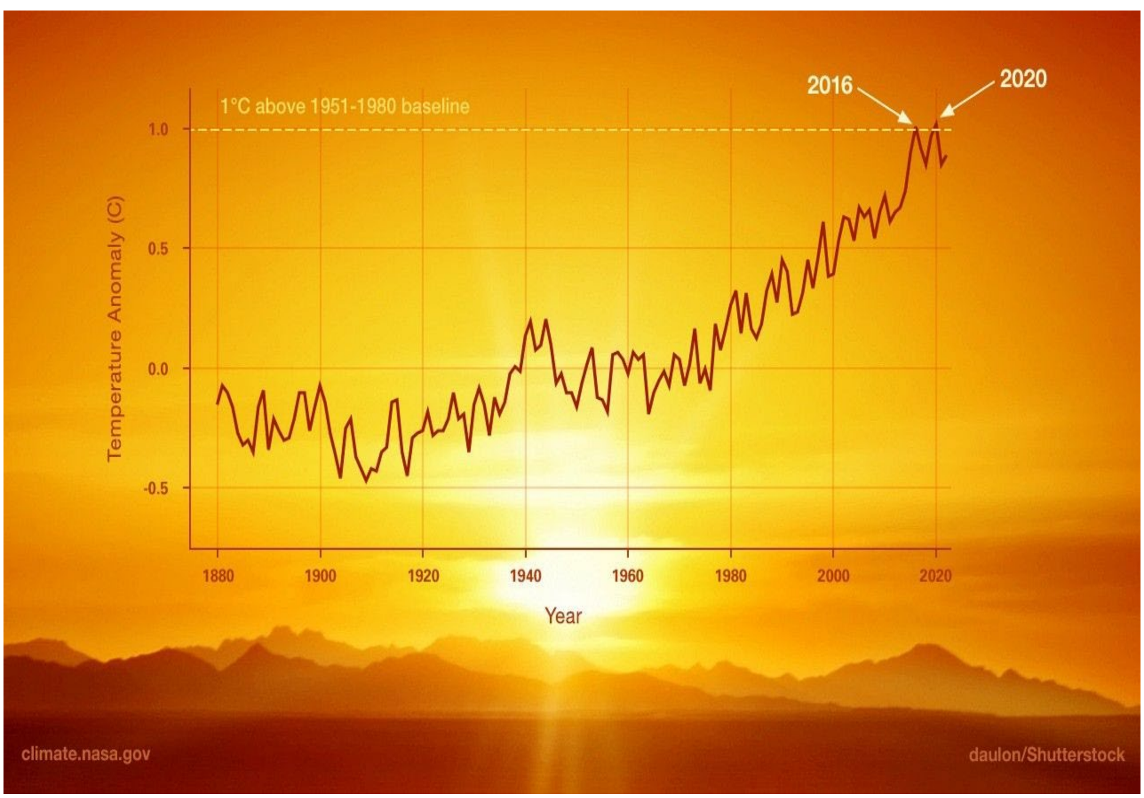
National Aeronautics and Space Administration

Math Problems Related to

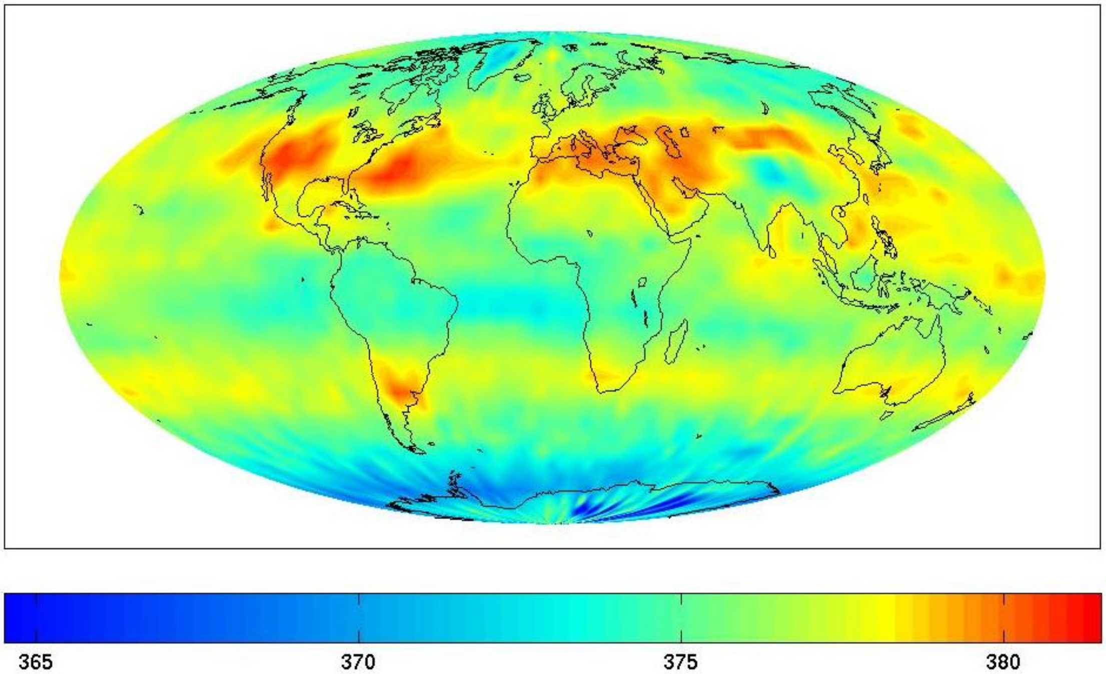
Climate Change



Introduction

This book provides an assortment of mathematics problems that deal with our changing climate. Since the 1950's, we now live in a world where the steady increase in global temperature is inexorably causing dramatic changes in the frequency and severity of storms, droughts and forest fires. The problems focus on using actual data to predict how global temperature, sea level and atmospheric carbon dioxide are changing. Simple models are explored to foster an understanding of how data is analyzed, and why constant monitoring is crucial in creating accurate forecasts. For more information, visit NASA's climate website at https://science.nasa.gov/climate-change/.

NASA Satellite 'Sees' Carbon Dioxide



The Atmospheric Infrared Sounder (AIRS) instrument on NASA's Aqua spacecraft has been used by scientists to observe atmospheric carbon dioxide. The above map shows the concentrations of atmospheric carbon dioxide in units of 'parts per million', and range from 363 ppm (dark blue) to 380 ppm (red). The data was obtained in July 2003, and the gas is at an altitude of 8 kilometers. The map shows that carbon dioxide is not evenly mixed in the atmosphere, but there are regional differences that change in time. For example, the red 'clouds' move in time and change size and shape.

Problem 1 - From the color bar, about what is the average concentration of carbon dioxide across the globe, in ppm, not including the orange or red areas?

Problem 2 - What is the difference in ppm between your answer to Problem 1, and the highest levels of concentration?

Problem 3 - At these altitudes, atmospheric winds generally blow from west to east (left to right on the map). What geographic regions are nearest the highest concentrations of carbon dioxide in this map?

Problem 4 - The average mass of carbon dioxide in the atmosphere, at a concentration of 1 ppm equals 15 tons per square kilometer. From an estimate of the areas contained in the red regions, and your answer to Problem 2, how much excess carbon dioxide is contained in these clouds in units of gigatons (1 billion tons)? (Note, the area of the United States is about 9 million square kilometers)

Answer Key

Problem 1 - From the color bar, about what is the average concentration of carbon dioxide across the globe, in ppm, not including the orange or red areas?
Answer: Most of the areas are yellowish, but the rest is light blue, so according to the color bar, the concentrations is about 375 ppm…very roughly.

Problem 2 - What is the difference in ppm between your answer to Problem 1, and the highest levels of concentration? Answer: The darkest reds are near 382 ppm, so the difference is about 7 ppm.

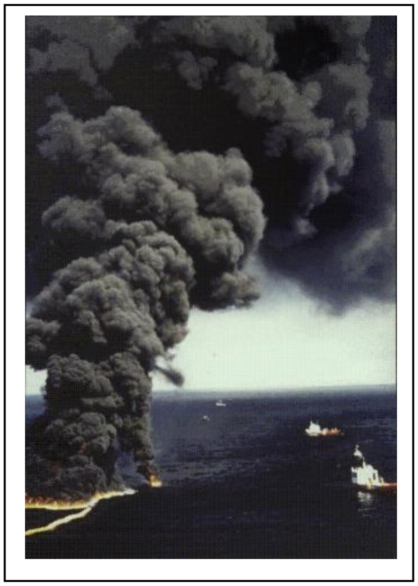
Problem 3 - At these altitudes, atmospheric winds generally blow from west to east (left to right on the map). What geographic regions are nearest the highest concentrations of carbon dioxide in this map? Answer: Western United States, the East Coast of the US, and regions in the Mediterranian and mid-East, some of which are 'downwind' from Europe.

Problem 4 - The average mass of carbon dioxide in the atmosphere, at a concentration of 1 ppm equals 15 tons per square kilometer. From an estimate of the areas contained in the red regions, and your answer to Problem 2, how much excess carbon dioxide is contained in these clouds in units of gigatons (1 billion tons)?

Answer: Students can use maps of these regions in which areas are preserved, called Equal-Area maps. The map in this problem is of this type. The area of the US is 9 million square km, so we estimate that the two US red zones are about 10 million square km, and the European-Asian areas are about the same area as the US. The region in South America is about 2 million square km. The total area is 22 million square km.

At 1 ppm, the total mass is 1 ppm x 22 million x 15 tons per square kilometer = 330 million tons. But since the additional carbon dioxide equals a concentration of 7 ppm, the total mass is 7 x 330 million tons or 2.3 gigatons.

