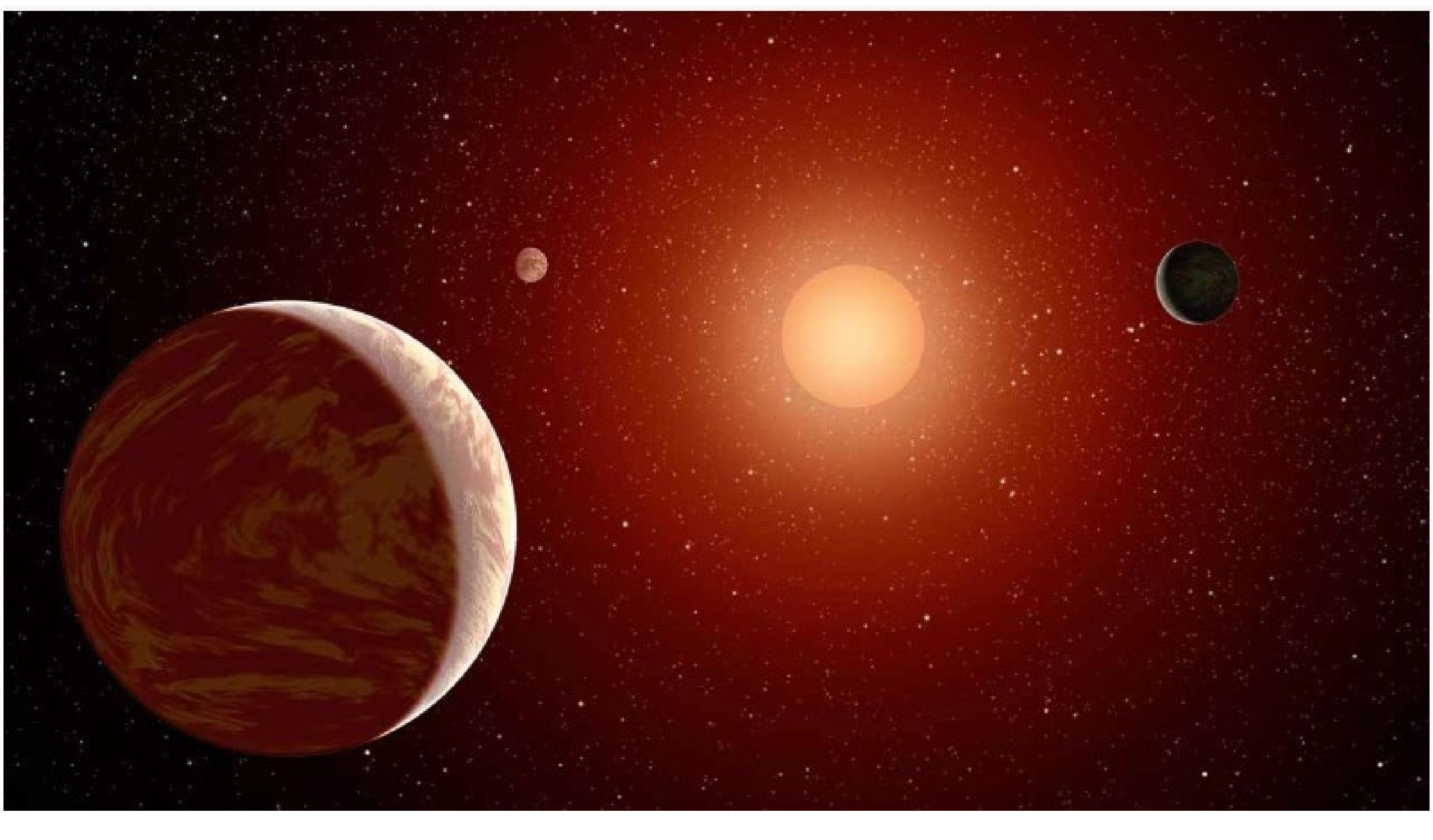
National Aeronautics and Space Administration

Math Problems Related to

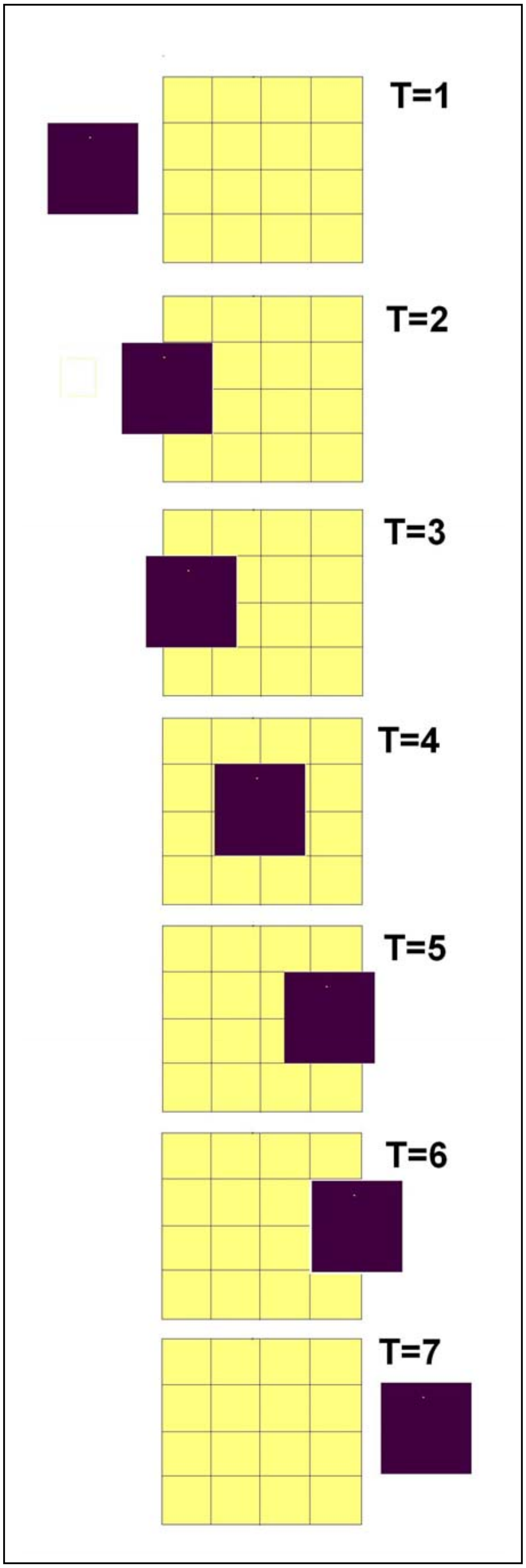
Exoplanets and Habitable Zones



Introduction

This book provides an assortment of mathematics problems that aspects of exoplanets including their statistics, classifications, the limits to their habitable zones and other quantitative aspects of their properties and discovery.

Kepler Spies Five New Planets

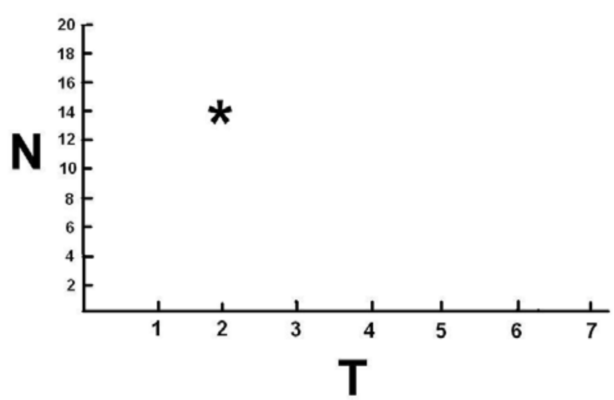


NASA's Kepler spacecraft recently announced the discovery of five new planets orbiting distant stars. The satellite measures the dimming of the light from these stars as planets pass across the face of the star as viewed from Earth. To see how this works, lets look at a simple model.

In the Bizarro Universe, stars and planets are cubical, hot spherical. Bizarro astronomers search for distant planets around other stars by watching planets pass across the face of the stars and cause the light to dim.

Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?



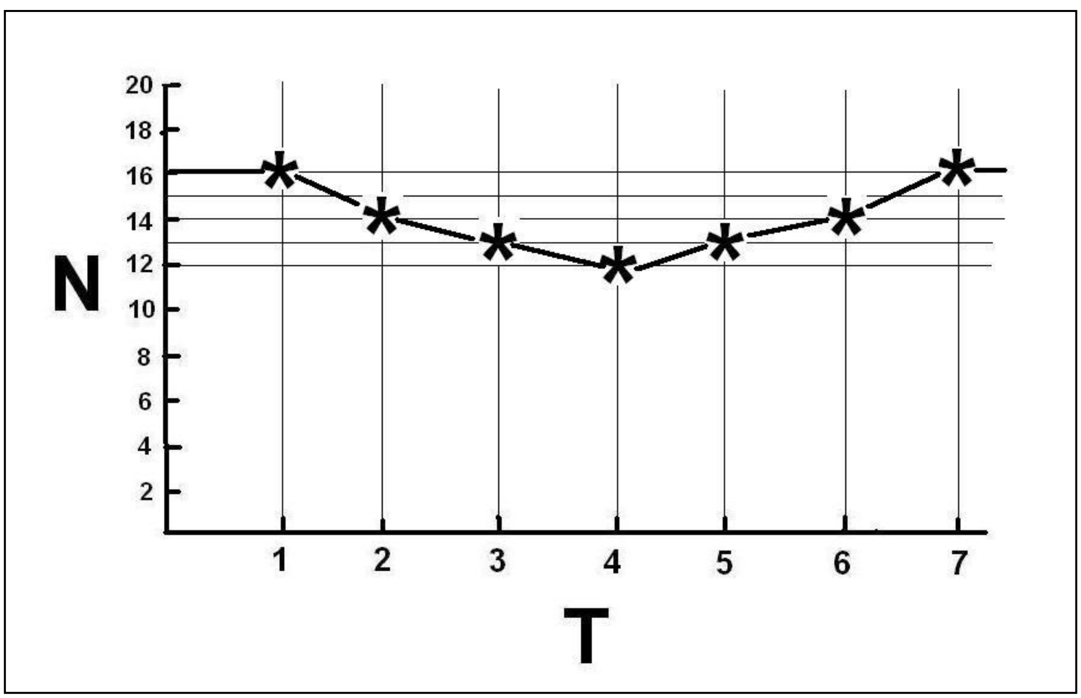
Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Answer: **Count the number of yellow squares in the star and plot these for each value of T in the graph as shown below. Note, for T= 3 and 5, the black square of the planet occupies 2 full squares and 2 half squares for a total of 2 + 1/2 + 1/2 = 3 squares covered, so there are 16 - 3 = 13 squares remaining that are yellow.**

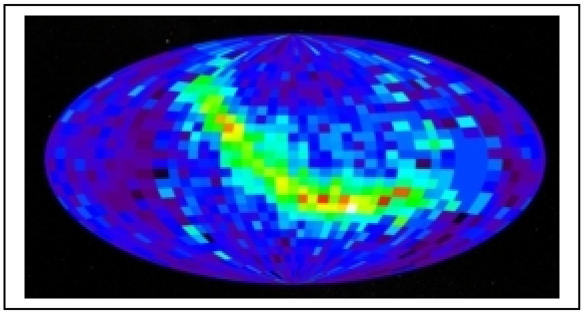
Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?

Answer: The light curve shows that the planet caused the light from the star to decrease from 16 units to 12 units because the planet blocked 16-12 = 4 units of the stars surface area. That means that the planet squares occupy 4/16 of the stars area as seen by the astronomers. The area of the star is just the area of a square, so the area of the square planet is 4/16 of the stars area or Ap = 4/16 x Astar. Since the star as a width of Wstar = 1 million kilometers, the planet will have a width of $Wp = Wstar\sqrt{\frac{4}{16}}$ or **500,000 kilometers**.

The amount of star light dimming is proportional to the ratio of the area of the planet and the star facing the observer. The Kepler satellite can detect changes by as little as 0.0001 in the light from a star, so the smallest planets it can detect have diameters about 1/100 the size of the stars that they orbit. For a star with a diameter of the sun, 1.4 million kilometers, the smallest planet detectable by the Transit Method has a diameter about equal to 14,000 kilometers or about the size of Earth.



IBEX Creates an Unusual Image of the Sky!



NASA's IBEX satellite recently made headlines by creating a picture of the entire sky, not using light but by using cosmic particles called ENAs (Energetic Neutral Atoms). These fast-moving atoms flow through the solar system. Some of them reach Earth, where they can be captured by the IBEX satellite. By counting how many of these ENAs the satellite sees in different directions in the sky, IBEX can create a unique 'picture' of where ENAs are coming from in space.

The big surprise was that they were not coming from all over the sky as expected. They were also coming from a specific band of directions as we see in the image to the left. This image has the same kind of geometry as the map of the Earth below it! It is called a Mollweide Projection, except that instead of graphing geographic points on Earth, the IBEX image shows points in outer space!

	A	B	C	D	E
1					
2					
3					
4					
5					

Data String:
A5, E2, B2, D4, C3, A1, E4, C3, D4, B2, D4, B3, C4, E5, D5, D4, C2, D3, B1, E5, A2, C3, D5, C5, D4, E4, D3, C4, B4, D2, E3, C1, B5, A3, E1, A4, D1, B3, C2, E3

The IBEX satellite detected a series of particles entering its ENA instrument, and was able to determine the direction that each particle came from in the sky. The grid above shows a portion of the sky as a 5x5 grid with columns labeled by their letter and rows by their number. The data string to the right shows the detections of individual ENA particles with their direction indicated by their cell. 'A5, E2, B2…' means that the first ENA particle came from the direction of cell 'A5', the second from cell 'E2' and the third from cell 'B2' and so on. In some ways this process is like the 'call out' during a Bingo game, except that you keep track of the particle 'tokens' in each square to build a picture! Let's look at an example of constructing an ENA image.

Problem 1 - From the hypothetical data string, tally the number of particles detected in each sky cell in the grid. Select colors to represent the number of ENAs to create an 'image' of the sky in ENAs! How many particles were reported by this data string?

Problem 2 - Suppose that cell B2 is in the direction of the constellation Auriga, cell C3 is towards Taurus and cell D4 is towards Orion, from which constellation in the sky were most of the ENAs detected?

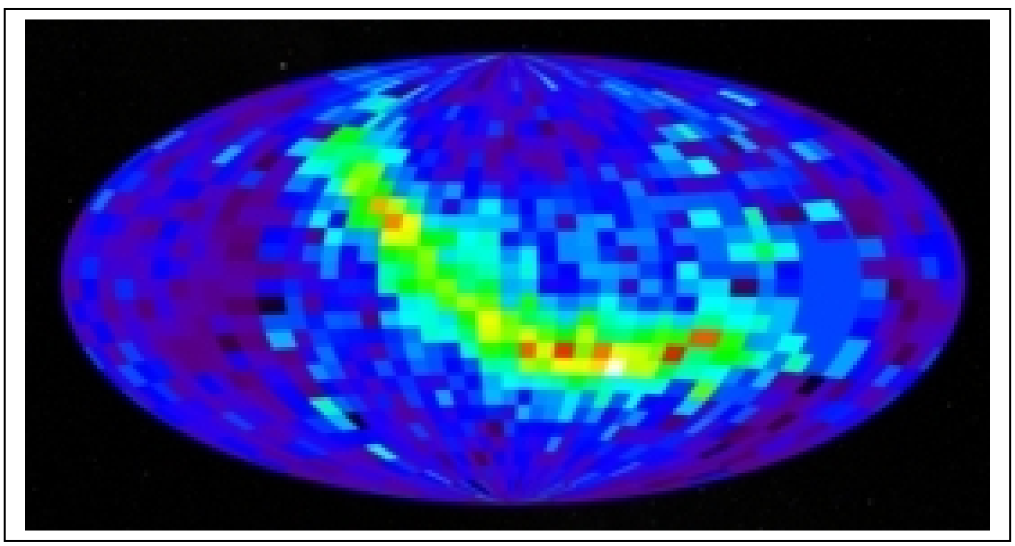
Answer Key

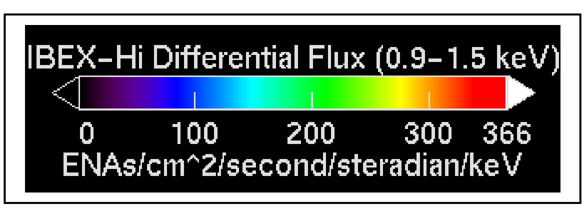
	A	B	C	D	E
1	1	1	1	1	1
2	1	2	2	1	1
3	1	2	3	2	2
4	1	1	2	5	2
5	1	1	1	2	2

Problem 1 - Answer above. Students can select yellow = 5, orange=3, red=2 and blue=1 as an example and colorize the table. This is an example of using 'false color' to highlight data in order to reveal patterns. This technique is used by scientists, but should never be confused with the 'actual' color of an object. If you add up the number of ENAS in the data stream you get 40 ENAs being reported.

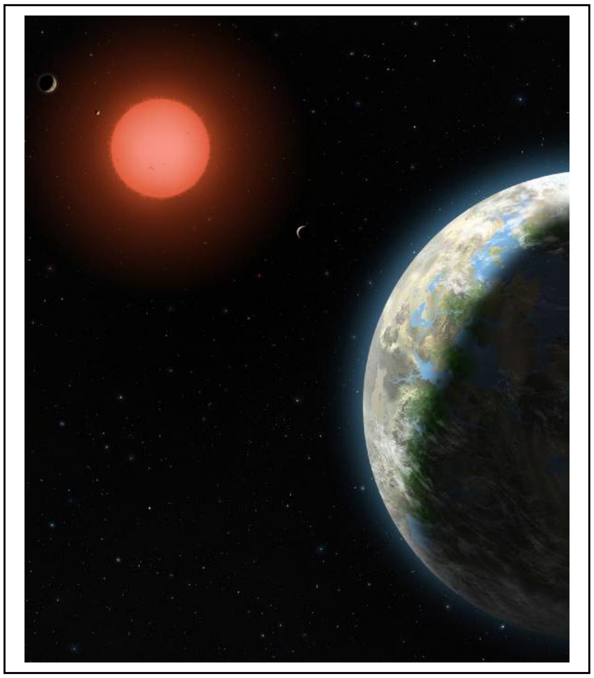
Problem 2 - The most ENAs recorded in any of the cells is '5' from Cell D4, which is in the direction of the constellation Orion.

The image below shows a close-up of actual IBEX data revealing the individual sky cells that make up the image. The color bar below shows the relationship between the color used, and the number of ENAs detected in each sky cell (called the particle flux).





The Earth-like Planet Gliese 581g



Professors Steven Vogt at UC Santa Cruz, and Paul Butler of the Carnegie Institution have just announced the discovery of a new planet orbiting the nearby red dwarf star Gliese 518. The star is located 20 light years from Earth in the constellation Libra. The planet joins five others in this crowded planetary system, and has a mass about three to four times Earth, making it in all likelihood a rocky planet, rather than a gas giant. The planet is tidally locked to its star which means that during its 37 day orbit, it always shows the same face to the star so that one hemisphere is always in daylight while the other is in permanent nighttime.

One of the most important aspects to new planets is whether they are in a distance zone where water can remain a liquid on the planets surface. The Habitable Zone (HZ) location around a star depends on the amount of light energy that the star produces. For the Sun, the HZ extends from about the orbit of Venus to the orbit of Mars. For stars that emit less energy, the HZ will be much closer to the star. Once an astronomer knows what kind of star a planet orbits, they can calculate over what distances the HZ will exist.

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system?

Problem 2 - One Astronomical Unit (AU) is the distance between Earth and the Sun (150 million kilometers). Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table below:

Planet	Discovery Year	Distance (AU)	Period (days)	Diameter (km)
Gliese 581 b	2005	0.04	5.4	50,000
Gliese 581 c	2007	0.07	13.0	20,000
Gliese 581 d	2007	0.22	66.8	25,000
Gliese 581 e	2009	0.03	3.1	15,000
Gliese 581 f	2010	0.76	433	25,000
Gliese 581 g	2010	0.15	36.6	20,000

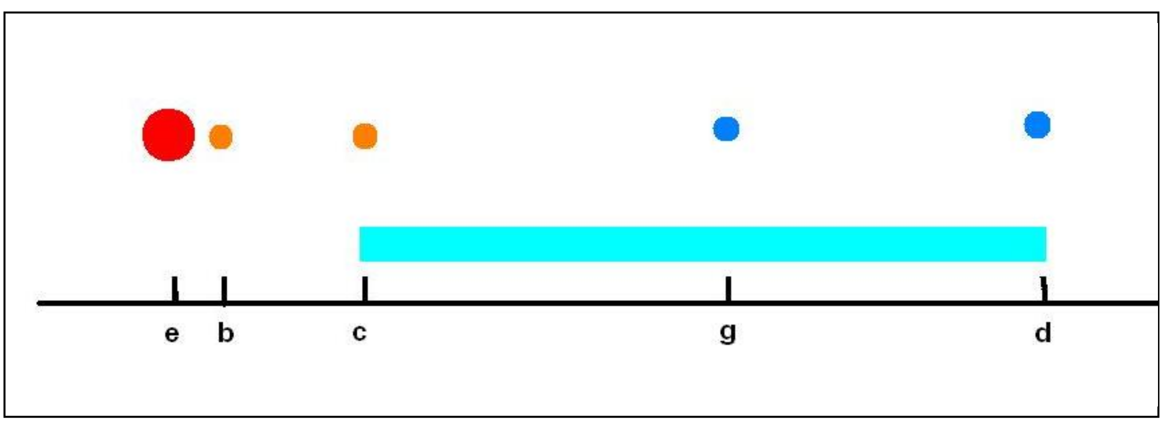
Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model diagram, shade-in the range of distances where the HZ exists for the Gliese 581 planetary system. Why do you think astronomers are excited about Gliese 581g?

