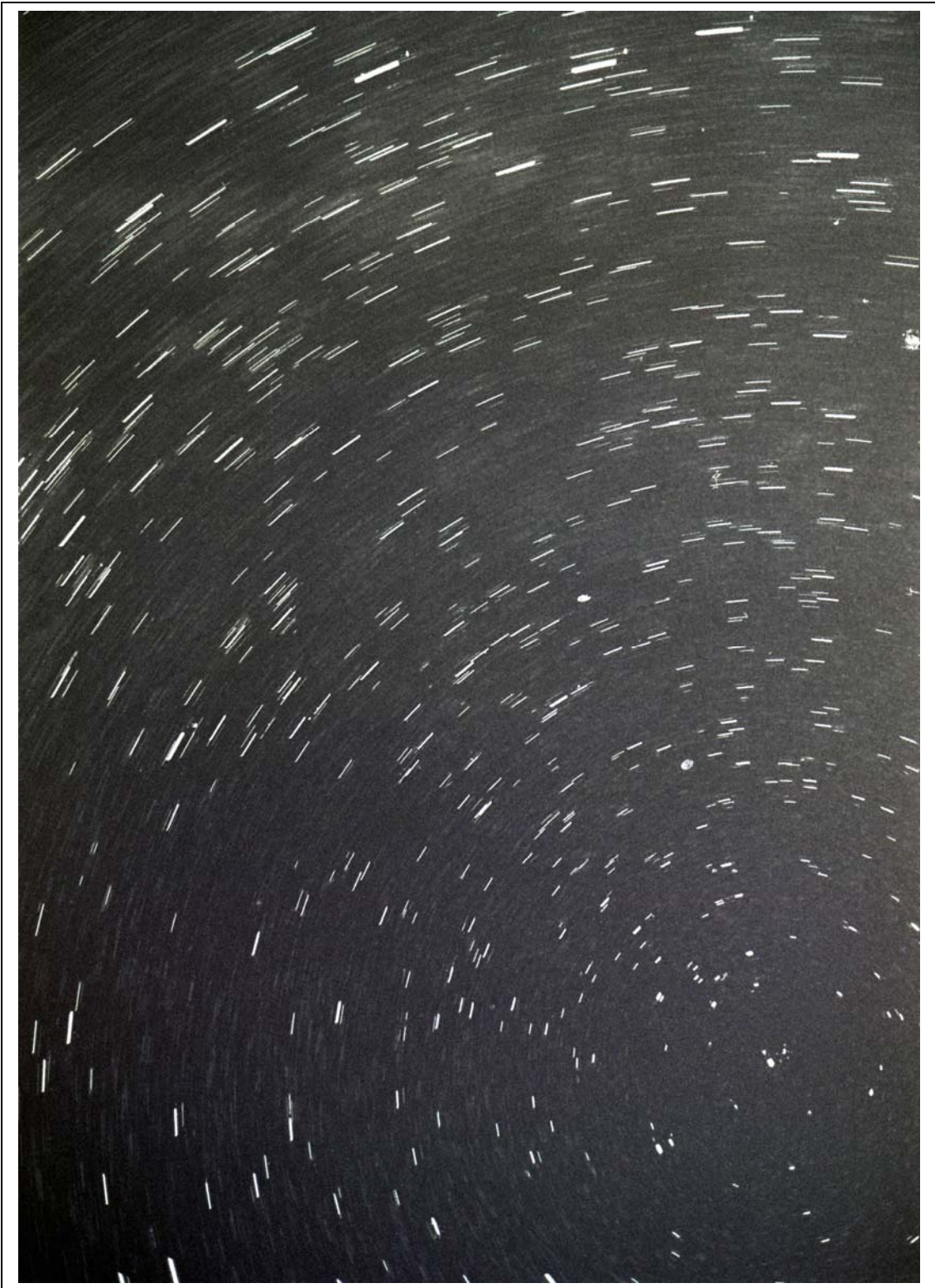
Alignment with Standards

The problems have been developed to meet specific math and science benchmarks as stated in the NSF Project 2061. Project 2061's benchmarks are statements of what all students should know or be able to do in science, mathematics, and technology by the end of grades 2, 5, 8, and 12.

(<http://www.project2061.org/publications/bsl/online/bolintro.htm>)

2.B. The Nature of Mathematics; Grades 3 - 5 The interaction should become more frequent and more sophisticated as students progress through the upper elementary and middle grades. Graphing, making tables, and making scale drawings should become commonplace in student inquiry and design projects, as should the use of geometric and mathematical concepts such as perpendicular, perimeter, volume, powers, roots, and negative numbers. Problems that are used to challenge students may take the form of contests and games, but at least some of the problems should stem directly from the science and technology being studied. **By the end of 5th grade, students should know that scale drawings show shapes and compare locations of things very different in size.**

11.D Common Themes; Scale; Grades 3 - 5 Children at this level tend to be fascinated by extremes. That interest should be exploited to develop student math skills as well as a sense of scale. Students may not have the mathematical sophistication to deal confidently with ratios and with differences among ratios but the observational groundwork and familiarity with talking about them can begin. At the very least, students can compare speeds, sizes, distances, etc., as fractions and multiples of one another. Students should now be building structures and other things in their technology projects. Through such experience, they can begin to understand both the mathematical and engineering relationships of length, area, and volume. They can be challenged to measure things that are hard to measure on account of being very small or very large, very light or very heavy. **By the end of the 5th grade, students should know that: 1) Almost anything has limits on how big or small it can be; 2) Finding out what the biggest and the smallest possible values of something are is often as revealing as knowing what the usual value is.**



A photograph of stars circling the North Celestial Pole (Author)



This QuickBird Satellite image was taken of downtown Las Vegas Nevada from an altitude of 450 kilometers. Private companies such as Digital Globe (<http://www.digitalglobe.com>) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellagio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter. Report your answer to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: How long is each of the three wings of the Bellagio Hotel in meters?

Question 2: What is the length of a car on the street in meters?

Question 3: How wide are the streets entering the main intersection?

Question 4: What is the smallest feature you can see, in meters?

Question 5: What kinds of familiar objects can you identify in this image?

Answer Key:

This QuickBird Satellite image was taken of downtown Las Vegas Nevada on October 14, 2005 from an altitude of 450 kilometers. Private companies such as Digital Globe (<http://www.digitalglobe.com>) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellagio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Answer: 150 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Answer: The information in the introduction says that the image is 700 meters long.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter.

Answer: $700 \text{ meters} / 150 \text{ millimeters} = 4.66 \text{ meters} / \text{millimeter}$. To two sig.fig this becomes 4.7 meters/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: How long is each of the three wings of the Bellagio Hotel in meters?

Answer: About 25 millimeters on the image or $25 \text{ mm} \times (4.7 \text{ meters/mm}) = 120 \text{ meters}$.

Question 2: What is the length of a car on the street in meters?

Answer: About 1 millimeter on the image or $1 \text{ mm} \times 4.7 \text{ meters/mm} = 4.7 \text{ meters}$.

Question 3: How wide are the streets entering the main intersection?

Answer: About 8 millimeters on the image or $8 \text{ mm} \times 4.7 \text{ meters/mm} = 37 \text{ meters}$.

Question 4: What is the smallest feature you can see, in meters?

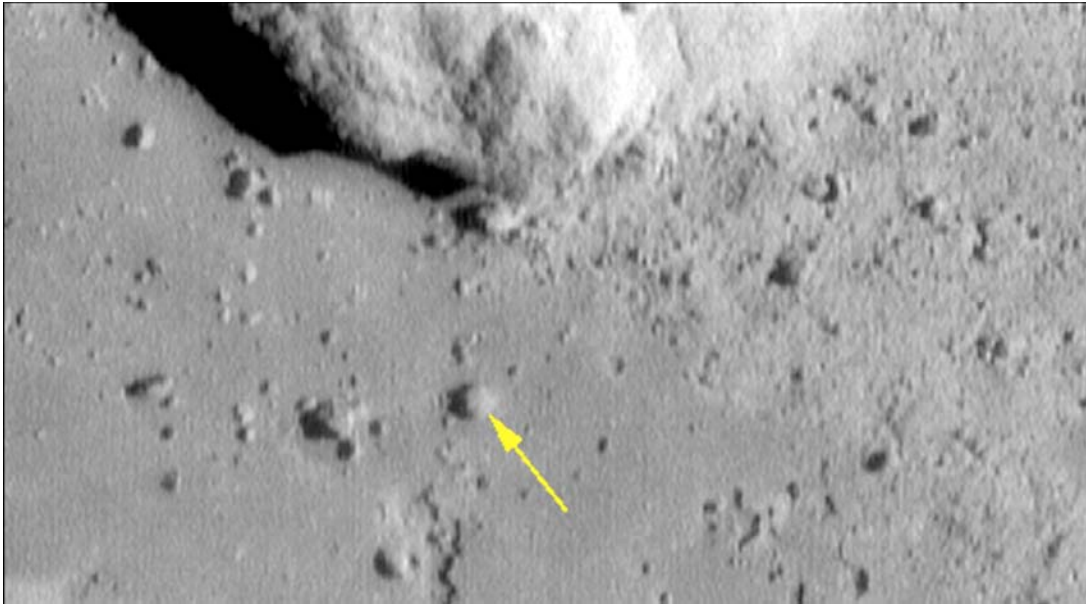
Answer: Some of the small dots on the roof tops are about 0.2 millimeters across which equals 1 meter.