Teacher Note: Most studies suggest that human activity add about 25 gigatons per year. Since the NASA map represents an average over a month, we see that our very crude estimate of 2.3 gigatons/month x 12 months = 28 gigatons/year which is close to more detailed estimates. This similarity may, however, be accidental since certain approximations had to be used in deriving our estimate that would not be made in the more careful studies. Also, the satellite only observed the carbon dioxide at an altitude of 8 kilometers, not all of the additional carbon dioxide down to sea-level.



On March 21, 2010 the Eyjafjalla Volcano in Iceland erupted, and the expanding ash cloud grounded over 3,000 flights in Europe. Then on April 20, a major oil spill began in the Gulf of Mexico. Although the preferred method for dealing with the oil spill is to collect it using skimmers, burning it is also a common option (see above left photo). A major concern in burning this oil is the addition of carbon dioxide to the atmosphere during the combustion process.

Problem 1 – The Gulf Oil Spill is predicted to generate 200,000 gallons of crude oil every day. If 50% of this is ultimately burned-off, how many tons/day of carbon dioxide are generated if the combustion of 1 gallon of oil generates 10 kg of carbon dioxide?

Problem 2 – Scientists have estimated that the Iceland volcano generated 15,000 tons of carbon dioxide per day, and this eruption continued for about 28 days. How many days will the Gulf Oil burn-off have to continue before its carbon dioxide contribution equals that of the total carbon dioxide generated by the Eyjafjalla Volcano?

Problem 3 – It has been estimated that the European aviation industry generates 344,000 tons of carbon dioxide each day. If 60% of this industry was shut down by the ash cloud from the Eyjafjalla Volcano, how many tons of carbon dioxide would have been produced by airline flights during the 5-day shut-down of the industry?

Problem 4 – What can you conclude by comparing your answers to Problem 1, 2 and 3?

Problem 1 – The Gulf Oil Spill is predicted to generate 200,000 gallons of crude oil every day. If 50% of this is ultimately burned-off, how many tons/day of carbon dioxide are generated if the combustion of 1 gallon of oil generates 10 kg of carbon dioxide?

Answer: 200,000 gallons/day x (0.50) x (10 kg/ 1 gallon) = 1,000,000 kg/day or **1,000 tons/day**

Problem 2 – Scientists have estimated that the Iceland volcano generated 15,000 tons of carbon dioxide per day, and this eruption continued for about 28 days. How many days will the Gulf Oil burn-off have to continue before its carbon dioxide contribution equals that of the total carbon dioxide generated by the Eyjafjalla Volcano?

Answer: The volcano generated 15,000 tons/day x 28 days = 420,000 tons of CO_2. The Gulf Oil burn-off generates 1,000 tons/day, so the Gulf Oil burn-off would have to continue for **420 days** before it equaled the emission of the volcano.

Problem 3 – It has been estimated that the European aviation industry generates 344,000 tons of carbon dioxide each day. If 60% of this industry was shut down by the ash cloud from the Eyjafjalla Volcano, how many tons of carbon dioxide would have been produced by airline flights during the 5-day shut-down of the industry?

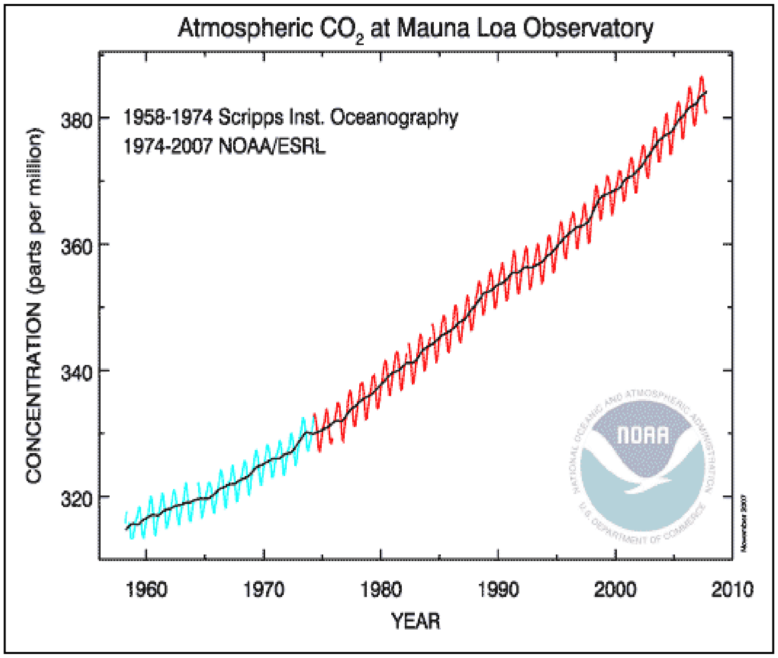
Answer: 344,000 tons/day x (0.6) x (5 days) = **1 million tons**.

Problem 4 – What can you conclude by comparing your answers to Problem 1, 2 and 3?

Answer: The total carbon dioxide generated by the volcano is only 40% of what was generated by the European airline industry during the time it was shut down, and the burn-off of the oil spill will only exceed what the volcano generated if the cleanup continues for over one year, which most experts say is very unlikely. The oil spill cleanup is a small part of the carbon dioxide generated by aviation or by the Icelandic volcano.

Note: Since the crude oil was destined to be burned to generate electricity, and combusted in cars, the fact that the clean-up is producing carbon dioxide is almost beside the point since this very same quantity of carbon dioxide would have been generated anyway once the crude oil is used under more controlled conditions.

Carbon Dioxide Increases



The Keeling Curve, shown to the left, shows the variation in concentration of atmospheric carbon dioxide since 1958-1974. It is based on continuous measurements taken at the Mauna Loa Observatory in Hawaii under the supervision of Dr. Charles Keeling of the Scripps Institution of Oceanography in San Diego.

Keeling's measurements beginning in 1958, showed the first significant evidence of rapidly increasing carbon dioxide levels in the atmosphere. Additional measurements by scientists working at NOAA have extended the Keeling Curve from 1974-2006.

Atmospheric scientists measure the concentration of gases in terms of parts-per-million (ppm). One ppm = 1 particle out of 1 million particles in a sample. It also represents a percentage: ppm/1million x 100%.

Problem 1 - In the graph, what was the concentration of carbon dioxide in 2005?

Problem 2 - What percentage of Earth's atmosphere, by volume, was carbon dioxide gas in 2005?

Problem 3 - If a concentration of 127 ppm of carbon dioxide in the atmosphere equals a total of 1,000 gigatons of carbon dioxide (1,000 billion tons), about what was the total mass of carbon dioxide gas in 2005?

Problem 4 - How many additional gigatons of carbon dioxide were added to the atmosphere between 1958 and 2005?

Problem 5 - What was the average rate of increase of carbon dioxide gas in gigatons per year between 1958 and 2005?

Problem 6 - The seasonal change in carbon dioxide is shown by the 'wavey' shape of the line. What is the width of this wave (range from maximum to minimum) in ppm, and about how many gigatons does this natural change correspond to?

Answer Key

Problem 1 - In the graph, what was the concentration of carbon dioxide in 2005? Answer: About 379 ppm.

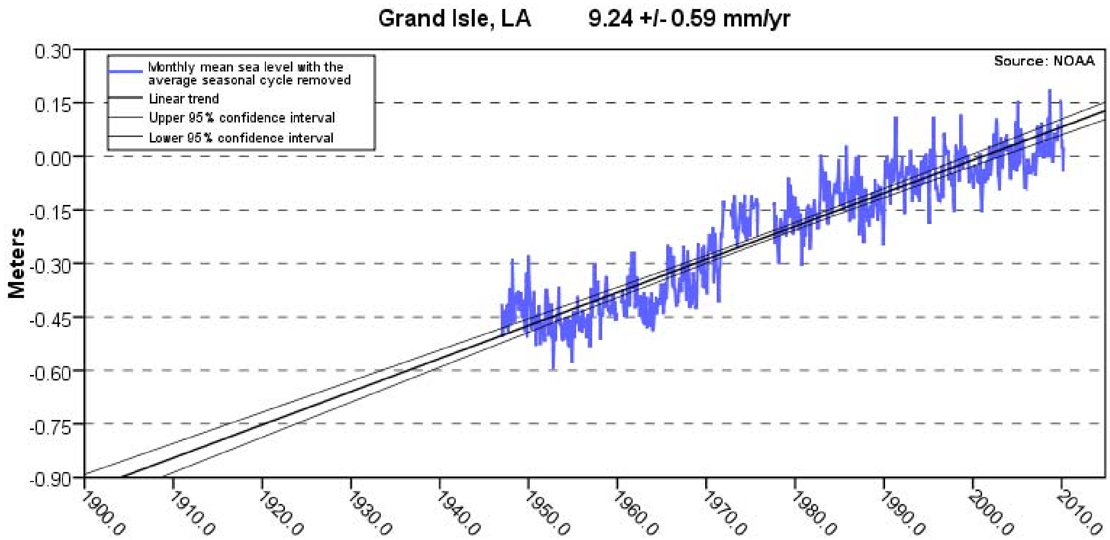
Problem 2 - What percentage of Earth's atmosphere, by volume, was carbon dioxide gas in 2005? Answer: 379 ppm is equal to 100% x 379/1 million = 0.0379 %

Problem 3 - If a concentration of 127 ppm of carbon dioxide in the atmosphere equals a total of 1,000 gigatons of carbon dioxide (1,000 billion tons), about what was the total mass of carbon dioxide gas in 2005? Answer: (379/127) x 1,000 gigatons = 2,984 gigatons, or to 3 significant figures, 2,980 gigatons.