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system? Answer: **According to the order of discovery date.** Note: Gliese 581 A is the designation given to the star itself.

Problem 2 - Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table. Answer: **The table below gives the dimensions on the scaled diagram. See figure below for an approximate appearance. On the scale of the figure below, Gliese 581f would be located about 32 centimeters to the right of Gliese 581d.**

Planet	Discovery Year	Distance (cm)	Period (days)	Diameter (mm)
Gliese 581 b	2005	4	5.4	10
Gliese 581 c	2007	7	13.0	4
Gliese 581 d	2007	22	66.8	5
Gliese 581 e	2009	3	3.1	3
Gliese 581 f	2010	76	433	5
Gliese 581 g	2010	15	36.6	4

Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model, shade-in the range of distances where the HZ exists. Why do you think astronomers are excited about Gliese 581g? Answer: **See the bar spanning the given distances. Note that Gliese c, g and d are located in the HZ of Gliese 581. Because Gliese 581 g is located near the center of this zone and is very likely to be warm enough for there to be liquid water, which is an essential ingredient for life. Gliese 581c may be too hot and Gliese 581 d may be too cold.**



Earth-like Planets by the Score!

Table of Candidate Planet Sizes

Size Class	Size (Earth Radius)	Number of candidates
Earths	R <1.25	68
Super-Earths	1.25 < R < 2.0	288
Neptunes	2.0 < R < 6.0	662
Jovians	6.0 < R < 15	165
Super-Jovians	15 < R < 22	19
Dwarf Stars, etc	R > 22	15

NASA's Kepler mission has just completed its first year of surveying 156,453 stars to detect the tell-tail signs of distant planets passing across the faces of their stars .This causes a slight dimming of the starlight, which can be detected by the satellite observatory. From the 1,235 transits detected so far, 33 were eliminated because the planets would have been far larger than Jupiter, and possibly dwarf stars. The table above shows the distribution of the remaining planet candidates among the interesting sizes ranges for planets in our own solar system.

About 30 percent of the candidates have been found to belong to multiple-planet systems, with several planets orbiting the same star. To find Earth-like planets where liquid water could be present, astronomers define a Habitable Zone (HZ) surrounding each star where planet surfaces could be warm enough for liquid water. This is roughly between temperatures of 270 to 300 Kelvin. A careful study of the orbits of the planetary candidates discovered 54 candidate planets in the HZs of their stars. Of these, five planets are roughly Earth-sized, and the other 49 planets range from twice the size of Earth to larger than Jupiter.

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Answer: There are 1,202 candidate planets in the larger survey, and 54 found in their HZ so the percentage is P = 100% x (54/1202) = **4.5%**.

Note: *The essay says that 33 candidates were eliminated because they were probably not planets, so 1,235 - 33 = 1,202 planet candidates.*

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Answer: There were 68 Earth-sized planets found from among 1,202 candidates, and 5 were Earth-sized, so the percentage of Earth-sized planets in their HZs is P = 100% x (5/68) = 7.4%.

So, if you find one Earth-sized planet orbiting a star in the Kepler survey, there is a 7.4% chance that it will be in its HZ so that liquid water can exist.

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

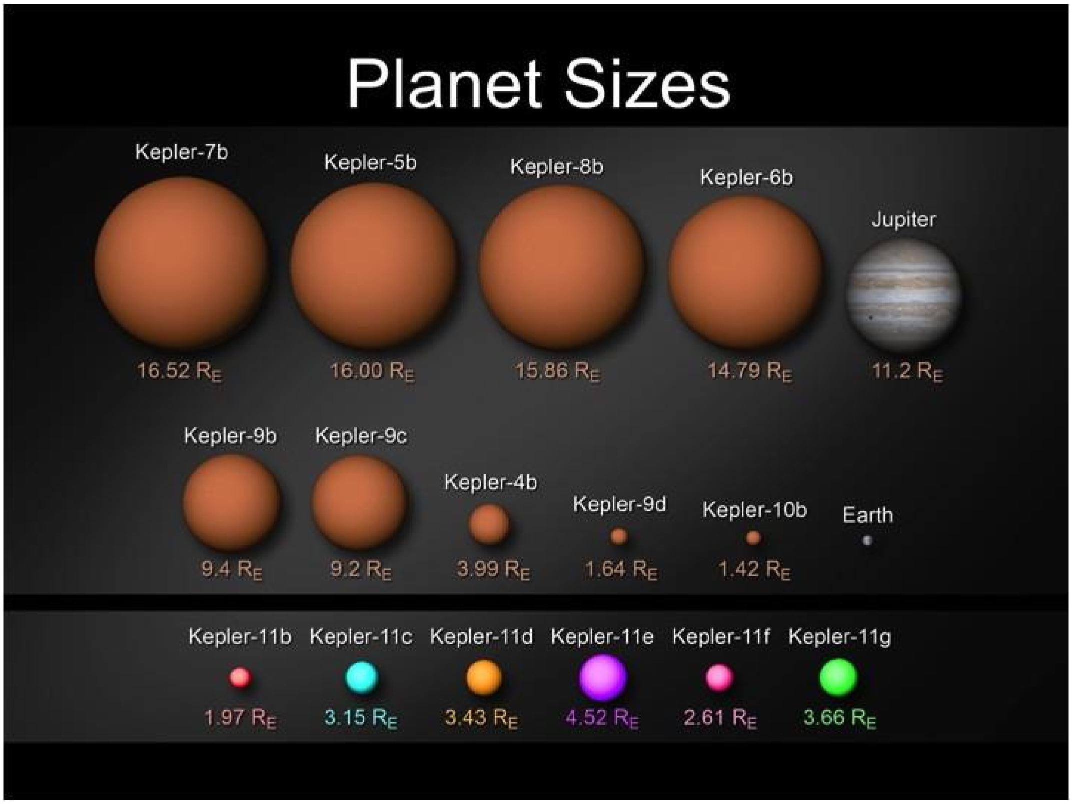
Answer: Students may use simple scaling. The Kepler star sample contains 156,453 stars. It resulted in the discovery, so far, of 5 Earth-sized planets in the Habitable Zones of their respective stars. So

$\frac{5}{156,453} = \frac{N}{40 billion}$ so

N = (40 billion / 156,453) x 5 planets

or about **1.3 million planets**.

Another way to think about this is that if you select 30,000 stars in the sky similar to our own sun, you could expect to find about 1 Earth-sized planet orbiting within the Habitable Zone of its star. (5 x 30,000/156453 = 0.96 or about 1.0 planets)



This diagram shows disks representing the planets discovered in orbit around 8 different stars all drawn to the same scale. Earth and Jupiter are also shown so that you can see how big they are in comparison. Solve the problems below using fraction arithmetic to find out how big these new planets are compared to Earth and Jupiter.

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b. Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Problem 2 – The planet Kepler-9c is 9./11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b. If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b .Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Answer: It helps to set up these kinds of problems as though they were unit conversion problems, and then cancel the planet names to get the desired ratio:

$$\frac{1 \times \text{Kepler5b}}{8 \times \text{Kepler11b}} \times \frac{1 \times \text{Kepler11b}}{2 \times \text{Earth}} = \frac{1\ \text{Kepler5b}}{8 \times \text{Earth}}$$

so Kepler 5b is **16 x Earth**

Note how the 'units' for Kepler-11b have canceled out.

Problem 2 – The planet Kepler-9c is 9/11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

$$\frac{11 \times \text{Kepler9c}}{9 \times \text{Jupiter}} \times \frac{2 \times \text{Kepler11e}}{1 \times \text{Kepler9c}} = \frac{22\text{x Kepler9c}}{9 \times \text{Jupiter}}$$

so Kepler 11e = **9/22 Jupiter**

From the figure we see Kepler11e = 4.52 Re and Jupiter = 11.2 Re so Kepler11e = 4.5/11 = 9/22 Jupiter.

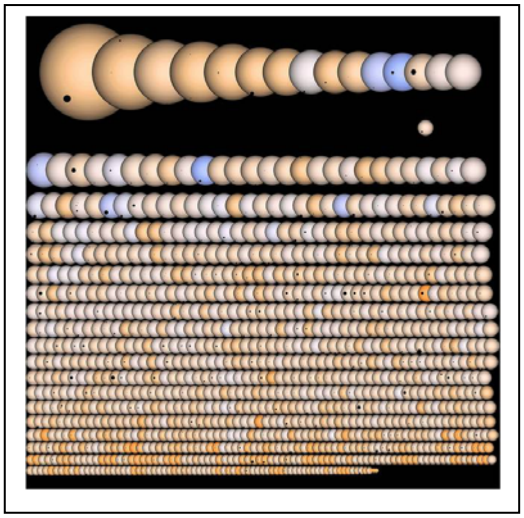
Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b .If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

$$\frac{10\text{xKepler10b}}{1\text{xKepler6b}} \times \frac{9\ \text{Kepler6b}}{15\ \text{Kepler9b}} \times \frac{4\ \text{Kepler9b}}{9\ \text{Kepler11g}} = \frac{40\ \text{Kepler10b}}{15\ \text{Kepler11g}}$$

So 15 x the diameter of Kepler 11g is equal to 40x the diameter of Kepler 10b
And so, Kepler 11g is 40/15 times the diameter of Kepler 10b

From the figure we see that Kepler 11g is 3.66 Re and Kepler 10b is 1.4 Re which is in about the same ratio as our fractions. 3.66/1.4 = 2.6 and 40/15 = 2.7.

Kepler's Latest Count on Goldilocks Planets



Scientists from around the world are gathered this week at NASA's Ames Research Center in Moffett Field, Calif., for the second Kepler Science Conference, where they will discuss the latest findings resulting from the analysis of Kepler Space Telescope data.

New Kepler data analysis and research also show that most stars in our galaxy have at least one planet. This suggests that the majority of stars in the night sky may be home to planetary systems, perhaps some like our solar system.

Problem 1 - Based on a total of 3538 candidate planets, Kepler announced in November 2013 that it had detected 674 objects smaller than 1.25 Re, 1076 objects between 1.25 and 2.0 Re, 1457 objects between 2.0 and 6.0 Re, 229 objects between 6.0 and 15 Re, and 102 objects larger than 15 Re. What are the percentages of each size of planet candidate, and create a histogram plot to show this data.

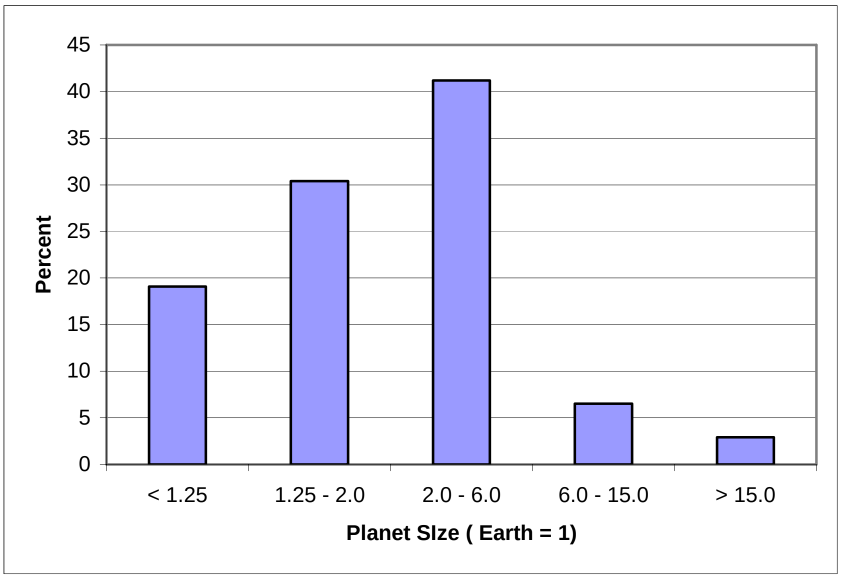
Problem 2 - The survey included 42,000 stars and detected 10 candidate planets that were about the same size as Earth, and located at a distance from their stars where liquid water could occur (Goldilocks Zone). Because of the way that Kepler detected these planets, they were only able to see about 1.3% of all planets orbiting the surveyed stars. If the Milky Way contains about 500 billion stars similar to the ones surveyed by Kepler, about how many Earth-like planets might you expect to find in the entire Milky Way galaxy?

NASA Kepler Results Usher in a New Era of Astronomy
November 4, 2013

http://www.nasa.gov/content/nasa-kepler-results-usher-in-a-new-era-of-astronomy/index.html

Problem 1 - Based on a total of 3538 candidate planets, Kepler announced in November 2013 that it had detected 674 objects smaller than 1.25 Re, 1076 objects between 1.25 and 2.0 Re, 1457 objects between 2.0 and 6.0 Re, 229 objects between 6.0 and 15 Re, and 102 objects larger than 15 Re. What are the percentages of each size of planet candidate, and create a histogram plot to show this data.

Answer: 1.25	=	674 / 3538	=	**19.1 %**
1.25 to 2.0	=	1076 / 3538	=	**30.4 %**
2.0 to 6.0	=	1457 / 3538	=	**41.2 %**
6.0 to 15.0	=	229 / 3538	=	**6.5 %**
> 15.0	=	102 / 3538	=	**2.9 %**



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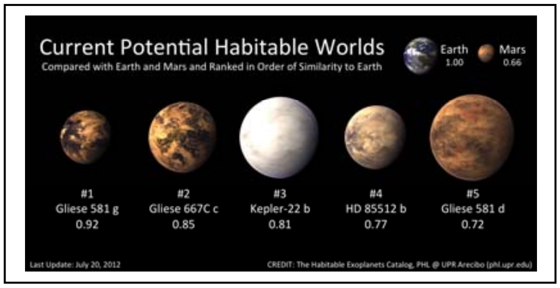
Answer: 500 billion x (10 detections /42,000 surveyed stars) x (100 stars/1.3 detection) = 9.1 billion planets. To the nearest billion this becomes **9 billion earth-like planets**.

A Number Puzzle about Planets Beyond our Solar System 7



By 2013, astronomers have detected over 941 confirmed planets, and 2500 additional candidates, which orbit stars beyond our solar system. Many of these are earth-like and could have liquid water on their surfaces.

Find the roots to the quadratic equations below and match the X-value to the Word Bank to find the word to use in the essay about exoplanets.

Since 1995, astronomers have discovered over 900 planets orbiting other stars. Because they are not planets orbiting our own sun, they are called 1)_____________. At first, astronomers could only detect planets that were larger than 2)____________. Those were often found orbiting their stars much closer than 3)_____________ orbits our own sun. Then as instruments improved in their 4)_____________, planets as large as the outer planet 5)__________ and as small as our own Earth were found. NASA's 6)_____________ observatory in space was launched in 2010 and has since discovered over 2000 new candidate planets, and many of these are Earth-like in size. What is also so exciting is that 7)________ of these earth-sized worlds orbit far enough from their stars that liquid 8)__________ can exist on their surfaces if the planets have dense-enough 9)_______________.

Some exoplanets are unbelievably strange. The world orbiting the star HD209458 looks like a Jupiter-sized comet. It is so close to its star that this planet is actually evaporating. Another exoplanet called 10)____________ is located in the constellation Monoceros. It is so hot that its clouds rain droplets of liquid 11)_________ onto its surface, which is covered by a liquid lava ocean. The Hubble and 12)__________ Space Telescopes have actually seen two planets orbiting the stars 13)___________ and 14)_________ in the constellations Pisces Austrinus and Vulpecula. They are about the size of Jupiter and orbit their stars far beyond the orbit of Neptune in our solar system. One star called 15)__________ located 127 light years from Earth in the constellation Hydrus, has seven planets but they all orbit closer to their star than Mars in our own solar system. Five of these planets are as big as Neptune!

1) x^2-2x-3
2) x^2-16
3) $x^2+3x-10$
4) $2x^2-6x$
5) $3x^2-12$
6) $5x^2+30x-35$
7) $x^2-13x+36$
8) $2x^2-12x+10$
9) $4x^2-8x-12$
10) $x^2+4x-12$
11) $x^2+11x+30$
12) $x^2-13x+42$
13) $x^2-16X+63$
14) $3x^2+24X-27$
15) $2x^2-12x-32$

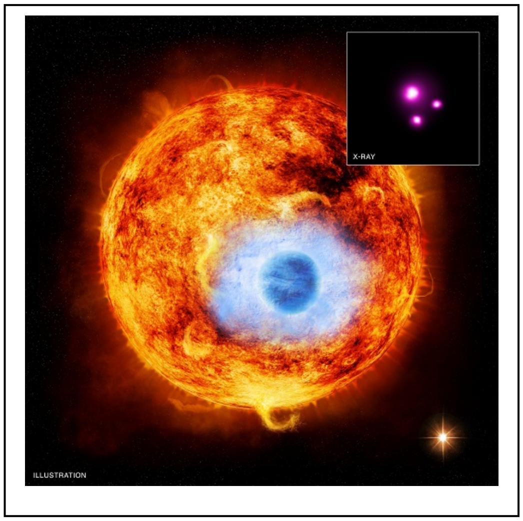
Word Bank

+3	exoplanets	0	sensitivity	-2	Neptune
-1	atmospheres	-7	Kepler	+1	water
+8	HD10180	+9	many	+4	few
-4	Jupiter	+5	oxygen	-6	CoRot7b
-5	iron	+6	Spitzer	+7	Fomalhaut
+2	Mercury	-9	HD189733	+10	methane

Since 1995, astronomers have discovered over 900 planets orbiting other stars. Because they are not planets orbiting our own sun, they are called 1)___**exoplanets**__. At first, astronomers could only detect planets that were larger than 2)_____**jupiter**__. Those were often found orbiting their stars much closer than 3)__**mercury**__ orbits our own sun. Then as instruments improved in their 4)__**sensitivity**__, planets as large as the outer planet 5)__**Neptune** and as small as our own Earth were found. NASA's 6)_**Kepler**__ observatory in space was launched in 2010 and has since discovered over 2000 new candidate planets, and many of these are Earth-like in size. What is also so exciting is that 7)__**many**__ of these Earth-sized worlds orbit far enough from their stars that liquid 8)__**water**_ can exist on their surfaces if the planets have dense-enough 9)__**atmospheres**__.

Some exoplanets are unbelievably strange. The world orbiting the star HD209458 looks like a Jupiter-sized comet. It is so close to its star that this planet is actually evaporating. Another exoplanet called 10)__**CoRot7b**__ is located in the constellation Monoceros. It is so hot that its clouds rain droplets of liquid 11)__**iron** onto its surface, which is covered by a liquid lava ocean. The Hubble and 12)__**Spitzer**__ Space Telescopes have actually seen two planets orbiting the stars 13)_**Fomalhaut**__ and 14)__**HD189733**__ in the constellations Pisces Austrinus and Vulpecula. They are about the size of Jupiter and orbit their stars far beyond the orbit of Neptune in our solar system. One star called 15)_**HD101080**__ located 127 light years from Earth in the constellation Hydrus, has seven planets but they all orbit closer to their star than Mars in our own solar system. Five of these planets are as big as Neptune!

1)	x^2-2x-3	: (x-3)(x+1)	x = +3, x= -1	**exoplanet**, atmospheres
2)	x^2-16	: (x-4)(x+4)	x= +4 , x=-4	few, **jupiter**
3)	x^2+3x-10	: (x+5)(x-2)	x=-5, x=+2	iron, **mercury**
4)	2x^2-6x	: x(2x-6)	x=0, x=+3	**sensitivity**, exoplanets
5)	3x^2-12	: (3x-6)(x+2)	x=+2, x=-2	mercury, **neptune**
6)	5x^2+30x-35	: (5x-5)(x+7)	x=-7, x=+1	**Kepler**, water
7)	x^2-13x+36	: (x-9)(x-4)	x=+9, x=+4	**many**, few
8)	2x^2-12x+10	: (2x-2)(x-5)	x=+1, x=+5	**water**, oxygen
9)	4x^2-8x-12	: (x+1)(4x-12)	x=-1, x=+3	**atmosphere**s, exoplanets
10)	x^2+4x-12	: (x+6)(x-2)	x=-6, x=+2	**CoRot7b**, mercury
11)	x^2+11x+30	: (x+5)(x+6)	x=-5, x=-6	**iron**, CoRot7b
12)	x^2-13x+42	: (x-6)(x+7)	x=+6, x=-7	**Spitzer**, Kepler
13)	x^2-16X+63	: (x-7)(x-9)	x=+7, x=+9	**Fomalhaut**, many
14)	3x^2+24x-27	: (3x+27)(x-1)	x=-9, x+1	**HD189733**, water
15)	2x^2-12X-32	: (2x-16)(x+2)	x=+8, x=-2	**HD10180**, neptune



At a distance of 63 light years from Earth, this Jupiter-sized planet orbits its star once every 2.2 days. With a mass only 13% greater than Jupiter, Astronomers have measured its light and found that it would appear like a giant blue marble similar to Neptune. Its atmosphere contains carbon dioxide, along with water vapor and methane. It is so close to its star, less than 3 million miles, that it is permanently locked so that the same side of the planet always faces its star. The daytime, sun-facing temperature is a sizzling 1000 C and nighttime temperature of 700 C.

