National Aeronautics and Space Administration

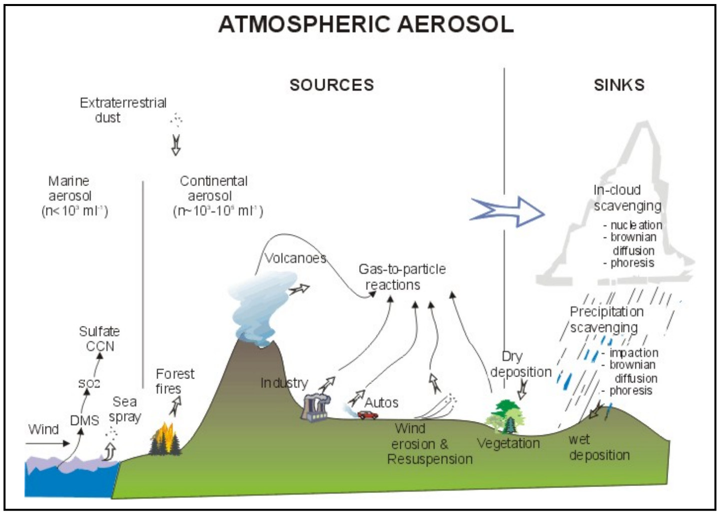
Math Problems Related to

The Atmosphere

Introduction

This book provides an assortment of mathematics problems that cover aspects of the atmosphere so that Learners can see how mathematical reasoning and data analysis enhance our understanding of planetary atmospheres, their properties, and their stability over time.

Atmospheric Aerosols by Percentage



Atmospheric aerosols can come in all sizes and types. The origin of these aerosols is also very complicated. Over the years scientists have been able to measure the percentages of the various types from different parts of the world.

The table below shows the results in percent compiled by NOAA's Earth Systems Research Laboratory.

Atmospheric Aerosols						
Location	Sea Salt	Dust	Water Droplets	Sulfate (SO_4)	Organic Particulates (PAH, etc)	Other
Marine	40%		30%	15%		15%
Europe	3%		23%	46%	10%	18%
Africa	5%	3%	13%	40%	20%	19%
India		4%	28%	44%	5%	19%
Asia	1%	22%	20%	27%	20%	10%
USA		1%	20%	19%	50%	10%

Problem 1 – In which direction (rows or columns) do you expect the percentages to add up to 100% and why?

Problem 2 – Draw a pie graph showing the percentages of the various aerosol contributions over the United States. What type of aerosol is the most abundant?

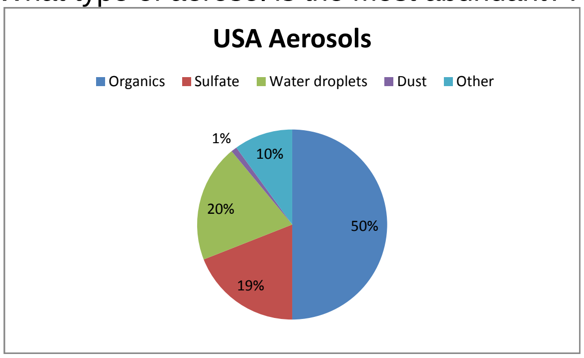
Problem 3 – Create a pie graph that shows the percentage abundance of sulfate aerosols over the 6 different regions. Which region has the highest abundance?

Problem 4 – Sulfate aerosols and organic particulates are typically man-made from industrial processes and the burning of fossil fuels. Which region has the highest concentration of industrial sources of aerosols, and why do you think this trend occurs?

NOAA aerosol data from Earth System Research Laboratory:
http://www.esrl.noaa.gov/research/themes/aerosols/#fig1

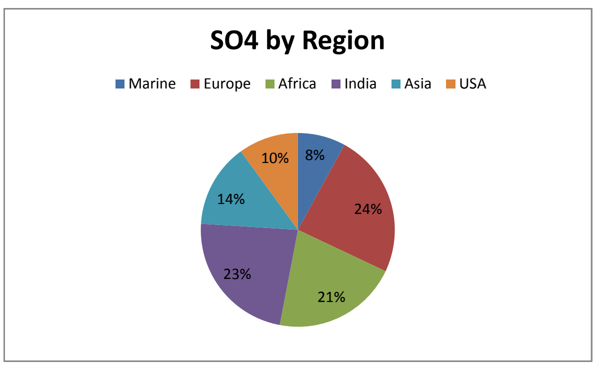
Problem 1 – In which direction (rows or columns) do you expect the percentages to add up to 100% and why? Answer: **The total percentage of aerosols has to add up to 100% because this table is indicating the types of aerosols found in the study. The only direction in which this happens in the table is along he rows, so each row adds up to 100% for the indicated location.**

Problem 2 – Draw a pie graph showing the percentages of the various aerosol contributions over the United States. What type of aerosol is the most abundant? Answer: See below.



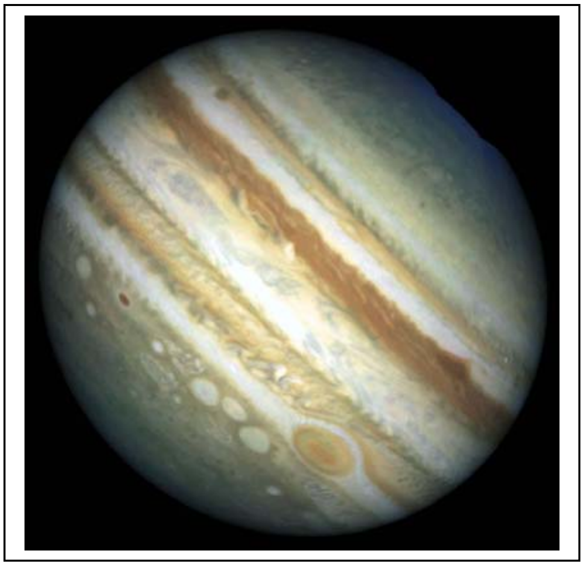
Problem 3 – Create a pie graph that shows the percentage abundance of sulfate aerosols over the 6 different regions. Which region has the highest abundance?

Answer: First you have to add up the percentages for each region: 15% + 46% + 40% + 44% + 27% + 19% = 191%. Then divide each percentage by 191% to get the percentage for each region. Marine = 100% x (15/191) = 8%; Europe = 24%; Africa = 21%; India = 23%; Asia= 14% and USA= 10%. Now create the pie graph. Europe has the highest concentration (24%).



Problem 4 – Sulfate aerosols and organic particulates are typically man-made from industrial processes and the burning of fossil fuels. Which region has the highest concentration of industrial sources of aerosols and why do you think this trend occurs?

Answer: From the table, if we combine the columns for Sulfate and Organic Particulates we see that the **USA with 69% is the largest**. This is probably because the USA is more 'industrialized' than other countries, especially along its East Coast.



All of the planets in our solar system, and some of its smaller bodies too, have an outer layer of gas we call the atmosphere. The atmosphere usually sits atop a denser, rocky crust or planetary core. Atmospheres can extend thousands of kilometers into space.

The table below gives the name of the kind of gas found in each object's atmosphere, and the total mass of the atmosphere in kilograms. The table also gives the percentage of the atmosphere composed of the gas.

Object	Mass (kilograms)	Carbon Dioxide	Nitrogen	Oxygen	Argon	Methane	Sodium	Hydrogen	Helium	Other
Sun	3.0×10^{30}							71%	26%	3%
Mercury	1000			42%			22%	22%	6%	8%
Venus	4.8×10^{20}	96%	4%							
Earth	1.4×10^{21}		78%	21%	1%					<1%
Moon	100,000				70%		1%		29%	
Mars	2.5×10^{16}	95%	2.7%		1.6%					0.7%
Jupiter	1.9×10^{27}							89.8%	10.2%	
Saturn	5.4×10^{26}							96.3%	3.2%	0.5%
Titan	9.1×10^{18}		97%			2%				1%
Uranus	8.6×10^{25}					2.3%		82.5%	15.2%	
Neptune	1.0×10^{26}					1.0%		80%	19%	
Pluto	1.3×10^{14}	8%	90%			2%				

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth.

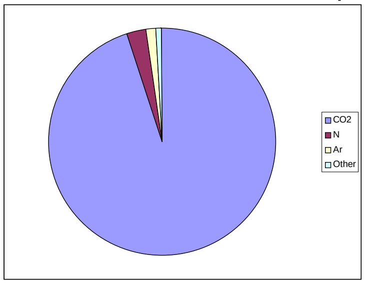
Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto.

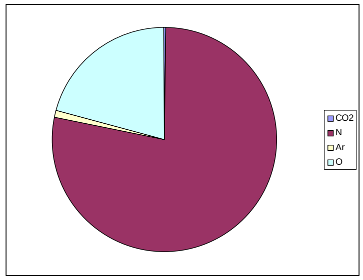
Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen?

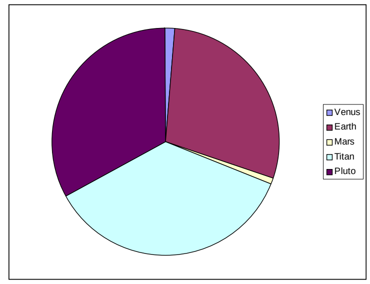
Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen?

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Problem 1 – Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth. Answer: Mars (left), Earth (middle)







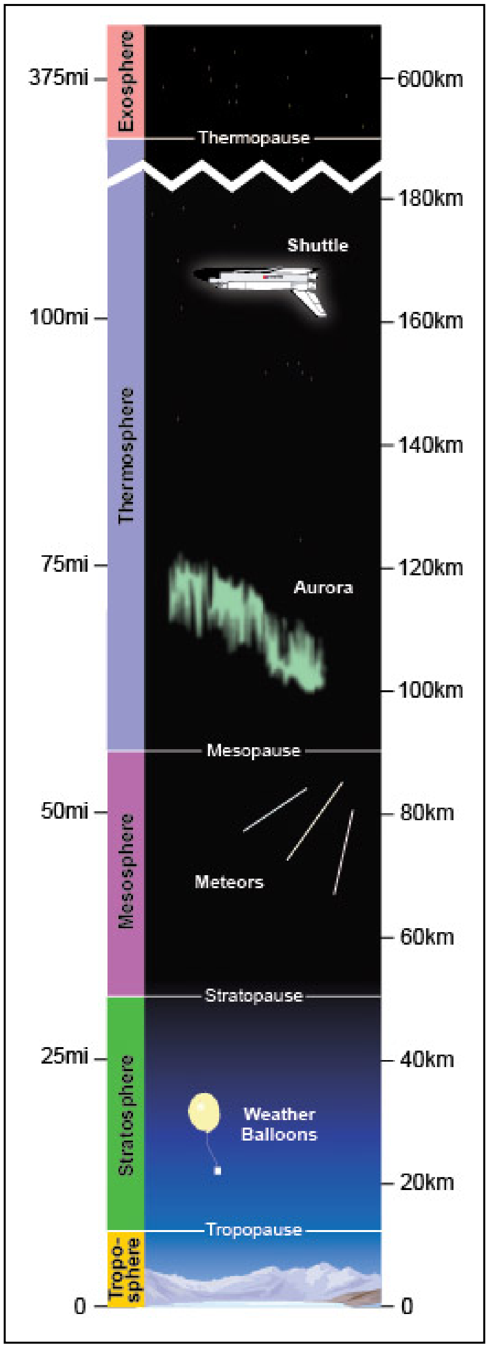
Problem 2 – Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto. Answer: First add up all the percentages for Nitrogen in the column to get 271.7%. Now divide each of the percentages in the column by 271.7% to get the percentage of nitrogen in the planetary atmospheres that is taken up by each of the planets: Venus = (4/271) = 1.5%; Earth = (78/271) = 28.8%, Mars = (2.7/271)= 1.0%, Titan = (97/271)=35.8%, Pluto=(90/271)=33.2%. Plot these new percentages in a pie graph (see above right). This pie graph shoes that across our solar system, Earth, Titan and Pluto have the largest percentage of nitrogen. In each case, the source of the nitrogen is from similar physical processes involving the chemistry of the gas methane (Titan and Earth) or methane ice (Pluto).

Problem 3 – Which planet has the atmosphere with the greatest percentage of Oxygen? Answer: From the table we see that **Mercury** has the greatest percentage of oxygen in its atmosphere.

Problem 4 – Which planet has the atmosphere with the greatest number of kilograms of oxygen? Answer: Only two planets have detectable oxygen: Earth and Mercury. Though mercury has the highest percentage of oxygen making up its atmosphere, the number of kilograms of oxygen is only 1000 kg x 0.42 = 420 kilograms. By comparison, **Earth** has a smaller percentage of oxygen (21%) but a vastly higher quantity: 1.4×10^{21} kg x 0.21 = **2.9×10^{20} kilograms**. (That's 290,000,000,000,000,000,000 kg)

Problem 5 – Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Answer: The objects with the highest percentage of hydrogen are the sun, Mercury, Jupiter, Saturn, Uranus and Neptune. The objects with the least percentage are Venus, Earth, Moon, Mars, Titan, Pluto. With the exception of Mercury, which has a very thin atmosphere, the high-percentage objects are the largest bodies in the solar system. The planet Jupiter, Saturn, Uranus and Neptune are sometimes called the Gas Giants because so much of the mass of these planets consists of a gaseous atmosphere. These bodies generally lie far from the sun. The low-percentage objects are among the smallest bodies in the solar system. They are called the 'Rocky Planets' to emphasize their similarity in structure, where a rocky core and mantel are surrounded by a thin atmosphere. Most of these bodies lie close to the sun.



NASA's Van Allen Probes spacecraft will be exploring a region of space near Earth where the atmosphere of Earth is almost non-existent, but it can still be measured. Scientists use density as a way to show just how much gas there is in a cubic meter of space if you were to collect all of the gas in such a box.

On Earth we often talk about a rock being dense, and measure density in kilograms per cubic meter. For most rocks, their densities are 3000 kg/m^3, so if you had a pick-up truck that could hold 1 cubic meter of rock, it would hold 3000 kilograms of mass or 3 metric tons!

Gas is so dilute that, instead of writing density as kilograms/m^3 we use atoms (or molecules) per cubic meter. This tells us how many particles of gas are in a cubic meter. Because the number of particles is so large, we sometimes have to use scientific notation to write them!

Problem 1 – At sea level, the average density of air molecules (oxygen and nitrogen) is 2.5×10^{25} molecules/m^3. Write this number in: A) decimal form, B) by using 'million', 'billion', 'trillion' etc.

Problem 2 – Imagine a piece of paper 1000 kilometers on a side. How many dots would you have to place in each square that is 1 cm on a side in order to fill up the page with this many dots?

Problem 3 – The mesosphere is one of the highest levels of the atmosphere and at 70 km has a density of 0.00001 kg/m^3. This is 100,000 times lower than the density of the atmosphere at sea level. How many dots would you have in each cell of the paper you used in Problem 2?

Problem 4 - In the Van Allen belts, which is located above the exosphere, the density of particles is about 900 atoms/m^3. If you used the same 1000 km wide piece of paper, how far apart would the atoms of the Van Allen belt be on this scale?

Problem 1 – At sea level, the average density of air molecules (oxygen and nitrogen) is 2.5×10^{25} molecules/m^3. Write this number in: A) decimal form, B) by using 'million', 'billion', 'trillion' etc.

Answer: A) 25,000,000,000,000,000,000,000,000 molecules/m^3.
B) 250 trillion trillion molecules/m^3.

Problem 2 – Imagine a piece of paper 1000 kilometer on a side. How many dots would you have to place in each square that is 1 cm on a side in order to fill up the page with this many dots?

Answer: First we have to find out how many 1 cm^2 squares there are in a 1000 km x 1000 km piece of paper. Since 1 km = 1000 meters and 1 meter = 100 cm, each side of the paper measures 100,000,000 cm, (10^8) so the area of the paper is $(100,000,000)^2 = 10^{16}$ = 10 thousand trillion cm^2. There are 10 thousand trillion squares on the page, so the number of dots we need to put in each square is
250 trillion trillion / 10 thousand trillion = $2.5\times10^{25}/10^{16}$ = 2.5×10^9 or **2,500,000,000 dots!**

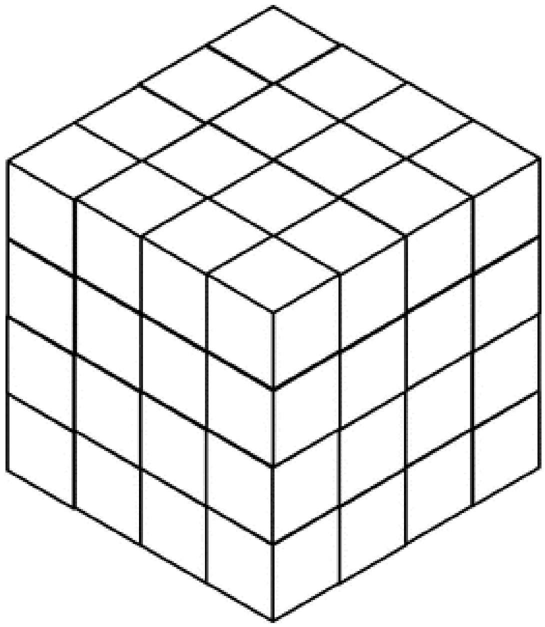
Problem 3 – The mesosphere is one of the highest levels of the atmosphere and at 70 km has a density of 0.00001 kg/m^3. This is 100,000 times lower than the density of the atmosphere at sea level. How many dots would you have in each cell of the paper you used in Problem 2?

Answer: If the density is 100,000 lower, then you only need a **total** of $2.5\times10^{25}/10^5$ = 2.5×10^{20} dots over the entire sheet. This means that each 1 cm^2 square will only need 100,000 times fewer dots or $2.5\times10^9/10^5 = 2.5\times10^4$ = **25,000 dots**.

Problem 4 - In the Van Allen belts, which is located above the exosphere, the density of particles is about 900 atoms/m^3. If you used the same 1000 km wide piece of paper, how far apart would the atoms of the Van Allen belt be on this scale?

Answer: You want 900 atoms scattered across a 1000 km x 1000 km piece of paper. Because 30x30 =900, that means that along each side of the paper we would mark off 30 atoms, and complete a grid with this spacing covering the 1000km x 1000km page to mark 900 points. Since 1000 km/30 = 33 1/3 kilometers, the atoms of the Van Allen belts on this scale would be about **33 kilometers apart!**

Exploring Gas Density in Space



Earth's atmosphere does not have a hard edge that says 'this is where space starts'. Instead, the density of the atmosphere gets smaller and smaller…but it never quite becomes zero!

Scientists measure gas density in space in terms of the number pf particles that you would find in any cubic meter of space. This is called the **Number Density, n,** and is measured in particles/m^3. Near the Earth, the gas densities are so large that we have to use scientific notation to write them. For instance, at Earth's surface, the number density of air is $n= 2.5 \times 10^{25}$ molecules/m^3. In the mesosphere at 70 km altitude, it is $n= 2.5 \times 10^{20}$ molecules/m^3.