Problem 4 - How many additional gigatons of carbon dioxide were added to the atmosphere between 1958 and 2005?
Answer: In 1958 the concentration was about 315 ppm ,so it gained 379-315 = 64 ppm of carbon dioxide. Since 1,000 gigatons corresponds to 127 ppm, we have (64/127) x 1,000 gigatons = 500 gigatons were added.

Problem 5 - What was the average rate of increase of carbon dioxide gas in gigatons per year between 1958 and 2005? Answer: For the 47 years that span the time interval, there was a 500 gigaton increase, so the rate was 500 gigatons/47 years = 11 gigatons/year.

Problem 6 - The seasonal change in carbon dioxide is shown by the 'wavey' shape of the line. What is the width (range from maximum to minimum) of this wave in ppm, and about how many gigatons does this natural change correspond to? Answer: Using a metric ruler and estimating the peak to peak variation of the amplitude, students should get about 5 ppm as the range. This equals (5 ppm/127 ppm) x 1,000 gigatons = 39 gigatons.



The NOAA 'Tides and Currents' website also provides an archive of sea level measurements for hundreds of places around the world based on tide gauges and satellite data for the past 100 years.

http://tidesandcurrents.noaa.gov/sltrends/sltrends_station.shtml?stnid=8761724

The plot above shows the sea level rise since 1950 for Grand Isle, Louisiana, which has an average elevation of 2-meters, and a population of 1,500 people. A storm surge from Hurricane Katrina on August 28, 2005 had an elevation of 1.5 meters, and did severe damage to hundreds of homes.

Problem 1 - Assume a linear sea level change of the form h = mt + b where h is the sea level height in meters and t is the number of years since 1950. What is the best-fit linear equation for the sea level rise?

Problem 2 - In what year will the sea level equal the storm surge from Hurricane Katrina?

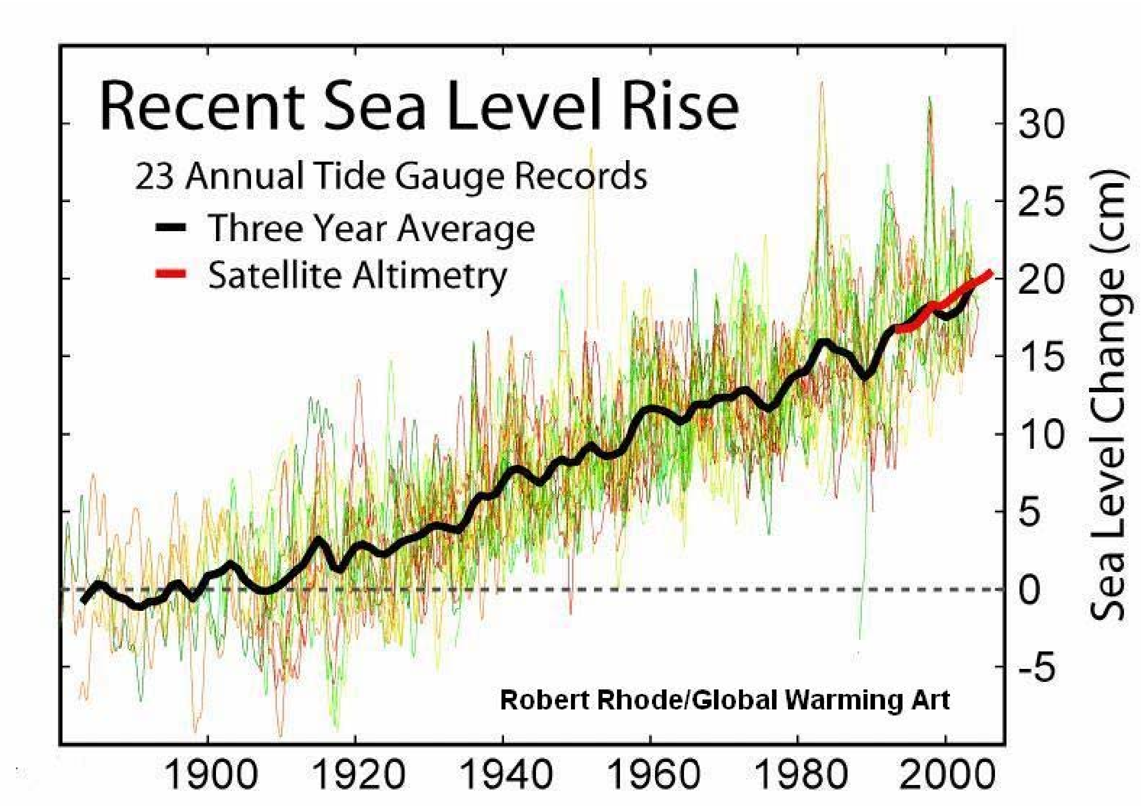
Problem 1 - Assume a linear change of the form h = mt + b where h is the sea level height in meters and t is the number of years since 1950. What is the best-fit linear equation for the sea level rise?

Answer: Point 1 = (1950, -0.45) Point 2 = (2010, +0.10) then
M = (0.10 + 0.45)/(2010-1950) = +0.0092 meters/year

H = 0.0092(t-1950) - 0.45

Problem 2 - In what year will the seal level equal the storm surge from Hurricane Katrina?

Answer: 1.5 = 0.0092 (T-1950) - 0.45 so **T = 2162**



The graph, produced by scientists at the University of Colorado and published in the IPCC Report-2001, shows the most recent global change in sea level since 1880 based on a variety of tide records and satellite data. The many colored curves show the individual tide gauge trends. The black line represents an average of the data in each year.

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line?

Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form?

Problem 3 - If the causes for the rise remained the same, what would you predict for the sea level rise in A) 2050? B) 2100? C) 2150?

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line? Answer; See figure below. First, selecting any two convenient points on this line, for example X= 1910 and Y = 0 cm (1910, +0) and X = 1980 Y= +15 cm (1980, +15). The slope is given by m = (y2-y1)/(x2-x1) = 15 cm / 70 years = 0.21 cm/year.

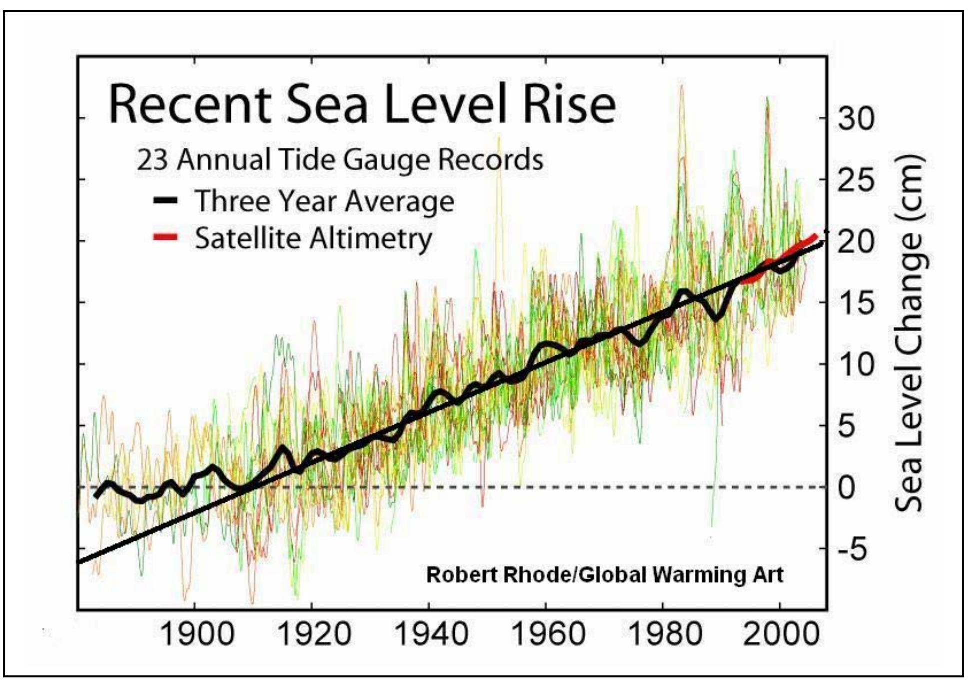
Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form? Answer:

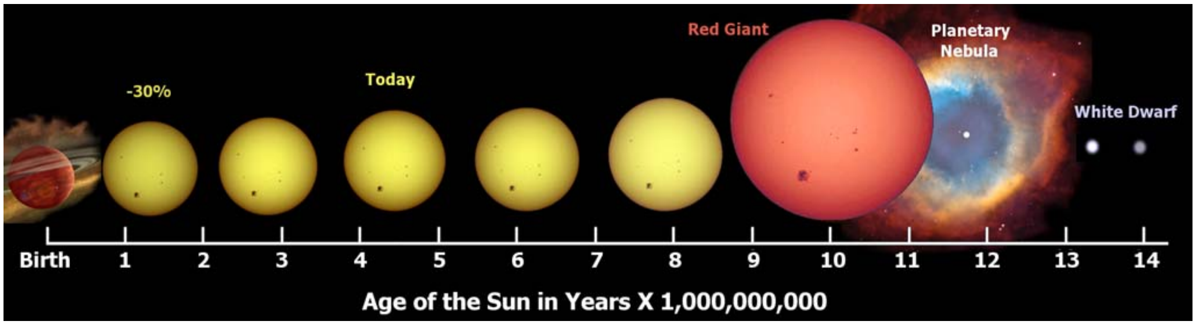
A) $y - y1 = \frac{(y2 - y1)}{(x2 - x1)} (x - x1)$ so $y - 0 = \frac{(15 - 0)}{(1980 - 1910)} (x - 1910)$

B) $y - y1 = m (x - x1)$ so $y - 0 = 0.21 (x - 1910)$

C) $y = 0.21 x - 0.21(1910)$ so $y = 0.21x - 401.1$

Problem 3 - If the causes for the rise remained the same, what would you predict for the sea level rise in A) 2050? B) 2100? C) 2150? Answer:
A) y = 0.21 (2050) - 401.1 = 29.4 centimeters. (Note; this equals 12 inches)
B) y = 0.21 (2100) - 401.1 = 39.9 centimeters (Note: this equals 16 inches)
C) y = 0.21 (2150) - 401.1 = 50.4 centimeters. (Note: this equals 20 inches)





Our sun was formed 4.6 billion years ago. Since then it has been steadily increasing its brightness. This normal change is understood by astronomers who have created detailed mathematical models of the sun's complex interior. They have considered the nuclear physics that causes its heating and energy, gravitational forces that compress its dense core, and how the balance between these processes change in time. The diagram above shows the major stages in our sun's evolution from birth to end-of-life after 14 billion years. A simple formula describes how the power of our sun changes over time:

$$L = \frac{L_0}{1 + \frac{2}{5}(1 - x)}$$

where $x = t/t_0$ t_0 = 4.6 billion yrs, L_0 = 1.0 for the luminosity of the sun today.

