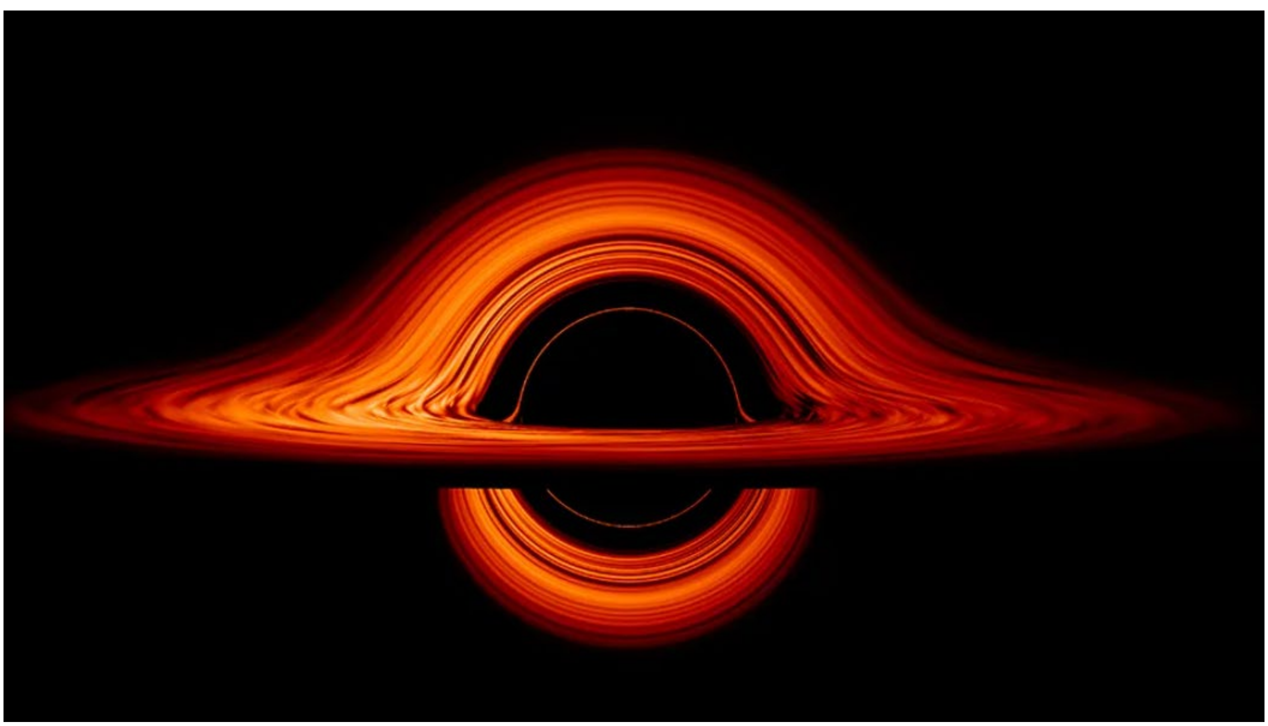
National Aeronautics and Space Administration

# Math Problems Related to

# Black Holes



## Introduction

Although stars like our sun eventually evolve into white dwarfs after 12 billion years, very massive stars end up as supernova and black holes. This book provides an assortment of 20 mathematics problems related to black holes including their sizes, gravity and their distortions to space and time.

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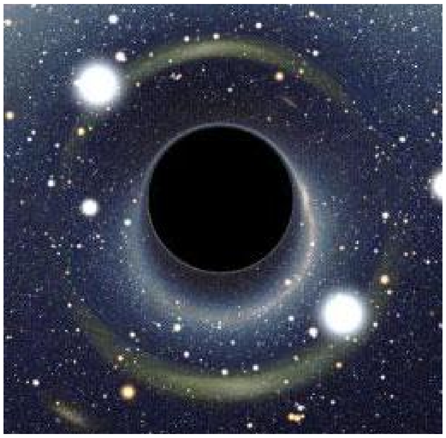
# The Moon as a Black Hole!





Suppose that a group of hostile aliens passed through our solar system and decided to convert our moon into a black hole!

A body with the mass of our moon (about 7 million trillion tons!) would be compressed into a black hole with a diameter of only 0.2 millimeters!

**Problem 1** – In the space below, draw a black disk 0.2 millimeters in diameter to represent the size of Black Hole Moon.

**Problem 2** - The Earth as a black hole would have a radius of 8.7 millimeters. In the space below, draw a circle the size of Black Hole Earth.

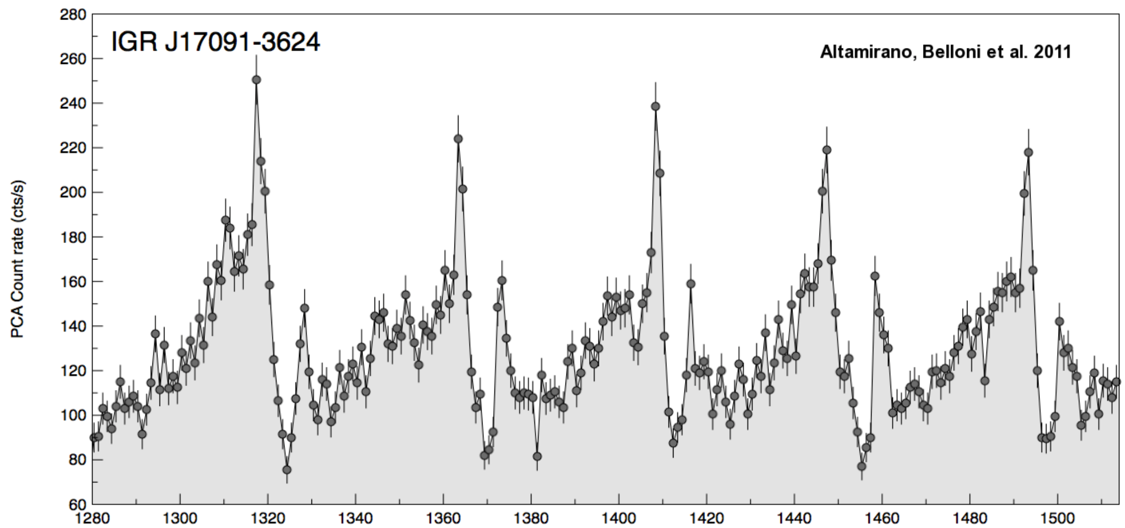
**Problem 3** - If the distance to the moon is 356,000 kilometers, how far from our Black Hole Earth would the new Black Hole Moon be located if its diameter were only 0.2 millimeters?

# Answer Key 1

**Problem 1 -** Moon black hole shown as the following dot .

**Problem 2** - Earth black hole shown above. It has a diameter of 17 millimeters, or about the same diameter as a dime.

**Problem 3** - Answer: It would still be 356,000 kilometers because this is NOT a scaled drawing of the black holes sizes, but an illustration of their actual sizes, so the distance between the black disks above would be 356,000 kilometers!




NASA's RXTE spacecraft recently recorded rhythmic x-ray flashes from the black hole candidate called IGR J17091-3624. The black hole is a member of a binary system that combines a normal star, with a black hole that may weigh three times the sun's mass. That is near the theoretical mass boundary where black holes become possible. The system is located in the direction of the constellation Scorpius between 16,000 light-years and 65,000 light-years from Earth.

Gas from the normal star streams toward the black hole and forms a disk around it. Friction within the disk heats the gas to millions of degrees, which is hot enough to emit X-rays. The very fast, cyclical variations in the x-ray light may be occurring near the black hole's event horizon - the point beyond which nothing, not even light, can escape.

**Problem 1** – The RXTE satellite measured the x-ray intensity of this star system hundreds of times an hour. The plot above (called a light curve) shows how this brightness changes over time. Each division is 10 seconds long. From this figure, what is the average period of the brightness changes in seconds?

**Problem 2** – The radius of a 3-solar mass black hole is about 10 kilometers. If the flickering has to do with the orbital motion of gasses at a distance of 15 kilometers, how fast is the gas orbiting the black hole in kilometers per second?

http://www.nasa.gov/topics/universe/features/black-hole-heartbeat.html

**NASA's RXTE Detects 'Heartbeat' of Smallest Black Hole Candidate**
**12.15.11**

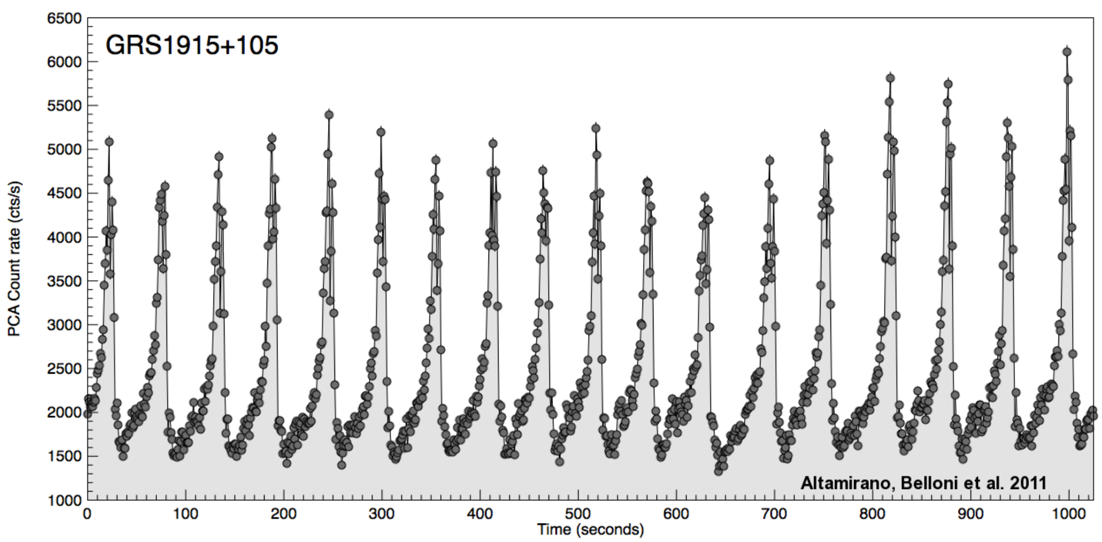
**Problem 1** – The RXTE satellite measured the x-ray brightess of this star system hundreds of times an hour, and the plot above shows how this brightness changes over time. Each division is 10 minutes long. From this figure, what is the average period of the brightness changes in seconds?

Answer: With the help of a millimeter ruler, the horizontal scale is about 1.7 seconds/mm so the peaks are separated by 27mm, 26mm, 22mm, 27mm or 46, 44, 37 and 46 seconds respectively. **The average period is 43 seconds.**

**Problem 2** – The radius of a 3-solar mass black hole is about 10 kilometers. If the flickering has to do with the orbital motion of gasses at a distance of 15 kilometers, how fast is the gas orbiting the black hole in kilometers per second?

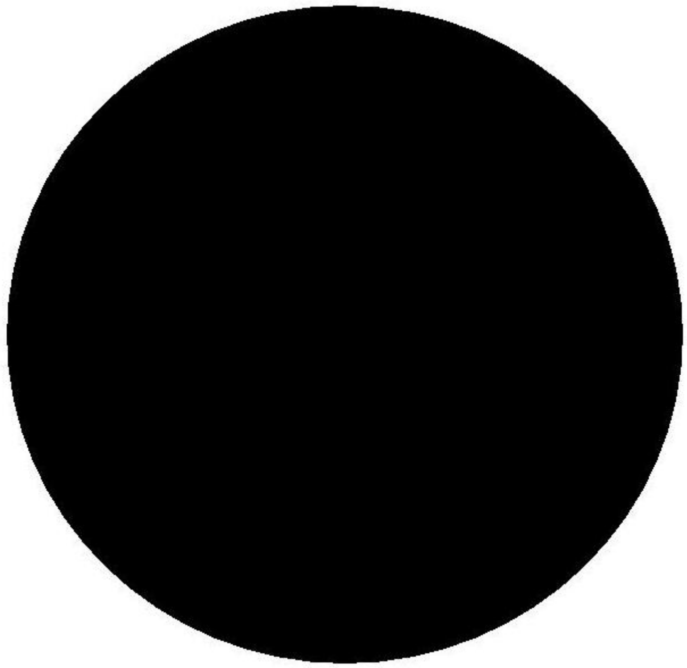
Answer: We know hoe long it takes the gas to go once-around so now we need the distance tralveled along the circumference of the orbit with a radius of 15 km.
C = 2 (3.14)(15 km) = 93.6 km. Then the speed is just the distance divided by time:
S = 94 km/43 seconds so **S = 2.2 km/sec**.

The graph below is another flickering black hole candidate. How fast does the gas travel around this black hole?