NASA's Chandra Observatory has detected the X-ray fadeout of this planet as it passes across its star. From this they estimate that the planet is losing about 600 million kg of its mass every second. The intense heat from its star is literally evaporating this planet!

Problem 1 – The mass of Jupiter is 1.9×10^{27} kg. What is the mass of HD189733b?

Problem 2 – The atmosphere of Jupiter is about 1000 km thick, and the radius of Jupiter is 70,000 km. What is the volume of the atmospheric shell of Jupiter?

Problem 3 – The average density of the Jovian atmosphere is about 50 grams/m^3. If the atmosphere of HD189733b is identical to Jupiter's in density and volume, how much mass is in the atmosphere of HD189733b?

Problem 4 – The Chandra observations suggest that the atmosphere of HD189733b is decreasing at a rate of 6.0×10^8 kg/second. How many years will it take for the entire atmosphere of this planet to be lost?

Problem 5 – How many years will it take for the entire planet to be evaporated?

NASA's Chandra Sees Eclipsing Planet in X-rays for First Time
July 29, 2013
http://www.nasa.gov/mission_pages/chandra/news/exoplanet-HD189733b.html

Problem 1 – The mass of Jupiter is 1.9×10^{27} kg. What is the mass of HD189733b?

Answer: The mass is 13% greater than Jupiter so M = $1.13 \times 1.9 \times 10^{27}$ kg = **2.1×10^{27} kg.**

Problem 2 – The atmosphere of Jupiter is about 1000 km thick, and the radius of Jupiter is 70,000 km. What is the volume of the atmospheric shell of Jupiter?

Answer: Volume = $4/3\ \pi\ (70000\text{km})^3 - 4/3\ \pi\ (69000\text{km})^3$
$= 1.44 \times 10^{15}\ \text{km}^3 - 1.38 \times 10^{15}\ \text{km}^3$
$= \mathbf{6.0 \times 10^{13}\ km^3}$

Problem 3 – The average density of the Jovian atmosphere is about 50 grams/m^3. If the atmosphere of HD189733b is identical to Jupiter's in density and volume, how much mass is in the atmosphere of HD189733b?

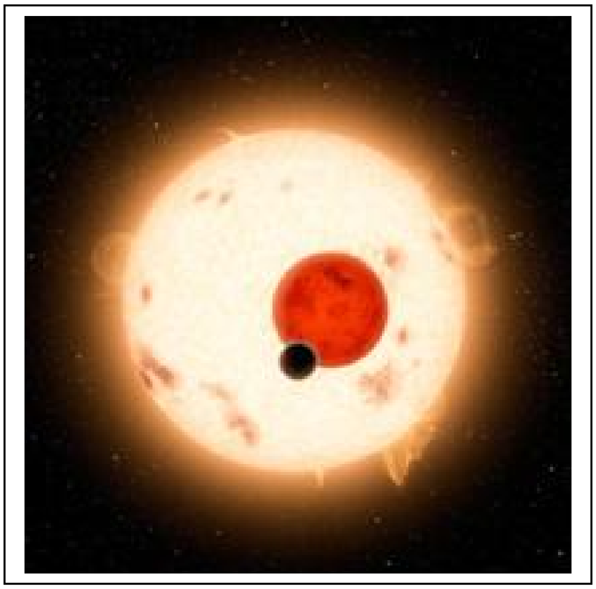
Answer: The volume of the atmosphere is 6.0×10^{13} km^3 or 6.0×10^{22} m^3.
Since mass = density x volume.
M = 50 gm/m^3 x 6.0×10^{22} m^3
= 3.0×10^{24} grams or **3.0×10^{21} kg**.

Problem 4 – The Chandra observations suggest that the atmosphere of HD189733b is decreasing at a rate of 6.0×10^{8} kg/second. How many years will it take for the entire atmosphere of this planet to be lost?

Answer : Time = amount/rate
= 3.0×10^{21} kg / (6.0×10^{8} kg/sec)
= 5.0×10^{12} seconds.
= 5.0×10^{12} seconds x (1 year/3.1×10^{7} sec) = **161,000 years**.

Problem 5 – How many years will it take for the entire planet to be evaporated?

Answer: Time = 2.1×10^{27} kilograms / 6.0×10^{8} kg/sec
= 3.5×10^{18} seconds
= **113 billion years**.



In 2011, the Kepler observatory detected a Saturn-sized planet orbiting the binary stars Kepler 16A and Kepler 16B. Nicknamed 'Tatooine', the view from this planet of its twin suns would be spectacular. The smaller star, Kepler-16B orbits once every 41 days at a distance of 30 million km from the larger star Kepler 16A, and the planet is in a circular orbit 108 million km from Kepler 16A which takes 229 days to complete. As seen from 'Tatooine', the small orange star Kepler-16B never gets more than 18 degrees from the much larger, yellow star Kepler 16A.

Predicting when Kepler 16B will pass across the face of Kepler 16A, called a transit, is made a bit more difficult because Tatooine is also moving along its orbit while Kepler 16B is in motion around the main star. To see a transit, Kepler 16B and Tatooine must be located in their orbits so that a line through their centers passes through the center of Kepler 16A at the center of the orbits. The circumferences of the two orbits are different, and they take different amounts of time to complete a full orbit. Given that you have just observed one transit, the time you must wait in order to see the next one depends on how long it takes for Kepler 16B and Tatooine to return to their 'straight line' configuration.

Problem 1 - What is the speed of the star Kepler 16B in its orbit in terms of degrees per day?

Problem 2 - What is the speed of Tatooine in its orbit in terms of degrees per day?

Problem 3 - What is the difference in angular speed between fast-moving Kepler 16B and slower-moving Tatooine?

Problem 4 - How many days will it take Kepler 16B to overtake Tatooine?

Problem 5 - Can you show that your answer to Problem 4 is just the time between transits?

Problem 1 - What is the speed of Kepler 16B in its orbit in terms of degrees per day?

Answer: SK = 360 degrees /41days = **8.78 degrees/day.**

Problem 2 - What is the speed of Tatooine in its orbit in terms of degrees per day?

Answer: ST = 360 degrees / 229 days = **1.57 degrees/day.**

Problem 3 - What is the difference in angular speed between fast-moving Kepler 16B and slower-moving Tatooine?

Answer: 8.78 - 1.57 = **7.21 degrees/day**

Problem 4 - How many days will it take Kepler 16B to overtake Tatooine?

Answer: T = 360 degrees / 7.21 = **50 days**.

Problem 6 - Can you show that your answer to Problem 4 is just the time between transits?

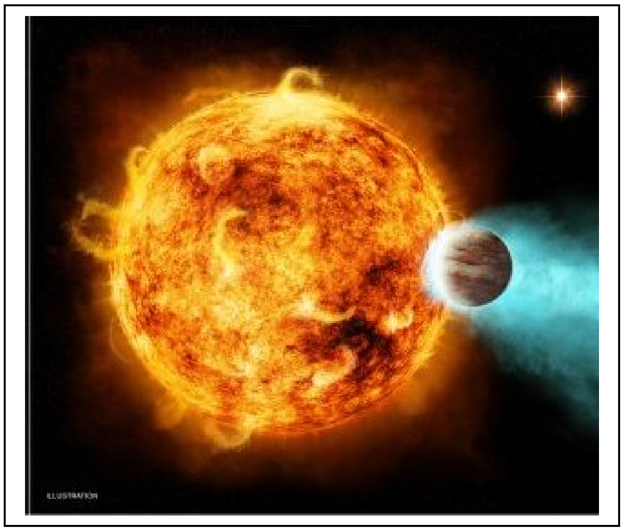
Answer: Students can sketch two concentric circles with Kepler 16B on the inner circle and Tatooine on the outer circle. Draw a line from Kepler 16A at the center of the circle to the location of Tatooine. Place Kepler 16B in its orbit on the line you drew to 'start' the clock. In 41 days, Kepler 16B will go once around its circle and return to this spot, while Tatooine will move 41 days further along its orbit. After 50 days, Kepler 16B and Tatooine will once again beon the same line to Kepler 16A.

Note: In astronomy, the synodic period, P, is related to the orbit periods of a fast-moving planet, t, and a slow-moving planet, T, by the formula

$$\frac{360}{t} - \frac{360}{T} = \frac{360}{P}$$

In our problem t = 41 days and T = 229 days so P = 50 days. The synodic period is a measure of the time taken for two planets to return to the same spatial configuration with respect to a central star. The synodic period of the moon is 29 days and this is the time between corresponding phases of the moon (full moon to full moon), when Earth, sun and moon are in the same orientation to produce the moons illumination.

Chandra 'Sees' a Distant Planet Evaporating



NASA's Chandra Observatory has discovered that the star CoRot-2a is a powerful X-ray source. This is unfortunate because it is also known that a planet orbits this star at a distance of only 5 million kilometers. Called CoRoT-2b, the planet is three times the mass of Jupiter, and probably has an atmosphere similar to that of Jupiter given its size. Every second, the planet loses about 5 million tons of matter due to its star's radiation.

The star CoRoT-2a is located 880 light years from Earth. The planet orbits its star once every 1.7 days.

The star, CoRoT-2a is similar to our own sun while the planet is about 1.4 times the diameter of Jupiter. The heating of its upper atmosphere to temperatures of 1,500 K have caused the planet to 'puff out' in size due to the added heat energy.

A simple model of this planet's interior suggests that its atmosphere might account for as much as 50% the mass of the planet. By comparison, the mass of Jupiter is about 1.9×10^{27} kilograms or 315 times the mass of Earth.

Problem 1 - About what is the mass of the atmosphere of CoRoT-2b in kilograms?

Problem 2 - Based on the rate at which the planet is being evaporated, about how many years might the planet survive before losing all of its atmosphere if the rate is constant the whole time? (1 ton = 1000 kg)

Problem 1 - About what is the mass of the atmosphere of CoRoT-2b in kilograms?

Answer: Mass = 0.50 x 3.0 Jupiters x (1.9 x 10^{27} kilograms / 1 Jupiter)
= **2.9 x 10^{27} kilograms**

Problem 2 - Based on the rate at which the planet is being evaporated, about how many years might the planet survive before losing all of its atmosphere if the rate is constant the whole time? (1 ton = 1000 kg)

Answer: The mass loss rate is stated as 5 million tons per second .In terms of its loss per year R = 5 x 10^{6} tons x (1000 kg/1 ton) x (3.1 x 10^{7} seconds/ 1 year)
= 1.6 x 10^{17} kilograms/year

The mass of the planets atmosphere is M = 2.9 x 10^{27} kilograms, so
Time = Amount / Rate and so
Time = 2.9 x 10^{27} kilograms / (1.6 x 10^{17} kilograms/year)
= **18 billion years**.

Note: Mass loss rates for planets can sound HUGE, but with few exceptions, planets can usually survive for a period of time longer than the lifetime of their star (10 - 20 billion years) even with such very high rates of mass loss!

Artist's rendition of GJ 1214b with a hypothetical moon.
(Courtesy: CfA/David Aguilar)

In December 2009, astronomers announced the discovery of the transiting super-Earth planet GJ 1214b located 42 light years from the sun, and orbits a red-dwarf star. Careful studies of this planet, which orbits a mere 2 million km from its star and takes 1.58 days to complete 'one year'. Its mass is known to be 6.5 times our Earth and a radius of about 2.7 times Earth's. Its day-side surface temperature is estimated to be 370 F, and it is locked so that only one side permanently faces its star.

When a planet passes in front of its star, light from the star passes through any atmosphere the planet might contain and travels onwards to reach Earth observers. Although the disk of the planet will temporarily decrease the brightness of its star by a few percent, the addition of an atmosphere causes an additional brightness decrease. The amount depends on the thickness of the atmosphere, the presence of dust and clouds, and the chemical composition. By studying the light dimming at many different wavelengths, astronomers can distinguish between different atmospheric constituents by using specific spectral 'fingerprints'. They can also estimate the thickness of the atmosphere in relation to the diameter of the planet.

Problem 1 – Assuming the planet is a sphere, from the available information, to two significant figures, what is the average density of the planet in kg/meter^3? (Earth mass = 6.0×10^{24} kg; Diameter = 6378 km).

Problem 2 – The average density of Earth is 5,500 kg/m^3. Suppose that GJ 1214b has a rocky core with Earth's density and a radius of R, and a thin atmosphere with a density of D. Let R = 1.0 at the surface of the planet and R=0 at the center, and assume the core is a sphere, and that the atmosphere is a spherical shell with inner radius R and outer radius R=1.0. The formula relating the atmosphere density, D, to the core radius, R, is given by:

$$1900 = (5500 - D)R^3 + D$$

A) Re-write this equation by solving for D.

B) Graph the function D(R) over the domain R:[0,1].

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Problem 1 – Answer: Volume = 4/3 (3.141) $(2.7x6378000)^3$ = 2.1 x 10^{22} $meter^3$.
Density = Mass/Volume
= (6.5x6.0 x 10^{24} kg) / (2.1 x 10^{22} $meter^3$)
= **1,900 kg/meter^3**

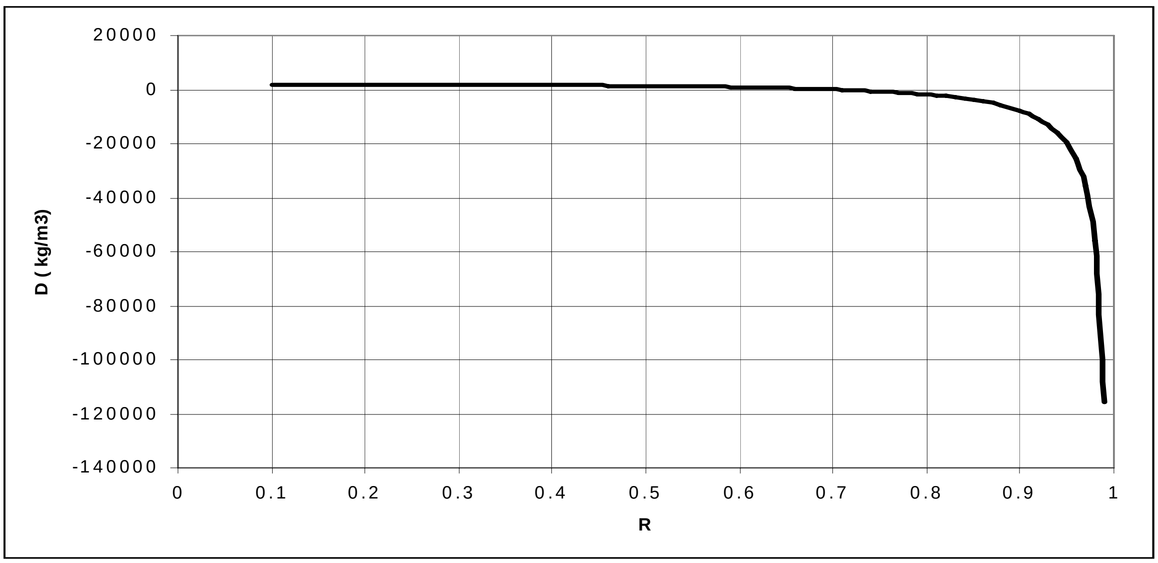
Problem 2 – A) Answer: $1900 = (5500 - D)R^3 + D$

$$1900 - D = 5500R^3 - DR^3$$

$$D\left(R^3 - 1\right) = 5500R^3 - 1900$$

$$D = \frac{5500R^3 - 1900}{R^3 - 1}$$

B) See below:



Note that for very thin atmospheres where D: [0, 100] the function predicts that the core has a radius of about R=0.7 or 70% of the radius of the planet. Since the planet's radius is 2.7 times earth's radius, the core is about 0.7x2.7 = 1.9 x earth's radius. Values for D < 0 are unphysical even though the function predicts numerical values. This is a good opportunity to discuss the limits of mathematical modeling for physical phenomena.

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Answer: **For D=100, R = 0.71, so the core occupies the inner 71 % of the planet, and the surrounding atmospheric shell occupies the outer 29% of the planet's radius.**

In 1961, astronomer Frank Drake devised an ingenious equation that has helped generations of scientists estimate how many intelligent civilizations may exist in the Milky Way. The 'Drake Equation' looks like this:

N = S x P x E x C x I x A x L

Where:

S = Number of stars in the Milky Way

P = Fraction of stars with planets

E = Number of planets per star in the right temperature zone

C = Fraction of planets in E actually able to support life

I = Fraction of planets in C where intelligent life evolves

A = Fraction of planets in I that communicate with radio wave technology

L = Fraction of a stars lifetime when communicating civilization exists

On Earth, bacteria have existed for nearly 4 billion years. Insects for 500 million years. Modern humans for 20,000 years. Which lifeform is the most likely to be found on a distant planet?

Known values:
S = 500 billion
P = 0.1

For our solar system:
E = 2 (Earth and Mars)
C = 0.5 (Earth)
I = 1.0 (Earth)
A = 1.0 (Earth)
L = 0.00000002 (100 yrs/4.5 billion yrs)

The great challenge is to determine from direct or indirect measurements what each of these factors might be. Fortunately, there are at least a few of these factors that we have pretty good ideas about, especially for our own planet. The estimate based on the solar system as a model is the sum of the products in the sample table N = 500 billion x 0.1 x 2 x 0.5 x 1.0 x 1.0 x 0.00000002 = 1000 civilizations existing right now.

Question 1: Based on your internet research, what do you think are the possible ranges for the factors P, E, and C? Remember to cite your sources and use only primary sources by astronomers or other scientists, not opinions by non-scientists.

Question 2: Which factors are the most uncertain and why?

Question 3: What kinds of astronomical observations might help decide what the values for E and C?

Question 4: Using the evolution of life on Earth as a guide, what could you conclude about I? What is the most likely form of life in the Milky Way?

Question 5: How would you try to estimate A using a radio telescope?

Question 6: What would the situation have to be for the value of L to be 0.000002 or 0.000000002?

Question 7: Using the Drake Equation, and a Milky Way in which 1 million intelligent civilizations exist, work backwards to create a 'typical' scenario for each factor so that a) N = 1 million. B) N = 1. Make sure you can defend your choices of the different factors.

Question 1: Based on your internet research, what do you think are the possible ranges for the factors P, E, and C?

Answer: Students may cite the following approximate ranges, or plausible variations after providing the appropriate bibliographic reference: P probably between 1% and 10% based on local planet surveys; E between 1 and 3 based on planet surveys and our own solar system; C between 0.1 and 0.5;

Question 2: Which factors are the most uncertain and why?

Answer: I , A and L. These depend on the details of evolution on non-Earth planets and it is basically anyone's guess what these numbers might be. Students may consult various internet resources or essays on the Drake Equation to get an idea of what ranges are the most talked about.

Question 3: What kinds of astronomical observations might help decide what the values for E and C?
Answer: Our best tool is to conduct surveys of nearby stars and attempt to detect earth-sized planets, not the much larger Jupiter-sized bodies that astronomers currently study.

Question 4: Using the evolution of life on Earth as a guide, what could you conclude about I? What is the most likely form of life in the Milky Way?

Answer: Bacteria were the first life forms on Earth and have survived for nearly 4 billion years. Statistically, they should be the most common life forms in the universe. Also, alien life forms are much more likely to be simple rather than complex. This is favored by the distribution of life on Earth, in which there are very few complex organisms compared to simple ones, especially if you rank them by the total mass of the species and use this to set probabilities. For example, for every billion pounds of bacteria, there are perhaps only 1 thousand pounds of species larger and more complex than insects.

Question 5: How would you try to estimate A using a radio telescope?