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years

Problem 2 – By what percentage will L increase when it is 2 billion years older than it is today?

Problem 3 – A simple formula for the temperature, in kelvins, of Earth is given by:

$$T = 284[(1 - A)L]^{\frac{1}{4}}$$

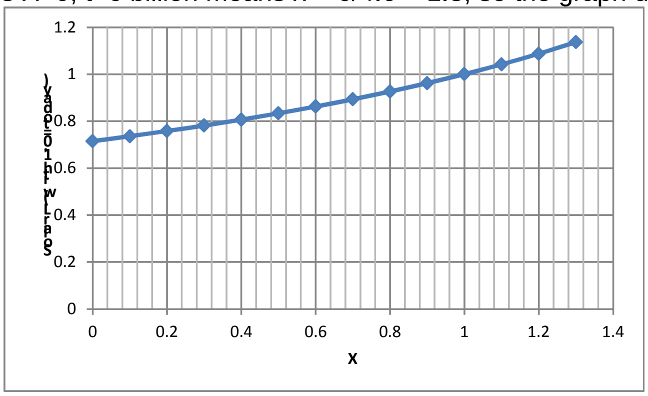
where L is the solar luminosity (today L=1.0), and A is the surface albedo, which is a number between 0 and 1, where asphalt is A=0 and A=1.0 is a perfect mirror   A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Problem 4 - Combine the two formulae above to define a new formula that gives Earth's temperature in kelvins only as a function of time, t, and albedo, A.

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 150° F (339 kelvins)? (Note: it is currently 60° F)

$$L = \frac{L_0}{1 + \frac{2}{5}(1-x)}$$

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years.
Answer: t = 0 means X=0, t=6 billion means x = 6/4.6 = 1.3, so the graph domain is [0,1.3]



Problem 2 - By what percentage will L increase when it is 2 billion years older than it is today?

Answer: X = (4.6 + 2.0)/4.6 = 1.43, then L = 1 / (1+0.4(1-1.43)) so L = 1.20 this is **20% brighter** than today.

Problem 3 – A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Answer: A) L = 1.0 today so $T = 284((1-0.4)\times1.0)^{1/4}$ = **250 kelvins** (or -23^{o} Celsius)
B) L = 1.05 so $T = 284(0.6\times1.05)^{1/4}$ = **253 kelvins** (or -20^{o} Celsius)

Problem 4 - Combine the formulae for L(x) and T to define a new formula, T(x,A) that gives Earth's temperature only as a function of time, x, and albedo, A, and assumes that L_0=1.0 today.

$$T(x,A) = 425\left(\frac{1-A}{7-2x}\right)^{\frac{1}{4}}$$

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 100^{o} F (310 kelvins)? (Note: it is currently 60^{o} F)

Answer: $310 = 425\,(0.4/(7-2x))^{1/4}$
$0.4\,(425/310)^4 = 7-2x$
1.4 = 7 – 2x
2x = 5.6 so x = 2.8
and the age of the sun will be t = 2.8 x 4.6 billion = **12.9 billion years**.
This occurs 12.9 – 4.6 = 8.3 billion years in the future.


Courtesy Aaron McNeeley
mmcneely@nd.edu

When a cloud is dense enough with water droplets it appears fleecy white, it is also dense enough that it can cause a shadow. Scientists use the terms albedo and transmission to describe how clouds and other materials reflect and transmit light.

Albedo: The amount of light a cloud reflects, making it appear white.

Transmission: The amount of light that passes through a cloud to the ground.

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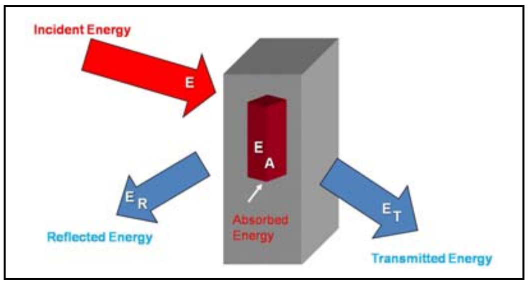


Albedo and transmission can be conveniently measured in percentages. For example, in the figure to the left, if 100% of the light energy falls on the cloud and 70% is reflected back into space, the cloud albedo is 70% and the percentage of transmitted energy is 100% - 70% = 30%.

Problem 1 – A cloud has an albedo of 65%, but a sensitive light meter registers only 30% transmitted light directly under the cloud. How much light energy has been absorbed by the cloud to heat it?

Problem 2 – A satellite view of a small area of Earth from space shows that 1/6 of the area had soil cover with an albedo of 20%, 1/3 of the area was covered by clouds with an albedo of 60%, and 1/2 of the area covered by water with an albedo of 10%. What is the average albedo of this area?

Instead of transmission, scientists prefer to use the term opacity, x, because it can be more easily calculated from the actual properties of the cloud. For example, x = kL, where L is the thickness of the cloud and k is a constant that describes the density of droplets in the cloud and droplet sizes. Transmission, T, and opacity are related by the formula:

$$T = 100\% \; 10^{-0.69x}$$

Problem 3 - Graph the function T(x) for opacities from 0.0 to 5.0. To the nearest percentage, what is the range of cloud transmission and albedo for opacities covered by your graph?

Problem 4 – A cumulus cloud is 2.5 kilometers thick and its opacity constant, k = 0.5, what is the albedo of this cloud, and how much light is transmitted through the cloud to the ground?

Common Core Math Standards:

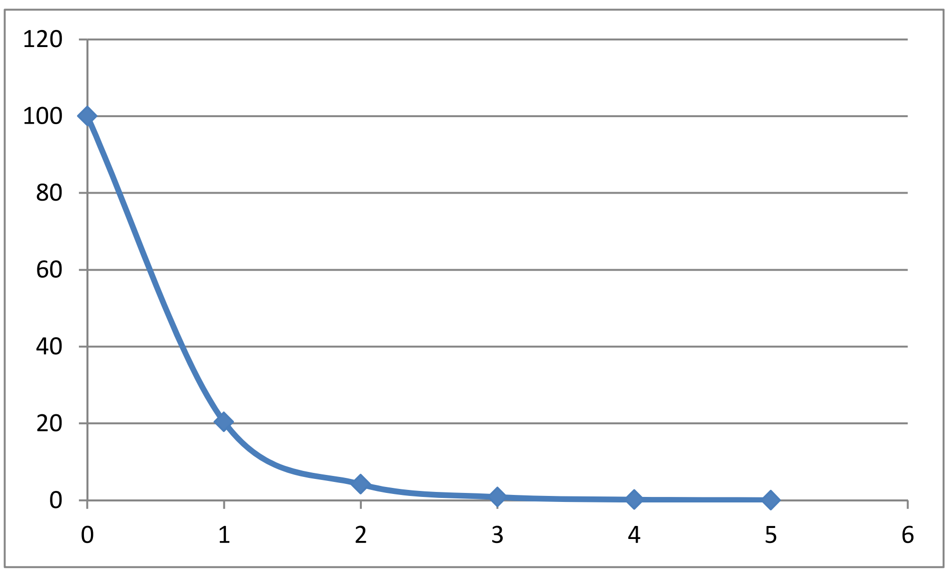
CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Problem 1 – A cloud has an albedo of 65%, but a sensitive light meter registers only 30% transmitted light directly under the cloud. How much light energy has been absorbed by the cloud to heat it? Answer: With an albedo of 65%, 35% of the light energy should have reached the ground. Since only 30% was detected, that means that **5% of the light energy** was absorbed by the cloud to heat it.

Problem 2 – A satellite view of a small area of Earth from space shows that 1/6 of the area had soil cover with an albedo of 20%, 1/3 of the area was covered by clouds with an albedo of 60%, and 1/2 of the area covered by water with an albedo of 10%. What is the average albedo of this area? Answer: A = 1/6 (20%) + 2/6(60%) +3/6(10%) = **28%**

Problem 3 - Graph the function T(x) for opacities from 0.0 to 5.0. To the nearest percentage, what is the range of cloud transmission and albedo for opacities covered by your graph?



Opacity 1 to 5 Transmission: 20% to 0% Albedo: 80% to 100%

Problem 4 – A cumulus cloud is 2.5 kilometers thick and its opacity constant, k = 0.5, what is the albedo of this cloud, and how much light is transmitted through the cloud to the ground?