# Exploring a Full-Sized Black Hole 3

This black ball shown below is the exact size of a black hole with a diameter of 9.0 centimeters. Such a black hole would have a mass of **5 times** the mass of our Earth. All of this mass would be INSIDE the ball below.



Although it looks pretty harmless, if this black hole were at arms-length, you would already be dead. In fact, if you were closer to it than the distance from New York to San Francisco, 1 150-pound person would weigh 3 tons and would be crushed by their own weight!

Suppose that you could survive being crushed to death as you got closer to the black hole shown above. To stay in an orbit around the black hole so that you did not fall in, you have to be traveling at a specific speed V, in kilometers per second, that depends on your distance R, in meters from the center of the black hole, is given below:

$$V = \frac{44,700}{\sqrt{R}}$$

**Problem 1** - If you were orbiting at the distance of the Space Shuttle (R=6,800 km) from the center of this black hole, what would your orbital speed be in A) kilometers/sec? B) kilometers/hour? C) miles per hour (1 mile = 1.6 km).

**Problem 2** - If a small satellite were orbiting 20 centimeters away from the center of the black hole shown above, how fast would it be traveling in A) km/second? B) percentage of the speed of light? (The speed of light = 300,000 km/sec).

**Problem 3)** If the orbit is a circle, how long: A) would the Space Shuttle in Problem 1 take to go once around in its orbit? B) would it take the satellite in Problem 2 to go once around in its orbit?

**Problem 1** - If you were orbiting at the distance of the Space Shuttle (R=6,800 km) from the center of this black hole, what would your orbital speed be in A) kilometers/sec? B) kilometers/hour? C) miles per hour (1 mile = 1.6 km).

Answer; A) The formula says that for R = 6,800,000 meters, **V = 17 km/sec**.
B) 1 hour = 3600 seconds, so V = 17 km/sec x (3600 sec/1 hour) = **61,200 km/hour.**
C) V = 61,200 km/sec x (1 mile / 1.6 km) = **38,250 miles/hr**

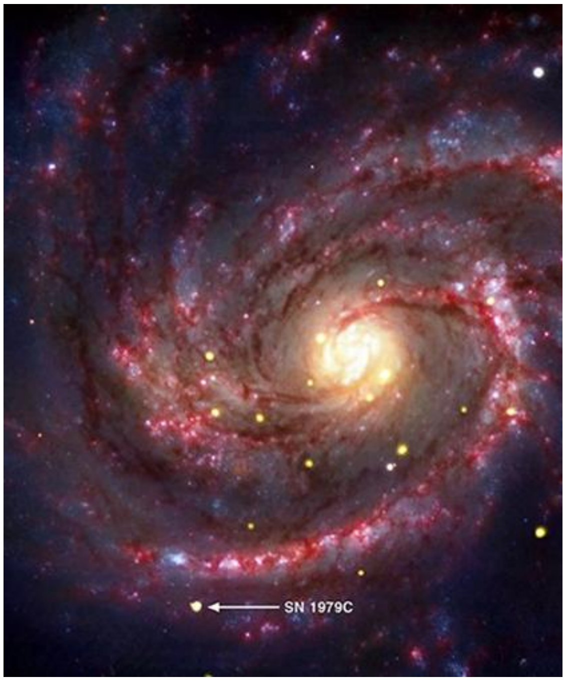
**Problem 2** - If a small satellite were orbiting 20 centimeters away from the center of the black hole shown above, how fast would it be traveling in A) km/second? B) percentage of the speed of light? (The speed of light = 300,000 km/sec).

**Answer;** A) R = 0.2 meters, so from the formula V = **100,000 km/sec**
B) Speed = 100% x (100000/300000) so speed = **33% the speed of light**.

**Problem 3)** If the orbit is a circle, how long: A) would the Space Shuttle in Problem 1 take to go once around in its orbit? B) would it take the satellite in Problem 2 to go once around in its orbit?

A) Orbit circumference, $C = 2\pi R$ so for R = 6,800 km, C = 40,000 kilometers. The Shuttle speed is V=17 km/sec, so the time is T = C/V or **2,353 seconds. This equals about 39 minutes.**

B) A) Orbit circumference, $C = 2\pi R$ so for R = 0.2 meters, C = 1.25 meters. The satellite speed is V=100,000 km/sec. Converting this to meters we get 100,000,000 meters/sec, so the time is T = C/V or **0.000000013 seconds. This is 13 billionths of a second!**




The Chandra X-Ray Observatory recently confirmed the discovery of an infant black hole in the nearby galaxy Messiar-100. The product of the supernova of a star with a mass of 20 times our sun, the resulting black hole may only involve about 8 times our sun's mass.

For black holes that do not rotate, called Schwarschild Black Holes, there are several different sizes for such black holes that all scale with the mass of the black hole. When referring to the size of a black hole, astronomers usually mention its mass, which is well defined, rather than its diameter, which depends on the specific kinds of physical processes involved.

**The Schwarschild Radius** – This is the distance from the center of the black hole at which an incoming person, or light signal, can enter the black hole interior, but cannot emerge back out into the universe. It is also called the Event Horizon. It is a perfectly spherical surface with a radius of Rs = 3.0 M kilometers, where M is the mass of the black hole in multiples of the sun's mass (1 M = 2.0 x $10^{30}$ kg). For the SN1979C black hole, with an estimated mass of 8 M, what is its Schwarschild Radius, Rs, in kilometers?

**Last Photon Orbit** – At this distance outside the Event Horizon, an incoming photon of light can enter into an exactly circular orbit, where it will stay until it is disturbed, at which time it will fall into the Event Horizon and never get back out. If Rp =1.5 Rs, what is the radius of the photon orbit for black hole SN1979C in kilometers?

**Last Stable Particle Orbit** – Inside this distance, a material particle cannot be in a stable circular orbit, but is relentlessly dragged to the Event Horizon and disappears. This occurs at a distance from the black hole center of Rl = 3.0 Rs. How close can a hydrogen atom, an asteroid or a planet remain in a stable circular orbit around the SN1979C black hole?

**Problem** – An asteroid is spotted at a distance of 700 km from a black hole with a mass of 120 solar masses. Can it escape or remain where it is?

NASA Press release 'Youngest Nearby Black Hole' November 15, 2010

*"Data from Chandra, as well as NASA's Swift, the European Space Agency's XMM-Newton and the German ROSAT observatory revealed a bright source of X-rays that has remained steady for the 12 years from 1995 to 2007 over which it has been observed. This behavior and the X-ray spectrum, or distribution of X-rays with energy, support the idea that the object in SN 1979C is a black hole being fed either by material falling back into the black hole after the supernova, or from a binary companion.*

*The scientists think that SN 1979C formed when a star about 20 times more massive than the Sun collapsed. It was a particular type of supernova where the exploded star had ejected some, but not all of its outer, hydrogen-rich envelope before the explosion, so it is unlikely to have been associated with a gamma-ray burst (GRB). Supernovas have sometimes been associated with GRBs, but only where the exploded star had completely lost its hydrogen envelope. Since most black holes should form when the core of a star collapses and a gamma-ray burst is not produced, this may be the first time that the common way of making a black hole has been observed.*

*The very young age of about 30 years for the black hole is the observed value, that is the age of the remnant as it appears in the image. Astronomers quote ages in this way because of the observational nature of their field, where their knowledge of the Universe is based almost entirely on the electromagnetic radiation received by telescopes."*
(**http://www.nasa.gov/mission_pages/chandra/multimedia/photoH-10-299.html)**

**The Schwarschild Radius** – This is the distance from the center of the black hole at which an incoming person, or light signal, can enter the black hole interior, but cannot emerge back out into the universe. It is also called the Event Horizon. It is a perfectly spherical surface with a radius of Rs = 3.0 M kilometers, where M is the mass of the black hole in multiples of the sun's mass (1 M = 2.0 x $10^{30}$ kg). For the SN1979C black hole, with an estimated mass of 8 M, what is its Schwarschild Radius, Rs, in kilometers?

Answer: M = 8, so Rs = 3.0 x 8 = **24 kilometers**.

**Last Photon Orbit** – At this distance outside the Event Horizon, an incoming photon of light can enter into an exactly circular orbit, where it will stay until it is disturbed, at which time it will fall into the Event Horizon and never get back out. If Rp =1.5 Rs, what is the radius of the photon orbit for black hole SN1979C in kilometers?

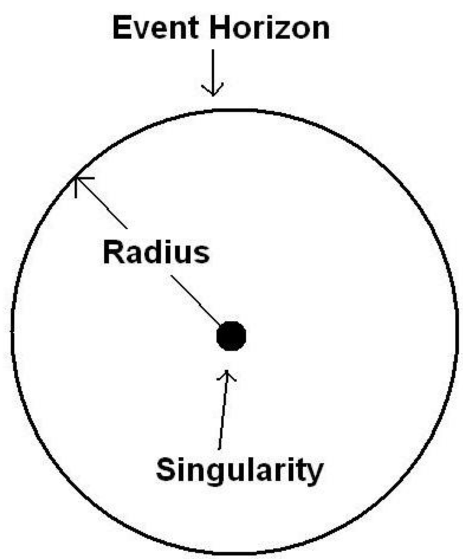
Answer: Rs = 24 kilometers so Rp = 1.5 x 24 km = **36 kilometers**.

**Last Stable Particle Orbit** – Inside this distance, a material particle cannot be in a stable circular orbit, but is relentlessly dragged to the Event Horizon and disappears. This occurs at a distance from the black hole center of Rl = 3.0 Rs. How close can a hydrogen atom, an asteroid or a planet remain in a stable circular orbit around the SN1979C black hole?

Answer: R = 3.0 x 24 km = **72 kilometers**.

**Problem** – An asteroid is spotted at a distance of 700 km from a black hole with a mass of 120 solar masses. Can it escape or remain where it is? Answer: Rs = 3.0 x 120 = 360 kilometers. Rp = 1.5 x 360 km = 540 km; Rl = 3.0 x 360 km = 1080 km. **Since the asteroid is at 700 km, it is inside the distance where it can remain in a stable orbit, so it is about to fall through the black hole's event horizon located some 700-360 = 340 kilometers inside its current position.**

# Black Holes.....The Event Horizon




$$\text{Radius} = \frac{2\,G\,M}{c^2}$$

where $c = 3 \times 10^{10}$ cm/sec

$G = 6.67 \times 10^{-8}$ dynes cm$^2$ / gm$^2$

you get

$$\text{Radius} = 1.48 \times 10^{-28}\ \mathbf{M}\ \text{centimeters}$$

Black holes are objects that have such intense gravitational fields, they do not allow light to escape from them. They also make it impossible for anything that falls into them to escape, because to do so, they would have to travel at speeds faster than light. No forms of matter or energy can travel faster than the speed of light, so that is why black holes are so unusual!

### There are three parts to a simple black hole:

**Event Horizon -** That's the part that we see from the outside. It looks like a black, spherical surface with a very sharp edge in space.

**Interior Space** - This is a complicated region where space and time can get horribly mangled, compressed, stretched, and otherwise a very bad place to travel through.

**Singularity -** That's the place that matter goes when it falls through the event horizon. It's located at the center of the black hole, and it has an enormous density. You will be crushed into quarks long before you get there!

Black holes can, in theory, come in any imaginable size. The size of a black hole depends on the amount of mass it contains. It's a very simple formula, especially if the black hole is not rotating. These 'non-rotating' black holes are called Schwarzschild Black Holes.

Problem 1 - The formula in red gives the Schwarzschild radius of a black hole, in centimeters, in terms of its mass, in grams. From the equation for the radius in terms of the speed of light, c, and the constant of gravity, G, verify the formula shown in red.

Problem 2 - Calculate the Schwarzschild radius, in centimeters, for Earth where M = $5.7 \times 10^{27}$ grams. (Express answer to two significant figures)

Problem 3 - Calculate the Schwarzschild radius, in kilometers, for the sun, where M = $1.98 \times 10^{33}$ grams. (Express answer to two significant figures)

Problem 4 - Calculate the Schwarzschild radius, in kilometers, for the entire Milky Way, with a mass of 250 billion suns. (Express answer to two significant figures)

Problem 5 - Calculate the Schwarzschild radius, in centimeters, for a black hole with a mass of an average human being with M = 60 kilograms. (Express answer to two significant figures)

## Answer Key:

**Problem 1** - The formula in red gives the Schwarzschild radius of a black hole, in centimeters, in terms of its mass, in grams. From the equation for the radius in terms of the speed of light, c, and the constant of gravity, G, verify the formula shown in red.

Answer: Radius = $2 \times (6.67 \times 10^{-8}) / (3 \times 10^{10})^2$ M

**= $1.5 \times 10^{-28}$ M centimeters**

where M is the mass of the black hole in grams.