Question 5: What kinds of familiar objects can you identify in this image?

Answer: Will vary depending on student.

1. Cars, busses
2. swimming pools and reflecting ponds
3. Trees
4. lane dividers
5. Shadows of people walking across the plaza to the Hotel.

Note: Ask the students to use image clues to determine the time of day (morning, afternoon, noon); Whether it is rush-hour or not; Time of year, etc.



This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6.0 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?

Question 2: What is the width, in centimeters, of the largest feature?

Question 3: What is the size of the smallest feature you can see?

Question 4: How big is the stone shown by the arrow?

Answer Key:

This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University)). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Answer: 144 millimeters

Step 2: Use clues in the image description to determine a physical distance or length.

Answer: 6.0 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter.

Answer: $6.0 \text{ meters} / 144 \text{ mm} = 600 \text{ cm} / 144 \text{ millimeters} = 4.2 \text{ cm/mm}$

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters.

Question 1: What are the dimensions, in meters, of this image?

Answer: Height = 80 mm = 336 cm or 3.4 meters so area is 6.0 m x 3.4 m

Question 2: What is the width, in centimeters, of the largest feature?

Answer: The big rock at the top of the image is about 60 mm across or 2.5 meters.

Question 3: What is the size of the smallest feature you can see?

Answer: The small pebbles are about 0.5 millimeters across or 2.1 centimeters (about 1 inch).

Question 4: How big is the stone shown by the arrow?

Answer: 4 millimeters or 17 centimeters (about 7 inches).



This is a picture taken by International Space Station astronauts of Washington, DC, and can be found among many other pictures at <http://eol.jsc.nasa.gov/Coll/EarthObservatory/PostedSort.htm>. The bridge at the bottom-center of the image is the George Mason Bridge (1) and it is 0.75 kilometers from end to end across the main part of the Potomac River (2).

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. It is the most important number to determine because without it, you don't know how big the objects in the image are!

Step 1: Measure the length of the George Mason Bridge with a metric ruler. How many millimeters long is the image of the bridge?

Step 2: The information in the introduction says that the bridge is actually 0.75 kilometers long. Convert this number into meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: About what is the distance between the US Capitol Building (3) and the Washington Monument (4)?

Question 2: About how wide are the major boulevards and roadways?

Question 3: About how wide is the Potomac River?

Question 4: How big is the smallest feature you can measure, and what do you think it is?

Question 5: How big is the area covered by this image in kilometers rounded to the nearest tenth?

Question 6: What other features can you recognize in this image?

You can use GOOGLE-Earth to help find other interesting landmarks in the image!

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. It is the most important number to determine because without it, you don't know how big the objects in the image are!

It is highly recommended that students use GOOGLE-Earth and dial-in 'Washington DC' to zoom-in on this area in higher resolution. They can use the various tools to bring up the labels for roads, buildings and geographic features.

Step 1: Measure the length of the Mason Bridge with a metric ruler. How many millimeters long is the image of the bridge? **Answer: 15 millimeters**

Step 2: The information in the introduction says that the bridge is actually 0.75 kilometers long. Convert this number into meters. **Answer: 750 meters**

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: The image scale is 50 meters/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: About what is the distance between the US Capitol Building and the Washington Monument?
Answer: 72 millimeters on the image x 50 meters/mm = 3,600 meters or 3.6 kilometers.

Question 2: About how wide are the major boulevards and roadways?
Answer: The thick black lines are about 1.0 millimeter wide or 1.0mm x 50 meters/mm = 50 meters.

Question 3: About how wide is the Potomac River?
Answer: The river banks are about 12 millimeters apart along most of the river, so their true width is 12 mm x 50 meters/mm = 600 meters or 0.6 kilometers.

Question 4: How big is the smallest feature you can measure...and what do you think it is?
Answer: Students should be able to find many buildings that look like white spots with barely a square shape. These would be about 1 millimeter wide or 50 meters in true physical size.

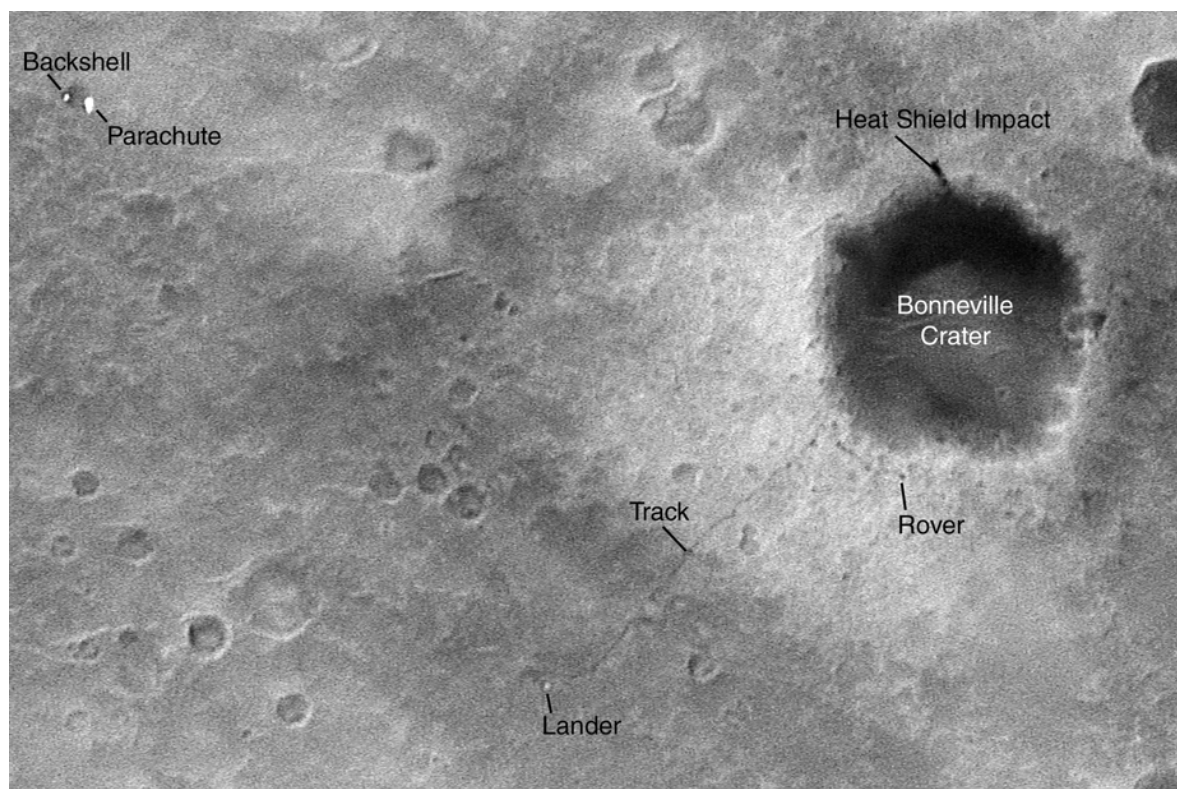
Question 5: How big is the area covered by this image in kilometers rounded to the nearest tenth?
Answer; The field is 169 millimeters by 97 millimeters which is 8.5 kilometers x 4.9 kilometers in true size.

Question 6: What other features can you recognize in this image?
Answer: Students should be able to figure out the following features without using GOOGLE:

1. Rivers and waterways
2. Large and small buildings
3. Major boulevards
4. Minor streets
5. Bridges
6. Areas with trees and plant life

Mars Rover Landing Site

4



This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the image is exactly 895 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: About what is the diameter of Bonneville Crater rounded to the nearest ten meters?

Question 2: How wide, in meters, is the track of the Rover?

Question 3: How big is the Rover?

Question 4: How small is the smallest well-defined crater to the nearest meter in size?

Question 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

Answer Key:

This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the crater is 200 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?

Answer: 157 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.

Answer: 895 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Answer: $895 \text{ m} / 157 \text{ mm} = 5.7 \text{ meters / millimeter}$.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

Question 1: About what is the diameter of Bonneville Crater rounded to the nearest 10 meters?

Answer: Students answers for the diameter of the crater in millimeters may vary, but answers in the range from 30-40 mm are acceptable. Then this equals $30 \times 5.7 = 170 \text{ meters}$ to $40 \times 5.7 = 230 \text{ meters}$. Students may average these two measurements to get $(170 + 230) / 2 = 200 \text{ meters}$.

Question 2: How wide, in meters, is the track of the Rover?

Answer: $0.2 \text{ millimeters} = 1 \text{ meter}$.

Question 3: How big is the Rover?

Answer: $0.3 \text{ millimeters} = 1.7 \text{ meters}$ but since the measurement is only 1 significant figure, the answer should be 2 meters.

Question 4: How small is the smallest well-defined crater in meters?

Answer: $2 \text{ millimeters} \times 5.7 = 11.4 \text{ meters}$, which to the nearest meter is 11 meters.