Imagine the each particle sits at the center of its own cube. The number of these cubes, N, in <u>one cubic meter</u> is just the gas number density: **N = n.** In the figure above n = 64 if the length of each cube edge is 1 meter.

Problem 1 - Suppose you had 64 cubes arranged in a cube with a side length of one meter. How far apart would the centers of each cube be?

Problem 2 – Suppose that the large cube had an edge length of 1 meter and it contained 1 million identical cubes. What would the distance between the cube centers be?

Problem 3 – In the Van Allen belts, the average number density is about 900 particles/m^3. What is the average distance between the atoms in the Van Allen belts?

Problem 4 – In the mesosphere, the average number density is about 2.5×10^{20} particles/m^3. What is the average distance between the atoms in the mesosphere in microns, where 1 micron = 10^{-6} meters?

Problem 1 - Suppose you had 64 cubes arranged in a cube with a side length of one meter. How far apart would the centers of each cube be?

Answer: 64 cubes arranged in a cube means that you have 4 cubes along each side so that 4 x 4 x 4 = 64 cubes total. If the length of each side is 1 meter, then the center to center distance for the cubes is just 1 meter/4 = **25 centimeters**.

Problem 2 – Suppose that the large cube had an edge length of 1 meter and it contained 1 million identical cubes. What would the distance between the cube centers be?

Answer: 1 million = 100 x 100 x 100 so there are 100 cubes along each edge, and since each edge measures 1 meter, the separation between the cubes would be 1 meter/100 = 1 centimeter.

Problem 3 – In the Van Allen belts, the average number density is about 1000 particles/m^3. What is the average distance between the atoms in the Van Allen belts?

Answer: The number of cubes along each side is (1000)1/3 = 10 so the distance is d = 1 meter/10 = **10 centimeters.**

Problem 4 – In the mesosphere, the average number density is about 2.5×10^{20} particles/m^3. What is the average distance between the atoms in the mesosphere in microns, where 1 micron = 10^{-6} meters?

Answer: The number of cubes along each 1-meter side is $(2.5\text{x}1020)^{1/3} = 6.3\times10^6$. The average separation between atoms is then 1 meter/$6.3\times10^6 = 1.6\times10^{-7}$ meters. Since 1 micron equals 10^{-6} meters so the atoms are separated by **0.16 microns**.

Note: The general formula for particle separation is

$$D = \frac{1 \text{ meter}}{n^{1/3}}$$ where n is the number density in particles/m^3

Snow to Water Ratios

The amount of snow from a storm can look impressive when it covers your house and cars, but if you melted the snow you would discover that very little water is actually involved. The 'snow to ice ratio' or Snow Ratio expresses how much volume of snow you get for a given volume of water. Typically a ratio of 10:1 (ten to one) means that every 10 inches of snowfall equals one inch of liquid water.

Problem 1 - During a winter storm called 'Snowmageddon' in 2010, the Washington DC region received about 24 inches of snow fall. If this was dry, uncompacted snow, about how many inches of rain would this equal if the Snow Ratio was 10:1 ?

Problem 2 - The Snow Ratio depends on the temperature of the air as shown in the table below:

Temp (F)	30^o	25^o	18^o	12^o	5^o	-10^o
Ratio	10:1	15:1	20:1	30:1	40:1	50:1

If 30 inches of snow fell in Calgary, Alberta at 18^oF, and 25 inches of snow fell in Denver, Colorado where the temperature was 25^o F, at which location would the most water have fallen?

Problem 1 - During a winter storm called 'Snowmageddon' in 2010, the Washington DC region received about 24 inches of snow fall. If this was dry, uncompacted snow, about how many inches of rain would this equal if the Snow Ratio was 10:1 ?

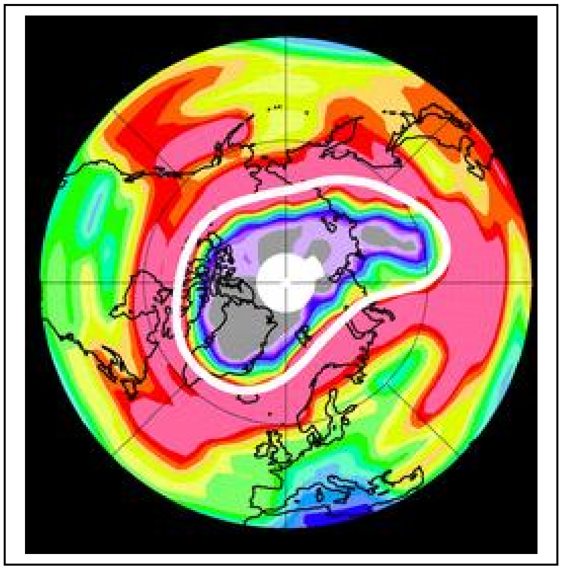
Answer: 24 inches of snow x (1 inch water/10 inches of snow) = **2.4 inches of water**.

Problem 2 - If 30 inches of snow fell in Calgary, Alberta at 18^{o}F, and 25 inches of snow fell in Denver, Colorado where the temperature was 25^{o} F, at which location would the most water have fallen?

Answer - In Alberta, the Snow Ratio for 18^{o} F is 20:1 and in Denver at 25^{o} F it is 15:1.

The amount of water that fell in Alberta is then 30 inches of snow x (1 inch water/20 inches snow) = 1.5 inches of water. In Denver it is 25 inches of snow x (1 inch water/15 inches snow) = 1.7 inches of water. **So more water fell in Denver, even though there was less snow on the ground!**

The Arctic Ozone Hole Continues to Grow in 2011



A NASA-led study has documented an unprecedented depletion of Earth's protective ozone layer above the Arctic during the winter and spring of 2011 caused by an unusually prolonged period of extremely low temperatures in the stratosphere. The amount of ozone destroyed in the Arctic in 2011 was comparable to that seen in some years in the Antarctic, where an ozone "hole" has formed each spring since the mid 1980s.

The figure to the left shows this 'Ozone Hole' as the area inside the white contour. The white circle indicates no data was available directly above the North Pole itself

The stratospheric ozone layer, extending from about 10 to 20 miles (15 to 35 kilometers) above the surface, protects life on Earth from the sun's harmful ultraviolet rays.

Problem 1 - If the diameter of Earth is 12,700 km about what is the approximate rectangular area of the Arctic ozone hole interior to the white oval contour shown in the above satellite imagery in units of millions of square kilometers?

Problem 2 - The study by Dr. Gloria Manney and her colleagues, which was published in the journal Nature on October 2, 2011, stated that the loss of ozone was most severe between altitudes of 16 to 22 kilometers. What is the total volume of the ozone layer involved in the Arctic depletion in 2011 in millions of cubic kilometers?

Problem 3 – The normal Arctic concentration of ozone is about 4 parts-per-million by volume (ppmV), but during the Arctic depletion it fell to 1.5 ppmV. If 1 ppmV represents a concentration of 1 cm^3 of material in a 1 million cm^3 volume, what is the volume occupied by the lost ozone?

Problem 1 - Answer: The diameter of the Earth disk is 65 mm, so the scale of the image is 12,700 km/65 mm = 195 km/mm. The approximate rectangular shape that fits inside the oval has dimensions of 30 mm x 15 mm, so the actual dimensions are 5950 km x 2925 km. The area is then A = 5850x2925 = **17 million square kilometers to 2 significant figures. Students answers will vary depending on the sizes assumed.**

Problem 2 - What is the total volume of the ozone layer involved in the Arctic depletion in 2011 in millions of cubic kilometers? Answer: From Problem 1, the area of the region was A = 17 million km^2. The thickness is 22-16 = 6 kilometers, so the volume of this rectangular slab of atmosphere is V = 17 million km^2x 6 km, V = 102 million cubic kilometers.

Problem 3 –What is the volume occupied by the lost ozone? Answer: The ozone layer has a volume of 102 million km^3. The difference in ozone volumes before and after the loss is 4 ppmV – 1.5 ppmV = 2.5 ppmV, so V= 2.5 x 102 million km^3/1 million and so C = 255 km^3. To check, C(ppmV) = 255 km^3/102 million km^3 = 2.5 ppmV. So, the volume of disappeared ozone equals **255 km^3**.

For more information, read the press release:

NASA Leads Study of Unprecedented Arctic Ozone Loss

Oct 2, 2011

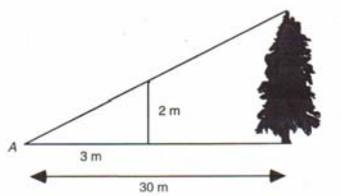
http://www.nasa.gov/topics/earth/features/arctic20111002.html

'A lonely cloud and its shadow!'
Courtesy: Henriette (2005)
The Cloud Appreciation Society

Sometimes, if you are lucky, you can see a single cloud and its shadow, perhaps while you were visiting the beach, standing in a meadow, or driving across the desert. By using the properties of similar triangles and a simple proportion, you can use this cloud and its shadow to figure out how high up the cloud is! You need a meter stick, and a bit of help from a friend to do this, though.

Let's see how this works for an example so that you can try this the next time you are at the beach...or the desert!

Similar Triangles and Proportions



The figure shows two similar right-triangles formed by the sides '3m' and '2m' and the sides '30m' and the unknown height of the tree. The basic geometric rule of similar triangles is that corresponding sides are in the same proportion. That means that the ratio formed by the sides '3m' and '30m' is the same numerical proportion as the sides '2m' and 'X' where X is the height of the tree. We can write this as:

$$\frac{30 \text{ meters}}{3 \text{ meters}} = \frac{X}{2 \text{ meters}}$$

and then solve this to get

$$X = 2 \text{ meters} \times (30/3)$$
$$= 20 \text{ meters.}$$

Suppose that in this problem, instead of the 2 meter long stick we used the height of your thumb, about 1.5 inches (3.8 cm), and placed it at arms-length from your eye, which is 20 inches (51 cm). Suppose also that if you were at a distance of 269 meters from the base of the tree the height of the tree would exactly equal the height of your thumb. How tall would the tree be? Again we set-up the proportion:

$$\frac{269 \text{ meters}}{51 \text{ cm}} = \frac{X}{3.8 \text{ cm}}$$

and solve for X: $X = 269 \text{ meters} \times (3.8 /51) = 20 \text{ meters.}$

So, using similar triangles and proportions is a very COOL way to figure out things about distant objects. In astronomy, you cannot even travel to these objects so using similar triangles and proportions is the only way to learn about their sizes and distances!

<u>Now let's apply this proportion method to studying clouds!</u>

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters is it from the base of the cloud down to the ground?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the vertical distance to the base of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Problem 3 – This next part is a bit tricky. As best you can, estimate how far it is from where you are standing to where the shadow of the cloud begins. You can also note some feature at this location like a tree or a rock formation, or a distant person sitting on a blanket! Count the number of paces it takes to get to this spot. Suppose that for this problem your pace was 2-feet long (0.7 meters) and you completed 3000 paces to get to the spot. How many meters did you travel?

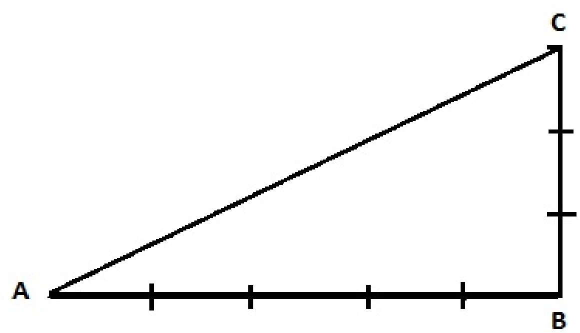
Problem 4 – It is now time to use proportional reasoning! Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual height of the cloud above the distant point! What would be your answer for the example we used?

Common Core Math Standards:
CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters is it from the base of the cloud down to the ground?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the vertical distance to the base of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Answer: We want all measurements to be in centimeters in order to draw the scaled triangle. AB = 20 inches x (2.5 cm/1 inch) = 50 cm. AC = 12 inches x (2.5 cm/ 1 inch) = 30 cm. The drawing looks like this. Each division is 10 centimeters.



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Answer: In this example, 3000 paces x (0.7 meters/1 pace) = **2,100 meters**.

Problem 4 – It is now time to use proportional reasoning. Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual height of the cloud above the distant point! What would be your answer for the example we used?

Answer: Let X be the actual height of the cloud, then from the similar triangle BC/AB = X/2100 meters, and since BC=30cm and AB=50 cm we have 30/50 = X/2100 and so X = 2100 (30/50) = 1260 meters. **So the cloud is about 1260 meters above the ground!**

You look up at the sky one day and see puffy little cumulus clouds hovering over the beach, a meadow, or over your town. Did you ever wonder just how much a cloud might weigh as it drifts by over your head?

Different clouds carry different amounts of water droplets and so they have different densities. Brilliant white cumulus clouds, for example, have densities of 0.3 grams/meter^3.

From the known cloud densities, we can estimate their masses once we know their volumes because Mass = Density x Volume.

Problem 1 – From the definition of density, what are the other two equations you can create that define mass and volume?

Problem 2 – A puffy cumulus cloud looks almost like a sphere. If its diameter is 3.0 kilometers, what is its volume in cubic meters? (use π = 3.14)

Problem 3 – What is the total mass of the cumulus cloud in kilograms and metric tons?

Problem 4 – You spot two clouds in the sky. The cumulus cloud is 1/5 the diameter of the cumulonimbus cloud, and the cumulonimbus cloud has 8 times the density of the cumulus cloud. What is the ratio of the mass of the cumulus cloud to the cumulonimbus cloud if both clouds are spherical in shape?

Grade 7 - Working with Density, mass and volume: Examples: 'California Mathematics Standards' - *Students can calculate the mass of a cylinder given its dimensions and density*. - Utah State Science Standards: I.2.c. "*Calculate the density of various solids and liquids.*"

Grade 8 - Common Core Math Standards:
CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Problem 1 – From the definition of density, what are the other two equations you can create that define mass and volume?

Answer: Density = Mass / Volume   Volume = Mass/Density

Problem 2 – A puffy cumulus cloud looks almost like a sphere. If its diameter is 3.0 kilometers, what is its volume in cubic meters? (use $\pi = 3.14$)

Answer: $V = 4/3\ \pi\ R^3$ and for D = 3.0 km, we have R = 1500 meters and so $V = 4/3\ \pi\ (1500\text{meters})^3 =$ **1.4×10^{10} meters^3**

Problem 3 – What is the total mass of the cumulus cloud in kilograms and metric tons?

Answer: Mass = Density x Volume so $M = 0.3\ \text{grams/m}^3 \times 1.4 \times 10^{10}\ \text{m}^3 = 4.2\times10^9$ grams. But 1 kg = 1000 grams, so **M = 4,200,000 kg**. This also equals **4200 metric tons**!

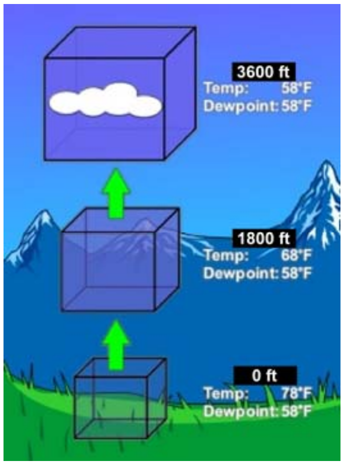
Problem 4 – You spot two clouds in the sky. The cumulus cloud is 1/5 the diameter of the cumulonimbus cloud, and the cumulonimbus cloud has 8 times the density of the cumulus cloud. What is the ratio of the mass of the cumulus cloud to the cumulonimbus cloud if both clouds are spherical in shape?

Answer: Mass = Density x Volume.

$V(\text{Cumulus})/V(\text{CN}) = (1/5)^3$ and D(Cumulus) = 1/8D(CN) so

$\text{Mass(Cumulus)} = (1/5)^3 \times (1/8) \times \text{Mass(CN)} = 1/1000\ \text{Mass(CN)}$ and so

Mass(Cumulus)/Mass(CN) = **1/1000**



Courtesy Katie R. Roussy (2006)
University of Illinois at Urbana

When it comes to your local weather report, it's easy to understand what the Weatherperson means by the terms temperature and wind speed. During the summertime, you will often hear about 'humidity', which is the amount of moisture carried by the air. This term also makes a lot of sense!

When the air is carrying a lot of moisture on a 'humid' hot summer's day, it feels very uncomfortable. Your body is trying to cool off by perspiring. Because the air is already carrying a lot of moisture, there is no place for your perspired water to easily go. So you start to over-heat and feel wet all over!

Air can carry water vapor, and warm air can carry a lot more water vapor than cold air. That's why your skin feels wet and clammy in the summer, and you often have problems with dry skin during the winter. Because the amount of water and air humidity depends on temperature, your Weatherperson will often use the term 'dew point' to tell you what to expect when you step outside!

When the air temperature is close to a critical temperature called the **dew point**, water vapor begins to condense out of the air as droplets. The windshield of your car will have beads of water all over, and if this happens inside your house, your windows will cloud up with drops of moisture. That is why for some locations, indoor air conditioners have to have a 'dehumidifier' to remove moisture from the air so that it doesn't condense on the cooler panes of window glass.

For large masses of air, millions of droplets of moisture can form in every cubic centimeter and you see a cloud begin to appear.

The diagram above shows what happens to an 'air mass' with a dew point temperature of 58^{o}F as it rises to cooler altitudes. Nothing happens if the dew point temperature is <u>below</u> the air temperature. But when the local air temperature equals or is larger than the dew point, the cloud appears.

Sometimes, the local temperature near the ground can be slightly above the dew point. When this happens, the air remains clear, but droplets of water can form on cooler windows or on cars. When the local ground temperature is <u>below</u> the dew point, droplets will condense in the air and you get ground fog!

Dew point temperature can be a confusing idea when you first work with it, but it is such a common and practical idea that you will hear about it on your local weather report, especially during the spring, summer and fall!

The dew point temperature is very complicated to calculate exactly because it depends on the local atmospheric pressure and temperature, and the amount of water vapor in the air. There are some ways to estimate dew point temperature that give a rough idea of what to expect. The following formula is one of these methods:

$$T_{\mathbf{dewpoint}} = T_{\mathbf{air}} - \frac{100 - P}{5}$$

A general rule-of-thumb is that, if the humidity of the air is 60% you will feel uncomfortable (P = 60). If the outside temperature on a hot summer's day is 90^{o}F (T_{air}=90^{o}F) then the dew point temperature is 82^{o}F. If the humidity of the air approaches 100%, then for this example the dew point temperature equals the air temperature and droplets of moisture will start to form.

Problem 1 – A Marathon runner finishes the race and gets into her car. The outdoor temperature is 75^{o}F and the humidity is 80%. Explain why her actions may be dangerous if she immediately drives away, and a simple remedy.

Problem 2 - It's a warm sunny day and the air is rather humid with a dew point of 75^{o}F .The ground temperature is 85^{o}F. If the air temperature decreases at a rate of 3.5^{o}F/1000 feet (called the wet lapse rate), at what altitude will a cloud begin to appear?