Answer: x= kL so x= (0.5)(2.5) = 1.25 then the transmission $T = 100\%\ 10^{-0.69(1.25)}$

Then T = 100%(0.137)

And so T = **13.7%** and the albedo = 100% - 13.7% = **86.3%**

Table of Global Temperature Anomalies

Year	Temperature (degrees C)	Year	Temperature (degrees C)
1900	-0.20	1960	+0.05
1910	-0.35	1970	0.00
1920	-0.25	1980	+0.20
1930	-0.28	1990	+0.30
1940	+0.08	2000	+0.45
1950	-0.05	2010	+0.63

A new study by researchers at the Goddard Institute for Space Studies determined that 2010 tied with 2005 as the warmest year on record, and was part of the warmest decade on record since the 1800s. The analysis used data from over 1000 stations around the world, satellite observations, and ocean and polar measurements to draw this conclusion.

The table above gives the average 'temperature anomaly' for each decade from 1900 to 2010. The Temperature Anomaly is a measure of how much the global temperature differed from the average global temperature between 1951 to 1980. For example, a +1.0 C temperature anomaly in 2000 means that the world was +1.0 degree Celsius warmer in 2000 than the average global temperature between 1951-1980.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000?

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/La Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature anomaly change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.

Problem 3 - From the data in the table, calculate the rate of change of the temperature anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900?

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Problem 1 - Answer: In 1900 it was -0.20 C and in 2000 it was +0.45, so it has changed by +0.45 - (-0.20) = **+0.65 C**.

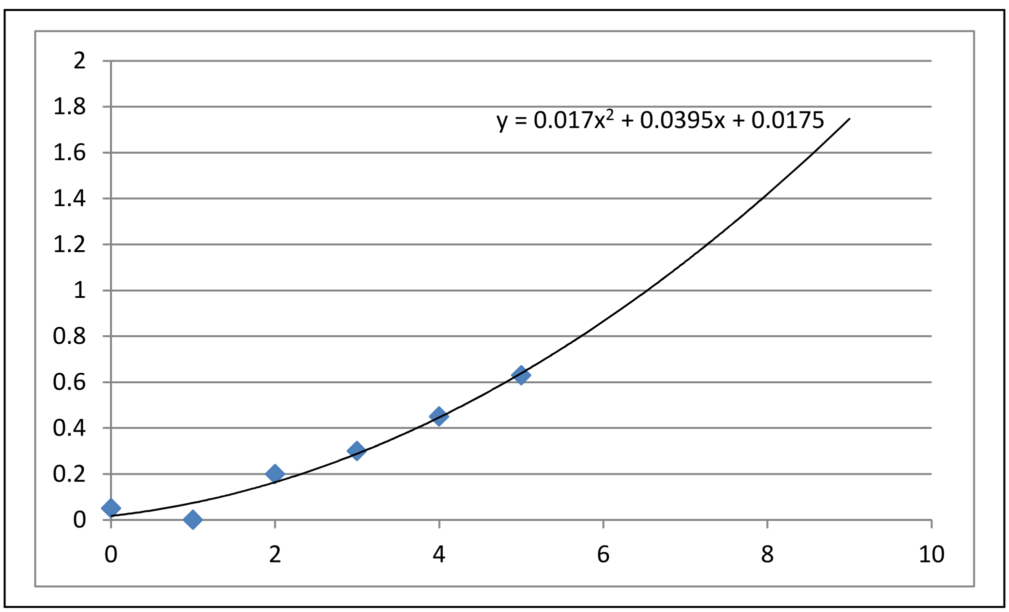
Problem 2 - Answer:
1900 to 1920: -0.25C - (-0.20C) = **-0.05 C a decrease (cooling) of 0.05 C**
1920 to 1950: - 0.05C - (-0.25C) = **+0.20 C an increase (warming) of 0.20 C**
1950 to 1980: +0.20C - (-0.05C) = **+0.25 C an increase (warming) of 0.25 C**
1980 to 2010: +0.63C - (+0.20C) = **+0.43 C an increase (warming) of 0.43 C**

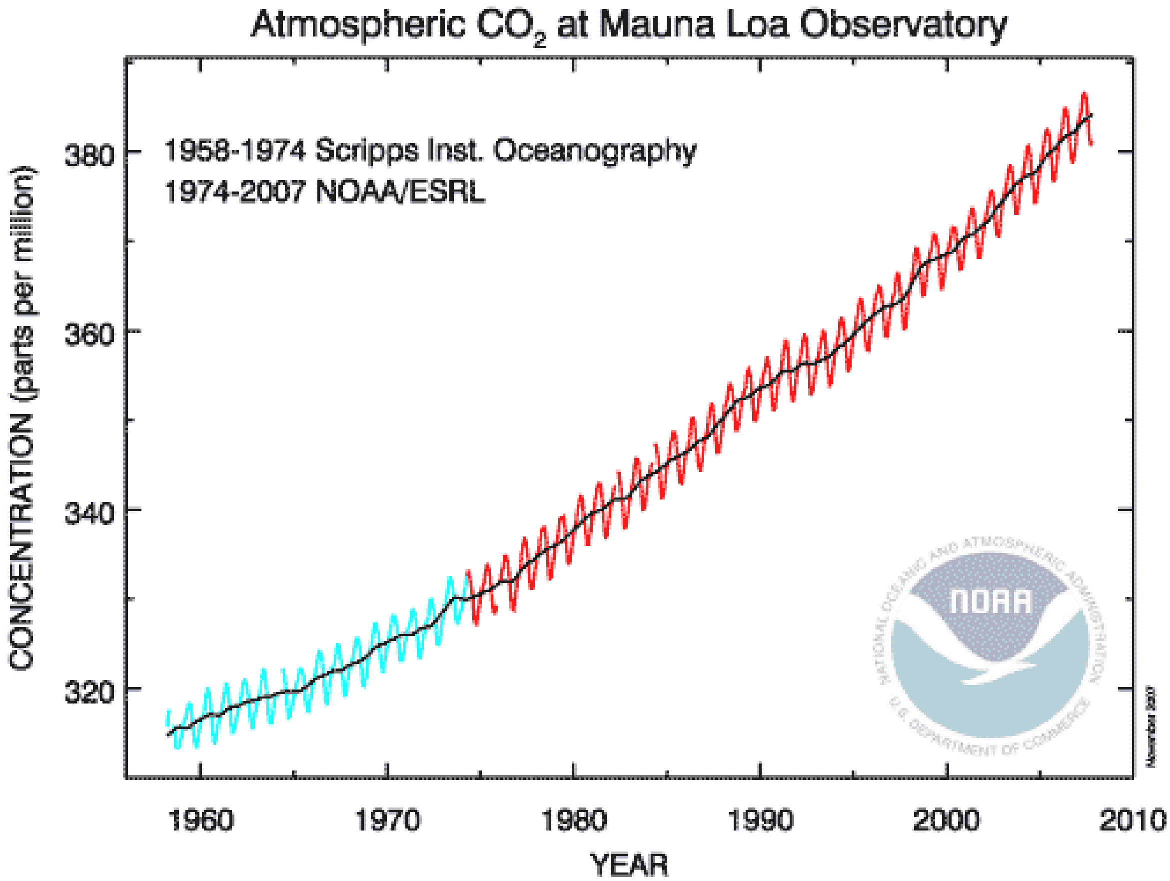
Problem 3 - Answer:
1900 to 1920: -0.05 C/3 decades = **-0.017 C per decade**
1920 to 1950: +0.20 C/ 3 decades = **+0.067 C per decade**
1950 to 1980: +0.25 C/3 decades = **+0.083 C per decade**
1980 to 2010: +0.43 C/3 decades = **+0.143 C per decade**.

The pace of global temperature change is **increasing in time**. It is almost doubling every 10 years.

Problem 4 - Answer: Students may graph the data in the table, then use a ruler to draw a line on the graph between 1960 and 2000, to extrapolate to the temperature anomaly in 2050. A linear equation, T = mx + b, that models this data is b = +0.05C m = (+0.45 - 0.05)/4 decades so m = +0.10 C/decade. Then T = +0.10x + 0.05. For 2050, which is 9 decades after 1960, x=9 so T = +0.1(9) + 0.05 = **+0.95 C**. So, the world will be, on average, about **+1 C warmer** in 2050 compared to its average temperature between 1950 and 1980. This assumes a linear change in T with time.

Note to Teacher: From Problem 3 we see that the temperature anomaly change is accelerating because the value for the increase in each 3-decade interval continues to increase from +0.067 per decade to +0.143 per decade from 1920 to 2010. A linear trend would have only a constant value for the change in each interval. If we use Excel spreadsheets and enter the data for 1960-2010, we get a more accurate 'quadratic' fit to the data since 1960 (See figure below): T = $+0.017\,x^2 +0.04x + 0.018$. For the year 2050, this quadratic prediction suggests $T = 0.017(9)^2 +0.04(9) + 0.018$ so T = +1.75 C.





This is the Keeling Curve, derived by researchers at the Mauna Kea observatory from atmospheric carbon dioxide measurements made between 1958 - 2005. The accompanying data in Excel spreadsheet form for the period between 1982 and 2008 is provided at

http://spacemath.gsfc.nasa.gov/data/KeelingData.xls

Problem 1 - Based on the tabulated data, create a single mathematical model that accounts for, both the periodic seasonal changes, and the long-term trend.

Problem 2 - Convert your function, which describes the carbon dioxide volume concentration in parts per million (ppm), into an equivalent function that predicts the mass of atmospheric carbon dioxide if 383 ppm (by volume) of carbon dioxide corresponds to 3,000 gigatons.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm) and mass for the years: A) 2020? B)2050, C)2100?