Question 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

Answer: Students answers may vary and lead to interesting discussions about what features are real, and which ones are flaws in the printing of the picture. This is an important discussion because 'image artifacts' are very common in space-related photographs. 5 meters is about 1 millimeter, and there are no obvious rounded objects this large or larger in this image.



This image was taken by NASA's Mars Reconnaissance Orbiter on February 19, 2008. It shows an avalanche photographed as it happened on a cliff on the edge of the dome of layered deposits centered on Mars' North Pole. From top to bottom this impressive cliff is over 700 meters (2300 feet) tall and reaches slopes over 60 degrees. The top part of the scarp, to the left of the image, is still covered with bright (white) carbon dioxide frost which is disappearing from the polar regions as spring progresses. The upper mid-toned (pinkish-brownish) section is composed of layers that are mostly ice with varying amounts of dust. The dust cloud extends 190 meters from the base of the cliff.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?

Question 2: What is the smallest detail you can see in the ice shelf?

Question 3: What is the average thickness of the red layers on the cliff?

Question 4: What is the total width of the reddish rock cliff?

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?

Answer: 60 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length.

Answer: 190 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Answer: $190 \text{ meters} / 60 \text{ mm} = 3.2 \text{ meters} / \text{mm}$

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?

Answer: $140 \text{ mm} \times 86 \text{ mm} = 448.0 \text{ meters} \times 275.2 \text{ meters}$, but to two significant figures this becomes $450 \text{ meters} \times 280 \text{ meters}$.

Question 2: What is the smallest detail you can see in the ice shelf?

Answer: $0.2 \text{ mm} = 0.6 \text{ meters}$

Question 3: What is the average thickness of the red layers on the cliff?

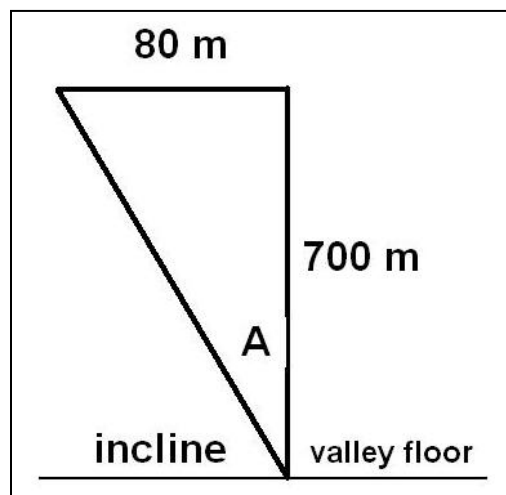
Answer: $1.0 \text{ millimeter} = 3.2 \text{ meters}$.

Question 4: What is the total width of the reddish rock cliff?

Answer: $25 \text{ millimeters} = 80 \text{ meters}$.

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?

Answer: Draw the triangle with the 700-meter side being vertical and the 80 meter side being horizontal. The tangent of the angle is $(80 / 700) = 0.11$ so the angle is 6.5 degrees. This is the angle from the vertical, so the incline angle from the floor of the valley is $90 - 6.5 = 84$ degrees. This is a nearly vertical wall!





This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of Mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?

Question 2: How wide, in meters, are the streams half-way down their flow channels?

Question 3: What is the smallest feature you can see in the image?

Question 4: How wide is the top of the crater wall at its sharpest edge?

Answer Key:

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of Mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?

Answer: 13 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Answer: 300 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Answer: $300 \text{ meters} / 13 \text{ mm} = 23 \text{ meters} / \text{millimeter}$.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?

Answer: $134 \text{ mm} \times 120 \text{ mm} = 3.1 \text{ km} \times 2.8 \text{ km}$

Question 2: How wide, in meters, are the streams half-way down their flow channels?

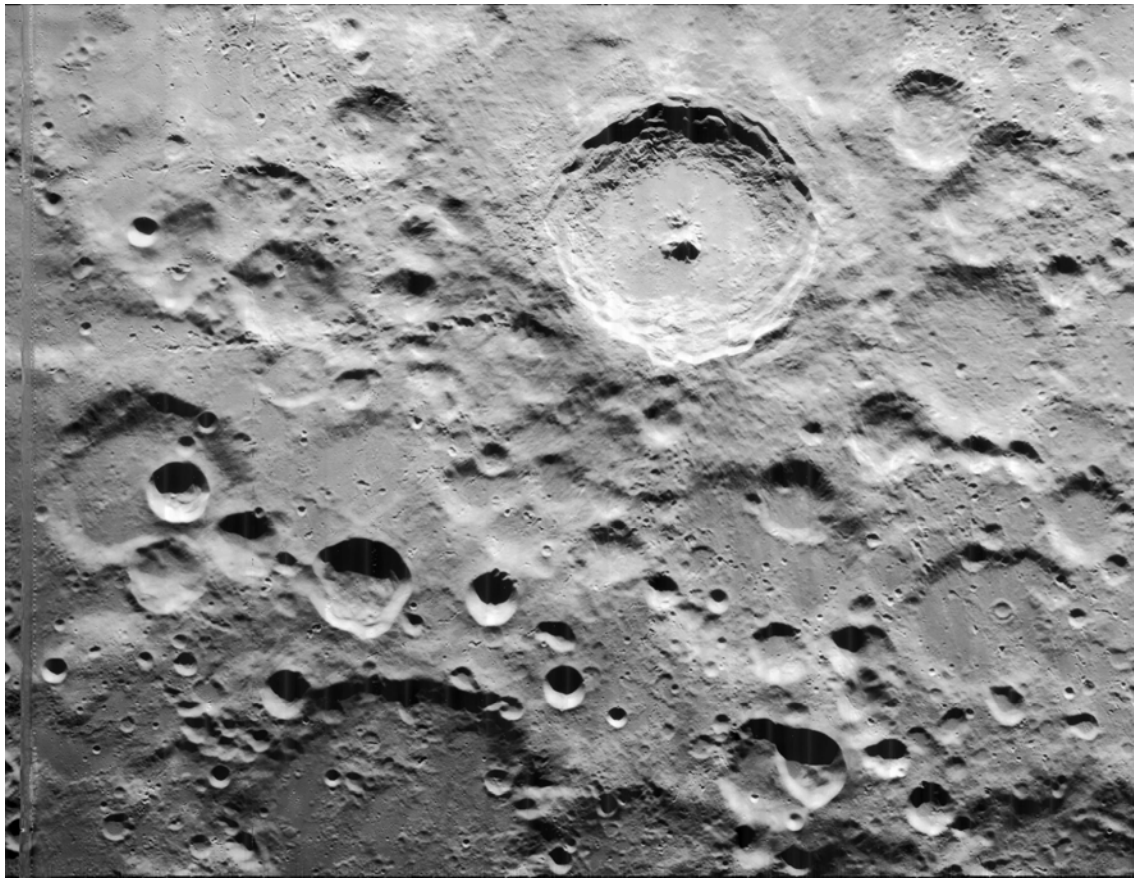
Answer: $0.5 \text{ millimeters} = 12 \text{ meters}$.

Question 3: What is the smallest feature you can see in the image?

Answer: Sand dunes in upper left of image = 0.3 millimeters or 7 meters wide .

Question 4: How wide is the top of the crater wall at its sharpest edge?

Answer: 0.2 millimeters or 4 meters wide .



This is a NASA image taken by the Lunar Orbiter IV spacecraft as it captured close-up images of the lunar surface in May, 1967. The large crater at the top-center is Tycho. Other images from the Lunar Orbiter spacecrafts can be found at the Lunar Orbiter Photo Gallery (<http://www.lpi.usra.edu/resources/lunarorbiter/>) The satellite was at an altitude of 3,000 kilometers when it took this image, which measures 350 km x 270 km.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 250 kilometers x 270 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What is the diameter of the crater Tycho in kilometers?

Question 2: How large is the smallest feature you can see?

Question 3: How large are some of the smaller hills at the floor of the crater, in meters?

Question 4: About how large are the most common craters in the field?

Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Answer: 150 millimeters.

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.

Answer: 350 kilometers.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Answer: $350 \text{ kilometers} / 150 \text{ millimeters} = 2.3 \text{ kilometers} / \text{millimeter}$.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What is the diameter of the crater Tycho in kilometers?

Answer: About 35 millimeters $\times 2.3 \text{ km/mm} = 80.5 \text{ kilometers}$ in diameter which is 80 kilometers to two significant figures.

Question 2: How large is the smallest feature you can see?

Answer: There are many small details in the image, pits, hills, etc, that students can estimate 0.1 to 0.3 millimeters for a physical size of 0.2 to 0.7 kilometers since the measurement is only good to one significant figure.

Question 3: How large are some of the smaller hills at the floor of the crater, in meters?

Answer: These small features are about 0.1 millimeters across or 200 meters in size.

