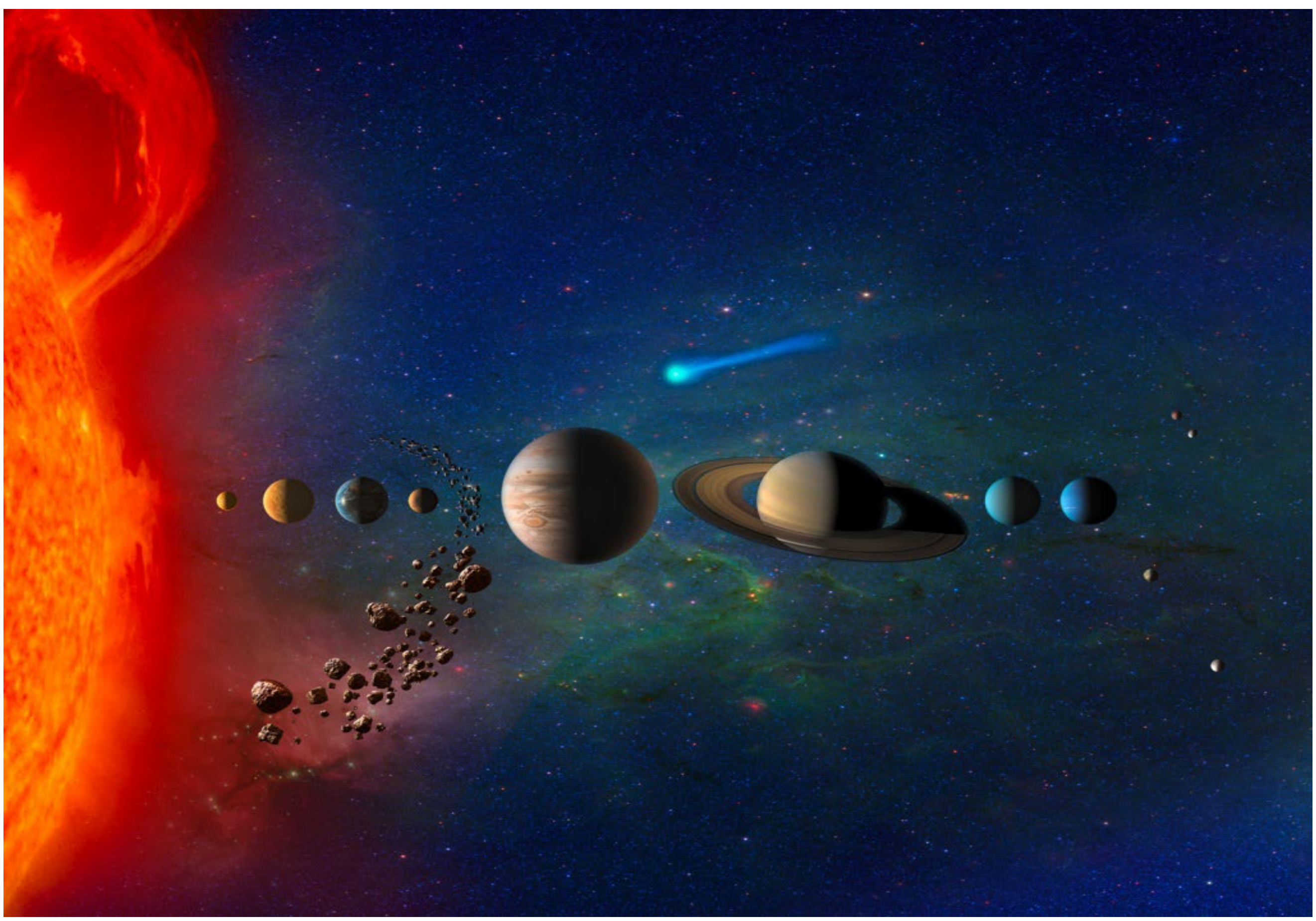
National Aeronautics and Space Administration

Math Problems Related to

The Solar System III



Introduction

This book provides an assortment of 47 mathematics problems that cover aspects of the solar system and its planets. Basic concepts such as orbits, mass loss, meteor impacts, and planet formation are presented among other topics.

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20	Advanced	Ratios, Percentages, Equations	Meteorite Compositions: A matter of density.
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23	Advanced	Data Analysis	Deep Impact: Approaching Comet Hartley-2
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37	Advanced	Logarithms	Exploring Marsquake Energy with the Moment Magnitude Scale
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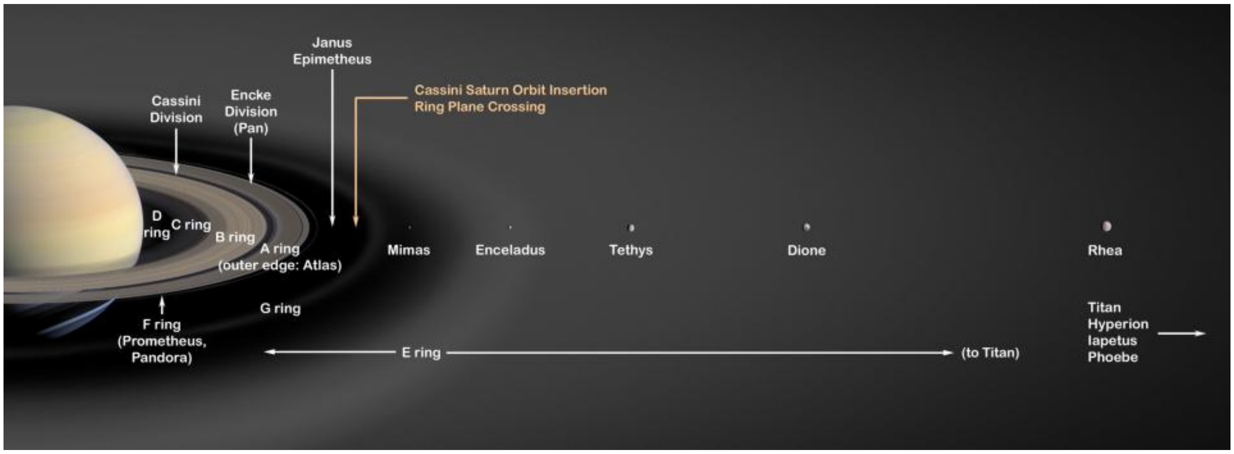
45	Advanced	Calculus	Estimating the mass of Comet Hartley 2 using calculus.
46	Advanced	Calculus	How to grow a planet or a Rain Drop
47	Advanced	Calculus	Deep Impact Comet Flyby

The Rings of Saturn



Saturn, the most beautiful planet in our solar system, is famous for its dazzling rings. Shown in the figure above, these rings extend far into space and engulf many of Saturn's moons. The brightest rings, visible from Earth in a small telescope, include the D, C and B rings, Cassini's Division, and the A ring. Just outside the A ring is the narrow F ring, shepherded by tiny moons, Pandora and Prometheus. Beyond that are two much fainter rings named G and E. Saturn's diffuse E ring is the largest planetary ring in our solar system, extending from Mimas' orbit to Titan's orbit, about 1 million kilometers (621,370 miles).

The particles in Saturn's rings are composed primarily of water ice and range in size from microns to tens of meters. The rings show a tremendous amount of structure on all scales. Some of this structure is related to gravitational interactions with Saturn's many moons, but much of it remains unexplained. One moonlet, Pan, actually orbits inside the A ring in a 330-kilometer-wide (200-mile) gap called the Encke Gap. The main rings (A, B and C) are less than 100 meters (300 feet) thick in most places. The main rings are much younger than the age of the solar system, perhaps only a few hundred million years old. They may have formed from the breakup of one of Saturn's moons or from a comet or meteor that was torn apart by Saturn's gravity.

Problem 1 – The dense main rings extend from 7,000 km to 80,000 km above Saturn's equator (Saturn's equatorial radius is 60,300 km). If the average thickness of these rings is 1 kilometer, what is the volume of the ring system in cubic kilometers? (use π = 3.14)

Problem 2 – The total number of ring particles is estimated to be 3×10^{16} . If these ring particles are evenly distributed in the ring volume calculated in Problem 1, what is the average distance in meters between these ring particles?

Problem 3 – If the ring particles are about 1 meter in diameter and have the density of water ice, 1000 kg/m^3, about what is the diameter of the assembled body from all of these ring particles?

Problem 1 – The dense main rings extend from 7,000 km to 80,000 km above Saturn's equator (Saturn's equatorial radius is 60,300 km). If the average thickness of these rings is 1 kilometer, what is the volume of the ring system in cubic kilometers? (use π = 3.14)

Answer: The area of a ring with an inner radius of r and an outer radius of R is given by $A = \pi (R^2 - r^2)$ and its volume for a thickness of h is just $V = \pi (R^2-r^2)h$.

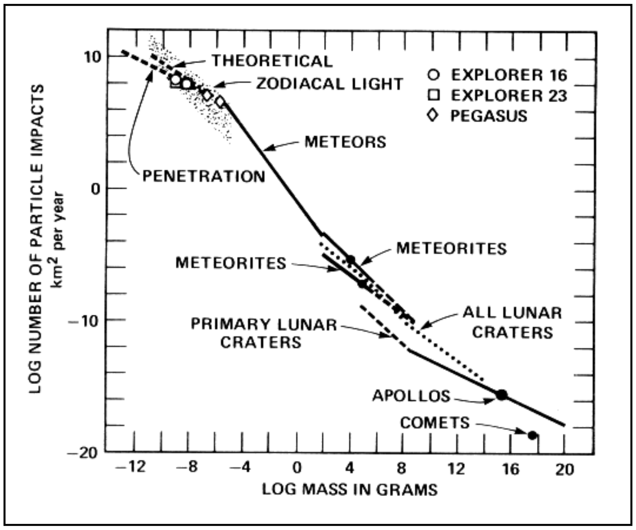
For Saturn's rings we have an inner radius r = 60300km+7000km =67300 km and an outer radius of R = 60300km+80000km = 140,300 km, and a volume of V = 3.14 $((140300)^2 - (67300)^2)$ (1.0) = **4.75×10^{10} km^3**.

Problem 2 – The total number of ring particles is estimated to be 3×10^{16} . If these ring particles are evenly distributed in the ring volume calculated in Problem 1, what is the average distance in meters between these ring particles?

Answer: 4.75×10^{10} km^3 / 3×10^{16} = 1.6×10^{-6} km^3/particle, so each particle is found in a volume of 1.6×10^{-6} km^3. For two cubes next to each other each with a volume of $V = s^3$, their centers are separated by exactly s. The distance to the nearest ring particle is D = $(1.6\times10^{-6}\ \text{km}^3)^{1/3}$ = 0.012 km, or **12 meters**!

Problem 3 – If the ring particles are about 1 meter in diameter and have the density of water ice, 1000 kg/m^3, about what is the diameter of the assembled body from all of these ring particles?

Answer: The volume of a single particle is $4/3\pi(1/2)^3$ = 0.5 meter^3. The total volume of all the 3×10^{16} particles is then V = 1.5×10^{16} meter^3. If this is a spherical body then $4/3\pi R^3 = 1.5\times10^{16}$ m^3, so R = 151185 meters or 150 kilometers in radius. The diameter is then **300 kilometers**.



The space between the planets is filled with fragments of asteroids, comets and material left over from the formation of the planets. These rocks and debris rain down upon exposed surfaces at speeds up to 30 km/sec.

The figure on the left summarizes the impact frequency of various sizes of particles in space. Note the graph is plotted in the Log-Log format due to the enormous range of masses and rates being described.

Understanding the data graph:

A meteorite with a density of 3 grams/cm^3 has a diameter of 4 centimeters (about 1 1/2 inches).

Problem 1 - What is the mass of this meteorite assuming it is a sphere?

Problem 2 - From the graph, where on the horizontal axis are objects of this mass located?

Problem 3 - What is the number of impacts per year you would expect over an area of 10,000 square kilometers?

A meteorite with a density of 3 grams/cm^3 has a diameter of 4 centimeters (about 1 1/2 inches).

Problem 1 - What is the mass of this meteorite assuming it is a sphere?

Answer: Mass = Density x Volume.
Radius of sphere = 2 cm, so
M = 3.0 x (4/3) (3.14) $(2)^3$ = **100 grams**.

Problem 2 - From the graph, where on the horizontal axis are objects of this mass located?

Answer: The horizontal axis is un units of Log(grams) so Log(100) = 2, and this is the location **half-way between 0' and '4' on the axis**.

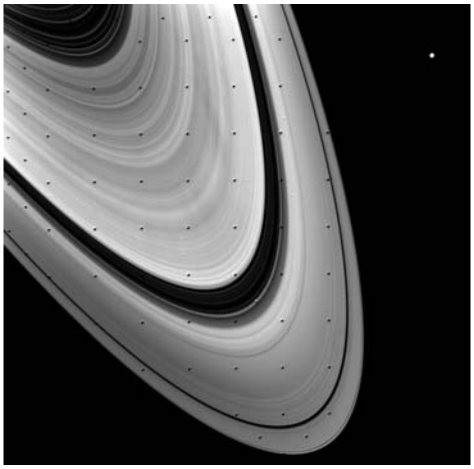
Problem 3 - What is the number of impacts per year you would expect over an area of 1000 square kilometers?

Answer: From 'x=2' ,a vertical line intercepts the data at about 'y=--3.5' on the vertical axis. This represents Log(n) = -3.5 so that N = 0.00032 impacts/km^2/year.

Over an area of 10000 km^2, there would be an estimated

0.00032 impacts/km^2/year x (10000 km^2) = **3.2 impacts per year.**

How Saturn's Moon Mimas created the Cassini Division



The Cassini Division is easily seen from Earth with a small telescope, and splits the rings of Saturn into two major groups. A little detective work shows that there may be a good reason for this gap that involves Saturn's nearby moon, Mimas.

Mimas orbits Saturn once every 22 hours, and would-be particles in the Cassini Division would orbit once every 11-12 hours, so that the ratio of the orbit periods is close to 2 to 1. This creates a resonance condition where the gravity of Mimas perturbs the Cassini particles and eventually ejects them.

Imagine a pendulum swinging. If you lightly tap the pendulum when it reaches the top of its swing, and do this every other swing, eventually the small taps add up to increasing the height of the pendulum.

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division from Mimas is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G\,\frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67\times10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

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$$\text{Acceleration} = G \frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67\times10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

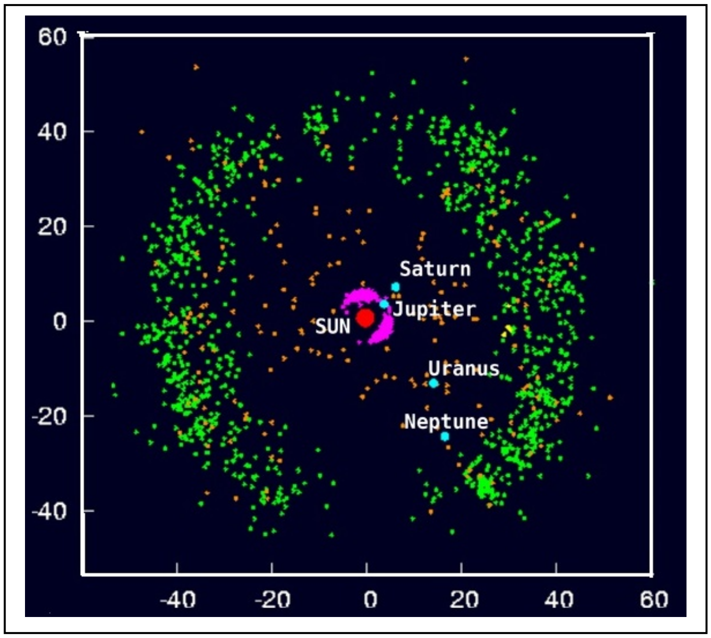
Answer: Acceleration = 6.67×10^{-11} (4.0×10^{19}) / $(6.7\times10^{7}\text{ meters})^2$
= **5.9×10^{-7} meters/sec^2**

Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Answer: 1 hour = 3600 seconds, so 2 hours = 7200 seconds and
speed = 5.9×10^{-7} m/sec^2 x 7200 sec
= 4.2×10^{-3} meters/sec per orbit.

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

Answer: (1000 m/s)/(0.0042 m/s) = 238000 orbits. Since 1 orbit = 12 hours, we have 12 x 238,000 = 2,856,000 hours or about **326 years**.



Since the belt was discovered in 1992, the number of known Kuiper belt objects (KBOs) has increased to over a thousand, and more than 100,000 KBOs over 100 km (62 mi) in diameter are believed to exist.

Pluto is the largest known member of the Kuiper belt. Originally considered a planet, Pluto's status as part of the Kuiper belt caused it to be reclassified as a "dwarf planet" in 2006

This figure shows the locations of the known KBOs with the X and Y positions given in terms of Astronomical Units (AUs), where 1 AU equals the distance from Earth to the Sun (93 million miles or 149 million km).

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by

$$V = 2\pi^2 r^2 R,$$

where R is the Kuiper Belts average distance from the sun and r is its radius?

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km^3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by $V = 2\pi^2 r^2 R$, where R is the Kuiper Belts average distance from the sun and r is its radius?

Answer: $R = (60+30)/2 = 45$ AU or $45 \times 149 \times 10^6$ km = 6.7×10^9 km
$r = (60-30)/2 = 15$ AU or $15 \times 149 \times 10^6$ km = 2.2×10^9 km

$V = 2 (3.141)2 (2.2\times10^9)^2 (6.7\times10^9)$ = **6.4×10^{29} km^3**

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km^3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

Answer: $N = 10^5/6.4\times10^{29}$ km^3 = **1.6×10^{-25} objects/km^3**.

Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

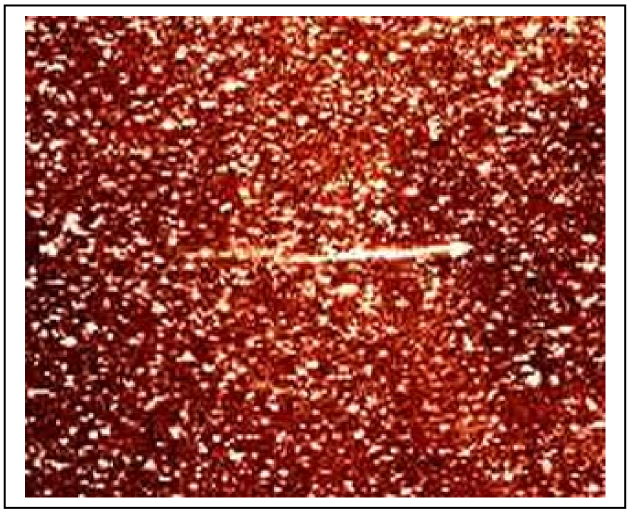
Answer: We just need to calculate the cube root of the density to get the reciprocal of this distance:

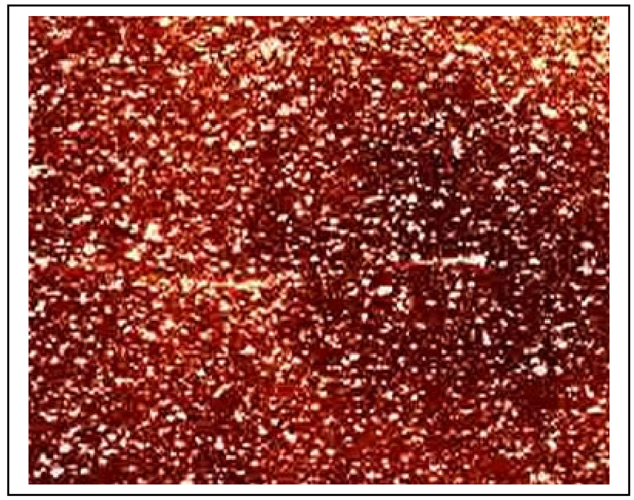
$D = 1 / (1.6\times10^{-25})^{1/3}$ = **183 million kilometers**.

Since the Earth-Sun distance is 149 million kilometers, the average distance between KBOs is about 1.2 AU or **1.2 times the Earth-Sun distance**.