Data from: C. D. Keeling, S. C. Piper, R. B. Bacastow, M. Wahlen, T. P. Whorf, M. Heimann, and H. A. Meijer, Exchanges of atmospheric CO_2 and $^{13}CO_2$ with the terrestrial biosphere and oceans from 1978 to 2000. I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, 88 pages, 2001. Excel data obtained from the Scripps CO2 Program website at http://scrippsco2.ucsd.edu/data/atmospheric_co2.html

Problem 1 - Answer: The general shape of the curve suggests a polynomial function of low-order, whose amplitude is modulated by the addition of a sinusoid. The two simplest functions that satisfy this constraint are a 'quadratic' and a 'cubic'… where 't' is the elapsed time in years since 1982

F1 = A sin (Bt + C) + (Dt2 + Et + F) and F2 = A sin(Bt + C) + (Dt3 + Et2 + Ft + G)

We have to solve for the two sets of constants A, B, C, D , E, F and for A, B, C, D, E, F, G. Using *Excel* and some iterations, as an example, the constants that produce the best fits appear to be: F1: (3.5, 6.24, -0.5, +0.0158, +1.27, 342.0) and F2: (3.5, 6.24, -0.5, +0.0012, -0.031, +1.75, +341.0). Hint: Compute the yearly averages and fit these, then subtract this polynomial from the actual data and fit what is left over (the residual) with a sin function.) The plots of these two fits are virtually identical. We will choose Fppm = F1 as the best candidate model because it is of lowest-order. The comparison with the data is shown in the graph below: red=model, black=monthly data. Students should be encouraged to obtain better fits.

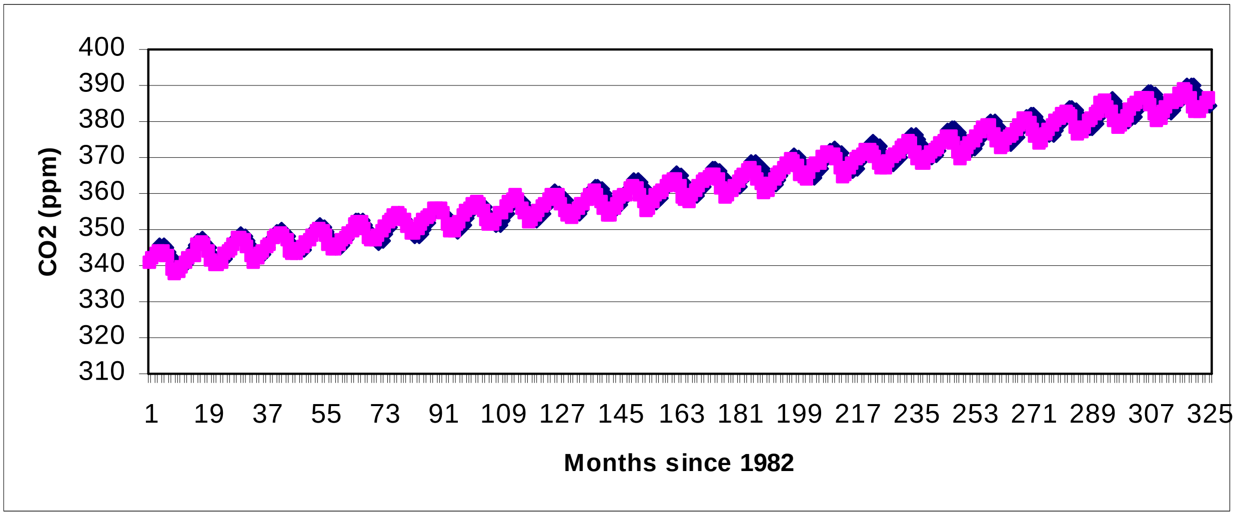
Problem 2 - Answer: The model function gives the atmospheric carbon dioxide in ppm by volume. So take Fppm and multiply it by the conversion factor (3,000/383) = 7.83 gigatons/ppm to get the desired function, Fco2 for the carbon mass.

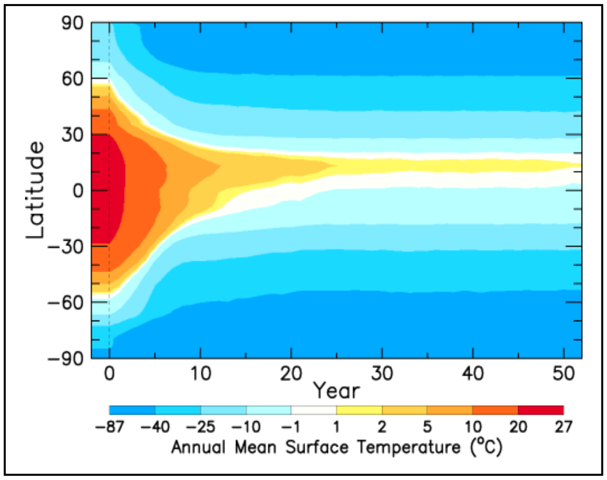
Problem 3 - What would you predict as the carbon dioxide concentration (ppm), and mass for the years: A) 2020? B)2050, C)2100? Answer:

A) t = 2020-1982 = 38, so Fco2(38) = 7.83 x 410 ppm = 3,200 gigatons

B) t = 2050-1982 = 68, so Fco2(68) = 7.83 x 502 ppm = 3,900 gigatons

C) t = 2100-1982 = 118, so Fco2(118) = 7.83 x 718 ppm = 5,600 gigatons





A computer model developed by NASA scientists at the Goddard Institute for Space Science shows that without carbon dioxide, the terrestrial greenhouse would collapse and plunge Earth into an icebound state. Today, the average temperature is +15°C. Within 50 years the average temperature would drop to -21°C without the warming provided by atmospheric carbon dioxide. The delicate link between the planet's temperature and carbon dioxide has also been proved by geologic records of CO_2 levels during ice ages and interglacial periods. The temperature difference between an ice age period and an interglacial period is only 5°C. During previous ice ages, CO_2 levels were near 180 parts per million (ppm). During the warm interglacial periods the levels were near 280 ppm. Today we are living in an interglacial period that started 12,000 years ago and may last another 40,000 years. Scientists continue to worry that, as CO_2 levels approach 400 ppm, we are in uncharted territory with no historical precedent as far back as 1 million years.

Although there is no known process that would instantly remove all CO_2 from the atmosphere, this computer model is important for another reason. It helps us predict how warm a planet would be if it had no greenhouse gases, even though it is close to its star.

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where T < 0°C?

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

http://www.giss.nasa.gov/research/news/20101014/
How Carbon Dioxide Controls Earth's Temperature
October 14, 2010

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where $T < 0^oC$?

Answer: From the formula, we need $A/4\pi R^2 = ½$
so $½=1-\sin\theta$ and so $\theta = 30^o$ latitude

From the model, the zone where $T = -1^oC$ to $+1^o$ C reaches a latitude of 30^o occurs after a time of **5 years.**

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

Answer: The average albedo is found by averaging the albedo of the area covered by the ice caps (A=0.7) with the albedo of the area covered by the ocean (a=0.2).

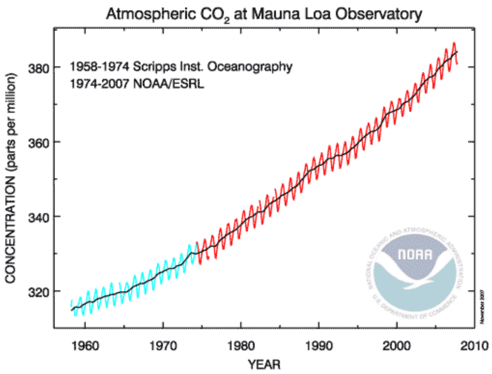
The area covered by ice caps = $4\pi R^2(1-\sin\theta)$
The remaining area covered by water = $4\pi R^2 - 4\pi R^2(1-\sin\theta) = 4\pi R^2(\sin\theta)$

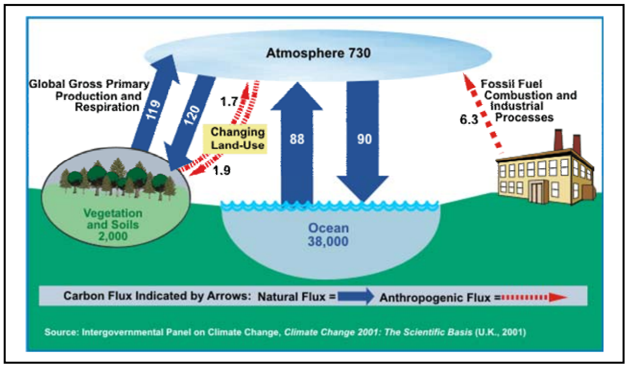
$$0.6\ (4\pi R^2) = 0.7\ (4\pi R^2)(1-\sin\theta) + 0.2(4\pi R^2)[\sin\theta]$$

Simplifying: $0.6 = 0.7(1-\sin\theta) + 0.2\sin\theta$
$0.6 = 0.7 -0.7\sin\theta + 0.2\sin\theta$
$-0.1 = -0.5\sin\theta$
$\sin\theta = 0.2$
So $\theta = 12^o$

So, when the zone $-1^oC < T < +1^oC$ reaches a latitude of $+12^o$, the albedo of Earth will have increased from its current value of 0.4 to a much higher reflectivity of 0.6. According to the model graph, this happens after 10 years.

Note: Although CO_2 loss is an important cooling process, the rapid increase in albedo is a significant cause for cooling and an important ‘feed back’ in the process. As the planet cools, more ice appears and the albedo increases, causing more cooling and more ice to appear…





The graph to the left shows the 'Keeling Curve' which plots the increase in atmospheric carbon dioxide between 1958-2005. The average net annual rate is about 11 gigatons/year. The figure to the right shows a simplified view of the sources and sinks of the element carbon on Earth. Note that, for every 44 gigatons of the carbon dioxide molecule, there are 12 gigatons of the element carbon.