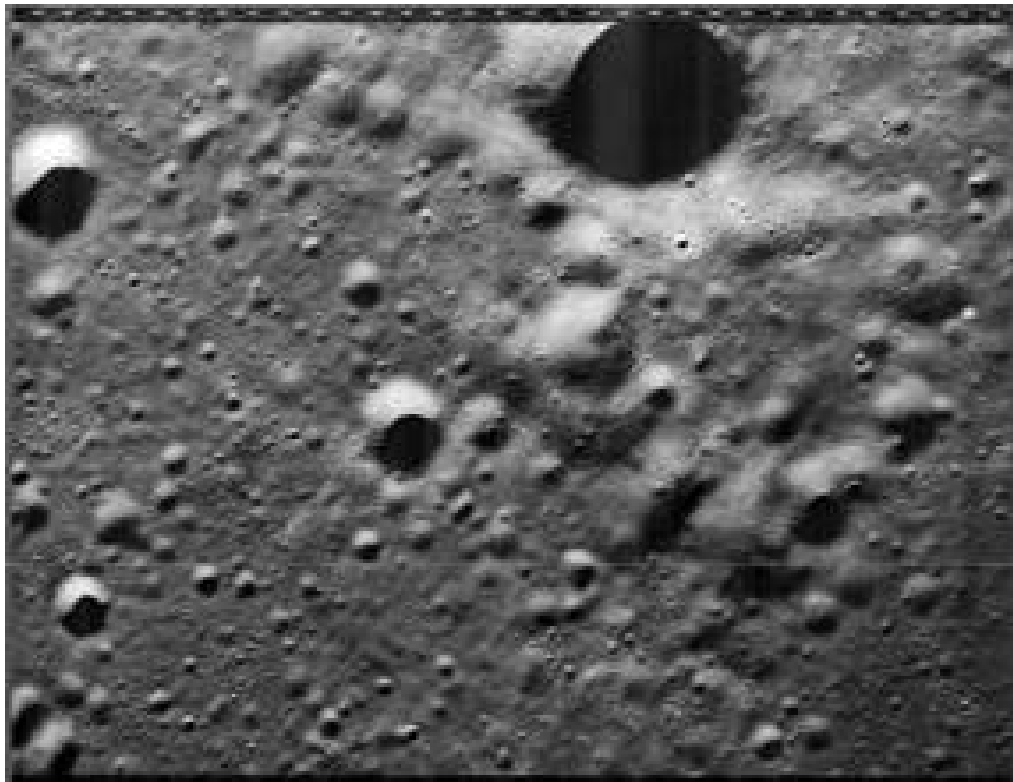
Question 4: About how large are the most common craters in the field?

Answer: The answer may vary a bit, but the small craters that are 0.5 millimeters across are the most common. These have a physical size of about 1 kilometer.

Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?

Answer: In order to fit Denver into one of these lunar craters, it will have to appear to be about $25 \text{ km} \times (1.0 \text{ millimeter} / 2.3 \text{ km}) = 11 \text{ millimeters}$ across. There are three craters just to the right of Tycho that are about this big. Students should not get 'lost' trying to exactly match up their estimate with a precise lunar feature. 'Close-enough' estimates are good enough! See below comparison as a guide.





This is a high resolution image of the lunar surface taken by NASA's Lunar Orbiter III spacecraft in February 1967 as it orbited at an altitude of 46 kilometers. It is located near the lunar equator. The field of view is 16.6 kilometers x 4.1 kilometers. Additional Orbiter images can be found at the Lunar Orbiter Gallery ([http:// www.lpi.usra.edu/resources/lunarorbiter/](http://www.lpi.usra.edu/resources/lunarorbiter/)). Because of the low sun angle, craters look like circles that are half-black, half-white inside!

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

Question 1: How big is the largest crater in the image?

Question 2: How big is the smallest crater in the image, in meters?

Question 3: About what is the typical distance between craters in the image?

Question 4: How far would you have to walk between the largest, and next-largest craters?

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Answer: 134 millimeters.

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.

Answer: 16600 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.

Answer: $16600 \text{ meters} / 134 \text{ millimeters} = 124 \text{ meters} / \text{millimeter}$.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

Question 1: How big is the largest crater in the image?

Answer: The one at the top is about 25 millimeters across. $25 \text{ mm} \times 124 \text{ meters/mm} = 3,100 \text{ meters}$ or 3.1 kilometers.

Question 2: How big is the smallest crater in the image, in meters?

Answer: The largest number of features are about 1 millimeter across or 100 meters to one significant figure. Students may also go for the recognizable craters which are about 2 millimeters across or about 200 meters.

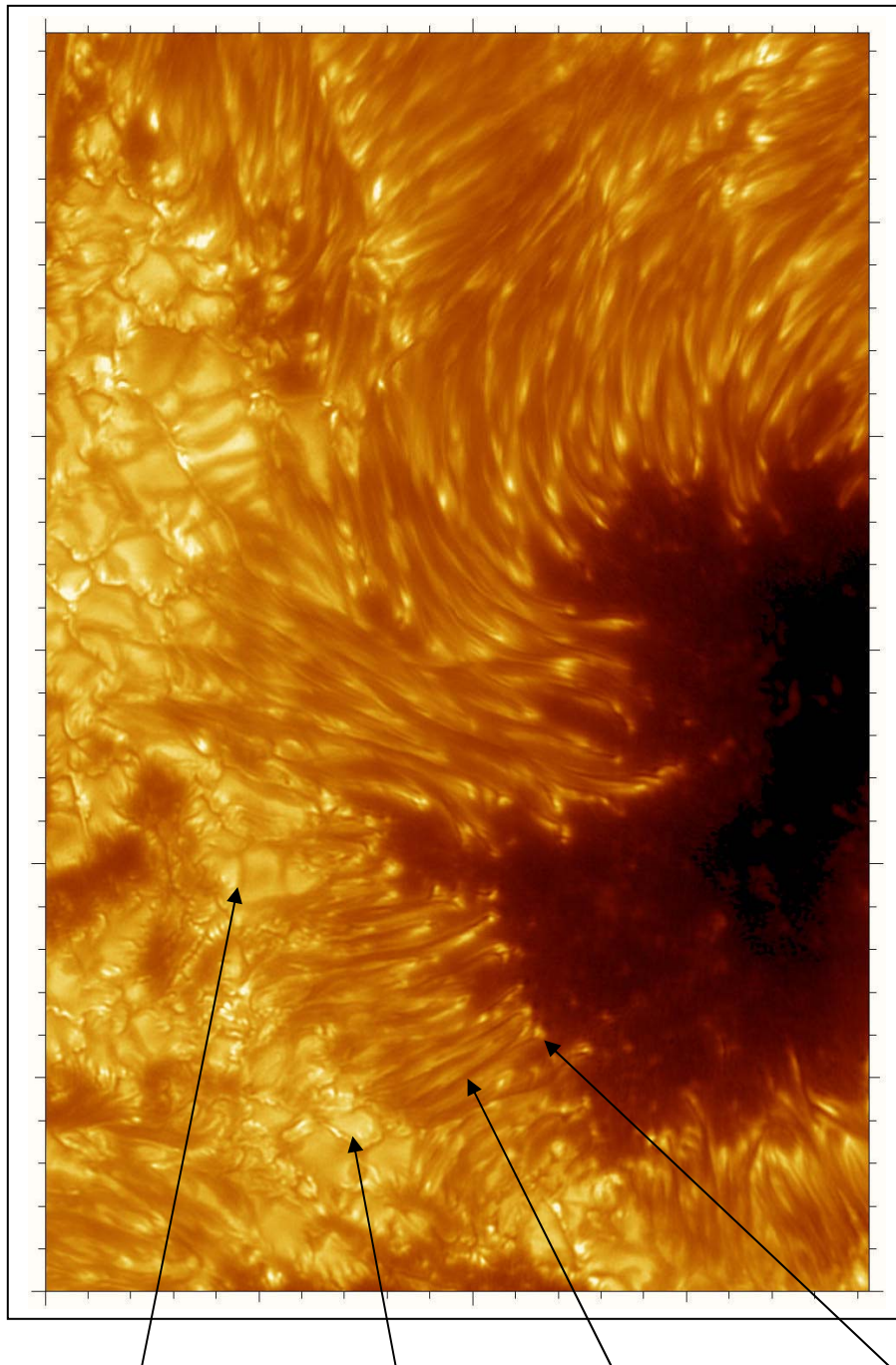
Question 3: About what is the typical distance between craters in the image?

Answer: The answer may vary, but the distance between obvious craters (about 2 mm in diameter) is about 5 millimeters or $5 \text{ mm} \times 124 \text{ meters/mm} = 600 \text{ meters}$ to one significant figure.

Question 5: How far would you have to walk between the largest, and next-largest craters?

Answer: The crater rims are about 35 millimeters apart or $35 \text{ mm} \times 124 \text{ meters/mm} = 4,340 \text{ meters}$ or 4.3 kilometers to two significant figures.

The sun is our nearest star. From Earth we can see its surface in great detail. The images below were taken with the 1-meter Swedish Vacuum Telescope on the island of La Palma, by astronomers at the Royal Swedish Academy of Sciences (<http://www.astro.su.se/groups/solar/solar.html>). The image to the right is a view of sunspots on July 15, 2002. The enlarged view to the left shows never-before seen details near the edge of the largest spot. Use a millimeter ruler, and the fact that the dimensions of the left image are 19,300 km x 29,500 km, to determine the scale of the photograph, and then answer the questions. See the arrows below to identify the various solar features mentioned in the questions.