The Comet Encke Tail Disruption Event

On April 20, 2007, NASA's STEREO satellite witnessed a rare solar system event. The Comet Encke had just passed inside the orbit of Venus and was at a distance of 114 million kilometers from STEREO-A ,when a Coronal Mass Ejection occurred on the sun. The cloud of magnetized gas passed over the comet's tail at 18:50 UT, and moments later caused the tail of the comet to break into two. The two images below show two images from the tail breakup sequence. The left image was taken at 18:10 UT and the right image was taken at 20:50 UT. Each image subtends an angular size of 6.4 degrees x 5.3 degrees. For comparison, the Full Moon would correspond to a circle with a diameter of 0.5 degrees.





Problem 1 - What is the scale of the images in arcminutes per millimeter? (1 degree=60 arcminutes)

Problem 2 - How many seconds elapsed between the time the two images were taken by the STEREO-A satellite?

Problem 3 - The left image shows the comet with an intact tail. The right image shows the tail separated from the head of the comet (the right-most bright feature along the comet's horizontal axis which we will call Point A), and flowing to the left. Meanwhile, you can see that the comet has already begun to reform a new tail. Carefully examine the right-hand image and identify the right-most end of the ejected tail (Call it Point B). Note that star images do not move, and are more nearly point-like than the tail gases. How far, in millimeters, is Point B from Point A?

Problem 4 - From the image scale, convert your answer to Problem 3 into arcminutes.

Problem 5 - The distance of the comet was 114 million kilometers, and at that distance, one arcminute of angular separation corresponds to 33,000 kilometers. How far did the tail fragment travel between the times of the two images?

Problem 6 - What was the speed of the tail fragment?

Problem 7 - If the comet's speed was about 40 km/sec and the CME speed was at least several hundred times faster, based on your answer to Problem 6, was the comet fragment 'left behind' or did the CME carry it off?

Answer Key:

Problem 1 - What is the scale of the images in arcminutes per millimeter? (1 degree=60 arcminutes)

Answer: horizontally, the image span 6.4 degrees x 60 minutes/degree = 384 arcminutes. The length is 77 millimeters, so the scale is 384/77 = **5.0 arcminutes/mm**

Problem 2 - How many seconds elapsed between the time the two images were taken by the STEREO-A satellite?

Answer: 20:50 - 18:10 = 2 hours and 40 minutes = 160 minutes or **9600 seconds.**

Problem 3 - The left image shows the comet with an intact tail. The right image shows the tail separated from the head of the comet (the right-most bright feature along the comets horizontal axis which we will call Point A), and flowing to the left. Meanwhile, you can see that the comet has already begun to reform a new tail. Carefully examine the right-hand image and identify the right-most end of the ejected tail (Call it Point B). Note that star images do not move, and are more nearly point-like than the tail gases. How far, in millimeters, is Point B from Point A?

Answer: An answer near **17 millimeters** is acceptable, but students may measure from 15 to 20 millimeters as reasonable answers.

Problem 4 - From the image scale, convert your answer to Problem 3 into arcminutes.

Answer: 17 millimeters x 5 arcminutes/mm = **85 arcminutes**.

Problem 5 - The distance of the comet was 114 million kilometers, and at that distance, one arcminute of angular separation corresponds to 33,000 kilometers. How far did the tail fragment travel between the times of the two images?

Answer: 85 arcminutes x 33,000 kilometers/arcminute = **2.8 million kilometers.**

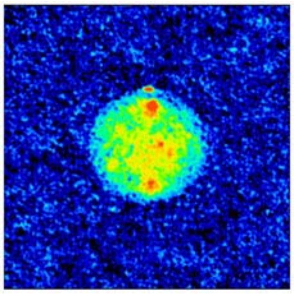
Problem 6 - What was the speed of the tail fragment?

Answer: 2.8 million kilometers/9600 seconds = **292 kilometers/second.**

Problem 7 - If the comet's speed was about 40 km/sec and the CME speed was at least several hundred times faster, based on your answer to Problem 6, was the comet fragment 'left behind' or did the CME carry it off?
Answer: The speed in Problem 6 is much closer to the CME speed than the comet speed, so the fragment was carried off by the CME and not ejected by the comet.

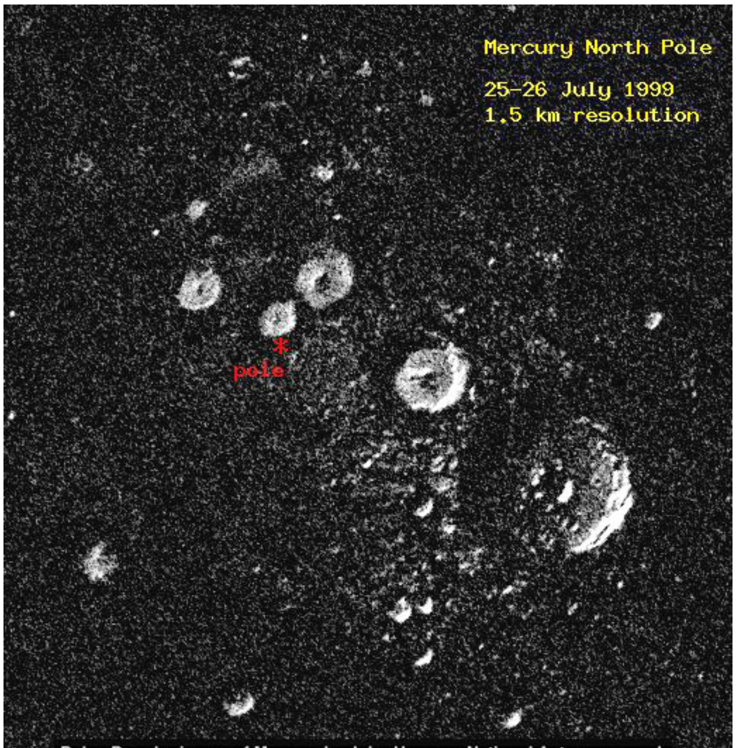
This collision was studied in detail by Dr. Angelos Vourlidas and his colleagues at the Naval Research laboratory in Washington, D.C and the Rutherford Laboratory in England. They deduced from a more careful analysis that the CME speed was about 500 km/sec and the solar wind speed was about 420 km/sec. The tail fragment was carried off by the CME. Details can be found in The Astrophysical Journal (Letters), vol. 668, pp L79-L82 which was published on October 10, 2007. A movie of the encounter may be seen at the STEREO web site (http://stereo.gsfc.nasa.gov) in their movie gallery.

Is there Ice on Mercury ?



The NASA MESSENGER spacecraft performed its first flyby of Mercury on January 14, 2008. In addition to mapping the entire surface of this planet, one of its goals is to shed new light on the existence of ice under the polar regions of this hot planet. Ice on Mercury? It's not as strange as it seems!

In 1991, Duane Muhleman and her colleagues from Caltech and the Jet Propulsion Laboratory, created the first radar map of Mercury. The image, shown here, contained a stunning surprise. The bright (red) dot at the top of the moon image to the left indicates strong radar reflection at Mercury's North Pole, resembling the strong radar echo seen from the ice-rich polar caps of Mars.



Delay-Doppler image of Mercury by John Harmon, National Astronomy and Ionosphere Center, Arecibo Observatory, Puerto Rico.

In 1999, astronomer John Harmon at the Arecibo Observatory in Puerto Rico, repeated the 1991 study, this time using the powerful microwave beam of the Arecibo Radio Telescope. The microwave energy reflected from mercury and was detected by the VLA radio telescope array in New Mexico, where a new image was made.

The radio-wavelength image to the left shows Mercury's North Polar Region at very high resolution. The image is 370 kilometers wide by 400 kilometers tall.

All the bright features are believed to be deposits of frozen water ice, at least several meters thick in the permanently shaded floors of the craters.

Reference: Harmon, Perillat and Slade, 2001, Icarus, vol 149, p.1-15

Problem 1 - From the information provided in the essay, what is the scale of the image in kilometers per millimeter?

Problem 2 - Measure the diameters of the craters, in kilometers, and estimate the total surface area covered by the large white patches in A) square kilometers and B) square meters.

Problem 3 - Suppose the icy deposit is mixed into the Mercurian surface to a depth of 10 meters. What is the total volume of the ice within the craters you measured in cubic meters?

Problem 4 - Suppose half of the volume is taken up by rock. What is the total remaining volume of ice?

Problem 5 - The density of ice is 917 kilograms/cubic meter. How many kilograms of ice are present?

Problem 6 - If this ice were 100% water ice, and 3.8 kilograms of water equals 1.0 gallons, how many gallons of water might be locked up in the shadowed craters of Mercury?

Answer Key:

Problem 1 - From the information provided in the essay, what is the scale of the image in kilometers per millimeter?

Answer; The image is 370 kilometers wide by 400 kilometers tall. The image is 95 millimeters wide by 104 millimeters tall. The scale is therefore about **4.0 kilometers / millimeter.**

Problem 2 - Measure the diameters of the craters, in kilometers, and estimate the total surface area covered by the large white patches in A) square kilometers and B) square meters.

Answer: Students should measure the diameters of at least the 5 large craters that form the row slanted upwards from right to left through the center of the image. Their diameters are about 90 km, 40 km, 30 km, 20 km and 25 km. The area of a circle is πR^2, so the crater areas are 6,400 km^2, 700 km^2, 314 km^2 and 490 km^2. The total area A) in square kilometers is about **7,900 km^2** or B) 7,900 x (1000 m/km) x (1000 m/km) = **7.9 x 10^9 meters^2**. Students may reasonably ask how to estimate the area of partially-filled craters such as the largest one in the image .They may use appropriate percentage estimates. For example, the largest crater is about 1/2 filled (white color in image) so its area can be represented as 6,400 x 0.5 = 3,200 km^2.

Problem 3 - Suppose the icy deposit is mixed into the Mercurian surface to a depth of 10 meters. What is the total volume of the ice within the craters you measured?

Answer: Volume = surface area x height = 7.9 x 10^9 meters^2 x 10 meters = 7.9 x 10^{10} meters^2.

Problem 4 - Suppose half of the volume is taken up by rock. What is the total remaining volume of ice?

Answer; 7.9 x 10^{10} meters^2 x 0.5 = **8.0 x 10^{10} meters^2**

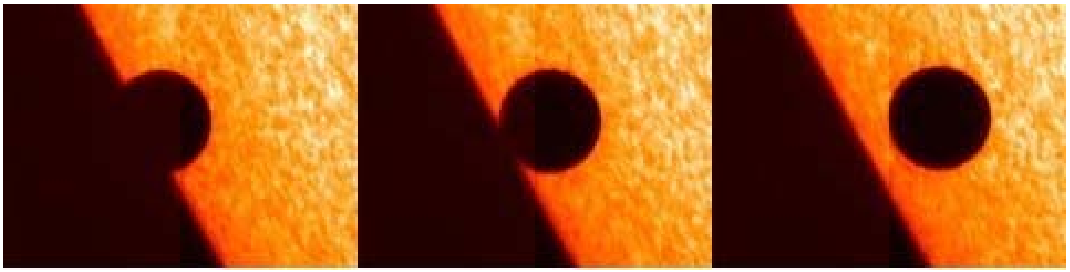
Problem 5 - The density of ice is 917 kilograms/cubic meter how many kilograms of ice are present?

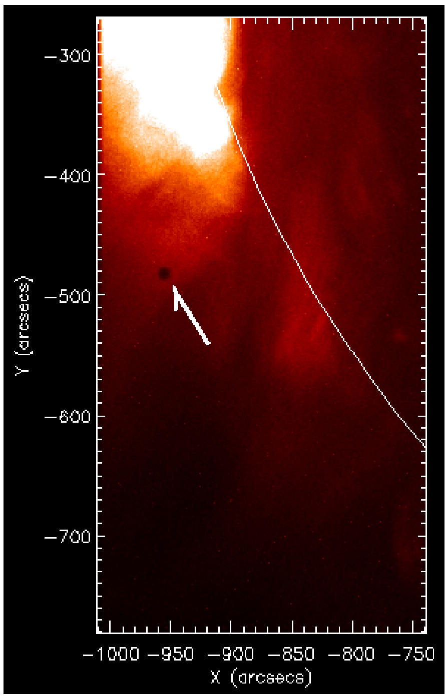
Answer: 8.0 x 10^{10} meters^2 x 917 kg/meters^3 = **7.3 x 10^{13} kilograms**

Problem 6 - If this ice were 100% water ice, and 3.8 kilogram of water equals 1.0 gallons, how many gallons of water might be locked up in the shadowed craters of Mercury?

Answer: 7.3 x 10^{13} kilograms / 3.8 kg/gallon = **1.9 x 10^{13} gallons or 19 trillion gallons!**

The Transit of Mercury





Every few times a century, the planet Mercury and Earth are lined up in such a way that Mercury passes across the disk of the sun as seen from Earth. The last time this happened was on November 8, 2006, and the next time this will happen will be on May 9, 2016. Since they were first observed in the 1600's, astronomers have studied them intently to learn more about Mercury, and to determine how far the sun is located from Earth. In recent times, astronomers no longer view these transits with much interest since the information that provide can be found by other more direct means. Still, when transits occur, astronomers turn their telescopes, now located in space, to watch the spectacle.

Top) Transit of Mercury obtained with Solar Optical Telescope (SOT) on the Hinode satellite on November 8, 2006. Left) Image obtained with the EUV Imaging Spectrometer (EIS). Solid curve indicates solar limb. The arrow shows the location of Mercury seen against the solar corona.

Problem 1: If the diameter of Mercury as viewed from Earth during the transit was 10 arcseconds, and the diameter of the sun at that time was 1900 arcseconds, what would be the diameter of the circle in the Hinode EIS image in centimeters that would represent the solar disk at this scale?

Problem 2: At the time of the transit, Mercury was about 55 million kilometers from the sun and about 92 million kilometers from Earth. How large, in arcseconds, would Mercury have appeared if it were at the distance of the Sun at this time?

Problem 3: How old will you be when the next Transit of Mercury happens?

Inquiry Problem: Why are transits of Mercury so rare?

Answer Key:

Problem 1: If the diameter of Mercury as viewed from Earth during the transit was 10 arcseconds, and the diameter of the sun at that time was 1900 arcseconds, what would be the diameter of the circle in the Hinode EIS image in centimeters that would represent the solar disk at this scale?

Answer: The tic marks on the lower image are 10 arcseconds apart. With a millimeter ruler, the separation is about 2 millimeters. The scale of the image is then 5 arcseconds/millimeter. If the solar diameter is 1900 arcseconds, its size on this page would be 1900 arcseconds / (5 arcseconds/mm) = 380 millimeters or 38 centimeters.

Problem 2: At the time of the transit, Mercury was about 55 million kilometers from the sun and about 92 million kilometers from Earth. How large, in arcseconds, would Mercury have appeared if it were at the distance of the Sun at this time?

Answer: The distance to the sun would be 55 + 92 = 147 million kilometers. At a distance of 92 million kilometers from Earth, mercury is 10 arcseconds in size, so at 147 million kilometers it would be 10 arcseconds x (92 million km /147 million km) = 6.2 arcseconds.

Problem 3: How old will you be when the next Transit of Mercury happens?

Answer: If you are 13 years old in 2007, you will be 13 + (2016 - 2007) = 13 + 9 = 22 years old, and will be graduating from college!!

Inquiry Problem: Why are transits of Mercury so rare?

Students may use GOOGLE to look up 'transit of Mercury' to find pages that discuss how transits occur.

They should deduce that the circumstances to not re-occur each year because Earth and mercury are on orbits that are tilted with respect to each other.

Extracting Oxygen from Moon Rocks

About 85% of the mass of a rocket is taken up by oxygen for the fuel, and for astronaut life support. Thanks to the Apollo Program, we know that as much as 45% of the mass of lunar soil compounds consists of oxygen. The first job for lunar colonists will be to 'crack' lunar rock compounds to mine oxygen.

NASA has promised $250,000 for the first team capable of pulling breathable oxygen from mock moon dirt; the latest award in the space agency's Centennial Challenges program.

Lunar soil is rich in oxides of silicon, calcium and iron. In fact, 43% of the mass of lunar soil is oxygen. One of the most common lunar minerals is *ilmenite*, a mixture of iron, titanium, and oxygen. To separate *ilmenite* into its primary constituents, we add hydrogen and heat the mixture. This hydrogen reduction reaction is given by the 'molar' equation:

$$FeTiO_3 + H_2 \text{ ---> } Fe + TiO_2 + H_2O$$

A Bit Of Chemistry - This equation is read from left to right as follows: One mole of *ilmenite* is combined with one mole of molecular hydrogen gas to produce one mole of free iron, one mole of titanium dioxide, and one mole of water. Note that the three atoms of oxygen on the left side (O_3) is 'balanced' by the three atoms of oxygen found on the right side (two in TiO_2 and one in H_2O). **One 'mole' equals 6.02×10^{23} molecules**.

The 'molar mass' of a molecule is the mass that the molecule has if there are 1 mole of them present. The masses of each atom that comprise the molecules are added up to get the molar mass of the molecule. Here's how you do this:

For H_2O, there are two atoms of hydrogen and one atom of oxygen. The atomic mass of hydrogen is 1.0 AMU and oxygen is 16.0 AMU, so the molar mass of H_2O is 2 (1.0) + 16.0 = 18.0 AMU. **One mole of water molecules will equal 18 grams of water by mass.**

Problem 1 -The atomic masses of the atoms in the *ilmenite* reduction equation are Fe = 55.8 and Ti = 47.9. A) What is the molar mass of ilmenite? B) What is the molar mass of molecular hydrogen gas? C) What is the molar mass of free iron? D) What is the molar mass of titanium dioxide? E) Is mass conserved in this reaction?

Problem 2 - If 1 kg of ilmenite was 'cracked' how many grams of water would be produced?

Inquiry Question - If 1 kg of ilmenite was 'cracked' how many grams of molecular oxygen would be produced if the water molecules were split by electrolysis into

$$2\, H_2O \text{ --> } 2H_2 + O_2?$$

Problem 1 -

A) What is the molar mass of ilmenite? 1(55.8) + 1 (47.9) + 3(16.0) = **151.7 grams/mole**
B) What is the molar mass of molecular hydrogen gas? 2(1.0) = **2.0 grams/mole**
C) What is the molar mass of free iron? 1(55.8) = **55.8 grams/mole**
D) What is the molar mass of titanium dioxide? 1(47.9) + 2(16.0) = **79.9 grams/mole**
E) Is mass conserved in this reaction? **Yes. There is one mole for each item on each side, so we just add the molar masses for each constituent. The left side has 151.7 + 2.0 = 153.7 grams and the right side has 55.8 + 79.9 + 18.0 = 153.7 grams so the mass balances on each side.**

Problem 2 -

Step 1 - The reaction equation is balanced in terms of one mole of ilmenite (1.0 x $FeTiO_3$) yielding one mole of water (1.0 x H_2O) . The molar mass of ilmenite is 151.7 grams which is the same as 0.1517 kilograms, so we just need to figure out how many moles is needed to make one kilogram.

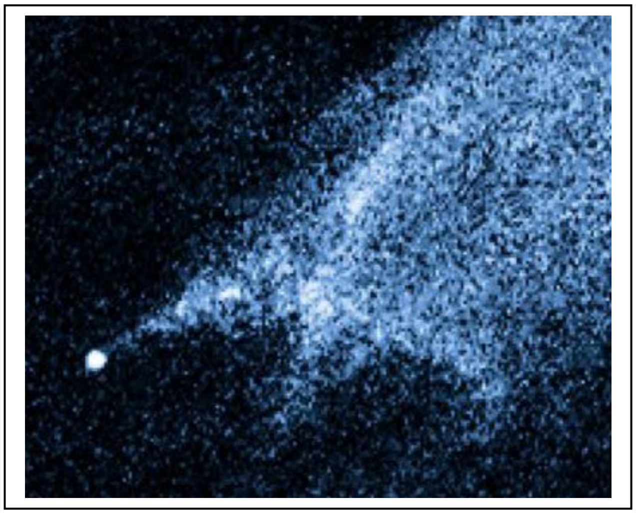
Step 2 - This will be 1000 grams/151.7 grams = 6.6 moles. Because our new reaction is that we start with 6.6 x $FeTiO_3$ that means that for the reaction to remain balanced, we need to produce 6.6 x H_2O, or in other words, 6.6 moles of water.