Problem 3 – On a comfortable summer day, the humidity is only 20% and the outside temperature is 80^{o}F. At what altitude might clouds start to form overhead if the air temperature is decreasing at a 'dry' lapse rate of 5.5^{o}F/1000 feet?

Common Core Math Standards:

Grade 6 – *CCSS.Math.Content.6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Grade 7 – *CCSS.Math.Content.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

Problem 1 – A Marathon runner finishes the race and gets into her car. The outdoor temperature is 75°F and the humidity is 80%. Explain why her actions may be dangerous if she immediately drives away, and a simple remedy.

Answer: The dew point temperature is T = 75 – (100-80)/5 = 71°F. As she expels moist air from her lungs, the humidity inside the car steadily increases and the dew point temperature rises from 71°F to 75°F. Then the inside of her windows begin to fog up as water condenses on their surfaces. This makes it harder for her to see outside and creates a dangerous situation.

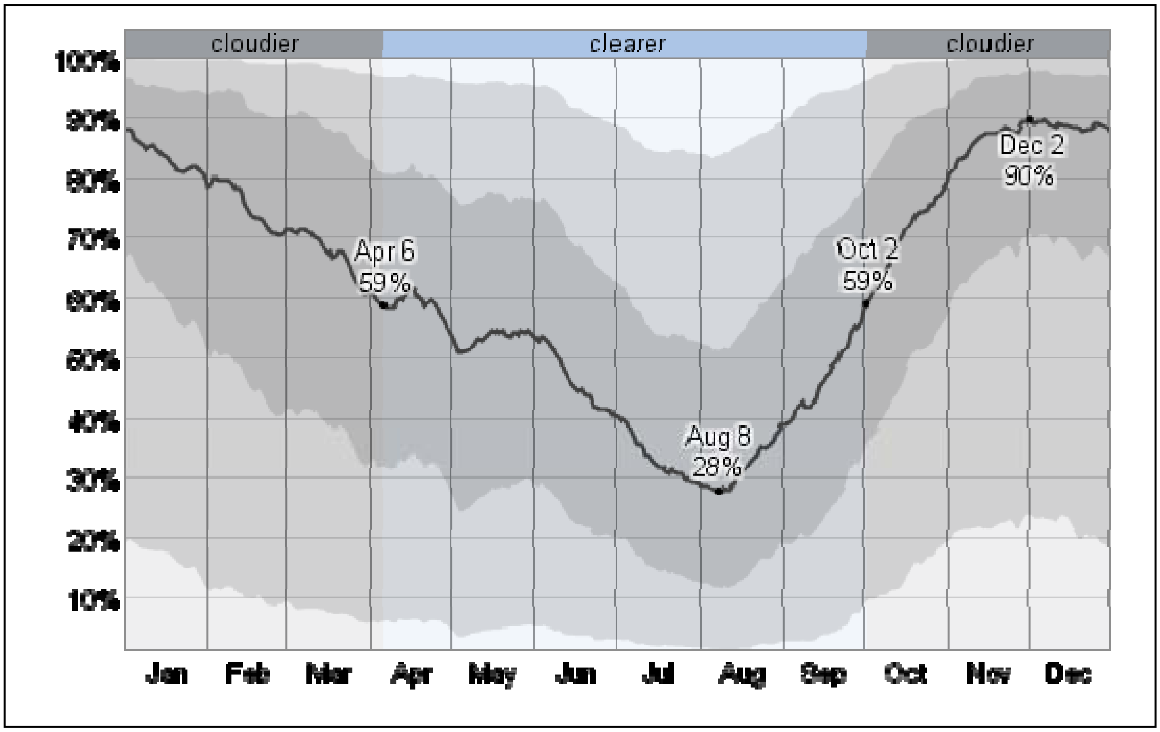
A remedy is to first turn on the car air conditioner, which will condense the remaining moisture in the air onto the cooling pipes. Then turn on the heater which will warm the windows and evaporate the moisture droplets on the windows. With the AC still on, this moisture will also condense on the cooling pipes, resolving the situation. Typically this takes only a minute or two for an average-sized car.

Problem 2 – It's a warm sunny day and the air is rather humid with a dew point of 75°F .The ground temperature is 85°F. If the air temperature decreases at a rate of 3.5°F/1000 feet, at what altitude will a cloud begin to appear? Answer: The rising air near the ground has to drop in temperature by 85°F – 75°F = 10°F. It is decreasing by 3.5°F every 1000 feet, so the dew point temperature of 75°F will be reached at an elevation of 10°F/3.5°F = **2857 feet**.

Problem 3 – On a comfortable summer day, the humidity is only 20% and the outside temperature is 80°F. At what altitude might clouds start to form overhead if the air temperature is decreasing at a 'dry' lapse rate of 5.5 °F/1000 feet?

Answer: First use the dew point formula to calculate $T_{\mathbf{dewpoint}}$. For P=20 and $T_{\mathbf{air}}$=80°F we get $T_{\mathbf{dewpoint}}$=80 – (100-20)/5 so $T_{\mathbf{dewpoint}}$=64°F.

Next, calculate the altitude from the lapse rate. The difference between the ground temperature and the dew point temperature is 80°F – 64°F = 16°F. The altitude will be A = 16°F/(5.5°F/1000feet) = **2909 feet**.



Many homeowners now use solar panels to collect sunlight and convert it into electricity on their rooftops. This is a good idea when there are no clouds in the sky, but what happens on a cloudy day? To get some idea of whether you should consider installing a solar power system, you need to understand how much cloud cover your area typically gets during the year. Too much cloud cover and you will not be able to support your house hold electrical needs by going 'off-grid'.

For example, the plot above (courtesy Weatherspark.com) describes the cloud cover percentage over Two Harbors, Minnesota during the course of an average year. This region has a humid continental climate with warm summers and no dry season. During a typical year, we can see that the cloud cover varies from nearly 90% in the winter to below 40% during the summer. Together with the high latitude of this location, it may be difficult to operate a year-round solar power system on your roof-top. At least during the winter, you will need to go back 'on grid' to run the electrical needs of your home because only 10% of the usable sunlight is available for your electrical system.

*You can investigate the cloud cover over your location and report what you see to other students and scientists by joining the S'COOL program (***http://scool.larc.nasa.gov/***). You can also use My NASA Data (***http://mynasadata.larc.nasa.gov/live-access-server-introduction/***) to download NASA cloud cover data and other Earth science resources.*

There is a simple formula to predict how much sunlight reaches the ground for different amounts of cloud cover:

$$P = 990\ (1-0.75F^3) \quad \text{watts/m}^2$$

where F is the fraction of sky cloud cover on a scale from 0.0 (0% no clouds) to 1.0 (100% complete coverage). Now let's see how to use this formula!

Problem 1 – For what percentage of the year are conditions considered cloudy in Two Harbors?

Problem 2 – Based upon the trend in the black line on the graph, what is the average cloud cover during the year?

Problem 3 – From the formula for solar power, for what percentage of sky cover will the homeowner get more than 50% of the maximum solar power from their electric system?

Problem 4 – On the cloud cover graph, shade in the region that represents the condition that the homeowner will get more than 50% of the available electrical power.

Problem 5 – About what percentage of the year will the homeowner be able to generate more than 50% of the available solar power?

Common Core Math Standards:
CCSS.Math.Content.6.RP.A.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
CCSS.Math.Content.8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

More about the cloud cover formula can be found at:
http://www.shodor.org/os411/courses/_master/tools/calculators/solarrad/

The data from Two Harbors is from
http://weatherspark.com/averages/31818/Two-Harbors-Minnesota-United-States

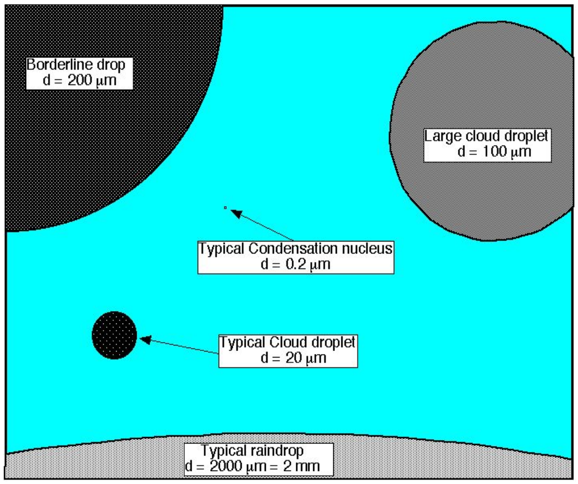
Problem 1 – For what percentage of the year are conditions considered cloudy in Two Harbors? Answer: January, February, March and October, November and December so P = 100% x (6/12) = **50%**.

Problem 2 – Based upon the trend in the black line on the graph, what is the average cloud cover during the year? Answer: The highest is 90% and the lowest is 28% so the average of these two is **59%**. Students may also calculate the average monthly coverage (January=85%, February=75%, March=67%, April=60%, May=55% June=45% July=32% August=32% September=45% October=70% November=87% December=87%) and get **62%**.

Problem 3 – From the formula for solar power, for what percentage of sky cover will the homeowner only get 50% of the maximum solar power from their electric system? Answer: The maximum solar power occurs for F=0 and equals 990 watts/m^2. Half of this is 495 watts/m2 so we want $495 = 990(1-0.75F^3)$. This means that $0.50 = 0.75F^3$ and so $F^3 = 0.67$ and so solving for F we get $F = (0.67)^{1/3} = 0.87$. **So 87% cloud cover produces a reduction of 50% in electrical power.**

Problem 4 – On the cloud cover graph, shade in the region that represents the condition that the homeowner will get more than 50% of the available electrical power. Answer: Draw a horizontal line across the graph at '87%'. And shade in all the area below this line to indicate 'more than 50% of available power'.

Problem 5 – About what percentage of the year will the homeowner be able to generate more than 50% of the available solar power? Answer: Only three months have more than 87% cloud cover: January, November and December, so there are 9 months producing more than 50% : P = 100% (9/12) = **75%**.



Without droplets of water, most clouds would be transparent! If you were to look inside a cloud you would see droplets of water of many different sizes because droplets constantly grow in size once they are formed.

The figure to the left shows some typical kinds of water droplets you might find in a cloud along with their diameters in micrometers (microns). Recall that one micrometer=1/1000000 or 10^{-6} meters.

The following exercises let you explore some of the properties of water droplets. In all cases, assume that the droplet is a perfect sphere!

Problem 1 – Water droplets are made out of water (of course!) and water has a density of 1000 kg/m^3. What is the mass, in grams, of each of the five types of droplets described in the figure?

Problem 2 – To the nearest 1000, about how many typical cloud droplets have to be combined to form one large cloud droplet?

Problem 3 – To the nearest 1000, about how many large cloud droplets have to combine to form one typical raindrop?

Problem 4 – Suppose that it takes about 2 minutes for a large cloud droplet to double in mass. How long does it take a large cloud droplet to grow into a raindrop and leave the cloud?

Common Core Math Standards:

<u>CCSS.Math.Content 6.RP.3.d</u> Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
<u>CCSS.Math.Content.8.G.C.9</u> Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
<u>CCSS.Math.Content.8.EE.A.4</u> Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology

Problem 1 – Water droplets are made out of water (of course!) and water has a density of 1000 kg/m^3. What is the mass, in grams, of each of the five types of droplets described in the figure?
Answer: Recall Volume = 4/3 π R^3 and Mass = Density x Volume.

Raindrop: r=2 mm = 0.002 meters
$V = 4/3\ \pi\ (0.002)^3 = 3.3\times10^{-8}\ m^3$
Mass = 1000 kg/m^3 x 3.3×10^{-8} m^3 x (1000gm/1kg) = **0.033 grams**

Borderline Drop: r = 200 microns. = 0.0002 meters
$V = 4/3\ \pi\ (0.0002)^3 = 3.3\times10^{-11}\ m^3$
Mass = 1000 x 3.3×10^{-11} x (1000gm/1kg) = **3.3×10^{-5} grams** (= 33 micrograms)

Large Cloud Droplet: r = 100 microns = 0.0001 meters
$V = 4/3\ \pi\ (0.0001)^3 = 4.2\times10^{-12}\ m^3$
Mass = 1000 x 4.2×10^{-12} x (1000gm/1kg) = **4.2×10^{-6} grams** (4.2 micrograms)

Typical Cloud Droplet: r = 20 microns = 0.00002 meters
$V = 4/3\ \pi\ (0.00002)^3 = 3.3\times10^{-14}\ m^3$
Mass = 1000 x 3.3×10^{-14} x (1000gm/1kg) = **3.3×10^{-9} grams** (3.3 nanograms)

Typical Condensation Nucleus: r = 0.2 microns = 0.0000002 meters
$V = 4/3\ \pi\ (0.0000002)^3 = 3.3\times10^{-20}\ m^3$
Mass = 1000 x 3.3×10^{-20} x (1000gm/1kg) = **3.3×10^{-14}** grams

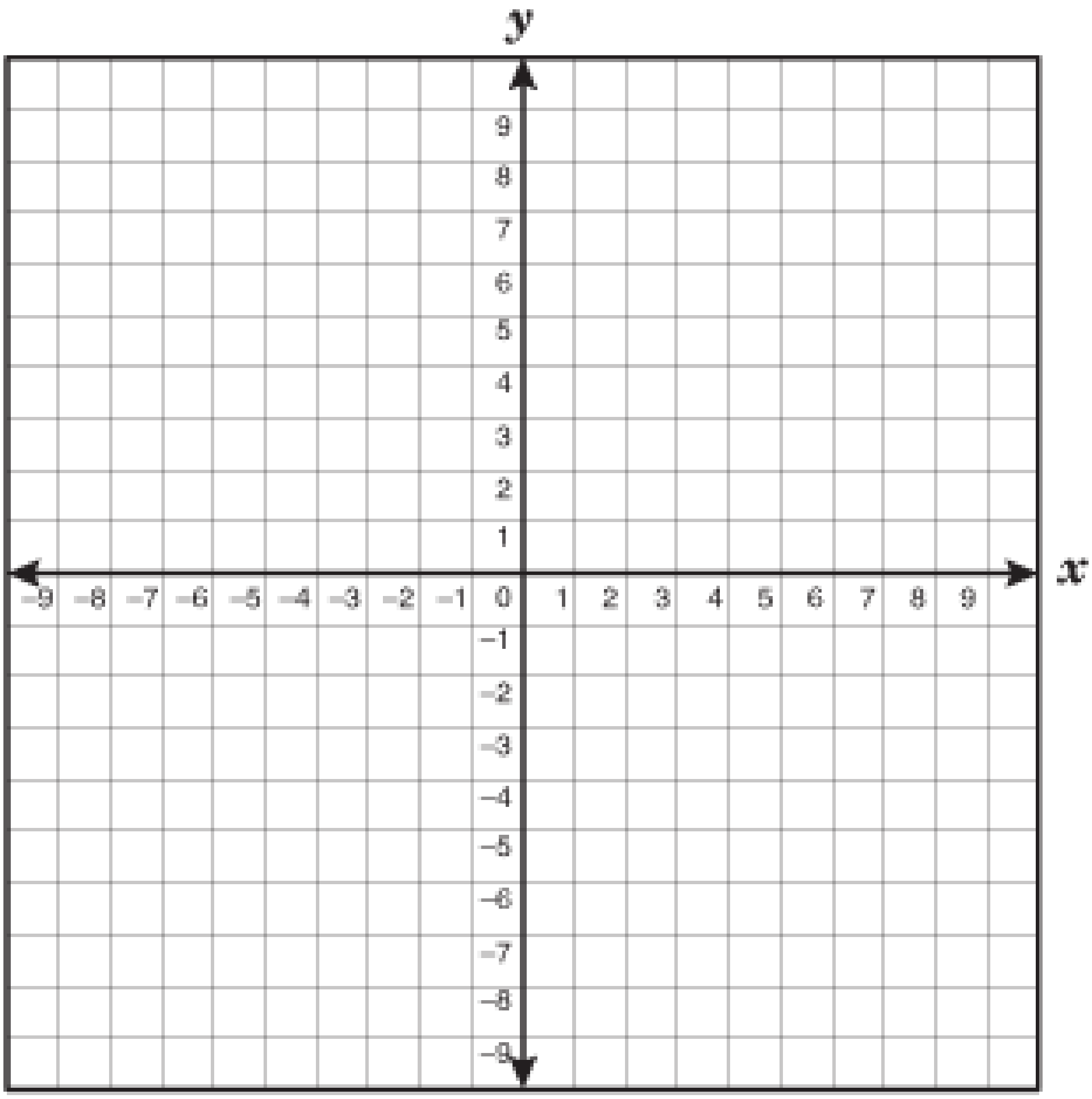
Problem 2 – To the nearest 1000, about how many typical cloud droplets have to be combined to form one large cloud droplet? Answer: N = Mass of large droplet / mass of typical cloud droplet = 4.2×10^{-6} gm/3.3×10^{-9} gm = 1272 typical droplets or **about 1000**.

Problem 3 – To the nearest 1000, about how many large cloud droplets have to combine to form one typical raindrop? Answer: N = Mass of raindrop/mass of large droplet = 0.033 grams/4.2×10^{-6} grams = 7857 or about **8000 large droplets**.

Problem 4 – Suppose that it takes about 2 minutes for a large cloud droplet to double in mass. How long does it take a large cloud droplet to grow into a raindrop and leave the cloud?

Answer: In Problem 3 we saw that about 8000 large cloud droplets equals a raindrop. Since 8000 is about 2^{13}, we need 13 doubling times to grow this large, which takes 13 x 2 minutes = **26 minutes**.

Note: Students may want to make a scaled model of droplet sizes using styrofoam balls or other round objects.



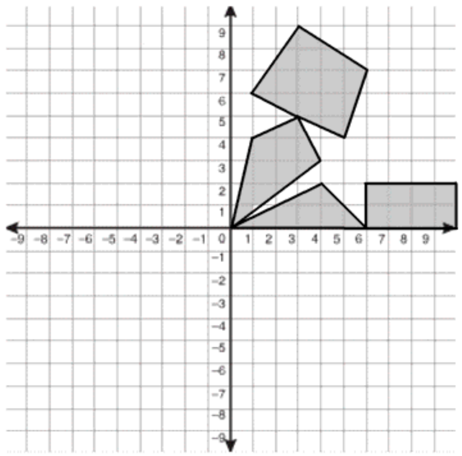
Snowflakes have a symmetrical shape that often follows a simple pattern that is replicated to form the full shape that you see.

