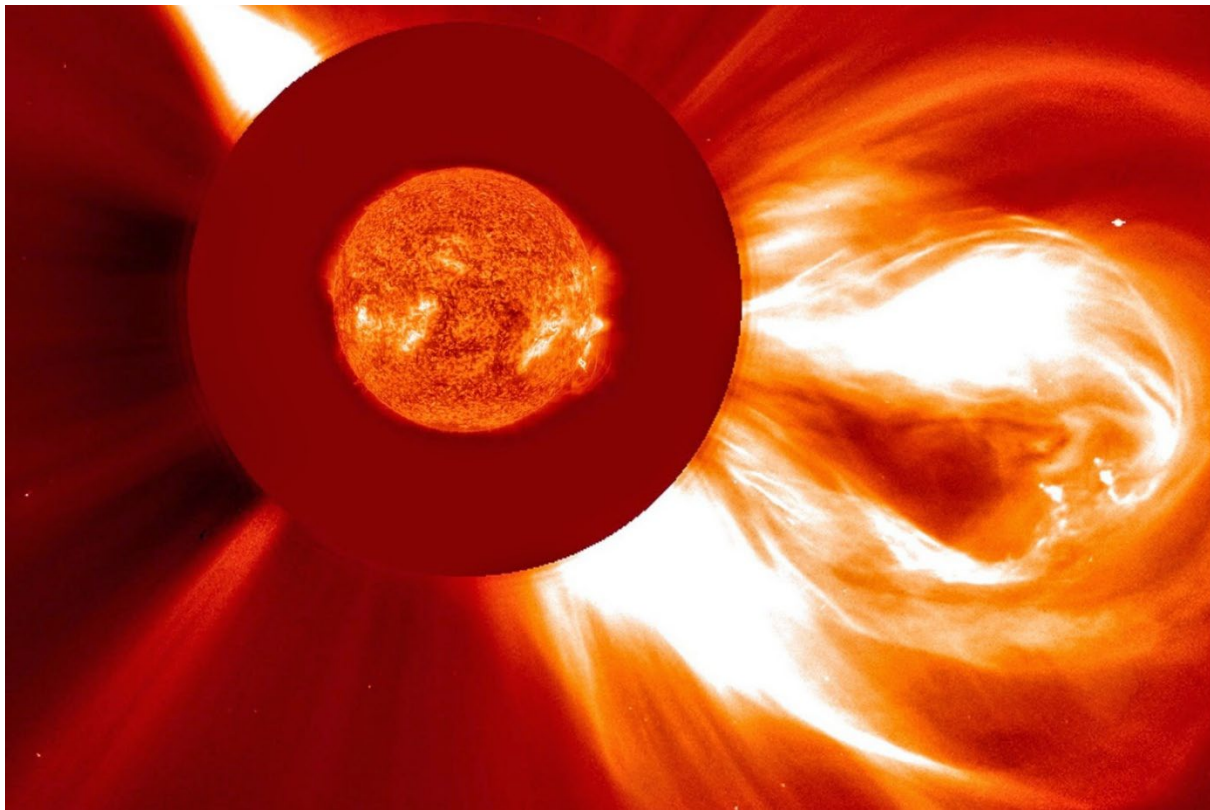




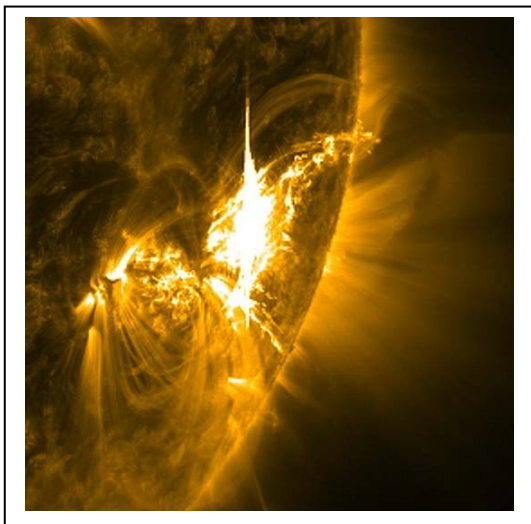
Math Problems Related to **CMEs and Flares**



Introduction

This book provides an assortment of mathematics problems that deal with coronal mass ejections and solar flares, which are the primary sources of mass and energy transfer to the Earth magnetosphere and are responsible for a variety of satellite outages, electric power grid problems, and also present a hazard to humans operating in space.

Page	Level	Math Skill	Topic
1	Beginning	Data Analysis, Percentages	How Common Are Solar Flares?
2	Beginning	Data Analysis, Percentages	Magnetic Storms; Aurora and the Kp Index.
3	Beginning	Arithmetic	A Magnetic Case for 'What Came First?'
4	Beginning	Scale, Ratios	Exploring the Solar Wind and Coronal Mass Ejections.
5	Beginning	Data Analysis	Solar Flares and the Stormy Sun.
6	Intermediate	Percentages, Formulas	IRIS Explores the Solar Transition Region.
7	Intermediate	Arithmetic, Equations	An Interplanetary Shock Wave.
8	Intermediate	Scale, Ratios, Rates	Hinode Sees a Solar Tsunami!
9	Intermediate	Venn Diagrams, Percentages	Do Fast CMEs Produce Solar Proton Events?
10	Intermediate	Graphing	The Sunspot Cycle: Endings and beginnings.
11	Intermediate	Scale, Ratios, Rates	Super-Fast Solar Flares!
12	Intermediate	Scale, Ratios, Conversions	The Mysterious Solar Micro-Flares!
13	Intermediate	Graphing	Solar Storm Energy and Pie Graphs.
14	Intermediate	Data Analysis, Percentages	How Common are X-Class Solar Flares?
15	Intermediate	Data Analysis, Percentages	Solar Flares and Sunspot Sizes.
16	Intermediate	Graphing	Magnetic Storms Plotted in Time.
17	Intermediate	Graphing	Magnetic Storms and the Kp Index.
18	Intermediate	Venn Diagrams	Solar Flares, CME's and Aurora.
19	Intermediate	Arithmetic, Equations	The November 8; 2004 Solar Storm.
20	Advanced	Calculus	Solar Flare Reconstruction.
21	Advanced	Equations, Algebra II	How Systems of Equations are Used in Space Science.
22	Advanced	Functions, Equations	CME Kinetic Energy and Mass.
23	Advanced	Scale, Rates, Conversions	A Star Sheds a Comet Tail!
24	Advanced	Measurement, Ratios, Conversions	STEREO Watches the Sun Kick Up a Storm.



This image was taken by NASA's Solar Dynamics Observatory on July 6, 2012 and shows a brilliant X-ray solar flare erupting from the sun.

Solar flares are not all the same. Some produce less energy than others, and so astronomers classify them by their X-ray energy using four different letters: B, C, M and X. C-class flares produce 10 times more X-ray energy than B-class flares. M-class flares produce 10 times more energy than C-class flares, and X-class flares produce 10 times more energy than M-class flares. One B-class flare can produce more energy than 240,000 million tons of TNT!

The table below lists all of the M and X-class flares detected between January 1, 2013 and August 15, 2013 at a time when solar activity was near its maximum. This period of time spans the first 227 days of 2013. Also during this time, there were about 690 C-class flares and 440 B-class flares. All of these flares were seen on the side of the sun facing Earth, which represents $\frac{1}{2}$ of the total surface area of the sun.

Day	Flare	Day	Flare	Day	Flare
1-5	M	5-2	M	5-20	M
1-11	M, M	5-3	M, M	5-22	M
1-13	M, M	5-5	M	5-31	M
2-17	M	5-10	M	6-5	M
3-5	M	5-12	M, M	6-7	M
3-15	M	5-13	X, M, X	6-21	M
3-21	M	5-14	X	6-23	M
4-5	M	5-15	X	7-3	M
4-11	M	5-16	M	8-12	M
4-22	M	5-17	M		

Problem 1 - What were the total number of M and X-class flares during this period of time?

Problem 2 – What were the total number of B, C, M and X-class flares detected during this period?

Problem 3 – What percentage of all flares were B, C, M and X?

Problem 4 – What was the average number of B and C-class flares seen each day?

Problem 5 – An astronaut wants to do a spacewalk on a particular day during this period. What are the odds that she will see an M or X-class flare?

The flare data was obtained from

http://www.swpc.noaa.gov/ftpmenu/warehouse/2013/2013_events.html

Problem 1 - What were the total number of M and X-class flares during this period of time?

Answer: By counting Ms in the table, there were **31 M-class and 4 X-class flares**.

Problem 2 – What were the total number of B, C, M and X-class flares detected during this period?

Answer: $690 + 440 + 31 + 4 = \mathbf{1165 \text{ flares}}$.

Problem 3 – What percentage of all flares were B, C, M and X?

Answer: $B = 100\% \times (440/1165) = 38\%$

$C = 100\% \times (690/1165) = 59\%$

$M = 100\% \times (31/1165) = 3\%$

$X = 100\% \times (4/1165) = 0.3 \%$

Problem 4 – What was the average number of B and C-class flares seen each day?

Answer: B: $440 \text{ flares} / 227 \text{ days} = \text{about } 2 \text{ flares}$

C: $690 \text{ flares} / 227 \text{ days} = \text{about } 3 \text{ flares}$

Problem 5 – An astronaut wants to do a spacewalk on a particular day during this period. What is the probability that she will see each an M or X-class flare?

Answer: There are 227 days in the sample and 35 M or X-class flares were seen, so the probability is $35/227 = \mathbf{0.15}$ which is also stated as 15%. The low probability means that it is not likely that on a random day the astronaut will see anything. In order to have a 50/50 chance, she would have to observe for at least 4 days ($4 \times 0.15 = 0.60$ which is greater than 0.50 or 50%).



When a solar coronal mass ejection collides with earth's magnetic field, it can produce intense aurora that can be seen from the ground. Geophysicists who study magnetic disturbances have created a 9-point scale that indicates how intense the storm is. Kp=9 is the most intense, and aurora can be seen near Earth's equator for many of these. Kp=7 and 8 are strong storms that can still cause aurora and upset electrical power systems. Kp=5 and 6 are mild storms that may or may not produce intense aurora.

The table below gives the dates when magnetic storms were detected that exceeded Kp=4. The multiple entries each day indicate consecutive 3-hour measurements of the storm intensity. The data are given for the 227 days from January 1, 2013 to August 15, 2013. Highlighted boxes in yellow indicate days when Halo CMEs were detected

Date	Kp measurements	Date	Kp Measurements
3-1	5	6-2	5
3-17	6, 5, 5, 6, 6, 5	6-7	5, 6, 5
3-29	5	6-29	6, 7, 5, 6
3-30	5, 5	7-10	5, 5
4-26	5	7-11	5
5-18	5, 5	7-15	5, 5
5-24	5	8-4	5
5-25	5, 5, 5	8-5	5
6-1	6, 6, 6, 5, 6		

Problem 1 – What percentage of the days during this time period had magnetic storm events?

Problem 2 – What percentage of the magnetic storm days were strong storms with Kp=6 or higher?

Problem 3 – During this same 227-day period of time, there were 12 Halo CMEs detected. What is the probability that these Halo events produce a magnetic storm with Kp=6 or higher?

Problem 4 – The Kp index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds Kp=5 during its entire duration?

Problem 1 – What percentage of the days during this time period had magnetic storm events?

Answer: 17 days out of 227 so $100\% \times (17/227) = 7\%$

Problem 2 – What percentage of the magnetic storm days were strong storms with $K_p=6$ or higher?

Answer: 4 out of 17 or $100\% \times (4/17) = 24\%$.

Problem 3 – During this same 227-day period of time, there were 12 Halo CMEs detected. What is the probability that these Halo events produce a magnetic storm with $K_p=6$ or higher?

Answer: Only 2 of the 12 Halo CMEs produced $K_p=6$ or stronger magnetic storms, so $P = 100\% \times (2/12) = 17\%$. The odds were 1 in 6.

Problem 4 – The K_p index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds $K_p=5$ during its entire duration?

Answer: There were 4 storms that exceeded $K_p=5$. These occurred on the dates: 3-17, 6-1, 6-7 and 6-29. The total hours for each storm were

$6 \times 3 = 18$ hours (for 3-17),

$5 \times 3 = 15$ hours (for 6-1);

$3 \times 3 = 9$ (for 6-7) and

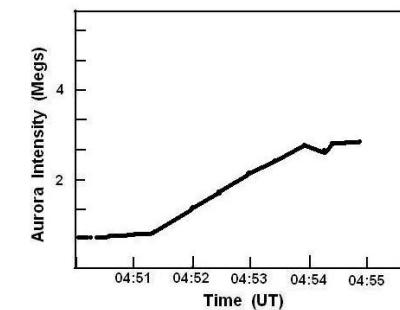
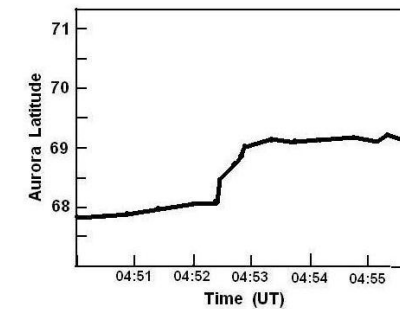
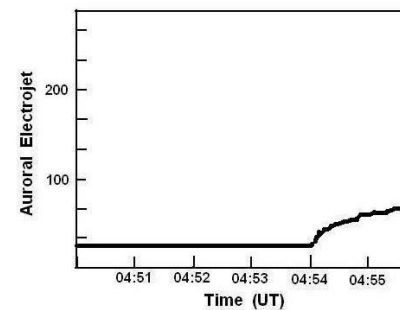
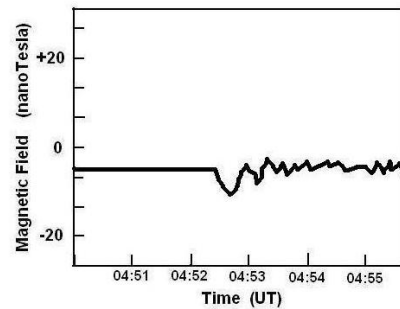
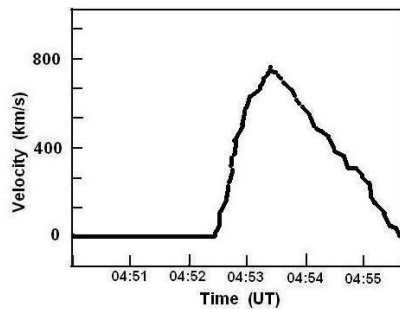
$4 \times 3 = 12$ (for 6-29).

The average for these hours is $(18+15+9+12)/4 = 14$ hours.

So, once a strong magnetic storm begins, it takes about one-half a day for it to reach its maximum intensity and then fade away. If this happens in the summertime during the day, you will not see an aurora borealis because of the daylight brightness.

THEMIS: A Magnetic Case of 'What came first?'

3



The NASA, THEMIS satellite constellation consists of five satellites, P1, P2, P3, P4 and P5, launched on February 17, 2007. The scientific goal was to determine the sequence of events connecting disturbances in Earth's distant magnetic field (a process called magnetic reconnection), with the start of magnetic storms and aurora near Earth.

The science team assembled the data shown in the graphs to the left. The event that triggered this sequence was a 'magnetic reconnection' in Earth's magnetic field that took place at about 4:50:03 at a location about 160,000 km from Earth. (Note these plots have been greatly simplified for clarity! See the original article in the journal *Science*, August 15, 2008, vol. 321, pp.931: Figure 2 and 3)

Problem 1 - At about what times do each of the plots note a significant change in the quantity being measured?

Problem 2 - What is the time sequence of events based on your answers to Problem 1?

Problem 3 - How long was the elapsed time between the increase in particle velocity at the P3 satellite, and the enhancement of the auroral electrojet?

Problem 4 - The P3 satellite was located 74,000 km from Earth. If the auroral electrojet is a stream of charged particles that flows in Earth's upper atmosphere (300 km from the surface) what was the speed of the event in km/sec between the P3 location and when the electrojet started to form?

Problem 5 - What was the time difference between the magnetic reconnection event at 04:50:03 and the start of the auroral change in latitude?

Problem 6 - How fast did the particles travel from the reconnection region to the Earth when the auroral 'substorm' began?

Problem 7 - Before the THEMIS observations, one theory said that the disturbances near the P3 satellite would come before the reconnection occurred. What does the data say about this theory?

Answer Key

Problem 1 - Answer: The times below were reported in the scientific journals based on the actual data. Student answers may vary slightly depending on their ability to interpolate the values on the horizontal axis. For best results, use a millimeter ruler to determine the scale of the axis in seconds/mm.

Plot 1: Particle velocity increase detected at P3: **4:52:27**

Plot 2: Magnetic field change at P3: **4:52:27**

Plot 3: Auroral electrojet amplification in the ionosphere: **4:54:00**

Plot 4: Substorm expansion; Latitude increase northwards: **4:52:21**

Plot 5: Auroral intensity change; arrival of particles at Earth that were generated by magnetic reconnection. **4:51:39**

Problem 2 - Answer:

4:51:39: Auroral intensity change; arrival of particles at Earth generated by the magnetic reconnection.

4:52:21: Substorm expansion: Latitude increase northwards

4:52:27: Particle velocity increase detected at P3 due to earthward flow of particles.

4:52:27: Magnetic field change at P3

4:54:00: Auroral electrojet amplification in the ionosphere.