Step 3 - Because the molar mass of water is 18.0 grams/mole, the total mass of water produced will be 6.6 x 18.0 = **119 grams of water.**

Inquiry Question - The reaction is: $2\ H_2O \rightarrow 2H_2 + O_2$

This means that for every 2 moles of water, we will get one mole of O_2. The ratio is 2 to 1. From the answer to Problem 2 ,we began with 6.59 moles of water not 2.0 moles. That means we will produce 6.6/2 = 3.3 moles of water. Since 1 molecule of oxygen has a molar mass of 2(16) = 32 grams/mole, the total mass of molecular oxygen will be 3.3 moles x 32 grams/mole = **106 grams. So, 1 kilogram of ilmenite will eventually yield 106 grams of breathable oxygen.**

Hubble Spies Colliding Asteroids!



It doesn't look like much, but this picture taken by the Hubble Space Telescope on January 25, 2010 shows all that remains of two asteroids that collided! The object, called P/2010 A2, was discovered in the asteroid belt 290 million kilometers from the sun, by the Lincoln Near-Earth Asteroid Research sky survey on January 6, 2010.

Hubble shows the main nucleus of P/2010 A2, about 150 meters in diameter, lies outside its own halo of dust. This led scientists to the interpretation that it is the result of a collision.

How often do asteroids collide in the asteroid belt? The collision time can be estimated by using the formula:

$$T = \frac{1}{NAV}$$

Where N is the number of bodies per cubic kilometer, v is the speed of the bodies relative to each other in kilometers/sec and A is the cross-sectional area of the body in square-kilometers. The answer, T, will be in the average number of seconds between collisions.

Estimating A: Assume that the bodies are spherical and 100 meters in diameter .What will be A, the area of a cross-section through the body?

Estimating the asteroid speed V: At the orbit of the asteroids, they travel once around the sun in about 3 years. What is the average speed of the asteroid in kilometers/sec at a distance of 290 million kilometers?

Estimating the density of asteroids N: - This quantity is the number of asteroids in the asteroid belt, divided by the volume of the belt in cubic kilometers. A) Assume that the asteroid belt is a thin disk 1 million kilometers thick, with an inner radius of 1.6 AU and an outer radius of 2.5 AU. If 1 AU = 150 million kilometers, want is the volume? B) Based on telescopic observations, an estimate for the number of asteroids in the belt that are larger than 100 meters across is about 30 billion. From this information, and your volume estimate, what is the average density of asteroids in the asteroid belt?

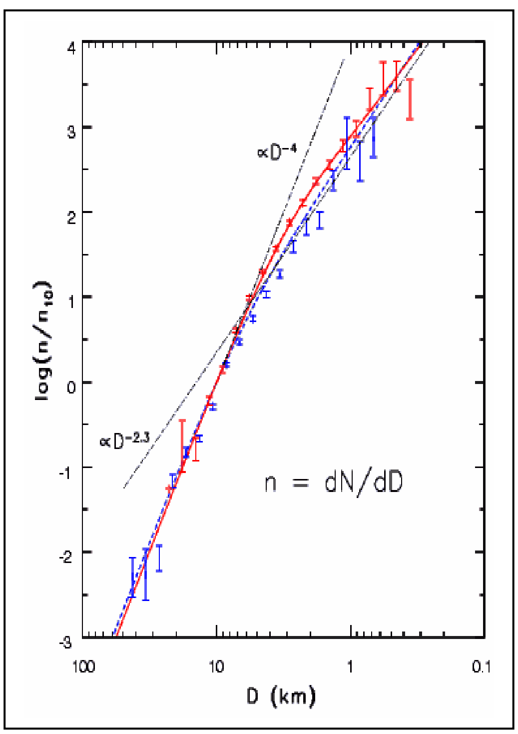
Estimating the collision time T: From the formula, A) what do your estimates for N, V and A imply for the average time between collisions in years? B) What are the uncertainties in your estimate?

Estimating A: Answer: The cross-section of a sphere is a circle, so $A = \pi R^2$ or for R = 100 meters, we have A = 3.14 $(0.05)^2 = 8\times10^{-3}$ km^2.

Estimating the asteroid speed V: Answer: The circumference of the orbit is C = 2 π (290 million km) = 1.8 x 10^9 kilometers, and the time in seconds is 3.0 years x (3.1 x 10^7 seconds/1 year) = 9.3 x 10^7 seconds, so the average speed V= C/T = 19 kilometers/sec. But what we want is the <u>relative</u> speed. If all the asteroids were going around in their orbits at a speed of 19 km/sec, their <u>relative</u> speeds would be zero. Example, although two cars are on the freeway traveling at 65 mph, their relative speeds are zero since they are not passing or falling behind each other.

Estimating the density of asteroids N: - Answer: R(inner) = 2.4 x 10^8 km .R(outer)= 3.4 x 10^8 km, so the volume of the disk is V = disk area x thickness = [$\pi(3.4\times10^8)^2 - \pi(2.4 \times 10^8)^2$] x 10^6 so the volume is **1.8 x 10^{23} km^3**. Then the average asteroid density is N= 3×10^{10} asteroids/1.8 x 10^{23} km^3 = **1.7 x 10^{-13} asteroids/km^3.**

Estimating the collision time T: Answer: A) T = 1/NAV so T = 1/(1.7 x 10^{-13} x 8×10^{-3} x 19) = 3.8 x 10^{13} seconds. Since 1 year = 3.1 x 10^7 seconds, we have about **1,200,000 years between collisions**. B) Students can explore a number of places where large uncertainties might occur such as: 1) the thickness of the asteroid belt; 2) the number of asteroids; 3) their actual relative speeds. 4) The non-uniform distribution of the asteroids…not smoothly distributed all over the volume of the asteroid belt, but may be in clumps or rings that occupy a smaller actual volume. Encourage them to come up with their own estimates and see how that affects the average time between collisions. For advanced students see the problem below.



Extra Credit with Calculus: Data from the Sloan Digital Sky Survey suggests that the number of asteroids in specific size ranges follow a piecewise power-law distribution:

$$N(D) = \begin{cases} 1.8x10^9 D^{-2.3} & \text{for D <70 km} \\ 2.4x10^{12} D^{-4.0} & \text{for D>70 km} \end{cases}$$

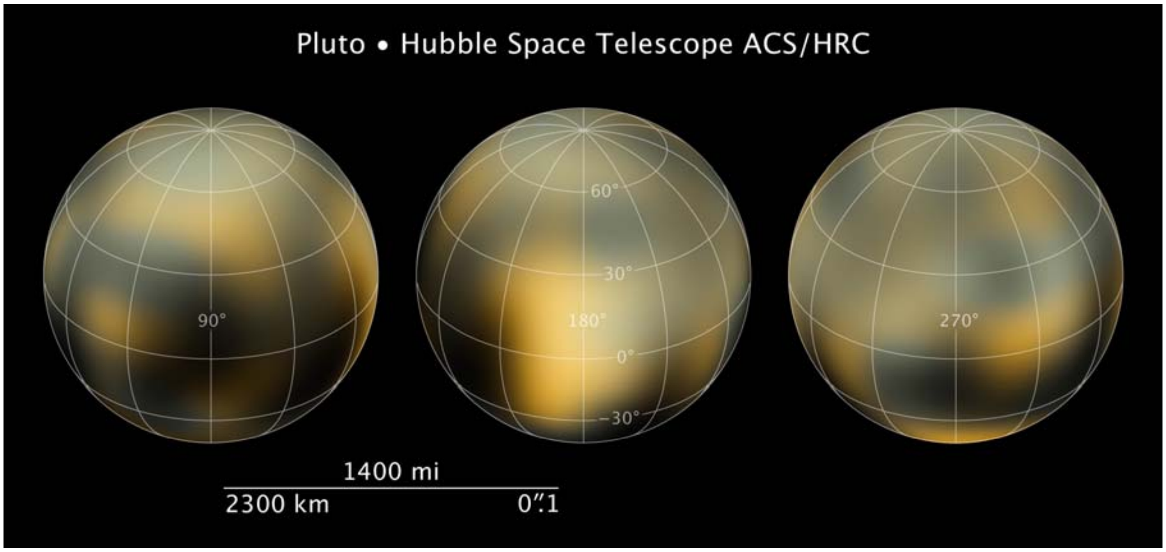
Integrate N(D) from 100 meters to infinity to determine the number of asteroids larger than 100 meters.

Answer: $n = \int_{0.1}^{70} N(D)dD + \int_{70}^{\infty} N(D)dD$

= 2.7×10^{10} + 2.32×10^6 = 2.7×10^{10} asteroids.

Note: the figure is scaled to the number of 10-km asteroids (n_{10}) which we take to be 10,000.

The Changing Atmosphere of Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

Problem 1 - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = 1.5×10^{11} meters).

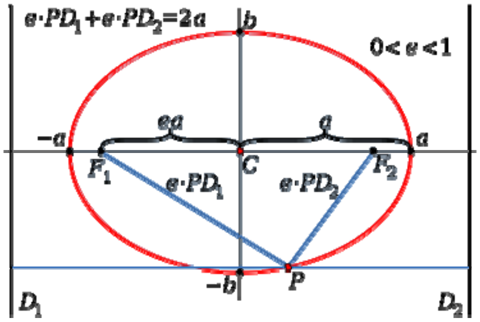
Problem 2 - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R) = \left(\frac{L(1-A)}{16\pi\sigma R^2} \right)^{\frac{1}{4}} \quad \text{degrees Kelvin (K)}$$

where L is the luminosity of the sun ($L=4 \times 10^{26}$ watts); σ is a constant with a value of 5.67×10^{-8}, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

Problem 3 - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?

Problem 1 - Answer:



In Standard Form $2433600=1521x^2+1600y^2$ becomes $1=\frac{x^2}{1600}+\frac{y^2}{1521} = \frac{x^2}{a^2}+\frac{y^2}{b^2}$

Then A) **a = 40 AU** and B) **b=39 AU**. C) The ellipticity $\mathbf{e = (a^2 - b^2)^{1/2}/a = 0.22}$. D) The longest distance from a focus is just a(1 + e) = 40(1+0.22) = **49 AU**. E) The shortest distance is just a(1-e) = (1-0.22)(40) = **31 AU**. Written out in meters we have a= 6×10^{12} meters; b= 5.8×10^{12} meters; aphelion = 7.35×10^{12} meters and perihelion = 4.6×10^{12} meters.

Problem 2 - Answer: For R in terms of AU, the formula simplifies to

$$T(R)=\left(\frac{4x10^{26}(1-0.6)}{16(3.14)(5.67x10^{-8})(1.5x10^{11})^2R^2}\right)^{\frac{1}{4}} \quad \text{so } T(R)=\frac{223}{\sqrt{R}} \text{ degrees K}$$

A) For a perihelion distance of 31 AU we have T = $223/(31)^{1/2}$ = 40 K; B) At an aphelion distance of 49 AU we have T = $223/(49)^{1/2}$ = 32 K. Note: The actual temperatures are about higher than this and average about 50K.

Problem 3 - Answer: At aphelion, the height of the atmosphere is about H=1.2x(32) = 38 kilometers, and at perihelion it is about H=1.2x(40) = 48 kilometers, so as Pluto orbits the sun its atmosphere increases and decreases in thickness.

Note: In fact, because the freezing point of methane is 91K, at aphelion most of the atmosphere freezes onto the surface of the dwarf planet, and at aphelion it returns to a mostly gaseous state. This indicates that the simple physical model used to derive H(T) was incomplete and did not account for the freezing-out of an atmospheric constituent.


Life-size model of the Webb Space Telescope.

In 2018, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, $\theta(R)$, for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on its surface is given by

$$T(R) = \frac{250}{\sqrt{R}} \text{ Kelvins}$$

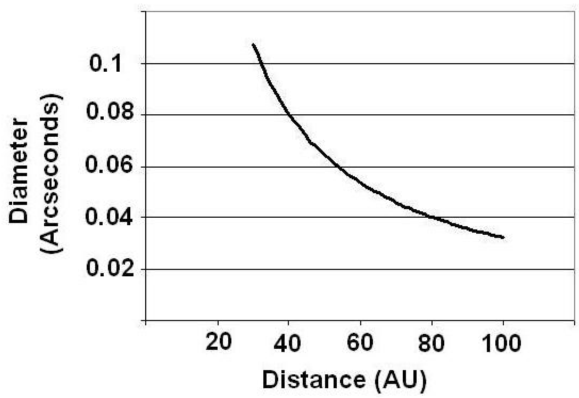
where R is the distance from the sun in AU. Create a graph that shows T(R) vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

Problem 3 - A body in other outer solar system with an angular size $\theta(R)$ emits most of its light energy in the infrared and has a temperature given by T(R) in Kelvins. Its brightness in units of Janskys, F, at a wavelength of 20 microns will be given by:

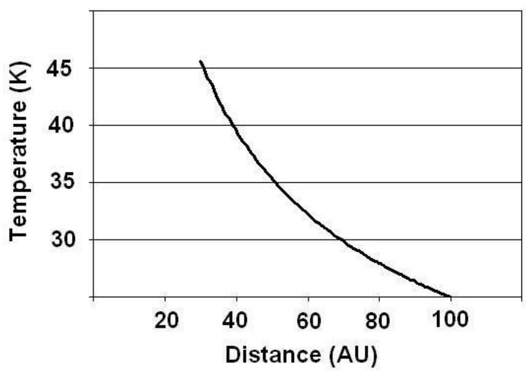
$$F(T) = \frac{120000}{(e^x - 1)}\theta(R)^2 \text{ Janskys} \qquad \text{where} \qquad x = \frac{720}{T(R)}$$

From the formula for $\theta(R)$ and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

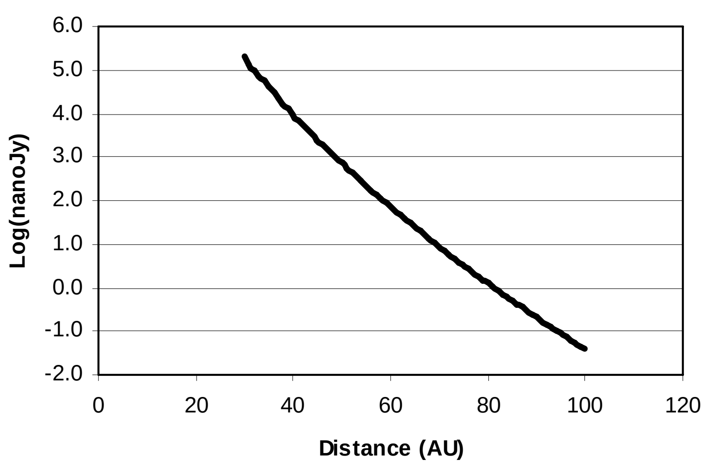
Problem 1 - Answer: At 90 AU, the disk of a Pluto-sized body will be 0.035 arcseconds in diameter.

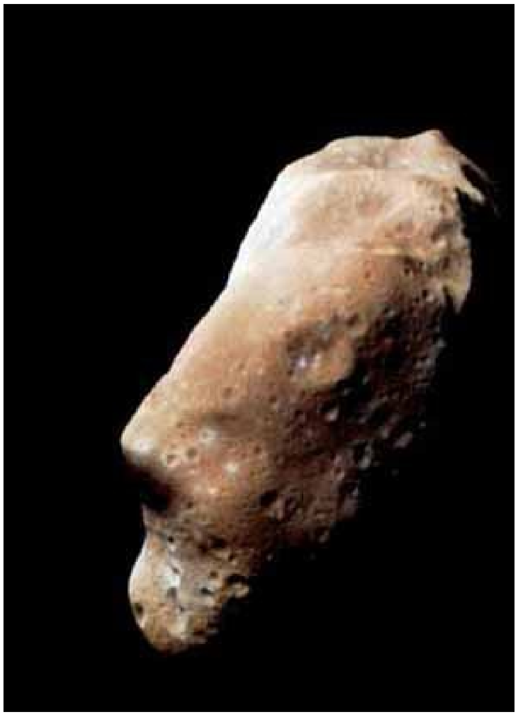


Problem 2 Answer: At 90 AU, the predicted temperature will be about 28 K.



Problem 3 - Answer: At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about 40 AU. More accurate estimates using more realistic emission properties for Pluto suggest 90 AUs as an actual limit.




Asteroid Gaspara

Astronomers studying the asteroid 24-Themis detected water-ice and carbon-based organic compounds on the surface of the asteroid.

NASA detects, tracks and characterizes asteroids and comets passing close to Earth using both ground and space-based telescopes.

NASA is particularly interested in asteroids with water ice because this resource could be used to create fuel for interplanetary spacecraft.

On October 7, 2009, the presence of water ice was confirmed on the surface of this asteroid using NASA's Infrared Telescope Facility. The surface of the asteroid appears completely covered in ice. As this ice layer is sublimated, it may be getting replenished by a reservoir of ice under the surface. The orbit of the asteroid varies from 2.7 AU to 3.5 AU so it is located withint the asteroid belt. The asteroid is 200 km in diameter, has a mass of 1.1 x 10^{19} kg, and a density of 2,800 kg/m^3 so it is mostly rocky material similar in density to Earth's.

By measuring the spectrum of infrared sunlight reflected by the object, the NASA researchers found the spectrum consistent with frozen water and determined that 24 Themis is coated with a thin film of ice. The asteroid is estimated to lose about 1 meter of ice each year, so there must be a sub-surface reservoir to constantly replace the evaporating ice.

Problem 1 – Assume that the asteroid has a diameter of 200 km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 km/$meter^3$? (Hint: Volume = Surface area x thickness)

Problem 2 – Suppose that only 1% by volume of the 1-meter-thick 'dirty' surface layer is actually water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

Problem 1 – Assume that the asteroid has a diameter of 200km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 km/meter^{3}? (Hint: Volume = Surface area x thickness)

Answer: Volume = surface area x thickness.

$$SA = 4 \pi r^2$$
$$= 4 (3.14) (100{,}000 \text{ meters})^2$$
$$= 1.3 \times 10^{11} \text{ meters}^2$$
$$\text{Volume} = 1.3 \times 10^{11} \text{ meters}^2 \times 1 \text{ meter}$$
$$= 1.3 \times 10^{11} \text{ meters}^3$$

Mass of water = density x volume

$$= 1{,}000 \text{ kg/meter}^3 \times 1.3 \times 10^{11} \text{ meter}^3$$
$$= \mathbf{1.3 \times 10^{14} \text{ kg}} \quad \text{(or 130 billion tons)}$$

Problem 2 – Suppose that only 1% by volume of the 'dirty' 1-meter-thick surface layer is water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

Answer: The mass of water in the outer 1-meter layer is 1% of 1.3×10^{14} kg or 1.3×10^{12} kg. Since 1 year = 365 days x 24h/day x 60m/hr x 60 sec/min = 3.1×10^{7} seconds, the mass loss is just 1.3×10^{12} kg / 3.1×10^{7} sec = **42,000 kg/sec or 42 tons/sec**.

This historic image of the nucleus of Halley's Comet by the spacecraft Giotto in 1986 reveals the gases leaving the icy body to form the tail of the comet.

Once astronomers discover a new comet, a series of measurements of its location allows them to calculate the orbit of the comet and predict when it will be closest to the Sun and Earth.

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years?

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

Answer: The standard formula for an ellipse is $x^2/a^2 + y^2/b^2 = 1$ so we can re-write this as $b^2x^2 + a^2y^2 = a^2b^2$.