Problem 1 - What are the sources of carbon increases to the atmosphere in the above diagram? What are the sinks of carbon?

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon in gigatons.

Problem 3 - Integrate the equation in Problem 2, assuming that C(2005) = 730 gigatons, and derive A) the simple equation describing the total amount of carbon in the atmosphere as a function of time; B) the equation for the total amount of carbon dioxide.

Problem 4 - What does your model predict for the amount of carbon in the atmosphere in 2050 if the above source and sink rates remain the same?

Problem 5 - If the current carbon dioxide abundance is 384 ppm, what does your model predict for the carbon dioxide abundance in 2050?

Problem 6 - Does your answer for the net change in Problem 2 match up with the Keeling Curve data that indicates a net annual increase of carbon dioxide of +11 gigatons/year?

Answer Key

Problem 1 - Answer: The sources of the carbon (arrow pointed into the atmosphere in the figure) are Vegetation (+119.6 gigatons/yr), oceans (+88 gigatons/yr), human activity (+6.3 gigatons/yr) and changing land use (+1.7 gigatons/yr). The sinks remove carbon (the arrows pointed down in the figure) and include vegetation (-120 gigatons/yr), oceans (-90 gigatons/yr), and changing land use (-1.9 gigatons/yr).

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon dioxide, C(t), in gigatons.

Answer:
$$\frac{dC(t)}{dt} = +119 + 88 + 6.3 + 1.7 - 120 - 90 - 1.9$$

so
$$\frac{d\,C(t)}{dt} = +3.1$$

Problem 3 - Answer: $C(t) = 3.1\,t + a$ where a is the constant of integration.
Since C(2005) = 730

$730 = 3.1\,(2005) + a$ and so a = -5500
$C(t) = 3.1\,t - 5500.$ for the total element carbon.

Since 44 gigatons of carbon dioxide contain 12 gigatons of carbon, the equation for the CO2 increase is 44/12 = 3.7x C(t) so that $CO_2(t) = 11.5\,t - 20{,}300$

Problem 4 - What does your model predict for the amount of carbon dioxide in the atmosphere in 2050 if the above source and sink rates remain the same?

Answer: $C(2050) = 3.1\,(2050) - 5500$
C(2050) = 860 gigatons of carbon.

Problem 5 - If the current carbon dioxide abundance is 384 ppm, what does your model predict for the abundance in 2050? Answer: Since we are interested in the carbon dioxide increase we use the equation

$$CO_2(t) = 11.4\,t - 20{,}300$$

For t = 2050 we get $CO_2(2050) = +3070$ gigatons of CO2.

Since 384 ppm corresponds to 730 gigatons of CO2, by a simple proportion, 3275 gigatons corresponds to 384 ppm x (3070/730) = 1,600 ppm of CO2.

Problem 5 - Does your answer for the net change in Problem 2 match up with the Keeling Curve data that indicates a net annual increase of carbon dioxide 11 gigatons/year?
Answer: Yes. The net change in Problem 2 was +3.1 gigatons/year of the element carbon. Since for every 44 gigatons of carbon dioxide there are 12 gigatons of the element carbon, we have + 3.1 x (44/12) = +11.4 gigatons of carbon dioxide increase, which is similar to the annual average of the increase implied by the Keeling Curve. This means that our simple model, based on the specified rates, provides a consistent picture of the atmospheric changes. Note: This simple, linear, model is only valid over the time span of the Keeling Curve.

The Declining Arctic Ice Cap during September

Survey Year	Ice area in millions of square km
1979	7.2
1980	7.9
1981	7.3
1982	7.5
1983	7.5
1984	7.2
1985	6.9
1986	7.5
1987	7.5
1988	7.5
1989	7.0
1990	6.2
1991	6.6
1992	7.6
1993	6.5
1994	7.2
1995	6.1
1996	7.9
1997	6.7
1998	6.6
1999	6.2
2000	6.3
2001	6.8
2002	5.9
2003	6.2
2004	6.1
2005	5.6
2006	5.9
2007	4.3
2008	4.7
2009	5.4
2010	4.9
2011	4.6

The data table to the left shows the minimum ice cap area for the Arctic during the month of September. At this time, the ice cap volume is at an annual minimum during the Arctic Summer. Global warming theory predicts that the polar regions will experience the largest impacts from continued planetary warming.

In this exercise, we will use regression techniques to examine the trends in this tabular data, and examine how reliable forcasts will be for the year 2030 given the current data.

In the problems below, perform the calculations using your choice of technology, or by hand.

Problem 1 - Graph the tabular data using convenient scaling of the horizontal (Year since 1979) and vertical (Area) axes.

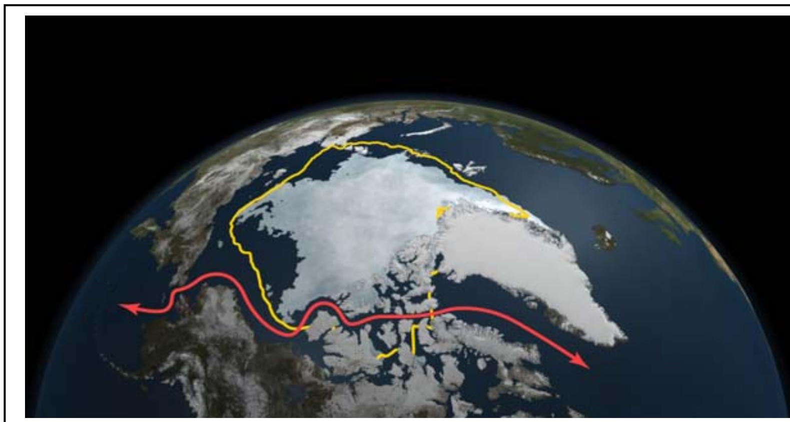
Problem 2 - Use a linear regression to model the data between 1979-2011 and compute the R^2 value of the fit.

Problem 3 - Use a quadratic regression to model the data between 1979-2011 and compute the R^2 value of the fit.

Problem 4 - To the nearest 100,000 km^2, what would you predict as the area of Arctic sea ice in the year 2030 using each regression model?

Problem 5 - Which of the two forecasts would you consider more statistically reliable? Which would you consider more believable?

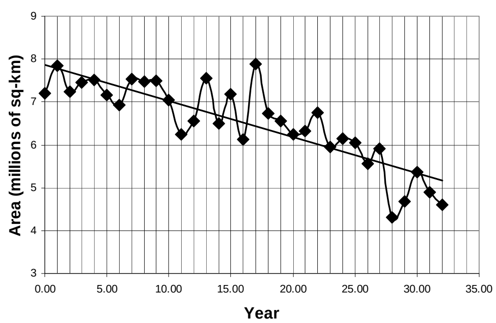
NASA satellite data reveals how this year's minimum sea ice extent, reached on Sept. 9, 2011 as depicted here, declined to a level far smaller than the 30-year average (in yellow) and opened up Northwest Passage shipping lanes (in red). ***(Credit: NASA Goddard's Scientific Visualization Studio)***

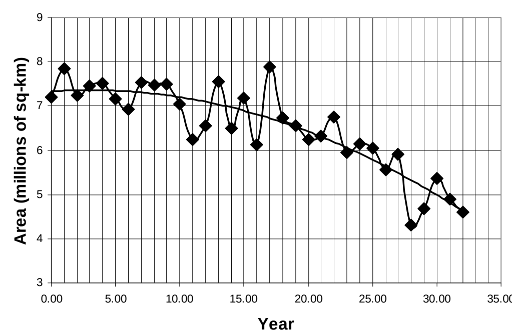


The tabular data was obtained from the NOAA, National Snow and Ice Data Center archive at ftp://sidads.colorado.edu/DATASETS/NOAA/G02135/Sep/
The archive of sea ice data can be assessed at
http://nsidc.org/data/seaice_index/archives/index.html

NASA Press Release:
Arctic Sea Ice Continues Decline, Hits 2nd-Lowest Level
October 4, 2011
URL = http://www.nasa.gov/topics/earth/features/2011-ice-min.html

Problem 1 - Graph the tabular data using convenient scaling of the horizontal (year since 1979) and vertical (Area) axes.





Note: For regression purposes, because the year numbers are so large, convert X axis to X = 'Years since 1979'.

Problem 2 - With Excel Spreadsheet, a linear regression (top left) gives **A = 7.9561 -0.0847 (X)** with $R^2 = 0.71$.

Problem 3 - With Excel Spreadsheet, a quadratic regression (top right) gives **A = -0.0033 $(X)^2$ + 0.0264(X) +7.308** with $R^2 = 0.79$

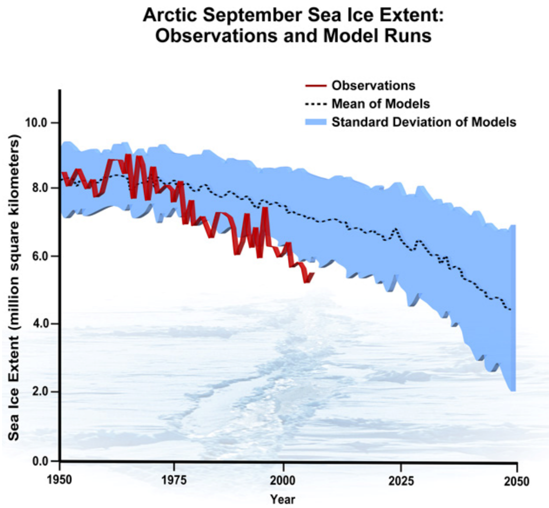
Problem 4 – To the nearest 100,000 km^2, what would you predict as the area of Arctic sea ice in the year 2030 using each regression model?