Problem 1 - Graph the following points to make a design in the First Quadrant:

(10,0), (10,2), (6,2), (6,0), (4,2), (0,0), (4,3), (3,5), (5,4), (6,7), (3,9), (1,6), (3,5), (1,4), (0,0)

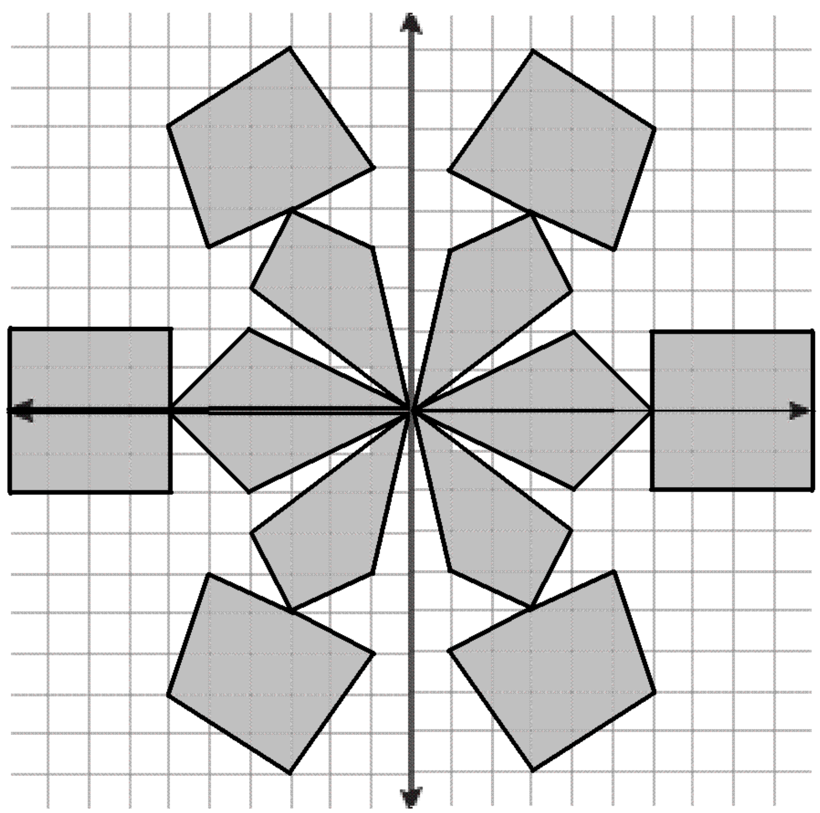
Problem 2 - Connect the points with line segments in the order given.

Problem 3 - Reflect the pattern that you drew into the Second Quadrant, then complete the pattern in Quadrants Three and Four to form the full snowflake shape!

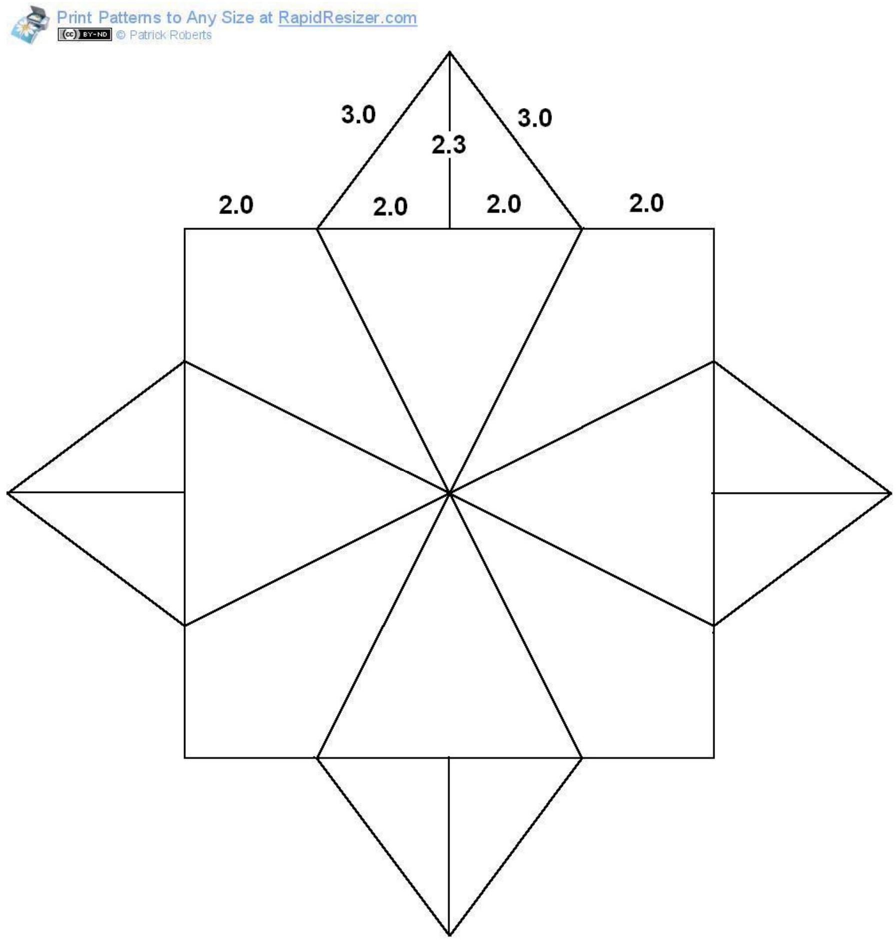


Problem 1 and 2 -

Problem 3 - Students may either place 'mirrors along the X and Y axis and redraw the shape in the First Quadrant, or use the following symmetry idea: To reflect the figure into Quadrant Two, plot the points in Quadrant One with the sign of the x coordinates replaced by their negative : (x,y) becomes (-x, y). For Quadrant Three use (x,y) becomes (-x,-y) and for Quadrant Four (x,y) becomes (x,-y). The full figure is shown below:



The Surface Area of a Snowflake



The diagram above shows the basic plan for one common type of snowflake. The detailed pattern within each polygonal area has been removed to show the regular areas. The numbers at the top are the measured line segments in millimeters.

Problem 1 - Using the geometric clues in the diagram, what is the total area of this pattern in square millimeters, rounded to the nearest integer?

Problem 2 - If all measurements were doubled in length, what would be the total area of the pattern to the nearest integer in square-millimeters?

Problem 1 - Using the geometric clues in the diagram, what is the total area of this pattern in square millimeters, rounded to the nearest integer?

Answer:

The pattern consists of a main square with a side length of 2.0mm+2.0mm+2.0mm+2.0mm = 8.0 mm and an area of $(8.0mm)^2$ = 64 mm^2.

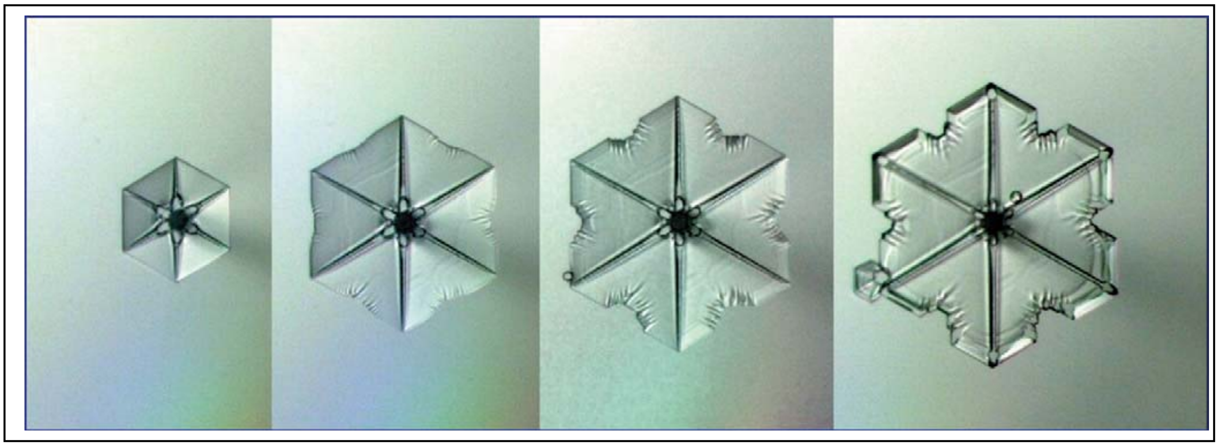
The four triangular points each have an area of 1/2(4.0mm)(2.3mm) = 4.6 mm^2, so the total area of the pattern is 64.0 mm^2+ 4(4.6mm^2) = 82.4 mm^2, which is rounded to **82 mm^2**.

Problem 2 - If all measurements were doubled in length, what would be the total area of the pattern to the nearest integer in square-millimeters?

Answer: Doubling the dimensions means that the area is increased by a factor of 2x2 = 4 so it now becomes 82.4mm^2 x 4 = 329.6 mm^2, which rounds to **330 mm^2**.

The side length of the square becomes 2 x 8.0mm = 16.0 mm and the area is then $(16mm)^2$ = 256 mm^2. The four triangles each have an area of 1/2 (8.0mm) (4.6mm) = 18.4 mm^2, so the total area is 256 mm^2+ 4(18.4mm^2) = 329.6 mm^2 or rounded to **330 mm^2**.

Snowflake Growth Rates and Surface Area



A snowflake is a flat figure whose area doubles over time as liquid droplets condense on its surface. For average cloud conditions, the area doubles every 2 hours.

No matter what the shape of a polygon, the area of a polygon will increase by a fixed amount as the size of the polygon increases.

Problem 1 - Suppose the time to double its area is 2 hours. How many doublings in area will have occurred in 8 hours?

Problem 2 – If the area of the snowflake at the start of its growth is 1 square millimeter, what will its area be after 8 hours? To organize your thinking about snowflake growth, create a table for the snowflakes size and area.

Problem 3 – If the size of the snowflake was 1 millimeter at the start of growth, what will be its size at the end of a snow storm that lasted 8 hours if the area doubling time is 2 hours? To organize your thinking about snowflake growth, create a table for the snowflakes size and area.

Problem 1 - Suppose the time to double its area is 2 hours. How many doublings in area will have occurred in 8 hours?

Answer: The snowflake has been growing for 8 hours which is 8/2 = **4 doubling times.**

Problem 2 – If the area of the snowflake at the start of its growth is 1 square millimeter, what will its area be after 8 hours?

Doubling	**1**	**2**	**3**	**4**	**5**	**6**
Area	**2**	**4**	**8**	**16**	**32**	**64**
Size	**1.4**	**2**	**2.8**	**4**	**5.7**	**8**

Answer: It will have an area that is 2 x 2 x 2 x 2 = 16 times larger or **16 square millimeters.**

Problem 3 - If the size of the snowflake was 1 millimeter at the start of growth, what will be its size at the end of a snow storm that lasted 8 hours if the area doubling time is 2 hours?

Answer: 8 hours = 4 doubling times so it has increased in area by 16 times. Because area = length x length, since 16 = 4 x 4, the snowflake has increased its size by 4 times so it is now 1 mm x 4 = **4 millimeters in diameter.**

Snow Density and Volume

This scientist is collecting cylindrical snow cores to study snow density from the wall of a snow pit. This pit was carefully dug into the Taku Glacier, in the Juneau Icefield of the Tongass National Forest, Alaska.

The density of snow tells scientists a lot about the history of the snow, and whether it is safe for skiers.

Density is defined as the amount of mass that an object has compared to the volume that it takes up. On average, a cubic meter of freshly-fallen snow has an average mass of about 50 kilograms. Snow that has been compacted by its own weight at a depth of 3 meters can have 200 kilograms in the same volume.

Density is defined as mass/volume. Freshly-fallen snow has a density of 50 kg/meter^3 = 50 kg/m^3, while the compressed snow described above has a higher density of 200 kg/m^3. Let's explore some other examples of estimating snow density!

Problem 1 – A scientist uses a cylindrical gauge to sample the snow in a trench wall. The cylinder has a radius of 5 centimeters and a length of 60 centimeters, and it has a mass of 50 grams. After filling the cylinder with snow, the cylinder is again weighed and now has a mass of 520 grams. What is the density of the snow that was sampled?

Problem 2 – Two scientists measure the snow density from two different mountain locations using two different snow gauges: A and B. Gauge A has a radius of 6.3 cm and a height of 40 cm, while Gauge B has a radius of 8.0 cm and a height of 40 cm. To the nearest cubic centimeter, what are the volumes of the two gauges? (use π = 3.141)

Problem 3 – If 500 grams is collected by Gauge A and 804 grams is collected by Gauge B, what are the snow densities to the nearest tenth. Are the scientists sampling different kinds of snow, or similar kinds of snow at the two locations?

Problem 1 – A scientist uses a cylindrical gauge to sample the snow in a trench wall. The cylinder has a radius of 5 centimeters and a length of 60 centimeters, and it has a mass of 50 grams. After filling the cylinder with snow, the cylinder is again weighed and now has a mass of 520 grams. What is the density of the snow that was sampled?

Answer: For a cylinder, the volume is given by the formula $V = \pi r^2 h$,where r is the radius and h is the height. The snow gauge volume is then

$V = (3.141)(5cm)^2 (60\ cm) = 4,711\ cm^3$.

When empty, the snow gauge had a mass of 50 grams and when full of snow it had a mass of 520 grams, so the actual mass of the snow was

M = 520 gm - 50 gm = 470 grams.

The density of the snow is then D = M/V = 470 gms/4711 cm^3 = **0.1 gm/cm^3**

Problem 2 – Two scientists measure the snow density from two different mountain locations using two different snow gauges: A and B. Gauge A has a radius of 6.3 cm and a height of 40 cm, while Gauge B has a radius of 8.0 cm and a height of 40 cm. To the nearest cubic centimeter, what are the volumes of the two gauges? (use $\pi = 3.141$)

Answer: The volume of a cylinder is given by $V = \pi R^2 h$, so

The volume of Gauge A is $V = (3.141) \times (6.3\ cm)^2 \times (40cm) = 4,987\ cm^3$.

The volume of Gauge B is $V = (3.141) \times (8.0\ cm)^2 \times (40cm) = 8,041\ cm^3$.

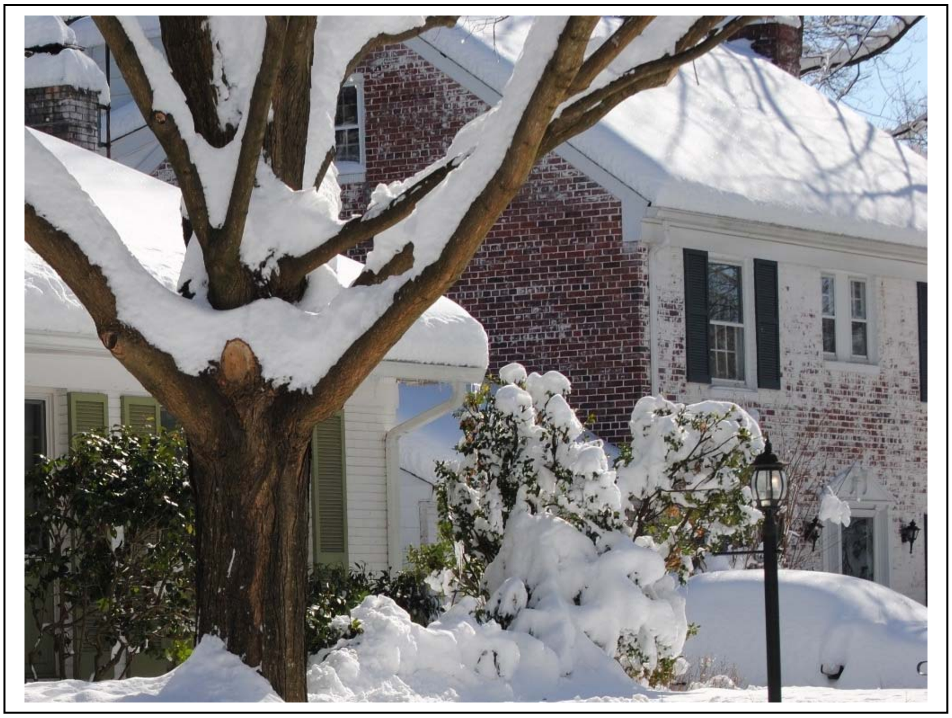
Problem 3 - If 500 grams is collected by Gauge A and 804 grams is collected by Gauge B, what are the snow densities to the nearest tenth, and are the scientists sampling different kinds of snow, or similar kinds of snow at the two locations?

Answer - The density measured by Gauge A is D = 500 gm/4987 cm^3 = **0.1 gm/cm^3**.

The density measured by Gauge B is D = 804 gm/8041 cm^3 = **0.1 gm/cm^3**

So the densities are the same and the kinds of snow are probably also the same at the two locations.

Snow Density, Mass and Roof Failure



During snowfalls, most children are excited by the accumulating snow, while many parents may worry if the weight of the snow will eventually cause their roofs to collapse. Although a small amount of snow weighs next to nothing, a few feet can weigh many pounds. How much snow is too much for the average roof on a house? Engineers estimate that 65 pounds per square foot (320 kg/m^2) is the average amount that a standard wood-framed roof can hold before it collapses. Dry snow has a density of about 50 kg/m^3 while wet snow has a density of 200 kg/m^3.

Problem 1 - Two houses are covered with a blanket of snow. House A has dry snow to a depth of 1 meter, and House B has a roof covered with wet snow to a depth of ½ meter. Which house is at greater risk of roof collapse?

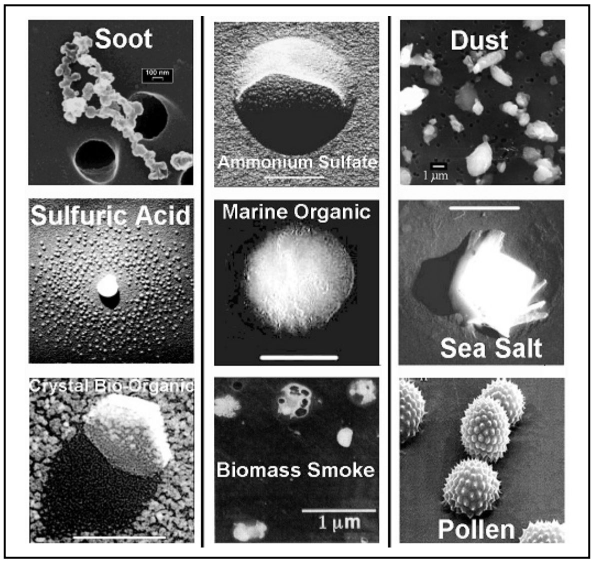
Problem 2 - A snow storm of wet snow began at 6:00 am and continued steadily all day at a rate of 20 cm/hour. At what time will the snow accumulating on the roof reach the critical load for roof collapse?

Problem 1 - Two houses are covered with a blanket of snow. House A has dry snow to a depth of 1 meter, and House B has a roof covered with wet snow to a depth of ½ meter. Which house is at greater risk of roof collapse?

Answer: House A has 1 meter of dry snow covering every square meter of surface, so the mass of this snow on the roof is 50 kg/m^3 x 1 meter = 50 kg/m^2. House B has wet snow to a depth of 1/2 meter so the mass is 200 kg/m^3 x 1/2 meter = 100 kg/m^2. **House B is at greater risk even though it appears to have much less snow cover.**

Problem 2 - A snow storm of wet snow began at 6:00 am and continued steadily all day at a rate of 20 cm/hour. At what time will the snow accumulating on the roof reach the critical load for roof collapse?