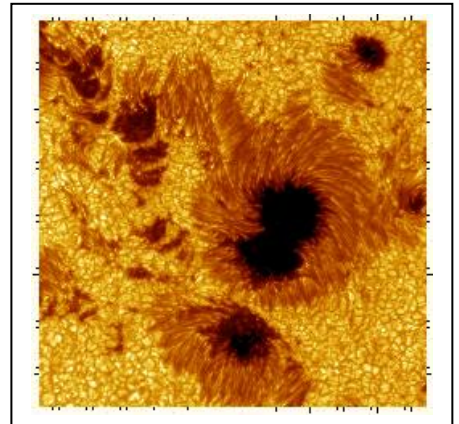


Granulation
Boundary

Solar Granulation

Dark Filament

Bright Spot



Question 1 - What is the scale of the image in km/mm?

Question 2 – What is the smallest feature you can see in the image?

Question 3 – What is the average size of a Solar Granulation region?

Question 4 – How long and wide are the Dark Filaments?

Question 5 – How large are the Bright Spots?

Question 6 – Draw a circle centered on this picture that is the size of Earth (radius = 6,378 km). How big are the features you measured compared to familiar Earth features?

Question 1 - What is the scale of the image in km/mm? **Answer:** the image is about 108mm x 164mm so the scale is $19300/108 = 179$ km/mm.

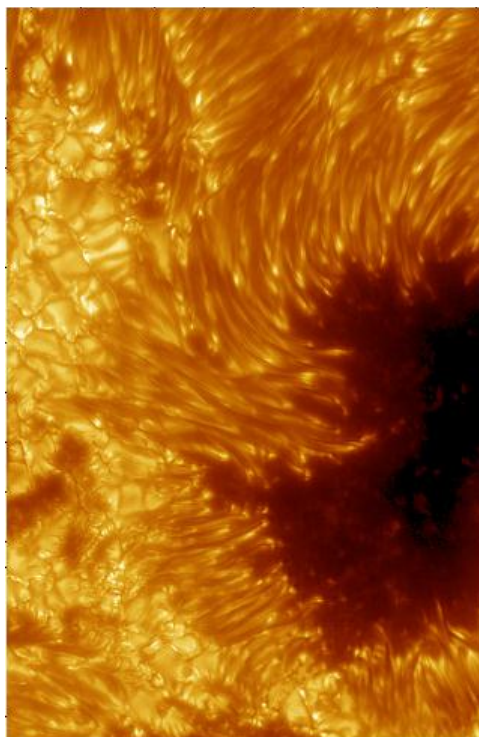
Question 2 – What is the smallest feature you can see in the image? **Answer:** Students should be able to find features, such as the Granulation Boundaries, that are only 0.5 mm across, or $0.5 \times 179 = 90$ km across.

Question 3 – What is the average size of a Solar Granulation region? **Answer:** Students should measure several of the granulation regions. They are easier to see if you hold the image at arms length. Typical sizes are about 5 mm so that 5×179 is about 900 km across.

Question 4 – How long and wide are the Dark Filaments? **Answer:** Students should average together several measurements. Typical dimensions will be about 20mm x 2mm or 3,600 km long and about 360 km wide.

Question 5 – How large are the Bright Spots? **Answer:** Students should average several measurements and obtain values near 1 mm, for a size of about 180 km across.

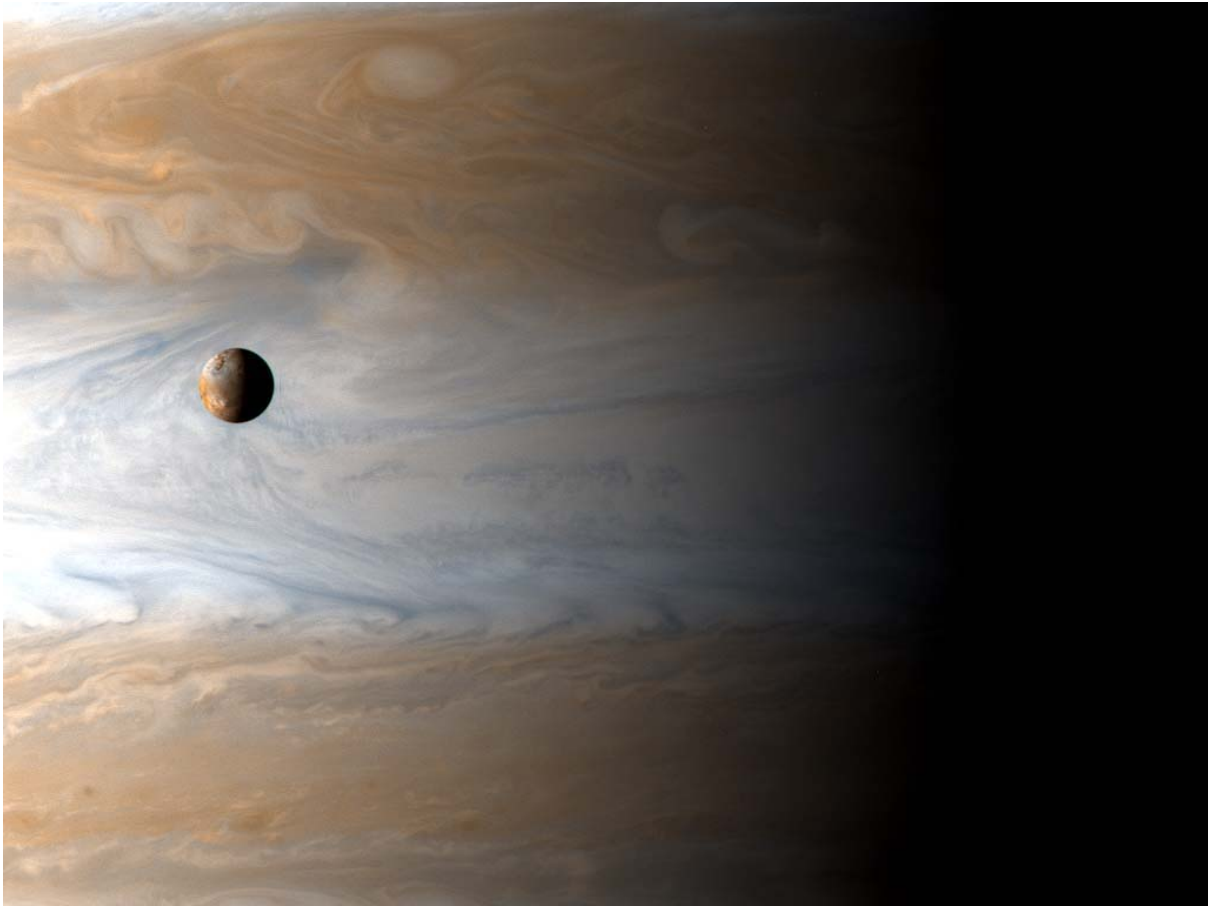
Question 6 – Draw a circle centered on this picture that is the size of Earth (radius = 6,378 km). How big are the features you measured compared to familiar Earth features? **Answer:** See below.



Granulation Region – Size of a large US state.

Bright Spot – Size of a small US state or Hawaii

Filament – As long as the USA, and as narrow as Baja California or Florida.



This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?

Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?

Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?

Question 4: What is the size of the smallest feature on Io that you can see?

This NASA image of Jupiter with its satellite Io was taken by the Cassini spacecraft. (Credit: NASA/Cassini Imaging Team). The satellite is 3,600 kilometers in diameter.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the diameter of Io is 3,600 kilometers.

Step 1: Measure the diameter of Io with a metric ruler. How many millimeters in diameter?

Answer: 10 mm

Step 2: Use clues in the image description to determine a physical distance or length.

Answer: 3,600 km

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Answer: $3600 \text{ km} / 10 \text{ mm} = 360 \text{ km/mm}$

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?

Answer: $160 \text{ mm} \times 119 \text{ mm} = 58,000 \text{ km} \times 19,000 \text{ km}$

Question 2: What is the width, in kilometers, of the largest feature in the atmosphere of Jupiter?

Answer: The width of the white equatorial band is 45 mm or 16,000 km

Question 3: What is the width, in kilometers, of the smallest feature in the atmosphere of Jupiter?

Answer: The faint cloud streaks are 0.5 mm wide or 200 km across to one significant figure.

Question 4: What is the size of the smallest feature on Io that you can see?

Answer: The white spots in the southern hemisphere are 0.5 mm across or 200 km to one significant figure. This is a good time to mention that some details in an image can be artifacts from the printing process or defects in the camera itself. Students may find photocopying artifacts at 0.5 mm or less.

Note to teachers: The correct scale for Io and Jupiter will be slightly different depending on how far away the camera was when taking the picture. If the camera was very close to Io, then the scale you will infer for Io will be very different than for the more distant Jupiter because Io will take up more of the field-of-view in the image. Geometrically, for a fixed angle of separation between features on Io, this angle will subtend a SMALLER number of kilometers than the same angle on the more-distant Jupiter. However, if the distance from the camera to Jupiter/Io is very large, then as seen from the camera, both objects are at essentially the same distance and so there will be little difference between the scales used for the two bodies. Students can check this result with an inquiry assignment.

Extra for Experts

11

Image Name - Stephan's Quintet
Instrument - Hubble Space Telescope

Image size = 60,000 LY x 90,000 LY
Distance - 270 million Light Years (LY)



From a GOOGLE search or other resource, what kind of object is this?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

See the last page of the booklet for the Answer Key.