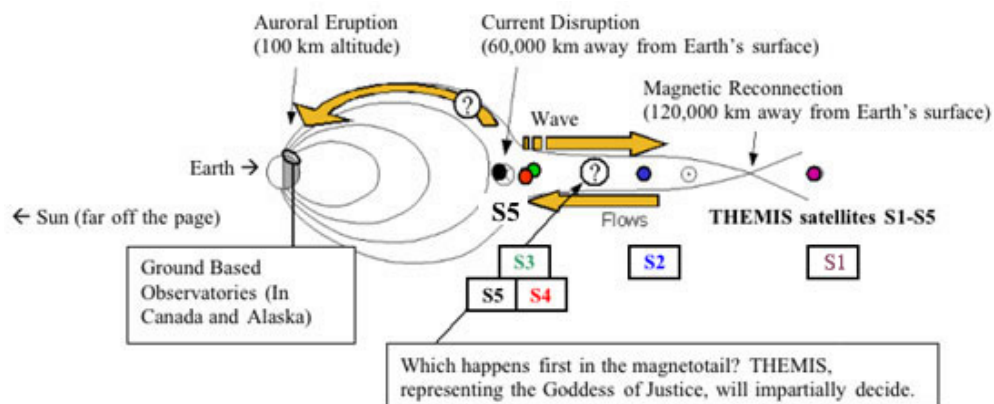
Problem 3 - Answer: $4:54:00 - 4:52:27 = 1\text{ minute and }33\text{ seconds} = \mathbf{93\text{ seconds.}}$

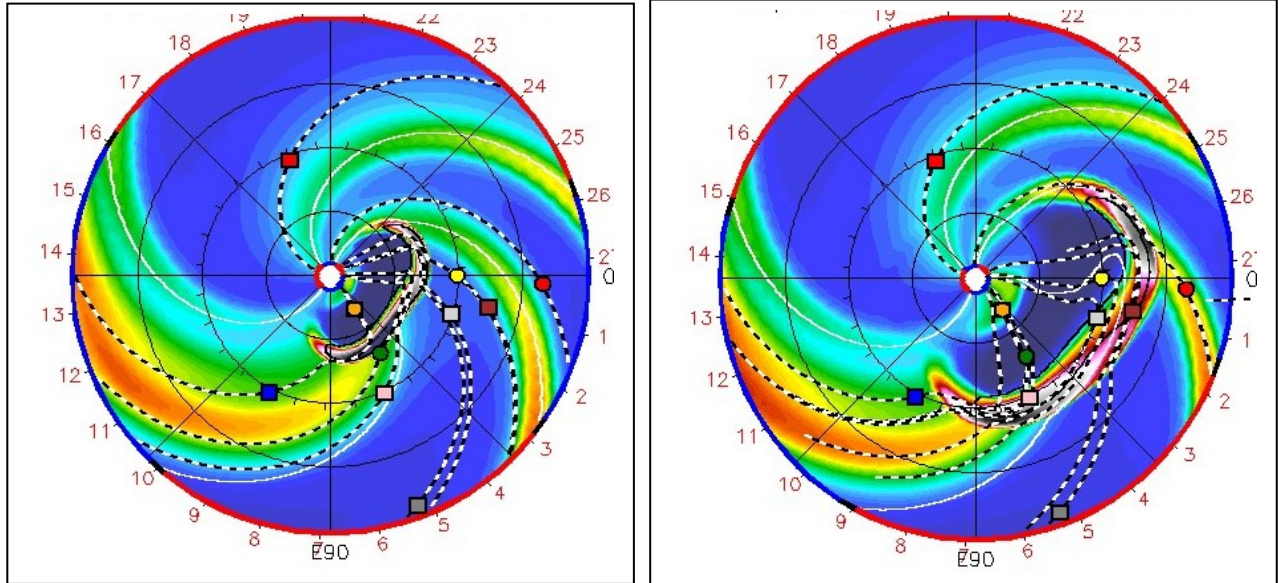
Problem 4 - Answer: $74,000\text{ km} / 93\text{ seconds} = \mathbf{796\text{ km/sec.}}$

Problem 5 - Answer: $4:52:21 - 4:50:03 = 2\text{m }18\text{s} = \mathbf{138\text{ seconds.}}$

Problem 6 -Answer: The distance traveled was 160,000 kilometers in 138 seconds, for an average speed of $160,000\text{ km} / 138\text{s} = \mathbf{1,160\text{ km/sec}}$ if no allowance is made for the radius of the earth (6378km) and the height of the ionosphere (300 km). If these allowances are made, then $(160,000 - 6378 - 300)\text{ km} / 138\text{s} = \mathbf{1,110\text{ km/sec.}}$

Problem 7 - Answer: Because the reconnection event happened at 04:50:03 ,this was about 2:27 before the disturbances at P3 in the particles speeds were recorded, so the theory in which P3 happened first is not consistent with the new data.





This pair of computed images shows the spiral pattern of the solar wind inside the orbit of Mars. It was created by the NASA Goddard, Coordinated Community Modeling Center to show the condition of the solar wind just after the March 6, 2012 solar storm. The images correspond to March 8 (00:00) and March 9 (00:00).

The planets are indicated at their correct positions by circles (yellow=Earth; red=Mars; orange=Mercury and green=Venus). Also shown are the positions of the STEREO A and B spacecraft (red and blue squares) and the Spitzer Space Telescope (pink square). The black concentric circles are drawn at intervals of 75 million kilometers.

The pinwheel pattern is formed from the high-speed gas streams leaving the sun through coronal holes. The crescent shaped cloud is the coronal mass ejection (CME) from the sun, which caused brilliant aurora on Earth.

Problem 1 - About what was the speed, in km/h, of the CME when it reached Earth?

Problem 2 - The dark cavity behind the CME represents a very low density region of space. How do you think this was created? Where did the gas go that once filled the cavity?

Problem 3 - At the orbit of Earth, about how fast do the high-speed gas streams sweep past the Earth in kilometers/hour?

Problem 4 - Assuming that the CME does not slow down, on what date will it arrive at Neptune, which is located 4.5 billion kilometers from the sun?

Problem 1 - About what was the speed, in km/h, of the CME when it reached Earth?

Answer: Students can estimate from the scaled figures that between the two days the crescent-shaped CME moves about one division or 75 million kilometers. This took 1 day or 24 hours, so the speed was 75 million km/24 h = **about 3 million km/h**.

Problem 2 - The dark cavity behind the CME represents a very low density region of space. How do you think this was created? Where did the gas go that once filled the cavity?

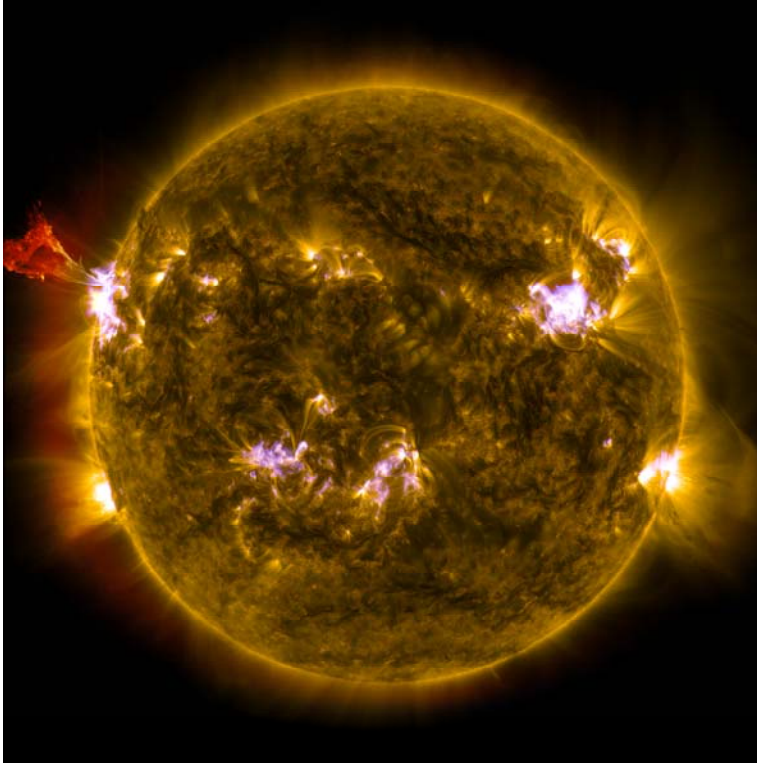
Answer: **The CME traveled through the gas and swept it up like a snow-plow in front of the CME.**

Problem 3 - At the orbit of Earth, about how fast do the high-speed gas streams sweep past the Earth in kilometers/hour?

Answer: It will be a challenge to find a feature in the spiral gas streams that can be tracked between the two days, but a reasonable answer would be that the pattern rotated by about 25 million km in 1 day, so the speed is about **1 million km/h**.

Problem 4 - Assuming that the CME does not slow down, on what date will it arrive at Neptune, which is located 4.5 billion kilometers from the sun?

Answer: The CME travels about 3 million km/h so it traveled about 4500 million km during the transit time, so $T = 4500/3 = 1500$ hours which equals 62.5 days. When this is added to the launch date of March 6 in the first frame, we get a date of about **May 7**.



This image was obtained by the Solar Dynamics Observatory on May 3, 2013. It shows a solar flare on the edge of the sun.

There are many other active areas on the solar surface near sunspots which have a white color in this image.

The year 2013 is called the Sunspot Maximum because in this year, the sun produces more sunspots and solar storms than any other time in its 11-year cycle.

The year 2009 was called Sunspot Minimum with the fewest number of sunspots counted during each month. For each month in 2009, the average number of sunspots counted was 2, 1, 1, 1, 3, 3, 4, 0, 4, 5, 4, 11. Although sunspots have only been counted for the first 4 months of 2013, the average monthly numbers are 63, 39, 58, 72.

Problem 1 - What is the average number of sunspots counted in 2009 rounded to the nearest integer?

Problem 2 – What is the average number of sunspots counted in 2013 rounded to the nearest integer?

Problem 3 – Suppose that after adding the sunspot counts for May 2013 that the new average became 60. What was the number of sunspots counted in May rounded to the nearest integer?

Problem 4 – In July 2013, suppose that the number of sunspots counted was 5 more than the average for May. The June counts were 6 more than the counts for July, and the May counts were 5 more than the average for January, February, March and April. What were the monthly sunspot counts for May, June and July, and what was the average sunspot number for January-July?

Sun Emits Mid-Level Flare, May 3, 2013

http://www.nasa.gov/mission_pages/sunearth/news/News050313-flare.html

Additional Sunspot Data: <http://www.ips.gov.au/Solar/1/6>

Problem 1 - What is the average number of sunspots counted in 2009 rounded to the nearest integer?

Answer: $(2+1+1+1+3+3+4+0+4+5+4+11)/12 = 39/12 = 3.25$ or **3**

Problem 2 – What is the average number of sunspots counted in 2013 rounded to the nearest integer?

Answer= $(63+39+58+72)/4 = 58$

Problem 3 – Suppose that after adding the sunspot counts for May 2013 that the new average became 60. What was the number of sunspots counted in May rounded to the nearest integer?

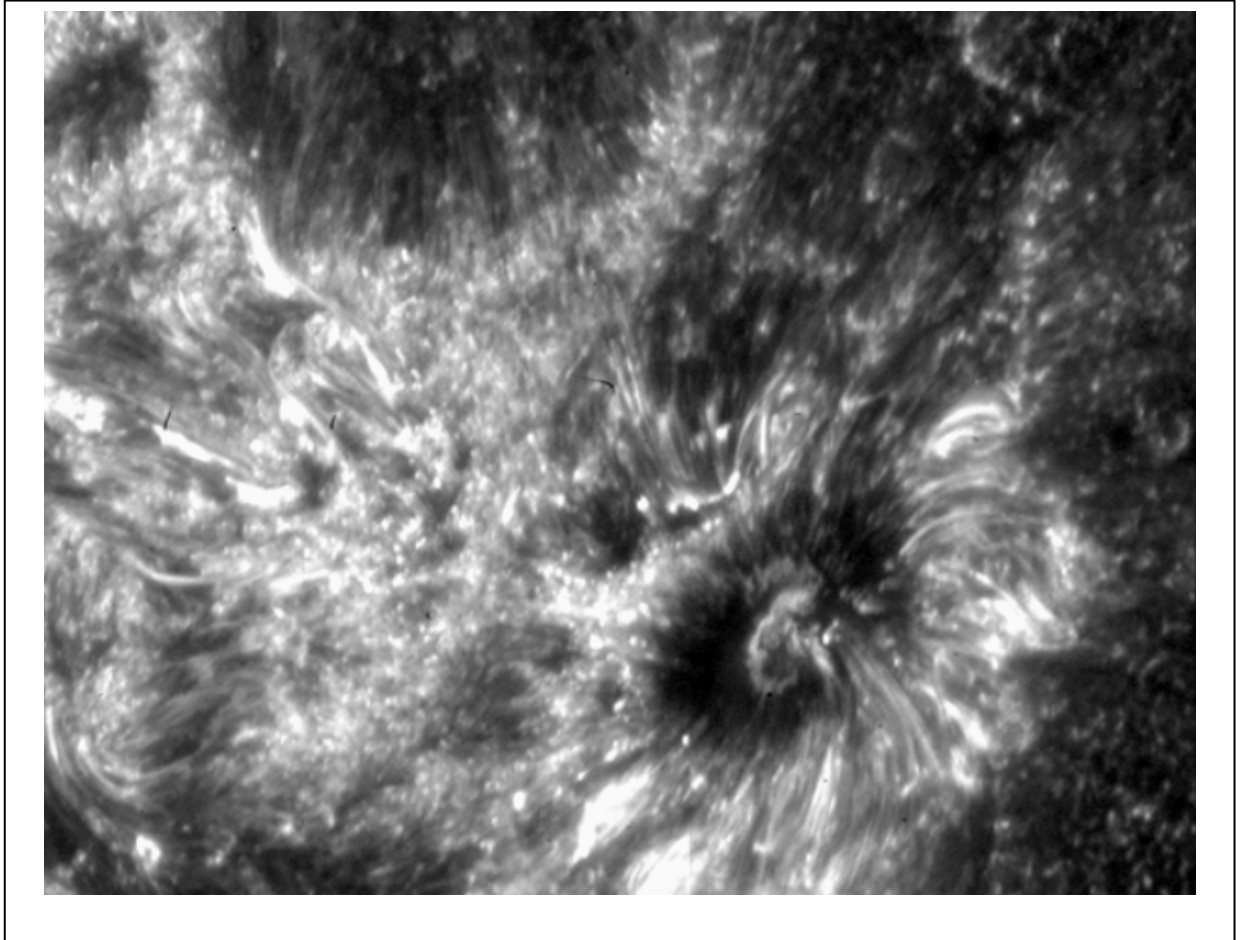
Answer: $(63+39+58+72 + X)/5 = 60$ then $(232+X) = 5 \times 60$ and $X = 68$.

Problem 4 – In July 2013, suppose that the number of sunspots counted was 5 more than the average for May. The June counts were 6 more than the counts for July, and the May counts were 5 more than the average for January, February, March and April. What were the monthly sunspot counts for May, June and July, and what was the average sunspot number for January-July?

Answer: In Problem 2, the average for the first four months was 58. The May counts were 5 more than this average or $58+5 = 63$. The July counts were 5 more than the average for May or $63+5 = 68$. The June counts were 6 more than the July counts or $68+6 = 74$. So we have

January.....63
 February.....39
 March.....58
 April.....72
May.....63
June.....74
July.....68

The average for the first seven months is then $(63+39+58+72+63+74+68)/7 = 62.4$ or rounded to the nearest integer we get **62 sunspots**.



This spectacular image of the solar atmosphere was obtained by NASA's IRIS satellite. The smallest details are 240 km (150 miles) across, and the image shows many small bright 'dots' where energy is being released and transported into the solar corona. The bright spots come and go in less than an hour, and the solar surface is peppered with millions of these regions. This image is 72,000 km across.

Problem 1 – The radius of the sun is 690,000 km. What is the surface area of the spherical sun in km^2 ?

Problem 2 – What percentage of the sun's surface area does the square IRIS image above represent?

Problem 3 – Suppose that each bright point is a cylinder 10,000 km tall with a diameter of 200 km in diameter. What is the volume of the cylinder in kilometers?

Problem 4 – The magnetic energy stored in this volume is 4.0×10^{10} Joules/ km^3 . If a single 10 megaton hydrogen bomb produces 4×10^{15} Joules, how many megatons of energy is produced by this region?

Problem 1 – The radius of the sun is 690,000 km. What is the surface area of the spherical sun in km^2 ?

Answer: $A = 4 \pi R^2$, so
 $A = 4 \times 3.141 \times (690,000)^2$
 $= 5.6 \times 10^{12} \text{ km}^2$

Problem 2 – What percentage of the sun's surface area does the square IRIS image above represent?

Answer: Image area = $(72,000)^2 = 5.2 \times 10^9 \text{ km}^2$.
 So $P = 100\% \times 5.2 \times 10^9 / 5.6 \times 10^{12} = 0.09\%$

Problem 3 – Suppose that each bright point is a cylinder 10,000 km tall with a diameter of 200 km in diameter. What is the volume of the cylinder in cubic meters?

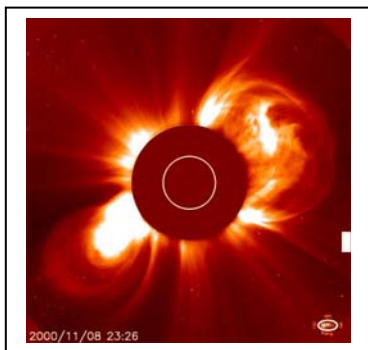
Answer: $V = \pi R^2 h$. $V = 3.141 \times (100 \text{ km})^2 \times 10,000 \text{ km} = 3.1 \times 10^8 \text{ km}^3$.