**Problem 2** - Calculate the Schwarzschild radius, in centimeters, for Earth where M = $5.7 \times 10^{27}$ grams.

Answer: R = $1.48 \times 10^{-28}$ ( $5.7 \times 10^{27}$ ) centimeters

**R = 0.84 centimeters !**

Problem 3 - Calculate the Schwarzschild radius, in kilometers, for the sun, where M = 1.98 x $10^{\mathbf{33}}$ grams.

Answer: R = $1.48 \times 10^{-28}$ ( $1.98 \times 10^{33}$ ) centimeters

R = 293,000 centimeters
**= 2.9 kilometers**

Problem 4 - Calculate the Schwarzschild radius, in kilometers, for the entire Milky Way, with a mass of 250 billion suns.

Answer: If a black hole with the mass of the sun has a radius of 2.93 kilometers, a black hole with 250 billion times the sun's mass will be 250 billion times larger, or

R = (2.93 km / sun) x 250 billion suns = **730 billion kilometers**.

Note: The entire solar system has a radius of about 4.5 billion kilometers!

Problem 5 - Calculate the Schwarzschild radius, in centimeters, for a black hole with a mass of an average human being with M = 60 kilograms.

Answer: R = $1.48 \times 10^{-28}$ ( 60,000 ) centimeters

**R = $8.9 \times 10^{-24}$ centimeters.**

Note: A proton is only about $10^{-14}$ centimeters in diameter.

# Black Holes....II



$$T = t\sqrt{1 - \frac{2GM}{Rc^2}}$$

T = the time measured by someone located on a planet (seconds)

t = the time measured by someone located in space (seconds)

M = the mass of the planet (grams)

R = the distance to the far-away observer from the planet (cm)

And the natural constants are:
$G = 6.67 \times 10^{-8}$
$C = 3 \times 10^{10}$

The modern theory of gravity developed by Albert Einstein in 1915 leads to some very unusual predictions, which have all been verified by experiments.

One of the strangest ones is that two people will experience the passage of time very differently if one is standing on the surface of a planet, and the other one is in space. This is because the rate of time passing depends on the strength of the gravitational field that the observer is in.

For example, at the surface of a very dense neutron star, R = 20 km and $M = 1.9 \times 10^{33}$ grams, so

$$T = t\ (1- 0.15)^{1/2} = 0.92\ t$$

This means that for every hour that goes by on the surface of the neutron star (T = 3600 seconds), someone in space will see t = 3600 / 0.92 = 3913 seconds pass from a vantage point in space.

Problem 1 - The GPS satellites orbit Earth at a distance of R = 26,560 kilometers. If the mass of Earth is $5.9 \times 10^{27}$ grams, use the formula to determine the time dilation factor. Be very careful with the small numbers in the 9th, 10th and 11th decimal places!

Problem 2 - What is the time dilation factor at Earth's surface?

Problem 3 - What is the ratio of the dilation in space to the dilation at earth's surface?

Problem 4 - At the speed of light ($3 \times 10^{10}$ cm/sec) how long does it take a radio signal from the GPS satellite to travel 26,560 kilometers to a hand-held GPS receiver?

Problem 5 - The excess time delay between a receiver at Earth's surface, and the GPS satellite is defined by the ratio computed in Problem 3, multiplied by the total travel time in Problem 4. What is the time delay for the GPS-Earth system?

Problem 6 - From your answer to Problem 5, how much extra time does the radio signal take compared to your answer to Problem 4?

Problem 7 - At the speed of light, how far will the radio signal travel during the extra amount of time?

Problem 8 - Is gravitational time delay an important phenomenon to include when using the GPS satellite system?

## Answer Key:

Problem 1 - The GPS satellites orbit Earth at a distance of R = 26,560 kilometers. If the mass of Earth is 5.9 x $10^{27}$ grams, use the formula to determine the time dilation factor. Be very careful with the small numbers in the 9th, 10th and 11th decimal places!

Answer: ( 1 - 0.84/2.65x$10^{9}$ cm)1/2 =
( 1 - 3.1 x $10^{-10}$) 1/2 =
(0.99999999969)1/2 = 0.99999999984

Problem 2 - What is the time dilation factor at Earth's surface?
( 1 - 0.84/6.38x$10^{8}$ cm)1/2 =
( 1 - 1.3 x $10^{-9}$) 1/2 =
(0.9999999987)1/2 = 0.99999999934

Problem 3 - What is the ratio of the dilation in space to the dilation at Earth's surface?
Answer - 0.99999999984 / 0.99999999934 = 1.0000000064

Problem 4 - How long does it take a radio signal from the GPS satellite to travel 26,560 kilometers to a hand-held GPS receiver?
Answer - Distance = 26,560 kilometers x (100,000 cm / kilometer) = 2.65 x $10^{9}$ centimeters.
Time = Distance / speed of light
= 2.65 x $10^{9}$ **cm** / 3 x $10^{10}$ cm/sec = 0.088 seconds.

Problem 5 - The excess time delay between a receiver at Earth's surface, and the GPS satellite is defined by the ratio computed in Problem 3, multiplied by the total travel time in Problem 4. What is the time delay for the GPS-Earth system?

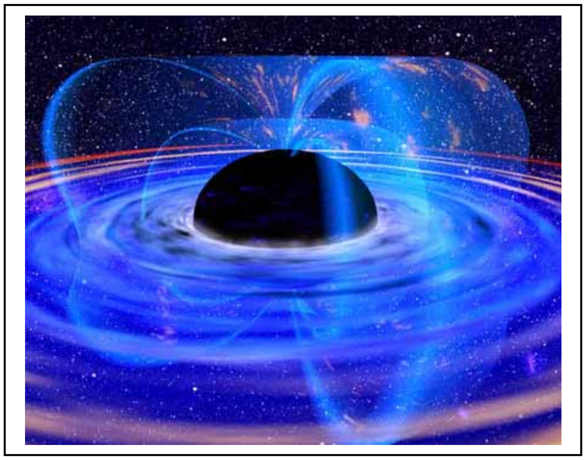
Answer - 0.088 seconds * 1.0000000064 = 0.08800000056 seconds.

Problem 6 - From your answer to Problem 5, how much extra time does the radio signal take compared to your answer to Problem 4?
Answer - 0.08800000056 - 0.088 seconds = 0.00000000056 seconds.

Problem 7 - At the speed of light, how far will the radio signal travel during the extra amount of time?
Answer = 3 x $10^{10}$ cm/sec x 5.6 x $10^{-10}$ sec = 16.8 centimeters.

Problem 8 - Is gravitational time delay an important phenomenon to include when using the GPS satellite system?
Answer: Yes!

# Black Holes....III



Artists illustration of a black hole with an orbiting disk of gas and dust. Friction in the disk causes matter to steadily flow inwards until it reaches the black hole event horizon. Magnetic forces in the disk cause matter to flow in complex jets and plumes. Time dilation causes delays in events taking place near the black hole compared to what distant observers will record.

Time dilation near a black hole is a lot more extreme than what the GPS satellite network experiences in orbit around Earth (See Problem 29).

$$\boxed{T = t\sqrt{1 - \frac{2GM}{Rc^2}}}$$

T = the time measured by someone located on a planet (seconds)

t = the time measured by someone located in space (seconds)

M = the mass of the planet (grams)

R = the distance to the far-away observer from the planet (cm)

And the natural constants are:
$G = 6.67 \times 10^{-8}$
$C = 3 \times 10^{10}$

Problem 1 - In the time dilation formula above, evaluate the quantity $2 G M /c^2$ for a black hole with a mass of one solar mass ($1.9 \times 10^{33}$ grams), and convert the answer to kilometers.

Problem 2 - Re-write the formula in a more tidy form using your answer to Problem 1.

Problem 3 - In the far future, a scientific outpost has been placed in orbit around this solar-mass black hole at a distance of 10 kilometers. What will the time dilation factor be at this location?

Problem 4 - A series of clock ticks were sent out by the satellite once each hour .What will be the time interval between the clock ticks by the time they reach a distant observer?

Problem 5 - If one tick arrived at 1:00 PM at the distant observer, when will the next clock tick arrive?

Problem 6 - A radio signal was sent by the black hole outpost to a distant observer. At the frequency of the signal, when transmitted from the outpost, the individual wavelengths take 0.000001 seconds to complete one cycle. From your answer to Problem 3, how much longer will they take by the time they arrive at the distant observer?

## Answer Key:

Problem 1 - In the time dilation formula above, evaluate the quantity 2 G M /$c^2$ for a black hole with a mass of one solar mass (1.9 x $10^{33}$ grams), and convert the answer to kilometers.

Answer - 2 x 6.67 x $10^{-8}$ x 1.9 x $10^{33}$ / (3 x $10^{10})^2$ = 281,600 centimeters or 2.82 kilometers.

Problem 2 - Re-write the formula in a more tidy form using your answer to Problem 1.

Answer -

$$T = t \ ( 1 - 2.82/R)^{1/2}$$

where R is in units of kilometers.

Problem 3 - In the far future, a scientific outpost has been placed in orbit around this solar-mass black hole at a distance of 10 kilometers. What will the time dilation factor be at this location?

Answer - $(1 - 2.82/10)^{1/2}$ = $(0.718)^{1/2}$ = 0.847

Problem 4 - A series of clock ticks were sent out by the satellite once each hour .What will be the time interval between the clock ticks by the time they reach a distant observer?

Answer - Time interval = 3600 / 0.847 = 4,250 seconds.

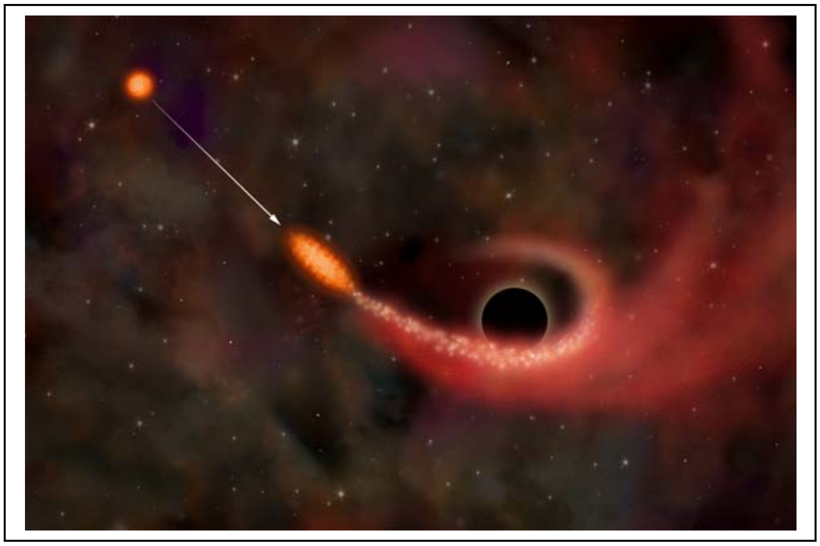
Problem 5 - If one tick arrived at 1:00 PM at the distant observer, when will the next clock tick arrive?

Answer - 1:00 PM + 4250 seconds = 1:00 PM + 1 Hour + (4250-3600) = 2:00 PM + 650 seconds = 2:10:50 PM

Problem 6 - A radio signal was sent by the black hole outpost to a distant observer. At the frequency of the signal, when transmitted from the outpost, the individual wavelengths take 0.000001 seconds to complete one cycle. From your answer to Problem 3, how much longer will they take by the time they arrive at the distant observer?

Answer - 0.000001 seconds / 0.847 = 0.00000118 seconds.

# Black Holes....IV



Thanks to two orbiting X-ray observatories, astronomers have the first strong evidence of a supermassive black hole ripping apart a star and consuming a portion of it. The event, captured by NASA's Chandra and ESA's XMM-Newton X-ray Observatories, had long been predicted by theory, but never confirmed ... until now. Giant black holes in just the right mass range would pull on the front of a closely passing star much more strongly than on the back. Such a strong tidal force would stretch out a star and likely cause some of the star's gasses to fall into the black hole. The infalling gas has been predicted to emit just the same blast of X-rays that have recently been seen in the center of galaxy RX J1242-11 located 700 million light years from the Milky Way, in the constellation Virgo. (News Report at: http://science.msfc.nasa.gov/headlines/y2004/18feb_mayhem.htm, see also NASA report at http://chandra.harvard.edu/photo/2004/rxj1242/

Problem 1 - The Schwarschild Radius of a black hole is given by the formula R = 2.83 M, where R is the radius in kilometers, and M is that mass of the black hole in units of the sun's mass. A supermassive black hole can have a mass of 100 million times the sun. What is its Schwarschild radius in: A) kilometers B) Multiples of the Earth orbit radius called an Astronomical Unit (1 AU = 147 million kilometers). C) Compared to the orbit of Mars ( 1.5 U)

Problem 2 - Black holes are one of the most efficient phenomena in converting matter into energy. As matter falls inward in an orbiting disk of gas, friction heats the gas up, and the energy released can be as much as 7% of the rest mass energy of the infalling matter. The quasar 3C273 has a power output of $3.8 \times 10^{45}$ ergs/second. If E = mc2 is the formula that converts mass (in grams) into energy (in ergs) and c = the speed of light, $3 \times 10^{10}$ cm/sec, how many grams/second does this quasar luminosity imply?