Answer: The wet snow density is 200 kg/m^3. It is accumulating at a rate of 0.2 meters/hour. To reach 320 kg/m^2, which engineers say is the critical loading for roof collapse, you need to accumulate a thickness of 320/200 = 1.6 meters. At a rate of 0.2 meters/hour this will take about 1.6 meters x (1 hour/0.2 meters) = 8 hours, so by about **2:00 pm**, the roof might collapse.



An aerosol is a mixture of fine solid particles or liquid droplets in air or another gas. Examples of aerosols include clouds, haze, and air pollution such as smog and smoke. The liquid or solid particles have diameter mostly smaller than 1 μm or so.

Aerosols are so small we have no real choice but to use scientific notation to determine their properties such as volume, mass or density! The graph to the left shows the sizes of different types of aerosols in terms of nanometers, where 1 nanometer is 1 one billionth of a meter or 10^{-9} meters.

Problem 1 – A human hair has a diameter of 100 micrometers (100 microns). If 1000 nanometers equals 1 micron, how many 250 nanometer aerosol particles can fit across the diameter of one human hair?

Problem 2 – If the density of a typical spherical sea salt aerosol particle is 2.0 grams/cm^3, and the particle has a diameter of 500 nanometers, what is the mass of a single aerosol particle in A) grams? B) micrograms?

Problem 3 - On an especially hazy day, the density of aerosol particles in the air is 10 million particles per cubic centimeter. If the particles have an average size of 900 nanometers and a density of 1.5 grams/cm^3, A) how much aerosol mass would there be in a cubic meter of air? B) If you breath-in 100 liters of air every minute, and 1 liter equals 1000 cm^3, how many grams of aerosols do you inhale every day?

Problem 1 – A human hair has a diameter of 100 micrometers (100 microns). If 1000 nanometers equals 1 micron, how many 250 nanometer aerosol particles can fit across the diameter of one human hair?

Answer: Diameter of human hair in nanometers = 100 micrometers x (1000 nanometers/1 micrometer) = 100000 nanometers. Since one aerosol is 250 nm in diameter, there would be 100000/250 = **400 aerosol particles** place end to end to cross the diameter of one human hair.

Problem 2 – If the density of a typical spherical sea salt aerosol particle is 2.0 grams/cm^3, and the particle has a diameter of 500 nanometers, what is the mass of a single aerosol particle in A) grams? B) micrograms?

Answer: A) The radius of the aerosol is 250 nanometers. Since 1 meter = 100 centimeters, the radius is 250 x 10^{-9} x (100 cm/1 meter) = 2.5×10^{-5} cm. Volume of a spherical particle = 4/3 π (2.5×10^{-5} meters)^3 = 6.5×10^{-14} cm^3. The mass in grams is then 2.0 grams/cm^3 x Volume in cm^3, so **M = 1.3×10^{-13} grams**.

B) 1 microgram = 10^{-6} grams, so **M = 1.3×10^{-7} micrograms**.

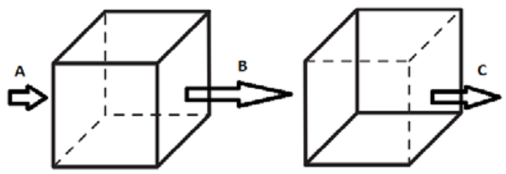
Problem 3 - On an especially hazy day, the density of aerosol particles in the air is 10 million particles per cubic centimeter. If the particles have an average size of 900 nanometers and a density of 1.5 grams/cm^3, A) how much aerosol mass would there be in a cubic meter of air? B) If you breath-in 100 liters of air every minute, and 1 liter equals 1000 cm^3, how many grams of aerosols do you inhale every day?

Answer: A) Each particle has a mass of

$$M = 1.5 \text{ gm/cm}^3 \times (4/3\ \pi\ (4.5\times10^{-5} \text{ cm}))^3 = 5.7\times10^{-13} \text{ grams.}$$

If the particle density is 10^7 particles/cm^3, in 1 cubic meter there would be 10^7particles/cm^3 x 10^6 cm^3= 10^{13} particles, and so the total mass per cubic meter would be 5.7×10^{-13} grams x 10^{13} particles = **5.7 grams**!

B) You breath in 100 liters/minute x (1000 cm^3/1 liter) x (60 minutes/1 hour) x (24 hours/1 day) = 1.44×10^8 cm^3. Since the aerosol mass is 5.7 grams/meter^3 we have 1.44×10^8 cm^3 x (1 meter^3/10^6 cm^3) x (5.7 grams/1 meter^3) = 144 x 5.7 = **821 grams/day**.



A ray of light passes through the two cubes from left to right. At **A**, the light ray has its full intensity of 100%.

After it leaves the first box, the aerosols have reduced the light intensity by 10% so that at B, the light intensity is 100% x (0.90) = 90%.

After passing through an identical aerosol region in the second box, the light intensity is reduced by an additional 10%. The final light intensity at C is then 100% x (0.90) x (0.90) = 81%.

Aerosols are small particles of liquids or solids that are light enough to be suspended in the air for long periods of time. Common aerosols can include dust from volcanoes, exhaust from jet plains, smog, or ash from the combustion of fossil fuels and wood. Most of the time you do not even notice they are there, unless they are present in large enough numbers.

When the concentration of aerosols is high enough, they can actually cause the dimming of sunlight, which is why the sky can appear darker on a foggy day (water aerosols) or very hazy on a day with lots of smog or distant forest fires.

The figure to the left shows how light dimming occurs, if the intensity of light is reduced by 10% as it passes through two boxes of air.

Problem 1 – Suppose in the above example, the light ray passes through 6 identical boxes. How bright will the light be after it leaves the sixth box?

Problem 2 – Suppose that each box is 1 kilometer on a side, and that the light is dimmed by1% through each box. If the total path is 20 kilometers, what will be the brightness of the light after it leaves this aerosol cloud to the nearest percent?

Problem 3 – A light ray passes through 5 kilometers of ordinary air, which reduces the light by 0.5% per kilometer, and then passes through a dense cloud that reduces the light by 5% per kilometer. If the cloud is 3 kilometers long, to the nearest percentage what will be the light intensity when the light leaves the cloud?

Problem 1 – Suppose in the above example, the light ray passes through 6 identical boxes. How bright will the light be after it leaves the sixth box?

Answer: 100% x 0.9 x 0.9 x 0.9 x 0.9 x 0.9 x 0.9 = 100% x $(0.9)^6$ = **53%**.

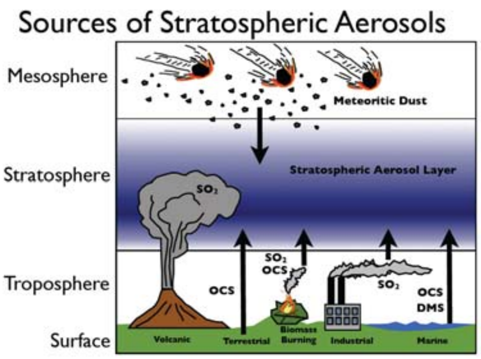
Problem 2 – Suppose that each box is 1 kilometer on a side, and that the light is dimmed by1% through each box. If the total path is 20 kilometers, what will be the brightness of the light after it leaves this aerosol cloud to the nearest percent?

Answer: There are 20 boxes along the light ray so 100% x $(0.99)^{20}$ = **82%**.

Problem 3 – A light ray passes through 5 kilometers of ordinary air, which reduces the light by 0.5% per kilometer, and then passes through a dense cloud that reduces the light by 5% per kilometer. If the cloud is 3 kilometers long, to the nearest percentage what will be the light intensity when the light leaves the cloud?

Answer: 100% x $(0.995)^5$ x $(0.95)^3$ = 100% x 0.975 x 0.857 = **84%**

Visibility and dimming: Typical morning fogs can attenuate light by 50% but there is still enough light to read a book, it's just that the light passing through the fog is scattered, which means that you cannot see an object clearly if it is more than a few hundred meters away.



(Credit: NOAA)

OCS = Carbonyl sulfide,
DMS = Dimethyl sulfide,
SO_2 = Sulfur dioxide,
CO2 = carbon dioxide.

SAGE-III is designed to measure the concentration of aerosols in the stratosphere, but where do these particles come from?

The figure to the left shows some of the common sources. Scientists are concerned about recent increases in stratospheric aerosols because they have an impact on climate change. In the stratosphere, miles above Earth's surface, aerosols can reflect sunlight back into space, which leads to a cooling influence at the ground. According to recent studies, this cooling effect may explain the changes in the pace of global warming measured since 2000.

According to recent measurements, carbonyl sulfide is produced at the following rates given in terms of millions of tons per year: Marine = 0.33 Mt/yr; Volcanism = 0.05 Mt/yr; Terrestrial = 0.02 Mt/yr; Biomass Burning =0.07 Mt/yr; Industrial= 0.33 Mt/yr.

Sulfur dioxide is produced at a rate of 10 Mt/yr from volcanos; Industrial = 146 Mt/yr; Biomass burning = 8 Mt/yr.

Problem 1 – What is the total rate of production of carbonyl sulfide by all sources?

Problem 2 – What is the percentage of each source of carbonyl sulfide compared to the total production rate?

Problem 3 – Draw a circle (pie) graph that illustrates the percentages of each carbonyl sulfide source compared to the total rate.

Problem 4 – Comparing carbonyl sulfide and sulfur dioxide, which of the two has the largest percentage contribution due to human activity?

OCS rates from C. Brühl1, J. Lelieveld1,3, P. J. Crutzen1, and H. Tost2 'The role of carbonyl sulphide as a source of stratospheric sulphate aerosol and its impact on climate', Atmos. Chem. Phys., 12, 1239-1253, 2012, http://www.atmos-chem-phys.net/12/1239/2012/acp-12-1239-2012.html'

According to recent measurements, carbonyl sulfide is produced at the following rates given in terms of millions of tons per year: Marine = 0.33 Mt/yr; Volcanism = 0.05 Mt/yr; Terrestrial = 0.02 Mt/yr; Biomass Burning =0.07 Mt/yr; Industrial= 0.33 Mt/yr. Sulfur dioxide is produced at a rate of 10 Mt/yr from volcanos; Industrial = 146 Mt/yr; Biomass burning = 8 Mt/yr.

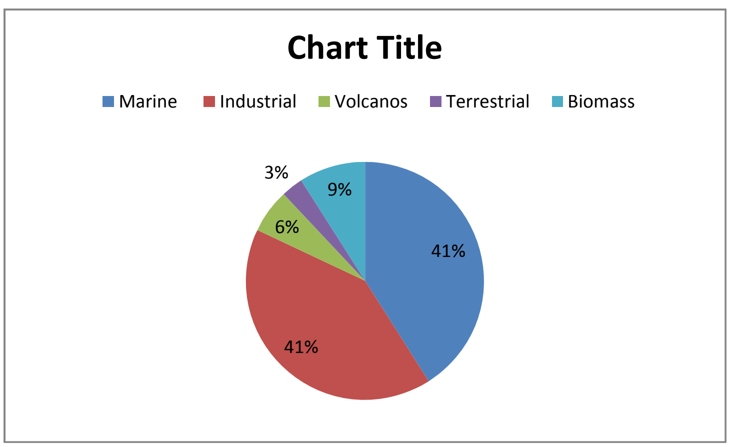
Problem 1 – What is the total rate of production of carbonyl sulfide by all sources?

Answer: 0.33 + 0.05 + 0.02 + 0.07 + 0.33 = **0.80 Mt/yr**.

Problem 2 – What is the percentage of each source of carbonyl sulfide compared to the total production rate?

Answer: **Marine: 41% Volcanism: 6% Terrestrial: 3% Biomass: 9% Industrial: 41%**

Problem 3 – Draw a circle (pie) graph that illustrates the percentages of each source compared to the total rate. Answer: See below.



Problem 4 – Comparing carbonyl sulfide and sulfur dioxide, which of the two has the largest percentage contribution due to human activity?

Answer: Total production = 164 Mt/yr. Percentages: Volcanism: 6%, Biomass burning: 5%; Industrial: 89%. **The production of sulfur dioxide by human activity is the highest: 89% vs 41%.**

Altitude (km)	Latitude		
	70N	30N	0
6	13.0	6.2	3.4
7	10.5	6.1	4.0
8	9.6	5.3	2.8
9	8.7	4.9	2.0
10	7.4	4.9	2.0
11	6.3	3.9	2.1
12	5.3	3.3	1.8
13	4.6	2.6	1.8
14	4.0	2.3	1.8
15	3.5	2.2	1.8
16	3.0	2.2	1.6
17	2.4	2.4	1.7
18	1.8	2.7	2.1
19	1.2	2.7	2.8
20	0.8	2.2	3.2
21	0.5	1.6	3.1
22	0.4	0.9	2.7
23	0.3	0.6	2.2
24	0.2	0.4	1.7
25	0.2	0.3	1.5

The SAGE-II experiment was flown on the International Space Station beginning October 5, 1984 and ended its investigations on August 26, 2005. The SAGE-III instrument will be launched in 2014 and complete programs begun by the SAGE-I and SAGE-II instruments.

Aerosols can block out sunlight reaching the ground, which is a process called extinction. This can causes changes in the heating of Earth and can alter its climate.

The SAGE instruments produce data tables like the one shown to the left. The table gives the altitude of the measurement. The columns give the extinction measured at different latitudes indicated in the top row. The numbers indicate the aerosol extinction in units of 0.0001 km^{-1}. For example, at 6 kilometers altitude at a latitude of 30N, the extinction was 6.2 x 0.0001 = 0.00062 km^{-1}.

Problem 1 – At what location is the extinction A) the highest? B) the lowest?

Problem 2 – Above an altitude of 20 kilometers, what is the average extinction at each latitude?

Problem 3 – Graph the extinction data for 30N between an altitude of 6 and 14 km and draw a straight line through the data. What is the slope of this line, and what are the units for this slope?

Problem 4 – Write the equation of the line that you drew in Problem 3, and use it to estimate the extinction at an altitude of 19 kilometers. Is our mathematical model a good fit to the actual data?

The SAGE-II Lesson Plan for graphing data can be found at
http://science-edu.larc.nasa.gov/EDDOCS/Aerosols/aer_sci.html

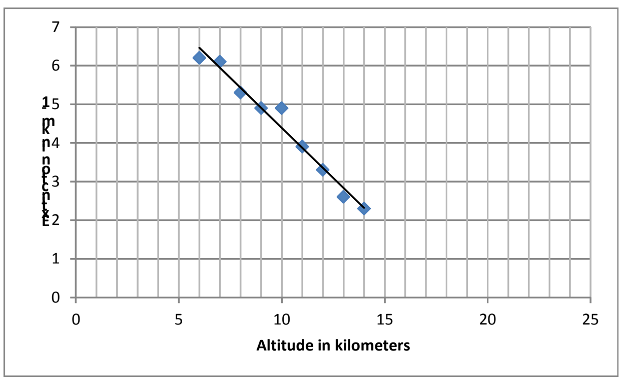
Problem 1 – At what location is the extinction A) the highest? B) the lowest?

Answer: A) **At 70N at an altitude of 6 km** above the ground where its value is 13.0 x 0.0001 = **0.0013** km^{-1}. B) **At an altitude of 24-25 km for 70N** where its value is 0.2 x 0.0001 = **0.00002 km**$^{-1}$.

Problem 2 – Above an altitude of 20 kilometers, what is the average extinction at each latitude?

Answer: 70N: (0.8+0.5+0.4+0.3+0.2+0.2)/6 = 0.4 or 0.4 x 0.0001 = **0.00004 km**$^{-1}$.
30N: (2.2+1.6+0.9+0.6+0.4+0.3)/6 = 1.0 or **0.0001 km**$^{-1}$**.**
0 N: (3.2+3.1+2.7+2.2+1.7+1.5)/6 = 2.4 or 2.4 x 0.0001 = **0.00024 km**$^{-1}$**.**

Problem 3 – Graph the extinction data for 30N between an altitude of 6 and 14 km and draw a straight line through the data. What is the slope of this line, and what are the units for this slope? Answer: See graph below. The slope is -0.52 and the units will be (km^{-1})/km = km^{-2} or 1/km^{2}



Alternate slope method using two-point formula and data table:
M = (y2-y1)/(x2-x1) so for the points at 6km and 14 km, M = (2.3-6.2)/(14 – 6) = -0.49 km^{-2}.

Students estimates should be close to M = -0.5 km^{-2}

Problem 4 – Write the equation of the line that you drew in Problem 3, and use it to estimate the extinction at an altitude of 19 kilometers. Is our mathematical model a good fit to the actual data?

Answer: The y-intercept for x=0 km is about +9.5, then **y = -0.5X + 9.5**. At an altitude of 19 km, the extinction would be y = -0.5(19)+9.5 = **0.0 km**$^{-1}$. The data show that the extinction is 0.00027 km^{-1} at this altitude, so we should not try to predict extinctions for altitudes outside the domain of our linear fit (6 to 14 km).

The Sources and Sinks of Carbonyl Sulfide

Sources and sinks of carbonyl sulfide

Source	Rate (Mt/yr)
Open ocean	**+0.10**
Coastal ocean, salt marshes	**+0.20**
Anoxic soils	**+0.02**
Wetlands	**+0.03**
Volcanism	**+0.05**
Precipitation	**+0.13**
DMS oxidation	**+0.17**
Anthropogenic CS2 oxidation	**+0.21**
Natural CS2 oxidation	**+0.21**
Biomass burning	**+0.07**
Anthropogenic production	**+0.12**
Oxic soils	**-0.92**
Vegetation	**-0.56**
Reactions with OH	**-0.24**
Reactions with oxygen	**-0.02**
Photodissociation	**-0.05**

The important source of aerosols in the stratosphere (altitude from 11 to 50 km) is the formation of carbonyl sulfide (COS) droplets. Although volcanism injects millions of tons of SO_2 into the atmosphere every few years, scientists have found that during other times, COS is by far the biggest source of sulfur compounds in the stratosphere, leading to the production of sulphuric acid aerosols. Just 1 kilogram of COS is over 720 times more damaging than the same amount of carbon dioxide in altering global climate.

The table to the left gives the known sources and sinks of COS in terms of millions of tons per year (Mt/yr).

Problem 1 - In the table, sources of COS are indicated by positive rates, and systems that remove COS from the atmosphere, called sinks, are indicated by negative rates. What are the total rates for the sources and sinks, and what is the net change in atmospheric COS in megatons/year?

Problem 2 – What percentage of the sources for COS are related to human activity (anthropogenic) according to the data?

Problem 3 – 6.0×10^{23} molecules of COS has a mass of 60 grams, and 1 year equals 3.1×10^{7} seconds. What is the net change each year in the number of COS molecules in one cubic meter of the atmosphere if the volume of the atmosphere is about 4.23 billion cubic kilometers?