Problem 4 – The magnetic energy stored in this volume is $4.0 \times 10^{10} \text{ Joules/km}^3$. If a single 10 megaton hydrogen bomb produces $4 \times 10^{15} \text{ Joules}$, how many megatons of energy is produced by this region?

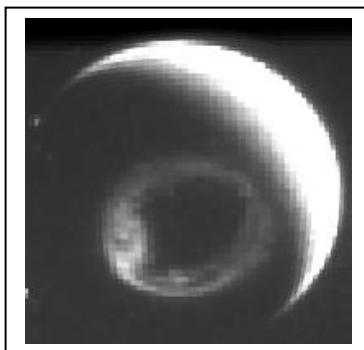
Answer: $E = 4.0 \times 10^{10} \text{ J/km}^3 \times 3.1 \times 10^8 \text{ km}^3 = 1.2 \times 10^{19} \text{ Joules}$.

$M = 1.2 \times 10^{19} \text{ Joules} / 4.0 \times 10^{15} = 3000 \text{ megatons}$.

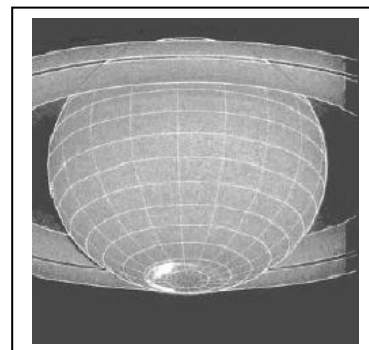
This equals 300, 10 megaton hydrogen bombs!



Sun - CME



Earth - Aurora



Saturn - Aurora

On November 8, 2000 the sun ejected a blast of plasma called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite. On December 8, the Hubble Space Telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

- 1 – How many days did the disturbance take to reach Earth and Saturn?
- 2 – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour?
- 3 – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour?
- 4 – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn?
- 5 – How long would the disturbance have taken to reach Jupiter as it passed Earth's orbit?
- 6 – On what date would you have expected to see aurora on Jupiter?

On November 8, 2000 the sun ejected a blast of plasma called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite. On December 8, the Hubble Space telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

1 – How many days did the disturbance take to reach Earth and Saturn?

Answer: Earth = 4 days; Saturn = 30 days.

2 – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour? **Answer:** Sun to Earth = 150 million km. Time = 4 days x 24 hrs = 96 hrs so Speed = 150 million km/96hr = 1.5 million km/hr.

3 – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour? **Answer:** Distance = 1,430 – 150 = 1,280 million km. Time = 30 days x 24h = 720 hrs so Speed = 1,280 million km/720hrs = 1.8 million km/hr.

4 – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn? **Answer:** The CME accelerated from 1.5 million km/hr to 1.8 million km/hr.

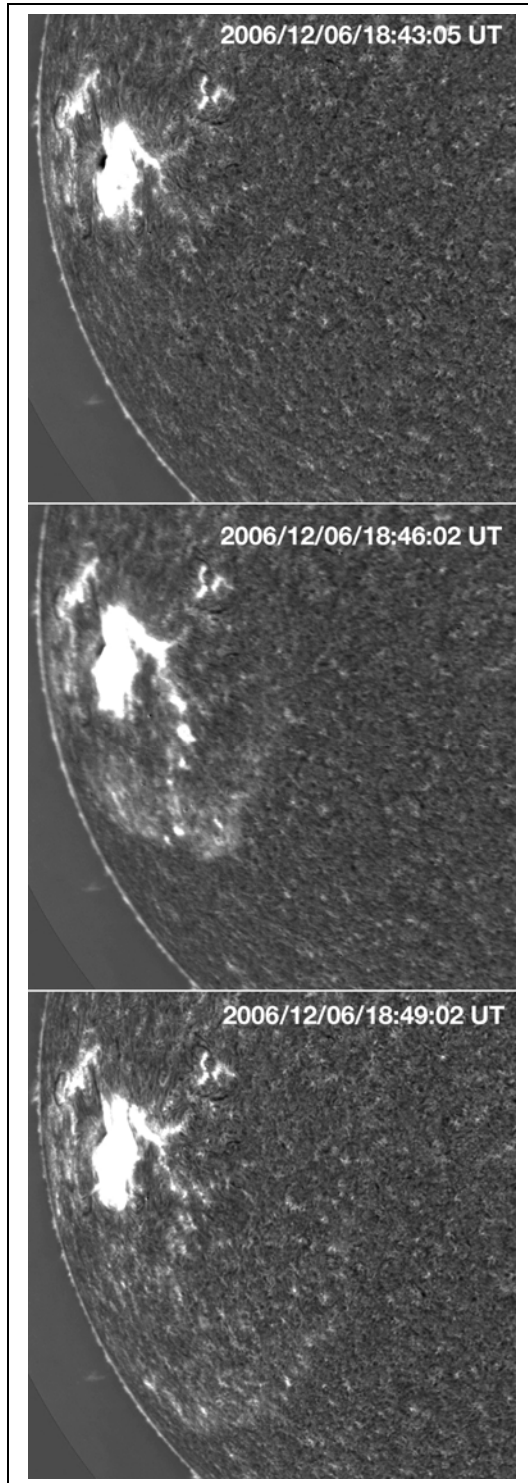
5 – How long would the disturbance have taken to reach Jupiter as it passed Earth's orbit? **Answer:** Jupiter is located 778 million km from the Sun or (778 – 150 =) 628 million km from Earth. Because the CME is accelerating, it is important that students realize that it is more accurate to use the average speed of the CME between Earth and Saturn which is $(1.8 + 1.5)/2 = 1.7$ million km/hr. The travel time to Jupiter is then $628/1.7 = 369$ hours.

6 – On what date would you have expected to see aurora on Jupiter? **Answer:** Add 369 hours (~ 15 days) to the date of arrival at Earth to get November 23. According to radio observations of Jupiter, the actual date of the aurora was November 20. Note: If we had used the Sun-Earth average speed of 1.5 million km/hr to get a travel time of $628/1.5 = 418$ hours, the arrival date would have been November 29, which is 9 days later than the actual storm. This points out that the CME was accelerating after passing Earth, and its speed was between 1.5 and 1.8 million km/hr.

For more details about this interesting research, read the article by Renee Prange et al. "An Interplanetary Shock Traced by Planetary Auroral Storms from the Sun to Saturn" published in the journal Nature on November 4, 2004, vol. 432, p. 78. Also visit the Physics Web online article "Saturn gets a shock" at <http://www.physicsweb.org/articles/news/8/11/2/1>

Measuring the Speed of a Solar Tsunami!

8



Moments after a major class X-6 solar flare erupted at 18:43:59 Universal Time on December 6, 2006, the National Solar Observatory's new Optical Solar Patrol Camera captured a movie of a shock wave 'tsunami' emerging from Sunspot 930 and traveling across the solar surface.

The three images to the left show the progress of this Morton Wave. The moving solar gasses can easily be seen. You can watch the entire movie and see it more clearly (<http://image.gsfc.nasa.gov/poetry/weekly/MortonWave.mpeg>).

Note: because the event is seen near the solar limb, there is quite a bit of fore-shortening so the motion will appear slower than what the images suggest.

Problem 1: From the portion of the sun's edge shown in the images, complete the solar 'circle'. What is the radius of the sun's disk in millimeters?

Problem 2: Given that the physical radius of the sun is 696,000 kilometers, what is the scale of each image in kilometers/millimeter?

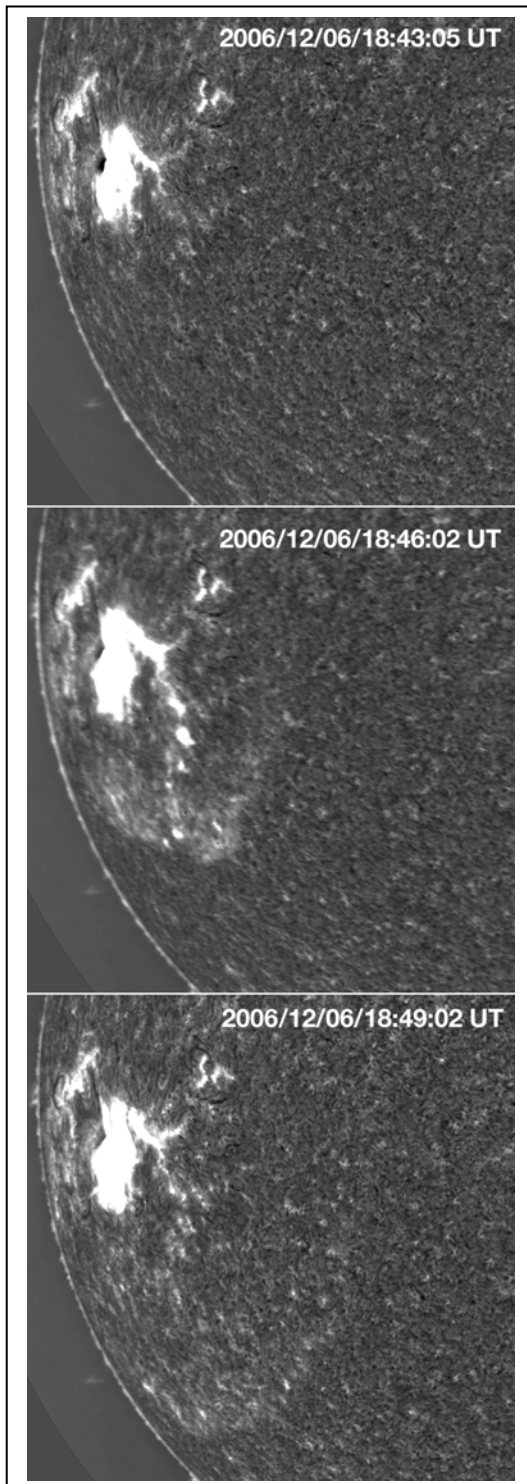
Problem 3: Select a spot near the center of the sunspot (large white spot in the image), and a location on the leading edge of the shock wave. What is the distance in kilometers from the center of the sunspot, to the leading edge of the shock wave in each image?

Problem 4: The images were taken at 18:43:05, 18:46:02 and 18:49:02 Universal Time. How much time has elapsed between these images?

Problem 5: From your answers to Problem 3 and 4, what was the speed of the Morton Wave in kilometers per hour between the three images? B) did the wave accelerate or decelerate as it expanded?

Problem 6: The speed of the Space Shuttle is 44,000 kilometers/hour. The speed of a passenger jet is 900 kilometers/hour. Would the Morton Wave have overtaken the passenger jet? The Space Shuttle?

Answer Key:



Problem 1: From the portion of the sun's edge shown in the images, complete the solar 'circle'. What is the radius of the sun's disk in millimeters?

Answer: About 158 millimeters using a regular dessert plate as a guide.

Problem 2: Given that the physical radius of the sun is 696,000 kilometers, what is the scale of each image in kilometers/millimeter?

Answer: $696,000/158 = 4,405$ kilometers/millimeter

Problem 3: What is the distance in kilometers from the center of the sunspot, to the leading edge of the shock wave in each image?

Answer:

$$\text{Image 2} = 27 \text{ mm} = 27 \times 4405 = 119,000 \text{ km}$$

$$\text{Image 3} = 38 \text{ mm} = 167,000 \text{ km}$$

Problem 4: The images were taken at 18:43:05, 18:46:02 and 18:49:02 Universal Time. How much time has elapsed between these images?

Answer: Image 1 - Image 2 = 2 minutes 57 seconds

Image 2 - Image 3 = 3 minutes

Problem 5: From your answers to Problem 3 and 4, A) what was the speed of the Morton Wave in kilometers per hour between the three images?

Answer:

$$V_{12} = 119,000 \text{ km} / 2.9 \text{ min} \times (60 \text{ min} / 1 \text{ hr}) \\ = 2.5 \text{ million kilometers/hour}$$

$$V_{23} = 167,000 / 3.0 \text{ min} \times (60 \text{ min} / 1 \text{ hr}) \\ = 3.3 \text{ million kilometers/hour}$$

B) Did the speed of the wave accelerate or decelerate?

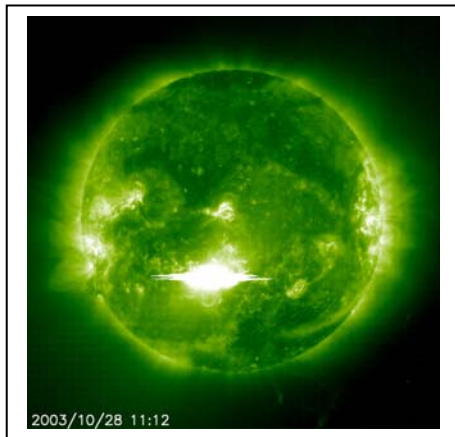
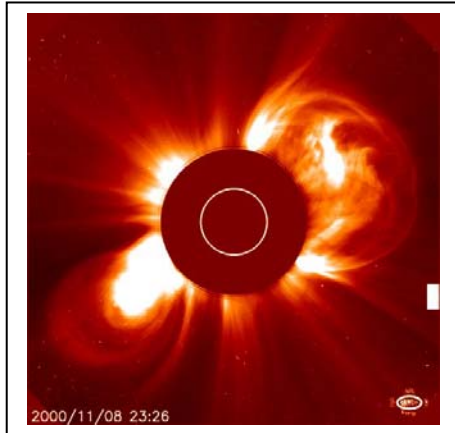
Answer: Because $V_{23} > V_{12}$ the wave accelerated.

Problem 6: The speed of the Space Shuttle is 44,000 kilometers/hour. The speed of a passenger jet is 900 kilometers/hour. Would the Morton Wave have overtaken the passenger jet? The Space Shuttle?

Answer: It would easily have overtaken the Space Shuttle! Because of fore-shortening, the actual speed of the wave was even higher than the estimates from the images, so the speed could have been well over 4 million km/hr.

Do Fast CMEs produce intense SPEs?

9



The sun produces two basic kinds of storms; coronal mass ejections (SOHO satellite: top left) and solar flares (SOHO satellite: bottom left). These are spectacular events in which billions of tons of matter are launched into space (CMEs) and vast amounts of electromagnetic energy are emitted (Flares). A third type of 'space weather storm' can also occur.

Solar Proton Events (SPEs) are invisible, but intense, showers of high-energy particles near Earth that can invade satellite electronics and cause serious problems, even malfunctions and failures. Some of the most powerful solar flares can emit these particles, which streak to Earth within an hour of the flare event. Other SPE events, however, do not seem to arrive at Earth until several days latter.

Here is a complete list of Solar Proton Events between 1976-2005: <http://umbra.nascom.nasa.gov/SEP/>

Here is a complete list of coronal mass ejections 1996 - 2006: http://cdaw.gsfc.nasa.gov/CME_list/

Between January 1, 1996 and June 30, 2006 there were 11,031 CMEs reported by the SOHO satellite. Of these, 1186 were halo events. Only half of the halo events are actually directed towards Earth. The other half are produced on the far side of the sun and directed away from Earth. During this same period of time, 90 SPE events were recorded by GOES satellite sensors orbiting Earth. On the next page, is a list of all the SPE events and Halo CMEs that corresponded to the SPE events. There were 65 SPEs that coincided with Halo CMEs. Also included is the calculated speed of the CME event.

From the information above, and the accompanying table, draw a Venn Diagram to represent the data, then answer the questions below.

- Question 1: A) What percentage of CMEs detected by the SOHO satellite were identified as Halo Events?
B) What are the odds of seeing a halo Event?
C) How many of these Halo events are directed towards Earth?

- Question 2: A) What fraction of SPEs were identified as coinciding with Halo Events?
B) What are the odds that an SPE occurred with a Halo CME?
C) What fraction of all halo events directed towards earth coincided with SPEs?

- Question 3: A) What percentage of SPEs coinciding with Halo CMEs are more intense than 900 PFUs?
B) What are the odds that, if you detect a 'Halo- SPE', it will be more intense than 900 PFUs?

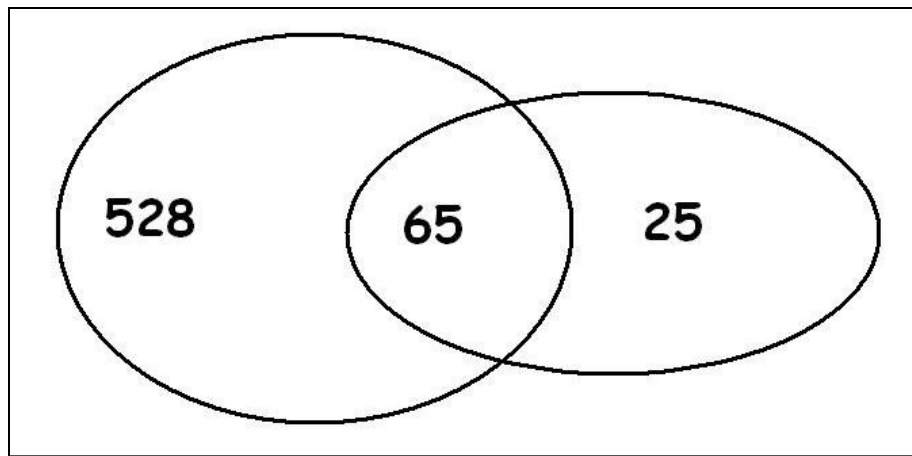
- Question 4: A) What percentage of Halo-SPEs have speeds greater than 1000 km/sec?
B) What are the odds that a Halo-SPE in this sample has a speed of > 1000 km/sec?

- Question 5: From what you have calculated as your answers above, what might you conclude about Solar Proton Events and CMEs? How would you use this information as a satellite owner and operator?