Problem 3 - If the mass of the sun is $1.9 \times 10^{33}$ grams, how many suns per year have to be consumed by the 3C273 supermassive black hole: A) at 100% conversion efficiency? B) At the black hole conversion efficiency of 7%? Note: 7% efficiency means that for every 100 grams involved, 7 grams are converted into pure energy ( by $E=mc^2$ )

## Answer Key:

Problem 1 - The Schwarschild Radius of a black hole is given by the formula R = 2.83 M, where R is the radius in kilometers, and M is that mass of the black hole in units of the sun's mass. A supermassive black hole can have a mass of 100 million times the sun. What is its Schwarschild radius in: A) kilometers B) Multiples of the Earth orbit radius called an Astronomical Unit (1 AU = 147 million kilometers). C) Compared to the orbit of Mars ( 1.5 U)

Answer: A) R = 283 million kilometers B) 283 million / 147 million = 1.9 AU. C) 1.9/1.5 = 1.3 Mars Orbit. The Event Horizon would be just beyond the orbit of Mars!

Problem 2 - Black holes are one of the most efficient phenomena in converting matter into energy. As matter falls inward in an orbiting disk of gas, friction heats the gas up, and the energy released can be as much as 7% of the rest mass energy of the infalling matter. The quasar 3C273 has a power output of $3.8 \times 10^{45}$ ergs/second. If $E = mc^2$ is the formula that converts mass (in grams) into energy (in ergs) and c = the speed of light, $3 \times 10^{10}$ cm/sec, how many grams per year does this quasar luminosity imply if 1 year = $3.1 \times 10^{7}$ seconds?

Answer: $3.8 \times 10^{45}$ ergs/second x ($3.1 \times 10^{7}$ seconds/year) / $(3 \times 10^{10})^2$
= $1.3 \times 10^{32}$ grams/year

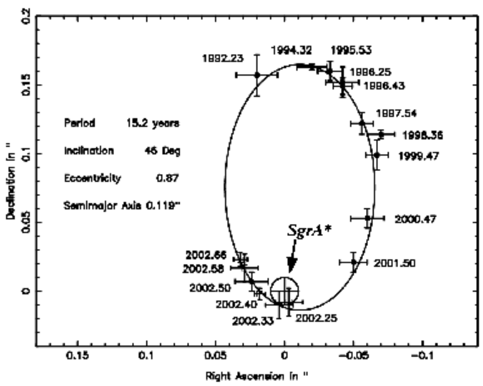
Problem 3 - If the mass of the sun is $1.9 \times 10^{33}$ grams, how many suns per year have to be consumed by the 3C273 supermassive black hole: A) at 100% conversion efficiency? B) At the black hole conversion efficiency of 7%?

Answer: A) ($1.3 \times 10^{32}$ grams/year) / ($1.9 \times 10^{33}$ grams/sun) = 0.07 suns per year

B) 7% efficiency means that for every 100 grams involved, 7 grams are converted into pure energy ( by $E=mc^2$ ). So,

0.07 suns per year / (7/100) = 1.0 suns per year for 7% efficiency.

# Black Holes.....V




At the center of our Milky Way Galaxy lies a black hole, called Sagittarius A*, with over 2.6 million times the mass of the Sun. Once a controversial claim, this astounding conclusion is now virtually inescapable and based on observations of stars orbiting very near the galactic center.

Astronomers patiently followed the orbit of a particular star, designated S2. Their results convincingly show that S2 is moving under the influence of the enormous gravity of an unseen object which must be extremely compact, and contain huge amounts of matter -- a supermassive black hole. The drawing shows the orbit shape.

The Chandra image above shows the x-ray light from a region of space a few light years across. The black hole is invisible, but is near the center of this image. The gas near the center produces x-ray light as it is heated. Many of the 'stars' in the field probably have much smaller black holes near them that are producing the x-ray light from the gas they are consuming.

Chandra image: http://chandra.harvard.edu/photo/2003/0203long/

Problem 1 - Kepler's Third Law can be used to determine the mass of a body by measuring the orbital period, T, and orbit radius, R, of a satellite. If R is given in units of the Astronomical Unit (AU) and T is in years, the relationship becomes $R^3 / T^2 = M$, where M is the mass of the body in multiples of the sun's mass. In these units, for Earth, R = 1.0 AU, and T = 1 year, so M = 1.0 solar masses. In 2006, the Hubble Space Telescope, found that the star Polaris has a companion, Polaris Ab, whose distance from Polaris is 18.5 AU and has a period of 30 years. What is the mass of Polaris?

Problem 2 - The star S2 orbits the supermassive black hole Sagittarius A*. Its period is 15.2 years, and its orbit distance is about 840 AU. What is the estimated mass of the black hole at the center of the Milky Way?

## Answer Key:

**Problem 1** - Kepler's Third Law can be used to determine the mass of a body by measuring the orbital period, T, and orbit radius, R, of a satellite. If R is given in units of the Astronomical Unit (AU) and T is in years, the relationship becomes $R^3 / T^2 = M$, where M is the mass of the body in multiples of the sun's mass. In these units, for Earth, R = 1.0 AU, and T = 1 year, so M = 1.0 solar masses. In 2006, the Hubble Space Telescope, found that the star Polaris has a companion, Polaris Ab, whose distance from Polaris is 18.5 AU and has a period of 30 years. What is the mass of Polaris?

Answer: $M = (18.5)^3 / (30)^2 = 7.0$ solar masses.

**Problem 2** - The star S2 orbits the supermassive black hole Sagittarius A*. Its period is 15.2 years, and its orbit distance is about 840 AU. What is the estimated mass of the black hole at the center of the Milky Way?

Answer: $M = (840)^3 / (15.2)^2 = 2.6 \times 10^6$ solar masses.

The infrared image below shows the central few light years of the Milky Way. The box contains the location of the supermassive black hole and Sagittarius A*. (Courtesy ESA - NAOS)

# Black Holes and Tidal Forces

$$a = \frac{2GMd}{R^3}$$

A tidal force is a difference in the strength of gravity between two points. The gravitational field of the moon produces a tidal force across the diameter of Earth, which causes the Earth to deform. It also raises tides of several meters in the solid Earth, and larger tides in the liquid oceans.

If the tidal force is stronger than a body's cohesiveness, the body will be disrupted. The minimum distance that a satellite comes to a planet before it is shattered this way is called its Roche Distance. The artistic image to the left shows what tidal disruption could be like for an unlucky moon.

A human falling into a black hole will also experience tidal forces. In most cases these will be lethal! The difference in acceleration between the head and feet could be many thousands of Earth Gravities. A person would literally be pulled apart! Some physicists have termed this process spaghettification!

**Problem 1** - The equation lets us calculate the tidal acceleration, **a**, across a body with a length of **d**. The acceleration of gravity on Earth's surface is 979 cm/sec$^2$. The tidal acceleration between your head and feet is given by the above formula. For M = the mass of Earth (5.9 x $10^{27}$ grams), R = the radius of Earth (6.4 x $10^{8}$ cm) and the constant of gravity whose value is G = 6.67 x $10^{-8}$ dynes cm$^2$/gm$^2$ calculate the tidal acceleration, a, if d = 2 meters.

**Problem 2** - What is the tidal acceleration across the full diameter of Earth?

**Problem 3** - A stellar-mass black hole has the mass of the sun (1.9 x $10^{33}$ grams), and a radius of 2.9 kilometers. A) What would be the tidal acceleration across a human at a distance of 100 kilometers? B) Would a human be spaghettified?

**Problem 4** - A supermassive black hole has 100 million times the mass of the sun (1.9 x $10^{33}$ grams), and a radius of 295 million kilometers. What would be the tidal acceleration across a human at a distance of 100 kilometers from the event horizon of the supermassive black hole?

**Problem 5** - Which black hole could a human enter without being spagettified?

## Answer Key:

**Problem 1** - The equation lets us calculate the tidal acceleration, **a**, across a body with a length of **d**. The acceleration of gravity on Earth's surface is 979 cm/sec$^{2}$. The tidal acceleration between your head and feet is given by the above formula. For M = the mass of Earth ($5.9 \times 10^{27}$ grams), R = the radius of Earth ($6.4 \times 10^{8}$ cm) and the constant of gravity whose value is G = $6.67 \times 10^{-8}$ dynes cm$^2$/gm$^2$ calculate the tidal acceleration, a, if d = 2 meters.

Answer: $a = 2 \times (6.67 \times 10^{-8}) \times (5.9 \times 10^{27}) \times 200 / (6.4 \times 10^{8})^{3}$
$= 0.000003 \times (200)$
$= 0.0006$ cm/sec$^{2}$

**Problem 2** - What is the tidal acceleration across the full diameter of Earth?
Answer: $d = 1.28 \times 10^{9}$ cm, so $a = 0.000003 \times 1.28 \times 10^{9}$ = 3,840 cm/sec$^{2}$

**Problem 3** - A stellar-mass black hole has the mass of the sun ($1.9 \times 10^{33}$ grams), and a radius of 2.9 kilometers. A) What would be the tidal acceleration across a human at a distance of 100 kilometers? B) Would a human be spaghettified?

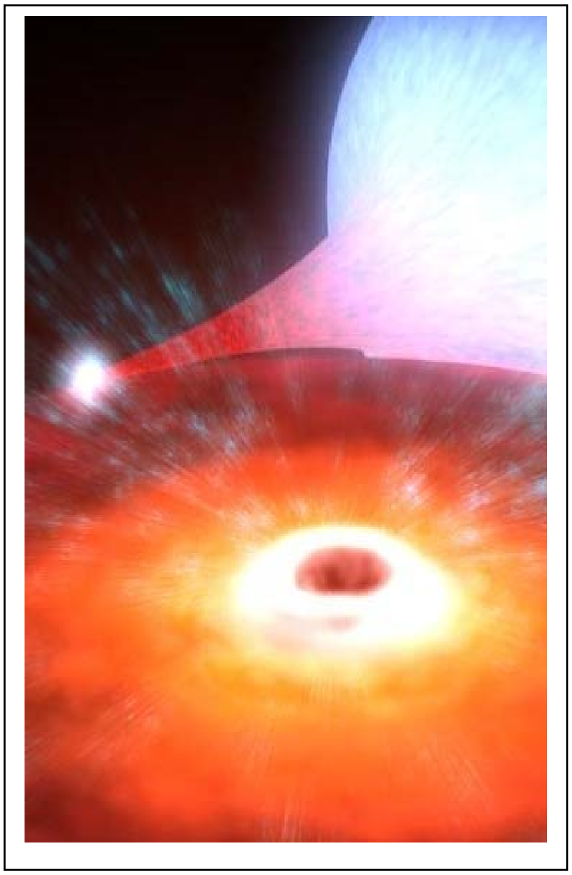
Answer: $a = 2 \times (6.67 \times 10^{-8}) \times (1.9 \times 10^{33}) \times 200 / (1.0 \times 10^{7})^{3}$
= 50,700,000 cm/sec$^{2}$
Yes, this is equal to 50,700,000/979 = 51,700 times the acceleration of gravity, and a human would be pulled apart and 'spaghettified'

**Problem 4** - A supermassive black hole has 100 million times the mass of the sun ($1.9 \times 10^{33}$ grams), and an event horizon radius of 295 million kilometers. What would be the tidal acceleration across a d=2 meter human at a distance of 100 kilometers from the event horizon of the supermassive black hole?

Answer: $a = 2 \times (6.67 \times 10^{-8}) \times (1.9 \times 10^{41}) \times 200 / (2.95 \times 10^{13})^{3}$
= 0.00020 cm/sec$^{2}$
Note that R + 2 meters is essentially R if R = 295 million kilometers.

**Problem 5** - Which black hole could a human enter without being spaghettified?
Answer: The supermassive black hole, because the tidal force is far less than what a human normally experiences on the surface of Earth. That raises the question that as a space traveler, you could find yourself trapped by a supermassive black hole unless you knew exactly what its size was before hand. You would have no physical sensation of having crossed over the black hole's Event Horizon before it was too late.