COS data from 'The role of carbonyl sulphide as a source of stratospheric sulphate aerosol and its impact on climate', C. Br¨uhl, J. Lelieveld, P. J. Crutzen, and H. Tost Journal of Atmospheric Chemistry and Physics, v.12, pp. 1239–1253, 2012 www.atmos-chem-phys.net/12/1239/2012/doi:10.5194/acp-12-1239-2012

Problem 1 - In the table, sources of COS are indicated by positive rates, and systems that remove COS from the atmosphere, called sinks, are indicated by negative rates. What are the total rates for the sources and sinks, and what is the net change in atmospheric COS in megatons/year?

Answer: **Total sources = +1.31 Mt/yr. Total sinks = -1.79 Mt/yr. The net change is the sum of the sources and sinks, or -0.48 Mt/yr**. This means that COS is being reduced in concentration each year at the current known rates.

Problem 2 – What percentage of the sources for COS are related to human activity (anthropogenic) according to the data?

Answer: Anthropogenic CS2 oxidation from wood burned in stoves, and direct production of this compound account for +0.21 and +0.12 Mt/year or a total of +0.33 Mt/yr. The total production is +1.31 Mt/yr, so anthropogenic sources are 100% x (0.33/1.31) = **25% of all sources.**

Problem 3 – 6.0×10^{23} molecules of COS has a mass of 60 grams, and 1 year equals 3.1×10^{7} seconds. What is the net change each year in the number of COS molecules in one cubic meter of the atmosphere if the volume of the atmosphere is about 4.23 billion cubic kilometers?

Answer: From Problem 1, the net change is a reduction by 0.48 Mt/yr.

Net reduction in tons = 480,000 tons / year x (1 year) = 480,000 tons.
Number of molecules:

$$480{,}000 \text{ tons} \times \frac{1000 \text{ kilograms}}{1 \text{ ton}} \times \frac{1000 \text{ grams}}{1 \text{ kilogram}} \times \frac{6.0 \times 10^{23} \text{ molecules}}{60 \text{ grams}} = 4.8 \times 10^{33} \text{ molecules}$$

Volume of atmosphere in cubic meters

$$= 4.23 \times 10^{9} \text{ km}^3 \times \frac{1.0 \times 10^{9} \text{ m}^3}{1 \text{ km}^3} = 4.23 \times 10^{18} \text{ m}^3$$

So: $4.8 \times 10^{33} / 4.23 \times 10^{18} \text{ m}^3$ = **1.1×10^{15} molecules <u>removed</u> each year.**

A Scale Model of Aerosol Sizes

Table of particle sizes

Type	Size
Atmospheric aerosol	0.015 microns
Volcanic aerosol	0.5 microns
Gasolene engine ash	20 nanometers
Diesel ash (small)	50 nanometers
Diesel ash (large)	0.4 microns
Smallpox virus	300 nanometers
E Coli bacterium	2.0 microns
Common cold virus	30 nanometers
Smog (small)	8 nanometers
Smog (large)	200 nanometers

An aerosol, or ‘aero-solution’, is a microscopic particle made from numerous atoms and molecules stuck together. They are usually produced from chemical reactions, and the burning of organic materials like wood and hydrocarbon fuels.

The table to the left shows some common aerosol sizes along with the sizes of other objects you may know about.

Problem 1 – if 1 micron=1000 nanometers, order the particles by increasing size.

Problem 2 - Create a scaled model showing the relative sizes of each type of particle so that 1 nanometer = 1 millimeter in your model.

Problem 3 – A red blood cell has a diameter of 10 microns. How many volcanic aerosol particles can you place side-by-side to span this diameter?

Problem 4 – How many atmospheric aerosol particles would span the width of an e. coli bacterium?

Problem 5 – Suppose that an aerosol particle were shaped like a cube. How many atmospheric aerosol particles could you fit inside the volume of a single large particle of smog?

Problem 1 – if 1 micron=1000 nanometers, order the particles by increasing size.

Type	Size (nanometers)	Scaled Size
Smog (small)	8	8 mm
Atmospheric aerosol	15	15 mm
Gasolene engine ash	20	20 mm
Common cold virus	30	30 mm
Diesel ash (small)	50	50 mm
Smog (large)	200	20 cm
Smallpox virus	300	30 cm
Diesel ash (large)	400	40 cm
Volcanic aerosol	500	50 cm
E Coli bacterium	2000	2 meters

Problem 2 - Create a scaled model showing the relative sizes of each type of particle so that 1 nanometer = 1 millimeter in your model. Answer: See table above. Students can draw circles for the objects less than 1 meter in diameter.

Problem 3 – A red blood cell has a diameter of 10 microns. How many volcanic aerosol particles can you place side-by-side to span this diameter?

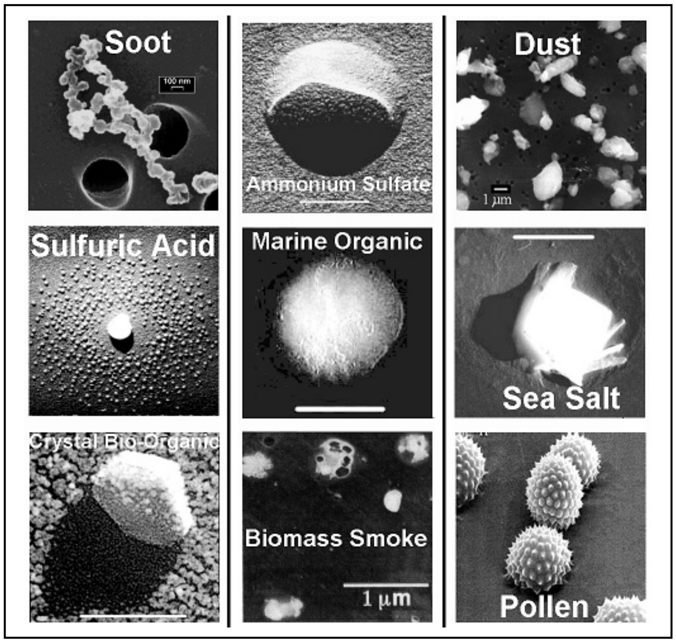
Answer: 10 microns / 0.5 microns = **20 volcanic aerosol particles**.

Problem 4 – How many atmospheric aerosol particles would span the width of an e. coli bacterium?

Answer: 2000 nanometers / 15 nanometers = **133 aerosol particles**.

Problem 5 – Suppose that an aerosol particle were shaped like a cube. How many atmospheric aerosol particles could you fit inside the volume of a single large particle of smog?

Answer: (Size of smog particle / size of aerosol)^3 = (200/15)^3 = **2370 particles**!



Aerosols are a complex ingredient to the atmosphere, which can result in both environmental and human health problems when their concentrations are too high. Smog and soot from burning fossil fuels or other organic materials can cause breathing difficulties, and perhaps even some forms of cancer.

The figure shows some common types of aerosol particles and their sizes. Most appear to be small spherical particles when seen under the microscope.

The formula for the volume of a sphere is given by $V = 4/3\ \pi\ R^3$. Also, 1 micron (µm)= 0.0001 centimeters, and 1 nanometer (nm) = 0.001 microns.

Problem 1 – A cubic meter container contains aerosols composed of soot produced from the combustion of diesel fuel. Each spherical aerosol particle has a density of 2 grams/cm^3. If the soot particle has a diameter of 20 nanometers, how much mass is in a single soot particle expressed in A) grams? B) micrograms?

Problem 2 – Scientists like to use two measurement units to indicate the density of aerosols in a sample of air: Particles/meter^3 or micrograms/meter^3 (µg/m^3). The average aerosol density for Los Angeles, California between October 2002 and September 2003 was 40 µg/m^3. 50% of these aerosols by mass were particulates with diameters of about 5 microns, while the remaining aerosols were mostly 500 nanometers in size. What was the density of the aerosols in particles/m^3 in each case, if the aerosols were small solid spheres with a density of 3.0 gm/cm^3?

Problem 1 – A cubic meter container contains aerosols composed of soot produced from the combustion of diesel fuel. Each spherical aerosol particle has a density of 2 grams/cm3. If the soot particle has a diameter of 20 nanometers, how much mass is in a single soot particle expressed in A) grams? B) micrograms?

Answer: First convert the particle diameter to centimeters:

$$D = 20 \text{ nanometers} \times \frac{1.0\times10^{-9} \text{ meters}}{1 \text{ nanometer}} \times \frac{100 \text{ cm}}{1 \text{ meter}} = 2.0\times10^{-6} \text{ cm.}$$

$V = 4/3\ \pi\ R^3$, and $R = D/2$, so $V = 1.333 \times 3.141 \times (1.0\times10^{-6} \text{ cm})^3 = 4.2 \times 10^{-18} \text{ cm}^3$.

Mass = density x volume, so

A) Mass = $2.0 \text{ gm/cm}^3 \times 4.2 \times 10^{-18} \text{ cm}^3 = 8.4\times10^{-18}$ grams.
B) Mass = 8.4×10^{-18} grams x (1 microgram/ 10^{-6} grams) = **8.4×10^{-12} micrograms**.

Problem 2 – Scientists like to use two measurement units to indicate the density of aerosols in a sample of air: Particles/meter^3 or micrograms/meter^3 (μg/m^3). The average aerosol density for Los Angeles, California between October 2002 and September 2003 was 40 μg/m^3. 50% of these aerosols by mass were particulates with diameters of about 5 microns, while the remaining aerosols were mostly 500 nanometers in size. What was the density of the aerosols in particles/m^3 in each case, if the aerosols were small solid spheres with a density of 3.0 gm/cm^3?

Answer: Aerosol masses:

5 micron case: $R = 2.5\times10^{-4}$ cm then $V = 4/3\ \pi\ (2.5\times10^{-4} \text{cm})^3 = 6.5\times10^{-11} \text{ cm}^3$.
Mass = $3.0 \text{ gm/cm}^3 \times 6.5\times10^{-11} \text{ cm}^3 = 2.0\times10^{-10}$ gm.

500 nanometer case: $R = 5.0\times10^{-5}$ cm. $V = 4/3\ \pi\ (5.0\times10^{-5} \text{ cm})^3 = 5.2\times10^{-13} \text{ cm}^3$.
Mass = $3.0 \text{ gm/cm}^3 \times 5.2\times10^{-13} \text{ cm}^3 = 1.6\times10^{-12}$ gm.

Since 50% of the aerosols were in each category and the total density was 40 μg/m^3,

5-micron case: $$\text{number} = 20\ \mu\text{g/m}^3 \times \frac{1 \text{ gm}}{10^6\ \mu\text{g}} \times \frac{1 \text{ particle}}{2.0\times10^{-10} \text{ gm}} = \mathbf{10^5 \text{ particles/m}^3}$$

500 nm case: $$\text{number} = 20\ \mu\text{g/m}^3 \times \frac{1 \text{ gm}}{10^6\ \mu\text{g}} \times \frac{1 \text{ particle}}{1.6\times10^{-12} \text{ gm}} = \mathbf{1.2\times10^6 \text{ particles/m}^3}$$

So although there were an equal amount of particles by their total mass, the small particles were 120 times more numerous as individual particles in the air samples.

AQI	$PM_{2.5}$ ($\mu g/m^3$)	PM_{10} ($\mu g/m^3$)	Air Quality Descriptor
0–50	0.0–15.4	0–54	Good
51–100	15.5–40.4	55–154	Moderate
101–150	40.5–65.4	155–254	Unhealthy for Sensitive Groups
151–200	65.5–150.4	255–354	Unhealthy
201–300	150.5–250.4	355–424	Very unhealthy

Because of their impacts to health, the US Environmental Protection Agency monitors the level of aerosols in the atmosphere (troposphere) for two categories: Large aerosols (PM_{10}) with diameters near 10 microns, and small aerosols ($PM_{2.5}$) with diameters near 2.5 microns (μm). The Air Quality Index (AQI) relates the density of each aerosol type (measured in micrograms per cubic meter or $\mu g/m^3$) to health risk as shown in the table above.

Problem 1 - Suppose the two types of aerosol particles have a density of 2000 kg/m^3. Assuming that each particle is a perfect sphere, what are the average masses of each type of aerosol particle in kilograms?

Problem 2 – Based on your estimate of the aerosol particle masses in Problem 1, how many aerosol particles of each type would be present in a 1 cubic meter volume of air of the AQI was 150?

Problem 1 - Suppose the two types of aerosol particles have a density of 2000 kg/m^3. Assuming that each particle is a perfect sphere, what are the average masses of each type of aerosol particle in kilograms?

Answer: Volume = $4/3 \pi R^3$,

$PM_{2.5}$ aerosols: For R = 1.3 microns, R = 1.3×10^{-6} meters so
V = $1.333 \times 3.141 \times (1.3 \times 10^{-6} m)^3$
= $9.2 \times 10^{-18} m^3$.

Mass = density x volume, so

M = $2000 \times 9.2 \times 10^{-18}$
= **1.8×10^{-14} kilograms.**

PM_{10} aerosols: R = 5 microns so

V = $1.333 \times 3.141 \times (5.0 \times 10^{-6} m)^3$
= $5.2 \times 10^{-16} m^3$, then

Mass = $2000 \times 5.2 \times 10^{-16}$
= **1.0×10^{-12} kilograms.**

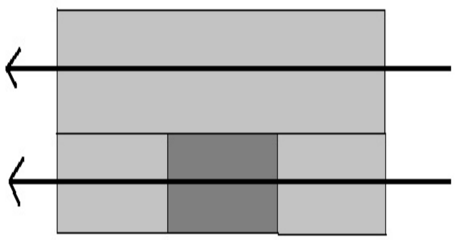
Problem 2 – Based on your estimate of the aerosol particle masses in Problem 1, how many aerosol particles of each type would be present in a 1 cubic meter volume of air of the AQI was 150?

Answer: The table indicates that for an AQI of 150, the density of the PM_{10} particles would be 254 $\mu g/m^3$. Since the mass of such an aerosol particle is about 1.0×10^{-12} kilograms, we have

N = 2.54×10^{-6} $\mu g/m^3$ x (1 kg/1000 gm) x (1 particle/1.0×10^{-12} kg)
= **2500 particles/meter^3.**

The table indicates that for an AQI of 150, the density of the $PM_{2.5}$ particles would be 65.4 $\mu g/m^3$. For $PM_{2.5}$ aerosols the density is 65.4 mg/m^3. The average mass is 1.8×10^{-14} kg, so

N = 65.4×10^{-6} $\mu g/m^3$ x (1 kg/1000 gm) x (1 particle/1.8×10^{-14} kg)
= **3.6×10^6 particles/meter^3**.



This figure shows two light rays from the Setting sun at two different altitudes above Earth. The top ray goes through a medium with fewer aerosols and a lower optical Depth.

The bottom ray passes closer to Earth's surface where a portion of the atmosphere is denser and has a different concentration of aerosols, making the dark region a higher optical depth.

The bottom atmosphere 'slab' is divided Into three equal sections, each 3,000 kilometers in length. The total width of the atmosphere being modeled in the example is 9,000 kilometers wide.

Opacity is a term used to describe the passage of light through a material, and is defined by the basic exponential formula

$$I = I_0 e^{-\tau}$$

Opaque materials have large positive values for τ, while transparent (translucent) materials have very low values for τ.

The quantity τ is also called the optical depth of a medium, and is proportional to both the length of the path taken by light through the medium, and the density of the absorbing particles. We can write this as

$$\tau = C N x$$

where x is in units of meters,
N is in units of particles/meter^3, and
C is a constant that is different for each kind of aerosol and has the units m^2/particle.

The Sage-III mission is designed to determine the product CN by looking at the extinction of sunlight along many different paths through the atmosphere given by x as shown in the figure to the left.

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9991 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34 \times 10^{-17}$ km^2 /particle and $C_B = 7.5 \times 10^{-18}$ km^2 /particle, what are the densities of the two aerosols in A) particles/kilometer^3? B) particles/meter^3?

Problem 1 – Suppose that in the figure shown above, the SAGE-III instrument measures an extinction of light by 0.9994 in the top layer and 0.995 in the bottom layer. What are the total optical depths of each layer?

Answer: $I=I_0 e^{-\tau}$ and

Top layer: $I/I_0 = 0.9994$ so $\ln(0.9994) = -\tau$ and so τ = **0.0006**

Bottom layer: $I/I_0 = 0.995$ so $\ln(0.995) = -\tau$ and so τ = **0.005**

Problem 2 - From the information given in the figure, write two equations that relate the total optical depth to the contributions from each aerosol component and solve them to find the product CN for each aerosol.

Answer:

Top Layer aerosol: $0.0006 = CN_A \times (9000\text{ km})$
so $CN_A = 0.0006/9000\text{km}$
$= 6.7\times10^{-8}\text{ km}^{-1}$

Bottom Layer aerosol:

$$
\begin{aligned}
0.005 &= CN_A\,(3{,}000\text{ km}) + CN_B\,(3{,}000\text{ km}) + CN_A\,(3{,}000\text{ km})\\
0.005 &= 2\,(6.7\text{x}10\text{-}8\text{ km-}1)(3{,}000\text{ km}) + CN_B\,(3{,}000\text{ km})\\
0.005 &= 0.0004 + CN_B\,(3{,}000\text{ km})\\
0.0046 &= CN_B\,(3{,}000\text{ km})\\
\text{So}\quad CN_B &= 0.0046/3000\text{km}\\
&= 1.5\times10^{-6}\text{ km}^{-1}
\end{aligned}
$$

So, **$CN_A = 6.7\times10^{-7}\text{ km}^{-1}$ and $CN_B = 1.5\times10^{-6}\text{ km}^{-1}$**

Problem 3 – Suppose that the constants, C , for each aerosol type are known from models of Earth's atmosphere and that they are $C_A = 1.34\times10^{-17}\text{ km}^2$ /particle and $C_B = 7.5\times10^{-18}\text{ km}^2$/particle, what are the densities of the two aerosols in A) particles/kilometer^3? And B) particles/meter^3?

Answer: Aerosol A: $(1.34\times10^{-17}\text{ km}^2/\text{particle})\ N_A = 6.7\times10^{-7}\text{ km}^{-1}$
so **$N_A = 5.0\times10^{10}$ particles/km^3**

Aerosol B: $(7.5\times10^{-18}\text{ km}^2/\text{particle})\ N_B = 1.5\times10^{-6}\text{ km}^{-1}$
so N_B = **2.0×10^{11} particles/km^3**

Aerosol A: 5.0×10^{10} particles/km^3 x (1km/1000meters)^3 = **50 particles/meter^3**
Aerosol B: 2.0×10^{11} particles/km^3 x (1 km/1000 meters)^3 = **200 particles/meter^3**

Some Basic Properties of the SAGE-III Instrument

The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014. SAGE III will be using the sun and moon as light sources to measure how well the ozone layer is recovering and replenishing itself.

The 76-kilogram instrument will be carried in a Dragon supply module, and launched on a SpaceX, Falcon 9 rocket from NASA's Kennedy Space Center. After installation on the ISS and a 30-day check out, it will start taking data at a rate of about 200 megabytes every day. SAGE-III has a pointing accuracy of 0.025 degrees and requires 700 kilowatt hours of electricity every year.