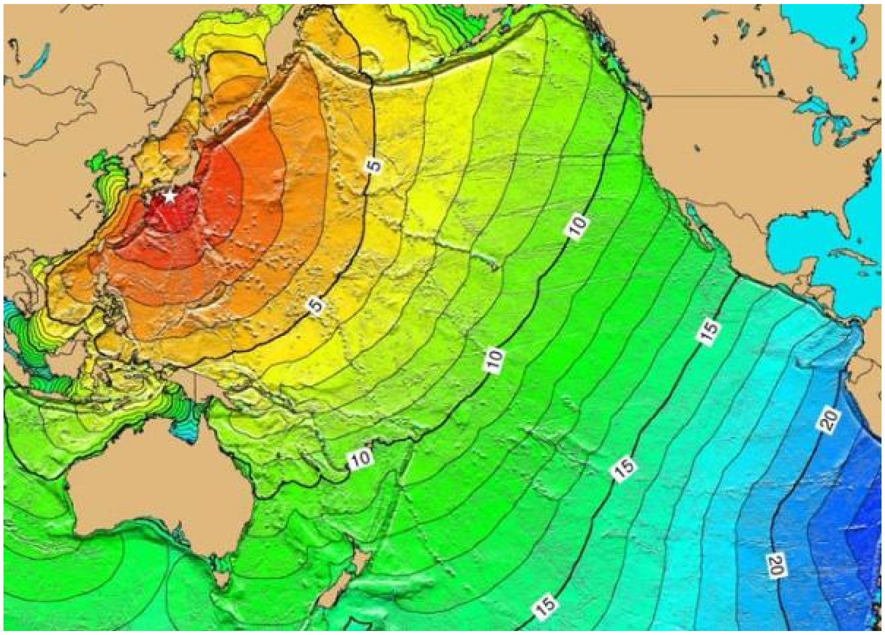
Answer, by measuring the radio output of thousands of stars every year and looking for signs of intelligent 'modulation' like a morse-code signal. Stars don't normally emit radio signals that vary in a precise way in time. The SETI program is continuing to conduct these kinds of surveys.

Question 6: What would the situation have to be for the value of L to be 0.000002 or 0.000000002?
Answer: A star like the sun lives for about 10 billion years. L would then be 10 billion x 0.000002 = 20,000 years or as little as 10 billion x 0.000000002 = 20 years. In the first case, a civilization has learned to survive its technological 'Childhood' and prosper. In the second case, the civilization may have perished after learning how to use radio technology, or it may still be a thriving civilization that no longer uses radio communication.

Question 7: Using the Drake Equation, and a Milky Way in which 1 million intelligent civilizations exist, work backwards to create a 'typical' scenario for each factor so that a) N = 1 million. B) N = 1.

Answer: This will depend on the values that students assign to the various factors. Make sure that each student can defend their choice. This may also be opened to class discussion as a wrap-up.

Estimating the Speed of a Tsunami



A massive 8.9/9.0 magnitude earthquake hit the Pacific Ocean near Honshu, Japan at around 2:46 PM (Japan Standard Time) on March 11, 2011 causing damage with blackouts, fire and tsunami. The tsunami wave then raced across the Pacific Ocean and arrived at many cities around the Pacific Rim. The map above, produced by the National Oceanic and Atmospheric Administration, shows the predicted arrival times in hours for the tsunami wave.

At 7:46 AM Pacific Standard Time on March 11, it resulted in 2.5 meter (8-foot) waves in some locations in Crescent City, California. Waves as tall as 3.5 meters (11-feet) were recorded in Kealakekua Bay, Hawaii at 3:24 AM Local Hawaii Time on March 11. The distance from the epicenter near Honshu, Japan to Crescent City is 8,020 km; to Kealakekua Bay, Hawaii is 6,640 km.

Because these destinations are not all in the same time zone, we have to also include this information: 6:00 AM (Japan Standard Time) on March 11, is the same as 11:00 AM March 10 (Hawaii Standard Time) in Hawaii, and 1:00 PM, March 10 (Pacific Standard Time) in Crescent City.

Problem 1 - At what local time and date did the earthquake occur in A) Hawaii? B) Crescent City?

Problem 2 - How long did it take the tsunami to reach each of the landfalls?

Problem 3 - What was the average speed of the tsunami based on the two estimates?

Problem 1 - At what local time and date did the earthquake occur in A) Hawaii? B) Crescent City?

Answer: Working with time zone calculations, especially across the International Date Line, can be very tricky, but it is an essential skill in today's world where many people travel around the world for vacations and for their careers.
The earthquake began at 2:46 PM JST on March 11. Using the '24-hour clock' makes the calculations easier, so this becomes 14:46 JST.
Then from
06:00 March 11 (Japan Standard Time) in Japan is the same as
11:00 March 10 (Hawaii Standard Time) in Hawaii, and the same as
13:00 March 10 (Pacific Standard Time) in Crescent City,

The table below shows the relevant calculations and answers converted back to the local 12-hour clock.

Location	Distance	Difference from JST	Local Time	Date
Japan	0	0	**2:46 PM (JST)**	**March 11**
Hawaii	6,640 km	-19 hours	**7:46 PM (HST)**	**March 10**
Crescent City	8,020 km	-17 hours	**9:46 PM (PST)**	**March 10**

Problem 2 - How long did it take the tsunami to reach each of the landfalls?
Answer: Using the local times for the earthquake calculated in Problem 1, we just take the difference in local time between the landfall time and the earthquake time:

Hawaii: 03:24 March 11 - 19:46 March 10 = **7.6 hours.**
Crescent City: 07:46 March 11 - 21:46 March 10 = **10 hours**

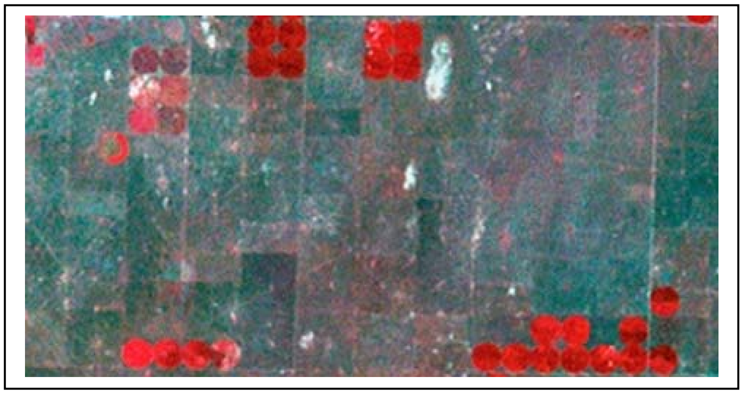
Problem 3 - What was the average speed of the tsunami based on the two estimates?

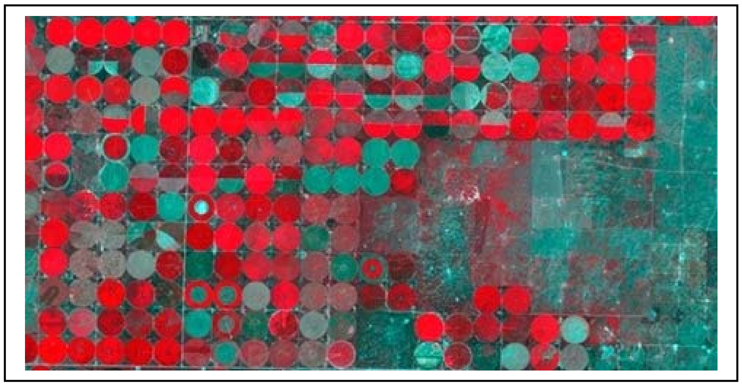
Answer:

Hawaii: 6,640 km / 7.6 hours = **873 km / hr.**
Crescent City: 8,020 km / 10 hours = **802 km/hr.**

The average speed is about (873 + 802)/2 = **837 km/hr**.

Note: The actual distance traveled by the tsunami wave is not exactly a straight line between the epicenter and the destinations. More accurate models by NOAA's Tsunami Forecasting Center predicted a wave speed closer to 800 km/hour.





The top image obtained by Landsat in 1972 shows arid farm land near Garden City, Kansas. The dimensions of the area are 2 miles wide (East-West) by 1 mile tall (North-south). The bottom image was taken of the same area in 2011.

Center-pivot 'sprinklers' use water pumped from the sub-surface Ogallala aquifer to irrigate circular crop areas 800 meters in diameter.

Because the Ogallala aquifer recharges from new rainwater slowly, some of the water used to irrigate these fields is actually water that's been trapped underground since the last Ice Age. Even with the rise of water-conserving center-pivot irrigation and other efforts to conserve, this aquifer is slowly going dry.

Problem 1 – Farmers measure crop areas in acres. If the radius of one crop circle is 400 meters, and 1 acre = 4047 meters^2, what is the area of a single irrigation circle in the Landsat images? (Use π = 3.14).

Problem 2 – How much additional acreage was irrigated in 2011 compared to 1972?

Problem 3 – Typical water application from center-pivot systems is about 8 gallons/minute per acre for corn, and suppose the irrigation is conducted for 3-hours each day for a normal 120-day growing season. How many extra gallons of water are being drawn out of the Ogallala Aquifer to irrigate the increased acreage of corn in this region of Kansas between 1972 and 2011?

Problem 1 – Farmers measure crop areas in acres. If the radius of one crop circle is 400 meters, and 1 acre = 4047 meters^2, what is the area of a single irrigation circle in the Landsat images? (Use π = 3.14).

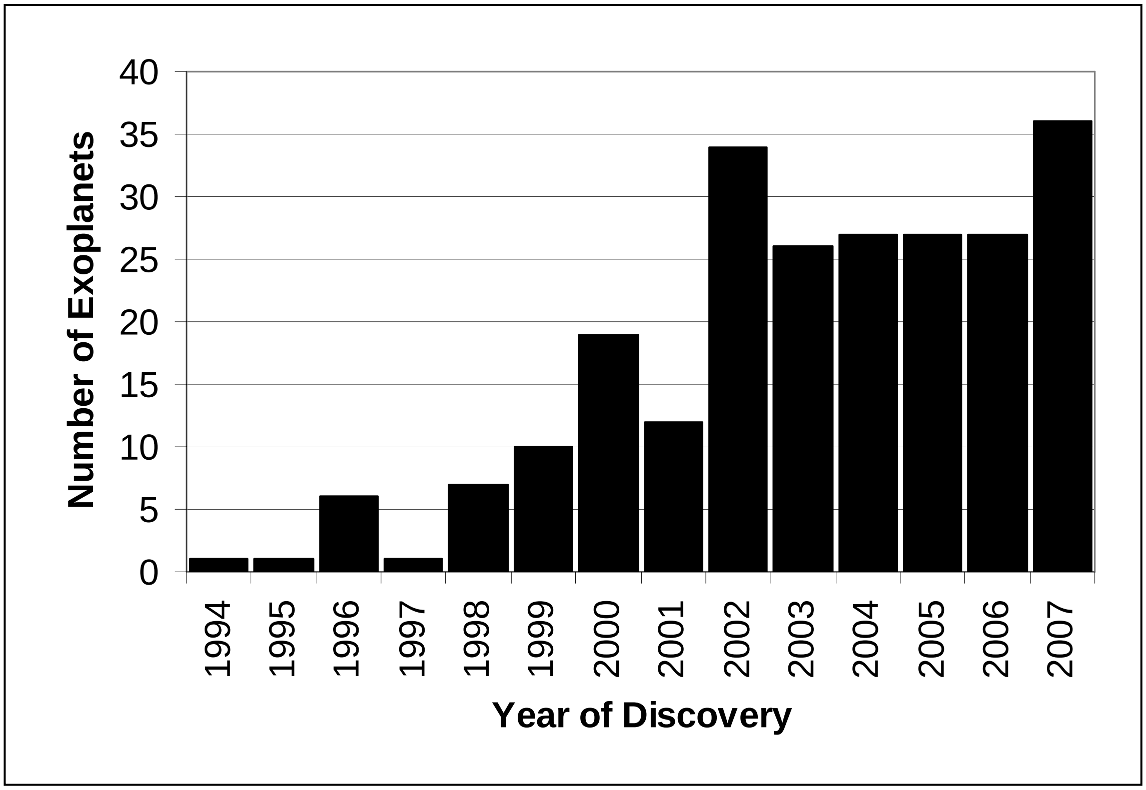
Answer: 3.14 $(400)^2$ = 502,560 m^2. Then 502,560 m^2 x (1 acre/4047 m^2) = **124 acres**.

Problem 2 – How much additional acreage was irrigated in 2011 compared to 1972?

Answer: Count the number of circles in each image and take the difference to get the increased number of irrigated areas. In 1972 there were 30 in 2011 there were about 189 full circles. Students may obtain slightly different numbers depending on how they count. In this example, the difference is 189-30 = 159. The total acreage added is now 159 areas x (124 acres/1 area) = **19,716 acres** added to the irrigation load of this region.

Problem 3 – Typical water application from center-pivot systems is about 8 gallons/minute per acre for corn, and suppose the irrigation is conducted for 3-hours each day for a normal 120-day growing season. How many extra gallons of water are being drawn out of the Ogallala Aquifer to irrigate the increased acreage of corn in this region of Kansas?

Answer: 8 gallons/min x (60minutes/1 hour) x (3 hours/1 day) x 120 days x 19,716 acres = **3.4 billion gallons** of water each growing season.



The 'area under a curve' is an important mathematical quantity that defines virtually all mathematical functions. It has many practical uses as well. For example, the function plotted above, call it P(X), determines the number of new planets, P, that were discovered each year, X, between 1994 through 2007. It was created by tallying-up the number of actual planet discoveries reported in research articles during each of the years. The actual curve representing the function P(X) is shown as a black line, and the columns indicate the number of discoveries per year.

Problem 1 - How would you calculate the total number of planets detected between 1994-2007?

Problem 2 - What is the total area under the curve shown in the figure?

Problem 3 - Suppose N(1995,2000) represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000. A) What does N(1994,2007) mean? B) What does N(1994,2007) - N(1994,2000) mean?

Problem 4 - Evaluate:

A) N(2002,2007)
B) N(1999,2002)

Problem 5 - Evaluate and re-write in terms of N (Example for a function defined for X = A,B,C and D : N(A,B) + N(C,D) is just N(A,D))

A) N(1994,2001) + N(2002,2007)
B) N(2001,2005) - N(2002,2005)
C) N(1994,2007) - N(1994,2001)

Answer Key

Problem 1 - How would you calculate the total number of planets detected between 1994-2007? Answer; You would add up the numbers of planets detected during each year, between 1994 and 2007. This is the same as adding up the areas of each of the individual columns.

Problem 2 - What is the total area under the curve shown in the figure? Answer; the total area is found by adding up the numbers for each column: 1 + 1 + 6+ 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36 = **235 planets.**

Problem 3 - Suppose N(1995,2000) represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000.

A) What does N(1994,2007) mean? Answer: It means the total number of planets detected between 1994 and 2007, which is **235 planets**

B) What does N(1994,2007) - N(1994,2000) mean? Answer: It means to subtract the number of planets detected between 1995-200 from the total number of planets detected between 1994-2007. N(1994,2000) =1 + 1 + 6+ 1 + 7 + 10 + 19 = 45, so you will get 235 - 45 = **190 planets.**

Problem 4 - Evaluate:

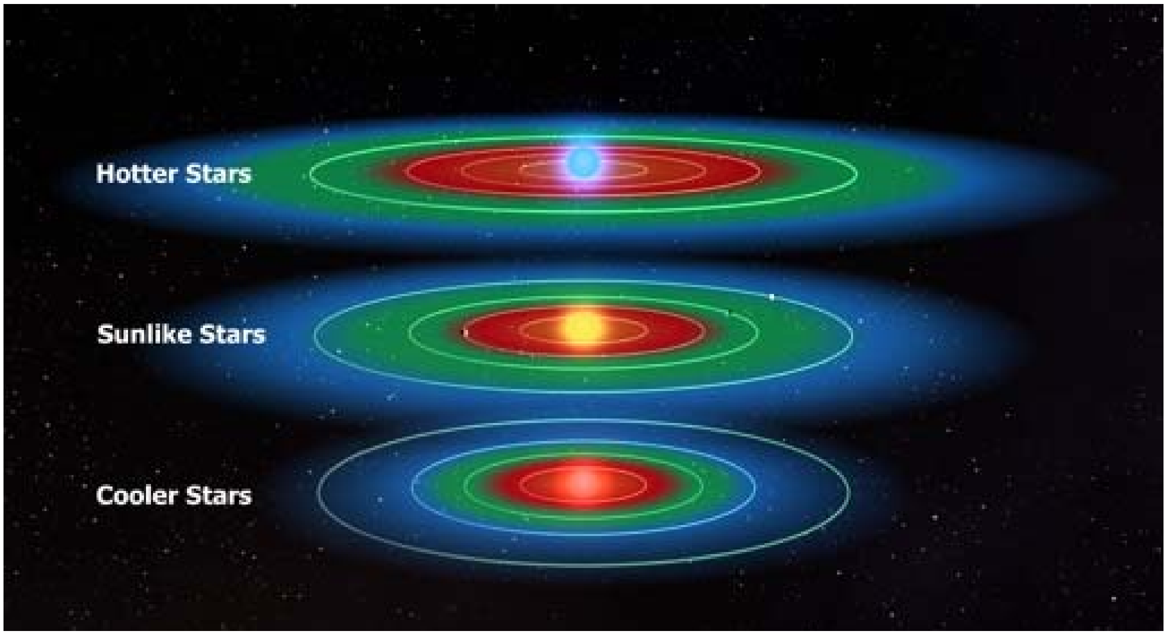
A) N(2002,2007) = 34 + 26 + 27 + 27 + 27 + 36 = **177 planets.**
B) N(1999,2002) = 10 + 19 + 13 + 34 = **76 planets.**

Problem 5 - Evaluate and re-write in terms of N:

A) N(1994,2001) + N(2002,2007) = (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13) + (34 + 26 + 27 + 27 + 27 + 36) = 58 + 177 = **235 planets which is just N(1994,2007)**

B) N(2001,2005) - N(2002,2005) = (13 + 34 + 26 + 27 + 27) - (34 + 26 + 27 + 27) = **13 planets, which is just N(2001, 2001).**

C) N(1994,2007) - N(1994,2001) = (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36) - (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13) = 235 - 58 = **177 planets which is just N(2002,2007)**



Once you have discovered a planet, you need to figure out whether liquid water might be present. In our solar system, Mercury and Venus are so close to the sun that water cannot remain in liquid form. It vaporizes! For planets beyond Mars, the sun is so far away that water will turn to ice. Only in what astronomers call the Habitable Zone (shown in green in the figure above) will a planet have a chance for being at the right temperature for liquid water to exist in large quantities (oceans) on its surface!

The Table on the following page lists the 54 planets that were discovered by NASA's Kepler Observatory in 2010. These planets come in many sizes as you can see by their radii. The planet radii are given in terms of the Earth, where '1.0' means a planet has a radius of exactly 1 Earth radius (1.0 Re) or 6,378 kilometers. The distance to each planet's star is given in multiples of our Earth-Sun distance, called an Astronomical Unit, so that '1.0 AU' means exactly 150 million kilometers.

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth?

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re?

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re?

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets?

Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

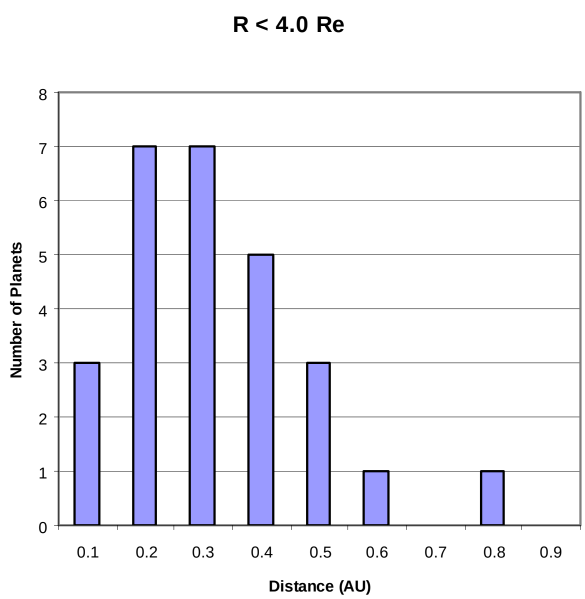
Answer Key

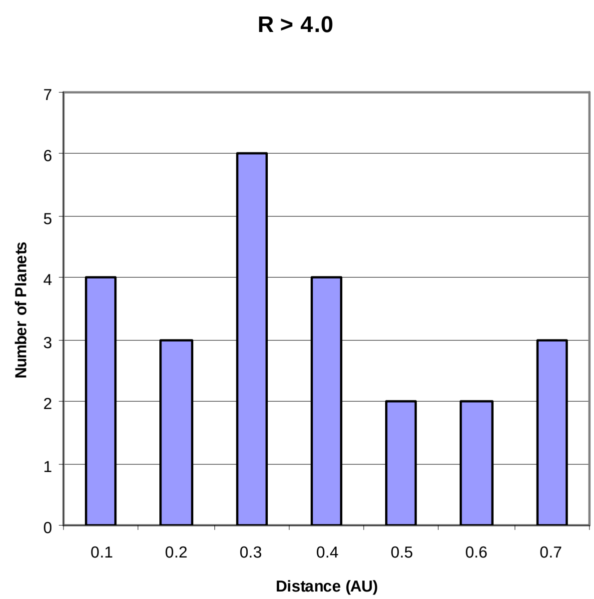
Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth? Answer: There are 28 planets for which R < 4.0 re, so P = 100% x (28/54) = **52%**

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re? Answer; Students will identify the 28 planets in the table that have R < 4.0, and then average the planet's temperatures in Column 6. **Answer: 317 K**.