# Falling Into a Black Hole



An object that falls into a black hole will cross the Event Horizon, and speed up as it gets closer. This is like a ball traveling faster and faster as it is dropped from a tall building. Suppose the particle fell 'from infinity', how fast would it be traveling? We can answer this question by considering the concepts of kinetic energy (K.E.) and gravitational potential energy (P.E.):

$$K.E. = 1/2\ m\ V^2$$

$$P.E. = G\ M\ m\ /\ R$$

The kinetic energy that the particle with mass, m, will gain as it falls, will depend on the total potential energy it has lost in traveling from infinity to a distance R. By setting the two equations equal to each other, we can relate the kinetic energy a particle gains as it falls to its current distance of R from the center of mass. The quantity, M, is the mass of the gravitating body the particle is falling towards. G is the constant of gravitation which equals $6.67 \times 10^{-8}$ dynes $cm^2/gm^2$

$$1/2\ m\ V^2 = G\ M\ m\ /\ R$$

We can then solve for the speed, V, in terms of R

$$V = (2GM / R)^{1/2}$$

**Problem 1** - Suppose a body falls to Earth and strikes the ground. How fast will it be traveling when it hits if $M = 5.9 \times 10^{24}$ kilograms and R = 6,378 kilometers? Explain why this is the same as Earth's escape velocity?

**Problem 2** - NASA's ROSSI satellite was used in 2004 to determine the mass and radius of a neutron star in the binary star system named EXO 0748-676, located about 30,000 light-years away in the southern sky constellation Volans, or Flying Fish. The neutron star was deduced to have a mass of 1.8 times the sun, and a radius of 11.5 kilometers. A) How fast, in km/sec, will a particle strike the surface of the neutron star if the mass of the sun is $1.9 \times 10^{33}$ grams? B) In percent, what will the speed be compared to the speed of light, 300,000 km/sec?

**Problem 3** - The star HD226868 is a binary star with an unseen companion. It is also the most powerful source of X-rays in the sky second to the sun - it's called Cygnus X-1. Astronomers have determined the mass of this companion to be 8.7 times the sun. As a black hole, its Event Horizon radius would be R = 2.95 x 8.7 = 26 kilometers. A) How fast, in km/sec, would a body be traveling as it passed through the Event Horizon? B) In percentage compared to the speed of light?

## Answer Key:

**Problem 1** - Suppose a body falls to Earth and strikes the ground .How fast will it be traveling when it hits? Explain why this is the same as Earth's escape velocity?

Answer - R = 6,378 kilometers. M = $5.9 \times 10^{24}$ kilograms.
V = $(2 \times 6.67 \times 10^{-8} \times 5.9 \times 10^{29}/6.4 \times 10^{8})^{1/2} = 1.1 \times 10^{6}$ cm/sec or 11 kilometers/second. The particle 'fell' from infinity, so this means that if you gave a body a speed of 11 km/sec at Earth's surface, it would be able to travel to infinity and 'escape' from Earth.

**Problem 2** - NASA's ROSSi satellite was used in 2004 to determine the mass and radius of a neutron star in the binary star system named EXO 0748-676, located about 30,000 light-years away in the southern sky constellation Volans, or Flying Fish. The neutron star was deduced to have a mass of 1.8 times the sun, and a radius of 11.5 kilometers. A) How fast, in km/sec, will a particle strike the surface of the neutron star if the mass of the sun is $1.9 \times 10^{33}$ grams? B) In percent, what will the speed be compared to the speed of light, 300,000 km/sec?

Answer: A) Mass = $1.8 \times 1.9 \times 10^{33}$ grams = $3.4 \times 10^{33}$ grams. V = $(2 \times 6.67\times10^{-8} \times 3.4 \times 10^{33}/1.15 \times 10^{6})^{1/2} = 1.98 \times 10^{10}$ cm/sec = 198,000 km/sec.

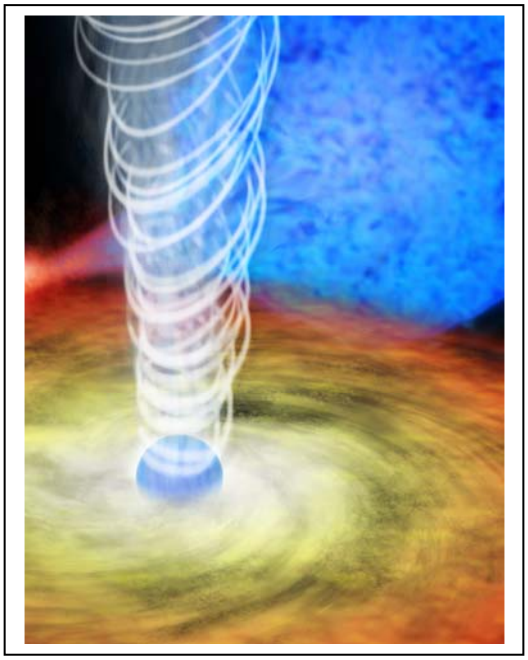
B) 198,000/300,000 = 66 % of the speed of light!

**Problem 3** - The star HD226868 is a binary star with an unseen companion. It is also the most powerful source of X-rays in the sky second to the sun - it's called Cygnus X-1. Astronomers have determined the mass of this companion to be 8.7 times the sun. As a black hole, its Event Horizon radius would be R = 2.95 km x 8.7 = 26 kilometers. A) How fast, in km/sec, would a body be traveling as it passed through the Event Horizon? B) In percentage compared to the speed of light?

Answer: A) Mass = $8.7 \times 1.9 \times 10^{33}$ grams = $1.7 \times 10^{34}$ grams. V = $(2 \times 6.67\times10^{-8} \times 1.7 \times 10^{34}/2.55 \times 10^{6})^{1/2} = 2.98 \times 10^{10}$ cm/sec = 298,000 km/sec.

B) 298,000/300,000 = 99.3% of the speed of light!

# Black Holes…Hot Stuff!



An artist's impression of a black hole orbiting a companion star, and gravitationally attracting gas from the star into an orbiting accretion disk. Through friction, the gas becomes hotter as it approaches the black hole, turning from red to yellow to white. Most of the gas is swallowed by the black hole, but some is magnetically launched in jets away from the black hole at almost the speed of light. (Credit: M. Weiss, NASA/Chandra)

The farther a particle falls towards a black hole, the faster it travels, and the more kinetic energy it has. Kinetic energy is mathematically defined as K.E. = 1/2 m $V^2$ where m is the mass of the particle and V is its speed.

Suppose all this energy is converted into heat energy by friction as the particle falls, and that this added energy causes nearby gases to heat up. How hot will the gas get? The equivalent amount of thermal energy, T.E., carried by a single particle is

$$T.E. = \frac{3}{2}kT$$

where Boltzman's Constant k = 1.38 x $10^{-23}$ Joules/deg. If we set K.E = T.E we get

$$T = \frac{mV^2}{3k}$$

If all the particles in a gas carried this same kinetic energy, then we would say the gas has a temperature of T in kelvins. We also know that the potential energy of the particle is given by

$$P.E. = \frac{GMm}{R}$$

So if we set P.E = T.E we also get the temperature

$$T = \frac{2GMm}{3kR}$$

**Problem 1** - The formula T = 2 G M m/(3kR) gives the approximate temperature of hydrogen gas (m = 1.6 x $10^{-27}$ kg) in an accretion disk around a black hole. To two significant figures, what is the temperature for the material at the distance of Earth's orbit for a solar-mass black hole? (R = 1.47 x $10^{11}$ m, M = 1.9 x $10^{30}$ kg, for the constant of gravity G = 6.67 x $10^{-11}$ Nt $m^2/kg^2$)?

**Problem 2** - How hot would the disk be at the distance of Neptune (R = 4.4 x $10^{12}$ meters)?

**Problem 3** - X-rays are the most common forms of energy produced at temperatures above 100,000 K. Visible light is produced at temperatures above 2,000 K. Infrared radiation is commonly produced for temperatures below 500 K. What would you expect to see if you studied the accretion disk around a solar-mass sized black hole?

# Answer Key:

**Problem 1** - The formula T = 2/3 G M m/kR gives the approximate temperature of hydrogen gas ($m = 1.6 \times 10^{-27}$ kg) in an accretion disk around a black hole. To two significant figures, what is the temperature for a solar-mass black hole disk near the orbit of Earth? ($R = 1.47 \times 10^{11}$ m, $M = 1.9 \times 10^{30}$ kg, for $G = 6.67 \times 10^{-11}$ Nt $m^2/kg^2$)?

Answer: $T = 2/3 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} \times 1.6 \times 10^{-27} / (1.38 \times 10^{-23} \times 1.47 \times 10^{11})$
**= 65,000 K.** to 2 significant figures

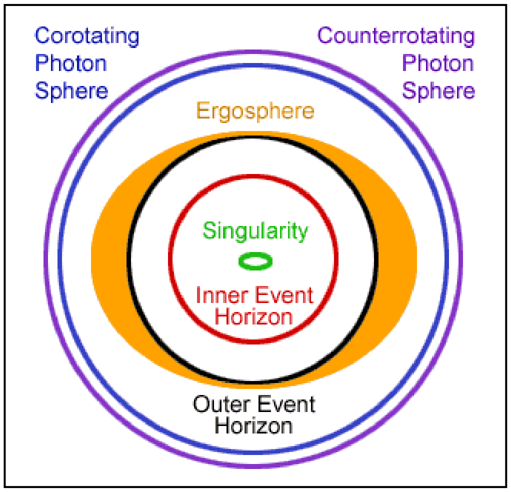
**Problem 2** - How hot would the disk be at the distance of Neptune ($R = 4.4 \times 10^{12}$ meters)?

Answer: $T = 2/3 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} \times 1.6 \times 10^{-27} / (1.38 \times 10^{-23} \times 4.4 \times 10^{12})$
= **2,200 K.**

**Problem 3** - X-rays are the most common forms of energy produced at temperatures above 100,000 K. Visible light is produced at temperatures above 2,000 K. What would you expect to see if you studied the accretion disk around a solar-mass sized black hole?

Answer: **The inner disk region would be an intense source of x-rays and visible light, because the gas is mostly at temperatures above 65,000 K. In the outer disk, the gas is much cooler and emits mostly visible or infrared light.**

# Black Holes.....What's inside IX




Outside a black hole, we have the normal universe of space, time, matter and energy we have all come to know. But inside, things are very different. We know this because the same mathematics that predicts black holes should exist, also predicts what to find inside them. One of the biggest surprises is the way that time and space, themselves, behave.

Inside a black hole, the roles played by time and space reverse themselves. Before we are crushed to death by the Singularity, we have limited freedom to intentionally move through space, but some freedom in how we travel through time...in our last moments of life!

Problem 1 - In relativity, space and time are part of a single 4-dimensional thing called spacetime. There are 3 dimensions to space and 1-dimension to time. Every point, called an Event, has three coordinates to describe its location in space, and one extra coordinate to describe its location in time. We write these as an ordered set like A(x,y,z,t) or B(x,y,z,t). Write the ordered set for the following event called A: I travel north 5 miles, east 3 miles, and up 1 miles, at 9:00AM on February 16, 2008.

Problem 2 - Suppose I travel from one event with coordinates A(3 km, 6 km, 2 km, 5:00PM) to another event B(5 km, 7 km, 3 km, 8:00PM). How far did I travel in space during the time interval from 5:00 PM to 8:00 PM?

Problem 3 - Use the Pythagorean Theorem to calculate the actual distance in space in Problem 2.

Problem 4 - The 4-dimensional, 'hyper' distance between the events is found by using the 'hyperbolic' Pythagorean Theorem formula $D^2 = -c^2T^2 + X^2 + y^2 + z^2$ where c is the speed of light (c = 300,000 km/sec). Calculate the hyper-distance, $D^2$, between Events A and B in Problem 2.

Problem 5 - Based on your answer, which part of $D^2$ makes the largest contribution to the hyper-distance, the time-like part, T, or the space-like part (x,y,z)?

Problem 6 - Inside a black hole, the formula for $D^2$ changes to $D^2 = c^2T^2 - x^2 - y^2 - z^2$ Suppose Events A and B are now happening inside the black hole. What is the hyper-distance between them?

## Extra for Experts:

Problem 7 - If an observer defines 'time' as the part of $D^2$ that has a negative sign, and 'space' as the part that has the positive sign, can you explain what happens as the traveler passes inside the black hole?