Data Tables showing dates and properties of Halo CMEs and Solar Proton Events.

Date	CME Speed (km/s)	SPE (pfu)	Date	CME Speed (km/s)	SPE (pfu)
November 4, 1997	785	72	January 8, 2002	1794	91
November 6, 1997	1556	490	January 14, 2002	1492	15
April 20, 1998	1863	1700	February 20, 2002	952	13
May 2, 1998	938	150	March 15, 2002	957	13
May 6, 1998	1099	210	March 18, 2002	989	19
May 3, 1999	1584	14	March 22, 2002	1750	16
June 1, 1999	1772	48	April 17, 2002	1240	24
June 4, 1999	2230	64	April 21, 2002	2393	2520
February 18, 2000	890	13	May 22, 2002	1557	820
April 4, 2000	1188	55	July 15, 2002	1151	234
June 6, 2000	1119	84	August 14, 2002	1309	24
June 10, 2000	1108	46	August 22, 2002	998	36
July 14, 2000	1674	24000	August 24, 2002	1913	317
July 22, 2000	1230	17	September 5, 2002	1748	208
September 12, 2000	1550	320	November 9, 2002	1838	404
October 16, 2000	1336	15	May 28, 2003	1366	121
October 25, 2000	770	15	May 31, 2003	1835	27
November 8, 2000	1738	14800	June 17, 2003	1813	24
November 24, 2000	1289	940	October 26, 2003	1537	466
January 28, 2001	916	49	November 4, 2003	2657	353
March 29, 2001	942	35	November 21, 2003	494	13
April 2, 2001	2505	1100	April 11, 2004	1645	35
April 10, 2001	2411	355	July 25, 2004	1333	2086
April 15, 2001	1199	951	September 12, 2004	1328	273
April 18, 2001	2465	321	November 7, 2004	1759	495
April 26, 2001	1006	57	January 15, 2005	2861	5040
August 9, 2001	479	17	July 13, 2005	1423	134
September 15, 2001	478	11	July 27, 2005	1787	41
September 24, 2001	2402	12900	August 22, 2005	2378	330
October 1, 2001	1405	2360			
October 19, 2001	901	11			
October 22, 2001	618	24			
November 4, 2001	1810	31700			
November 17, 2001	1379	34			
November 22, 2001	1437	18900			
December 26, 2001	1446	779			

Note: Solar Proton Event strengths are measured in the number of particles that pass through a square centimeter every second, and is given in units called Particle Flux Units or PFUs.



Question 1: A) What percentage of CMEs detected by the SOHO satellite were identified as Halo Events?
 $1186/11031 = 11\%$

B) What are the odds of seeing a halo Event?
 $1 / 0.11 = 1 \text{ chance in } 9$

C) How many of these Halo events are directed towards Earth?
 From the text, only half are directed to Earth so $1186/2 = 593$ Halos.

Question 2: A) What fraction of SPEs were identified as coinciding with Halo Events?
 $65 \text{ table entries} / 90 \text{ SPEs} = 72\%$

B) What are the odds that an SPE occurred with a Halo CME?
 $1 / 0.72 = 1 \text{ chance in } 1.38 \text{ or about } 2 \text{ chances in } 3$

C) What fraction of all halo events directed towards Earth coincided with SPEs?
 $65 \text{ in Table} / (528+65) \text{ Halos} = 11\%$

Question 3: A) What percentage of SPEs coinciding with Halo CMEs are more intense than 900 PFUs?
 From the table, there are 12 SPEs out of 65 in this list or $12/65 = 18\%$

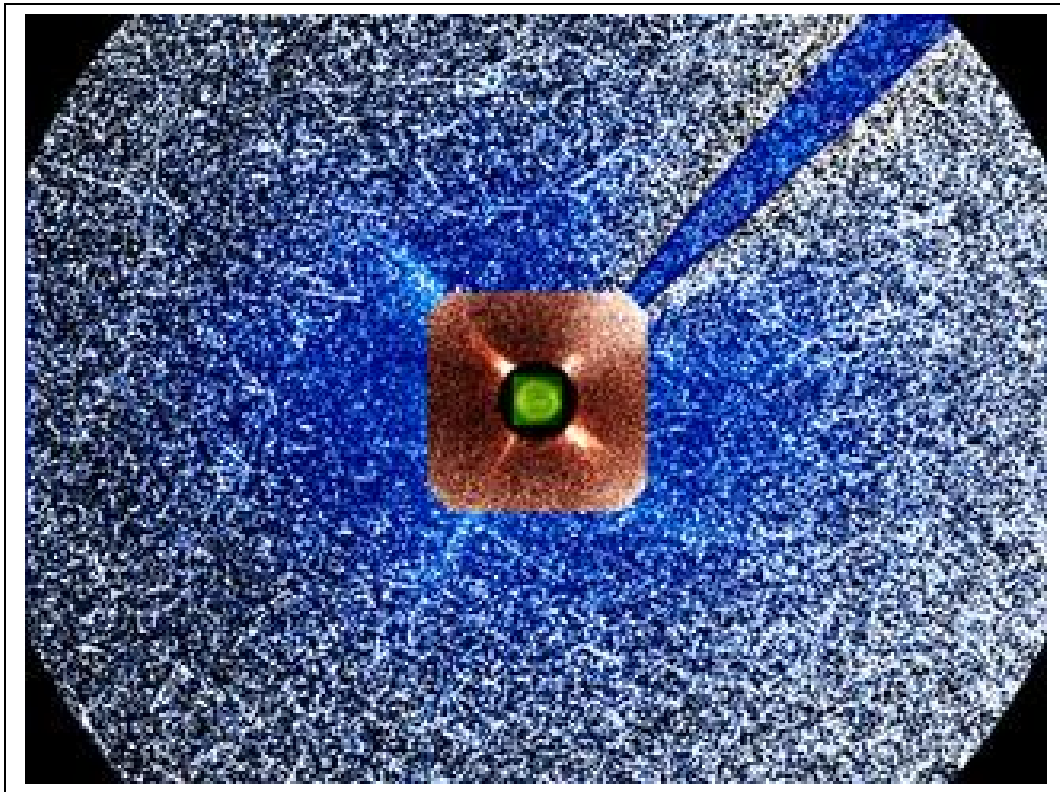
B) What are the odds that, if you detect a 'Halo- SPE', it will be more intense than 900 PFUs?
 $1 / 0.18 = 1 \text{ chance in } 5.$

Question 4: A) What percentage of Halo-SPEs have speeds greater than 1000 km/sec?
 There are 50 out of 65 or $50/65 = 77\%$

B) What are the odds that a Halo-SPE in this sample has a speed of $> 1000 \text{ km/sec}$?
 $1 / 0.77 = 1 \text{ chance in } 1.3 \text{ or } 2 \text{ chances in } 3.$

Question 5: From what you have calculated as your answers above, what might you conclude about Solar Proton Events and CMEs? How would you use this information as a satellite owner and operator?

A reasonable student response is that Halo CMEs occur only 11% of the time, and of the ones directed towards Earth only 1 out of 9 coincide with SPEs. However, in terms of SPEs, virtually all of the SPEs coincide with Halo events (2 out of 3) and SPEs are especially common when the CME speed is above 1000 km/sec. As a satellite owner, I would be particularly concerned if scientists told me there was a halo CME headed towards Earth AND that it had a speed of over 1000 km/sec. Because the odds are now 2 chances out of 3 that an SPE might occur that could seriously affect my satellite. I would try to put my satellite in a safe condition to protect it from showers of high-energy particles that might damage it.



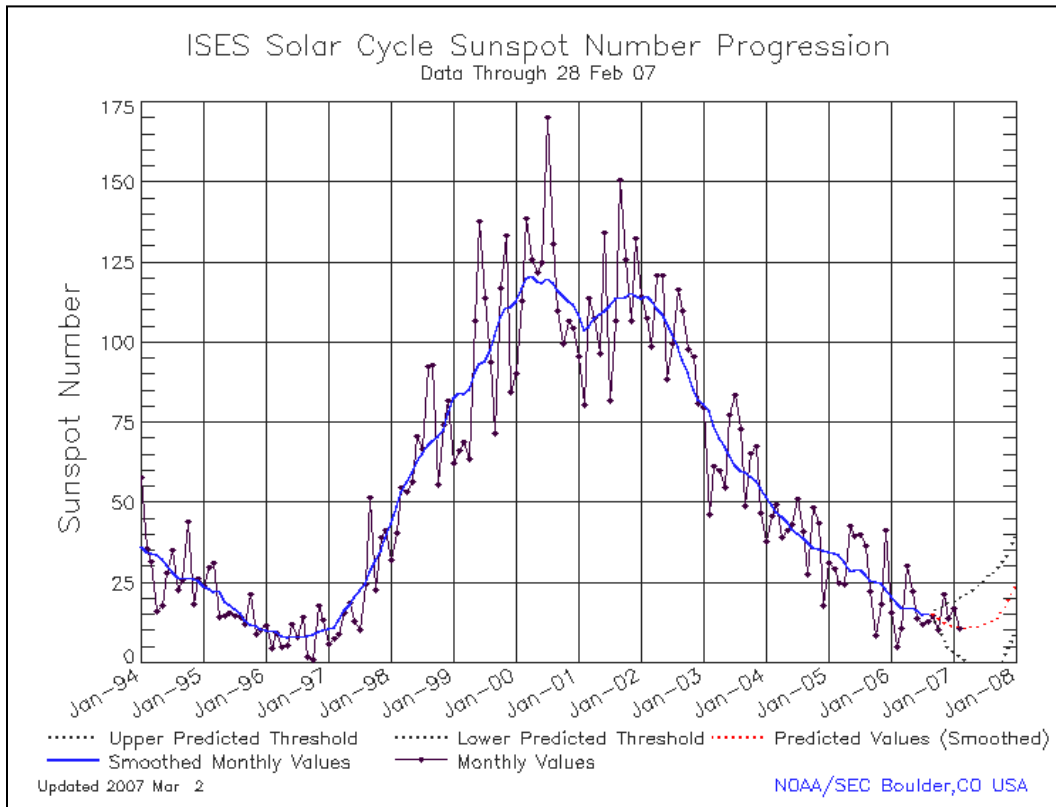
The January 20, 2005 solar proton event (SPE) was by some measures the biggest since 1989. It was particularly rich in high-speed protons packing more than 100 million electron volts (100 MeV) of energy. Such protons can burrow through 11 centimeters of water. A thin-skinned spacesuit would have offered little resistance, and the astronaut would have been radiation poisoned, and perhaps even killed.

The above image was taken by the SOHO satellite during this proton storm. The instrument, called LASCO, was taking an image of the sun in order for scientists to study the coronal mass ejection (CME) taking place. Each of the individual white spots in the image is a track left by a high-speed proton as it struck the imaging CCD (similar to the 'chip' in your digital camera). As you see, the proton tracks corrupted the data being taken.

The high-speed particles from these proton storms also penetrate satellites and can cause data to be lost, or even false commands to be given by on-board computers, causing many problems for satellite operators.

The Sunspot Cycle - endings and beginnings

10



The above plot shows the current sunspot cycle (Number 23) based on the average monthly sunspot counts since January, 1994.

Problem 1 - About when (month and year) did Sunspot Cycle 23 begin?

Problem 2 - About when (month and year) did Sunspot Cycle 23 reach its maximum?

Problem 3 - A) What was the average minimum sunspot count during the years of the previous sunspot minimum? B) What do you think the average sunspot count will be during the current sunspot minimum?

Problem 4 - What is the number of years between sunspot minima to the nearest tenth of a year?

Problem 5 - How long did Cycle 23 take to reach sunspot maximum?

Problem 6 - When (year, month) do you predict we will reach sunspot maximum during the next cycle (Cycle 24)?

Problem 7 - When (year, month) do you think the next sunspot minimum will occur?

Problem 8 - During which part of the sunspot cycle is there A) the greatest month-to-month variation in the number of sunspots counted? B) The least variation in the number counted?

Answer Key:

Problem 1 - When (month and year) did Sunspot Cycle 23 begin?

Answer: Around July, 1996

Problem 2 - When (month and year) did Sunspot Cycle 23 reach its maximum?

Answer: Around July, 2000 and a second maximum near September, 2001

Problem 3 - A) What was the average minimum sunspot count during the previous sunspot minimum?

Answer: From the graph the monthly numbers are 5,8,6,6,12,8,13,1,0,16,13,6 for an average of 8 sunspots during 1996.

B) What do you think the average sunspot count will be during the current sunspot minimum?

Answer: About 12.

Problem 4 - What is the number of years between sunspot minima to the nearest tenth of a year?

Answer: The first minimum was on July, 1996 and the current minimum seems to be around March, 2007 so the difference is $2007.25 - 1996.58 = 10.7$ years.

Problem 5 - How long did Cycle 23 take to reach sunspot maximum?

Answer: The first maximum occurred on July 2000, the minimum was July 1996, so it took 4 years.

Problem 6 - When (year, month) do you predict we will reach sunspot maximum during the next cycle (Cycle 24)?

Answer: If we add 4 years to the current minimum on March, 2007 we get March, 2011.

Problem 7 - When (year, month) do you think the next sunspot minimum will occur?

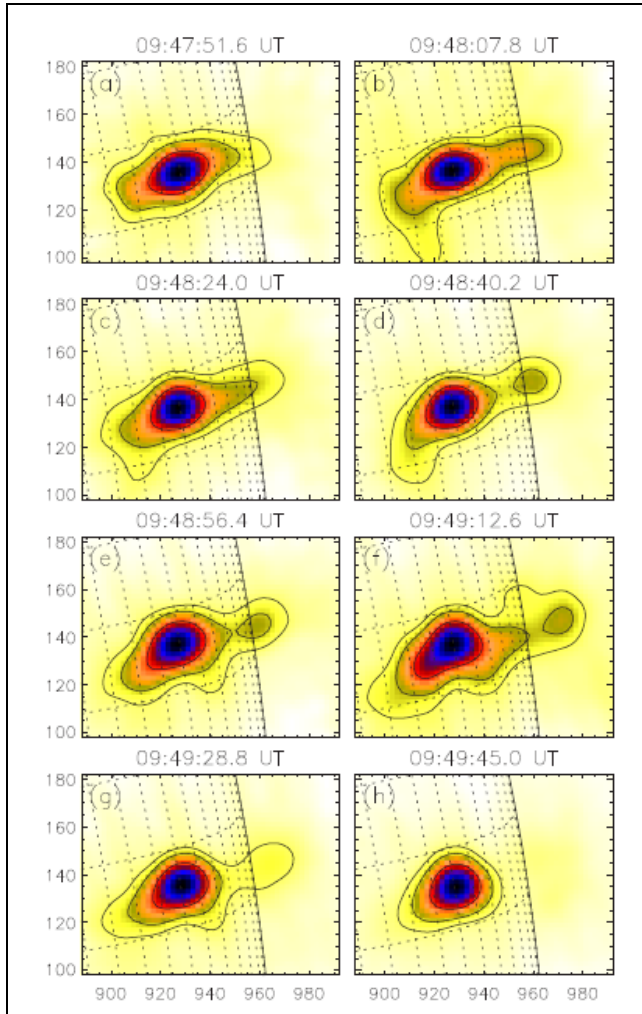
Answer: From our answer to Problem 4, if we add 10.7 years to March, 2007 we get $2007.25 + 10.7 = 2017.95$ or December, 2017.

Problem 8 - During which part of the sunspot cycle is there A) the greatest month-to-month variation in the number of sunspots counted? B) The least variation in the number counted?

Answer: Looking at the graph, the largest variations from month to month occur near sunspot maximum, and the least variations occur near sunspot minimum.

Super-fast solar flares !

11



NASA's Ramaty High Energy Solar Spectroscopic Imager (RHESSI) satellite has been studying solar flares since 2002. The sequence of figures to the left shows a flaring region observed on November 3, 2003. This flare was rated as 'X3.9' making it an extremely powerful event. A detailed study of this flare by astronomer Dr. Astrid Veronig and her colleagues at the Institute of Physics of the University of Graz in Austria allowed scientists to determine the physical properties of this event.

During the 4-minute flaring event, gas temperatures of over 45 million degrees Kelvin were reached in a plasma with a density of 400 billion atoms/cc.

The figures each have a field of view of 80 seconds of arc x 100 seconds of arc. The diameter of the sun in these angular units is 1950 seconds of arc, and its physical diameter is 1,392,000 kilometers.

Each image shows the main flare region (blue) and Images D, E and F show a second 'blob' being ejected by the flaring region.

"X-ray sources and magnetic reconnection in the X3.9 flare of 2003 November 3" A. Veronig et al., Astronomy and Astrophysics, 2005 vol. 446, p.675.

Problem 1 - From the information in the text, what is the size of each box in kilometers?

Problem 2 - What is the scale of each image in kilometers per millimeter?

Problem 3 - Between Image D and Image F, how much time elapsed?

Problem 4 - Between Image D and Image F, how far did the plasma Blob travel in kilometers?

Problem 5 - Between Image D and Image F, what was the average speed of the Blob in kilometers per second?

Problem 6 - The SR-71 Blackbird holds the official Air Speed Record for a manned airbreathing jet aircraft with a speed of 3,529.56 km/h (2,188 mph). It was capable of taking off and landing unassisted on conventional runways. The record was set on July 28, 1976 by Eldon W. Joersz near Beale Air Force Base in California. Would the SR-71 have been able to out-run the plasma blob?

Answer Key:

Problem 1 - From the information in the text, what is the size of each box in kilometers?