Extra for Experts

Image Name - Ikotawa

Image size = 535 meters long

Instrument - Hayabusa Spacecraft

Distance - 20 kilometers



From a GOOGLE search or other resource, what kind of object is this, and why is this image interesting?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

See the last page of the booklet for the Answer Key.

Extra for Experts

Image Name - Mercury Craters
Instrument - MESSENGER spacecraft

Image size = 563 kilometers wide
Distance - 19,760 kilometers



From a GOOGLE search or other resource, what kind of object is this, and what other object does it look similar to?

What is the image scale?

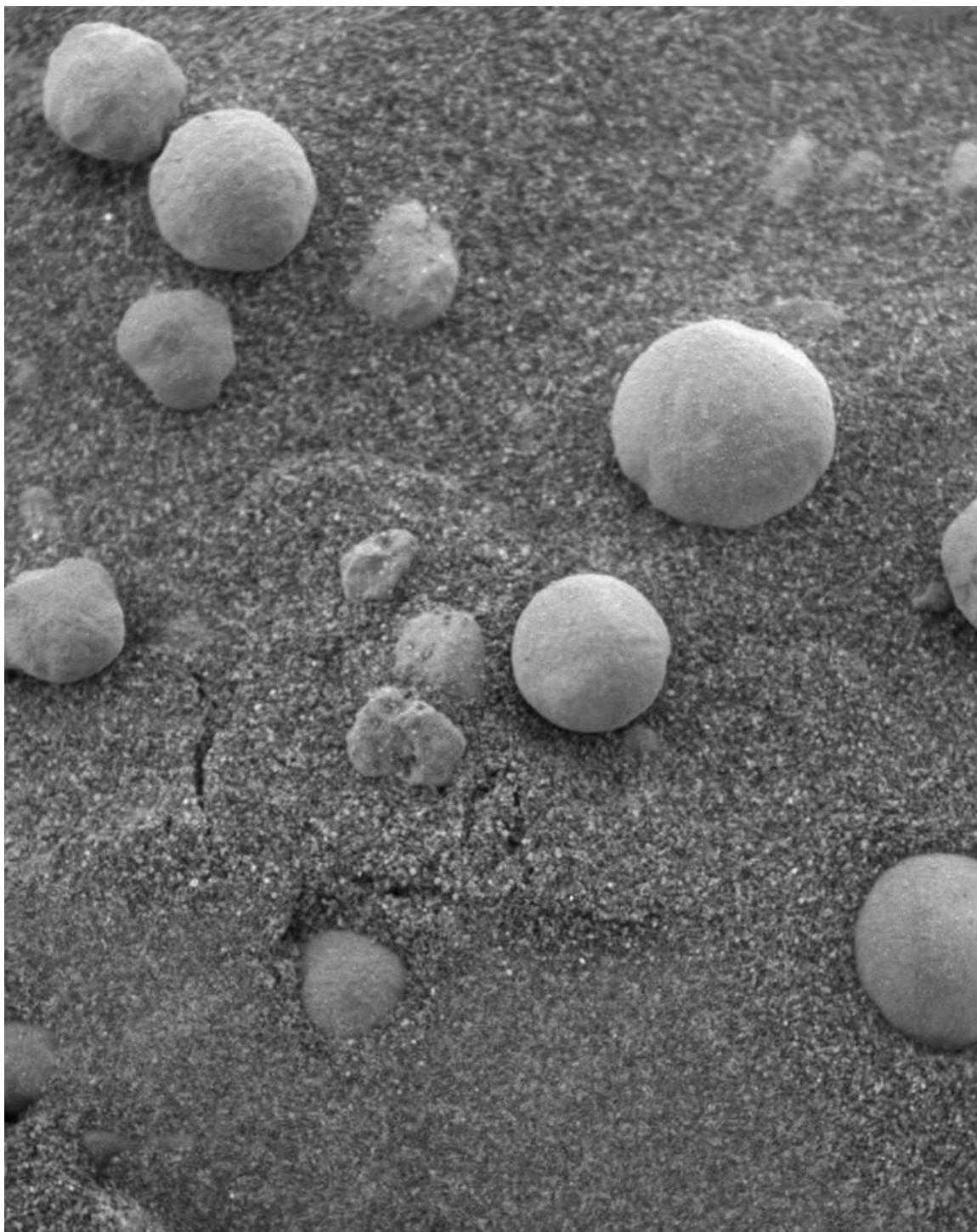
Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

See the last page of the booklet for the Answer Key.

Extra for Experts

Image Name - Hematite spheres
Instrument - Mars Rover Opportunity

Image size = 20 millimeters wide
Distance - 1 centimeter above surface



From a GOOGLE search or other resource, what kind of object is this, and how did scientists get this picture?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

See the last page of the booklet for the Answer Key.

Extra for Experts

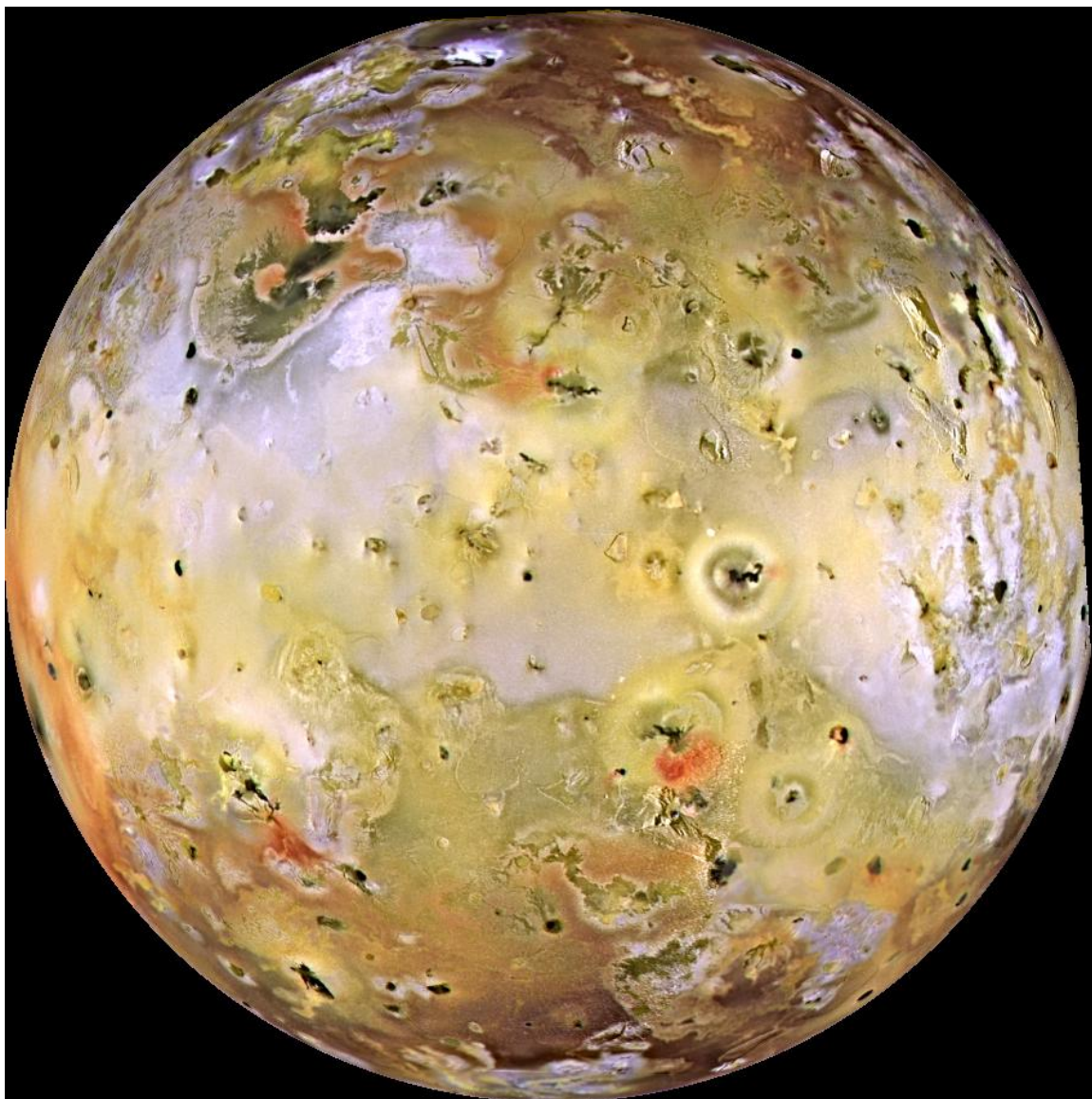
15

Image Name - Io

Instrument - Galileo Satellite

Image size = 3660 kilometers diameter

Distance - 130,000 kilometers altitude



From a GOOGLE search or other resource, what kind of object is this, and what are all those dark spots?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

See the last page of the booklet for the Answer Key.