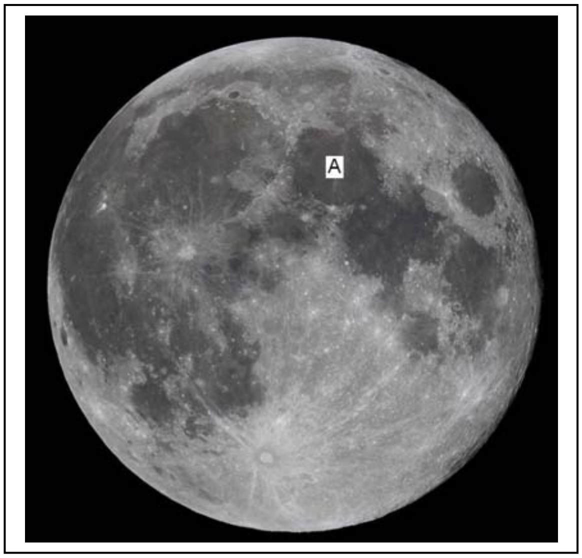
Then for Point 1 we have
$10^2b^2 + 4^2a^2 = (ab)^2$ so $\mathbf{100b^2 + 16a^2 = (ab)^2}$. Similarly for Point 2 we have
$14^2b^2 + 3^2a^2 = (ab)^2$ so $\mathbf{196b^2 + 9a^2 = (ab)^2}$

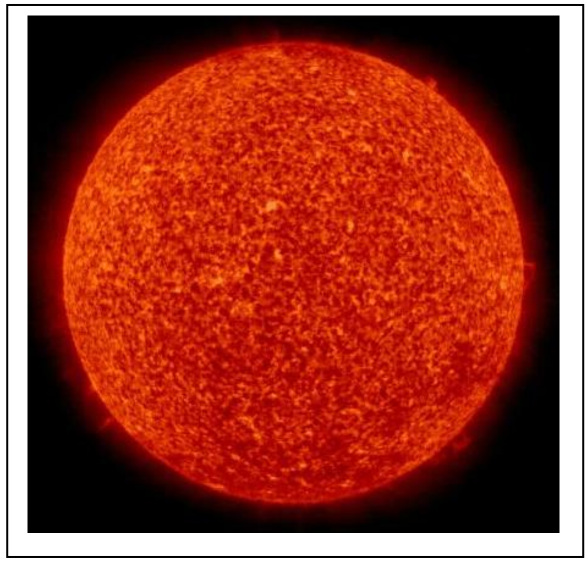
Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.
Answer:
$100b^2 + 16a^2 = (ab)^2$
$196b^2 + 9a^2 = (ab)^2$
Difference the pair to get $7a^2 = 96b^2$ so $a^2 = (96/7)b^2$.
Substitute this into the first equation to eliminate b^2 to get
$(700/96) + 16 = (7/96)a^2$ or $a^2 = 2236/7$ and so **a = 17.8 AU**.
Then substitute this value for a into the first equation to get
$5069 = 217\ b^2$ and so **b = 4.8 AU**.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years? Answer: $P = a^{3/2}$ so for a = 17.8 AU we have **P = 75.1 years**.

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the Sun in this orbit in kilometers?
Answer: From the definition for c as $c = (a^2 - b^2)^{1/2}$ we have c = 17.1 AU and so the perihelion distance is just d = 17.8 – 17.1 = 0.7 AU. Since 1 AU = 150 million km, it comes to within **105 million km** of the Sun. This is near the orbit of Venus.

Getting an Angle on the Sun and Moon





The Sun (Diameter = 1,400,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon?

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

Problem 5 - What is the area of Mare Serenitatis in square kilometers?

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

Answer Key

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter? Answer: Moon diameter = 65 mm and sun diameter = 61 mm so the lunar image scale is 1,865 asec/65mm = **28.7 asec/mm** and the solar scale is 1865 asec/61 mm = **30.6 asec/mm.**

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon? Answer: the smallest feature is about 0.5 mm or 0.5 x 28.7 asec/mm = **14.4 asec for the Moon** and 0.5 x 30.6 asec/mm = **15.3 asec for the Sun.**

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon? Answer: The diameter of the mare is 1 centimeter, so the radius is 5 mm or 5 mm x 28.7 asec/mm = 143.5 asec. Assuming a circle, the area is $A = \pi \times (143.5\ asec)^2$ = **64,700 $asec^2$**.

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle? Answer: The angular scale at the sun would correspond to 400 x 1.9 km = **760 kilometers per arcsecond**.

Problem 5 - What is the area of Mare Serenitatis in square kilometers? Answer: We have to convert from square arcseconds to square kilometers using a two-step unit conversion 'ladder'.

$$64{,}700\ asec^2 \times (1.9\ km/asec) \times (1.9\ km/asec) = \mathbf{233{,}600\ km^2.}$$

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun? Answer: The angular area is 400-times further away, so we have to use the scaling of 760 kilometers/asec deduced in Problem 4. The unit conversion for the solar area becomes:

$$64{,}700\ asec^2 \times (760\ km/asec) \times (760\ km/asec) = \mathbf{37{,}400{,}000{,}000\ km^2.}$$

On July 4, 2005 at 5:45 UT the 362-kilogram Impactor from NASA's Deep Impact mission, collided with the nucleus of the comet Tempel 1, causing a bright flash of light and a plume of ejected gas (see photo).

Traveling at 10.3 km/sec, the Impactor created a crater on the nucleus and ejected about 10,000 tons of material.

The average density of the comet nucleus is 400 kg/m^3 and its size can be approximated as a sphere with a radius of 3 kilometers.

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Problem 2 - If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Problem 3 – How far, in kilometers, will the comet nucleus have drifted 'sideways' to its orbit after 1 million years?

Problem 4 – Suppose that the comet had been headed toward Earth, and it was predicted that in 50 years it would collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5 x 10^8 kilogram kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Answer: The spherical volume was $V = 4/3 \pi (3000 \text{ meters})^3 = 1.1 \times 10^{11}$ meters^3. The density was 400 kg/m^3, so Mass = Density x Volume = $400 \times 1.1 \times 10^{11}$ = 45 trillion kilograms.

Problem 2 -- If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Answer: A) Vc = mi Vi/Mc = (362 kg) x (10.3 km/sec)/45 trillion kg = 8×10^{-11} kilometers/sec.

B) 8×10^{-11} km/s x (1000 m/km) x (3600 s/hr) x (24 hr/day) x (365 d/yr) = 2.5 meters/year.

Problem 3 – How far, in kilometers, will the comet nucleus have drifted sideways to its orbit after 1 million years?

Answer: From Problem 2, the drift is 2.5 meters/year, so after 1 million years the nucleus will have drifted about 2,500,000 meters or 2,500 kilometers.

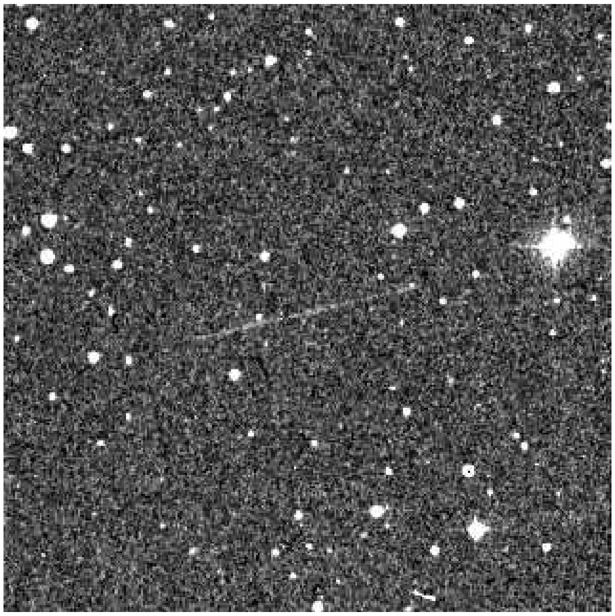
Problem 4 – Suppose that the comet had been headed towards Earth, and it is predicted that in 50 years it will collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5×10^8 kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Answer: Using miVi = mcVc, we get

$$Vc = (7.5 \times 10^8 \text{ kilograms}) \times (10.3\text{km/sec})/45 \text{ trillion kg}$$
$$= 0.00017 \text{ kilometers/sec.}$$

A) In 20 years ($20 \times 3.1 \times 10^7$ seconds) it travels 100,000 kilometers.
B) Yes, since it only needs to travel 12,000 kilometers sideways to avoid hitting Earth, the detonation did help to avoid the collision…assuming the comet wasn't fragmented into a large cloud of debris!

Spotting an Approaching Asteroid or Comet



An asteroid, or comet, viewed from Earth will be either bright or faint depending on many quantifiable factors. Of course the size of the body and its reflectivity make a big difference. So does its distance from the sun and earth at the time you see it. The brightness also depends on whether, from Earth, it is fully-illuminated like the full moon, or only partly-illuminated like the crescent moon.

Astronomers can put all of these variables together into one single equation which works pretty well to predict a body's brightness just about anywhere inside the solar system!

The streak in the photo above is the asteroid 1999AN10 (Courtesy Palomar Digital Sky Survey). Orbit data suggest that on August 7, 2027 it will pass within 37,000 kilometers of Earth. The formula for the brightness of the asteroid is given by:

$$R = 0.011\, d\ 10^{-\frac{1}{5}(m)}$$

where: R is the asteroid radius in meters, d is the distance to Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The photograph above shows stars as faint as m = +20. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 37,000 kilometers, what is the formula R(m) for the asteroid?

Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses?

Answer Key

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 37,000 kilometers, what is the formula R(m) for the asteroid? Answer: Substitute the given values into the equation and simplify. The formula will give the radius of the asteroid in meters as a function of its apparent brightness (called apparent magnitude by astronomers) given by m.

$R(m) = 0.011\ (37000)10^{-0.2m}$

$\mathbf{R(m)= 407\ 10^{-0.2m}}$

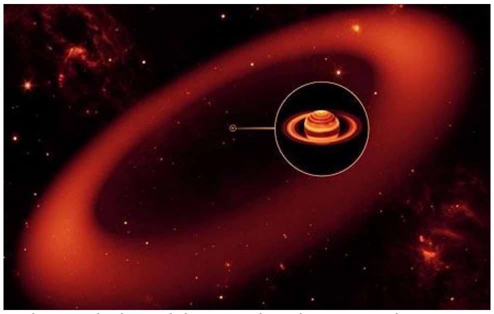
Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses? Answer: Solve the formula for R(m) for m(R) and evaluate for R = 200 meters and R = 1000 meters to obtain the range of the function.

$m(R) = -5 \log_{10}(R/4224)$

so m(R) for R = 200 yields m(200) = -5 (-11.3) so m(200) = +1.5
and m(1000) = -5(0.39) so m(1000) = -2.0

so Domain R: [200,1000]
and Range m: [+1.5, -2.0]

Note: The planet Venus can be as bright as m = -2.5 so this asteroid should be easily visible if it is in this size domain.



Artist rendering of the new ice ring around Saturn detected by the Spitzer Space Telescope.

"This is one supersized ring," said one of the authors, Professor Anne Verbiscer, an astronomer at the University of Virginia in Charlottesville. Saturn's moon Phoebe orbits within the ring and is believed to be the source of the material.

The thin array of ice and dust particles lies at the far reaches of the Saturnian system. The ring was very diffuse and did not reflect much visible light but the infrared Spitzer telescope was able to detect it. Although the ring dust is very cold -316F it shines with thermal 'heat' radiation. No one had looked at its location with an infrared instrument until now.

"The bulk of the ring material starts about 6.0 million km from the planet, extends outward about another 12 million km, and is 2.6 million km thick. The newly found ring is so huge it would take 1 billion Earths to fill it." (CNN News, October 7, 2009)

Many news reports noted that the ring volume was equal to 1 billion Earths. Is that estimate correct? Let's assume that the ring can be approximated by a washer with an inner radius of r, an outer radius of R and a thickness of h.

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Problem 3 - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Problem 6 - How does your answer compare to the Press Release information? Why are they different?

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Answer: The area of the large circle is given by πR^2 minus area of small circle πr^2 equals $A = \pi (R^2 - r^2)$

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Answer: Volume = Area x height so $V = \pi (R^2 - r^2) h$

Problem 3 - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Answer: $V = \pi (R^2 - r^2) h$

$= (3.141) [(1.2 \times 10^7)^2 - (6.0 \times 10^6)^2] \, 2.4 \times 10^6$

` $= 8.1 \times 10^{20}$ km^3

Note that the smallest number of significant figures in the numbers involved is 2, so the answer will be reported to two significant figures.

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Answer: Volume of a sphere $V = 4/3 \pi R^3$ so for Earth,

$V = 1.33 \times (3.14) \times (6.378 \times 10^3)^3$

$= 1.06 \times 10^{12}$ km^3

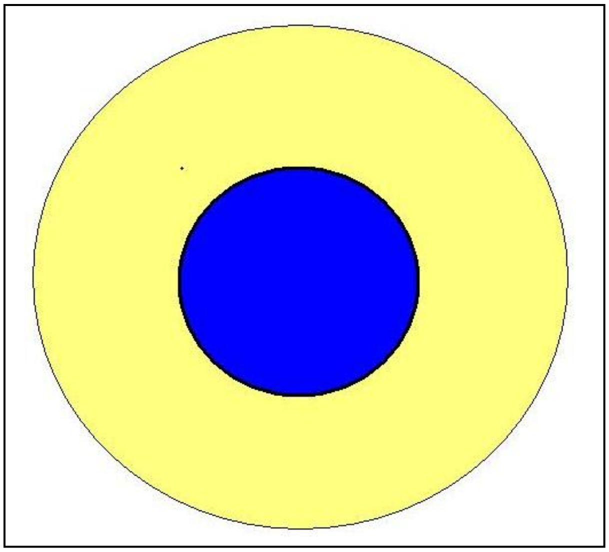
Note that the smallest number of significant figures in the numbers involved is 3, so the answer will be reported to three significant figures.

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Answer: Divide the answer for Problem 3 by Problem 4 to get

8.1×10^{20} km^3 / $(1.06 \times 10^{12}$ km^3) $= 7.6 \times 10^8$ times

Problem 6 - How does your answer compare to the Press release information? Why are they different? Answer: The Press Releases say 'about 1 billion times' because it is easier for a non-scientist to appreciate this approximate number. If we rounded up 7.6×10^8 times to one significant figure accuracy, we would also get an answer of '1 billion times'.



The planet Osiris orbits 7 million kilometers from the star HD209458 located 150 light years away in the constellation Pegasus. The Spitzer Space Telescope has recently detected water, methane and carbon dioxide in the atmosphere of this planet. The planet has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter.

By knowing the mass, radius and density of a planet, astronomers can create plausible models of the composition of the planet's interior. Here's how they do it!

Among the first types of planets being detected in orbit around other stars are enormous Jupiter-sized planets, but as our technology improves, astronomers will be discovering more 'super-Earth' planets that are many times larger than Earth, but not nearly as enormous as Jupiter. To determine whether these new worlds are Earth-like, they will be intensely investigated to determine the kinds of compounds in their atmospheres, and their interior structure. Are these super-Earths merely small gas giants like Jupiter, icy worlds like Uranus and Neptune, or are they more similar to rocky planets like Venus, Earth and Mars?

Problem 1 - A hypothetical planet is modeled as a sphere. The interior has a dense rocky core, and on top of this core is a mantle consisting of a thick layer of ice. If the core volume is 4.18×10^{12} cubic kilometers and the shell volume is 2.92×10^{13} cubic kilometers, what is the radius of this planet in kilometers?

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet? B) How many Earths could fit inside the mantle of this hypothetical planet?

Problem 3 - Suppose the astronomer who discovered this super-Earth was able to determine that the mass of this new planet is 8.3 times the mass of Earth. The mass of Earth is 6.0×10^{24} kilograms. What is A) the mass of this planet in kilograms? B) The average density of the planet in kilograms/cubic meter?

Problem 4 - Due to the planet's distance from its star, the astronomer proposes that the outer layer of the planet is a thick shell of solid ice with a density of 1000 kilograms/cubic meter. What is the average density of the core of the planet?

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?

Problem 1 - The planet is a sphere whose total volume is given by $V = 4/3\ \pi\ R^3$. The total volume is found by adding the volumes of the core and shell to get $V = 4.18 \times 10^{12} + 2.92 \times 10^{13} = 3.34 \times 10^{13}$ cubic kilometers. Then solving the equation for R we get $R = (3.34 \times 10^{13} / (1.33*3.14))^{1/3}$ = 19,978 kilometers. Since the data are only provided to 3 place accuracy, the final answer can only have three significant figures, and with rounding this equals a radius of R = **20,000 kilometers.**

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet?
Answer: V = 4.18×10^{12} cubic kilometers / 1.1×10^{12} cubic kilometers = **4 Earths.**

B) How many Earths could fit inside the mantle of this hypothetical planet?
Answer: V = 2.92×10^{13} cubic kilometers / 1.1×10^{12} cubic kilometers = **27 Earths.**

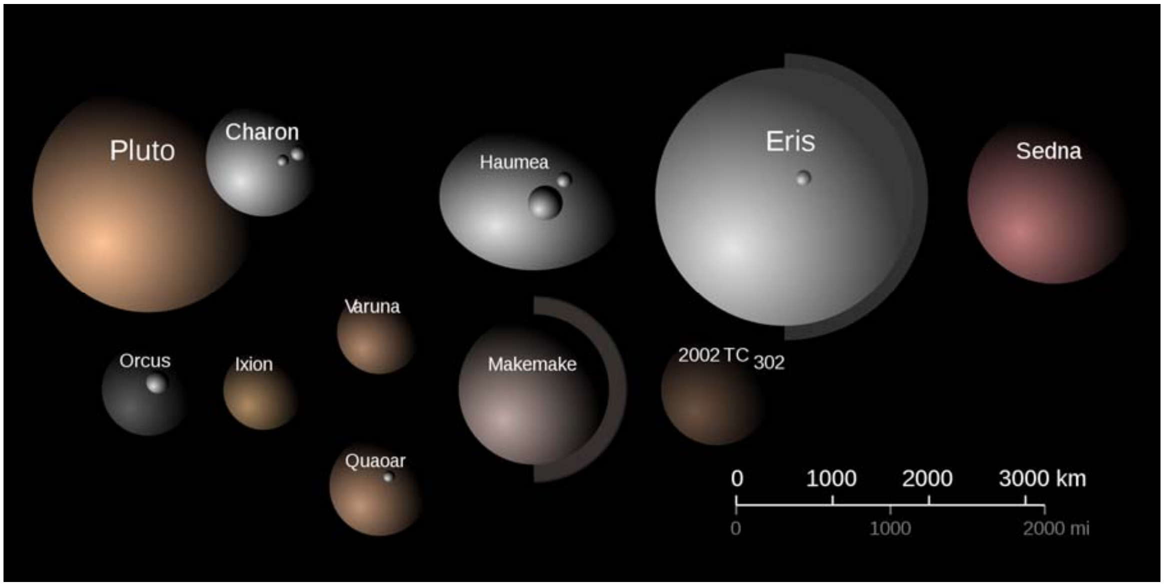
Problem 3 - What is A) the mass of this planet in kilograms? Answer: $8.3 \times 6.0 \times 10^{24}$ kilograms = **5.0×10^{25} kilograms.**
B) The average density of the planet in kilograms/cubic meter?
Answer: Density = total mass/ total volume
= 5.0×10^{25} kilograms/3.34×10^{13} cubic kilometers
= 1.5×10^{12} kilograms/cubic kilometers.
Since 1 cubic kilometer = 10^9 cubic meters,
= 1.5×10^{12} kilograms/cubic kilometers x (1 cubic km/10^9 cubic meters)
= **1,500 kilograms/cubic meter.**

Problem 4 - We have to subtract the total mass of the ice shell from the mass of the planet to get the mass of the core, then divide this by the volume of the core to get its density. Mass = Density x Volume, so the shell mass is 1,000 kg/ $m^3 \times 2.92 \times 10^{13}\ km^3 \times (10^9\ m^3/km^3) = 2.9 \times 10^{25}$ kg. Then the core mass = 5.0×10^{25} kilograms - 2.9×10^{25} kg = 2.1×10^{25} kg. The core volume is $4.18 \times 10^{12}\ km^3 \times (10^9\ m^3/km^3) = 4.2 \times 10^{21}\ m^3$, so the density is $D = 2.1 \times 10^{25}$ kg / $4.2 \times 10^{21}\ m^3$= **5,000 kg/ m^3.**

Problem 5 - The densities of some common ingredients for planets are as follows:
Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3
From your answer to Problem 4, what is the likely composition of the core of this planet?
Answer: **Basalt.**

Note that, although the average density of the planet (1,500 kg/m^3) is not much more than solid ice (1,000 kg/m^3), the planet has a sizable rocky core of higher density material. Once astronomers determine the size and mass of a planet, these kinds of 'shell-core' models can give valuable insight to the composition of the interiors of planets that cannot even be directly imaged and resolved! In more sophisticated models, the interior chemistry is linked to the temperature and location of the planet around its star, and proper account is made for the changes in density inside a planet due to compression and heating. The surface temperature, which can be measured from Earth by measuring the infrared 'heat' from the planet, also tells which compounds such as water can be in liquid form.