Answer: $(100 \text{ arc-sec}/1950\text{-arcsec}) \times 1,392,000 \text{ km} = 71,400 \text{ km}.$
 $(80 \text{ arcsec}/1950 \text{ arcsec}) \times 1,392,000 \text{ km} = 57,100 \text{ km}.$
 The boxes are 71,400 x 57,100 km in size.

Problem 2 - What is the scale of each image in kilometers per millimeter?

Answer: The 100-arcsec edge of a box measures 34 millimeters, so the scale is $(71,400 \text{ km}/34 \text{ mm}) = 2,100 \text{ km/mm}$

Problem 3 - Between Image D and Image F, how much time elapsed?

Answer: $09:49:12.6 \text{ UT} - 09:48:40.2 \text{ UT} = 72.6 - 40.2 = 30.4 \text{ seconds}.$

Problem 4 - Between Image D and Image F, how far did the plasma Blob travel in kilometers?

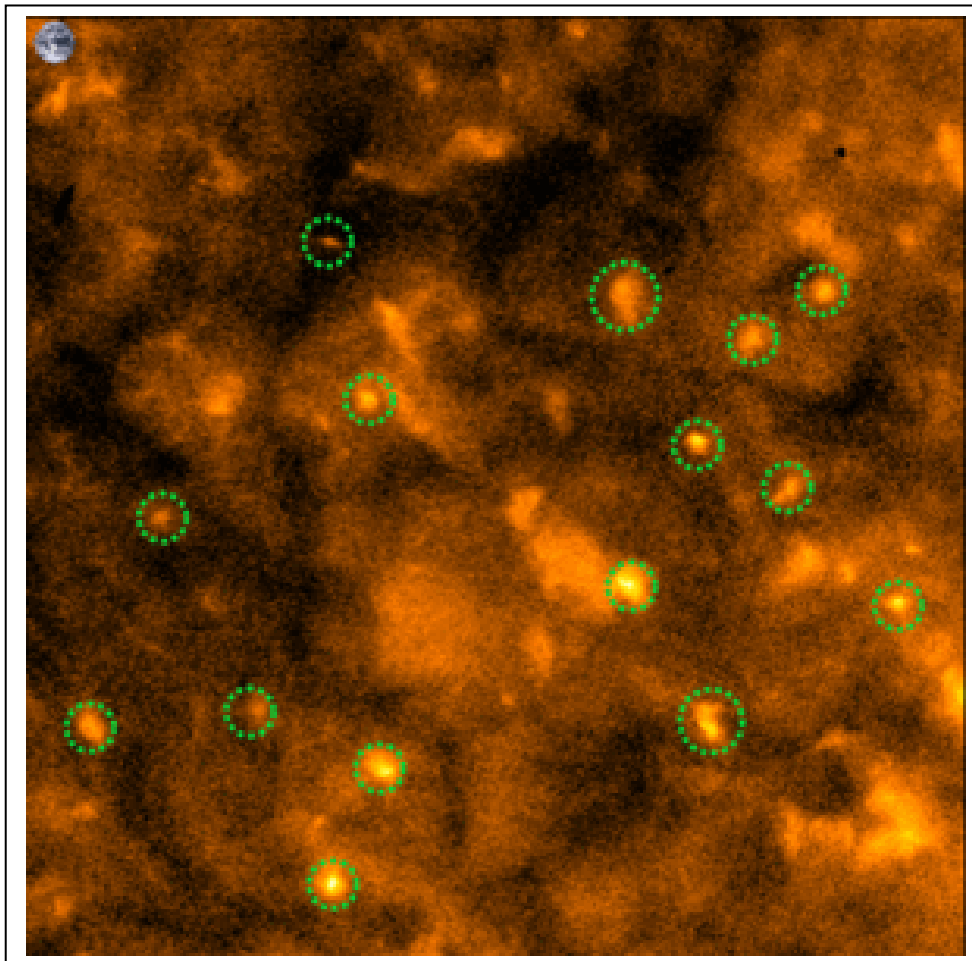
Answer: In Image D it was 12 millimeters from the flare center. In Image F it was 15 millimeters from the flare center, for a net change of 3 millimeters or $3 \text{ mm} \times 2,100 \text{ km/mm} = 6,300 \text{ kilometers}.$

Problem 5 - Between Image D and Image F, what was the average speed of the Blob in kilometers per second?

Answer: The speed was $6,300 \text{ kilometers}/30.4 \text{ seconds}$ or $207 \text{ kilometers/sec}.$

Problem 6 - The SR-71 Blackbird holds the official Air Speed Record for a manned air-breathing jet aircraft with a speed of 3,529.56 km/h (2,188 mph). It was capable of taking off and landing unassisted on conventional runways. The record was set on July 28, 1976 by Eldon W. Joersz near Beale Air Force Base in California. Would the SR-71 have been able to out-run the plasma Blob?

Answer: The SR-71 traveled at a speed of 3,530 km/hour. There are 3,600 seconds in an hour, so the speed was $3,530 \text{ km/hr} \times 1 \text{ hr}/3600 \text{ sec} = 0.98 \text{ kilometers/sec}.$ The solar flare blob was traveling at 207 kilometers per second or nearly 210 times faster! The Blob Wins!!!



The Sun's surface is not only speckled with sunspots, it is also dotted with intense spots of X-ray light called 'X-ray Bright Points'. Although sunspots can be over 100,000 kilometers across and easily seen with a telescope, X-ray Bright Points are so small even the largest solar telescope only sees a few of them with enough detail to reveal their true shapes. X-ray Bright Points release their energy by converting tangled magnetic fields into smoother ones. This liberates large quantities of stored magnetic energy. For that reason, these Bright Points can be thought of as micro-flares.

Hinode's X-ray Telescope (XRT) can now see the details in some of the Bright Points and allow scientists to see small magnetic loops. In the image above, individual bright points are circled in green. A few of them can be resolved into tiny magnetic loops. These data were taken on March 16, 2007. The image is 300 x 300 pixels in size. Each pixel views an area on the sun that is 1 arcsecond x 1 arcsecond on a side.

Problem 1: If the diameter of the Sun is 1800 arcseconds, and has a radius of 696,000 km, what is the scale of the above image in A) kilometers per arcsecond? B) kilometers/millimeter?

Problem 2: What are the dimensions, in kilometers, of the smallest circled Bright Point in the image?

Problem 3: How many Bright Points cover the solar surface if the above picture is typical?

Answer Key:

Problem 1: If the diameter of the sun measures 1800 arcseconds and has a radius of 696,000 km, what is the scale of the above image in kilometers per arcsecond?

Answer: A) The solar radius is $1800 \text{ arcseconds} / 2 = 900 \text{ arcseconds}$ which physically equals 696,000 km, so the scale is $696,000 / 900 = 773 \text{ kilometers/arcsecond}$.

B) The image is 300 pixels across, which measures 115 millimeters with a ruler. Each pixel is 1 arcsecond in size, so this represents $773 \text{ km/arcsec} \times 300 = 232,000 \text{ km}$. The ruler says that this equals 115 mm, so the image scale is $232,000 \text{ km} / 115 \text{ mm} = 2,020 \text{ km/mm}$.

Problem 2: What are the dimensions of the smallest circled Bright Point in the image?

Answer: With a ruler, the circled Bright Point at the top of the picture seems to be the smallest. It measures about 2 millimeters across and 1 millimeter wide. This corresponds to about $4000 \times 2000 \text{ km}$.

Problem 3: How many Bright Points cover the solar surface if the above picture is typical?

Answer:

The sun is a sphere with a radius of 696,000 kilometers. The area of a sphere is given by $4 \pi R^2$, so the surface area of the sun is $4 \times 3.141 \times (696,000 \text{ km})^2 = 6.1 \times 10^{12} \text{ kilometers}^2$.

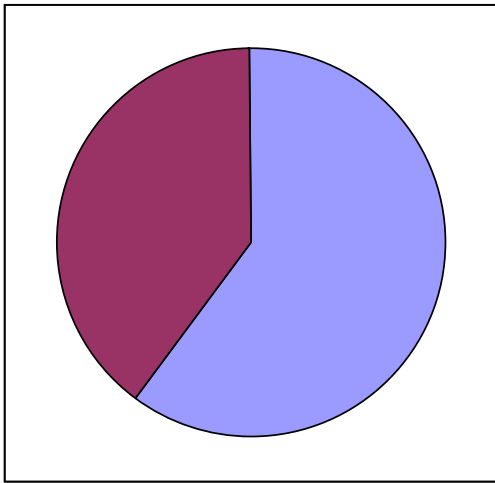
The size of the Hinode image is $300 \text{ pixels} \times 773 \text{ km/pixel} = 232,000 \text{ km}$ on a side. The area covered is about $(232,000 \text{ km} \times 232,000 \text{ km}) = 5.4 \times 10^{10} \text{ km}^2$. Note, this is an approximation because of the distortion of a flat image attempting to represent a curved spherical surface. The actual solar surface area covered is actually a bit larger.

The solar surface is about $6.1 \times 10^{12} \text{ km}^2 / 5.4 \times 10^{10} \text{ km}^2 = 113$ times larger than the Hinode image.

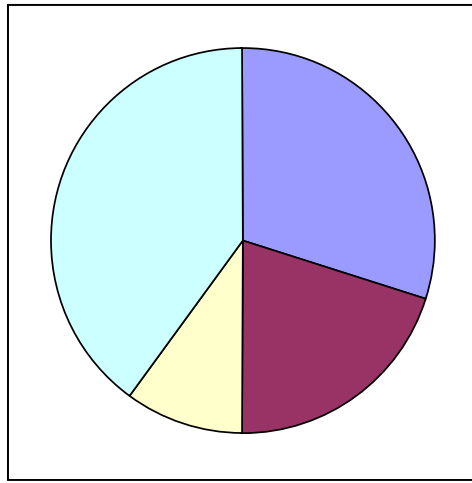
There are 16 Bright Points in the Hinode image, so there would be $15 \times 113 = 1,695$ Bright Points covering the full solar surface if the Hinode image is typical.

Solar Storm Energy and Pie Graphs

The pie charts below show approximately how various forms of energy are involved in a solar flare. Flares occur when stored magnetic energy is suddenly released. The chart on the left shows how much of this magnetic energy is available for creating a flare (purple) and how much is lost (blue). The chart on the right shows how much of the available magnetic flare energy goes into four different phenomena: Light green represents forms of radiation such as visible light and x-rays. Blue represents (kinetic) energy in ejected clouds of gas called Coronal Mass Ejections. Purple represents flare energy that goes into heating local gases to millions of degrees Centigrade, and white is the portion of the flare energy that is lost to working against gravity.



Graph of stored magnetic energy



Graph of solar flare energy forms

Problem 1 - About what percentages of each of the four forms of energy are represented in the right-hand chart?

Problem 2 - About what percentage of the original, stored magnetic energy is available for flares?

Problem 3 - About what fraction of the original magnetic energy ends up as solar flare radiation, assuming all forms of energy can be interchanged with each other?

Problem 4 - About what fraction of the original magnetic energy ends up in CME ejection?

Problem 5 - A typical large flare has enough total energy to meet the world-wide power demands of human civilization for 10,000 years. How many years would be equivalent to A) causing the flare to shine and B) ejecting a CME?

Problem 1 – What percentages of each of the four forms of energy are represented in the right-hand chart?

Answer: **Radiation = 40%, CME = 30%, Gas heating = 20 % and Gravity = 10%**

Problem 2 - What percentage of the original, stored magnetic energy is available for flares?

Answer: The size of the purple sector is **40%**

Problem 3 – What fraction of the original magnetic energy ends up as solar flare radiation?

Answer: 40% of the original magnetic energy is available for a flare, and 40% of the flare energy ends up as radiation, so the fraction of the original magnetic energy involved is $0.40 \times 0.40 = \mathbf{0.16}$

Problem 4 – What fraction of the original magnetic energy ends up in CME ejection?

Answer: 40% of the original magnetic energy is available for a flare, and 30% of the flare energy ends up as CME (kinetic) energy, so the fraction of the original magnetic energy involved is $0.40 \times 0.30 = \mathbf{0.12}$

Problem 5 – A typical large flare has enough total energy to meet the power demands of human civilization for 10,000 years. How many years would be equivalent to A) causing the flare to shine and B) ejecting a CME?

Answer: A) 40% of the flare energy ends up as radiation so this is equivalent to $0.40 \times 10,000 \text{ years} = \mathbf{4,000 \text{ years}}$ of human energy consumption.

B) 30% of the flare energy is available for CME kinetic energy, so this equals an equivalent of $0.30 \times 10,000 \text{ years} = \mathbf{3,000 \text{ years}}$ of human energy consumption.

	CY	J	F	M	A	M	J	J	A	S	O	N	D
1996	1							1					
1997	2											3	
1998	3				2	2			5			5	
1999	4								2		1	1	
2000	5		1	3			4	3		1		5	
2001	6			1	8		1		1	1	4	2	3
2002	7				1	1		5	4		1		
2003	8			2		3	4				7	4	
2004	9							6	2		1	2	
2005	10	6						1		10			
2006	11												2

X-Class solar flares are among the most powerful, explosive events on the solar surface. They can cause short-wave radio interference, satellite malfunctions and can even cause the premature re-entry of satellites into the atmosphere.

The table above lists the number of X-class flares detected on the sun during the last sunspot cycle which lasted from 1996 to about 2008. The second column also gives the year from the start of the 11-year sunspot cycle in 1996. The counts are listed by year (rows) and by month (columns). Study this table and answer the following questions to learn more about how common these flares are.

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1996,1997,1998,1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005,2006). For each group, calculate A) The percentage of months with no flares; and B) The average number of weeks between flares.

Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares?

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle?

Problem 5 – We are currently in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year?

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain. Answer; If you shaded in all the months with flares you would see that most occur between 1999-2002 so the distribution is not random, and is not uniform.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1996,1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005,2006). For each group, calculate A) the percentage of months with no flares, and B) The average number of weeks between flares. Answer: A) Pre-Maximum, $N = 39$ months so $P = 100\% \times 39/48 = 81\%$. Maximum; $N = 17$ months so $P = 100\% \times 17/36 = 47\%$; post-maximum $N = 48$ months so $P = 100\% \times 35/48 = 73\%$. B) pre-maximum $N = 22$ flares so $T = 48 \text{ months}/22 \text{ flares} = 2.2 \text{ months}$. Maximum: $N = 50$ flares so $T = 36 \text{ mo}/50 = 0.7 \text{ months}$; post-maximum: $N = 50$ flares so $T = 48 \text{ mo}/50 = 1.0 \text{ months}$.

Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares? Answer: Pre-maximum: 1,1,1,2,2,2,3,5,5 median = 1.5
Maximum: 1,1,1,1,1,1,1,1,2,3,3,3,4,4,4,5,8 median = 4; post-maximum 1,1,2,2,2,2,3,4,6,6,7 median = 6

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle? Answer: There were 122 flares detected during the 132 months of the sunspot cycle, so the average is $122 \text{ flares}/132 \text{ months} = 0.9$, which can be rounded to **1 flare/month**.

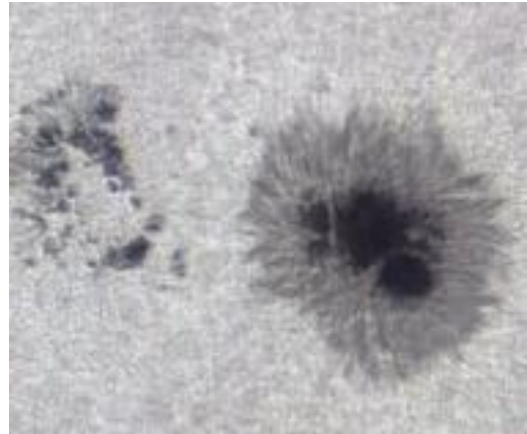
	CY	J	F	M	A	M	J	J	A	S	O	N	D	
1996	1							1						1
1997	2											3		3
1998	3				2	2			5			5		14
1999	4								2		1	1		4
2000	5		1	3			4	3		1		5		17
2001	6			1	8		1		1	1	4	2	3	21
2002	7				1	1		5	4		1			12
2003	8			2		3	4				7	4		20
2004	9							6	2		1	2		11
2005	10	6						1		10				17
2006	11												2	2
		6	1	6	11	6	9	16	14	12	14	22	5	

Problem 5 – We are in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year? Answer: From the table we find for Year 3 that there were 14, X-class flares. Since there are 12 months in a year, this means that the average time between flares is about $14/12 = 1.2 \text{ months}$.

Name _____

Sunspots are some of the most interesting, and longest studied, phenomena on the sun's surface. The table below shows the area of a sunspot. In comparison, the surface area of Earth is '169' units on the sunspot scale. The table also shows the brightest flare seen from the vicinity of the sunspot s beginning November 6, 2004. Flares are ranked by their brightness 'C', 'M' and 'X' with M-class flares being 10x more luminous that C-class flares, and X-class flares being 10x brighter than M-class flares.

Date	Spot #	Area	Flare
Nov 6	#696	820	M
Nov 7	#696	910	M
Nov 8	#696	650	X
Nov 10	#696	730	M
Nov 11	#696	470	X
Dec 2	#708	130	M
Dec 3	#708	150	M
Dec 9	#709	20	C
Dec 29	#713	150	M
Dec 30	#715	260	M
Dec 31	#715	350	M
Jan 1	#715	220	M
Jan 2	#715	180	X
Jan 4	#715	130	C
Jan 10	#719	100	M
Jan 14	#718	160	C
Jan 15	#720	1540	M
Jan 16	#720	1620	X
Jan 17	#720	1630	M
Jan 18	#720	1460	X
Jan 19	#720	1400	M



During the 75 day time period covered by this table, there were a total of $(720-696=)$ 24 catalogued sunspots. The table shows only those catalogued sunspots that were active in producing flares during this time. Sunspot areas are in terms of millionths of the solar hemisphere area, so '1630' means 0.163% of the Sun's face. Earth's area = 169 millionths by comparison!