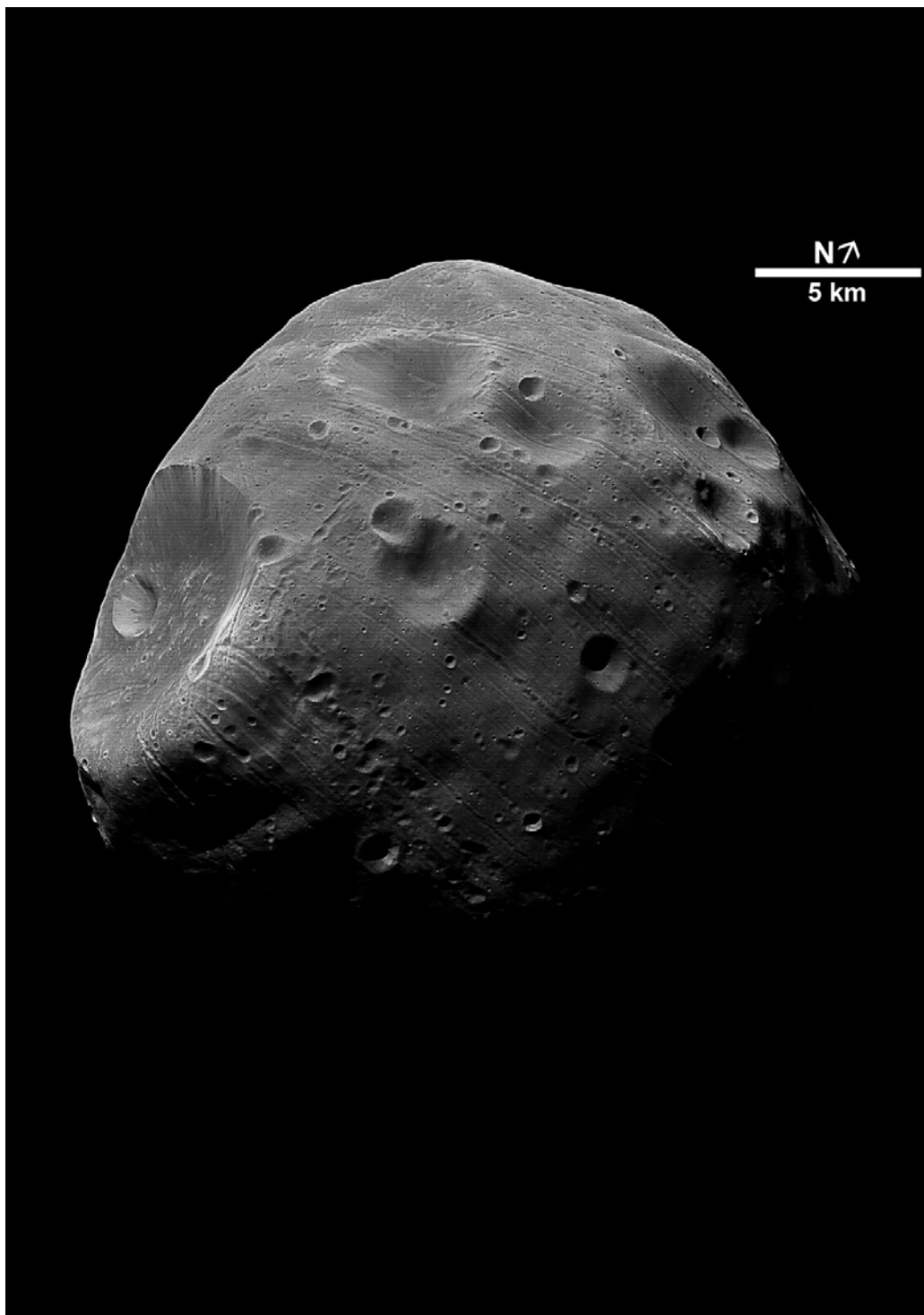
Extra for Experts

Image Name - Phobos

Image size = Use clues!

Instrument - Mars Express satellite (ESA)

Distance - 200 kilometers



From a GOOGLE search or other resource, what kind of object is this, and what do scientists think those lines might be?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

See the last page of the booklet for the Answer Key.

Extra for Experts

Image Name - Cluster CL0024+17
Instrument - Hubble Space Telescope

Image size = 2.6 million light years wide
Distance - 5 billion light years



From a GOOGLE search or other resource, what kind of object is this, and what major discovery did astronomers make by studying it?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

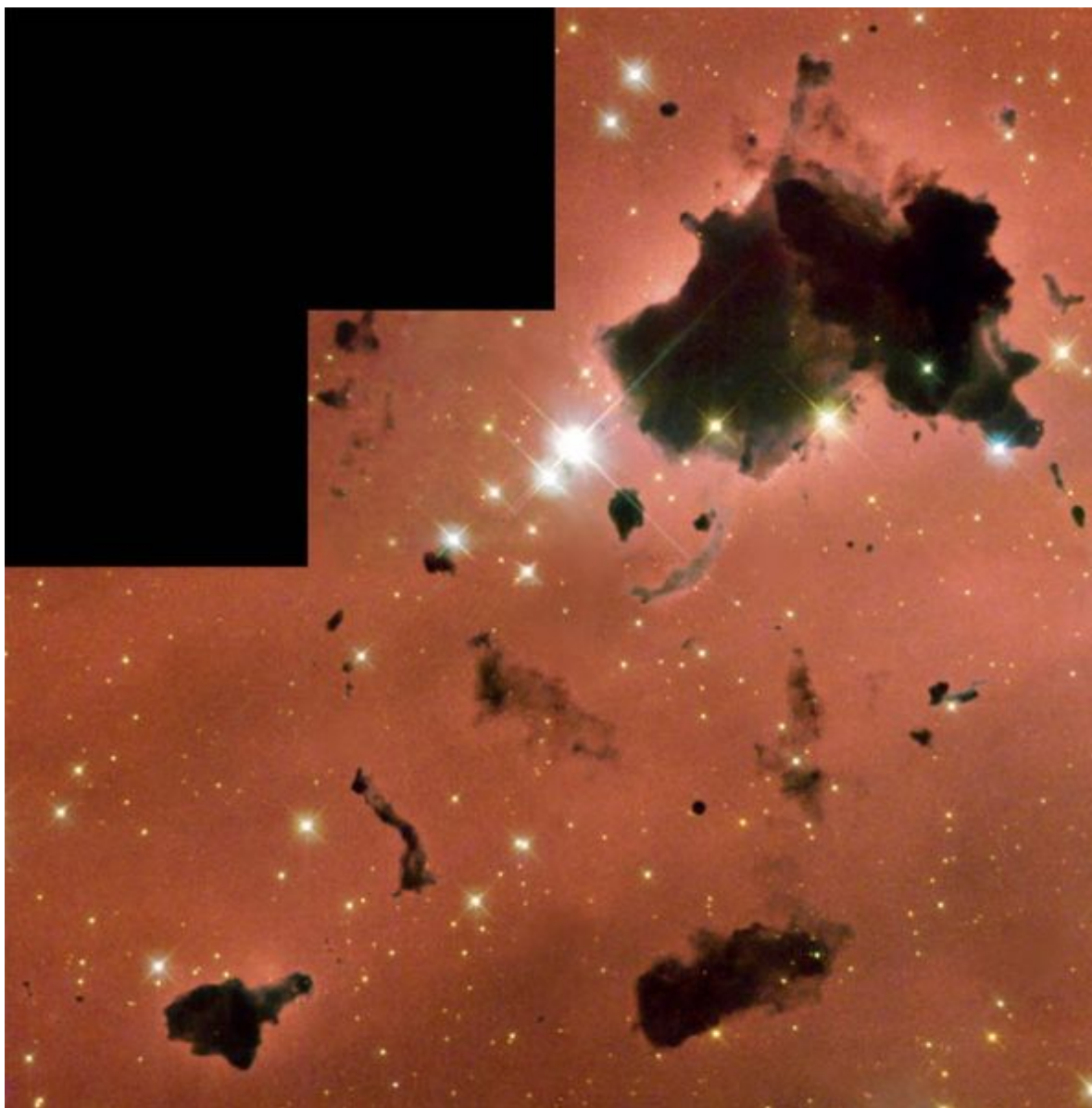
See the last page of the booklet for the Answer Key.