Dwarf Planets and Kepler's Third Law



Object	Distance (AU)	Period (years)
Mercury	0.4	0.24
Venus	0.7	0.61
Earth	1.0	1.0
Mars	1.5	1.88
Ceres	2.8	4.6
Jupiter	5.2	11.9
Saturn	9.5	29.5
Uranus	19.2	84.0
Neptune	30.1	164.8
Pluto	39.4	247.7
Ixion	39.7	
Huya	39.8	
Varuna	42.9	
Haumea	43.3	285
Quaoar	43.6	
Makemake	45.8	310
Eris	67.7	557
1996-TL66	82.9	
Sedna	486.0	

Astronomers have detected over 500 bodies orbiting the sun well beyond the orbit of Neptune. Among these 'Trans-Neptunian Bodies (TNOs) are a growing number that rival Pluto in size. This caused astronomers to rethink how they should define the term 'planet'.

In 2006 Pluto was demoted from a planet to a dwarf planet, joining the large asteroid Ceres in that new group. Several other TNOs also joined that group, which now includes five bodies shown highlighted in the table. A number of other large objects, called Plutoids, are also listed.

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit: A) Polynomial function? B) Power-law function?

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers?

Problem 3 - Using your best-fit model, what would you predict for the periods of the TNOs in the table?

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit:

A) Polynomial function? **The N=3 polynomial $D(x) = -0.0005x^3 + 0.1239x^2 +2.24x - 1.7$**

B) Power-law function? **The N=1.5 powerlaw: $D(x) = 1.0x^{1.5}$**

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers? The two predictions are shown in the table:

Object	Distance	Period	N=3	N=1.5
Mercury	0.4	0.24	-0.79	0.25
Venus	0.7	0.61	-0.08	0.59
Earth	1	1	0.66	1.00
Mars	1.5	1.88	1.93	1.84
Ceres	2.8	4.6	5.53	4.69
Jupiter	5.2	11.9	13.22	11.86
Saturn	9.5	29.5	30.33	29.28
Uranus	19.2	84	83.44	84.13
Neptune	30.1	164.8	164.34	165.14
Pluto	39.4	247.7	248.31	247.31
Ixion	39.7		251.21	250.14
Huya	39.8		252.19	251.09
Varuna	42.9		282.94	280.99
Haumea	43.3	285	286.99	284.93
Quaoar	43.6		290.05	287.89
Makemake	45.8	310	312.75	309.95
Eris	67.7	557	562.67	557.04
1996-TL66	82.9		750.62	754.80
Sedna	486		-27044.01	10714.07

Answer: The N=3 polynomial gives negative periods for Mercury, Venus and Sedna, and poor answers for Earth, Mars, Ceres and Jupiter compared to the N=3/2 power-law fit. The N=3/2 power-law fit is the result of Kepler's Third Law for planetary motion which states that the cube of the distance is proportional to the square of the period so that when all periods and distances are scaled to Earth's orbit, Period = Distance$^{3/2}$

Problem 3 - See the table above for shaded entries

Most people have heard about meteorites, and have seen meteors streaking across the night sky. These 'rocks' travel through space at thousands of kilometers per hour and can strike any other object in their way.

Astronomers have studied these recovered meteorites for over a century and have learned about their ages, compositions and in some cases their origins in the solar system.

The above meteorite sample is called the Esquel Pallasite, and was part of a 1000 kilogram 'fall' that occurred over Esquel, Argentine in 1951. The sample has a mass of 350 grams and was sliced, polished and stained to reveal the details of its composition. Pallasites contains small stone 'inclusions' of the mineral olivine, that are embedded in an iron-nickel alloy called a matrix.

It is thought that pallesites are small pieces of rock left over from the destruction of a large asteroid, which was not large enough for the iron-nickel and stony materials to segregate. For Earth, it was massive enough for its iron and nickel to form a dense core surrounded by the lighter, stony materials.

Meteorite samples fall into several dozen different families that seem to indicate that there were several dozen ancient bodies that disintegrated during collisions to form these remnants that we can now collect when they strike Earth.

Problem 1 – The density of the iron-nickel matrix is 8.2 grams/cm^3 and the olivine density is 3.3 grams/cm^3. If the total mass of the Esquel sample is 350 grams, and 20% of the sample is composed of olivine, what was the mass of each of the two ingredients to the pallasite?

Problem 2 - Meteorite collectors find and sell samples by the gram. The price of a gram of the Esquel pallasite is about $106. What was the price of the sample in Problem 1?

Problem 1 – The density of the iron-nickel matrix is 8.2 grams/cm^3 and the olivine density is 3.3 grams/cm^3. If the total mass of the Esquel sample is 350 grams, and 20% of the sample is composed of olivine, what was the mass of each of the two ingredients to the pallasite?

Answer: This is a very typical problem that also appears in chemistry classes in various different guises.

First, what are the volumes of the two ingredients?
Olivine = 20%
and so Iron/nickel = 80%.

Now, the product of density and volume = mass, so we can write the equation for the mass as:

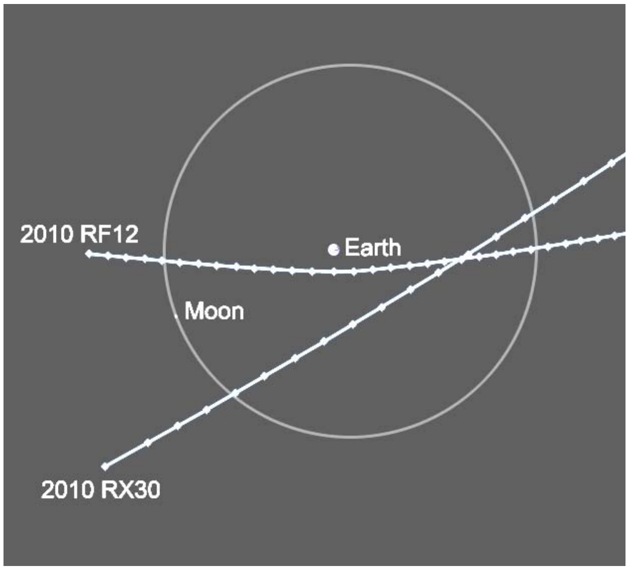
$8.2 \times 0.8V + 3.3 \times 0.2V = 350$ where V is the total volume of the sample.

Simplify and solve for V to get $(6.56 + 0.66)\ V = 350$ so $V = 48.5\ cm^3$.

Then for the **olivine** we have $3.3 \times 0.2 \times (48.5)$ = **32 grams**
And for the **iron/nickel** we have $8.2 \times 0.8 \times (48.5)$ = **318 grams**
for a total of 350 grams

Problem 2 - Meteorite collectors find and sell samples by the gram. The price of a gram of the Esquel pallasite is about $106. What was the price of the sample in Problem 1?

Answer: 350 grams x ($106 / 1 gram) = **$37,100**.



The Catalina Sky Survey near Tucson, Ariz., discovered two small asteroids on the morning of Sunday, September 5, 2010 during a routine monitoring of the skies.

Asteroid 2010RX30 is about 15 meters in diameter and will pass within 248,000 kilometers of Earth. Asteroid 2010RF12, about 10 meters in diameter, will pass within 79,000 kilometers of Earth.

Both asteroids should be observable near closest approach to Earth with moderate-sized amateur telescopes.

Neither of these asteroids has a chance of hitting Earth. A 10-meter- sized near-Earth asteroid from the undiscovered population of about 50 million would be expected to pass almost daily within a lunar distance, and one might strike Earth's atmosphere about every 10 years on average. The last asteroid that was observed to enter Earth's atmosphere in this size range was the *Great Daylight Fireball of 1972* which streaked above the Grand Tetons. It was about 5 meters in diameter but skipped out of the atmosphere and never struck ground.

Small asteroids appear very faint in the sky, not only because they are small in size, but because their surfaces are very dark and reflect very little sunlight. The formula for the brightness of a typical asteroid that is spotted within a few million kilometers of Earth is given by:

$$R = 0.011\, d\ 10^{-\frac{1}{5}(m)}$$

where: R is the asteroid radius in meters, d is the distance to the asteroid from Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The planet Venus when it is brightest in the evening sky has a magnitude of m = -2.5. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid (10 km/sec) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

Answer Key

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?
Answer:

2010RX30, R=15 meters, d = 248,000 kilometers

$15 = 0.011\ (248{,}000)10^{-.2m}$ then $0.0055 = 10^{-.2(m)}$

$\text{Log}(0.0055) = -0.2m$ so **m = +11.3 magnitudes**

2010RF12, R=10 meters, d= 79,000 kilometers

$10 = 0.011\ (79{,}000)10^{-.2m}$ then $0.011 = 10^{-.2(m)}$

$\text{Log}(0.011) = -0.2m$ so **m = +9.7 magnitudes**

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid (10 km/sec) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

Answer: First evaluate the equation for d = 1 million km and solve for m®

$R = 0.011\ (1.0\text{x}10^{6})\ 10^{-.2m}$
$R = 1.1 \times 10^{4}\ \ 10^{-.2m}$

so $m(R) = -5 \log10(0.000091R)$

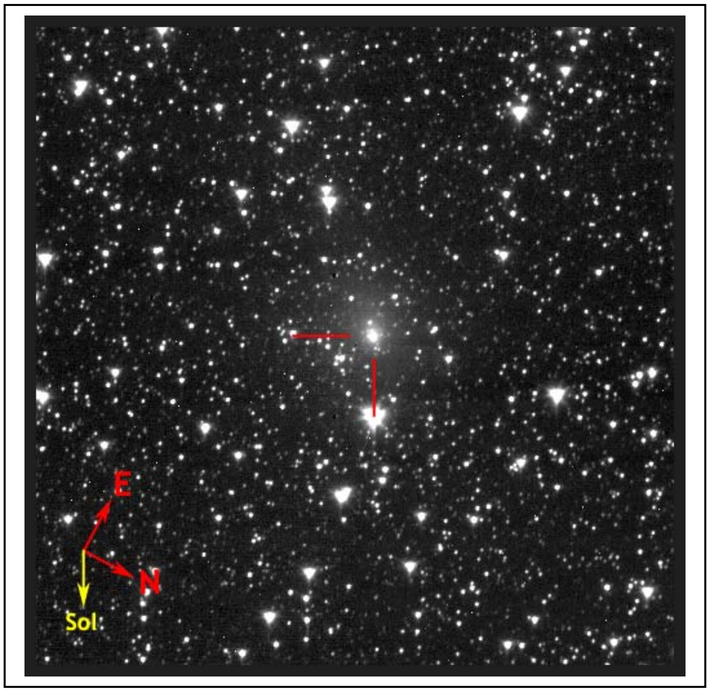
For R = 1 to 500 meters, **m = +20.2 to +6.7**

The most common asteroids have sizes between 1 meter and 50 meters, so the detection of such small, faint, and rapidly moving asteroids with ground based telescopes is a major challenge and may be a matter of luck in most cases.

For more information, read the NASA press release at

"Two Asteroids to Pass By Earth Wednesday"

http://www.nasa.gov/topics/solarsystem/features/asteroid20100907.html



This image was taken by the Deep Impact spacecraft on September 20 as it approaches Comet Hartley 2 from a distance of 46 million kilometers.

On November 4, the spacecraft will approach to within 700 kilometers of the surface of the comet.

The image area is 0.29 degrees on a side, and the nucleus of the comet is about 1 kilometer in diameter. The comet can be seen as the fuzzy spot near the center.

Whenever you look at a distant object, you see the object's angular size not its actual physical size. There is a simple formula that relates the actual size and distance to an object, to its apparent angular size:

$$Tan\left(\frac{\theta}{2}\right)=\frac{d}{2R}$$

where d is the object's width or diameter in kilometers, and R is its distance from the obeserver in kilometers. For example, a DVD disk (d=12 cm) held at arms-length (R=24 cm) will subtend an angle of θ = 28 degrees.

Problem 1 - Deep Impact will pass to within 700 km of the nucleus. How big will the comet nucleus appear to the Deep Impact spacecraft if its diameter is 1 kilometer?

Problem 2 - The Deep Impact High-Resolution Imager (HRI) has a format of 1024 x 1024 pixels and a field of view of 0.118 degrees. A single pixel sees an angular field of 0.118 deg/1024 pix = 0.000115 degrees. At a distance of 700 km, what linear distance will a pixel resolution of 0.000115 degrees represent?

Problem 3 - How many pixels across will the comet nucleus appear in the image taken at closest approach with the HRI?

Problem 1 - Deep Impact will pass to within 700 km of the nucleus. How big will the comet nucleus appear to the Deep Impact spacecraft if its diameter is 1 kilometer?

Answer: d = 1 kilometer, and R = 700 km, so solving for θ in the formula we have

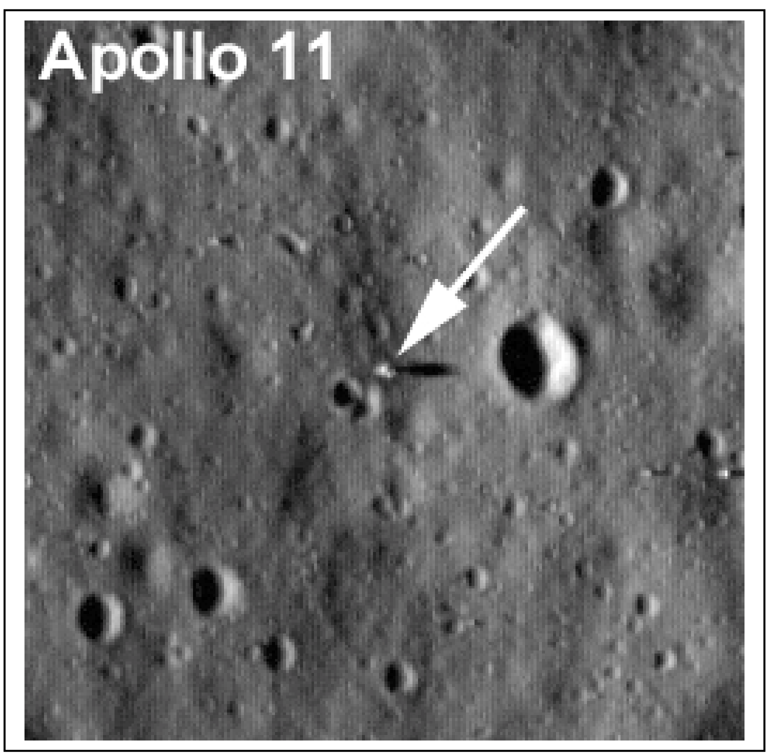
$Tan(\theta / 2) = \frac{d}{2R}$ so Tan $(\theta/2)$ = 1 kilometer / (2 x 700), and so θ = **0.08 degrees**

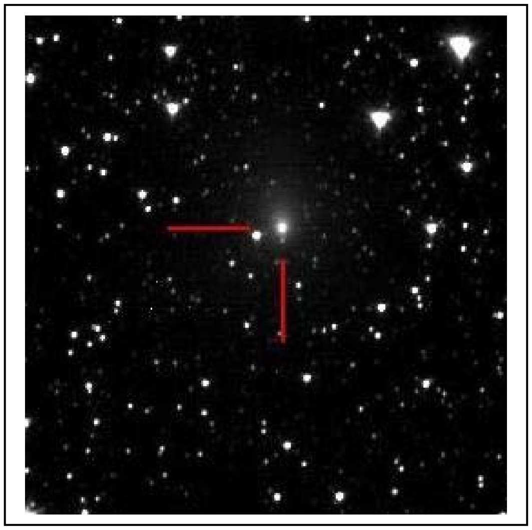
Problem 2 - The Deep Impact High-Resolution Imager (HRI) has a format of 1024 x 1024 pixels and a field of view of 0.118 degrees. A single pixel sees an angular field of 0.118 deg/1024 pix = 0.000115 degrees. At a distance of 700 km, what linear distance will a pixel resolution of 0.000115 degrees represent?

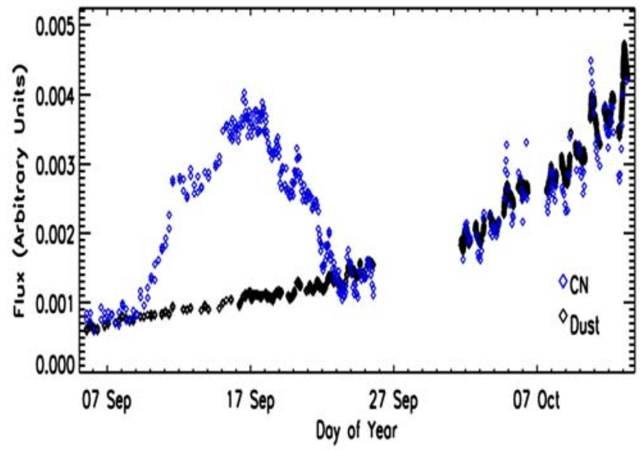
Answer: Tan(0.000115) = L/700 km so
L = 700 Tan(0.000115) km **L = 1.4 meters/pixel**.

Problem 3 - How many pixels across will the comet nucleus appear in the image taken at closest approach with the HRI?

Answer: The resolution of the imager is 1.4 meters/pixel. The diameter of the nucleus is believed to be about the 1 kilometer, so the comet nucleus will subtend about 1,000 meters x (1 pixel/1.4 meters) = **714 pixels**. Below is an image taken by the Lunar Reconnaissance Orbiter at a resolution of 1 meter showing the Apollo 11 landing area on the moon. A similar-resolution image will be possible for Comet Hartley 2 using HRI!







On September 25, 2010 at a distance of 41 million km, the spacecraft Deep Impact took this image of the comet Hartley 2 about 40 days before closest approach on November 4. The graph to the right indicates the brightness of the comet as measured by the spacecraft between 5 September and 13 October. There are two curves plotted, one that depicts dust (black) released from the nucleus and one taken with the cyanogen-gas filter, sensitive to both the dust continuum and CN gas (blue), The gap in data from 25 September through 1 October was due to a scheduled break in the observations for calibration observations.

Problem 1 - From the graph, A) how long did the ejection burst of cyanogen gas last, B) on what date did it reach its maximum brightness, and C) in terms of its flux, by what factor did the burst increase the brightness of the comet compared to its dust intensity?

Problem 2 - The distance to the comet is given by the table below:

Date	Day Number	Distance (km)
September 5	0	60 million
September 20	15	46 million
September 25	20	41 million
September 29	24	37 million
October 27	52	9 million

Write a linear equation that gives the distance to the comet in terms of the day number. What is the average speed of the comet in millions of km/day?

Problem 3 - Assume that the brightness of the comet follows the inverse-square law so that if you halve the distance to the comet, its brightness will be 4-times greater. If the comet's brightness was B = 0.0006 units on September 5, and the physical conditions in the comet did not change, what would you predict its brightness would be on October 13?

Problem 1 - From the graph, A) how long did the ejection burst of cyanogen gas last, B) on what date did it reach its maximum brightness, and C) in terms of its flux, by what factor did the burst increase the brightness of the comet compared to its dust intensity?

Answer: A) The ejection lasted between September 9 and 23 for a total of **14 days**.
B) The maximum brightness occurred on about **September 17th**.
C) The dust brightness on Sept 17 was 0.0012 .The total brightness was 0.0039, so the cyanogen outburst increased the comet's brightness by a factor of 0.0039/0.0012 = **3.25 times**.

Problem 2 - The distance to the comet is given by the table below:

Date	Day Number	Distance (km)
September 5	0	60 million
September 20	15	46 million
September 25	20	41 million
September 29	24	37 million
October 27	52	9 million

Write a linear equation that gives the distance to the comet in terms of the day number. What is the average speed of the comet in millions of km/day?

Answer: From the tabulated data, the slope over the time interval is m = (9-60)/(52) = -1.0, so the linear equation would be **D(t) = -1.0(T) + 60** where D(t) is in millions of kilometers and T is the number of days since September 5. The average approach speed is **1 million km/day**.

Problem 3 - Assume that the brightness of the comet follows the inverse-square law so that if you half the distance to the comet, its brightness will be 4-times greater. If the comet's brightness was B = 0.0006 units on September 5, and the physical conditions in the comet did not change, what would you predict its brightness would be on October 13?