Sunspot and solar flare data from NOAA
SWN data archive at
<http://www.sec.noaa.gov/Data/index.html>

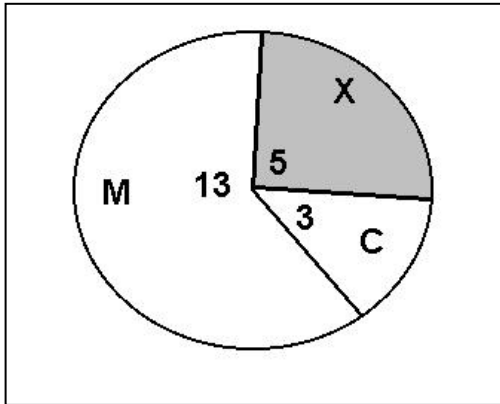
Question 1: Construct a pie chart for the X, M and C-class flare data. During this 75-day period, what percentage of flares are X-class?

Question 2: What percentage of sunspots produce X-class flares?

Question 3: What percentage of sunspots did not produce any flares during this time?

Question 4: What seems to be the minimum size for a sunspot that produces an X-class flare? An M-class flare? A C-class flare?

Question 5: If the area of Earth is '169' in the sunspot units used in the above tables, what are the maximum and minimum size of the sunspots compared to the area of Earth?



Note, there are 21 flares in the table.

X = 5 flares

M = 13 flares

C = 3 flares.

The pie chart angles are

21 = 360 degrees

$X = (5/21) \times 360 = 86$ degrees

$M = (13/21) \times 360 = 223$ degrees

$C = (3/21) \times 360 = 51$ degrees.

And to check: $223 + 86 + 51 = 360$.

Question 1: With a pie chart, what percentage of flares were X-type?

Answer: 5 out of 21 or $(5/21) \times 100\% = 24\%$

Question 2: During this 75-day period, what percentage of flares are X-class flares?

Answer: There are 24 sunspots in the sample because the catalog numbers run from 720 to 698 as stated in the table caption (A 'reading to be informed' activity). There were three sunspots listed in the table that produced X-class flares: #696, #715, #720. The percentage is $(3/24) \times 100\% = 12.5\%$ which may be rounded to 13%.

Question 3: What percentage of sunspots did not produce flares during this time?

Answer: There were only 8 sunspots in the table that produced flares, so there were 16 out of 24 that did not produce any flares. This is $(16/24) \times 100\% = 67\%$. An important thing for students to note is that MOST sunspots do not produce any significant flares.

Question 4: What seems to be the minimum size for a sunspot that produces an X-class flare? An M-class flare? A C-class flare?

Answer: Students may reasonably answer by saying that there doesn't seem to be any definite correlation for the X and M-class flares! For X-class flares, you can have them if the area is between 180 and 1620. For M-class flares, spots with areas from 130 to 1630 can have them. The two possibilities overlap. For C-class flares, they seem to be most common in the smaller spots from 20 – 130 in area, but the sample in the table is so small we can't really tell if this is a genuine correlation or not. Also, we have only shown in the table the largest flares on a given day, and smaller flares may also have occurred for many of these spots.

Question 5: If the area of Earth is '169' in the sunspot units used in the above tables, what are the maximum and minimum size of the sunspots compared to the area of Earth?

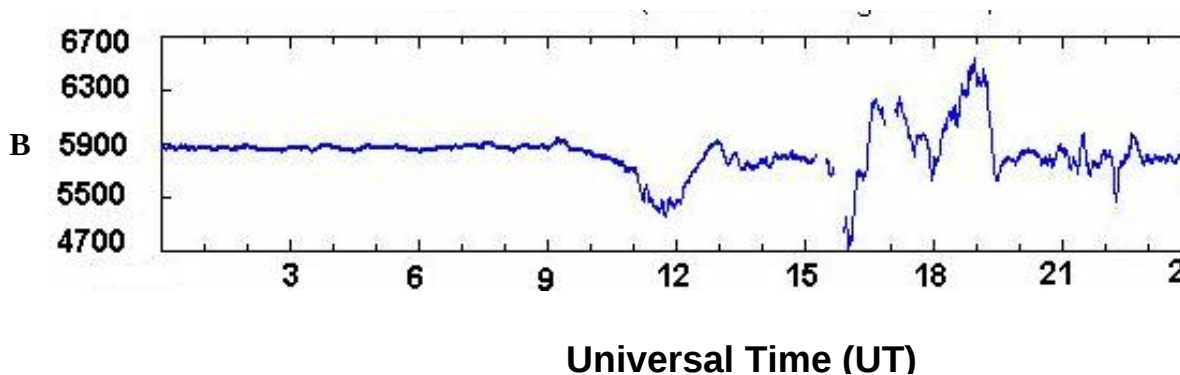
Answer: The smallest spot size occurred for #709 with an equivalent size of $(20/169) \times 100\% = 11\%$ of Earth's area. The largest spot was #720 with a size equal to $(1630/169) = 9.6$ times Earth's area.

Name _____

Magnetic Storms are disturbances in Earth's magnetic field that can be detected from the ground using sensitive instruments called magnetometers. Dozens of these instruments located at 'Magnetic Observatories' around the world keep track of these disturbances. The graph below shows these changes during a 24-hour period on October 24, 2003.

The vertical axis in the plot gives the magnitude, B , of this magnetic change on Earth that is in the East to West direction. The strength of a magnet can be described in terms of a unit called a Tesla. On this plot, the vertical axis gives the magnetic strength in units of nano-Teslas (nT). One nano-Tesla is one billionth of a Tesla (so 1 billion nano-Teslas = 1 Tesla). We can see that on this day, Earth's magnetic field varied between 4700 and 6700 nano-Teslas.

The horizontal axis is the time measured in Universal Time (UT). When scientists study events that change in time, they often use Universal Time, which is also known as Greenwich Mean Time. All scientific measurements are referred to this standard of time keeping to avoid problems converting from one time zone to another. Universal Time or 'UT' follows a 24-hour clock, so that 6:00 PM is written as 18:00 and 1:00 PM is written as 13:00. Use the time calculator at <http://www.indiana.edu/~animal/fun/conversions/worldtime.html> to convert your local time to Universal Time!



Question 1 – If a magnetometer measured a magnetic field of 137,000 nT, how many Teslas would that correspond to?

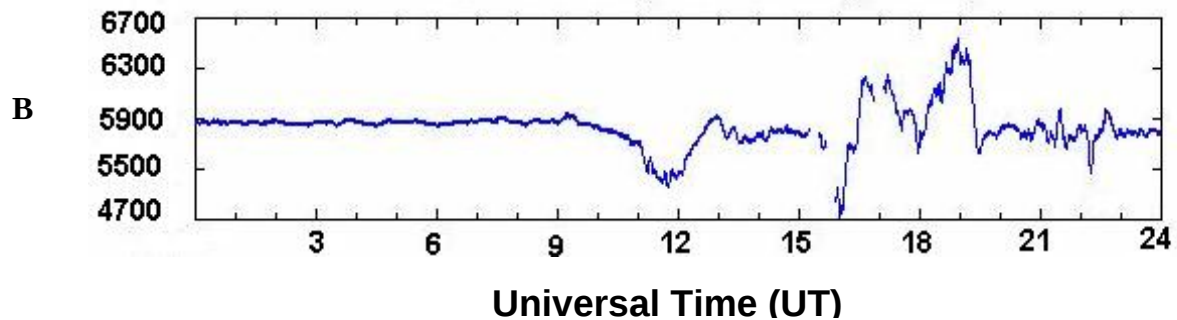
Question 2 – What is the corresponding Universal Time for 9:45 PM?

Question 3 - From the above graph, at what time was the magnetic storm most severe in terms of the absolute magnitude of its change in B ?

Question 4 – At what time did the storm episode begin and end?

Question 5 – As a percentage of 5900 nT, what was the largest change in the magnetic field?

Question 6 - How long did the magnetic storm last?



Question 1 – If a magnetometer measured a magnetic field of 137,000 nT, how many Teslas would that correspond to?

Answer :

$$\frac{137,000 \text{ nT}}{1} \times \frac{1 \text{ Tesla}}{1,000,000,000 \text{ nT}} = 0.000137 \text{ Tesla}$$

Question 2 – What is the corresponding Universal Time for 9:45 PM?

Answer : 21:45 UT.

Question 3 - From the above graph, at what time was the magnetic storm most severe?

Answer: Students should look for the largest 'absolute magnitude' change in the graph from the 'average' level of 5900 nT. That occurs at about 16:00 UT.

Question 4 – At what time did the storm episode begin and end?

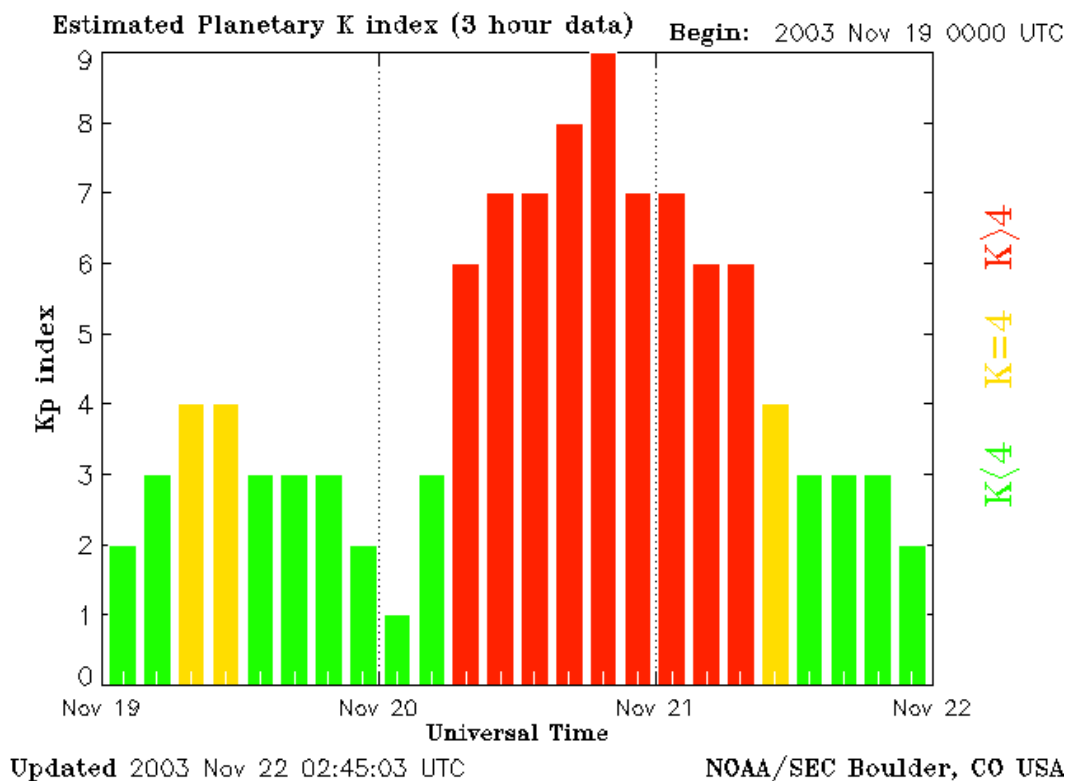
Answer: The 'calm' periods occurred between 00:00 and 09:00 and from 23:00 to 24:00. The storm period occurred between 09:00 and 21:00 UT

Question 5 – As a percentage of 5900 nT, what was the largest change in the magnetic field?

Answer: The largest change in terms of absolute magnitude occurred at 16:00 when the magnetic field went from 5900 nT to 4700 nT. This is a decrease of 1200 nT. As a percentage this was $(1200/5900) \times 100\% = 20\%$ from the non-storm conditions.

Question 6 - How long did the magnetic storm last?

Answer: The entire storm lasted $21:00 - 09:00 = 12$ hours.



Magnetic observatories generate a huge amount of data – far too much for anyone to digest easily. To help scientists take a quick measure of Earth's magnetic storminess, they invented the 'Planetary Variability' index. The magnitude of the disturbance in Earth's magnetic field is measured at 13 observatories and then averaged together. This average value is then reported every three hours as the **Kp Index**.

The bar graph above shows the changes in this index during the time of a major magnetic storm on November 20, 2003. Prior to the storm, Earth's magnetic field was in a typically disturbed state with variations between Kp 2 to 4. But after a solar disturbance collided with the magnetic field, the variations jumped to Kp 7 and higher within a few hours. This particular storm caused spectacular Northern Lights seen all across North America and Northern Europe.

Question 1 – If each bar is 3-hours wide, how long did the storm last above a level of Kp = 4?

Question 2 – At what time did the storm reach its maximum Kp value?

Question 3 - If New York City is 4 hours behind Universal Time, what time was it in New York during the height of the storm?

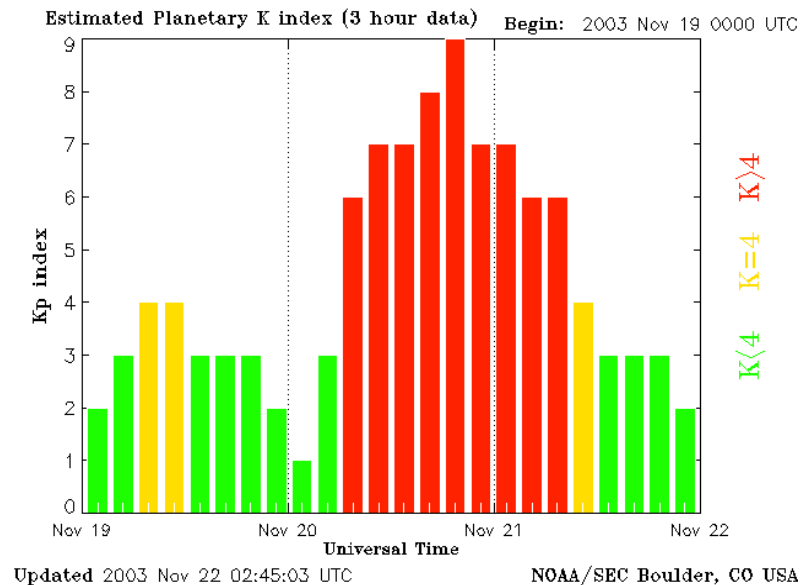
Answer - Extra Credit Problem

17

In this activity, students will:

1. Extract information from a bar graph
2. Extract basic information from a graph.
3. Calculate a percentage
4. Learn about magnetism

Benchmark:



Question 1 – If each bar is 3-hours wide, how long did the storm last?

Answer: The red portion of the bar graph which covers the most intense phase of the storm extends 9 bars or $9 \times 3\text{h} = 27$ hours!

Question 2 – At what time did the storm reach its maximum Kp value?

Answer: This occurred at the bar which spans the times 19:00 to 21:00 UT so you can take the start time as 19:00 UT, or the end time 21:00 UT or the mid-point time of the bar of 20:30 UT.

Question 3 - If New York City is 4 hours behind Universal Time, what time was it in New York during the height of the storm?

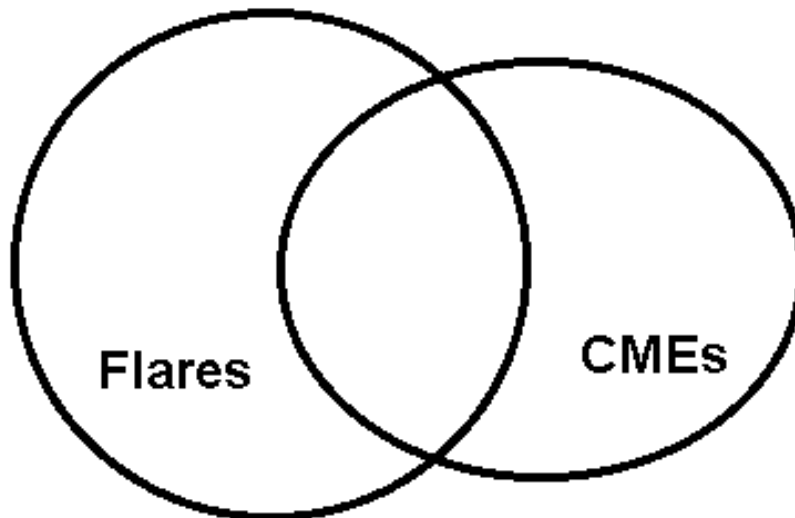
Answer: Taking the mid-time of 20:30 UT, the Eastern Standard Time in New York would be $20:30 - 4:00 = 16:30$ EST or 4:30 PM.

Solar flares are violent releases of energy from the sun that last 10 to 20 minutes and produce intense flashes of x-rays. Some of these flares can cause radio interference, and problems with communication satellites. Coronal mass ejections (CMEs) are enormous releases of matter (plasma) from the sun that travel at nearly a million miles per hour to Earth. CMEs can cause the Northern Lights and sometimes result in electrical power blackouts.

During a particular month of solar activity, the following events were recorded:

Solar Flares	22
CME s	12
Both flares and CMEs	7

A scientist decides to analyze the results to come up with an early-warning alert system. He wants to use CMEs to predict when solar flares will happen, and he wants to use flares to predict when CMEs will happen. His first step is to construct a Venn Diagram to display the data. Place the data in the correct locations.