Problem 1 - A single DVD can store 4.4 gigabytes of data. How many DVDs of data will the SAGE-III experiment generate during its 3-year mission onboard the ISS if 1 gigabyte = 1000 megabytes?

Problem 2 - If 1 kilogram equals 2.2 pounds, what is the weight of the SAGE-III instrument?

Problem 3 - A single 100-watt light bulb that is turned on for 10 hours will consume 1000 watt-hours of electricity, which is called 1 kilowatt-hour. How many watts will the SAGE-III instrument use if it is turned on for one full year?

Problem 4 - The SAGE-III instrument can change its direction of pointing by as little as 0.025 degrees. This is the same angle as the width of a dime (1 cm in diameter) if it were viewed at a distance of 23 meters. If two people stood one meter apart, how far away would they have to be standing from you to subtend the same angle? (Hint: Use proportions)

Problem 1 - A single DVD can store 4.4 gigabytes of data. How many DVDs of data will the SAGE-III experiment generate during its 3-year mission onboard the ISS if 1 gigabyte = 1000 megabytes?

Answer: The instrument produces 200 Mby of data every day. In 3 years it will produce 3 years x (365 days/year) x (200 Mby/day) = 219,000 Mby. Then:

219,000 Mby x (1 Gby/1000 Mby) x (1 DVD/4.4 Gby) = 49.8 DVDS. In terms of the total needed, we have **50 DVDs**.

Problem 2 - If 1 kilogram equals 2.2 pounds, what is the weight of the SAGE-III instrument?

Answer: 76 kg x (2.2 pounds/1 kg) = 167.2 pounds or **167 pounds**.

Problem 3 - A single 100-watt light bulb that is turned on for 10 hours will consume 1000 watt-hours of electricity, which is called 1 kilowatt-hour. How many watts will the SAGE-III instrument use if it is turned on for one full year?

Answer: The SAGE-III instrument uses 700 kWh of electricity. Then:

700 kWh x (1000 W/kW) x (1 year/ 365 days) x (1 day/ 24 hours) = **80 watts.**

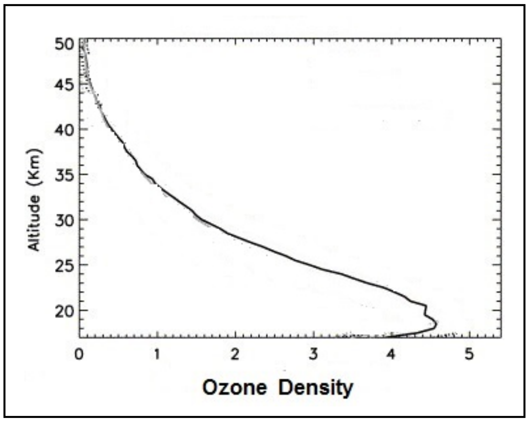
Problem 4 - The SAGE-III instrument can change its direction of pointing by as little as 0.025 degrees. This is the same angle as the width of a dime (1 cm in diameter) if it were viewed at a distance of 23 meters. If two people stood one meter apart, how far away would they have to be standing from you to subtend the same angle?

Answer:

$$\frac{1\text{ cm}}{23\text{ meters}} = \frac{1\text{ meter}}{X}$$

convert all units to meters: $\frac{0.01\text{ m}}{23\text{ m}} = \frac{1\text{ m}}{X}$ then X = 23/0.01 = **2300 meters**

Measuring Stratospheric Ozone with SAGE-III



The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014. An earlier version of the SAGE-III instrument was flown in 2001 on the Russian Meteop-3M spacecraft. The new SAGE III will be using the sun and moon as light sources to measure how well the ozone layer is recovering and replenishing itself.

The ozone layer is located at an altitude of about 20 kilometers and blocks solar UV rays that would otherwise burn skin and cause cancer.

The data plot shows the density of ozone molecules at a range of altitudes as measured by the earlier SAGE-III instrument. The density of ozone molecules found in the stratosphere is presented in multiples of 1 trillion molecules per cubic centimeter.

Problem 1 – The ozone layer has the highest concentration of ozone molecules in the stratosphere. From the graph, over what altitude range does the concentration exceed 4 trillion molecules per cubic centimeter?

Problem 2 - If the density of the atmosphere in this region of the stratosphere is 400,000 trillion molecules, what fraction of the molecules are ozone if the ozone density is 4 trillion molecules/cm^3?

Problem 3 – For every million atmosphere molecules in the ozone layer, how many ozone molecules do you expect to find? (Scientists use the term parts-per-million to indicate this number.)

Data plot from
Lunar occultation with SCIAMACHY: First retrieval results
L.K. Amekudzi , A. Bracher, J. Meyer, A. Rozanov, H. Bovensmann, and J.P. Burrows
Advances in Space Research, Volume 36, Issue 5, 2005, Pages 906–914

Problem 1 – The ozone layer has the highest concentration of ozone molecules in the stratosphere. From the graph, over what altitude range does the concentration exceed 4 trillion molecules per cubic centimeter?

Answer: **Between 17 and 23 kilometers**.

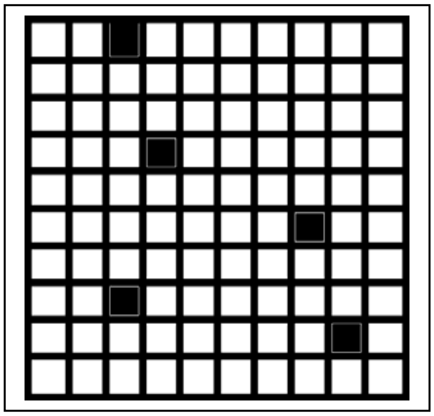
Problem 2 - If the density of the atmosphere in this region of the stratosphere is 400,000 trillion molecules, what fraction of the molecules are ozone if the ozone density is 4 trillion molecules/cm^3?

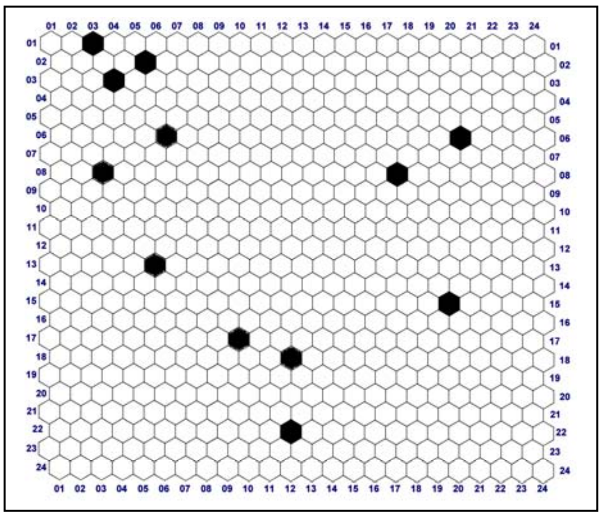
Answer: 4 trillion ozone molecules/400,000 trillion atmosphere molecules = **1/100,000**

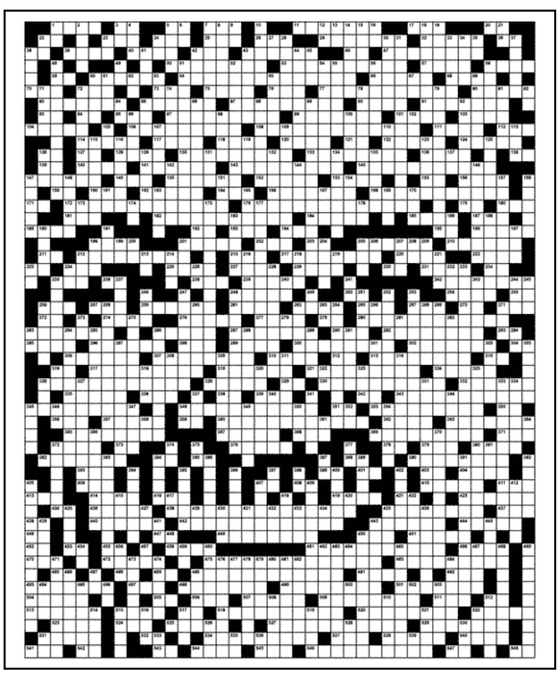
Problem 3 – For every million atmosphere molecules in the ozone layer, how many ozone molecules do you expect to find? (Scientists use the term parts-per-million to indicate this number.)

Answer: The ozone molecules are 1/100000 of the atmosphere molecules, so for every million atmosphere molecules, 1/100000 are ozone and so **10 ozone molecules** should be found for every 1 million atmosphere molecules.

This is written as 10 parts-per-million or 10 ppm.







The Stratospheric Aerosol and Gas Experiment III (SAGE III), the sensor will be installed on the International Space Station (ISS) sometime in 2014.

Aerosols are small particles that are suspended in the air. Examples include fog and smog, but also includes dust, soot and ash particles. These particles can affect climate, and can also cause health problems such as emphysema, asthma or even lung cancer.

Instead of measuring the amount of aerosols or pollutants by percentage, scientists often use units such as parts-per-million. This exercise helps you work with these units.

Problem 1 - Suppose you are 15 years old. How many parts-per-hundred is this of one century?

Problem 2 – You have a bag of 200 blue marbles, 280 green marbles and 20 red marbles. How many parts-per-thousand are the red marbles compared to the whole?

Problem 3 – Which of the figures to the left shows a concentration of black spots equal to
A) 243000 parts-per-million?
B) 50000 parts-per-million?
C) 20833 parts per million?

Problem 1 - Suppose you are 15 years old. How many parts-per-hundred is this of one century?

Answer: 15 years is 15/100 = **15 pph** of 1 century.

Problem 2 – You have a bag of 200 blue marbles, 280 green marbles and 20 red marbles. How many parts-per-thousand are the red marbles compared to the whole?

Answer: The total number of marbles is 200+280+20 = 500. So the 20 red marbles is 20/500 = 4/100 or **4 pph** of the total number of marbles.

Problem 3 – Which of the figures to the left shows a concentration of black spots equal to A) 243000 parts-per-million? B) 50000 parts-per-million? C) 20833 parts per million?

Answer: First let's find out how many squares we have.

Top = 10x10 = 100 total and 5 black so 5/100 are black

Middle = 24 x 24 = 576 squares and there are 12 black so 12/576 are black

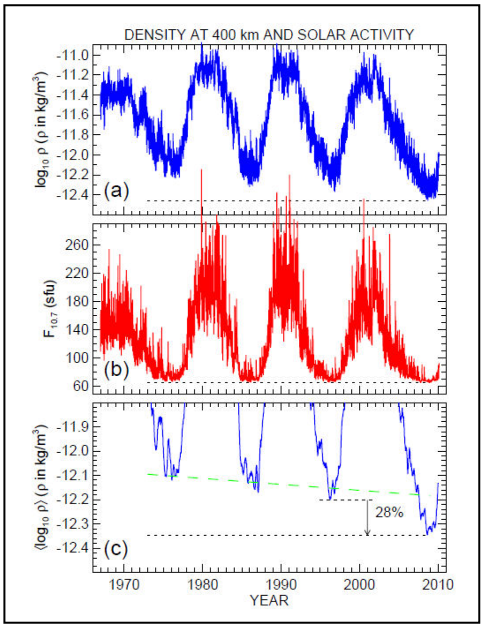
Bottom = 40 x 50 = 2000 squares and there are 486 black so 486/2000 are black

Lets use pph = parts-per-hundred
ppt = parts per thousand and
ppm = parts per million.

We can write the top as 5 pph = 50 ppt = 50,000 ppm
Middle as 12/576 = 0.0208333 = 2.0833 pph = 20.833 ppt = 20833 ppm
Bottom as 486/2000 = 0.243 = 24.3 pph = 243 ppt = 243000 ppm

So the answers are
A) Is the bottom figure
B) Is the top figure
C) Is the middle figure

Note: The concentration of carbon dioxide in our atmosphere has increased from 335 ppm to nearly 390 ppm since 1975!



The height of Earth's thermosphere is determined, in part, by the amount of solar ultraviolet radiation reaching it, and this varies during the sunspot cycle as the graphs show. According to recent work by Dr. John Emmett at the Naval Research Laboratory, the prolonged sunspot minimum we have just passed through caused an historic shrinkage in the thermosphere density. The temperatures can range from 700K at sunspot minimum to 1,600 K at maximum. This causes the density to rise and fall as the atmosphere expands and contracts. A simple formula relates the 'scale height' of an atmosphere to its temperature:

$$H = \frac{kT}{mg}$$

In this formula, k is Boltzman's Constant, which has a value of 1.38 $\times 10^{-23}$ Joules/K, g is the acceleration of gravity, which has a value of 9.8 meters/sec^2; m is the average mass of the gas particles in kilograms, and T is the temperature of the gas in Kelvins. The scale height, h, in meters represents the distance over which the gas will decrease in density by 2.7 times (a factor of e^{-1}).

Problem 1 - Suppose the gas consists of atoms of hydrogen for which m = 1.7 x 10^{-27} kg. What is the scale height, in meters for an atmosphere with a temperature of A) 700 K at sunspot minimum? B) 1,600 K at sunspot maximum?

Problem 2 - Suppose that during sunspot maximum, a satellite orbited at a constant altitude of 250 km, where the density of the gas was 1.0×10^{-12} kg/meter^3 and the temperature was 1,600 K. If the density, D, at a given altitude, z, is given by the formula

$$D(z) = D_0 e^{-\frac{Z}{H}}$$

what is the density of the atmosphere at 250 km when the temperature cools to 700 K at sunspot minimum?

Problem 1 - Suppose the gas consists of atoms of hydrogen for which m = 1.7 x 10^{-27} kg. What is the scale height, in meters for an atmosphere with a temperature of A) 700 K at sunspot minimum? B) 1,600 K at sunspot maximum?

Answer: At 700K, $H = \frac{(1.38x10^{-23})(700)}{(1.7x10^{-27})9.8}$ so **H = 580 km**

At 1,600K, H = 580 km x (1600/700) = **1,300 km**.

Problem 2 - Suppose that during sunspot maximum, a satellite orbited at a constant altitude of 250 km, where the density of the gas was $1.0x10^{-12}$ kg/meter^3 and the temperature was 1,600 K. If the density, D, at a given altitude, z, is given by the formula

$$D(z) = D_0 e^{-\frac{Z}{H}}$$

What is the density of the atmosphere at 250 km when the temperature cools to 700 K at sunspot minimum?

Answer: First you have to solve for D0 using H(1,600) = 1300 km and D(250) = $1.0x10^{-12}$ kg/meter^3 so

$$1.0x10^{-12} = D_0 e^{-\frac{250}{1300}}$$

and so D0 = $1.2x10^{-12}$ kg/meter^3

Now use this equation with the new H value to calculate D(250) for T = 700 K using H(700) = 580 km.

$$D(z) = 1.2x10^{-12} e^{-\frac{250}{580}}$$

so D(250) = **7.8 x 10^{-13} kg/meter^3**

So at the orbit of the satellite, during sunspot minimum, the density is lower that it was during sunspot maximum by about 100% x (0.78/1.0) = 78%.

Exponential Functions and the Atmosphere

A very common way to describe the atmosphere of a planet is by its 'scale height'. This quantity represents the vertical distance above the surface at which the density or pressure if the atmosphere decreases by exactly 1/e or $(2.718)^{-1}$ times (equal to 0.368).

The scale height, usually represented by the variable **H**, depends on the strength of the planet's gravity field, the temperature of the gases in the atmosphere, and the masses of the individual atoms in the atmosphere. The equation to the left shows how all of these factors are related in a simple atmosphere model for the density P. The variables are:

$$P(z) = P_0 \, e^{-\frac{z}{H}} \quad \text{and} \quad H = \frac{kT}{mg}$$

z: Vertical altitude in meters
T: Temperature in Kelvin degrees
m: Average mass of atoms in kilograms
g: Acceleration of gravity in meters/sec^2
k: Boltzmann's Constant 1.38×10^{-23} J/deg

Problem 1 - For Earth, g = 9.81 meters/sec^2, T = 290 K. The atmosphere consists of 22% O_2 (m= $2 \times 2.67 \times 10^{-26}$ kg) and 78% N_2 (m= $2 \times 2.3 \times 10^{-26}$ kg). What is the scale height, H?

Problem 2 - Mars has an atmosphere of nearly 100% CO_2 (m=7.3×10^{-26} kg) at a temperature of about 210 Kelvins. What is the scale height H if g= 3.7 meters/sec^2?

Problem 3 - The Moon has an atmosphere that includes about 0.1% sodium (m=6.6×10^{-26} kg). If the scale height deduced from satellite observations is 120 kilometers, what is the temperature of the atmosphere if g= 1.6 meters/sec^2?

Problem 4 - At what altitude on Earth would the density of the atmosphere P(z) be only 10% what it is at sea level, P_0?

Problem 5 - Calculate the total mass of the atmosphere in a column of air, below a height h with integral calculus. At what altitude, h, on Earth is half the atmosphere below you?

Answer Key

Problem 1 - Answer: First we have to calculate the average atomic mass. <m> = 0.22 ($2 \times 2.67 \times 10^{-26}$ kg) + 0.78 ($2 \times 2.3 \times 10^{-26}$ kg) = 4.76×10^{-26} kg. Then,

$$H = \frac{(1.38 \times 10^{-23})(290)}{(4.76 \times 10^{-26})(9.81)} = \textbf{8,570 meters or about 8.6 kilometers.}$$

Problem 2 - Answer:

$$H = \frac{(1.38 \times 10^{-23})(210)}{(7.3 \times 10^{-26})(3.7)} = \textbf{10,700 meters or about 10.7 kilometers.}$$

Problem 3 - Answer

$$T = \frac{(6.6 \times 10^{-26})(1.6)(120000)}{(1.38 \times 10^{-23})} = \textbf{918 Kelvins.}$$

Problem 4 - Answer: $0.1 = e^{-(z/H)}$, Take ln of both sides, $\ln(0.1) = -z/H$ then $z = 2.3\,H$ so for H= 8.6 km, **z = 19.8 kilometers**.

Problem 5 - First calculate the total mass:

$$M = \int_0^\infty P(z)dz \qquad M = P_0 \int_0^\infty e^{-\frac{z}{H}} dz \qquad M = P_0 H \int_0^\infty e^{-x} dx \qquad M = P_0 H$$

Then subtract the portion above you:

$$m = \int_h^\infty P(z)dz \qquad M = P_0 \int_h^\infty e^{-\frac{z}{H}} dz \qquad M = P_0 H \int_h^\infty e^{-x} dx \qquad M = P_0 H \left[e^{-h} - 1 \right]$$

To get: $dm(h) = P_0 H e^{-h}$

So $1/2 = e^{-h/H}$ and $h = \ln(2)(8.6 \text{ km})$ and so the height is **6 kilometers!**

The Arctic's Changing Ozone Layer

Table of Ozone Column Changes

Year	Column (Dobson Units)
1970	470
1972	466
1974	463
1976	460
1978	458
1980	455
1982	430
1984	445
1986	430
1988	445
1990	390
1992	395
1994	400
1996	375
1998	420
2000	370

This past spring, scientists using a variety of instruments witnessed an unprecedented depletion in Arctic stratospheric ozone levels. In a *Nature* paper published online on October 2, 2011, Dr. Gloria Manney of NASA's Jet Propulsion Laboratory in Pasadena and more than two dozen coauthors described the 2011 loss as "an Arctic ozone hole."