## Answer Key:

Problem 1 - In relativity, space and time are part of a single 4-dimensional thing called spacetime. There are 3 dimensions to space and 1-dimension to time. Every point, called an Event, has three coordinates to describe its location in space, and one extra coordinate to describe its location in time. We write these as an ordered set like A(x,y,z,t) or B(x,y,z,t). Write the ordered set for the following event called A: I travel north 5 miles, east 3 miles, and up 1 miles, at 9:00AM on February 16, 2008.
Answer - A(5,3,1, 9:00AM 2/16/2008)

Problem 2 - Suppose I travel from one event with coordinates A(3 km, 6 km, 2 km, 5:00PM) to another event B(5 km, 7 km, 3 km, 8:00PM). How far did I travel in space during the time interval from 5:00 PM to 8:00 PM?
Answer: Take the difference in the x, y and z coordinates to get x = 5-3 = 2 km; y=6-3 = 3 km and z=3-2 = 1 km.

Problem 3 - Use the Pythagorean Theorem to calculate the actual distance in space in Problem 2.
Answer: This only involves the x, y and z coordinate differences we found in Problem 2: Distance = $( 2^2 + 3^2 + 1^2 )^{1/2} = (14)^{1/2}$ or 3.7 kilometers.

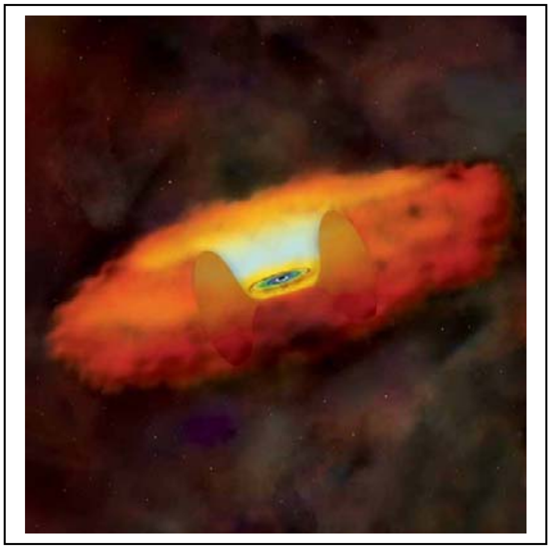
Problem 4 - The 4-dimensional, 'hyper' distance between the events is found by using the 'hyperbolic' Pythagorean Theorem formula $D^2 = -c^2T^2 + X^2 + y^2 + z^2$ where c is the speed of light (c = 300,000 km/sec). Calculate the hyper-distance, $D^2$, between Events A and B in Problem 2.
Answer: First, the time difference is 8:00PM - 5:00 PM = 3 hours. This equals 10,800 seconds. Then from the formula, where all units are in kilometers, we get $D^2 = -(300{,}000 \text{ km/sec})^2 (10{,}800 \text{ sec})^2 +2^2 + 3^2 + 1^2 =$ $- 1.05 \times 10^{19}$ kilometers.

Problem 5 - Based on your answer, which part of $D^2$ makes the largest contribution to the hyper-distance, the time-like part, T, or the space-like part (x,y,z)?
Answer - $D^2$ is negative, so it is the time-like part that makes the biggest difference.

Problem 6 - Inside a black hole, the formula for D2 changes to $D^2 = c^2T^2 - X^2 - y^2 - z^2$
Suppose Events A and B are now happening inside the black hole. What is the hyper-distance between them?
Answer - $D^2 = (300{,}000 \text{ km/sec})^2 (10{,}800 \text{ sec})^2 - 2^2 - 3^2 - 1^2 =$ $+1.05 \times 10^{19}$ kilometers.

Problem 7 - If an observer defines 'time' as the part of $D^2$ that has a negative sign, and 'space' as the part that has the positive sign, can you explain what happens as the traveler passes inside the black hole?
Answer - Outside the black hole, the T variable we defined to be time is part of the negative-signed component to $D^2$, and x,y,z are the space variables and are the positive part of $D^2$. When we enter the black hole, the T variable becomes part of the positive space-like part of $D^2$, and x, y and z are part of the negative part of $D^2$. This means that the roles of space and time have been reversed inside a black hole!

# Black Hole Power...X



Black holes are sometimes surrounded by a disk of orbiting matter. This disk is very hot. As matter finally falls into the black hole from the inner edge of that disk, it releases about 7% of its rest-mass energy in the form of light. Some of this energy was already lost as the matter passed through, and heated up, the gases in the surrounding disk. But the over-all energy from the infalling matter is about 7% of its rest-mass in all forms (heat+ light).

The power produced by a black hole is phenomenal, with far more energy per gram being created than by ordinary nuclear fusion, which powers the sun.

Illustration of black hole accretion disk cut-away, showing the central black hole. Courtesy NASA/Chandra/ M.Weiss (CXC)

Problem 1 - The Event Horizon of a black hole has a radius of 2.93 M kilometers, where M is the mass of the black hole in multiples of the sun's mass. Assume the Event Horizon is a spherical surface, so its surface area is $S = 4 \pi R^2$. What is the surface area of A) a stellar black hole with a mass of 10 solar masses? B) a supermassive black hole with a mass of 100 million suns?

Problem 2 - What is the volume of a spherical shell with the surface area of the black holes in Problem 1, with a thickness of one centimeter?

Problem 3 - If the density of gas near the horizon is $10^{10}$ atoms/cc of hydrogen, how much matter is in each black hole shell, if the mass of a hydrogen atom is $1.6 \times 10^{-24}$ grams?

Problem 4 - If $E = m c^2$ is the rest mass energy, E, in ergs, for a particle with a mass of m in grams, what is the rest mass energy equal to the masses in Problem 3 if $c = 3 \times 10^{10}$ cm/sec is the speed of light and only 7% of the mass produced energy?

Problem 5 - Suppose the material was traveling at 1/2 the speed of light as it crossed the horizon, how much time does it take to travel one centimeter if $c = 3 \times 10^{10}$ cm/sec is the speed of light?

Problem 6 - The power produced is equal to the energy in Problem 4, divided by the time in Problem 5. What is the percentage of power produced by each black hole compared to the sun's power of $3.8 \times 10^{33}$ ergs/sec?

## Answer Key:

Problem 1 - The Event Horizon of a black hole has a radius of 2.93 M kilometers, where M is the mass of the black hole in multiples of the sun's mass. Assume the Event Horizon is a spherical surface, so its surface area is $S = 4 \pi R^2$. What is the surface area of A0 a stellar black hole with a mass of 10 solar masses? B) a supermassive black hole with a mass of 100 million suns?

Answer; A) The radius is 2.93 x 10 = 29.3 kilometers. The surface area S = 4 x 3.14 x (2.93 x $10^6)^2$ = 1.1 x $10^{14}$ cm$^2$, B) 2.93 x $10^{13}$ cm S = 4 x 3.14 x (2.93 x $10^{13})^2$ = 1.1 x $10^{28}$ cm$^2$,

Problem 2 - What is the volume of a spherical shell with the surface area of the black holes in Problem 1, with a thickness of one centimeter?
Answer: Stellar black hole, V = Surface area x 1 cm = 1.1 x $10^{14}$ cm$^3$; Supermassive bklack hole, V = 1.1 x $10^{28}$ cm$^3$.

Problem 3 - If the density of gas near the horizon is $10^{10}$ atoms/cc of hydrogen, how much matter is in each black hole shell, if the mass of a hydrogen atom is 1.6 x 10-24 grams?

Answer - Stellar: M = (1.1 x $10^{14}$ cm$^3$ ) x (1.0 x $10^{10}$ atoms/cm$^3$) x (1.6 x $10^{-24}$ grams/atom = 1.76 grams. Supermassive: M = (1.1 x $10^{28}$ cm$^3$ ) x (1.0 x $10^{10}$ atoms/cm$^3$) x (1.6 x $10^{-24}$ grams/atom = 1.76 x $10^{14}$ grams.

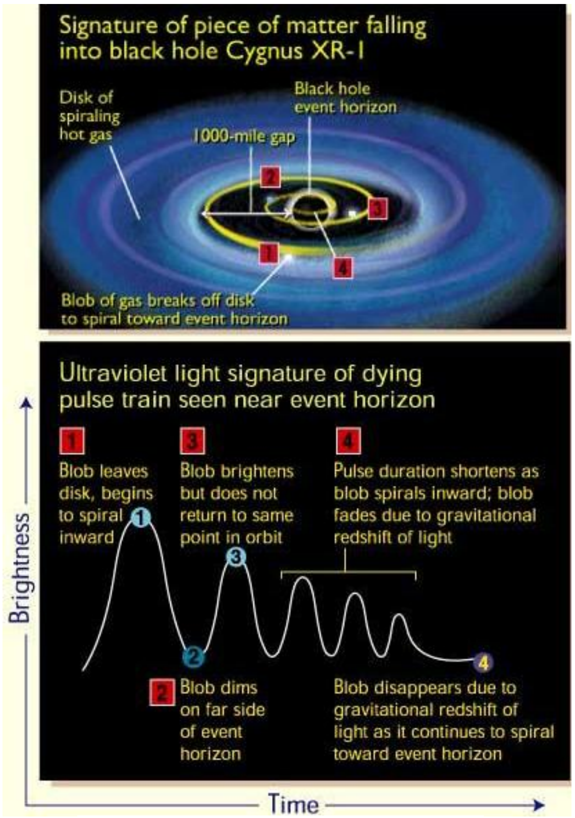
Problem 4 - If $E = m c^2$ is the rest mass energy, E, in ergs, for a particle with a mass of m in grams, what is the rest mass energy equal to the masses in Problem 3 if c = 3 x $10^{10}$ cm/sec is the speed of light and only 7% of the mass produced energy?
Answer: Stellar: E = 0.07 x ( 1.76 grams x ( 3 x $10^{10})^2$ ) = 1.1 x $10^{20}$ ergs.;
Supermassive E = 0.07 x (1.76 x $10^{14}$ grams x ( 3 x $10^{10})^2$ ) = 1.1 x $10^{34}$ ergs

Problem 5 - Suppose the material was traveling at 1/2 the speed of light as it crossed the horizon, how much time does it take to travel one centimeter if c = 3 x $10^{10}$ cm/sec is the speed of light?
Answer; 1 cm / 3 x $10^{10}$ cm/sec = 3.3 x $10^{-11}$ seconds.

Problem 6 - The power produced is equal to the energy in Problem 4, divided by the time in Problem 5. What is the percentage of power produced by each black hole compared to the sun's power of 3.8 x $10^{33}$ ergs/sec?
Answer Stellar: 1.1 x $10^{20}$ ergs / 3.3 x $10^{-11}$ seconds = 3.3 x $10^{30}$ ergs/sec
Percent = 100% x (3.3 x $10^{30}$ / 3.8 x $10^{33}$) = 0.08 %

Supermassive: 1.1 x $10^{34}$ ergs / 3.3 x $10^{-11}$ seconds = 3.3 x $10^{44}$ ergs/sec
= (3.3 x $10^{44}$ / 3.8 x $10^{33}$) = 86 billion times the sun's power!

# Black Hole...Fade-out!




$$L = L_0 e^{\frac{-2T}{3\sqrt{3}\ 2M}}$$

As seen from a distance, not only does the passage of time slow down for someone falling into a black hole, but the image fades to black!

This happens because, during the time that the object reaches the event horizon and passes beyond, a finite number of light particles (photons) will be emitted. Once these have been detected to make an image, there are no more left because the object is on the other side of the event horizon and the photons cannot escape. A star, collapsing to a black hole, will be going very fast as it collapses, then appear to slow down as time 'dilates'. Meanwhile, the image will become redder and redder, until it literally fades to black!

Photographs taken by the Hubble Space Telescope of the black-hole candidate called Cygnus XR-1 detected two instances where a hot gas blob appeared to be slipping past the event horizon for the black hole. Because of the gravitational stretching of light, the fragment disappeared from Hubble's view before it ever actually reached the event horizon. The pulsation of the blob, an effect caused by the black hole's intense gravity, also shortened as it fell closer to the event horizon. Without an event horizon, the blob of gas would have brightened as it crashed onto the surface of the accreting body. See The Astrophysical Journal, 502:L149–L152, 1998 August 1

Diagram courtesy Ann Field (STScI)

Problem 1 - The simple formula predicts the exponential decay of the light from matter falling in to a black hole. T is the time in seconds measured by distant observer, and M is the mass of the black hole in units of solar masses. How long does it take for the light to fall to half its initial luminosity (i.e. power in units of watts) given by $L_0$ for a M =1.0 solar mass, stellar black hole?

Problem 2 - How long will your answer be, in years, for a supermassive black hole with M = 100 million times the mass of the sun?

Problem 3 - A supermassive black hole 'swallows' a star. If the initial luminosity, $L_0$, of the star is 2.5 times the sun's, how long will it take before the brightness of the star fades to 0.0025 Suns, and can no longer be detected from Earth?

## Answer Key:

Problem 1 - The simple formula predicts the exponential decay of the light from matter falling in to a black hole. T is the time in seconds measured by distant observer, and M is the mass of the black hole in units of solar masses. How long does it take for the light to fall to half its initial luminosity (i.e. power in units of watts) given by $L_0$ for a M =1.0 solar mass, stellar black hole??