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re? Students will identify the 26 planets in the table that have R > 4.0, and then average the planet's temperatures in Column 6. **Answer: 306 K**.

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets? Answer; They tend to be found slightly closer to their stars, which is why in Problem 2 their average temperatures were slightly hotter than the larger planets.



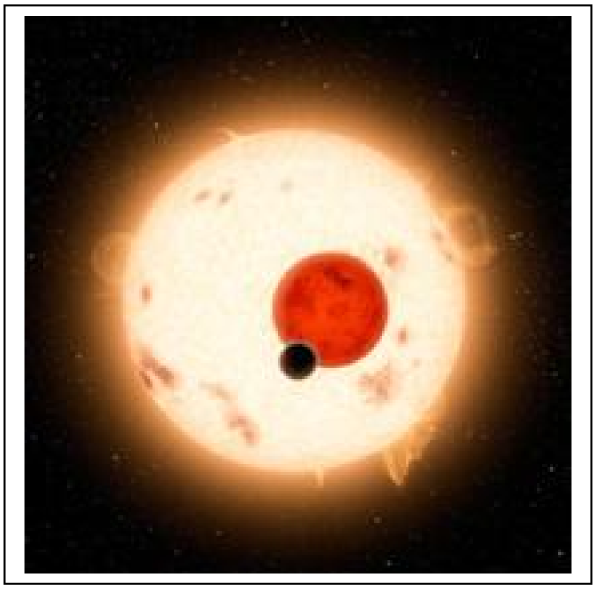


Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

Answer: Students may come up with a number of different strategies and estimates. For example, they might create Venn Diagrams for the data in the table that meet the criteria given in the problem. Then, from the number of planets in the intersection, find their proportion in the full sample of 54 planets, then multiply this by the ratio of 40 billion to 157,453. Estimates near 1 million are in the right range.

Table of Habitable Zone Candidates

	Planet Name (KOI)	Orbit Period (days)	Distance To Star (AU)	Planet Radius (Re)	Planet Temp. (K)	Star Temp. (K)
1	683.01	278	0.84	4.1	239	5,624
2	1582.01	186	0.63	4.4	240	5,384
3	1026.01	94	0.33	1.8	242	3,802
4	1503.01	150	0.54	2.7	242	5,356
5	1099.01	162	0.57	3.7	244	5,665
6	854.01	56	0.22	1.9	248	3,743
7	433.02	328	0.94	13.4	249	5,237
8	1486.01	255	0.80	8.4	256	5,688
9	701.03	122	0.45	1.7	262	4,869
10	351.01	332	0.97	8.5	266	6,103
11	902.01	84	0.32	5.7	270	4,312
12	211.01	372	1.05	9.6	273	6,072
13	1423.01	124	0.47	4.3	274	5,288
14	1429.01	206	0.69	4.2	276	5,595
15	1361.01	60	0.24	2.2	279	4,050
16	87.01	290	0.88	2.4	282	5,606
17	139.01	225	0.74	5.7	288	5,921
18	268.01	110	0.41	1.8	295	4,808
19	1472.01	85	0.37	3.6	295	5,455
20	536.01	162	0.59	3.0	296	5,614
21	806.01	143	0.53	9.0	296	5,206
22	1375.01	321	0.96	17.9	300	6,169
23	812.03	46	0.21	2.1	301	4,097
24	865.01	119	0.47	5.9	306	5,560
25	351.02	210	0.71	6.0	309	6,103
26	51.01	10	0.06	4.8	314	3,240
27	1596.02	105	0.42	3.4	316	4,656
28	416.02	88	0.38	2.8	317	5,083
29	622.01	155	0.57	9.3	327	5171
30	555.02	86	0.38	2.3	331	5,218
31	1574.01	115	0.47	5.8	331	5,537
32	326.01	9	0.05	0.9	332	3,240
33	70.03	78	0.35	2.0	333	5,342
34	1261.01	133	0.52	6.3	335	5,760
35	1527.01	193	0.67	4.8	337	5,470
36	1328.01	81	0.36	4.8	338	5,425
37	564.02	128	0.51	5.0	340	5,686
38	1478.01	76	0.35	3.7	341	5,441
39	1355.01	52	0.27	2.8	342	5,529
40	372.01	126	0.50	8.4	344	5,638
41	711.03	125	0.49	2.6	345	5,488
42	448.02	44	0.21	3.8	346	4,264
43	415.01	167	0.61	7.7	352	5,823
44	947.01	29	0.15	2.7	353	3,829
45	174.01	56	0.27	2.5	355	4,654
46	401.02	160	0.59	6.6	357	5,264
47	1564.01	53	0.28	3.1	360	5,709
48	157.05	118	0.48	3.2	361	5,675
49	365.01	82	0.37	2.3	363	5,389
50	374.01	173	0.63	3.3	365	5,829
51	952.03	23	0.12	2.4	365	3,911
52	817.01	24	0.13	2.1	370	3,905
53	847.01	81	0.37	5.1	372	5,469
54	1159.01	65	0.30	5.3	372	4,886



In 2011, the Kepler observatory detected a Saturn-sized planet orbiting the binary stars Kepler 16A and Kepler 16B. Nicknamed 'Tatooine', the view from this planet of its twin suns would be spectacular. The smaller star, Kepler-16B orbits once every 41 days at a distance of 30 million km from the larger star Kepler 16A, and the planet is in a circular orbit 108 million km from Kepler 16A which takes 229 days to complete. As seen from 'Tatooine', the small orange star Kepler-16B never gets more than 18 degrees from the much larger, yellow star Kepler 16A.

Predicting when Kepler 16B will pass across the face of Kepler 16A, called a transit, is made a bit more difficult because Tatooine is also moving along its orbit while Kepler 16B is in motion around the main star. To see a transit, Kepler 16B and Tatooine must be located in their orbits so that a line through their centers passes through the center of Kepler 16A at the center of the orbits. The time it takes this to happen is called the Synodic Period and is calculated using the formula

$$\frac{360}{t} - \frac{360}{T} = \frac{360}{P}$$

In our problem t = 41 days and T = 229 days so P = 50 days.

Step 1 - Draw a circle representing the orbit of Tatooine.

Step 2 - Place a dot on the circle to mark the location during the planet's orbit when the first transit was observed. Label this dot A

Step 3 - Place a second dot, B, at a position 50 days later where the next transit would be observed.

Problem 1 - How many days will elapse before someone on Tatooine sees a transit within 10 days of the first dot, A, that you placed on the circle?

Problem 2 - How many days will elapse before someone on Tatooine sees a transit within 5 days of the first dot that you placed on the circle?

Problem 3 - How many days will elapse before someone on Tatooine sees a transit within 1 day of the first dot that you placed on the circle?

Problem 4 - How many days will elapse before someone on Tatooine sees a transit on the same day as the first dot that you placed on the circle?

Problem 1 - How many days will elapse before someone on Tatooine sees a transit within 10 days of the first dot that you placed on the circle?

Answer: Students will need to compare two number series, one in intervals of 229 the orbit period of Tatooine, and one with a period of 50 days, which is when transits will occur.

229, **458**, **687**, 916, 1145, 1374, 1603, 1832, 2061
50, 100, 150, 200, 250, 300, 350, 400, **450**, 500, 550, 600, 650, **700**, 750, 800, 850, 900, 950

We see that 458 and 450 are within 8 days of each other, while the next match, 687 and 700 are 13 days apart, so after **458 days or two Tatooine 'years'** a transit will happen within 10 days of Point A.

Problem 2 - How many days will elapse before someone on Tatooine sees a transit within 5 days of the first dot that you placed on the circle?

Answer: Using the same method as in Problem 1, students should find the first such event after 1145 days when the two series yield 1150 and 1145. This will happen after 5 Tatooine years.

Problem 3 - How many days will elapse before someone on Tatooine sees a transit within 1 day of the first dot that you placed on the circle?

Answer: The first time this happens is after 4351 days, which is 19 Tatooine years.

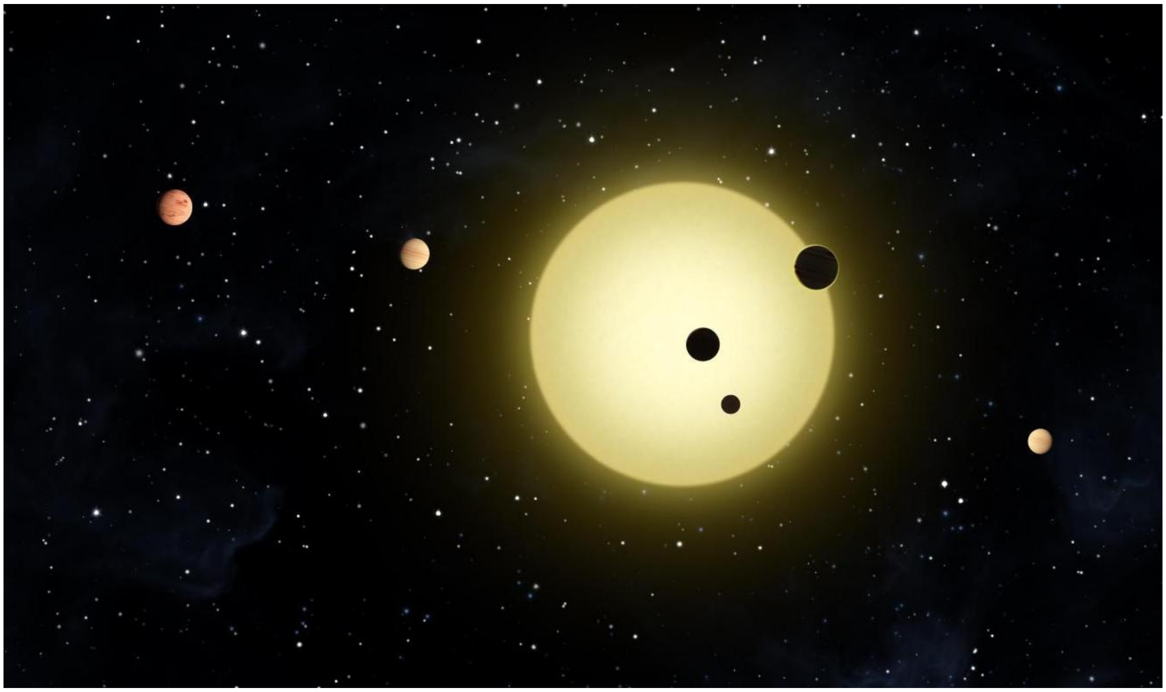
Problem 4 - How many days will elapse before someone on Tatooine sees a transit on the same day as the first dot that you placed on the circle?

Answer: This requires that students solve M x 50 = N x 229 to find N.

The first time this happens is after 11450 days where N = 50 and M = 229 transits. **So, a transit will occur on the same day of the year on Tatooine every 50 years!**

This can be done by finding the Least Common Multiple…since 229 is a prime number, the LCM is just 50 x 229 = 11450.

Earth-sized Planets by the Score!



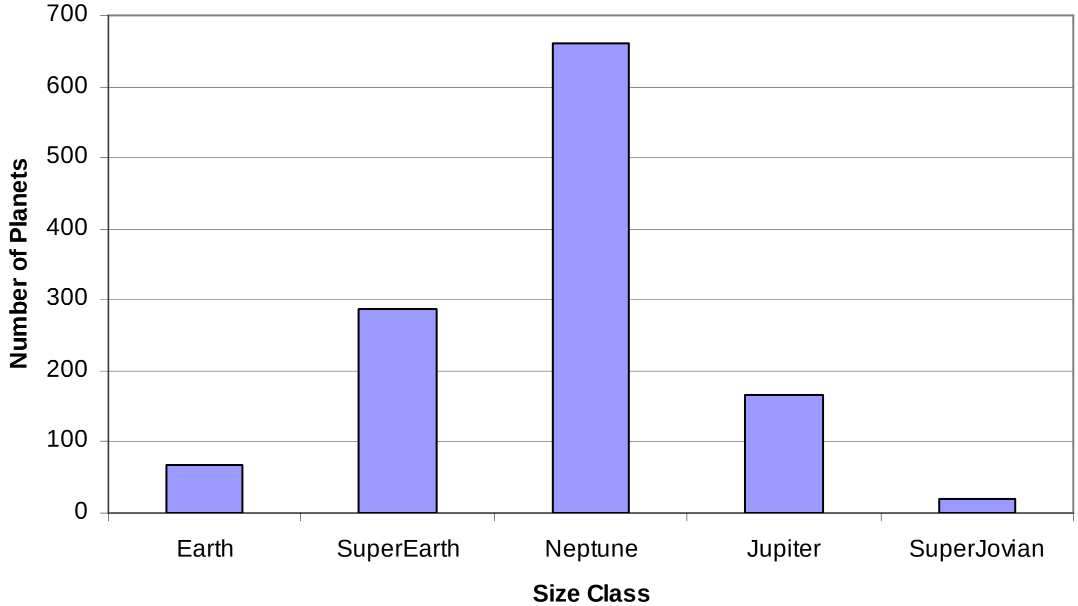
NASA's Kepler Space Observatory recently announced the results of its continuing survey of 156,453 stars in the search for planet transits. Their survey, in progress for just under one year, has now turned up 1,235 transits from among this sample of stars of which 33 were eliminated because they were too big to be true planets. About 30 percent of the remaining candidates belong to multiple-planet systems in which several planets orbit the same star. Among the other important findings are the numbers of planet candidates among the various planet types summarized as follows: Earth-sized = 68; superEarths = 288; Neptune-sized = 662; Jupiter-sized=165; superJovians=19.

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes.

Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized?

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by NASA's Kepler Space Observatory?

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes. Answer:



Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized? Answer: P = 100% x (68/1202) = **5.7 %**

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by Kepler?

Answer: There are 40 billion candidate stars in the Milky Way, so by using simple proportions and re-scaling the survey to the larger sample size

$$\frac{68}{157,453} = \frac{X}{40billion}$$

we get about (40 billion/156,453) x 68 planets or about **17 million Earth-sized planets**.

Note: *The Kepler survey has not been conducted long enough to detect planets much beyond the orbit of Venus in our own solar system, so in time many more earth-sized candidates farther away from their stars will be reported in the years to come. This means that there may well be considerably more than 17 million Earth-sized planets orbiting stars in the Milky Way similar to our own sun.*

Earth-like Planets by the Score!

Table of Candidate Planet Sizes

Size Class	Size (Earth Radius)	Number of candidates
Earths	R <1.25	68
Super-Earths	1.25 < R < 2.0	288
Neptunes	2.0 < R < 6.0	662
Jovians	6.0 < R < 15	165
Super-Jovians	15 < R < 22	19
Dwarf Stars, etc	R > 22	15

NASA's Kepler mission has just completed its first year of surveying 156,453 stars to detect the tell-tail signs of distant planets passing across the faces of their stars .This causes a slight dimming of the starlight, which can be detected by the satellite observatory. From the 1,235 transits detected so far, 33 were eliminated because the planets would have been far larger than Jupiter, and possibly dwarf stars. The table above shows the distribution of the remaining planet candidates among the interesting sizes ranges for planets in our own solar system.

About 30 percent of the candidates have been found to belong to multiple-planet systems, with several planets orbiting the same star. To find Earth-like planets where liquid water could be present, astronomers define a Habitable Zone (HZ) surrounding each star where planet surfaces could be warm enough for liquid water. This is roughly between temperatures of 270 to 300 Kelvin. A careful study of the orbits of the planetary candidates discovered 54 candidate planets in the HZs of their stars. Of these, five planets are roughly Earth-sized, and the other 49 planets range from twice the size of Earth to larger than Jupiter.

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Answer: There are 1,202 candidate planets in the larger survey, and 54 found in their HZ so the percentage is P = 100% x (54/1202) = **4.5%**.

Note: *The essay says that 33 candidates were eliminated because they were probably not planets, so 1,235 - 33 = 1,202 planet candidates.*

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Answer: There were 68 Earth-sized planets found from among 1,202 candidates, and 5 were Earth-sized, so the percentage of Earth-sized planets in their HZs is P = 100% x (5/68) = 7.4%.

So, if you find one Earth-sized planet orbiting a star in the Kepler survey, there is a 7.4% chance that it will be in its HZ so that liquid water can exist.

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

Answer: Students may use simple scaling. The Kepler star sample contains 156,453 stars. It resulted in the discovery, so far, of 5 Earth-sized planets in the Habitable Zones of their respective stars. So

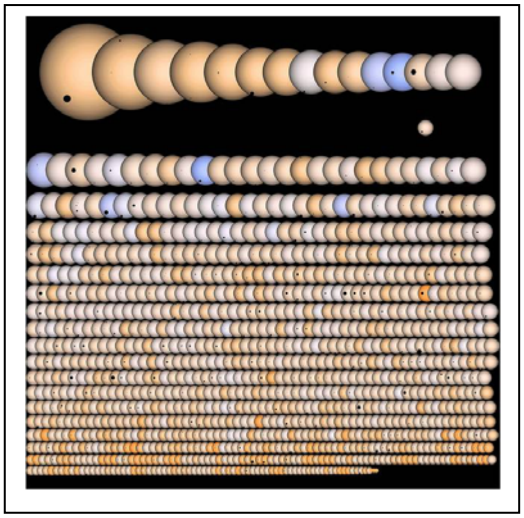
$$\frac{5}{156,453} = \frac{N}{40 billion}$$ so

N = (40 billion / 156,453) x 5 planets

or about **1.3 million planets**.

Another way to think about this is that if you select 30,000 stars in the sky similar to our own sun, you could expect to find about 1 Earth-sized planet orbiting within the Habitable Zone of its star. (5 x 30,000/156453 = 0.96 or about 1.0 planets)

Kepler's Latest Count on Goldilocks Planets



Scientists from around the world are gathered this week at NASA's Ames Research Center in Moffett Field, Calif., for the second Kepler Science Conference, where they will discuss the latest findings resulting from the analysis of Kepler Space Telescope data.

New Kepler data analysis and research also show that most stars in our galaxy have at least one planet. This suggests that the majority of stars in the night sky may be home to planetary systems, perhaps some like our solar system.

Problem 1 - Based on a total of 3538 candidate planets, Kepler announced in November 2013 that it had detected 674 objects smaller than 1.25 Re, 1076 objects between 1.25 and 2.0 Re, 1457 objects between 2.0 and 6.0 Re, 229 objects between 6.0 and 15 Re, and 102 objects larger than 15 Re. What are the percentages of each size of planet candidate, and create a histogram plot to show this data.

Problem 2 - The survey included 42,000 stars and detected 10 candidate planets that were about the same size as Earth, and located at a distance from their stars where liquid water could occur (Goldilocks Zone). Because of the way that Kepler detected these planets, they were only able to see about 1.3% of all planets orbiting the surveyed stars. If the Milky Way contains about 500 billion stars similar to the ones surveyed by Kepler, about how many Earth-like planets might you expect to find in the entire Milky Way galaxy?

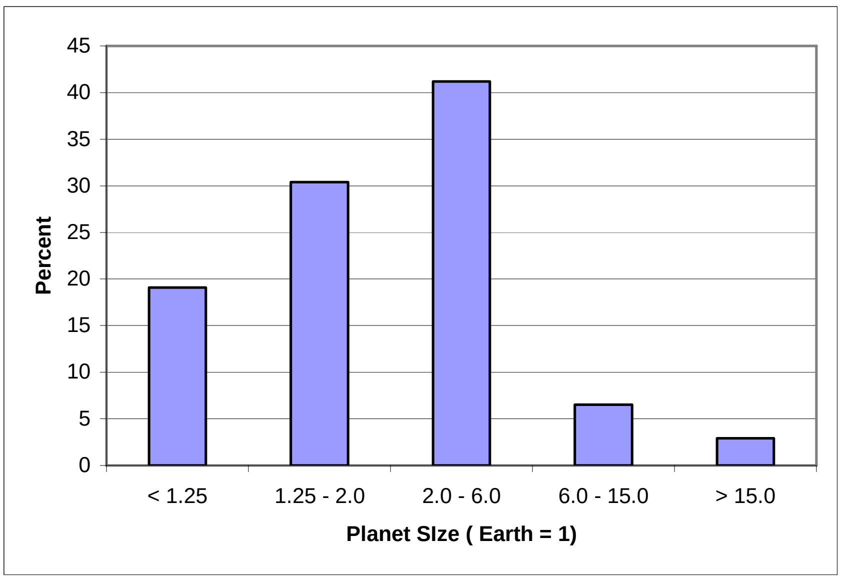
NASA Kepler Results Usher in a New Era of Astronomy
November 4, 2013

http://www.nasa.gov/content/nasa-kepler-results-usher-in-a-new-era-of-astronomy/index.html

Problem 1 - Based on a total of 3538 candidate planets, Kepler announced in November 2013 that it had detected 674 objects smaller than 1.25 Re, 1076 objects between 1.25 and 2.0 Re, 1457 objects between 2.0 and 6.0 Re, 229 objects between 6.0 and 15 Re, and 102 objects larger than 15 Re. What are the percentages of each size of planet candidate, and create a histogram plot to show this data.