This table shows the changes in the Arctic ozone abundance for the month of March between 1970 and 2000 based on data provided by Dr. Paul Newman (NASA/GSFC). March is the coldest month of the Arctic year, and the time when the chemistry of the ozone layer favors the destruction of ozone molecules. The Antarctic ozone hole appears in September when it is mid-winter in the southern hemisphere.

Problem 1 - Graph the data in the table, and explain what kind of trend you see in the data. Is the amount of Arctic ozone detected in March generally increasing with time, decreasing with time, or remaining about the same?

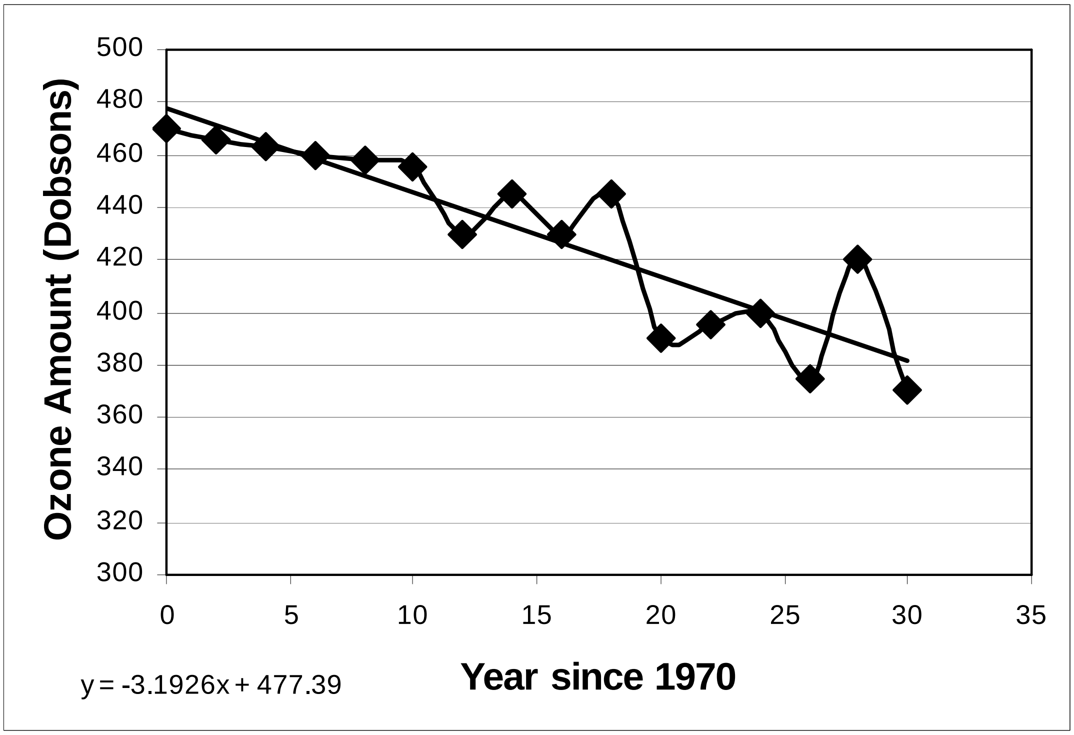
Problem 2 - Draw a slanted line through the plotted data so that half of the points are above the line and half are below the line. This represents a 'linear' regression model for the data. What does the slope of the line represent physically? What are the physical units for this slope?

Problem 3 - What is the formula for the line that you drew in Problem 2? What does the y-intercept for this line represent physically? What does the x-intercept for this line represent physically?

Problem 4 - From your linear model for the ozone data, what would you predict as the amount of ozone above the Arctic region in the year 2010?

Tabulated data from:
1970 - 1997: Paul Newman/NASA Goddard Space Flight Center. The figure is taken from P.A. Newman, J.F. Gleason, R.D. McPeters, and R.S. Stolarski, "Anomalously low ozone over the Arctic," Geophysical Research Letters, *Vol. 24, No. 22, pp. 2689-2692, November 15, 1997.*

1998-2010:



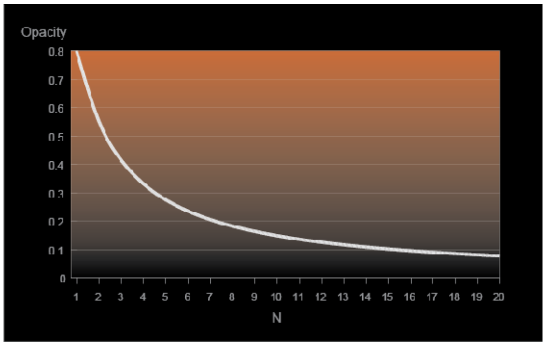
Problem 1 - Answer: See above. The amount of ozone is generally decreasing with time.

Problem 2 - Answer: Students can use a straight-edge and measure the slope of the line from its endpoints. In the example above the points are (30, 380) and (0, 480) so s = (380-480)/(30-0) = -3.3. The slope represents how fast the amount of ozone is decreasing over time. It is a negative number, so the amount is decreasing in time. The units for the slope are Dobsons/year.

Problem 3 - Answer: D = 477 - 3.3T where T is the year. The Y intercept, 477 Dobsons, is the amount of ozone predicted for 'Year 0', which is 1970, while the x-intercept, 145, (or 1970+145 = 2115 is the year when the amount of ozone will be 0.0 Dobsons.

Problem 4 - Answer: D = 477 - 3.3(41) so D = **342 Dobsons**.

Note: The actual amount of ozone detected in March, 2011 was found to be about 250 Dobsons! This is why it is called an ' Ozone hole' since similar readings are found over the Antarctic region.



This curve shows how much light is transmitted through a stack of 20 filters if each filter reduces the transmitted light by 20%.

After the first filter, N=1, the light intensity has been reduced from 100% to 80%.

After passing through the second filter, N=2, the light intensity is reduced to
100% x 0.8 x 0.8 = 100% x $(0.8)^2$ = 64%.

The reduction of light brightness as it passes through an aerosol cloud is not an additive process, but a multiplicative one. The figure to the left shows how light brightness changes as it passes through stacks of filters of varying lengths from 1 to 20. Note that the curve is not a straight line as it would be if the dimming were additive.

Scientists measure the dimming of light by the distance at which the light brightness is reduced by exactly 2.718 times or from 100% to 36.8%. Because aerosols are very dilute, the distance to which the light intensity falls to 36.8% is typically measured in kilometers. A basic mathematical function that describes light attenuation is given by

$$I(x) = 1.0\, e^{-x/L}$$

where L is the attenuation distance in kilometers and X is the length of the path through the aerosols.

Problem 1 – Suppose that for a particular cloud of aerosols the attenuation distance is 2 kilometers and the actual thickness of the cloud is 0.5 kilometers. What will be the light intensity to the nearest percent for a light ray passing through this cloud?

Problem 2 – For convenience, the attenuation distance L is usually reported as the extinction coefficient C = 1/L in units of km^{-1}. A) What is the equation for I(x) in terms of C? B) What is the value for I(x) in percent for a cloud with C = 0.20 km^{-1} and a cloud thickness of x=20 km?

Problem 3 – The SAGE-III instrument will measure the stratospheric aerosols, which have an average extinction of 1.2×10^{-4} km^{-1}. Light from the rising and setting sun will be measured along a path through the stratosphere that is about 3,000 km in length. What is the intensity of sunlight reaching the SAGE-III instrument?

Answer Key

Problem 1 – Suppose that for a particular cloud of aerosols the attenuation distance is 2 kilometers and the actual thickness of the cloud is 0.5 kilometers. What will be the light intensity to the nearest percent for a light ray passing through this cloud?

Answer: L = 2 km and x = 0.5 km so I = 100% x e(-0.5/2) = **78%**

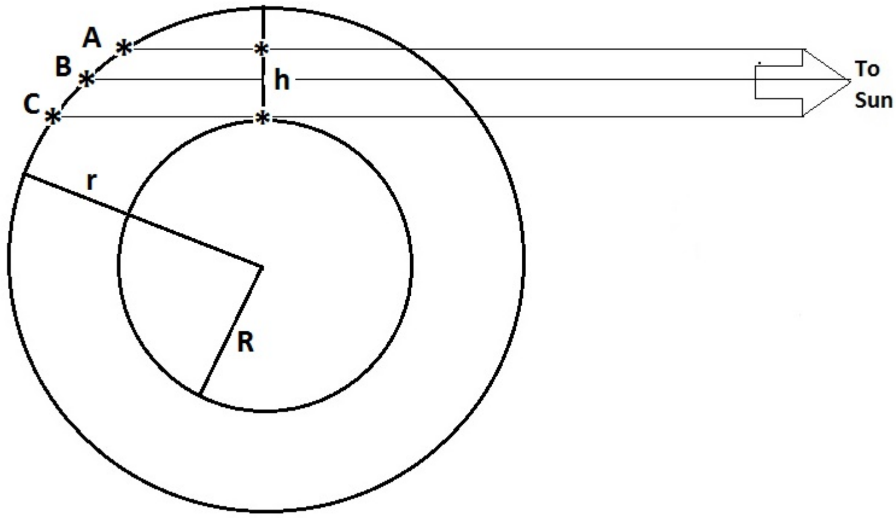
Problem 2 – For convenience, the attenuation distance L is usually reported as the extinction coefficient C = 1/L in units of km^{-1}. A) What is the equation for I(x) in terms of C? B) What is the value for I(x) in percent for a cloud with C = 0.20 km^{-1} and a cloud thickness of x=20 km?

Answer: A) $\mathbf{I(x) = 1.0\ e^{-Cx}}$ B) $I(20) = 100\%\ e^{(-0.2x20)}$ = **1.8%**

Problem 3 – The SAGE-III instrument will measure the stratospheric aerosols, which have an average extinction of $1.2x10^{-4}$ km^{-1}. Light from the rising and setting sun will be measured along a path through the stratosphere that is about 3,000 km in length. What is the intensity of sunlight reaching the SAGE-III instrument?

Answer: $I(3000km) = 100\%\ e^{(-0.00012)(3000)}$ = **69.7%**.

SAGE-III Sunset and Sunrise Geometry



The SAGE-III instrument on the International Space Station orbits Earth at a distance of r = 6,730 km from the center of Earth. The radius of Earth is R = 6,378 km. The time for one complete orbit is about 90 minutes. As it travels from Point A to C in the figure, the height of the sun, h, above the edge of Earth decreases to zero and astronauts observe a sunset. Each time SAGE-III observes a sunrise or sunset, its instruments measure the brightness of the sun. From this sun-dimming information scientists can determine the aerosol content of the stratosphere above an altitude of 10 km.

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Problem 1 - Use the Pythagorean Theorem to determine the length of a chord for a given value of h for h < 100 km.

Answer: $l^2 = r^2 - (R+h)^2$ so

Length = $2l = 2\,(r^2 - R^2 - 2Rh - h^2)^{1/2}$

Since R = 6378 and r = 6730 we have by simplifying

$L = 2\,(6730^2 - 6378^2 - 2(6378)h - h^2)^{1/2}$

Factor out 6730^2

Then $L = 2\,(6730)(1 - 0.90 - 0.00028h - (h/6730)^2)^{1/2}$

But h/6730 is never more than 100/6730 = 0.015 so we can ignore the h^2 term entirely!

So, $\mathbf{L = 13460(0.10 - 0.00028h)^{1/2}}$

Problem 2 - Most of the sunlight extinction will happen within a height of h = 40 km. About how long is the total length of the chord near the sunset point in the orbit?

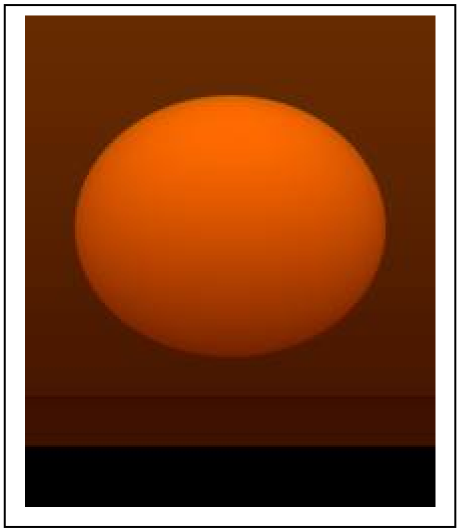
Answer: h = 0 at sunset so $L = 13460\,(0.10)^{1/2}$ = **4256 km**.

Problem 3 – About how many seconds will it take for the sunset to progress from h=40km to h=0 km?

Answer: The ISS will travel about 40 km in its orbit. Since the circumference of the circular orbit is C = 2 π (6730 km) = 42280 km, and this takes 90 minutes, the sunset range of 40 km will be traversed in

$$\frac{40\text{ km}}{42280\text{ km}} \times 90\text{ minutes} \times (60\text{ sec}/1\text{ minute}) = \mathbf{5\ seconds}.$$

Three Mathematical Ways to Describe Light Extinction



When light passes through a medium it can lose some of its intensity. Scientists call this extinction. Depending on what properties they want to highlight in a calculation or a measurement, different ways of expressing extinction by a medium have arisen.

Opacity - Symbol τ : $I = I_0 e^{-\tau}$

Decibels - Symbol D : $I = I_0 10^{-D/10}$

Extinction Coefficient - Symbol C : $I = I_0 e^{-Cx}$

Problem 1 - If $e = 10^{0.434}$, and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

Problem 2 – In base-10, what is the relationship between τ, D and C?

Problem 3 – In base-e, what is the relationship between τ, D and C?

Problem 4 - The SAGE-III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Problem 1 - If $e = 10^{0.434}$ and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

A) $I = I_0 e^{-\tau}$ $I = I0\,(10^{0.434})^{\tau}$ so $\mathbf{I = I_0\, 10^{-0.434\tau}}$

$I = I_0\, 10^{-D/10}$ unchanged so $\mathbf{I = I_0\, 10^{-D/10}}$

$I = I_0 e^{-Cx}$ $I = I_0\,(10^{0.434})^{(-Cx)}$ so $\mathbf{I = I_0\, 10^{-0.434Cx}}$

B) $\mathbf{I = I_0 e^{-\tau}}$ unchanged

$I = I_0\,(e^{2.3})^{-D/10}$ so $\mathbf{I = I_0 e^{-0.23D}}$

$\mathbf{I = I_0 e^{-Cx}}$ unchanged

Problem 2 – In base-10, what is the relationship between τ, D and C?

Answer: Just set the exponential factors equal to each other in Problem 1 A:

$-0.434\tau = -D/10 = -0.434Cx$ so after simplifying we get $\mathbf{\tau = 0.23D = Cx}$

Problem 3 – In base-e, what is the relationship between τ, D and C?

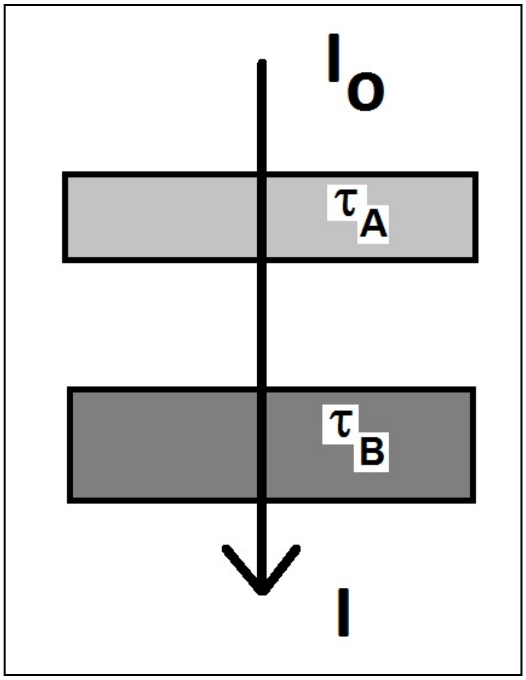
Answer: Set the exponential factors equal to each other in Problem 1 B: $\mathbf{\tau = 0.23D = Cx}$

Problem 4 - The SAGE-III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Answer: For 1 dB, and from Problem 2 (or 3!) we have

$\tau = 0.23$ D so
$\tau = 0.23 \times 1$ dB
$\tau =$ **0.23**.

For 1 dB and for x = 2000 km, we have
0.23 dB = Cx and so
0.23 = 2000C and so
$\mathbf{C = 0.000115\ km^{-1}}$.



Opacity is a term used to describe the difficulty for light to travel through a medium. A rain cloud with high opacity appears almost black as it hangs in the sky above your head. On the other hand, frosted glass lets some light through and has low opacity.

Extinction is a term that describes how much the intensity of light has been reduced as it passes through a medium.

The terms opacity and extinction are often used interchangeably, but mathematically, scientists who work with light define them differently. One basic equation that relates them together is:

$$I = I_0 \, e^{-\tau}$$

$\mathbf{I_0}$ is the initial intensity of the light as it strikes the front surface of the medium. **I** is the intensity of the light after it has left the medium, and τ is the opacity of the medium that the light has passed through. A high opacity (opaque) medium is one for which τ is large, and this causes the light leaving the medium, I, to be reduced in intensity, which we call extinction.

When light passes through two different materials, one after the other, the final intensity is just

$$I = (I_0 \; e^{-A})e^{-B} \quad \text{or} \quad I = I_0 \, e^{-(A+B)}$$

where A is the opacity (τ) of the first medium and B is the opacity (τ) of the second medium.

Problem 1 – Show that for three different mediums, the total opacity of the materials is given by the formula

$$\tau = \tau_A + \tau_B + \tau_C$$

Problem 2 - A photographer is given three different filters with opacities of $\tau_A = 5.2$, $\tau_B = 1.3$ and $\tau_C = 0.5$. He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

Problem 1 – Show that for three different mediums, the total opacity of the materials is given by the formula

$$\tau = \tau_a + \tau_b + \tau_c$$

Answer: From our example, and extended for three medii:

$$I = ((I0\ e^{-A})e^{-B})e^{-C})\ \text{ or }\ I = I0\ e^{-(A+B+C)}$$

But the total opacity is just

$$I = I_0 e^{-\tau},$$

so since $A = \tau_A$, $B=\tau_B$ and $C=\tau_C$ we have

$$I0e^{-\tau} = I0e^{-(\tau_A+\tau_B+\tau_C)}$$

And so

$$\tau = \tau_A + \tau_B + \tau_C$$

Problem 2 - A photographer is given three different filters with opacities of τ_A= 5.2, τ_B = 1.3 and τ_C = 0.5. He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

Answer: This will not work because the final opacity is the sum of the opacities of the three filters, and the order in which you add the filters does not matter in the sum. You will still get a final opacity of 5.2+1.3+0.5 = 7.0 and a drop in brightness by a factor of

$$I = I_0\ e^{-7} \qquad \text{or}\ \ 0.00091$$