Extra for Experts

18

Image Name - Thackeray's Globules in IC2944
Instrument - Hubble Space Telescope

Image size = 4.5 light years wide
Distance - 5,900 light years



From a GOOGLE search or other resource, what kind of object is this, and what role does it play in forming new stars?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

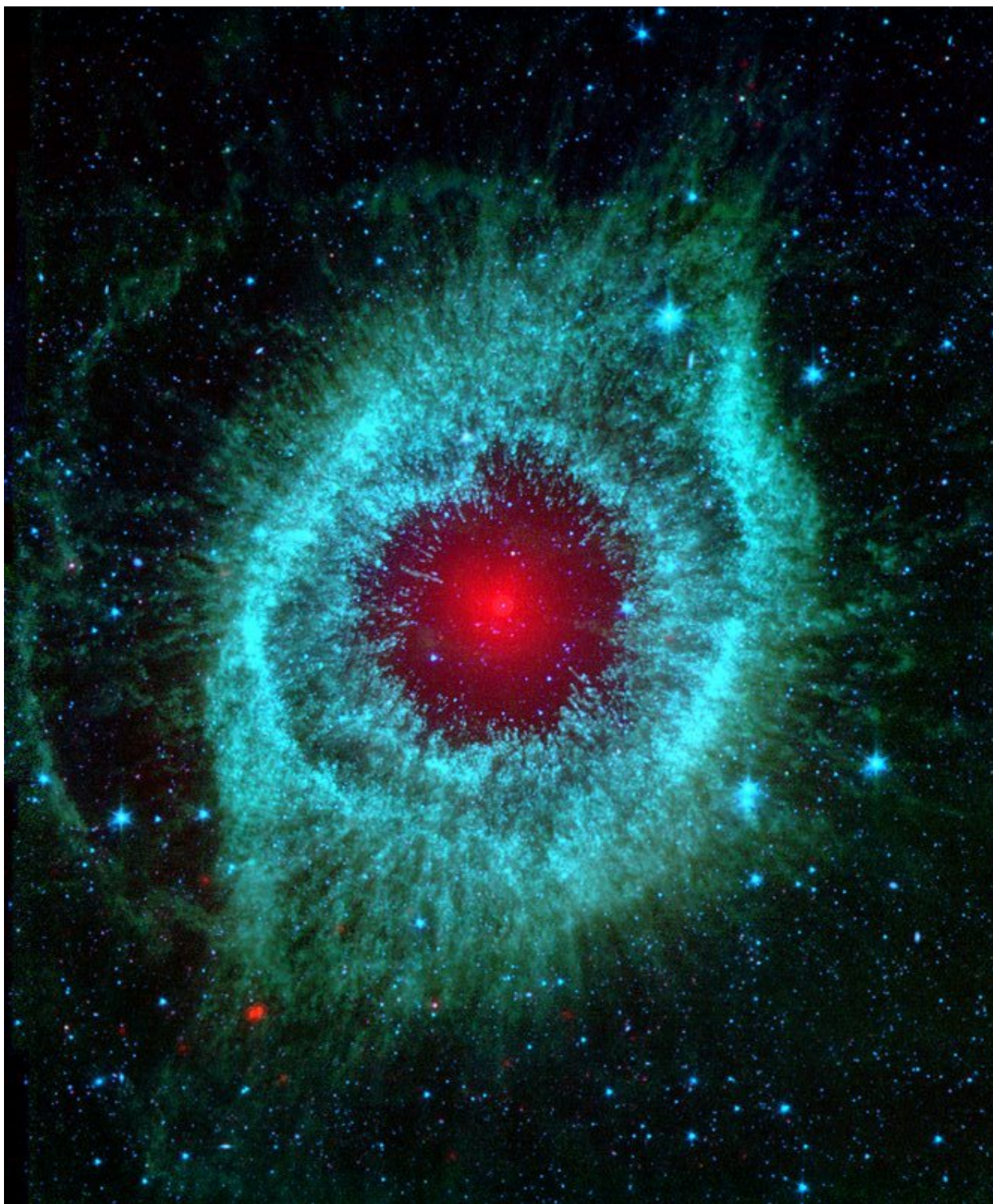
See the last page of the booklet for the Answer Key.

Extra for Experts

19

Image Name - Helix Nebula
Instrument - Spitzer Space Telescope

Image size = 2.5 light years diameter
Distance - 650 light years



From a GOOGLE search or other resource, what kind of object is this, and what kinds of stars produce them?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

See the last page of the booklet for the Answer Key.

Extra for Experts Answer Key

Stephan's Quintet - A cluster of galaxies - image scale = $60,000 \text{ ly}/153\text{mm} = 392 \text{ lightyears/mm}$

Ikatowa - An asteroid with a surface strewn by boulders and no craters! - scale = $20 \text{ km}/195 \text{ mm} = 103 \text{ meters/mm}$.

Mercury Craters - The surface of the planet mercury resembles our very own moon! - Scale = $563 \text{ km}/153\text{mm} = 3.7 \text{ km/mm}$

Hematite Spheres - Found on a rock on the surface of mars by instruments on the Mars Rover - Scale = $20 \text{ millimeters}/153 \text{ mm} = 0.13 \text{ millimeters/mm}$

Io - The volcanically active satellite of Jupiter .The black spots are volcanic and geyser vents that spew out sulfur compounds. Scale = $3660 \text{ km}/153 \text{ mm} = 24.0 \text{ km/mm}$

Phobos - One of two satellites of mars. The numerous straight lines are probably stress fractures caused by the gravitational distortion of Phobos by mars. The scale can be deduced from the line in the photo that says '5 km' which is 25 mm long so Scale = $5\text{km}/25\text{mm} = 0.2 \text{ km/mm}$ or 200 meters/mm.

Cluster CL0024+17 - A cluster of galaxies that is held together by Dark Matter - One of the first clusters of galaxies where Dark Matter has been clearly detected by its gravitational influences. Scale = $2.6 \text{ million light years}/153 \text{ mm} = 17,000 \text{ light years/mm}$.

Thackeray's Globules - These dark clouds are dense collections of interstellar gas from which new stars are formed as the gas collapses. Scale = $4.5 \text{ light years}/153 \text{ mm} = 0.03 \text{ light years/mm}$

Helix Nebula - This is a planetary nebula formed after a star like the sun reaches the end of its evolution, ejecting much of its outer layers of gas into space as an expanding cloud. Students estimates for the diameter of this nebula will vary within a probable range from 80 to 170 millimeters, so the scale will be $2.5 \text{ light years}/80 \text{ mm} = 0.03 \text{ light years/mm}$ to $2.5/170 = 0.01 \text{ light years/mm}$.

There are a vast number of interesting questions that students may propose having to do with the sizes of the various details that they find in the images, the distances between these elements, especially compared to the sizes of other things they know that are more familiar. Students should be encouraged to compare the scales of the images to each other and may find it helpful to order the images from the highest scale to the lowest scale to study how the nature of objects change as you go from one scale to the next. This is similar to the 'Powers of Ten' approach to studying the structure of the universe.

Additional Resources

The scale of an image is similar to the scale of a map in that it determines the relationship between the actual physical size of objects in the image, or their separations, based on a measure of their size using a ruler or other measuring tool. Often for digital pictures, the scale can be given in terms of kilometers per pixel or similar units.

The issue of 'scale' is related to the more familiar terms Low-resolution and High-resolution. A low-resolution image of the moon's surface may have a scale of 10 kilometers per pixel or 10-kilometers per millimeter in the photograph, while a high-resolution version of the same scene may have a scale of 100 meters per pixel or 100 meters per millimeter.

There are a number of resources online that discuss mapmaking, and which are similar to issues that occur in image analysis. For example,

Map Scale and Accuracy - is a document created by the Indiana Geographic Information Council in 2001 that has interesting information on map scales and frequently asked questions

<http://www.in.gov/igic/standards/mapscaleaccuracystandard.pdf>

Map Scales - The US Geological Service has a document that details the scales used in its many different maps

<http://egsc.usgs.gov/isb/pubs/factsheets/fs01502.html>

Mathematics of Cartography - Cynthia Lanius has a short lesson on map scales at Rice University

<http://egsc.usgs.gov/isb/pubs/factsheets/fs01502.html>

CHANDRA Images with Scale Bars - Is a gallery of images provided by this NASA observatory, and an intriguing look at the universe from many different scales!

http://chandra.harvard.edu/photo/scalebar_images.html

Note: Any student that has tried to 're-size' a digital photograph to make a screen-saver image has had to deal with image scaling. A computer monitor has a fixed width and height in centimeters, but an image at a resolution of 72 pixels/inch and one with 200 pixels/inch look very different. This is also true for the new HD TV screens which come in two resolutions 1080p and 720p. If the screen is 30 inches across its scale will be 36 pixels/inch or 24 pixels/inch. The former will provide a higher-resolution rendition of the picture displayed compared to the later resolution.

A note from the Author,

One of the most fun things I recall doing when I was in grade school is looking at a picture and working out how big things were. This turned out to be an important skill as I became a professional astronomer, even though it only required the ability to use a millimeter ruler and do a simple division.

Every time you pick up a map, or look at a picture, your first challenge is to decide how big things are. Most of the time, our brains do that silently when we look at pictures in our family photo album. We have a sense for how big things are in the photos that we take. But what happens when you are taking pictures of something completely unfamiliar? This happens all the time in astronomy, and there is no way we can intuitively gauge how big things are unless we actually work out the math. Luckily, this is the same math that you see when you look at a map of your home town. The information is located in the map Legend as a bar measured out in miles, kilometers or feet and meters.

This collection of satellite images show familiar (Las Vegas!) and unfamiliar (craters on the moon) images, and the challenge is to work out how big things are in the images by calculating the image scale. The math is really quite simple. Students will divide two whole numbers to get a third decimal number, which is the image scale in kilometers per millimeter. Challenge the students to find the smallest object photographed among all 10 images. Is it a car? A crater? A boulder? You might also organize the 10 images in order of increasing image scale as though you were looking through a mathematical microscope and cranking up the magnifying power. When you go from 100 meters per millimeter to 1 meter per millimeter, you are actually magnifying the scene by 100-times!

But don't get too lost in the mathematics. Let the student enjoy exploring the surface of another planet and appreciate the 'sense of scale' as they look for the biggest, smallest or strangest features they can find. This activity also helps them become better, critical, observers and interpreters of image data; a skill that will serve them well as they read x-rays and CAT scans as doctors, or become architects.

Sten Odenwald



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