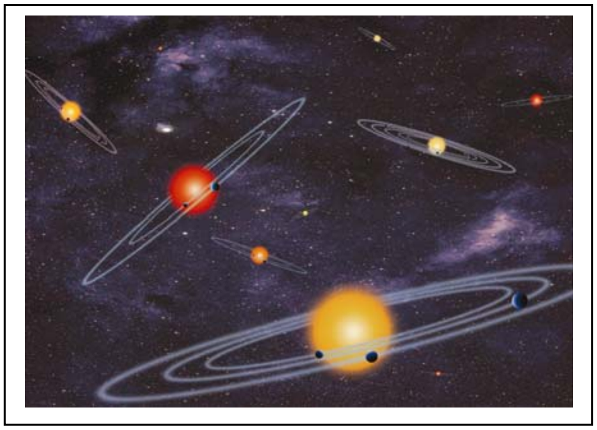
Answer:

1.25	=	674 / 3538	=	**19.1 %**
1.25 to 2.0	=	1076 / 3538	=	**30.4 %**
2.0 to 6.0	=	1457 / 3538	=	**41.2 %**
6.0 to 15.0	=	229 / 3538	=	**6.5 %**
> 15.0	=	102 / 3538	=	**2.9 %**



Problem 2 - The survey included 42,000 stars and detected 10 candidate planets that were about the same size as Earth, and located at a distance from their stars where liquid water could occur (Goldilocks Zone). Because of the way that Kepler detected these planets, they were only able to see about 1.3% of all planets orbiting the surveyed stars. If the Milky Way contains about 500 billion stars similar to the ones surveyed by Kepler, about how many Earth-like planets might you expect to find in the entire Milky Way galaxy?

Answer: 500 billion x (10 detections /42,000 surveyed stars) x (100 stars/1.3 detection) = 9.1 billion planets. To the nearest billion this becomes **9 billion earth-like planets**.



NASA's Kepler mission announced Wednesday the discovery of 715 new planets. These worlds orbit 305 stars, revealing multiple-planet systems much like our own solar system.

Nearly 95 percent of these planets are smaller than Neptune, which is almost four times the size of Earth. This discovery marks a significant increase in the number of known planets similar in size to Earth.

The Kepler mission surveyed 190,751 stars similar to our own sun in the constellation Cygnus the Swan. It searched for the tell-tail dimming of light that means a planet has 'eclipsed' the face of the star as viewed from Earth. As of February 2014, the Kepler spacecraft and its follow-up observations has detected 3670 stars with transit events, of these 1140 were later identified as binary stars, and 779 were eliminated because their transits were misidentified. The remaining transits included 340 stars that had more than one transit event suggesting that they were multi-planet systems (called Multis) with a total of 851 individual planet transits. Thirty four of these Multis were already catalogued stars which had 83 planet transits.

Problem 1 - Draw a branching diagram that shows how each of different groups and numbers are related to each other. How many single star/single transits were discovered among the 3670 stars with transits? What was the total number of new stars and planets announced by Kepler in the Multi systems?

Problem 2 - A subsample of 739 of the 851 discovered Kepler planets could be measured for their sizes. The result was 102 planets that were Earth-sized, 215 planets that were super-Earth sized up to twice the diameter of Earth, 373 that were Neptune-sized, and 13 that were Jupiter-sized or larger. Draw a histogram of this ensemble. What percentage of multi-planet system detected by Kepler have Earth-sized planets?

Problem 3 - If the amount of solar energy falling on Earth to keep it warm is defined as 1.0, for Venus it is 2.0 and for Mars it is 0.4. From the 851 planets detected in orbit around the 340 stars, there were 18 planets orbiting 16 stars found to be in the solar energy 'habitable zone' from 0.4 to 2.0 so that liquid water could exist on their surfaces. What is the probability that, given a multi-planet system you will find an Earth-sized planet in its Habitable Zone?

Problem 4 - The Milky Way is estimated to have about 1 trillion stars from red dwarfs to brilliant super giants. From various surveys, about 20% are similar to our sun. From the Kepler survey data, A) how many multi-planet systems would you expect to find if the Kepler survey could be conducted for all the stars in the Milky Way? B) How many Earth-sized planets in multi-planet systems would you expect to find? C) How many Earth-like planets in their Habitable Zones would you expect to find?

Answer Key

NASA's Kepler Mission Announces a Planet Bonanza, 715 New Worlds
February 26, 2014
http://www.nasa.gov/press/2014/february/nasas-kepler-mission-announces-a-planet-bonanza-715-new-worlds/

Teacher Note: The number '715' is a revised version of the 768 new planets reported by the Kepler Team in the paper by Rowe et al. 'Validation of Kepler's Multiple Planet Candidates: III (Table III), based on combining other surveys after the paper was published, but before the press conference public announcement on February 26.

Problem 1 -

		190,751 stars		
		3670 stars with transits		
779 false positive		1751 valid transits	1140 binary stars	(total=3670)
	1411 Single Stars	340 Multis (851 planets)		(total=1751)
		306 stars 768 planets	34 stars 83 planets	(total=340)

Answer: There were 1751-340=1411 single star transits. The total number of new stars and planets was 340-34 = 306 stars and 851-83 = 768 planets.

Problem 2 - A subsample of 739 of the 851 discovered Kepler planets could be measured for their sizes. The result was 102 planets that were Earth-sized, 215 planets that were super-Earth sized up to twice the diameter of Earth, 373 that were Neptune-sized, and 13 that were Jupiter-sized or larger. Draw a histogram of this ensemble. What percentage of multi-planet system detected by Kepler have Earth-sized planets? Answer: 102/739 = 13.8 %

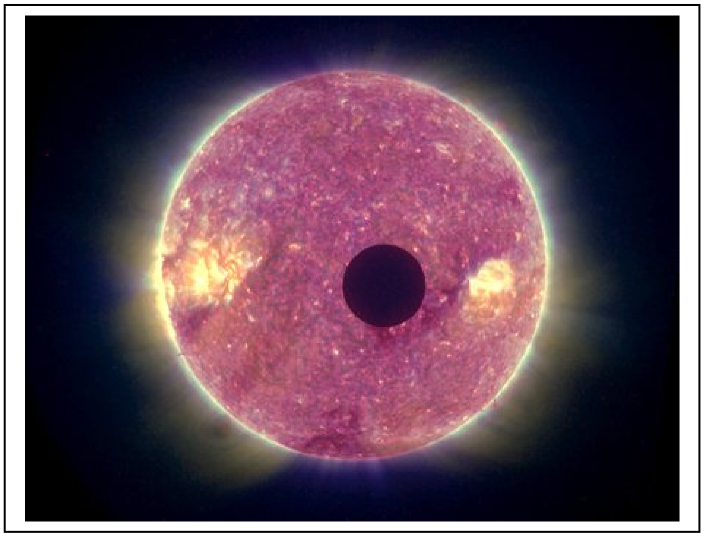
Problem 3 - If the amount of solar energy falling on Earth to keep it warm is defined as 1.0, for Venus it is 2.0 and for Mars it is 0.4. From the 851 planets detected in orbit around the 340 stars, there were 18 planets orbiting 16 stars found to be in the solar energy 'habitable zone' from 0.4 to 2.0 so that liquid water could exist on their surfaces. What is the probability that, given a multi-planet system you will find an Earth-sized planet in its Habitable Zone? Answer: **There are 340 multiplanet systems, so for 16 stars having such a planet, the probability is 16/340 = 0.047 or 4.7%. Alternatively, for 851 planets in multiplanet systems we have 18/851 = 0.021 or 2.1%.**

Problem 4 - The Milky Way is estimated to have about 1 trillion stars from red dwarfs to brilliant super giants. From various surveys, about 20% are similar to our sun. From the Kepler survey data, A) how many multi-planet systems would you expect to find if the Kepler survey could be conducted for all the stars in the Milky Way? B) How many Earth-sized planets in multi-planet systems would you expect to find? C) How many Earth-like planets in their Habitable Zones would you expect to find?

Answer: A) 1 trillion x 0.2 x (340 multiplanets /190,751 stars searched) = **357 million**.
B) 357 million x (102/739) = **49 million**.
C) 49 million x (16 / 340) = **2.3 million Earth-sized planets in their HZ in the Milky Way.**

Note: The Kepler survey only sees planetary systems if their orbit planes are along the line of sight to the star and Earth. Statistically this only happens about 5% of the time, so we can multiply the Kepler results by 20 to account for planetary systems we miss using this technique. That means for Answer C, there could be nearly **50 million planets like Earth**. Also, in our solar system Neptune and Jupiter-sized planets can have moons the size of Earth, so that means ANY planet found in the HZ could be potentially habitable if we include its moons, so we eliminate the step in Answer B and so have 357 million x (16/340) x 20 = **336 million possible Earth-sized planets (or moons) in the Habitable Zone!**

Kepler – The Hunt for Earth-like Planets



On March 11, 2009, NASA launched the Kepler satellite. Its 3-year mission is to search 100,000 stars in the constellation Cygnus and detect earth-sized planets. How can the satellite do this?

The image to the left shows what happens when a planet passes across the face of a distant star as viewed from Earth. In this case, this was the planet Mercury on February 25, 2007.

The picture was taken by the STEREO satellite. Notice that Mercury's black disk has reduced the area of the sun. This means that, on Earth, the light from the sun dimmed slightly during the Transit of Mercury. Because Mercury was closer to Earth than the Sun, Mercury's disk appears very large. If we replace Mercury with the Moon, the lunar disk would exactly cover the disk of the Sun and we would have a total solar eclipse.

Now imagine that the Sun was so far away that you couldn't see its disk at all. The light from the Sun would STILL be dimmed slightly. The Kepler satellite will carefully measure the brightness of more than 100,000 stars to detect the slight changes caused by 'transiting exoplanets'.

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the sun is 696,000 kilometers, what is the scale of your sun disk in kilometers/millimeter?

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)?

Problem 3 – What is the area of the Sun disk in square millimeters?

Problem 4 – What is the area of Earth and Jupiter in square millimeters?

Problem 5 – By what percent would the area of the Sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk?

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer Key

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the Sun is 696,000 kilometers, what is the scale of your Sun disk in kilometers/millimeter? **Answer: 4,350 km/mm**

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)? **Answer: 1.5 mm and 16.4 mm respectively.**

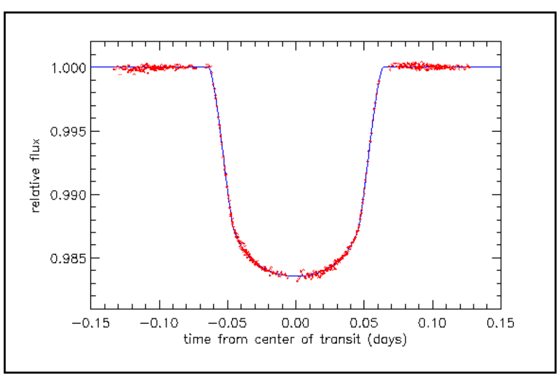
Problem 3 – What is the area of the sun disk in square millimeters? Answer; π x $(160)^2$ **= 80,400 mm^2.**

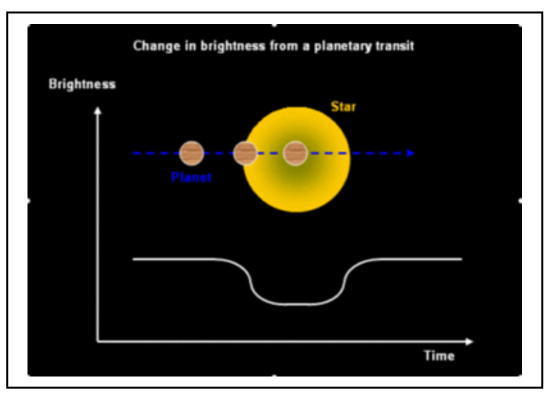
Problem 4 – What is the area of Earth and Jupiter in square millimeters? Answer: Earth = π x $(1.5)^2$ = **7.1 mm^2**. Jupiter = π x $(16.4)^2$ = **844.5 mm^2**.

Problem 5 – By what percent would the area of the sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk? Answer A) 100% x (7.1 mm^2 / 80400 mm^2) = 100% x 0.000088 **= 0.0088 %**
B) Jupiter: 100% x (844.5 mm^2 / 80400 mm^2) = 100% x 0.011 = **1.1%.**

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer: Students should note from their answer to Problem 5 that when the planet disk is fully on the star disk, the star's brightness will dim from 100% to 100% - 1.1% = 98.9%. Students should also note that as the transit starts, the stars brightness will dim as more of the planet's disk begins to cover the star's disk. Similarly, as the planet's disk reaches the edge of the star's disk, the area covered by the planet decreases and so the star will gradually brighten to its former 100% level. The figures below give an idea of the kinds of graphs that should be produced. The left figure is from the Hubble Space Telescope study of the star HD209458 and its transiting Jupiter-sized planet.





Period (days)	F	G	K
0-10	11	138	20
11-20	7	53	16
21-30	4	25	6
31-40	2	13	0
41-50	1	7	0
51-60	1	1	0
61-70	0	1	0
71-80	0	1	0
> 81	0	3	2
Total:	26	242	44

On June 16, 2010 the Kepler mission scientists released their first list of stars that showed evidence for planets passing across the face of their stars. Out of the 156,097 target stars that were available for study, 52,496 were studied during the first 33 days of the mission. Their brightness was recorded every 30 minutes during this time, resulting in over 83 million high-precision measurements.

700 stars had patterns of fading and brightening expected for planet transits. Of these, data were released to the public for 306 of the stars out of a sample of about 88,000 target stars.

The surveyed stars for this study were distributed by spectral class according to F = 8000, G = 55,000 and K = 25000. For the 306 stars, 43 were K-type, 240 were G-type, and 23 were K-type. Among the 312 transits detected from this sample of 306 stars, the table above gives the number of transits detected for each stellar type along with the period of the transit.

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?

Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?
Answer: Of the 88,000 stars F = 8000/88000 = 9%; G = 55000/88000 = 63% and K=25000/88000 = 28%.

For the 306 stars: F = 43/306 = 14%; G = 240/306 = 78% and K = 23/306 = 8%....so there were significantly fewer transits detected for K-type stars (8%) compared to the general population (28%).

Note: Sampling error accounts for s = $(306)^{1/2}$ = +/-17 stars or a +/- 6% uncertainty which is not enough to account for this difference in the K-type stars.

Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

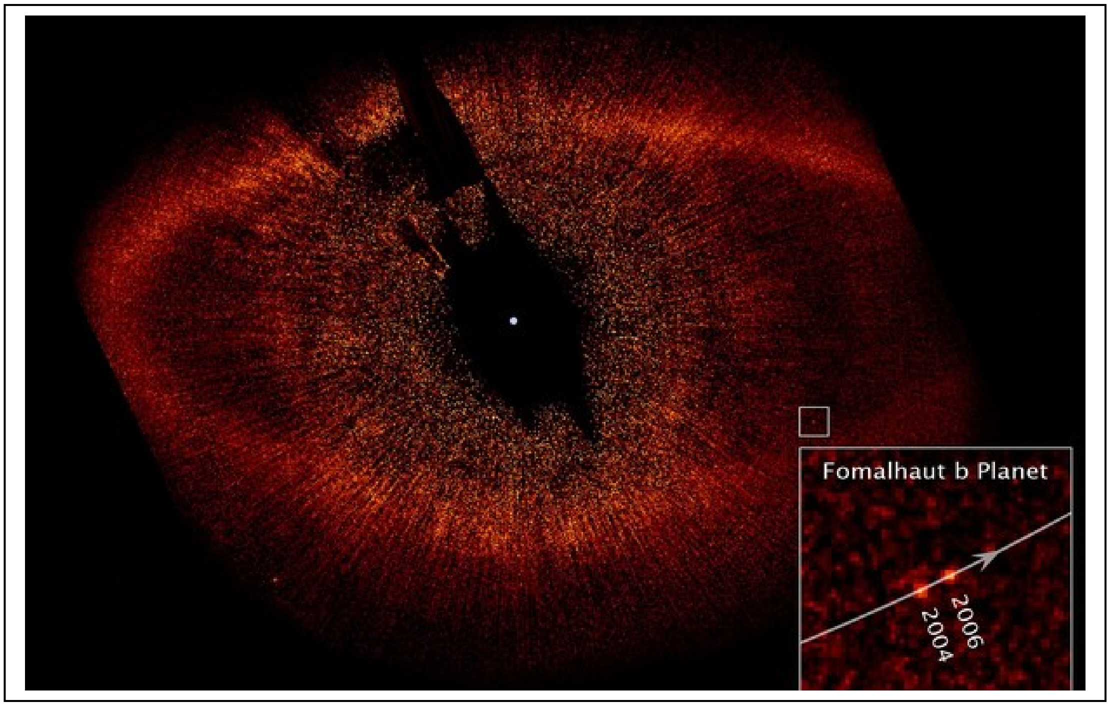
Answer: A) T = 88 days/365 days = 0.24 Earth Years, than $D^3 = 0.24^2$, $D^3 = 0.058$, so $D = (0.058)^{1/3}$ and so D = **0.39 times Earth's orbit distance**.
B) The time range is 0.014 to 0.22 Earth Years, and so D is in the range from 0.058 to 0.36 Earth distances. Since Mercury has D = 0.39, in terms of the orbit distance of Mercury, the transiting planets span a range from 0.058/0.39 = **0.15** to 0.36/0.39 = **0.92 Mercury orbits**.

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Answer: The amount of dimming is equal to the ratio of the areas of the planet's disk to the star's disk, so $0.001 = \pi R^2/\pi(500,000)^2$ so **R = 15,800 kilometers**, which equals a diameter of 31,600 kilometers. Since Earth's diameter = 13,000 km, the transiting planet is about **2.4 times the diameter of Earth**.

The bright star Fomalhaut, in the constellation Piscis Austrinus (The Southern Fish) is only 25 light years away. It is 2000^{o} K hotter than the Sun, and nearly 17 times as luminous, but it is also much younger: Only about 200 million years old. Astronomers have known for several decades that it has a ring of dust (asteroidal material) in orbit 133 AU from the star and about 25 AU wide. Because it is so close, it has been a favorite hunting ground in the search for planets beyond our solar system. In 2008 such a planet was at last discovered using the Hubble Space Telescope. It was the first direct photograph of a planet beyond our own solar system.

In the photo below, the dusty ring can be clearly seen, but photographs taken in 2004 and 2006 revealed the movement of one special 'dot' that is now known to be the star's first detected planet. The small square on the image is magnified in the larger inset square in the lower right to show the location of the planet in more detail.



Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer Key

Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Answer: The distance from the center of the ring (location of star in picture) to the center of the box containing the planet is 42 millimeters, then 42 x 2.7 AU/mm = 113 AU. Since 1 AU = 150 million km, the distance is 113 x 150 million **= 17 billion kilometers.**

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Answer: On the main image, the box has a width of 4 millimeters which equals 4 x 2.7 = 11 AU. The inset box showing the planet has a width of 36 mm which equals 11 AU so the scale of the small box is 11AU/36 mm = 0.3 AU/mm. The planet has shifted in position about 4 mm, so this corresponds to 4 x 0.3 = **1.2 AU or 180 million km**.

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Answer: The average speed is 180 million km/17520 hours = **10,273 km/hr.**

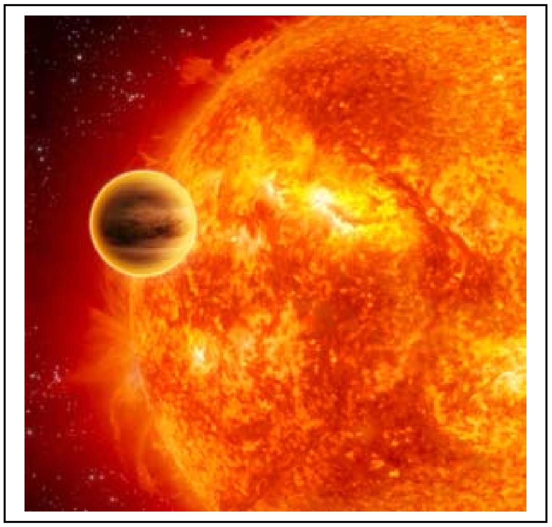
Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer: The radius of the circle is 113 AU so the circumference is $2 \pi R = 2 (3.141) (113 AU) = 710 AU$. The distance traveled by the planet in 2 years is, from Problem 2, about 1.2 AU, so in 2 years it traveled 1.2/710 = 0.0017 of its full orbit. That means a full orbit will take 2.0 years/0.0017 = **1,176 years.**

Note - Because we are only seeing the 'projected' motion of the planet along the sky, the actual speed could be faster than the estimate in Problem 3, which would make the estimate of the orbit period a bit smaller than what students calculate in Problem 4.