Answer - Set L = 1/2 $L_0$, and M = 1.0, then solve for T. The formula is 0.5 = e (-0.19 T) Tale the natural logarithm of both sides to get -0.69 = -0.19 T so T = 3.6 seconds.

Problem 2 - How long will your answer be, in years, for a supermassive black hole with M = 100 million times the mass of the sun?

Answer, The formula will be 0.5 = e (- 1.9 x $10^{-9}$ T) so -0.69 = -1.9 x $10^{-9}$ T, and T = 3.6 x $10^{8}$ seconds. If there are 3.1 x $10^{7}$ seconds in 1 year, T = 11.5 years.

Problem 3 - A supermassive black hole 'swallows' a star. If the initial luminosity, $L_0$, of the star is 2.5 times the sun's, how many years will it take before the brightness of the star fades to 0.0025 Suns, and can no longer be detected from Earth?

Answer:

$0.0025\ L_{sun} = 1.0\ L_{sun}\ e\ (-1.9x10-9\ T)$

$Ln(0.0025) = -1.9x10^{-9}\ T$

$-6.0 = -1.9x10^{-9}\ T$

$T = 6.0/1.9x10^{-9}$

$T = 3.2 \times 10^{9}$ seconds

$T = 3.2 \times 10^{9}$ seconds x (1.0 year /3.1 x $10^{7}$ seconds )

T = 103 years.

# Calculating Black Hole Power

This image shows the large galaxy, NGC-6872, interacting with a smaller galaxy, IC-4970, located just above the center of NG-6872. These galaxies are located in the southern constellation Pavo, and about 300 million light years from the Sun. From tip to tip, NGC-6872 measures about 700,000 light years, making it nearly 3 times as big as the Milky Way. The image is a composite made by NASA's Spitzer and Chandra satellites, and a ground-based telescope. Although NGC-6872 is dramatically bigger, IC-4970 is the real 'player' in this collision because it contains a super-massive black hole, which is emitting energy as it absorbs interstellar gas and dust that has been gravitationally ripped from the larger galaxy. The X-ray power, alone, is about 450 million times the sun's total light output!

When black holes 'digest' gas, stars and other forms of matter that enter their event horizons, the energy that is produced as the matter falls in can be converted into heat in the orbiting 'accretion disk' that surrounds a black hole. The amount of heat energy that is emitted by the matter each second (power) can be detected at great distances as a brilliant source of light or other forms of electromagnetic radiation. The formula that approximately relates the rate of in-falling matter into a super-massive black hole (**R** in solar masses per year), to the emitted power (**L** in multiples of the sun's power) is given by $L = 1.1 \times 10^{12}$ R solar luminosities. (Note 1 solar mass = $2 \times 10^{33}$ grams, and 1 solar luminosity = $4 \times 10^{33}$ ergs/sec).

**Problem 1** - What is the minimum accretion rate that is needed to account for the x-ray power of the black hole in the core of IC-4970?

**Problem 2** - How much mass would have to be accreted in order for the supermassive black hole to have the same power as an average quasar with a luminosity of about 2 trillion times the luminosity of our sun?

**Problem 3** - The supermassive black hole in the center of the Milky Way has an estimated output equal to 2,500 suns. About how fast is it accreting matter?

**Problem 1** - What is the minimum accretion rate that is needed to account for the x-ray power of the black hole in the core of IC-4970?
Answer: $L = 1.1 \times 10^{12}$ R solar luminosities, and since Lx = 450 million solar luminosities, solving for R we get $R = 4.5 \times 10^{8} / 1.1 \times 10^{12}$ = **0.00041 solar masses per year**.

**Problem 2** - How much mass would have to be accreted in order for the supermassive black hole to have the same power as an average quasar with a luminosity of about 2 trillion times the luminosity of our sun?
Answer: $R = 2.0 \times 10^{12} / 1.1 \times 10^{12}$ = **1.8 solar masses per year**.

**Problem 3** - The supermassive black hole in the center of the Milky Way has an estimated output equal to 2,500 suns. About how fast is it accreting matter?
Answer: $R = 2{,}500 / 1.1 \times 10^{12}$ = **$2.3 \times 10^{-9}$ solar masses per year**

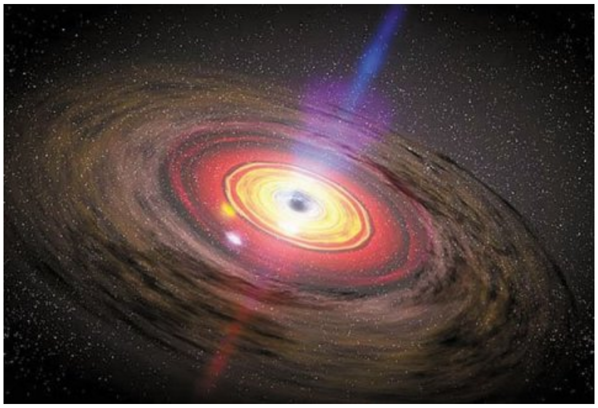
**Note:** The formula is derived by using $E = mc^2$ to convert the in-falling mass into energy, and for non-rotating 'Schwarschild' black holes, the conversion efficiency is 7%, so that only 7% of the available 'rest mass' energy of the in-falling material actually is converted into energy. If R is given in solar masses per year, then the energy liberated is equal to:

L = R (solar mass/year) x ($2 \times 10^{33}$ grams/solar mass) x $(3 \times 10^{10})^2 \times 0.07$
= 1.3 x 1053 R ergs/year

Since 1 solar luminosity =$3.8 \times 10^{33}$ ergs/sec, and 1 year = $3.1 \times 10^{7}$ seconds we have:

L = $1.3 \times 10^{53}$ ergs/year x (1 solar luminosity/ $3.8 \times 10^{33}$) x ( 1 year/$3.1 \times 10^{7}$ seconds) R

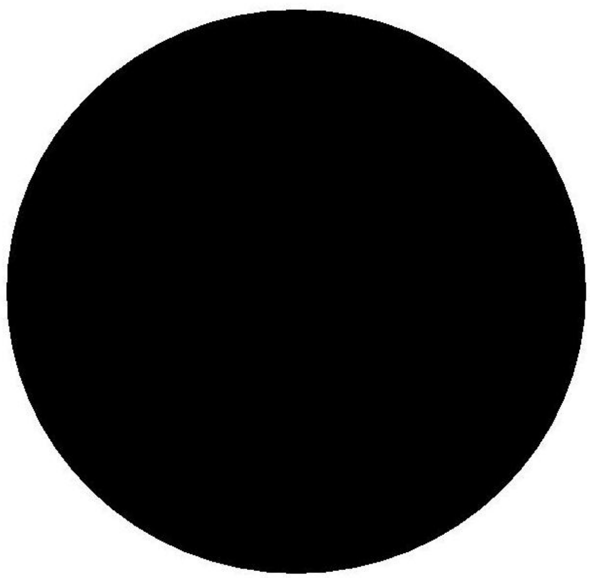
**L = $1.1 \times 10^{12}$ R solar luminosities**.
where R is the accretion rate in solar masses per year.



Artist's impression of gas falling into a black hole Image credit: NASA / Dana Berry, SkyWorks Digital

When gas flows into a black hole, it gets very hot and emits light. The gas is heated because the atoms collide with each other as they fall into the black hole. Far away from the black hole, the atoms do not travel very fast so the gas is cool. But close to the black hole, the atoms can be moving at millions of kilometers/hour and the gas can be thousands of degrees hot!

The circle below represents the spherical shape of a black hole with a mass of about 5 times our Earth. Its diameter is 9 centimeters.



The formula that gives the gas temperature, T in Kelvins, at a distance of R in meters from the center of the black hole, is given by:

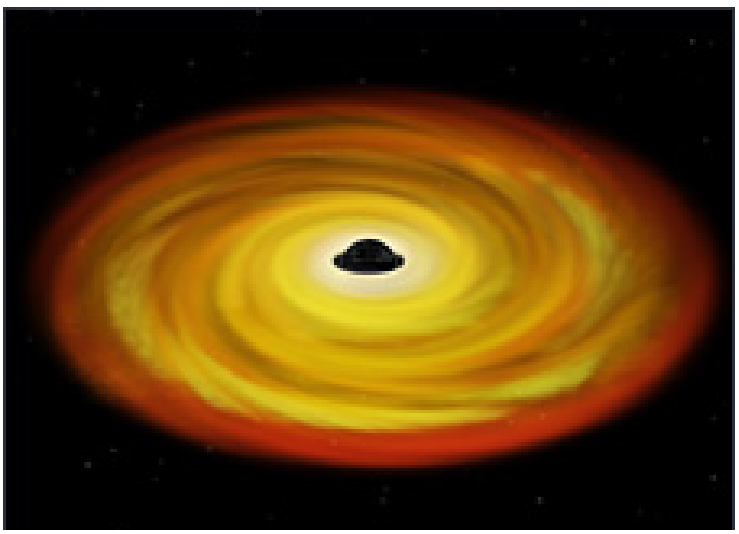
$$T = \frac{35,000}{R^{\frac{3}{4}}}$$

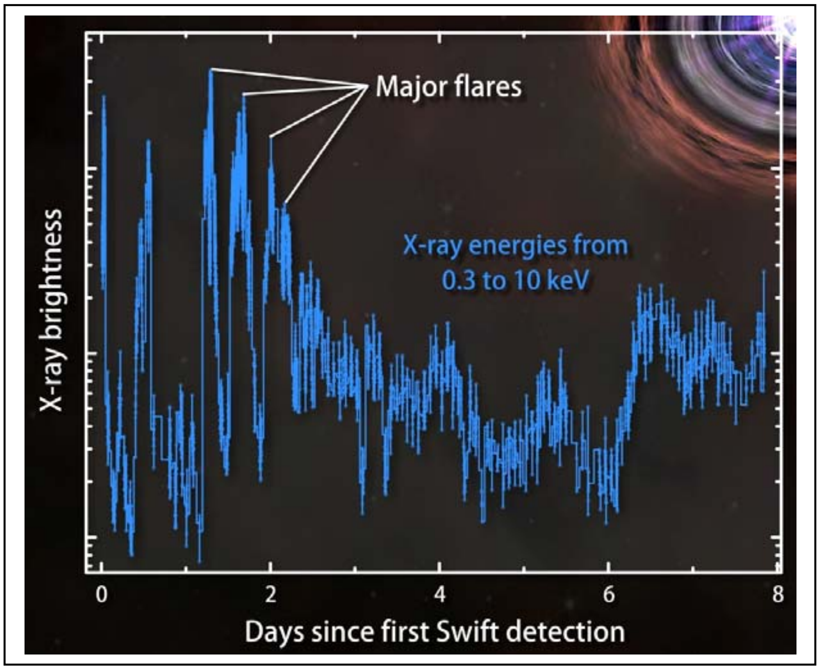
**Problem 1** - Sketch a life-sized illustration of the gas surrounding the above black hole and give the temperature at a distance of 1 meter, 50 centimeters and 5 centimeters from the center of the black hole.

**Problem 1** - Sketch a life-sized illustration of the gas surrounding the above black hole and give the temperature at a distance of 1 meter, 50 centimeters and 5 centimeters from the center of the black hole.

Answer: At 1 meter, **T = 35,000 K,** which is 6 times the surface temperature of our sun.
At 50 cm or 0.5 meters, **T = 59,000 K**.
At 5 centimeters or 0.05 meters, **T = 331,000 K**.

Students may color many different versions, but they should all show that the most distant gas is cooler (35,000 K) than the gas near the black hole (331,000 K) which could be temperature coded using some plausible scheme like the 'yellow to white' scheme below.






On March 28, 2011 Swift's Burst Alert Telescope discovered the source in the constellation Draco when it erupted with the first in a series of powerful X-ray blasts. The satellite determined a position for the explosion, now cataloged as gamma-ray burst (GRB) 110328A, and informed astronomers worldwide. As dozens of telescopes turned to study the spot, astronomers quickly noticed that a small, distant galaxy appeared very near the Swift position. A deep image taken by Hubble on April 4 pinpoints the source of the explosion at the center of this galaxy, which lies 3.8 billion light-years away.

That same day, astronomers used NASA's Chandra X-ray Observatory to make a four-hour-long exposure of the puzzling source. The image, which locates the object 10 times more precisely than Swift can, shows that it lies at the center of the galaxy Hubble imaged.

The duration of the x-ray bursts tells astronomers approximately how large the emitting region is, and since the source is a black hole, it gives the approximate diameter of the black hole. The radius of a black hole is related to its mass by the simple formula R = 3 M ,where M is the mass of the black hole in units of the sun's mass, and R is the radius of the Event Horizon in kilometers.