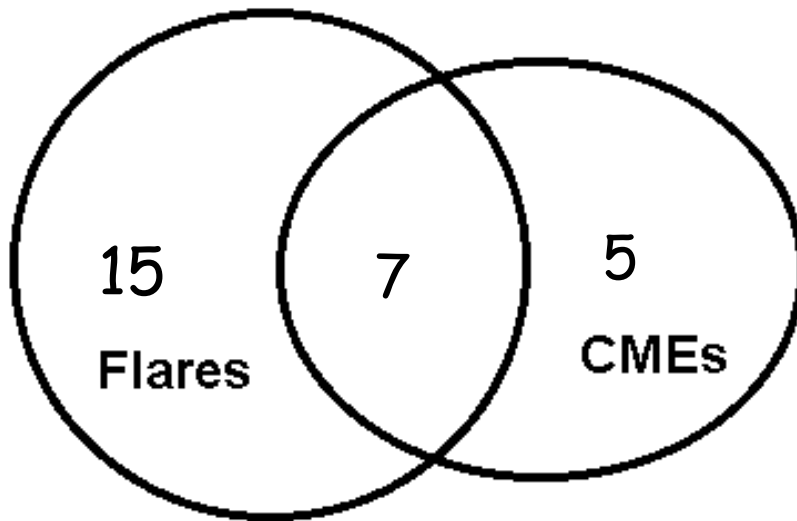


Problem 1 - What do the non-overlapping parts of the diagram represent?

Problem 2 - Based on this data, if you see a flare on the sun, what is the probability that a CME will also occur?

Problem 3 - Based on this data, if you see a CME, what is the probability that a flare will also occur?

Problem 4 - In your own words, explain which early warning system gives you the best prediction of what a particular kind of solar storm will occur.



Problem 1 - What do the non-overlapping parts of the diagram represent?

Answer: They represent solar storm events where only CMEs are seen, or only flares are seen but not both happening at the same time.

Problem 2 - Based on this data, if you see a flare on the sun, what is the probability that a CME will also occur?

Answer: The number of flares that occur at the same time as a CME is 7, so $7/12$ is the probability that if you see a flare, you will also see a CME.

Problem 3 - Based on this data, if you see a CME, what is the probability that a flare will also occur?

Answer: There are 22 CMEs so the probability is $7/22$ that, given you see a CME you will also see a flare.

Problem 4 - In your own words, explain which early warning system gives you the best prediction of what a particular kind of solar storm will occur.

Answer: Because $7/12$ is larger than $7/22$, an early warning system that uses flares to predict CMEs will be more reliable than a system that uses CMEs to predict flares. This means that you have a better chance of predicting when an electrical outage (caused by a CME) will occur than when a satellite problem (caused by a flare) will occur using these warning systems.

Name _____

The last few days from November 6 - 9, 2004 have been very active days for solar and auroral events. A major sunspot group, AR 0696, with a complex magnetic field produced several Coronal Mass Ejections and flares during this time. The latest ones were an X2.0-class solar flare on Nov 7, 16:06 UT and a CME ejection. On November 9 and 12:00 UT, the beginnings of a major geomagnetic storm started. The NOAA Space Weather Bulletin announced that:

The Geomagnetic field is expected to be at unsettled to major storm levels on 09 November due to the arrival of a CME associated with the X2.0 flare observed on 07 November. Unsettled to minor storm levels are expected on 10 November. Quiet to active levels are expected on 11 November.

Many observers as far south as Texas and Oklahoma reported seeing beautiful aurora on Sunday night from an earlier CME/flare combination on Saturday, November 6th. In the space provided below, calculate the speed of the CME as it traveled to Earth between November 7th - 9th assuming that the distance to Earth is 93 million miles, or 147 million kilometers.

Question 1 - How long did it take for the CME to arrive?

Question 2 - What is the speed of the CME in miles per hour?

Question 3 - What is the speed of the CME in kilometers per hour?

Question 4 - What is the speed of the CME in miles per second?

Question 5 - What is the speed of the CME in kilometers per second?

Extra-Extra Credit : Many observers were able to see the Sunday night aurora across much of North America. On a separate piece of paper, describe what you saw on Sunday night November 7th, or on Tuesday night November 9th from your location. On the back of the paper, create a drawing of what you saw. To have your essay and drawing posted on the NASA-IMAGE web site send your drawing and essay to:

Dr. Sten Odenwald
Code 630
NASA Goddard Space Flight Center
Greenbelt, MD 20771

We will post the first 25 submissions.

This activity uses the following math concepts:

- 1 - Addition and subtraction of time units
- 2 - Working with speed = distance/time
- 3 - Conversion of miles to kilometers
- 4 - Time conversion from seconds to hours
- 5 - Writing descriptive sentences based on observations.
- 6 - Rendering observations into a visual medium.

The distance to the Earth is 93 million miles or 147 million kilometers.

Start time = November 7 at 16:06 UT

Arrival time = November 9 at 12:00 UT

Question 1 - How long did it take for the CME to arrive?

Answer: November 7 at 16:06 UT to November 8 at 16:06 UT is 24 hours.

From Nov 8 at 16:06 to Nov 9 at 12:00 UT is

$$(24:00 - 16:06) + 12:00$$

$$= 7:54 + 12:00$$

$$= 19 \text{ hours and } 54 \text{ minutes.}$$

$$\text{Total time} = 24 \text{ hours} + 19 \text{ hours and } 54 \text{ minutes} = 43 \text{ hours and } 54 \text{ minutes.}$$

Question 2 - What is the speed of the CME in miles per second?

Answer: In decimal units, the travel time from question 1 equals 43.9 hours.

The distance is 93 million miles so the speed is 93 million miles/43.9 hours

Or 2.1 million miles per hour.

Question 2 - What is the speed of the CME in kilometers per second?

Answer: Use the conversion that 1.0 miles = 1.6 kilometers, then

$$2.1 \text{ million miles/hour} \times 1.6 \text{ km/mile} = 3.4 \text{ million kilometers per hour}$$

Question 3 - What is the speed of the CME in miles per second?

Answer: Convert hours to seconds by

$$1 \text{ hour} \times 60 \text{ minutes/hour} \times 60 \text{ seconds/minute} = 3,600 \text{ seconds.}$$

Then from question 2: 2.1 million miles/hour divided by 3600 seconds/hour

$$= 583 \text{ miles/second.}$$

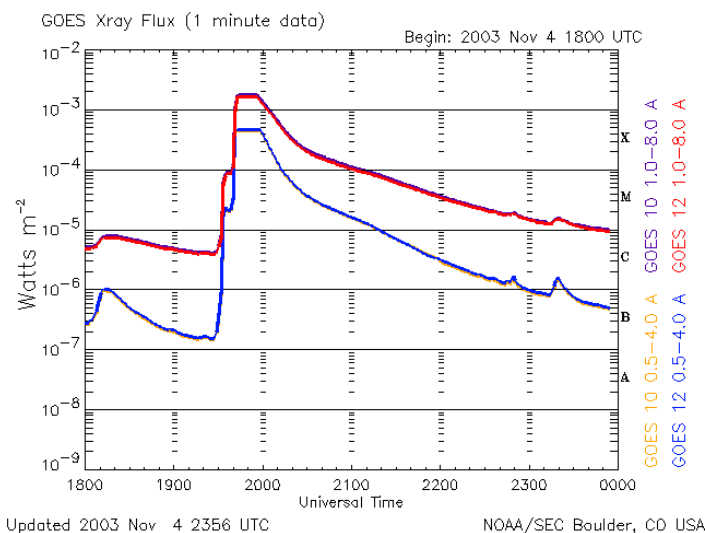
Question 4 - What is the speed of the CME in kilometers per second?

Answer: From question 3 and the conversion of 1 hour = 3600 seconds:

$$3.4 \text{ million kilometers / second divided by } 3600 \text{ seconds/hour}$$

$$= 944 \text{ kilometers/sec.}$$

Solar Flare Reconstruction



X-ray Flare Data.

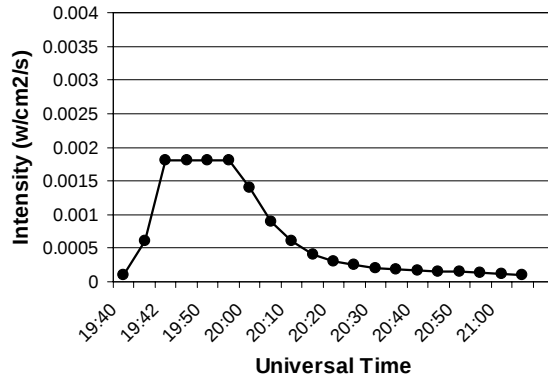
Universal Time (UT)	Intensity (Watts/m ² /sec)
19:40	0.00010
19:41	0.00060
19:42	0.00180
19:45	saturated
19:50	saturated
19:55	0.00180
20:00	0.00140
20:05	0.00090
20:10	0.00060
20:15	0.00040
20:20	0.00030
20:25	0.00025
20:30	0.00020
20:35	0.00019
20:40	0.00017
20:45	0.00016
20:50	0.00015
20:55	0.00014
21:00	0.00012
21:05	0.00010

During the November 4, 2003 solar flare, the GOES satellite measured the intensity of the flare as its light increased to a maximum and then decreased. The problem is that the solar flare was so bright that it could not record the most intense phase of the brightness evolution - what astronomers call its light curve. The figure above shows the light curve for two different x-ray energies, and you can see how its most intense phase near 19:50 UT has been clipped. This is a common problem with satellite detectors and is called 'saturation'. To work around this problem to recover at least some information about the flare's peak intensity, scientists resorted to mathematically fitting the pieces of the light curve that they were able to measure, and interpolated the data using their mathematical model, to estimate the peak intensity of the flare.

Problem 1 (Pre-Algebra): Re-plot the data in the table and from the trend on either side of the saturated region, estimate the peak intensity.

Problem 2 (Algebra): Re-plot the data, and from the information on either side of the saturation region, create two exponential functions that fit the data. Use the elapsed time since 19:40 as the independent variable. Find the intersections of these two functions to estimate the peak intensity and time.

Problem 3 (Calculus): Integrate the piecewise function in Problem 2 to determine the area under the light curve to 21:05. Note: 1 Watt equals 1 Joule of energy per second. Given that the sun is 147 million kilometers from the GOES satellite, calculate the surface area, in square meters, of a sphere of this radius. Calculate the total energy, in Joules, radiated by the flare that passed through the surface area of the sphere.



Re-plotted data to left, allowing extra space for interpolation.

Problem 1 (Pre-Algebra):

Answer: The curves, drawn free-hand, intersect between 0.0035 to 0.004

Watts/m²/sec

Problem 2 (Algebra): Create two exponential functions that fit the data. Use the elapsed time since 19:40 as the independent variable.

Rising: From (0.0, 0.0001), (1.0, 0.0006) and (2.0, 0.00018) a best-fit exponential curve is $R(T) = 0.0001 e^{(+1.44T)}$

Falling: From (20.0, 0.0018), (25.0, 0.0014), (30.0, 0.0009) and (35.0, 0.0006) a best-fit exponential curve is $R(T) = 0.0074 e^{(-0.07T)}$

Find the intersections of these two functions to estimate the peak intensity and time.

$$0.0001 e^{1.44T} = 0.0074 e^{-0.07T}$$

Taking $\log_e$ of both sides : $\log_e(0.0001) + 1.44 T = \log_e(0.0074) - 0.07T$
 solve for T to get: $T = (+9.21 - 4.91)/1.51 = 2.84$ minutes
 So the peak UT is $19:40 + 2.84 = 19:42.84$ or 19:42:50

The peak intensity is then $0.0001 e^{(1.44 \times 2.84)} = 0.006 \text{ Watts/m}^2$

Problem 3 (Calculus): Integrate the piecewise function in Problem 2 to determine the area under the light curve. Note 1 watt x 1 second = 1 Joule.

Rising-side: From 0 to 2.84 minutes: $0.0001 \times (1/1.44) [e^{(1.44 \times 2.84)} - 1] = 0.0041 \text{ Joules/m}^2$

Falling side from 2.84 to 85 minutes:

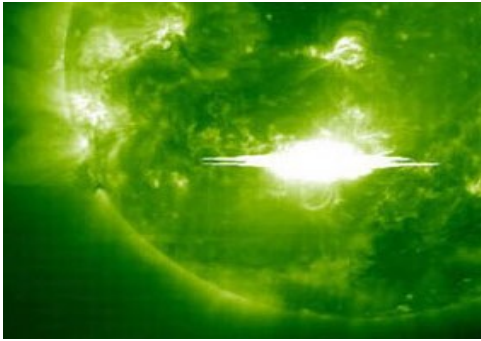
$$\begin{aligned} & 0.0074 \times (1/0.07) [e^{(-0.07 \times 2.84)} - e^{(-0.07 \times 85.0)}] \\ &= 0.106 [0.820 - 0.0026] \\ &= 0.0867 \text{ Joules/m}^2 \end{aligned}$$

Combining we get a total 'flux' of 0.091 Joules/m^2

Given that the sun is 147 million kilometers from the GOES satellite, calculate the surface area of a sphere of this radius. Calculate the total energy radiated by the flare in ergs that passed through the surface area of the sphere.

$$\text{Area} = 4 \pi (147 \times 10^6 \text{ km})^2 = 2.71 \times 10^{17} \text{ km}^2 \times 1.0 \times 10^6 \text{ meter}^2/\text{km}^2 = 2.71 \times 10^{23} \text{ m}^2$$

$$\text{Total energy} = 0.091 \text{ Joules/m}^2 \times 2.71 \times 10^{23} \text{ m}^2 = 2.5 \times 10^{22} \text{ Joules}$$



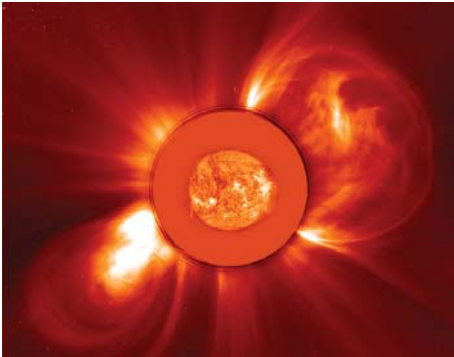
Solving a system of three equations in three unknowns can commonly be found in several space science and astronomy applications.

Solar flares are a frequent phenomenon on the sun, especially during the peaks of solar activity cycles. Over 21,000 can occur during an average solar cycle period of 11 years! In our first problem, you will determine the average intensity of three classes of flares ('C', 'M' and 'X') by using statistical information extracted from three solar activity (sunspot!) cycles.

During February 4 - 6, 2000 the peak month of Cycle 23 solar scientists tallied 37 C-class, 1 M-class and 1 X-class flares, for a total x-ray intensity of 705 mFU (1 mFU = 10^{-6} watts/m²).

During March 4 - 6, 1991 scientists tallied 15 C-class, 14 M-class and 4 X-class flares for a total x-ray intensity of 2775 mFU

During April 1 - 3, 2001 scientists tallied 5 C-class, 9 M-class and 4 X-class flares for a total x-ray intensity of 2475 mFU.



Problem 1: Use the above data to create a system of equations, solve them, and determine the average intensity of flares, to the nearest tenth, in each category (C, M and X) in units of mFU.



Communications satellites use electrical devices called transponders to relay TV and data transmissions from stations to satellite subscribers around the world. There are two basic types: K-band transponders operate at frequencies of 11-15 GHz and C-band transponders operate at 3-7 GHz. Satellites come in a variety of standard models, each having its own power requirements to operate its pointing and positioning systems. The following satellites use the same satellite model:

Satellite 1 : Anik F1
Total power = 15,000 watts
Number of K-band transponders = 48
Number of C-band transponders = 36

Satellite 2 : Galaxy IIIc
Total power = 14,900 watts
Number of K-band transponders = 53
Number of C-band transponders = 24

Satellite 3 : NSS-8
Total power = 16,760 watts
Number of K-band transponders = 56
Number of C-band transponders = 36

Problem 2: Use the data to determine the average power, to the nearest integer, of a K-band and a C-band transponder, and the satellite operating power, F, in watts.