A careful study of this system by its discoverer, Dr. Paul Kalas (UC Berkeley) suggests an orbit distance of 119 AU, and an orbit period of 872 years.

Estimating the Temperatures of Exoplanets



Because many exoplanets orbit their stars in elliptical paths, they experience large swings in temperature. Generally, organisms can not survive if water is frozen (0 C = 273 K) or near its boiling point (100 C or 373 K). Due to orbital conditions, this very narrow 'zone of life' may not be possible for many of the worlds detected so far.

Problem 1 - Complete the table below by calculating
A) The semi-minor axis distance B = $A(1-e^2)$
B) The perihelion distance Dp = A(1-e)
C) The aphelion distance, Da = B(1+e)

Problem 2 - Write the equation for the orbit of 61 Vir-d in Standard Form.

Problem 3 - The temperature of a planet similar to Jupiter can be approximated by the formula below, where T is the temperature in Kelvin degrees, and R is the distance to its star in AU. Complete the table entries for the estimated temperature of each planet at the farthest 'aphelion' distance Ta, and the closest 'perihelion;' distance Tp.

$$T(R) = \frac{250}{\sqrt{R}}$$

Problem 4 - Which planets would offer the most hospitable, or the most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39		0.22	2190				
61 Vir-d	0.47		0.35	123				
HD106252-b	2.61		0.54	1500				
HD100777-b	1.03		0.36	383				
HAT-P13c	1.2		0.70	428				

Problem 1 See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**		
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**		
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**		
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**		
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**		

Problem 2 Write the equation for the orbit of 61 Vir-d in Standard Form.
Answer: A = 0.47 and B = 0.4

So $1 = \frac{x^2}{0.47} + \frac{y^2}{0.40}$ and also $188 = 40x^2 + 47y^2$

Problem 3 - See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**	**154**	**123**
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**	**452**	**314**
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**	**228**	**125**
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**	**308**	**211**
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**	**417**	**175**

Problem 4 - Which planets would offer the most hospitable, or most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?
Answer: For the habitable 'water' range between 273K and 373K, none of these planets satisfy this minimum and maximum condition. They are either too hot at perihelion 'summer' or too cold at 'winter' aphelion.

Only HD100777-b during perihelion is in this temperature range during 'summer', at a temperature of 308 K (35 C). During 'winter' at aphelion, it is at -62 C which is below the freezing point of water, and similar to the most extreme temps in Antarctica.

Note: These temperature calculations are only approximate and may be considerably different with greenhouse heating by the planetary atmosphere included.

Artist view of planet G. Bacon (STScI/NASA)

Every 4 days, this planet orbits a sun-like star located 153 light years from Earth. Astronomers using NASA's Hubble Space Telescope have confirmed that this gas giant plane is orbiting so close to its star its heated atmosphere is escaping into space.

Observations taken with Hubble's Cosmic Origins Spectrograph (COS) suggest powerful stellar winds are sweeping the cast-off atmospheric material behind the scorched planet and shaping it into a comet-like tail. COS detected the heavy elements carbon and silicon in the planet's super-hot, 2,000-degree-

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3-times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b?

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Answer: A) 4×10^{11} grams/sec x (10^{-6} kg/gm x 86,400 sec/day = **3.5×10^{10} tons/day**

B) 3.5×10^{10} tons/day x 365 days/year = **1.3×10^{13} tons/year**

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3-times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

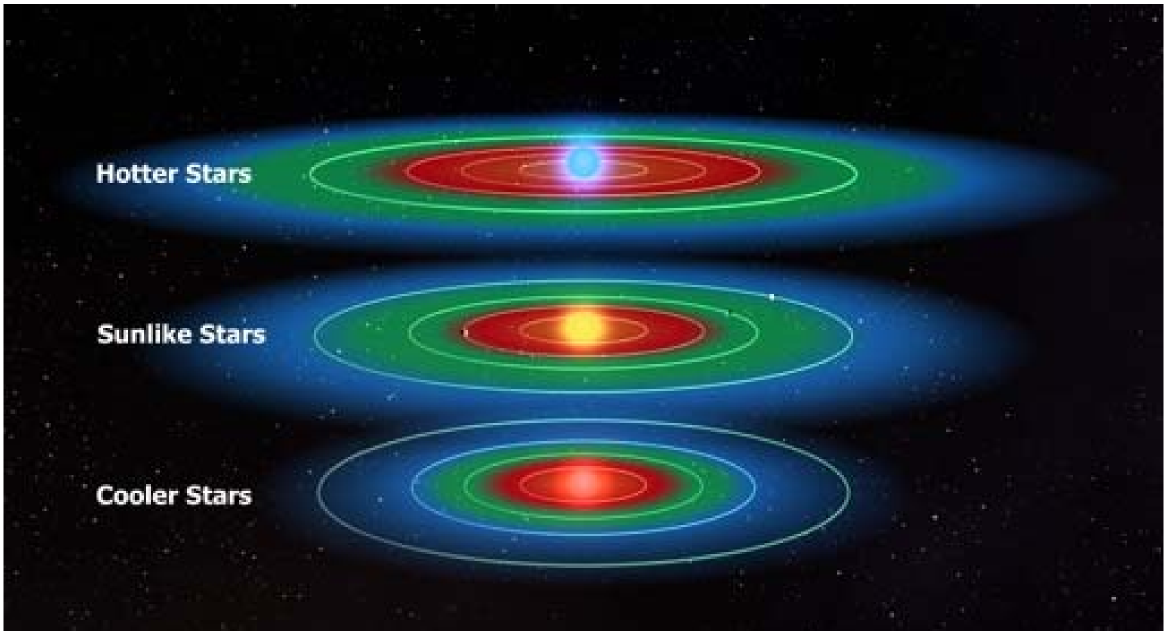
Answer: A) $V = 4/3\ \pi\ R^3$ so V(Jupiter) = 1.5×10^{24} meters^3. Density = mass/volume so Density (Jupiter) = 1.9×10^{27} kg/1.5×10^{24} meters^3 = **1266 kg/meter^3**.
B) Mass = 0.6 M(Jupiter) and volume = $(1.3)^3$ V(Jupiter) so density = $0.6/(1.3)^3$ x 1266 kg/meter^3 =0.27x1266 = **342 kg/meter^3**.

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b? Answer: M(HD209458b) = 0.6x Jupiter = 1.1×10^{27} kg so M(atmosphere) = 1.1×10^{27} kg – 18x(5.9×10^{24} kg) = **9.9×10^{26} kg**.

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Answer: Time = Mass/rate
= 9.9×10^{26} kg / (1.3×10^{16} kg/year)
= **7.6×10^{10} seconds**
= 2,456 years!

The Goldilocks Planets - Not too hot or cold!



Once you have discovered a planet, you need to figure out whether liquid water might be present. In our solar system, Mercury and Venus are so close to the sun that water cannot remain in liquid form. It vaporizes! For planets beyond Mars, the sun is so far away that water will turn to ice. Only in what astronomers call the Habitable Zone (shown in green in the figure above) will a planet have a chance for being at the right temperature for liquid water to exist in large quantities (oceans) on its surface!

The Table on the following page lists the 54 planets that were discovered by NASA's Kepler Observatory in 2010. These planets come in many sizes as you can see by their radii. The planet radii are given in terms of the Earth, where '1.0' means a planet has a radius of exactly 1 Earth radius (1.0 Re) or 6,378 kilometers. The distance to each planet's star is given in multiples of our Earth-Sun distance, called an Astronomical Unit, so that '1.0 AU' means exactly 150 million kilometers.

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth?

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re?

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re?

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets?

Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

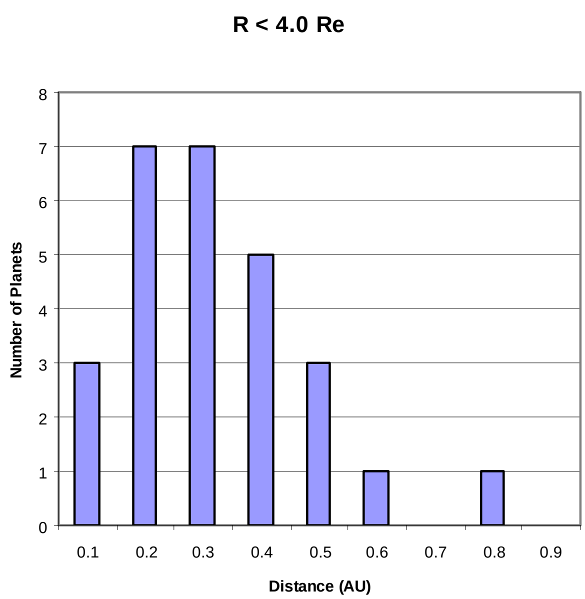
Answer Key

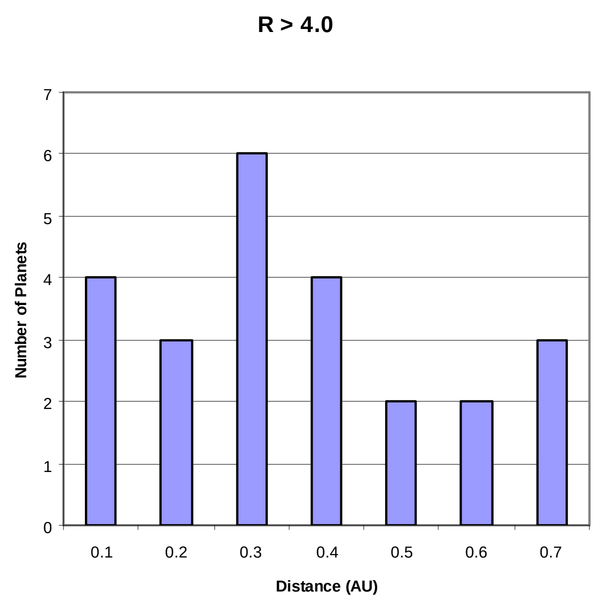
Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth? Answer: There are 28 planets for which R < 4.0 re, so P = 100% x (28/54) = **52%**

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re? Answer; Students will identify the 28 planets in the table that have R < 4.0, and then average the planet's temperatures in Column 6. **Answer: 317 K**.

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re? Students will identify the 26 planets in the table that have R > 4.0, and then average the planet's temperatures in Column 6. **Answer: 306 K**.

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets? Answer; They tend to be found slightly closer to their stars, which is why in Problem 2 their average temperatures were slightly hotter than the larger planets.



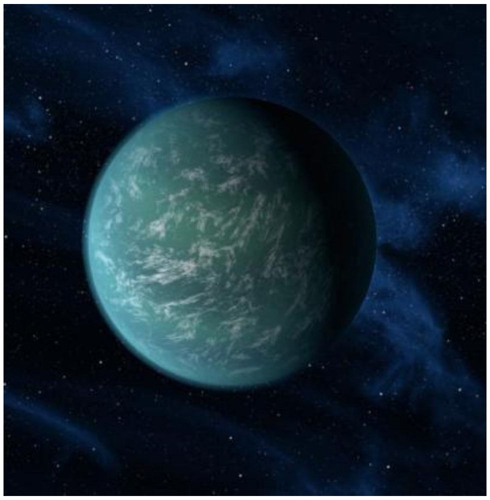


Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

Answer: Students may come up with a number of different strategies and estimates. For example, they might create Venn Diagrams for the data in the table that meet the criteria given in the problem. Then, from the number of planets in the intersection, find their proportion in the full sample of 54 planets, then multiply this by the ratio of 40 billion to 157,453. Estimates near 1 million are in the right range.

Table of Habitable Zone Candidates

	Planet Name (KOI)	Orbit Period (days)	Distance To Star (AU)	Planet Radius (Re)	Planet Temp. (K)	Star Temp. (K)
1	683.01	278	0.84	4.1	239	5,624
2	1582.01	186	0.63	4.4	240	5,384
3	1026.01	94	0.33	1.8	242	3,802
4	1503.01	150	0.54	2.7	242	5,356
5	1099.01	162	0.57	3.7	244	5,665
6	854.01	56	0.22	1.9	248	3,743
7	433.02	328	0.94	13.4	249	5,237
8	1486.01	255	0.80	8.4	256	5,688
9	701.03	122	0.45	1.7	262	4,869
10	351.01	332	0.97	8.5	266	6,103
11	902.01	84	0.32	5.7	270	4,312
12	211.01	372	1.05	9.6	273	6,072
13	1423.01	124	0.47	4.3	274	5,288
14	1429.01	206	0.69	4.2	276	5,595
15	1361.01	60	0.24	2.2	279	4,050
16	87.01	290	0.88	2.4	282	5,606
17	139.01	225	0.74	5.7	288	5,921
18	268.01	110	0.41	1.8	295	4,808
19	1472.01	85	0.37	3.6	295	5,455
20	536.01	162	0.59	3.0	296	5,614
21	806.01	143	0.53	9.0	296	5,206
22	1375.01	321	0.96	17.9	300	6,169
23	812.03	46	0.21	2.1	301	4,097
24	865.01	119	0.47	5.9	306	5,560
25	351.02	210	0.71	6.0	309	6,103
26	51.01	10	0.06	4.8	314	3,240
27	1596.02	105	0.42	3.4	316	4,656
28	416.02	88	0.38	2.8	317	5,083
29	622.01	155	0.57	9.3	327	5171
30	555.02	86	0.38	2.3	331	5,218
31	1574.01	115	0.47	5.8	331	5,537
32	326.01	9	0.05	0.9	332	3,240
33	70.03	78	0.35	2.0	333	5,342
34	1261.01	133	0.52	6.3	335	5,760
35	1527.01	193	0.67	4.8	337	5,470
36	1328.01	81	0.36	4.8	338	5,425
37	564.02	128	0.51	5.0	340	5,686
38	1478.01	76	0.35	3.7	341	5,441
39	1355.01	52	0.27	2.8	342	5,529
40	372.01	126	0.50	8.4	344	5,638
41	711.03	125	0.49	2.6	345	5,488
42	448.02	44	0.21	3.8	346	4,264
43	415.01	167	0.61	7.7	352	5,823
44	947.01	29	0.15	2.7	353	3,829
45	174.01	56	0.27	2.5	355	4,654
46	401.02	160	0.59	6.6	357	5,264
47	1564.01	53	0.28	3.1	360	5,709
48	157.05	118	0.48	3.2	361	5,675
49	365.01	82	0.37	2.3	363	5,389
50	374.01	173	0.63	3.3	365	5,829
51	952.03	23	0.12	2.4	365	3,911
52	817.01	24	0.13	2.1	370	3,905
53	847.01	81	0.37	5.1	372	5,469
54	1159.01	65	0.30	5.3	372	4,886



An artists illustration of Kepler-22b
(Credit NASA/Ames/JPL-Caltech)

NASA's Kepler mission has confirmed its first planet in the "habitable zone," the region where liquid water could exist on a planet’s surface.

The newly confirmed planet, Kepler-22b, is the smallest yet found to orbit in the middle of the habitable zone of a star similar to our sun.

The planet is about 2.4 times the radius of Earth. Scientists don't yet know if Kepler-22b has a rocky, gaseous or liquid composition, but its discovery is a step closer to finding Earth-like planets.

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth’s mass (5.97x1024 kg)?

Problem 2 - The acceleration of gravity on a planetary surface is given by the formula

$$a = \frac{GM}{R^2}$$

where M is in kilograms, R is in meters and G is the Newtonian Constant of Gravity with a value of 6.67×10^{-11} m^3 kg^{-1} sec^{-2} What is the surface acceleration of Kepler-22b A) In meters/sec^2? B) In multiples of Earth’s surface gravity 9.8 meters/sec^2?

Problem 3 - The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b?

Problem 4 - The dimensions of a typical baseball park are determined by the farthest distance that an average batter can bat a home-run. This in turn depends on the acceleration of gravity, which is the force that pulls the ball back to the ground to shorten its travel distance. For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the minimum distance to the back-field fence from Home Plate along one of the two foul lines?

B) What are the dimensions of the baseball diamond?

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97×10^{24} kg)?

Answer: A) First find the volume of the spherical planet in cubic meters, then multiply by the density of the planet to get the total mass.

$$R = 15{,}000 \text{ km} \times (1000 \text{ m}/1 \text{ km}) = 1.5 \times 10^7 \text{ meters.}$$

$V = 4/3\, \pi\, R^3$
$= 1.33 \times 3.14 \times (1.5 \times 10^7 \text{ meters})^3 \quad = 1.41 \times 10^{22} \text{ meters}^3$

Then M = density x volume
$= 5{,}500 \text{ kg/m}^3 \times (1.41 \times 10^{22})$
$= 7.75 \times 10^{25}$ kg

B) $M = 7.75 \times 10^{25} \text{ kg} / 5.97 \times 10^{24} \text{ kg}$ = **12.9 Earths**.

Problem 2 - A) In meters/sec^2? B) In multiples of Earth's surface gravity 9.8 meters/sec^2?

Answer: A) $a = 6.67 \times 10^{-11} (7.75 \times 10^{25}) / (1.5 \times 10^7)^2$ = **23.0 meters/sec^2**
B) 23.0 / 9.8 = **2.3 times earth's surface gravity**

Note: From the formula for M and a, we see that the acceleration varies directly with the radius change, which is a factor of 2.4 times Earth, so a = 2.4xa(earth)

Problem 3 – The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b?

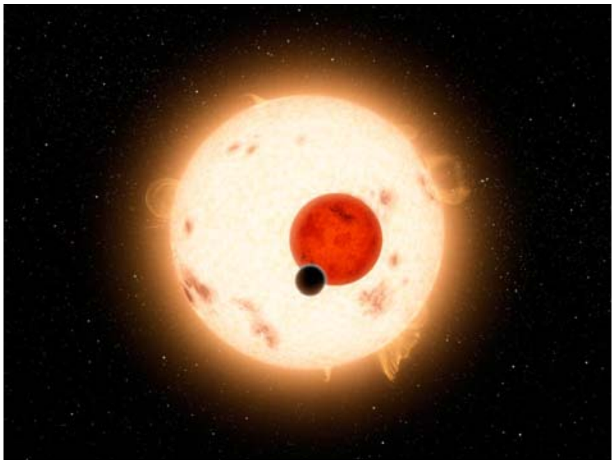
Answer: By a simple proportion: X/150 = 2.3/1.0 so x = 2.3x150 = **345 pounds**.

Problem 4 - For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the distance to the back-field fence from Home Plate along one of the two foul lines? Answer: 325 / 2.3 = **141 feet**.

B) What are the dimensions of the baseball diamond? Answer: 90/2.3 = **39 feet** on a side.

Exploring the New Planet 'Tatoine'



NASA's Kepler mission has discovered a world where two suns set over the horizon instead of just one. The planet, called Kepler-16b, is the most "Tatooine-like" planet yet found in our galaxy. Tatooine is the name of Luke Skywalker's home world in the science fiction movie *Star Wars*. In this case, the planet it not thought to be habitable. It is a cold world, with a gaseous surface, but like Tatooine, it circles two stars.

The planet orbits two stars, Kepler 16A and Kepler-16B located 200 light years from Earth. (Image credit: NASA/JPL-Caltech/R. Hurt)

The Saturn-sized planet Kepler-16b, orbits Kepler-16A at a distance of 0.7 Astronomical Units or 105 million kilometers. Its orbit is nearly a perfect circle. The smaller star Kepler-16B orbits its more massive companion at a distance of 0.2 Astronomical Units or 30 million kilometers. The diameters of the two stars are 890,000 kilometers and 300,000 kilometers respectively.

Problem 1 – Suppose you were standing on the surface of 'Tatooine' and watching (safely!) these two stars in the sky, perhaps much like Luke Skywalker was watching his twin suns near sunset. In terms of angular degrees, and by using the definition of the tangent of an angle, what would be the greatest separation between these two stars as they orbited one another to the nearest degree?

Problem 2 - At the time the stars are farthest apart in the sky, what would be the angular diameters of each star as viewed from Tatooine?

Problem 1 – Suppose you were standing on the surface of 'Tatooine' and watching (safely!) these two stars in the sky, perhaps much like Luke Skywalker was watching his twin suns near sunset. In terms of angular degrees, and by using the definition of the tangent of an angle, what would be the greatest separation between these two stars as they orbited one another to the nearest degree?

Answer: The geometry of this viewing triangle is that the adjacent side is the distance of Tatooine from Kepler-16A, which is 105 million kilometers, and the opposite side is the distance between Kepler-16A and 16B, which is 30 million kilometers.

The maximum separation angle is then

$\mathrm{Tan}(\theta) = 30/105$
$= 0.286$, so arctangent(0.286) = **16 degrees**.

Problem 2 - At the time the stars are farthest apart in the sky, what would be the angular diameters of each star as viewed from Tatooine?