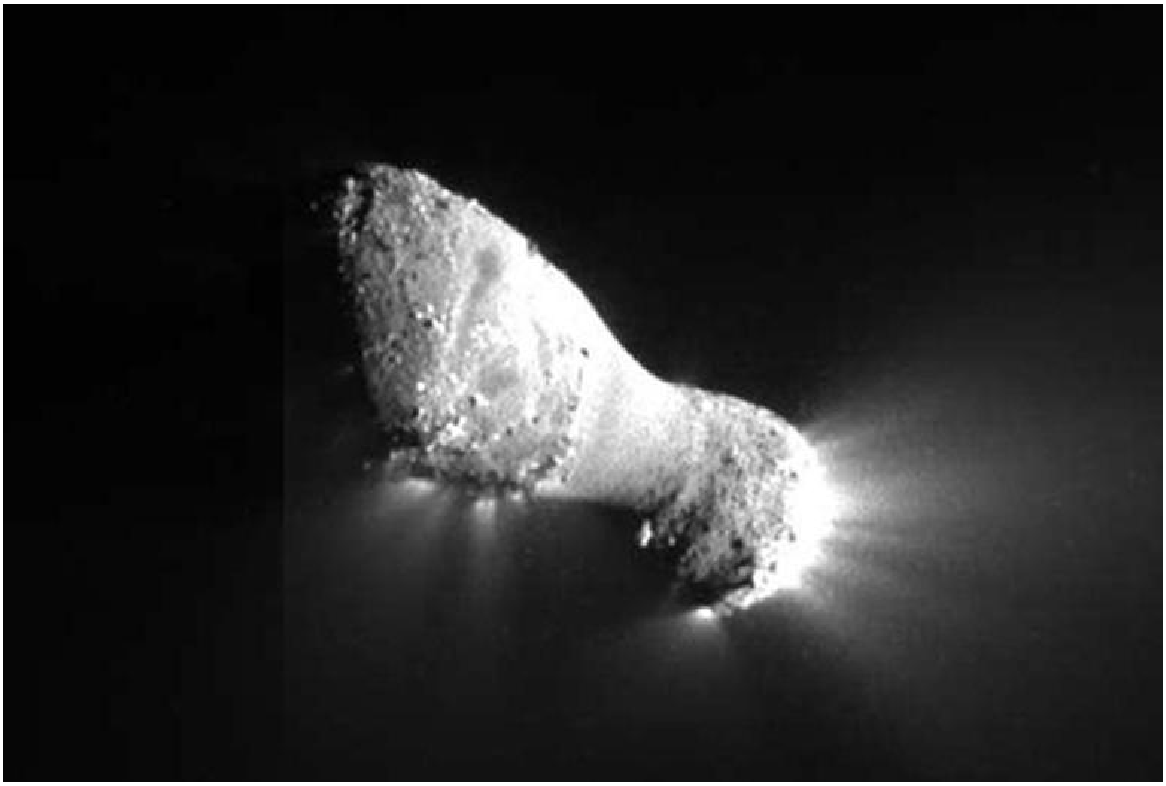
Answer: October 13 occurs at T = 38 days, so its distance is D = -1.0(38) + 60 = 22 million km. On September 5 its distance was 60 million km, so the factor of its distance change is 38/60 = 0.63 .The comet's brightness from the inverse-square law is just

$$B = 0.0006\frac{1}{(0.63)^2}$$

so the brightness on October 13 is just **B=0.0015 flux units**.

Note: Students will notice from the light curve graph that the actual comet brightness on October 13 was B = 0.0042 units because the comet ejected a cloud of dust, which increased its brightness significantly - by 0.0042/0.0015 = 2.8 times its normal brightness without the dust and cyanogen eruption.

Estimating the Volume and Mass of Comet Hartley-2



Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The scale of this image is 25 meters/millimeter. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Assume that the volume of the nucleus can be approximated by a dumbbell-shaped model consisting of two spherical 'end-caps' connected by a cylindrical bar. To two significant figures, about what is the total volume of the nucleus in cubic meters? (Note: the spheres need not have the same diameters)

Problem 2 - The densities of only a few cometary nuclei have been determined from their mass and volume: Halley's Comet (0.6 gm/cm^3); Comet Tempel-1 (0.62 gm/cm^3); Comet Borrelly (0.3 gm/cm^3) and Comet Wild (0.6 gm/cm^3). The low density indicates large quantities of water and other ices make up the composition of these bodies. Assuming that the density of Comet Hartley-2 is similar to the median density of these comets, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Problem 1 - Assume that the volume of the nucleus can be approximated by a dumbbell-shaped model consisting of two spherical 'end-caps' connected by a cylindrical bar. To two significant figures, about what is the total volume of the nucleus in cubic meters? (Note: the spheres need not have the same diameters)

Answer: Although it appears that the nucleus was viewed at an oblique angle and not face-on, we will not include this perspective effect in the size estimates. Students may attempt to make this correction by, for example, assuming that the end-cap spheres are of equal diameter, with a diameter that is the average of the widths of the two ends of the nucleus.
Using a millimeter ruler, the diameter of the left end-cap is about 40mm and the right endcap is about 30 mm, so at a scale of 25 meters/mm, the actual diameters are 1,000 meters and 750 meters respectively. The diameter of the cylindrical bar is about 20 mm or 500 meters. The length of the cylinder is 2000 meters - 1000 meters - 750 meters = 250 meters based on the assumed length of 2 km and subtracting the two spheres.
The volume of a sphere is $4/3\ \pi\ R^3$ and the volume of a cylinder is $\pi\ R^2\ h$ so we have
Right endcap $V = 1.33\ (3.14)\ (750/2)^3 = 2.2 \times 10^8$ meters^3
Left endcap $V = 1.33\ (3.14)\ (1000/2)^3 = 5.2 \times 10^8$ meters^3
Cylinder $V = 3.14\ (250)^2\ (250) = 4.9 \times 10^7$ meters^3

So the total volume using this geometric model is just V = **7.9×10^8 meters^3**

Problem 2 - The densities of only a few cometary nuclei have been determined from their mass and volume: Halley's Comet (0.6 gm/cm^3); Comet Tempel-1 (0.62 gm/cm^3); Comet Borrelly (0.3 gm/cm^3) and Comet Wild (0.6 gm/cm^3). The low density indicates large quantities of water and other ices make up the composition of these bodies. Assuming that the density of Comet Hartley-2 is similar to the median density of these comets, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Answer: The median density is **0.6 gm/cm^3.**
Mass = Density x Volume

First convert the volume to cubic centimeters from cubic meters:

$V = 7.9 \times 10^8$ meters^3 x (100 cm/1 meter)^3 = 7.9×10^{14} cm^3.

Then, Mass = 0.6 gm/cm^3 x 7.9×10^{14} cm^3
= 4.7×10^{14} gm

Convert grams to megatons:

Mass = 4.7×10^{14} gm x (1 kg/1000 gm) x (1 ton/1000 kg) = 4.7×10^8 tons or **470 megatons**.

As a comet orbits the sun, it produces a long tail stretching millions of kilometers through space. The tail is produced by heated gases leaving the nucleus of the comet.

This image of the head of Comet Tempel-1 was taken by the Hubble Space Telescope on June 30, 2005. It shows the 'coma' formed by these escaping gases about 5 days before its closest approach to the sun (perihelion). The most interesting of these ingredients is ordinary water.

Problem 1 – The NASA spacecraft Deep Impact flew-by Temple-1 and measured the rate of loss of water from its nucleus. The simple quadratic function below gives the number of tons of water produced every minute, W, as Comet Tempel-1 orbited the sun, where T is the number of days since its closest approach to the sun, called perihelion.

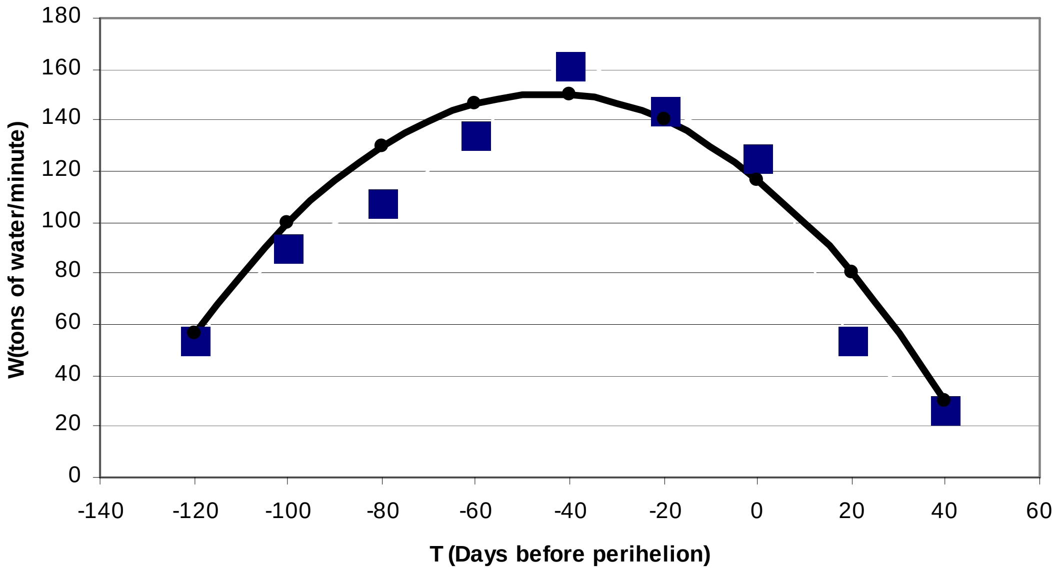
$$W(T) = \frac{(T+140)(60-T)}{60}$$

A) Graph the function W(T). B) For what days, T, will the water loss be zero? C) For what T did the comet eject its maximum amount of water each minute?

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Problem 1 – A) The graph below was created with Excel. The squares represent the actual measured data and are shown as an indicator of the quality of the quadratic model fit to the actual data.



Answer: B) The roots of the quadratic equation, where W(T)=0 are for **T=-140** days and **T=+60** days after perihelion. C) The maximum (vertex of the parabola) occurs half-way between the two intercepts at T = (-140+60)/2 or **T = -40** which indicates 40 days before perihelion.

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Answer: W(-130) = (-130+140)(60+130)/60 = **32 tons/minute**

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Answer: The fitting function W(T) predicts that W(+70) = (70+140)(60-70)/60 = -35 tons per minute. Although this value smoothly follows the prediction curve, it implies that instead of ejecting water (positive answer means a positive rate of change) the comet is absorbing water (negative answer means a negative rate of change), so the prediction is not realistic.

The image of Mercury's surface on the left was taken by the MESSENGER spacecraft on March 30, 2011 of the region near crater Camoes near Mercury's south pole. In an historic event, the spacecraft became the first artificial satellite of Mercury on March 17, 2011. The image on the right is a similar-sized area of our own Moon near the crater King, photographed by Apollo 16 astronauts.

The Mercury image is 100 km wide and the lunar image is 115 km wide.

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Answer: Mercury image width is 80 mm, scale is 100 km/80mm = **1.3 km/mm**
Moon image width = 68 mm, scale is 115 km/68mm = **1.7 km/mm**

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Answer: Mercury = 0.5 mm x (1.3 km/mm) = **700 meters**.
Moon = 1.0 mm x (1.7 km/mm) = **1,700 meters**.

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Answer: **On Mercury, less of the material ejected by the impact gets away, and so more of it falls back to the surface near the crater. For the Moon, the escape velocity is so low that ejected material can travel great distances, or even into orbit and beyond, so less of it falls back to the surface to make additional craters**.

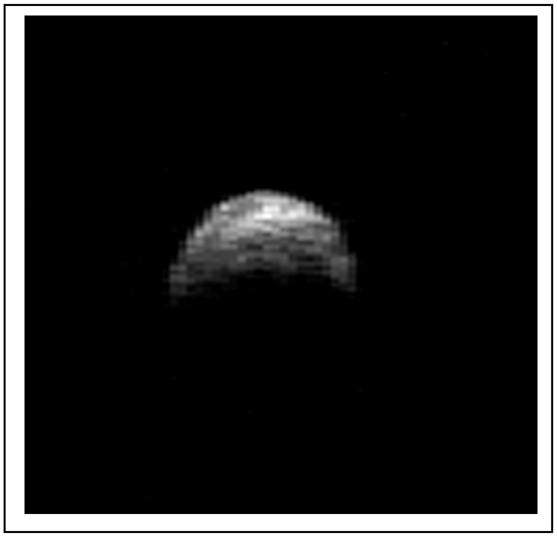
Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Answer: The volume of a sphere is given by $4/3\ \pi\ R^3$, so if you increase the radius of a sphere by a factor of 1.4, you will increase its volume by a factor of 1.4^3 = 2.7 times, so the volume of Mercury is **2.7 times** larger than the volume of the Moon.

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Answer: The density of Mercury is a factor of 5400/3400 = 1.6 times that of the Moon. Since the volume of Mercury is 2.7 times larger than the Moon, the mass of Mercury will be density x volume or 1.6 x 2.7 = **4.3 times than of the Moon**.

Note: Actual masses for Mercury and the Moon are 3.3×10^{23} kg and 7.3×10^{22} kg respectively, so that numerically, Mercury is 4.5 times the Moon's mass...which is close to our average estimate.



NASA scientists will be tracking asteroid 2005 YU55 with antennas of the agency's Deep Space Network at Goldstone, Calif., as the space rock safely flies past Earth slightly closer than the moon's orbit on Nov. 8.

Scientists are treating the flyby of the 1,300-foot-wide (400-meter) asteroid as a science target of opportunity – allowing instruments on "spacecraft Earth" to scan it during the close pass.

The image above was made at radio wavelengths by the Arecibo Radio Telescope in Puerto Rico in April, 2011. (Credit NASA/Cornell/Arecibo). The trajectory of asteroid 2005 YU55 is well understood.

On a standard Cartesian grid with Earth at the Origin, the Asteroid is predicted to be located at Point A (-247, -543) on November 8.438, and at Point B (+506, +1360 on November 9.438. The Moons orbit is represented by a circle with a radius of 346. All units are in thousands of kilometers so '346' is 346,000 kilometers.

Problem 1 - What is the slope-intercept form of the linear equation that represents the trajectory of the Asteroid between November 8 and November 9?

Problem 2 - What is the equation representing the orbit of the moon?

Problem 3 - What are the coordinates of the points that represent the intersection of the asteroid's trajectory and the lunar orbit?

Problem 4 - Over how many hours will the asteroid be inside the orbit of the moon if the asteroid was traveling at a speed of 41,700 km/hr?

Problem 5 - If the closest distance to Earth occurs when the asteroid is at the mid-point of its time inside the orbit of the Moon, A) about when does this happen and B) what is the distance to Earth at that time?

Problem 1 - Answer: Start with the two-point formula and simplify to the slop-intercept form.

$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$ so $y + 543 = \frac{(136 + 543)}{(506 + 247)}(x + 247)$ and so **y = 0.902x - 320**

Problem 2 - Answer: r = 346 so $x^2 + y^2 = (346)^2$

Problem 3 - Answer: Eliminate y in the equation for the lunar orbit by substituting the linear equation formula. Then solve the quadratic equation for the intersection points.

$x^2 + (0.902x - 320)^2 = (346)^2$ so $1.814x^2 - 577x - 17316 = 0$

use the quadratic formula with a = 1.814, b=-577 and c = -17316 to get:

$x = \frac{577 \pm \sqrt{(577^2 - 4(1.814)(-17316))}}{2(1.814)}$ so x = 159 +/- 187 x1 = +346 and x2 = -28

Find the corresponding y coordinates
Y1 = 0.902(346) - 320 = -8.0 so **First Point = (+346,-8.0)**
Y2 = 0.902(-28) - 320 = -345. so **Second Point = (-28, -345)**

Note, if you use the equation for the circle, you get y1 = 0.0 and y2 = -345. so **First Point (+346,0.0)** and **Second Point (-28, -345)**. The differences can be attributed to rounding error.

Problem 4 - Answer: Using the linear equation result, the distance between the First and Second Points is

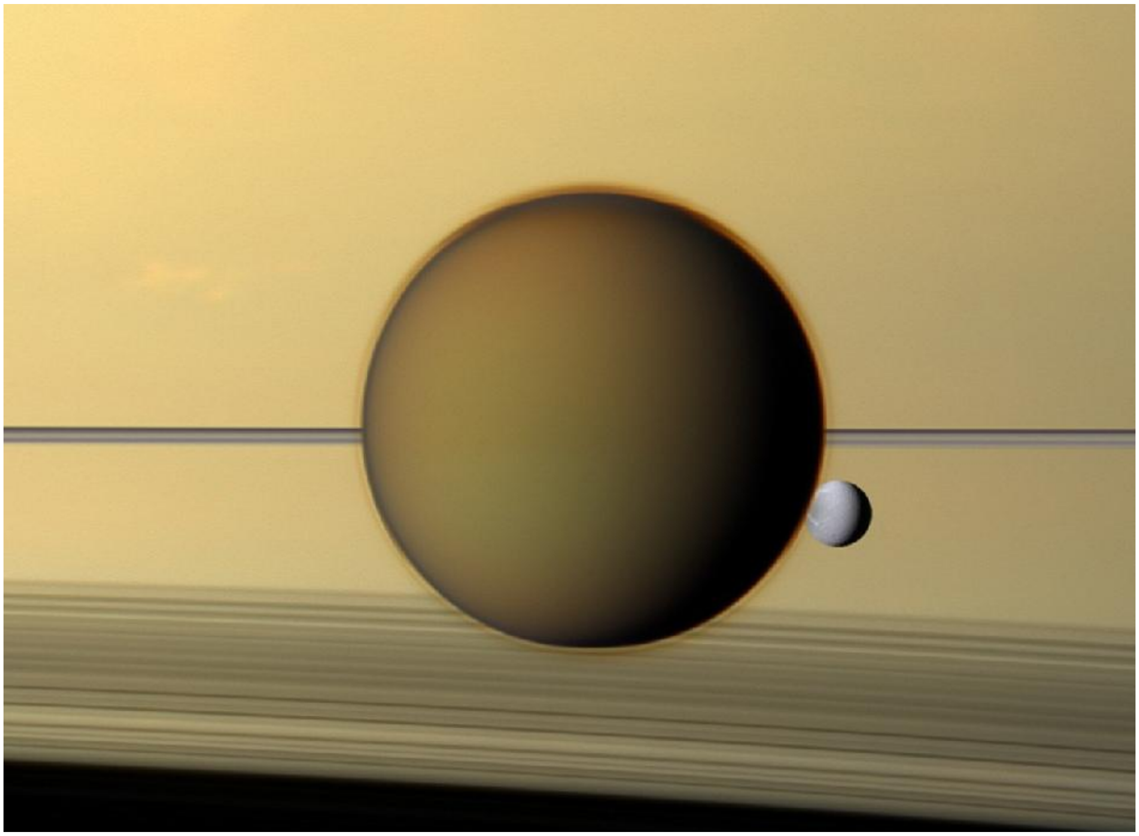
$d = \sqrt{(-28 - 346)^2 + (-345 + 8)^2}$ so d = 503 units or 503,000 km. At a speed of 41,700 km/h, the asteroid travels this distance in about **12.1 hours**.

Problem 5 - If the closest distance to Earth occurs when the asteroid is at the mid-point of its time inside the orbit of the Moon, A) about when does this happen and B) what is the distance to Earth at that time?
Answer: T(mid) = (12.1)/2 = 6.05 hours.
Distance from Point A (-247, -543) to the Second Point at (-28, -345) is

$d = \sqrt{(-28 + 247)^2 + (-345 + 543)^2}$ so d = 295 or 295,000 km. At a speed of 41,700 km/h this time interval is just 7.07 hours. Adding the two times together we get 5.25+7.07 = 12.32 hours or 0.51 days after Point A. Since Point A occurs on November 8.438d we add 0.51d to this and get
A) **November 8.95 or November 8 at 22:48 Universal Time**.

B) The point that is half way between the
First point (+346,-8.0) and the Second point (-28, -345) is at
x=(346-28)/2= +159, and y = (-345 - 8)/2 = -177.
This point is located at a distance of $d = \sqrt{(159)^2 + (177)^2}$ = **238 units or 238,000 km from Earth**.

Note: The actual answer is 324,600 km when more accurate modeling is performed in 3-dimensions. The asteroid is off-set by an additional +221,000 km in the 'Z' direction.



Saturn's third-largest moon Dione can be seen through the haze of its largest moon, Titan, in this view of the two posing before the planet and its rings from NASA's Cassini spacecraft. This view looks toward Titan (about 5000 kilometers across) with Saturn in the background, and the smaller moon Dione as it is abpout to be eclipsed by Titan. The images were obtained with the Cassini spacecraft narrow-angle camera on May 21, 2011 at a distance of approximately 1.4 million miles (2.3 million kilometers) from Titan.

Problem 1 – From the image above, what is the ratio of the apparent diameter of Dione to the diameter of Titan rounded to the nearest, simple fraction (example: 0.34 = 1/3)?