Answer Key:

After setting up the problems as a matrix, you might want to use the spiffy online matrix calculator at <http://www.bluebit.gr/matrix-calculator/>

Problem 1:

The system of equations is

$$\begin{aligned} 31C + 1M + 1X &= 705 \\ 15C + 14M + 4X &= 2775 \\ 5C + 9M + 4X &= 2475 \end{aligned}$$

Matrix:

$$\begin{bmatrix} 31 & 1 & 1 \\ 15 & 14 & 4 \\ 5 & 9 & 4 \end{bmatrix}$$

Inverse:

$$\begin{bmatrix} 0.031 & 0.008 & -0.016 \\ -0.062 & 0.184 & -0.169 \\ 0.101 & -0.425 & 0.650 \end{bmatrix}$$

Solution:

$$\begin{aligned} C: & 0.031 \times 705 + 0.008 \times 2775 - 0.016 \times 2475 = 4.5 \text{ mFU} \\ M: & -0.062 \times 705 + 0.185 \times 2775 - 0.169 \times 2475 = 51.4 \text{ mFU} \\ X: & 0.101 \times 705 - 0.425 \times 2775 + 0.650 \times 2475 = 500.2 \text{ mFU} \end{aligned}$$

Problem 2. Solving for satellite transponder power, K and C, and satellite operating power, F using 3 equations in three variables. From the satellite data

$$\begin{aligned} 48K + 36C + F &= 15,000 \\ 53K + 24C + F &= 14,900 \\ 56K + 36C + F &= 16,760 \end{aligned}$$

Matrix:

$$\begin{bmatrix} 48 & 36 & 1 \\ 53 & 24 & 1 \\ 56 & 36 & 1 \end{bmatrix}$$

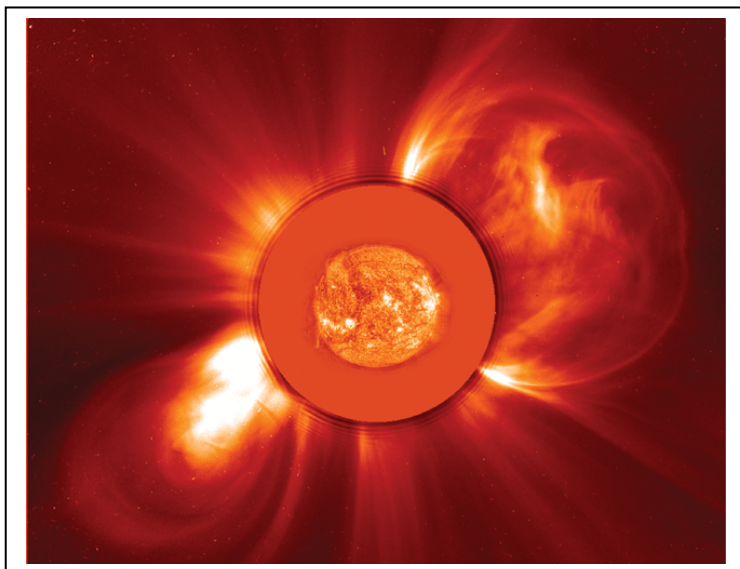
Inverse:

$$\begin{bmatrix} -0.125 & 0.0 & 0.125 \\ 0.031 & -0.083 & 0.052 \\ 5.875 & 3.00 & -7.875 \end{bmatrix}$$

$$\text{Solution} = -0.125 \times 15000 + 0.125 \times 16,760 = K = 220 \text{ watts per K-band transponder}$$

$$0.031 \times 15000 - 0.083 \times 14,900 + 0.052 \times 16,760 = C = 100 \text{ watts per C-band transponder}$$

$$5.875 \times 15000 + 3 \times 14,900 - 7.875 \times 16,760 = F = 840 \text{ watts for the satellite operating power}$$



Kinetic energy is the energy that a body has by virtue of its mass and speed. Mathematically, it is expressed as one-half of the product of the mass of the object (in kilograms), times the square of the objects speed (in meters/sec).

$$K.E. = 0.5 m v^2$$

Between October 1996 and May 2006, the SOHO satellite detected and catalogued 11,031 coronal mass ejections like the one seen in the figure to the left. There was enough data available to determine the properties for 2,131 events. The table below gives values for ten of these CMEs.

Date	Speed (km/s)	K.E. (Joules)	Mass (kilograms)
4/8/1996		1.1×10^{20}	2.2×10^9
8/22/2000	388	1.3×10^{22}	
6/10/2001	731	8.2×10^{23}	
1/18/2002	64		2.6×10^{10}
5/16/2002	1,310		7.8×10^{10}
10/7/2002		7.8×10^{21}	3.0×10^{10}
1/24/2003	387	9.1×10^{18}	
10/31/2003	2,198	1.6×10^{24}	
11/2/2003		9.3×10^{25}	4.5×10^{13}
11/10/2004	3,387		9.6×10^{12}

Problem 1: Complete the table by determining the value of the missing entries using the formula for Kinetic Energy.

Problem 2: What is the minimum and maximum range for the observed kinetic energies for the 10 CMEs? The largest hydrogen bomb ever tested was the Tsar Bomba in 1961 and was equivalent to 50 megatons of TNT. It had a yield of 5×10^{23} Joules. What is the equivalent yield for the largest CME in megatons, and 'Tsar Bombas'?

Problem 3: What are the equivalent masses of the smallest and largest CMEs in metric tons?

Problem 4: Compare the mass of the largest CME to the mass of a small mountain. Assume that the mountain can be represented as a cone with a volume given by $\frac{1}{3} \pi R^2 H$ where R is the base radius and H is the height in meters, and assume the density of rock is 3 grams/cm³.

Date	Speed (km/s)	K.E. (Joules)	Mass (kilograms)
4/8/1996	316	1.1×10^{20}	2.2×10^9
8/22/2000	388	1.3×10^{22}	1.7×10^{11}
6/10/2001	731	8.2×10^{23}	3.1×10^{12}
1/18/2002	64	5.3×10^{19}	2.6×10^{10}
5/16/2002	1,310	6.7×10^{22}	7.8×10^{10}
10/7/2002	721	7.8×10^{21}	3.0×10^{10}
1/24/2003	387	9.1×10^{18}	1.2×10^8
10/31/2003	2,198	1.6×10^{24}	6.6×10^{11}
11/2/2003	2,033	9.3×10^{25}	4.5×10^{13}
11/10/2004	3,387	5.5×10^{25}	9.6×10^{12}

Problem 1: Complete the table by determining the value of the missing entries using the formula for Kinetic Energy.

Answer: See above answers in red.

Problem 2: What is the minimum and maximum range for the observed kinetic energies for the 10 CMEs?

Answer: Maximum = 9.3×10^{25} Joules. Minimum = 5.3×10^{19} Joules

The largest hydrogen bomb ever tested was the *Tsar Bomba* in 1961 and was equivalent to 50 megatons of TNT. It had a yield of 5×10^{23} Joules. What is the equivalent yield for the largest CME in megatons, and 'Tsar Bombas'?

Answer: The CME on November 2, 2003 was equal to

$(9.3 \times 10^{25} / 5 \times 10^{23}) = 186$ Tsar Bombas,

and an equivalent TNT yield of 186×50 megatons = 9,300 megatons!

Problem 3: What are the equivalent masses of the smallest and largest CMEs in metric tons?

Answer: One metric ton is 1,000 kilograms. The smallest mass was for the January 24, 2003 CME with about 120,000 tons. The largest mass was for the November 2, 2003 'Halloween Storm' with about 45 billion metric tons.

Problem 4: Compare the mass of the largest CME to the mass of a small mountain. Assume that the mountain can be represented as a cone with a volume given by $V = 1/3 \pi R^2 H$ where R is the base radius and H is the height in meters, and assume the density of rock is 3 grams/cm^3 .

Answer: One possibility is for a mountain with a base radius of $R = 1$ kilometers, and a height of 50 meters. The cone volume is $0.33 \times 3.14 \times (1000)^2 \times 50 = 5.2 \times 10^7$ cubic meters. The rock density of 3 gm/cm^3 converted into kg per cubic meters is $0.003 \text{ kg}/(.01)^3 = 3000 \text{ kg/m}^3$. This yields a mountain with a mass of 156 billion tons, which is close to the largest CME mass in the table. So CMEs, though impressive in size, carry no more mass than a small hill on Earth!

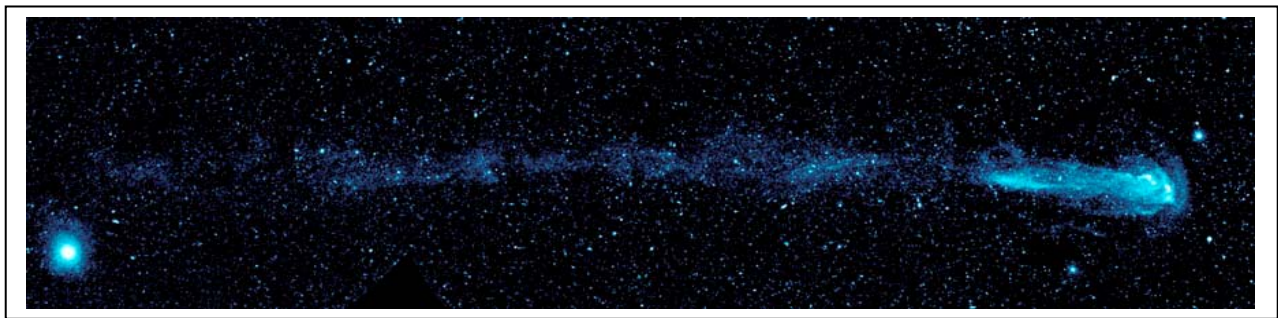
A Star Sheds a Comet Tail!

23

For 400 years, astronomers have carefully watched the star Mira in the constellation Cetus the Whale, made famous for its variable brightness. This red giant star is very old, and it is in its last few million years of life before evolving onto a planetary nebula and white dwarf 'combo'. Our own sun will reach a similar stage in about 7 billion years. While astronomers thought they knew quite a lot about this star after decades of satellite and ground-based telescopic study, the NASA, Galaxy Evolution Explorer (GALEX) satellite uncovered a surprising new twist to this famous star. Using its sensitive ultraviolet telescope while routinely mapping the light from stars across the entire sky, GALEX detected faint light from an enormous tail of dust stretching over 13 light years from Mira.

Red giant and red supergiant stars are known to be factories for producing dust grains in their cooling outer atmospheres where temperatures can reach a chilling 1,500 K - enough for molecules like silicon dioxide to condense out like raindrops, solidifying into small dust grains. These are then driven away from the star by radiation pressure. Some stars form spherical shells that can be so dusty that the star literally fades from optical view behind a dense obscuring wall of dust. But Mira does something different. It seems to be ejecting large quantities of oxygen, carbon and nitrogen!

Mira is known to be a fast-moving star traveling at a speed of 130 km/sec (or about 300,000 miles per hour). As its atmosphere sheds these gases in vast clouds, these clouds are left behind the star as it plows through the gases in the Interstellar Medium. Mira sheds about 10 Earth masses worth of dust and gas every ten years. The distance to Mira is 350 light years, and at this distance, the picture below spans an area 15 light years long and 4 light years wide.



Problem 1 - From the speed of Mira, how long did it take for it to travel 13 light years?

Problem 2 - What is the scale of the image in units of light years per centimeter?

Problem 3 - From your answer to Problem 1, what is the scale of the gas tail in thousands of years per centimeter, rounded to the nearest thousands?

Problem 4 - Did Mira emit gas in a steady rate during the period of time estimated in Problem 1?

Problem 5 - Can you create a chronology for Mira that tells about the sequence of events, in time, of its history of ejecting gas during the time that the dust tail was produced?

Problem 1 - From the speed of Mira, how long did it take for it to travel 13 light years?

Answer: First, a light year is the distance light travels in one year at a speed of 300,000 km/sec. There are 31 million seconds in a year, so $(3 \times 10^5 \text{ k/s}) \times (3.1 \times 10^7 \text{ sec/yr}) = 9.3 \times 10^{12}$ kilometers. The speed of Mira is 350 kilometers/s, so the time required is just

$$\begin{aligned} T &= (13 \text{ light years} \times 9.3 \times 10^{12} \text{ km/LY}) / 130 \text{ km/sec} \\ &= 9.3 \times 10^{11} \text{ seconds or} \\ &= 30,000 \text{ years.} \end{aligned}$$

Problem 2 - What is the scale of the image in units of light years per centimeter?

Answer: 14.5 centimeters = 13 Light years so $13 \text{ LY} / 14.5 \text{ cm} = 0.9 \text{ light years/cm}$

Problem 3 - From your answer to Problem 1, what is the scale of the gas tail in thousands of years per centimeter, rounded to the nearest thousands?

Answer: 30,000 years / 14.5 centimeters = **2000 years/cm**

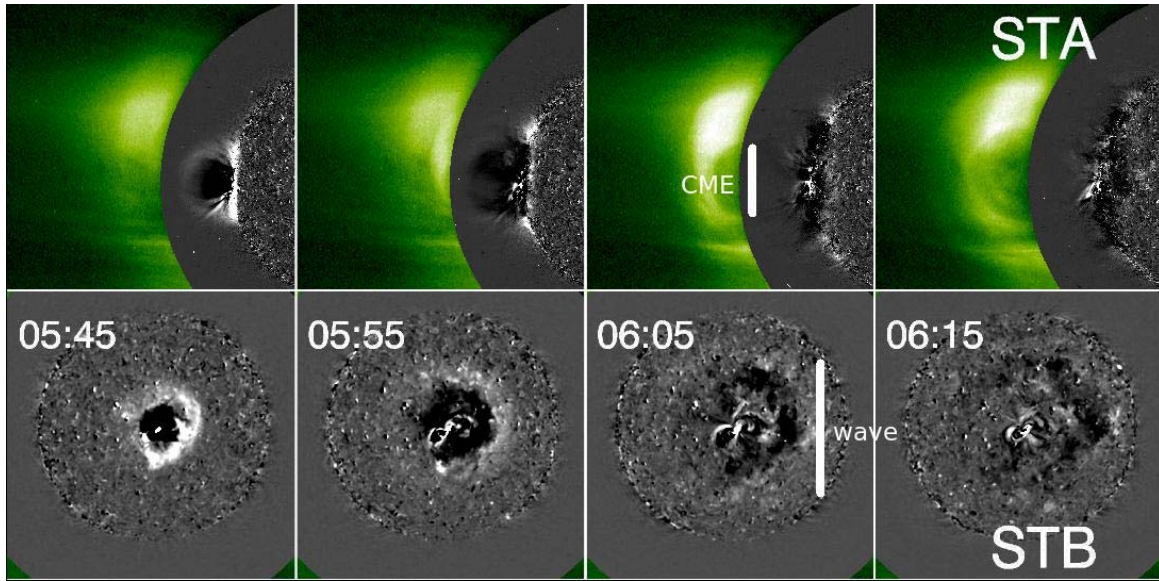
Problem 4 - Did Mira emit gas in a steady rate during the period of time estimated in Problem 1?

Answer: No, because the gas ejected by the star during the last 30,000 years is clumpy with many gaps. This means there were periods when the star was ejecting gas, and periods where this process seems to have temporarily stopped.

Problem 5 - Can you create a chronology for Mira that tells about the sequence of events in its history of ejecting gas during the time that the tail was produced?

Answer: The current period of gas loss seems to extend back to about 3000 years ago. There is then a gap of about 1,000 years when there seemed to be little gas production. Then about 4,000 years ago and extending back to about 23,000 years ago, the gas production was occurring but at a reduced level from recent times (more intense during the last 3,000 years). There is then another gap between about 23,000 years ago and 27,000 years ago with reduced gas production, followed by an earlier brief phase of gas production extending from about 27,000 to 30,000 years ago.

Note: Some of the fading of the tail may be due to the interaction of the gas with the interstellar medium which may be trying to dissipate the tail. Some of this interaction may also be partly responsible for the detailed clumpiness in the tail, so it is probably not a good idea to analyze the tail structure finer than centimeter differences.



A solar tsunami that occurred in February 13, 2009 has recently been identified in the data from NASA's STEREO satellites. It was spotted rushing across the Sun's surface. The blast hurled a billion-ton CME into space and sent a tsunami racing along the sun's surface. STEREO recorded the wave from two positions separated by 90 degrees, giving researchers a spectacular view of the event. Satellite A (STA) provided a side-view of the explosion, called a Coronal Mass Ejection (CME), while Satellite B (STB) viewed the explosion from directly above. The technical name is "fast-mode magnetohydrodynamic wave" – or "MHD wave" for short. The one STEREO saw raced outward at 560,000 mph (250 km/s) packing as much energy as 2,400 megatons of TNT.

Problem 1 - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Problem 2 - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? (Note '05:45' means 5:45 o'clock Universal Time).

Problem 3 - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

Problem 4 - Kinetic Energy is defined by the equation $K.E. = \frac{1}{2} m V^2$ where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

Problem 5 - If 1 kiloton of TNT has the explosive energy of 4.1×10^{12} Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

Problem 1 - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Answer: The diameter is 31 millimeters, which corresponds to $2 \times 696,000 \text{ km}$ or 1,392,000 km. The scale is then $1,392,000 \text{ km}/31\text{mm} = 45,000 \text{ km/mm}$.

Problem 2 - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? (Note '05:45' means 5:45 o'clock Universal Time).

Answer: From the scale of 45,000 km/mm, the difference in the ring radii is $12\text{mm} - 5 \text{ mm} = 7\text{mm}$ which corresponds to $7 \text{ mm} \times (45,000 \text{ km}/1 \text{ mm}) = 315,000 \text{ kilometers}$. Students answers may vary depending on where they defined the outer edge of the ring.

Problem 3 - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

Answer: The time difference is $06:15 - 05:45 = 30 \text{ minutes}$. The speed was about $315,000 \text{ km} / 30 \text{ minutes} = 11,000 \text{ kilometers/minute}$, which is $11,000 \text{ km/minute} \times (1 \text{ minute}/60 \text{ seconds}) = 180 \text{ kilometers/sec}$.

Problem 4 - Kinetic Energy is defined by the equation $K.E. = 1/2 m V^2$ where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

Answer: The mass of the CME was 1 billion metric tons. There are 1,000 kilograms in 1 metric ton, so the mass was 1.0×10^{12} kilograms. The speed is 180 km/sec which is 180,000 meters/sec. The kinetic energy is then about $0.5 \times 1.0 \times 10^{12} \times (180,000)^2 = 1.6 \times 10^{22} \text{ Joules}$.

Problem 5 - If 1 kiloton of TNT has the explosive energy of 4.1×10^{12} Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

Answer: $1.6 \times 10^{22} \text{ Joules} \times (1 \text{ kiloton TNT}/4.1 \times 10^{12} \text{ Joules}) = 3.9 \times 10^9 \text{ kilotons TNT}$. Since 1 megaton = 1,000 kilotons, we have an explosive yield of **3,900,000 megatons TNT**. (Note; this answer differs from the STEREO estimate because the speed is approximate, and does not include the curvature of the sun).

Teacher Note: Additional information, and movies of the event, can be found at the STEREO website: <http://stereo.gsfc.nasa.gov/news/SolarTsunami.shtml>. Also published in the Astrophysical Journal Letters (*ApJ* 700 L182-L186)