Answer: The distance to **Kepler-16A** is just 105 million kilometers. Its diameter is 890,000 km, then

$\mathrm{Tan}(\theta) = 0.89/105$
$= 0.00848$ and arcTan(0.00848) = **0.49 degrees**.

For **Kepler-16B**, the problem is a bit tricky because the distance from Tatooine to Kepler-16B is now the hypothenuse of the triangle in Problem 1, which is $d^2 = (105)^2 + (30)^2$ so d = 109 million km. Then the angular diameter of Kepler-16B is just

$\mathrm{Tan}(\theta) = 0.3/109$
$= 0.00275$ so arcTan(0.00275) = **0.16 degrees**.

As seen from Earth, our sun (and moon) have angular diameters of 0.5 degrees by comparison.

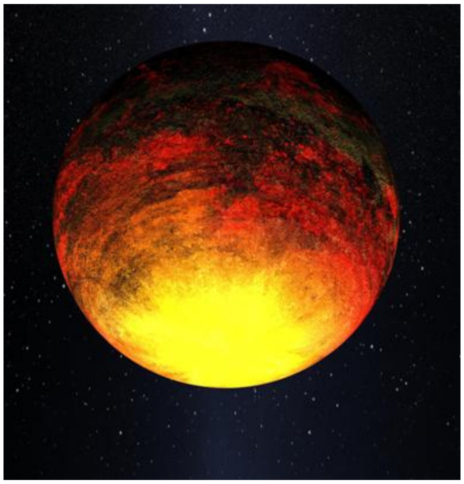
Note, as a preliminary activity, have students sketch the two stars in the afternoon sky at what they think might be their sizes as seen from Tatooine. After completing these two problems, students may sketch the same scene in which the two stars are located in the sky at their proper angular scales, and compare with their first guesses.

For more details read the Press Release on September 15, 2011:

NASA's Kepler Mission Discovers a World Orbiting Two Stars

http://www.nasa.gov/mission_pages/kepler/news/kepler-16b.html

Planet Kepler-10b: A matter of gravity



NASA Artist rendition of the sizzling-hot, Earth-like world: Kepler 10b.

The Kepler Space Observatory recently detected an Earth-sized planet orbiting the star Kepler-10. The more than 8 billion year old star, located in the constellation Draco, is 560 light years from Earth. The planet orbits its star at a distance of 2.5 million km with a period of 20 hours, so that its surface temperature exceeds 2,500 F.

Careful studies of the transit of this planet across the face of its star indicates a diameter 1.4 times that of Earth, and an estimated average density of 8.8 grams/cc, which is about that of solid iron, and 3-times the density of Earth's silicate-rich surface rocks.

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \quad \text{meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Answer: The planet is 1.4 times the radius of Earth, so its radius is 1.4 x 6,378 km = 8,929 kilometers. Since we need to use units in terms of meters because we are given the density in cubic meters, the radius of the planet becomes 8,929,000 meters.

Volume = $\frac{4}{3}\pi R^3$

so V = 1.33 x (3.141) x (8,929,000 meters)^3

V = 2.98 x 10^{21} meter^3

Mass = Density x Volume

= 8,800 x 2.98 x 10^{21}

= **2.6x10^{25} kilograms**

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \text{ meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

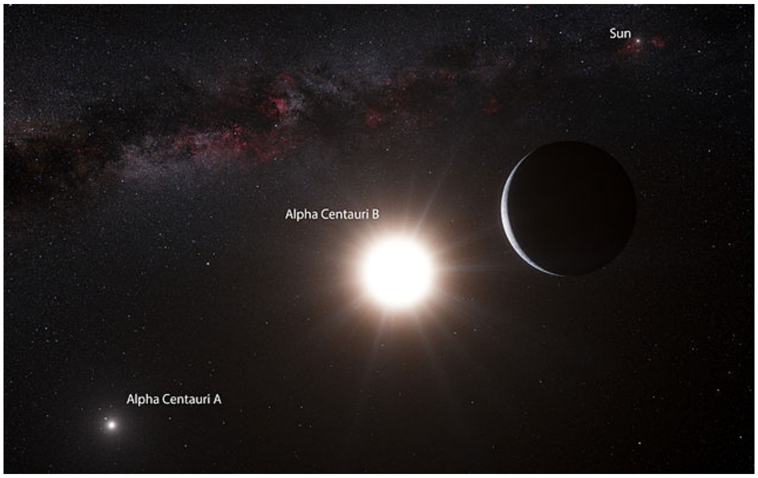
Answer: a = 6.67 x 10^{-11} (2.6 x 10^{25})/(8.929x10^6)^2

= **21.8 meters/sec^2**

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Answer: The acceleration is 21.8/9.8 = 2.2 times Earth's gravity, and since weight is proportional to gravitational acceleration we have the proportion:

$\frac{21.8}{9.8} = \frac{X}{150lb}$ and so the human would weigh 150 x 2.2 = **330 pounds!**



Artist rendition courtesy ESO/L. Calçada/N. Risinger (skysurvey.org)

Alpha Centauri is a binary star system located 4.37 light years from our sun. The two stars, A and B, are both sun-like stars, but they are older than our sun by about 1.5 billion years.

Astronomers have used the European Space Agency's 3.6 meter telescope at La Silla in Chile to detect the tell-tail motion of Alpha Centauri B caused by an earth-sized planet in close orbit around this star.

The planet, called Alpha Centauri Bb, orbits at a distance of only six million kilometers from its parent star – closer than Mercury is to the sun. The planet is bathed in unbearable heat, and has a surface temperature of 1,200 Celsius (2,200 F or 1,500 Kelvin). This is hot enough that its surface must be mostly molten lava. Its tight orbit means a year passes in only 3.2 Earth days.

The astronomers made hundreds of measurements of the speed of the Alpha Cen B star to search for a periodic change it its speed through space. They found a change (amplitude) of about 50 cm/sec that increased and decreased with a precise period, which would only be expected from an orbiting object.

This discovery is still being confirmed through independent observations by other astronomers.

We can check the numbers in the information box ourselves. Here are a few of the measurements made of the star's speed:

Time (hours)	Speed (cm/sec)	Time (hours)	Speed (cm/sec)
6	170	48	50
10	150	56	70
21	110	71	130
33	60	83	170

Problem 1 – Graph the speed data. Draw a smooth curve through the data (which need not go through all the points) and estimate the period (in days) of the speed curve to get the orbit period of the proposed planet.

Problem 2 – Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star (D in Astronomical Units) using the formula

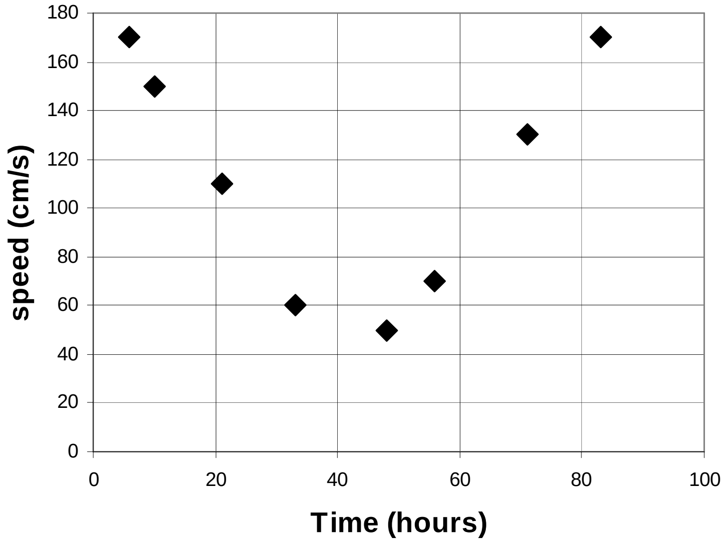
$$T^2 = D^3$$

where 1 Astronomical Unit equals the distance from Earth to our sun (150 million km). Using your estimated planet period, what is the orbit distance of the new planet from Centauri B in A) Astronomical Units? B) kilometers?

Problem 3 – What is the temperature T (in kelvins) of the new planet if its average temperature at a distance of D Astronomical Units is given by the formula:

$$T = \frac{310}{\sqrt{D}}$$

Problem 1 – Graph the speed data. Draw a smooth curve through the data and estimate the period of the speed curve to get the orbit period of the proposed planet. Answer should be about 77 hours or **3.2 days.**



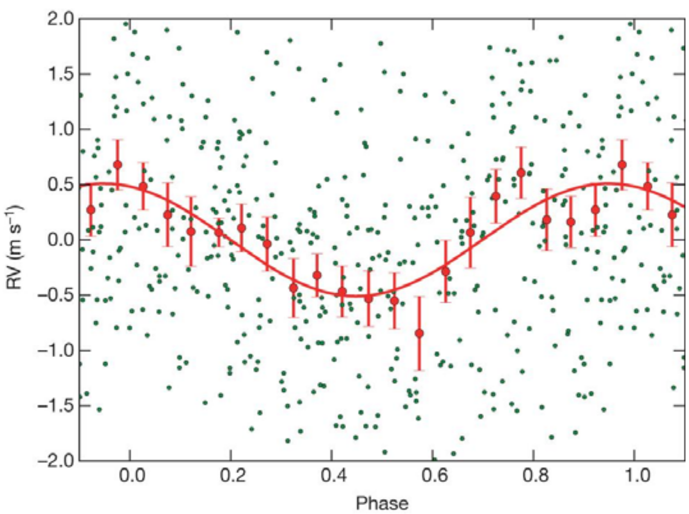
Problem 2 – Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star (D in Astronomical Units) using the formula

$$T^2 = D^3$$

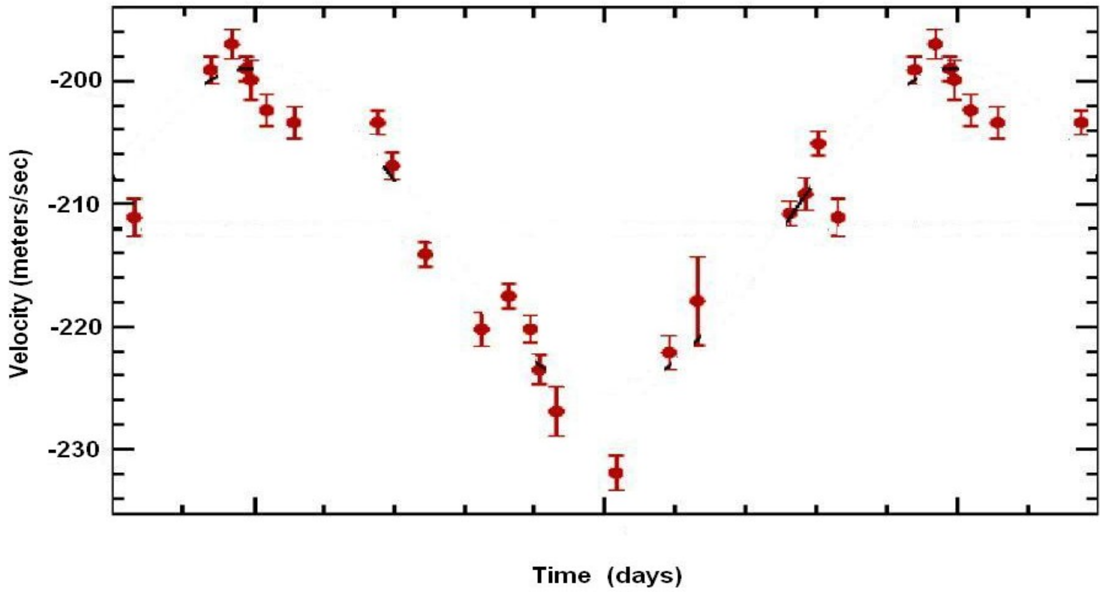
where 1 Astronomical Unit equals the distance from the earth to our sun (150 million km). Using your estimated planet period, what is the orbit distance from Centauiri B in A) Astronomical Units? B) kilometers? Answer: T = 3.2 days or in terms of earth-years T = 3.2/365 = 0.00877 years. Then $D^3 = (0.00877)^2$, $D^3 = 7.69\text{x}10^{-5}$ $D = (7.69\text{x}10^{-5})^{1/3}$ **D = 0.043 Astronomical Units**. B) In kilometers, this is 0.043 AU x (150 million km/1 AU) = **6.4 million kilometers**.

Problem 3 – What is the temperature T (in kelvins) of the planet if its average temperature at a distance of D Astronomical Units. Answer: $T = 310 / (0.043)^{1/2}$ so T = **1,500 kelvins**.

The actual graph of the data published by the astronomers is shown below:



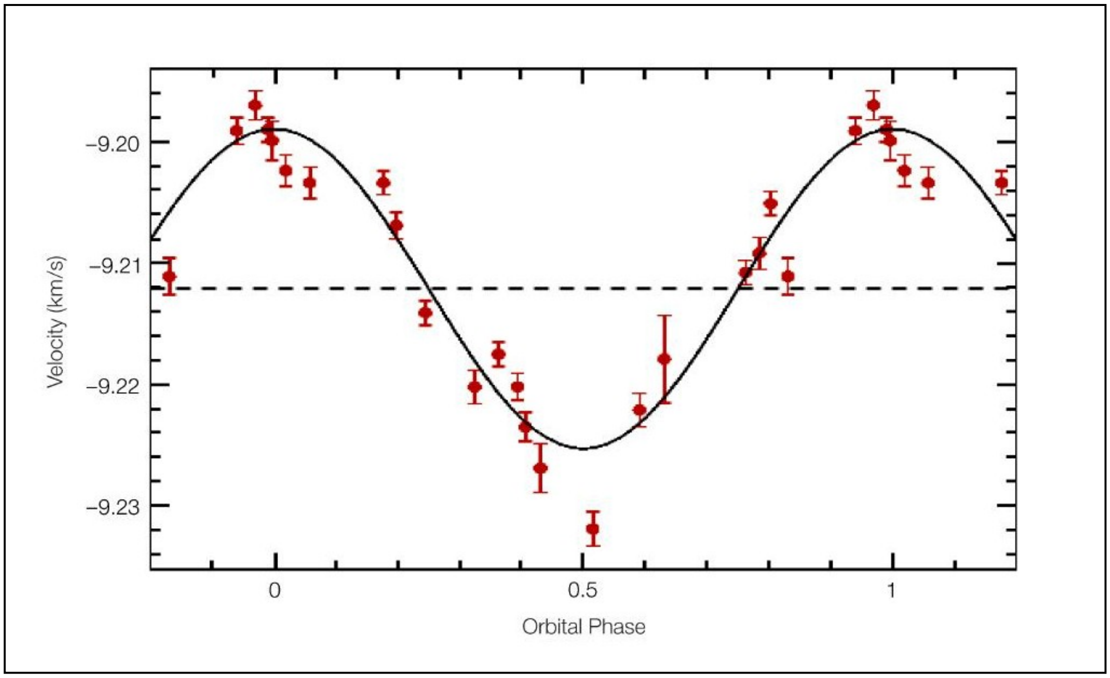
Fitting Periodic Functions - Distant Planets



A team of French and Swiss astronomers have discovered one of the lightest exoplanets ever found using the HARPS instrument on ESO's 3.6-m telescope at La Silla (Chile). They measured the speed of the star, Gleise-581 that the planet orbits, and plotted the data as shown above. The marks on the horizontal axis are spaced every 0.54 days apart starting at 0 which occurred on June 8, 2004.

Problem 1: From the data, create an estimate of the best-fit periodic function that follows the trend in the data. Calculate the amplitude, offset (vertical shift) phase and the formula for the angle in terms of the elapsed time in days since the start of the plot.

Problem 2: What would you predict as the velocity of the star on June 19, 2004?



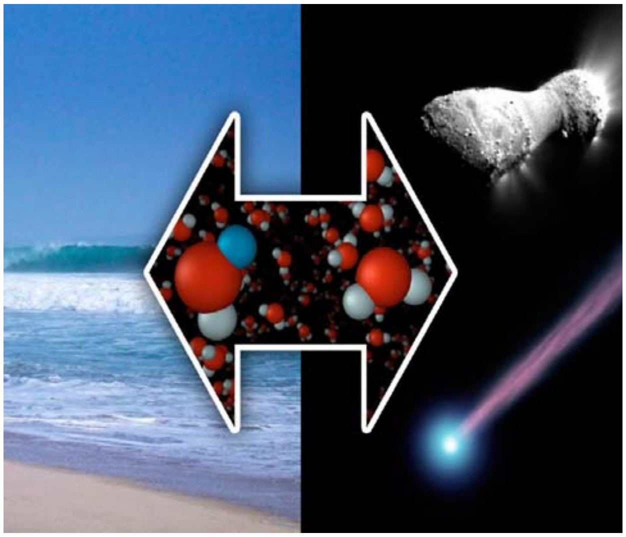
The figure above shows the actual cosine fit proposed by the astronomers for the star's periodic motion. It is of the form V = amplitude x cos($\theta - \phi$) + v0

Problem 1: The two peaks occur at T = 2 x 0.54 = 1.08 days and T = 12 x 0.54 = 6.48 days. The first peak occurs at a velocity of -198 m/s. The first minimum occurs at -226 km/sec so the average value (dotted line) is v0 = –(198 + 226)/2 = **-212 km/sec.** The amplitude is found by taking the difference between the maximum and minimum and dividing by 2 to get (-198 – (-226))/2 = **14 m/sec.** The period between the peaks is 6.48 – 1.08 = 5.4 days, so the orbital period is 5.4 days during which time the angle goes through 2π radians. That means that for all other times, the angle will be $\theta = 2\pi T/5.4$ or **0.37 π T** where T is the elapsed time in days since June 8, 2004. The first peak starts at 1.08 days, (where cos(0) = 1) so the phase-shift of the first peak from T=0 (June 8) is $\phi = 2\pi\, 1.08/5.4 =$ **0.4 π.** The best-fit equation is as follows:

$$\mathbf{V = 14 \cos(0.37\pi T - 0.4 \pi) - 212 \text{ km/s}}$$

Problem 2: T = June 19-June 8 = 11 days, so (0.37 π 11 – 0.4 π) = 11.53 radians. Then V = 14 cos(11.53) – 212 **= -205 m/sec**

The scientific results were published in 2005, in the journal *Astronomy and Astrophysics*, vol 443, page L15. For more information, see the Press Release at **http://www.eso.org/public/outreach/press-rel/pr-2005/pr-30-05.html**



The abundance of heavy-water in Earth's oceans is about 0.015%. The abundance of heavy-water in Hartley-2 is about 0.016%, so comets like Hartley-2 could have impacted Earth and deposited over time Earth's ocean water. (Image courtesy NASA/JPL-Caltech)

New measurements from the Herschel Space Observatory show that comet Hartley 2, which comes from the distant Kuiper Belt, contains water with the same chemical signature as Earth's oceans. This remote region of the solar system, some 30 to 50 times as far away as the distance between Earth and the sun, is home to icy, rocky bodies including Pluto, other dwarf planets and innumerable comets.

Herschel detected the signature of vaporized water in this coma and, to the surprise of the scientists, Hartley 2 possessed half as much "heavy water" as other comets analyzed to date. In heavy water, one of the two normal hydrogen atoms has been replaced by the heavy hydrogen isotope known as deuterium. The amount of deuterium is similar to the abundance of this isotope in Earth's ocean water.

The deposition of Earth's oceans probably occurred between 4.2 and 3.8 billion years ago. Suppose that the comet nuclei consisted of three major types, each spherical in shape and made of pure water-ice: Type 1 consisting of 2km in diameter bodies arriving once every 6 months, Type-2 consisting of 20 km diameter bodies arriving once every 600 years and Type-3 consisting of 200 km diameter bodies arriving every one million years.

Problem 1 - What are the volumes of the three types of comet nuclei in km^3?

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Space Observatory Provides Clues to Creation of Earth's Oceans

http://www.nasa.gov/mission_pages/herschel/news/herschel20111005.html

Problem 1 - What are the volumes of the three types of comet nuclei in km^3?

Answer: $V = 4/3 \pi R^3$ so

Type 1 Volume = **$4.2\ km^3$**

Type 2 Volume = **$4,200\ km^3$**

Type 3 Volume = **$4.2 \times 10^6\ km^3$**

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Answer: $1.33 \times 10^9\ km^3$ of water requires
$6 \times (1.33 \times 10^9\ km^3) = 8.0 \times 10^9\ km^3$ of ice.

The average delivery rate would be about
$R = 8.0 \times 10^9\ km^3$ of ice / 400 million years
= **$20\ km^3$ of ice per year**.

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Type 1: $R1 = 4.2\ km^3/0.5$ yrs = **$8.4\ km^3/yr$**

Type 2: $R2 = 4200\ km^3/600$ yrs = **$7.0\ km^3/yr$**

Type 3: $R3 = 4.2 \times 10^6\ km^3$ / 1000000 yrs = **$4.2\ km^3/yr$**

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Answer: The total deposition rate is $R1+R2+R3 = 20\ km^3/yr$, so it would take
$T = 8.0 \times 10^9\ km^3$ of ice / ($20\ km^3/yr$) = **400 million years**.