**Problem 1** - What is the average duration of the three flare events seen in the X-ray plot above?

**Problem 2** - Light travels at a speed of 300,000 km/s. How many kilometers across is the x-ray emitting region based on the average time of the three x-ray flares?

**Problem 3** - The size of the x-ray emitting region from Problem 2 is a crude estimate for the diameter of the black hole. For reasons having to do with relativity, a better black hole size estimate will be 100 times smaller than your answer for Problem 2. From this better-estimate, about what is the mass of the black hole GRB110328A in solar masses?

**Problem 1** - What is the average duration of the three flare events seen in the X-ray plot above?

Answer: There were about 3 flares in one day, so the average flare duration is about **8 hours**.

**Problem 2** - Light travels at a speed of 300,000 km/s. How many kilometers across is the x-ray emitting region based on the average time of the three x-ray flares?

Answer: Distance = speed x time, so D = 300,000 km/s x 8 hours x (3600 sec/1 hour) = **8.6 billion kilometers.**

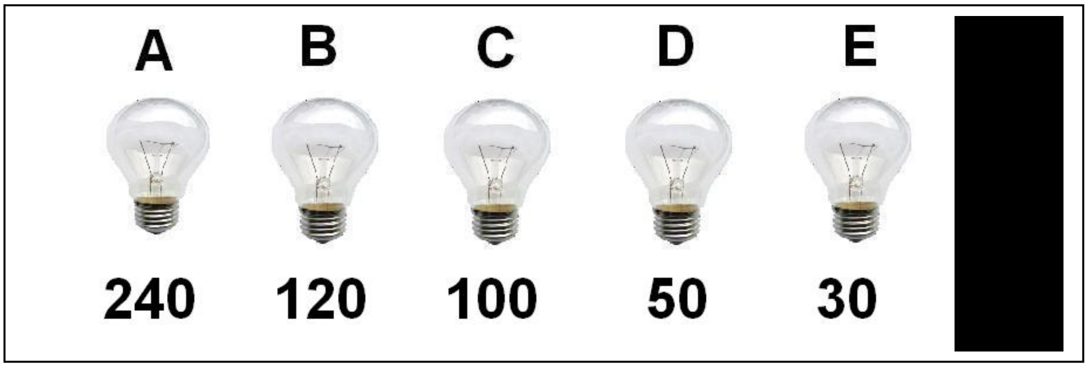
**Problem 3** - The size of the x-ray emitting region from Problem 2 is a crude estimate for the diameter of the black hole. For reasons having to do with relativity, a better black hole size estimate will be 100 times smaller than your answer for Problem 2. From this better-estimate, about what is mass of the black hole in solar masses?

Answer: If 8.6 billion kilometers is the width of the emitting region, then the radius of the region is about 4.3 billion kilometers, and the estimated radius of the black hole is about 100 times smaller than this or 43 million kilometers. Since the radius of a black hole is R = 3 x M, the mass of the black hole is 43 million = 3 x M, or M = **14 million solar masses**.

*Note: Astrophysicists have studied and modeled these kinds of events for decades, and it is generally agreed that gamma-ray bursts are probably caused by beams of particles and radiation leaving the vicinity of the black hole. Because of this, the estimated light-travel size of the emitting region from the changes in the gamma ray or x-ray brightness will greatly over-estimate the actual size of the emitting region. The 'factor of 100' is added to this calculation to account for this 'beaming' effect. Actual astrophysical models of these regions that take into account relativity physics are still in progress and will eventually lead to much better estimates for the black hole size and mass. Also, the relationship between black hole radius and mass that we used only works for black holes that do not rotate, called 'Schwarschild Black Holes'. In actuality, we expect most black holes to be rotation, at speeds that are perhaps even near the speed of light, and these will be significantly larger in size. These are called Kerr Black Holes.*

The sketch below shows the edge of a black hole on the right hand-side. The distance in centimeters from the edge of the black hole, called the **event horizon**, increases from right to left to a maximum distance of 240 centimeters from the event horizon (Bulb A). In this figure, the radius of the black hole is about 1 meter. This corresponds to a black hole with a mass equal to 120 times the mass of our Earth.

Although all of the light bulbs would be destroyed this close to an actual event horizon, we will pretend, for simplicity, that they can survive.




Someone far away from a black hole will see things very differently than someone close to a black hole. Because of the intense gravitational forces, ordinary light emitted close to a black hole will have its wavelength stretched as viewed by someone far away. The closer the light source, the more wavelength -stretching will be seen by the distant observer.

Suppose all of these light bulbs in the above figure are emitting light at a wavelength of 450 nanometers (nm), and are shining brightly with a color near yellow like our sun. A distant observer will see this light stretched to longer wavelengths. The wavelength they will observe, W, in nanometers depends on the distance of the light source, R to the center of the black hole in centimeters according to the formula:

$$W = \frac{450}{\sqrt{1 - \frac{100}{R}}}$$

For example, the event horizon for this black hole is at R=100 centimeters. If the light bulb is 50 centimeters to the left of the horizon, R = 150 centimeters, and so W = 780 nanometers. The middle of the Visible Band is at about 500 nm, so instead of yellow light, you would see this light bulb emitting very deep red color!

**Problem 1** - Suppose the bulbs were located at the distances from the event horizon shown in the figure above. What would be the wavelengths you would observe for Bulbs B, C, D and E?

**Problem 2** - The human eye can only detect light at a wavelength shorter than about 650 nm. Which of the light bulbs would appear to be invisible to you and 'black'?

**Problem 3** - How close to the event horizon would the light bulb have to be in order for you to only detect it as an invisible heat source emitting at a wavelength of just 14,000 nm?

**Problem 1** - Suppose the bulbs were located at the distances indicated above. What would be the wavelengths you would observe for Bulbs B, C, D and E?

| | | | |
|---|---|---|---|
| Bulb A | 240 cm | R = 340 cm | **W = 535 nm** |
| Bulb B | 120 cm | R = 220 cm | **W = 609 nm** |
| Bulb C | 100 cm | R = 200 cm | **W = 636 nm** |
| Bulb D | 50 cm | R = 180 cm | **W = 780 nm** |
| Bulb E | 30 cm | R = 130 cm | **W = 937 nm** |

Note: In the visible spectrum
Bulb A = yellow
Bulb B = orange
Bulb C = red
Bulb D = deep crimson or dull red and nearly invisible
Bulb E = infrared and invisible to the eye.

**Problem 2** - The human eye can only detect light at a wavelength shorter than about 650 nm. Which of the light bulbs would appear to be invisible to you and 'black'?

Answer: From your location far from the black hole, you see that the Bulbs D and E are not visible to your eyes. **Note: With the proper light detectors you could still see them shining at these longer wavelengths!**

**Problem 3** - How close to the event horizon would the light bulb have to be in order for you to only detect it as an invisible heat source emitting at a wavelength of just 14,000 nm?

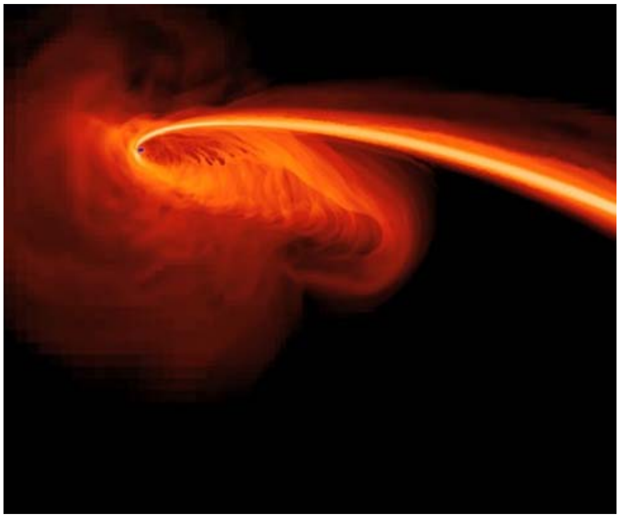
Answer: We need to solve for R the equation:

$$14{,}000 = \frac{450}{\sqrt{1-\frac{100}{R}}}$$ from this we get $$1-\frac{100}{R} = \left(\frac{450}{14{,}000}\right)^2$$

$$1-0.001 = \frac{100}{R}$$ so R = 100.1 centimeters.

This means that the bulb is located **0.1 centimeters or 1 millimeter** just outside the event horizon!

**Note: A black hole with a radius of 100 centimeters would have a mass of about 120 times that of Earth, or a little bit more than the planet Saturn.**



Simulation of black hole and tidal shredding of a star. (Courtesy Gezari and Guillochon: NASA)

$$r = 2.44R\left(\frac{D}{d}\right)^{\frac{1}{3}} \qquad r = R\left(\frac{M}{m}\right)^{\frac{1}{3}}$$

The gravitational force between two objects varies as the inverse-square of the distance between them. On the surface of Earth, we do not notice that the gravitational force of Earth pulling on our feet is slightly larger than the force pulling on our head. For astronomical bodies, however, the difference in gravity can be so great that it pulls the body apart! This is called the tidal gravitational force.

Around every body, there is a distance called the tidal radius within which an object will be gravitationally torn apart if the body is being held together by its own gravitational forces. This distance can be calculated using the formulae to the left. Let's explore what this distance is for some common astronomical bodies.

**Problem 1** – The Moon has a density of d = 3400 kg/m$^3$. The Earth has an average density of D = 5500 kg/m$^3$. If the radius, R, of Earth is 6378 km, how close to Earth would the moon have to get in order to be tidally disrupted? If the Moon is moving away from Earth at a speed of 3 cm/year, and its present distance is about 340,000 km, will it ever be tidally disrupted?

**Problem 2** - Saturn has a density of D=687 kg/m$^3$, and the average density of its innermost satellite Pan is about d=400 kg/m$^3$. Saturn has a radius of R=120,000 km. If Pan's distance from Saturn is about 134,000 km, is it in any danger of being tidally disrupted? Can a satellite gravitationally assemble itself from a collection of gas and dust if the orbit of this material is inside the tidal limit?

**Problem 3** - A red, supergiant star with a mass of m=20 times that of our sun, and a radius equal to the orbit of Earth (R=150 million km). If the mass of the black hole is M=10 times the mass of our sun, and its radius is 60 kilometers, what is the closest distance, r, that this star can come to the black hole before it is disrupted? Suppose, instead, that the black hole were like the one in the center of the Milky Way with a mass of M=3 million suns. What would be the tidal distance, r?

# Answer Key

**Problem 1** – The Moon has a density of 3400 $kg/m^3$. The Earth has an average density of 5500 $kg/m^3$. If the radius, R, of Earth is 6378 km, how close to Earth would the moon have to get in order to be tidally disrupted? If the Moon is moving away from Earth at a speed of 3 cm/year, and its present distance is about 340,000 km, will it ever be tidally disrupted?

Answer: From Equation 1, $r = 2.44\ (6378)(5500/3400)^{1/3}$ = **18,300 km**. Because the Moon is moving away from Earth and is already at a distance of 340,000 km, it will never be gravitationally disrupted by Earth to form a ring like Saturn's.

**Problem 2** - Saturn has a density of 687 $kg/m^3$, and the average density of its innermost satellite Pan is about 400 $kg/m^3$. Saturn has a radius of 120,000 km. If Pan's distance from Saturn is about 134,000 km, is it in any danger of being tidally disrupted? Can a satellite assemble itself from a collection of gas and dust if the orbit of this material is inside the tidal limit?

Answer: From Equation 1, $r = 2.44 \times (120{,}000)\ (687/400)^{1/3}$ = **351,000 km**. This is larger than Pan's distance so Pan could be tidally disrupted, however, unlike our moon, it is not a body held together by its own gravity but instead is held together by the stronger forces between atoms and molecules, called tensile forces. A body cannot be gravitationally assembled from local materials if its formation is occurring inside the tidal radius of a nearby body.

**Problem 3** - A red, supergiant star with a mass of 20 times that of our sun, and a radius equal to the orbit of Earth (150 million km). If the mass of the black hole is 10 times the mass of our sun, and its radius is 60 kilometers, what is the closest distance that this star can come to the black hole before it is disrupted? Suppose, instead, that the black hole were like the one in the center of the Milky Way with a mass of 3 million suns. What would be the tidal distance?

Answer: From Equation 2, we have m = 20 and M = 10 and R = 150 million km, so

$r = 150 \text{ million } (10/20)^{1/3}$

= **119 million kilometers**.

For a M=3 million solar mass black hole, the tidal distance would be

$R = 150 \text{ million } (3 \text{ million}/20)^{1/3}$

= **7.9 billion kilometers**, or just beyond the orbit of Pluto!