Problem 2 - Suppose that Dione were exactly the same diameter, in kilometers, as Titan. From the vantage point of Cassini, about what would be the distance to Dione in kilometers?

Problem 3 – The actual diameter of Dione is known to be about 1000 kilometers. What is its actual distance from Cassini at the time this image was taken? Explain your reasoning.

. Press Release: **NASA's Cassini Delivers Holiday Treats From Saturn**

December 22, 2011

http://www.nasa.gov/mission_pages/cassini/whycassini/cassini20111222.html

Problem 1 – From the image above, what is the ratio of the apparent diameter of Dione to the diameter of Titan rounded to the nearest, simple fraction (example: 0.34 = 1/3)?

Answer: Use a millimeter ruler to measure the apparent diameters of the two moons. Titan is about 60 millimeters and Dione is about 9 millimeters in diameter. The ratio is 9/60 or 3/20. Students should use **1/7** as 'simplist fraction' approximation.

Problem 2 - Suppose the Dione were exactly the same diameter, in kilometers, as Titan. From the vantage point of Cassini, about what would be the distance to Dione in kilometers?

Answer: To appear as large as it seems to Cassini, Titan is at a distance of 2.3 million kilometers. If Dione were the same physical diameter as Titan (5000 kilometers) it would have to be 7 times farther from Cassini in order for the apparent diameters to be in the ration 1/7. The distance to Dione is therefore 7 x 2.3 million km or **16 million km**.

Problem 3 – The actual diameter of Dione is known to be about 1000 kilometers. What is its actual distance from Cassini at the time this image was taken? Explain your reasoning.

Answer: If Titan and Dione were of comparable physical size, Dione would have to be 16 million km from Cassini in order for Dione to appear to be 1/7 the diameter of Titan.

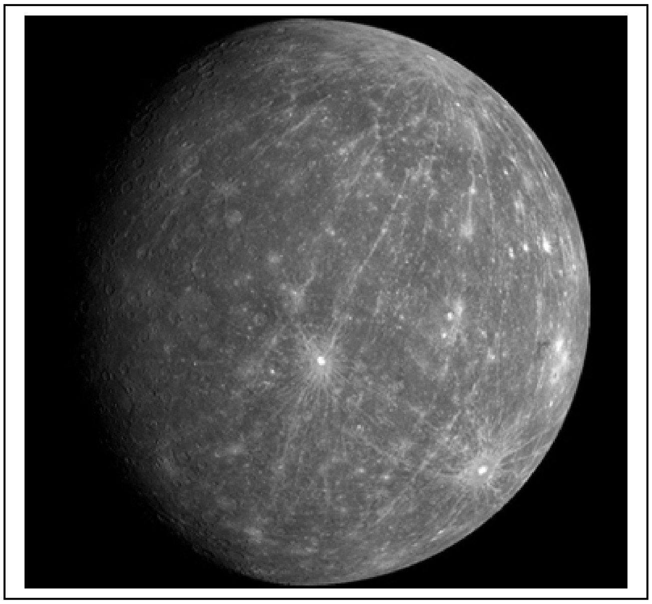
But, Dione's physical diameter compared to Titan is 1000 km compared to 5000 km for Titan, to their actual diameter ratio when seen at the same distance from Cassini is 1000/5000 = 1/5. If Dione and Titan were sitting next to each other, the ratio of their physical diameters alone would make the disk of Dione appear to be 1/5 the diameter of Titan or 12 millimeters instead of the 9 millimeters we measured.

To make the apparent ratio equal the slightly smaller ratio of 1/7 at a distance of 2.3 million km, we have to move Dione slightly further away from Cassini than Titan by an amount equal to the proportion:

$$\frac{(1/7)}{2.3million} = \frac{(1/5)}{X}$$ so x = 2.3 million x (1/5)/(1/7) = 2.3 million x 7/5 = **3.2 million km**

Note: even though Titan and Dione <u>appear</u> close together in the picture, they are actually over (3.2-2.3) million = 900,000 kilometers apart...about 3 times the distance from Earth to the Moon!

MESSENGER Explores the Mass of Mercury



According to Kepler's Third Law, the time it takes a satellite to go once around its planet is given by the formula

$$\frac{R^3}{T^2} = 1.69x10^{-12} M$$

where M is the mass of the planet in kilograms, T is the orbit period in seconds, and R is the radius of the orbit in meters.

For example, the International Space Station orbits Earth with R = 6738 km, T= 92.5 minutes so the mass of Earth is just M = 5.9 x10^{24} kg.

The MESSENGER spacecraft orbits the planet Mercury, but has changed its orbit several times since it arrived in April 2011. The following problems explore how these orbit changes affect the estimate for the mass of Mercury using the Kepler formula.

Problem 1 - On April 25, 2011 the orbit period of MESSENGER was 12 hours and 2 minutes, and its distance was 10,124 km from the center of Mercury. To three significant figures, what was the estimated mass of Mercury?

Problem 2 - On September 14, 2011 the orbit was changed to a distance of 10,085 kilometers and a period of 11hours 58 minutes. To three significant figures, what was the mass of Mercury?

Problem 3 - On May 25, 2012 the final orbit had a period of 8.0 hours and a distance of 7,715 kilometers. To three significant figures, what was the mass of Mercury?

Problem 4 - Explain what the formula is telling us about the properties of the orbit of a satellite and the mass of the body?

Problem 1 - On April 25, 2011 the orbit period of MESSENGER was 12 hours 2 minutes and its distance was 10,124 km from the center of Mercury. To three significant figures, what was the estimated mass of Mercury?

Answer: We first need to convert R and T to meters and seconds so that R = 10,124,000 meters and T = 12h x (3600 s/1hr) +120 sec = 43,320 seconds. Then from the formula:

$$M = 5.92x10^{11}\frac{(10124000)^3}{(43320)^2} = \mathbf{3.27 \times 10^{23}\ kg}$$

Problem 2 - On September 14, 2011 the orbit was changed to a distance of 10,085 kilometers and a period of 11 hours 58 minutes. To three significant figures, what was the mass of Mercury?

Answer: R = 10,085,000 meters and T = 43,080 seconds and so

$$M = 5.92x10^{11}\frac{(10085000)^3}{(43080)^2} = \mathbf{3.27 \times 10^{23}\ kg}$$

Problem 3 - On May 25, 2012 the final orbit had a period of 8.0 hours and a distance of 7715 kilometers. To three significant figures, what was the mass of Mercury?

Answer: R = 7715000 meters and T = 28,800 seconds and so

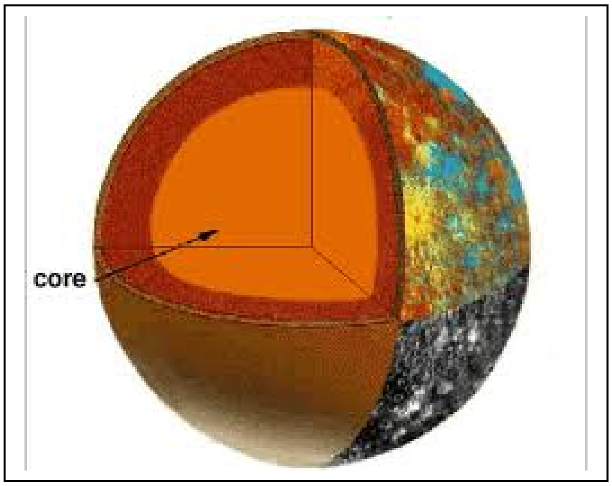
$$M = 5.92x10^{11}\frac{(7715000)^3}{(28800)^2} = \mathbf{3.27 \times 10^{23}\ kg}.$$

Problem 4 - Explain what the formula is telling us about the properties of the orbit of a satellite and the mass of the body?

Answer: Although the orbit properties change, the mass of Mercury remains the same because the spacecraft is orbiting the same body and the values for R ,T and M must lead to consistent solutions.

Note: From more careful orbital studies, astronomers use the adopted mass of Mercury of 3.31×10^{23} kg.

MESSENGER Explores the Interior of Mercury



This MESSENGER spacecraft is in orbit around the planet Mercury, with an elliptical orbit period of 8 hours at an average distance of 7715 kilometers. At its closest distance it is only 320 km above the surface of Mercury, which allows the spacecraft to obtain high-resolution images of the mysterious surface of Mercury.

A detailed study of the spacecrafts speed and altitude also lets astronomers study the gravity field of Mercury and deduce something about its interior.

We can create a simple model of the interior of Mercury by dividing it into a spherical core region, and an overlying shell of matter that reaches to the observed surface of the planet. Here is how we do this!

Problem 1 - The mass of Mercury is 3.31×10^{23} kilograms. If the planet is a perfect sphere with a radius of 2,425 kilometers, what is the average density of the planet Mercury in $kg/meter^3$ defined as density = mass/volume?

Problem 2 - Astronomers believe that the crust of the planet has an average density of 3,000 kg/m^3 and the iron-rich core has a density of 7,800 kg/m^3.

A) What is the formula that gives the total mass of the core, if the core has a radius of Rc in meters?

B) What is the formula for the outer shell of Mercury if the density equals the density of the crust, and its inner radius is Rc and its outer radius is the actual radius of the planet of 2,425 kilometers?

Problem 3 - If the sum of the core and shell masses must equal the mass of the planet, what is the value for Rc, the radius of the core in kilometers, that leads to a solution for this simple model?

Problem 1 - The mass of Mercury is 3.31 x 10^{23} kilograms. If the planet is a perfect sphere with a radius of 2,425 kilometers, what is the average density of the planet Mercury in kg/meter^3 defined as density = mass/volume?

Answer: Volume = $\frac{4}{3}\pi R^3$

so Volume = $\frac{4}{3}(3.141)(2,425,000)^3$ = 5.97 x 10^{19} meters^3

Density = 3.31 x 10^{23} kilograms / 5.97 x 10^{19} meters^3 = **5542 kg/m^3**.

Problem 2 - Astronomers believe that the crust of the planet has an average density of 3,000 kg/m^3 and the iron-rich core has a density of 7,800 kg/m^3. A) What is the formula that gives the total mass of the core, if the core has a radius of Rc in kilometers? B) What is the formula for the outer shell of Mercury if the density equals the density of the crust, and its inner radius is Rc and its outer radius is the actual radius of the planet of 2,425 kilometers?

Answer A) M(core) = $\frac{4}{3}\pi(1000Rc)^3(7800)$ = 3.26x10^{13}Rc^3 kilograms

B) Volume of the spherical shell = $\frac{4}{3}\pi(2,425,000)^3 - \frac{4}{3}\pi(1000Rc)^3$ cubic meters

Then M(shell) = 3,000 x V(shell)

$= 2,200\left(\frac{4}{3}\pi(2,425,000)^3 - \frac{4}{3}\pi(1000Rc)^3\right)$ kilograms

$= 1.3x10^{23} - 9.21x10^{12}Rc^3$ kilograms

Problem 3 - If the sum of the core and shell masses must equal the mass of the planet, what is the value for Rc, the radius of the core in kilometers, that leads to a solution for this simple model?

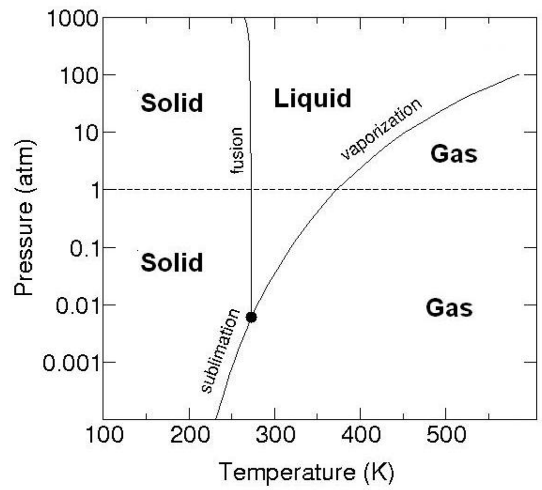
Answer: 3.31 x 10^{23} = 3.26x10^{13}Rc^3 + 1.3x10^{23} - 9.21x10^{12} Rc^3

So 2.01x10^{23} = 2.34x10^{13} Rc^3

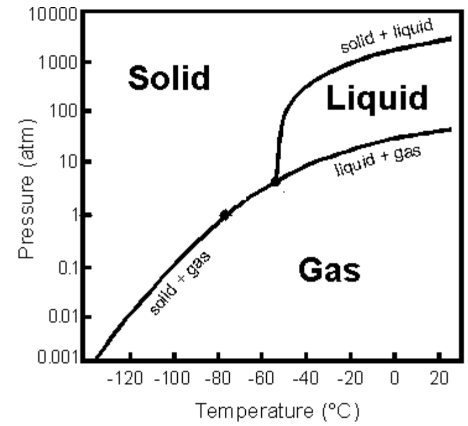
And so Rc = **2,046 kilometers!**

So the dense iron core of Mercury occupies 100%x(2046/2425) = 84% of the radius of Mercury! By comparison, Earth's core is only 30% of its radius.

What is a Snowball's Chance on Mars?



The H_2O Phase Diagram



The CO_2 Phase Diagram

These diagrams above are called **phase diagrams**. The one to the left shows all of the phases for matter for water as you change the temperature and pressure of the water in your sample. A pressure of 1.0 'atmospheres' is what we experience at sea level. This equals 14 pounds/inch^2 (or in metric units about 100 kiloPascals). As you move horizontally across the diagram towards increasing temperatures (measured in Kelvin units) at a constant pressure of 1.0 atm, the state of your water will change from solid ice, to liquid water at 273 Kelvin, to water vapor at 373 Kelvin.

Snow balls require that you create some liquid water by compressing the snow crystals so that they can glue together as the water refreezes. This will happen along the curve marked 'fusion' which is the boundary between the solid ice and liquid water phases.

The diagram to the right shows all of the phases for carbon dioxide as you change its pressure and temperature. For convenience we use the Celsius temperature scale. Note that 0^o Celsius = +273 on the Kelvin scale, and that a difference of 1^o C equals a change by 1 K on the Kelvin scale.

Problem 1 - We can make snow balls because the pressure (close to 1.0 atm) we apply with our hands at the ambient temperature (close to 273 K) is just enough to melt the ice into water and refreeze it to form a glue holding the snowflakes together. The temperature in Antarctica is typically 250 K. Can you make snowballs in Antarctica with normal hand pressure?

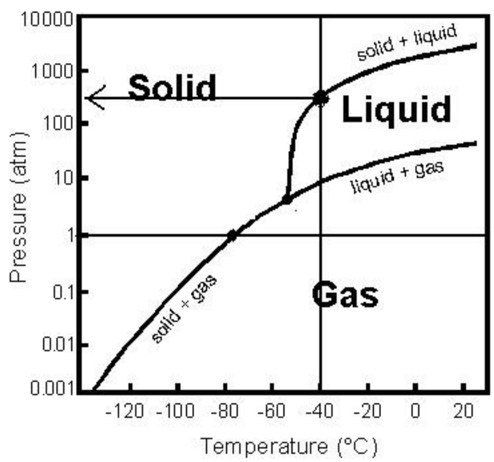
Problem 2 - On Mars, the majority of the ice is carbon dioxide ice. To make a carbon dioxide snowball, imagine applying 1 atm of hand pressure. The average temperature where the carbon dioxide snow falls is about -40^o Celsius in the daytime. Use the phase diagrams to explain why making a showball on Mars may be difficult or easy?

Problem 1 - We can make snow balls because the pressure (close to 1.0 atm) we apply at the ambient temperature (close to 273 K) is just enough to melt the ice into water and refreeze it to form a glue holding the snowflakes together. The temperature in Antarctica is typically 250 K. Can you make snowballs in Antarctica with normal hand pressure?

Answer - At normal winter temperatures near 273 K (0^oC) and 1 atm, the diagram shows that we are very, very close to the conditions needed to make solid ice turn to liquid water with a bit of extra pressure. The vertical line, which represents the ice to liquid transition crosses a pressure of 1 atm at a temperature of just +0.010 C! The diagram also shows that at 250 K (-17^oC) we are <u>far to the left</u> of the vertical line where solid ice can turn to liquid. At this temperature, we will need hand pressures higher that 1000 atm to make snowballs! So you can not make snowballs in Antarctica.

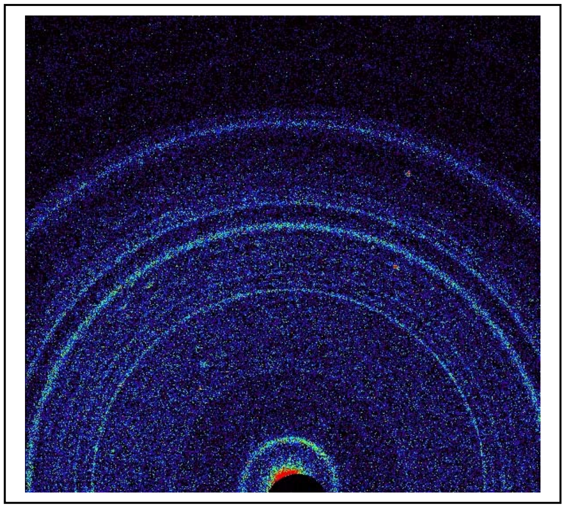
Problem 2 - On Mars, the majority of the ice is carbon dioxide ice. To make a carbon dioxide snowball, imagine applying 1 atm of hand pressure. The average temperature where the carbon dioxide snow falls is about -40^{o} Celsius in the daytime. Use the phase diagrams to explain why making a showball on Mars may be difficult or easy?

Answer: The main ingredient for a snowball on Mars would be carbon dioxide. The figure below shows the horizontal line representing a hand pressure of 1.0 atm and the vertical line representing the temperature of -40^o C on Mars where snowfall might occur.

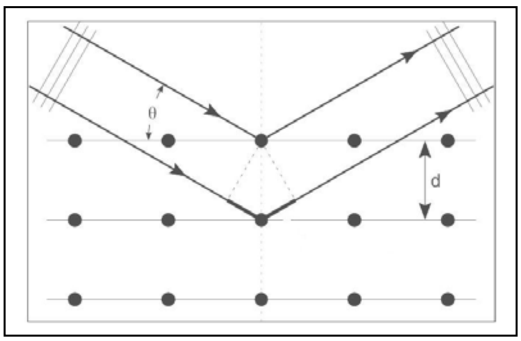


The temperature is -40 C, so draw a vertical line on the CO_2 phase diagram until it intersects the solid+liquid line. This is where CO_2 can be in both the solid and liquid phases. You need the liquid CO_2 to supply the glue to hold the solid snowflakes together in the CO_2 snowball. But if you draw a horizontal line to the left, you will see that for you to get this liquid+solid phase at this temperature, you need a pressure of over 100 atmospheres. That about 1500 pounds per square inch of hand pressure, which will be impossible for you to achieve!

This diagram also shows that, on Earth, where the atmospheric pressure is 1 atmosphere, if you move from right to left along the 'pressure = 1 atm' line, a solid piece of CO_2 that you buy at the store will immediately start evaporating into the gas phase. Only if you could freeze this 'dry ice' below -80 C will it stop evaporating into a gas phase (called sublimation) and remain a stable solid.



The Curiosity Rover recently used a technique called X-ray Diffraction Crystallography to determine the identity of compounds found in a rock sample on the surface of Mars. The image to the left shows what this data looks like. The exact radii of these rings, and the locations of spots along these rings, serve as a fingerprint of the shape of the mineral compound in space. We all know how human fingerprints work, and even 'DNA' fingerprinting is commonly mentioned in TV programs like NCIS or CSI. But how does this technique work?



The figure to the left shows a beam of light striking the surface of a crystal with 15 atoms arranged into three parallel planes. The light strikes the atoms and is 'diffracted' into a new direction defined by the angle θ.

If two beams of light are out-of-phase by 90 degrees, when they are added together, the crests of one wave interfere with the troughs of the other wave and you end up with no light. If they are in-phase, they will add together, you get the light intensified and you also get a ring of light!

The diagram shows the added distance that the lower ray gains by being diffracted through the angle θ.

Problem 1 – From the information in the diagram, what is the extra distance, s, traveled by the x-ray light in a crystal lattice where the planes are separated by a distance d?