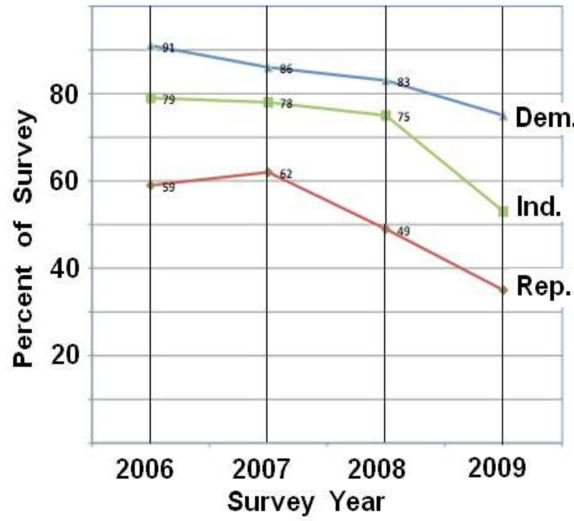
X = (2030-1979) = 51, then

Linear: A = 7.9561 – 0.0847 (51) so **A = 3.6 million km^2**.
Quadratic: A = -0.0033 $(X)^2$ + 0.0264(X) +7.308 so **A = 0.1 million km^2.**

Problem 5 - Which of the two forecasts would you consider more statistically reliable? Which would you consider more believable?

Answer: The quadratic regression has the better regression coefficient, but if we had removed the last three points (2009, 2010 and 2011) the regression coefficient would be similar to the one for the linear regression, so the quadratic regression is not as 'believable'. More data beyond 2011 is needed.





The graph above, based upon research by the National Sea Ice Data Center (Courtesy Steve Deyo, UCAR), shows the amount of Arctic sea ice in September (coldest Arctic month) for the years 1950-2006, based on satellite data (since 1979) and a variety of direct submarine measurements (1950 - 1978). The blue region indicates model forecasts based on climate models. Meanwhile, the figure on the right shows the results of polls conducted between 2006 and 2009 of 1,500 adults by the Pew Research Center for the People & the Press. The graph indicates the number of people, in all three major political parties, believing there is strong scientific evidence that the Earth has gotten warmer over the past few decades.

Problem 1 - Based on the red curve in the sea ice graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006?

Problem 2 - Based on the polling data, what are the three linear equations that model the percentage of Democrats (Dem.), Independents (Ind.) and Republicans (Rep.) who believed that strong evidence existed for global warming?

Problem 3 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Problem 4 - From your model for the polling data, by about what years will the average American in the Pew Survey, who identifies themselves as Democrats, Independents or Republicans, no longer believe that there is any scientific evidence at all for global warming?

Problem 1 - Based on the red curve in the graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006? Answer: The linear equation will be of the form y = mx + b. From the graph, the y-intercept for the actual data is 8.5 million km^2 for 1950. The value for 2006 is 5.5 million km^2. The slope is m = (5.5 - 8.5) / (2006 - 1950) = -0.053, so the equation is given by **Y = -0.053(x-1950) + 8.5** in millions of km^2.

Problem 2 - Based on the polling data, what are the three linear equations that model the percentage of Democrats (Dem.), Independents (Ind.) and Republicans (Rep.) who believed that strong evidence existed for global warming?

Answer:

Dems: m = (75% - 90%)/(2009-2006) = -5.0, so the model becomes

y = -5.0(x - 2006) + 90 percent ;

Ind. m = (52% - 79%)/(2009-2006) = -9.0, so the model becomes

y = -9.0(x - 2006) + 79 percent.

Rep. ; (35% - 60%)/(2009-2006) = -8.3, so the model becomes

y = -8.3(x - 2006) + 60 percent

Problem 3 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Answer: In 1950-1975 there were about 8.5 million km2 of sea ice in September. Half of this is 4.3 million km2. Set y = 4.3 and solve for x:

Solve 4.3 = -0.053(x-1950)+ 8.5 to get

-4.2 = -0.053(x-1950)

4.2 = 0.053(x-1950)

4.2/0.053 = x-1950

79 = x - 1950

And so x = 2029. So, during the year **2029 AD** there will only be half as much sea Ice in the Arctic in September.

Note: If we use only the slope data since 1975 when the ice cover was 8.0 million km^2, the slope would be m = (5.5 - 8.0)/(2006-1975) = -0.083, and linear equation is y = - 0.083(x-1975) + 8.0. The year when half the ice is present would then be about 2023 AD, because the slope is steeper during the most resent 30 years. If the slope continues to steepen with time, the year when only half the ice is present will move closer to the current year.

Problem 4 - From your model for the polling data, by about what years will the Democrats, Independents and Republicans no longer believe that there is any scientific evidence at all for global warming?

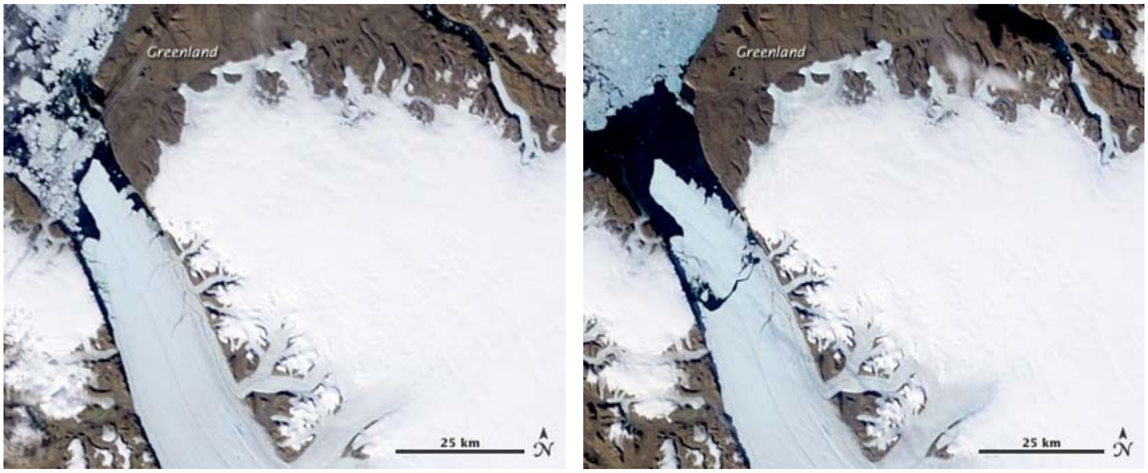
Answer: Solve each linear model in Problem 2 for X, given that y=0:

Democrats: 0 = -5.0(x - 2006) + 90 so x = **2024 AD**.

Independents: 0 = -9.0 (x-2006) + 79 so x = **2015 AD**

Republicans: 0 = - 8.3(x-2006) + 60 so x = **2013 AD**.

Terra Spies a Major Glacier Break-up



On Aug. 5, 2010, an enormous chunk of ice broke off the Petermann Glacier along the northwestern coast of Greenland. The Moderate Resolution Imaging Spectroradiometer (MODIS) on NASA's Terra satellite captured these natural-color images of Petermann Glacier 18:05 UTC on August 5, 2010 (left), and 17:15 UTC on July 28, 2010 (right). The Terra image of the Petermann Glacier on August 5 was acquired almost 10 hours after the Aqua observation that first recorded the calving event. By the time Terra took this image, the oblong iceberg had broken free of the glacier and moved a short distance down the fjord.

Problem 1 - From the scale of the two images, what is the approximate surface area of the portion of the glacier that broke-off in A) square kilometers? B) square miles if 1 km = 0.62 miles?

Problem 2 - From the width of the break line in the August image, what was the speed of drift of the glacier fragment in kilometers/hour?

Problem 3 - Assuming that the fragment is 1000 meters thick, what is the total volume of the fragment in cubic meters?

Problem 4 - If one cubic meter of ice = 917 kilograms 1 gallon of water = 3.8 kg, what is the total amount of fresh water, in gallons, that will eventually be added to the ocean after it melts?

Problem 1 - From the scale of the two images, what is the approximate surface area of the portion of the glacier that broke-off in A) square kilometers? B) square miles if 1 km = 0.62 miles?

Answer: The length of the '25 km' line is about 17 mm so the scale is 1.5 km/mm. Students may approximate the area of the glacier as a triangular portion with sides of 10 and 15 mm (15 km and 22 km) with an area of 1/2 (15)(22) = 165 km^2, and a rectangular part that has a long side on the hypotenuse of the triangle with dimensions of 2 mm x 20 mm (3 km x 30 km) and an area of **90 km^2**. The total area is then approximately 255 km2 . B) the area in square miles is 255 km^2x (0.62 mi/km)(0.62 mi/km) = **98 square miles**.

Problem 2 - From the width of the break line in the August image, what was the speed of drift of the glacier fragment in kilometers/hour?
Answer: Students may have difficulty estimating the width of the 'hairline' fracture in the image, but answers near 0.2 mm are acceptable. Students may use a photocopy machine to increase the magnification of this image, and the resulting scale of the image to get a better estimate.

The information says that this image was taken 10 hours after the separation began, so the speed is 0.2 mm x 1.5 km/mm x (1/10 hours) = **30 meters/hour**.

Problem 3 - Assuming that the fragment is 1000 meters thick, what is the total volume of the fragment in cubic meters?

Answer: 255 km^2 x 1 km = 255 km^3 since 1 km = 1000 meyters, we have 255 km^2 x $(1000 \text{ meters/km})^3$ = **255 billion $meters^3$ of ice.**

Problem 4 - If one cubic meter of ice = 917 kilograms 1 gallon of water = 3.8 kg, what is the total amount of fresh water, in gallons, that will eventually be added to the ocean after it melts?

Answer: 255 billion m^3 x (917 kg/1 m^3) x (1 gallon/3.8 kg) = **62 trillion gallons**.