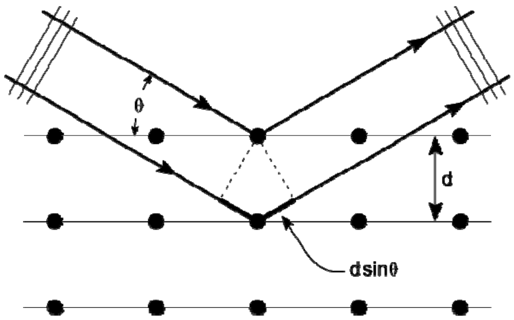
Problem 2 - If the light struck the crystal exactly face-on ($\theta=90^{o}$) how much extra distance would the second beam travel compared to the first beam that was reflected only from the top surface?

Problem 3 – If the wavelength of the x-ray light is L, what is the relationship between L and s so that the wave crests exactly match up?

Problem 4 – Suppose that in the Curiosity data, a diffraction ring is detected at an angle of incidence $\theta=2^{o}$. The Curiosity instrument uses X-rays with an energy of 6.929 keV, which have a wavelength of 1.79×10^{-10} meters. What is the separation, d, of the crystal planes in the mineral sample?

Problem 1 – From the information in the diagram, what is the extra distance, L, traveled by the x-ray light in a crystal lattice where the planes are separated by a distance d?

Answer: From the diagram below, **s = 2d sin** θ



Problem 2 - If the light struck the crystal exactly face-on ($\theta=90^{o}$) how much extra distance would the second beam travel compared to the first beam that was reflected from the top surface?

Answer: **s = 2d**

Problem 3 – If the wavelength of the x-ray light is L, what is the relationship between L and s so that the wave crests exactly match up?

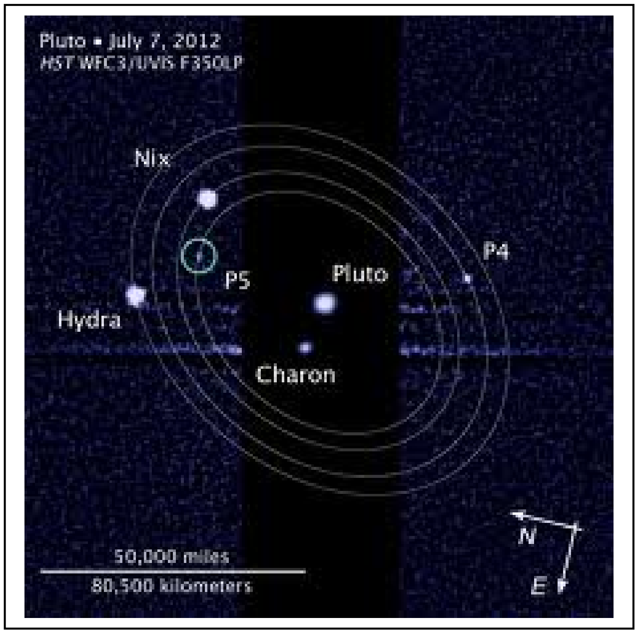
Answer: The wavelength is L and for the waves to exactly match up, the two waves can either be shifted by s=0, or by **one full wavelength** s = L. So the first non-zero condition is that L = 2 d sinθ and so

$$\mathbf{d = \frac{L}{2\sin\theta}}$$

Problem 4 – Suppose that in the Curiosity data, a diffraction ring is detected at an angle of incidence $\theta=2^{o}$. The Curiosity instrument uses x-rays with an energy of 6.929 keV, which have a wavelength of 1.79×10^{-10} meters. What is the separation, d, of the crystal planes in the mineral sample?

Answer: d = 1.79×10^{-10} meters/($2\sin2^{o}$) so d = **2.56×10^{-9} meters**.

Pluto's Fifth Moon!



The most distant well-known object in our solar system, Pluto, is an irresistible object for Hubble Space Telescope investigations.

In July, 2012, Hubble scientists released an image of Pluto showing a new moon called P5. It is irregularly shaped and about 15 km in diameter, and probably made from ice.

Its average orbit radius is 47,000 km and appears to lie in the same orbit plane as the other four moons, and takes about 20 days to make one orbit.

The table below gives the details for the four new moons of Pluto as of 2012.

Name	Discovery	Diameter (km)	Distance (km)	Period (hours)
Pluto V	2012	10 to 25	42,000	485
Nix	2005	46 to 137	48,700	598
Pluto IV	2011	13 to 34	59,000	770
Hydra	2005	61 to 167	65,000	917

Problem 1 – Compute for each moon the cube of the distance,D^3, and the square of the period, P^2. Calculate the value $R = D^3/P^2$ for each moon. What do you notice about the values for R? What is the average value for R using the data from the five moons?

Problem 2 – Suppose future observations discover a new moon, P6, orbiting at a distance of 35,000 km from Pluto. What would you predict as the orbit period for this satellite?

Problem 3 - The mass of a body can be determined from Kepler's Third Law, which you verified in Problem 1. By using the formula $M = 6.9\times10^{10}$ R, where R is in units of $km^3/days^2$, what is the mass of Pluto in kilograms?

Name	Discovery	Diameter	Distance	Period	R
		(km)	(km)	(hours)	
Pluto V	2012	10 to 25	42,000	485	3.15×10^{8}
Nix	2005	46 to 137	48,700	598	3.23×10^{8}
Pluto IV	2011	13 to 34	59,000	770	3.46×10^{8}
Hydra	2005	61 to 167	65,000	917	3.27×10^{8}

Problem 1 – Compute for each moon the cube of the distance,D^3, and the square of the period, P^2. Calculate the value $R = D^3/P^2$ for each moon. What do you notice about the values for R? What is the average value for R using the data from the five moons?

Answer: The values for R are very similar to each other even though there is a large range in orbit distances and periods**. The average value is 3.28×10^{8} km^3/hours^2**

Problem 2 – Suppose future observations discover a new moon, P6, orbiting at a distance of 35,000 km from Pluto. What would you predict as the orbit period for this satellite in days?

Answer: $P^2 = (35000)^3/3.28\times10^8 = 130{,}716$ so
P = 361 hours or **15.1 days**.

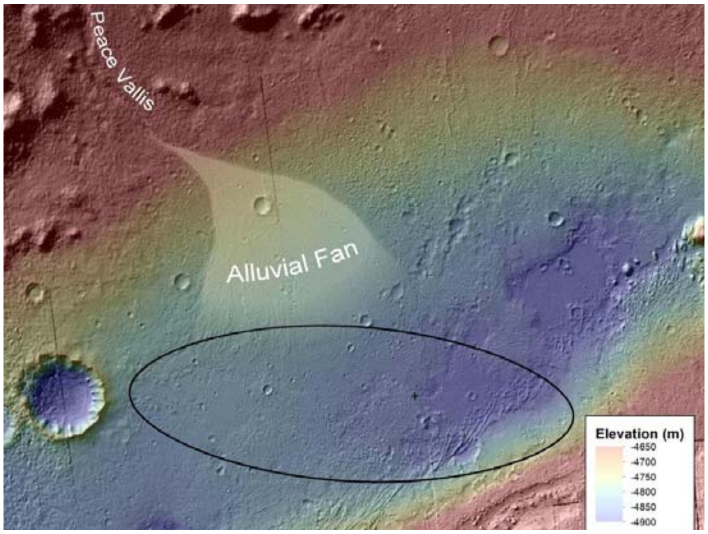
Problem 3 - The mass of a body can be determined from Kepler's Third Law, which you verified in Problem 1. By using the formula $M = 6.9\times10^{10}$ R, where R is in units of km^3/days^2, what is the mass of Pluto in kilograms?

Answer: First convert R into the correct units:

$R = 3.28\times10^{8}$ km^3/hours^2 x (24 hours/1 day)^2 = 1.88×10^{11} km^3/day^2

Then $M = 6.9\times10^{10} \times 1.88\times10^{11}$ so M = **1.3×10^{22} kg**.

Note, the mass of Earth is 6.0×10^{24} kg, so Pluto has a mass equal to about
1.3×10^{22} kg/6.0×10^{24} = 0.002 Earths!



The Curiosity Rover discovered rounded pebbles near its original landing site marked with the 'X' in the figure. The figure also shows the elevation changes in this area. Here is what the pebbles looked like! The white bar is 1 cm long.

Geologists studying the pebbles and the landscape believe that the water flow that moved and rounded the pebbles was at least ankle deep and perhaps waist deep. As on Earth, the pebbles were carried by fast moving water and over time became rounded by the constant scraping and bouncing. How fast was the water moving?

Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Vallis?

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.?

Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan?

ADVANCED) Calculating the stream flow speed:

Problem 4 – On Mars, the stream flow can be approximated by $V = (2gh)^{1/2} \sin(\theta)$ where $\tan(\theta) = SG$, $g = 3.8$ meters/sec^2 is the acceleration of gravity on Mars, and h is the difference in elevation of the top and bottom of the stream. About how fast was the water flowing past the Curiosity landing area to create the pebbles?

Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Vallia? Answer: **About 15 km**.

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.? Answer: -4650 m – (-4900 m) so h = **250 meters.**

Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan? 250 meters / 15 km = **17 meters/kilometer or SG=0.017 meters/meter.**

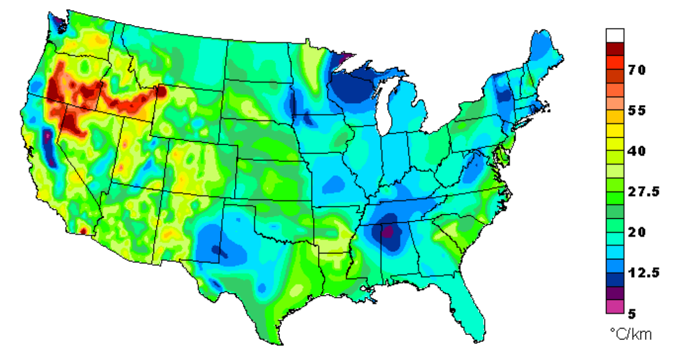
Calculating the stream flow speed:

Problem 4 – On Mars, the stream flow can be approximated by $V = (2gh)^{1/2} \sin(\theta)$ where $\tan(\theta)$ = SG, and g = 3.8 meters/sec^2 is the acceleration of gravity on Mars. About how fast was the water flowing past the Curiosity landing area to create the pebbles?

Answer: h = 650 meters, SG=0.017, g = 3.8 meters/sec^2 . Then θ = 1.0 degrees.
$V = (2 \times 3.8 \times 650)^{1/2} \sin(1.0)$
V = 1.2 meters/sec.

This is about as fast as a human walking very slowly (about 0.3 miles/hr or 4.3 km/hr).

Comparing the Heat Output of Mars and Earth



Temperature gradient map of the United States.
Courtesy Southern Methodist University, Geothermal Lab,
(http://smu.edu/geothermal/heatflow/heatflow.htm)

The amount of heat that a planet produces at its surface gives us clues about its interior.

For Earth, geologists have measured the heat flow at its surface to be about 80 milliWatts/m^2. For Mars the value is about 20 milliWatts/m^2.

Earth's interior is producing almost 4 times more heat flow than Mars, suggesting that the interior of Earth is significantly hotter.

When these measurements are combined with a knowledge of the properties of the crust of Earth and Mars, we can estimate what the value of the thermal gradient is near the surface of each planet! The basic formula is

$$F = K\frac{DT}{Z}$$

where F = heat flux in watts/m^2, K = heat diffusion coefficient of the crust, z = thickness of the crust in meters and DT = temperature difference in Celsius.

Problem 1 - For Earth, the crust is granite with K=2.0 watts/meter ^{o}C. For Mars, the crust is loosely packed rock with K = 0.08 Watts/meter ^{o}C. What are the temperature gradients near the surface of each planet in ^{o}C/meter?

Problem 2 - For a given value of F, what happens to the temperature gradient as the material becomes a better insulator (K smaller)? Explain what this means in words.

Problem 3 - The radius of Earth is 6378 km and Mars is 3389 km. What is the total heat power emitted by Earth and Mars in teraWatts? (1 TW = 1 trillion watts).

Problem 4 - Two planets, A and B, have the same diameter. If $F_B = 1/4F_A$ and K_B=8 K_A, which planet has the largest crustal temperature gradient?

Problem 1 - For Earth, the crust is granite with K=2.0 watts/meter ^oC. For Mars, the crust is loosely packed rock with K = 0.08 Watts/meter ^oC . What are the temperature gradients near the surface of each planet in ^oC/meter?

Answer: Mars: 0.020 watts/m^2 = (0.08 watts/meter ^oC) x DR/Z so

DR/Z = 0.25 ^{o}C/meter.

Earth: 0.080 watts/m^2 = (4.0 watts/meter ^oC) x DR/Z so

DR/Z = 0.02 ^{o}C/meter.

Problem 2 - For a given value of F, what happens to the temperature gradient as the material becomes a better insulator (K smaller)? Explain what this means in words.

Answer: As K becomes smaller, the temperature gradient becomes larger. That means that for every meter you travel through the material, the temperature difference decreases by a large value. In other words, most of the warmth remains close to the hottest side of the material.

Problem 3 - The radius of Earth is 6378 km and Mars is 3389 km. What is the total heat power emitted by Earth and Mars in teraWatts? (1 TW = 1 trillion watts).

Answer:
Mars surface area = $4 \pi (3378000)^2 = 1.4\text{x}10^{14}$ m^2
Power = 0.020 watts/m^2 x $1.4\text{x}10^{14}$ m^2 = $2.9\text{x}10^{12}$ watts = **2.9 teraWatts**.

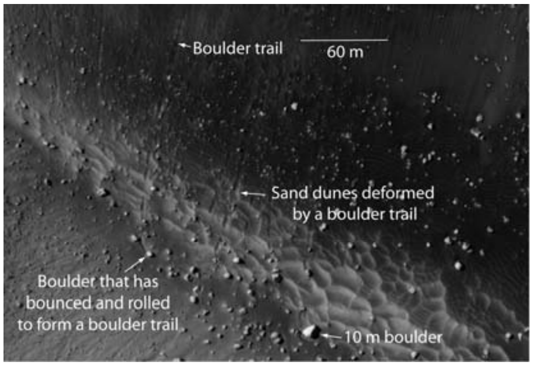
Earth surface area = $4 \pi (6378000)^2 = 5.1\text{x}10^{14}$ m^2
Power=0.080 watts/m^2 x $5.1\text{x}10^{14}$ m^2 = $4.1\text{x}10^{13}$ watts = **41 teraWatts**.

Problem 4 - Two planets, A and B, have the same diameter. If $F_B = 1/4F_A$ and K_B=8 K_A, which planet has the largest crustal temperature gradient?

Answer: For Planet B: $(DT/Z)_B = F_B/K_B$ so by substitution:

$$(DT/Z)_B = 1/4F_A / 8K_A$$
$$= 1/16\ (F_A/K_A)$$
$$= 1/16\ (DT/Z)_A$$

So Planet B has a temperature gradient 1/16 of Planet A. **Planet A has the largest**.



The InSight seismometer (SEIS) will measure vibrations caused by distant meteor impacts and 'marsquakes'. Scientists measure these impacts in three ways: By their energy in Joules, by their frequency in impacts per year, and by their magnitude on a Richter-like 'M' scale.

It is estimated that M=5.6 impacts deliver about 2×10^{17} Joules of energy, and occur about once each year. This amount of impact energy is equal to 48 million tons (48 megatons) of TNT!

The image above was taken by the Mars Reconnaissance Orbiter and shows boulders dislodged by marsquakes near the region known as Cerberus Fossae. Tracks made by the rolling boulders can be easily seen in the sand dunes.

Problem 1 – On Mars, scientists have predicted that there will be 100 events per year with M=3.5, 10 events per year with M=4.5 and 1 event per year with M=5.5. If an M=4.5 quake is 10 times more violent than an M=3.5 quake, write a mathematical formula that gives the impact rate, R, as a function of the quake severity, M.

Problem 2 – An impact with M=5.0 is strong enough that if it occurred closer than 100 km from the InSight seismometer, it would overpower the sensors and not be recorded accurately; a condition called data saturation. How long would scientists have to wait for such an impact to occur? (The radius of Mars is 3,376 km).

Problem 3 – For what magnitude of marsquake, M, would you expect to detect one event within 100km of the InSight seismometer?

Problem 4 – If an M=5.6 delivers 2.0×10^{17} Joules of energy, and M=4.6 delivers 30x less energy, about how many tons of TNT will the earthquake described in Problem 3 deliver?

Answer Key

Problem 1 – On Mars, scientists have predicted that there will be 100 events per year with M=3.5, 10 events per year with M=4.5 and 1 event per year with M=5.5. If an M=4.5 quake is 10 times more violent than an M=3.5 quake, write a mathematical formula that gives the impact rate, R, as a function of the quake severity, M.

Answer: $R(M) = 100 \times 10^{(3.5-M)}$

Problem 2 – An impact with M=5.0 is strong enough that if it occurred closer than 100 km from the InSight seismometer, it would overpower the sensors and not be recorded. How long would scientists have to wait for such an impact to occur? (The radius of Mars is 3,376 km).

Answer: For M=5.5, the rate is once per year over the entire planet. The surface area of Mars is $4 \times 3.141 \times (3376)^2 = 1.4 \times 10^8\ km^2$, and the area of the zone near the seismometer is $3.141 \times (100km)^2 = 3.1 \times 10^4\ km^2$. The ratio of the areas is about 1/4500, so the rate of impacts inside the InSight zone is 1/4500 per year or an average of 4500 years between impacts.

Problem 3 – For what magnitude of marsquake, M, would you expect to detect one event within 100km of the InSight seismometer?

Answer: The ratio of the martian surface area to the area inside 100km is 1/4500. We need 4500 events per year over all of Mars in order to get 1 event inside our detection zone. From our rate function:

4500 = 100 x 10(3.5-M)

Solving for M we get 45 = 10(3.5-M)
Taking Log10 on both sides we get Log10(45) = 3.5-M
Then 1.6 = 3.5-M
And so M = 3.5-1.6 **M = 1.9**

Problem 4 – If an M=5.6 delivers 2.0×10^{17} Joules of energy, and M=4.6 delivers 30x less energy, about how many tons of TNT will the earthquake described in Problem 3 deliver?

Answer: 2×10^{17} Joules equals 48 megatons, so we use that unit directly.
The difference between M=5.6 and M=1.9 is 3.7. Rounding this to 4.0, we have $30^{4.0}$ = 810,000 times less energy, so 48,000,000/810,000 = 59 tons of TNT.

Artist rendition of impact on Mars courtesy William Hartman (Planetary Science Institute)

A more useful earthquake severity scale is the Moment Magnitude scale represented by the logarithmic number Mw. Instead of giving the amount of ground displacement, it is directly related to the amount of seismic energy released by the earthquake. On this scale, Mw = 5.5 represents $E=2.0\times10^{17}$ Joules of energy released.

The InSight seismometer experiment (called SEIS) will be able to detect marsquakes with magnitudes from M=3.5 to 6. The relationship between E and M_W is given by the formula:

$$M_W = 2/3 \text{Log}E - 6.0$$

Problem 1 - What is the seismic energy range for the InSight SEIS instrument in Joules? How many megatons of TNT does this range equal if 1 megaTon TNT = 4.2×10^{15} Joules?

Problem 2 – The energy, E, of an earthquake (or marsquake) can be estimated from the formula E = R x A x d where A is the area of the fault slippage, d is the amount of slippage by the fault and R is the rigidity of the crust. Typically, R = 3.3×10^{10} Newton/m^2 and A and d are given in square-meters and meters respectively and E is in Joules of energy. A) What is the energy of an earthquake for which the area is 2000 km^2 and the slippage was 2 meter? B) What is the Moment Magnitude for this event?

Problem 3 - Most of the marsquakes detected by InSight will be due to meteors of various masses impacting the surface of Mars. Each impact will deliver an amount of energy equal to its kinetic energy or $E = ½ mV^2$ where E is in Joules if v is in meters/sec and m is in kilograms. Typical impact speeds will be about 15 km/sec. What is the formula that relates the Moment Magnitude to the mass of the meteorite in kilograms? What is the range of InSight sensitivites in terms of the mass range of the meteors in metric tons?

Problem 1 - What is the seismic energy range for the InSIght SEIS instrument in Joules? How many megatons of TNT does this range equal if 1 megaTon TNT = 4.2x1015 Joules?

Answer: $3.5 < M_w < 6.0$ then solving for E in the above equation we get

$E = 10^{(1.5M_w + 9.0)}$

and so for $M_w = 3.5$ we have **E = 1.8x10^{14} Joules** and for M_w=6.0 we have **E = 1.0x10^{18} Joules**. This range of energy corresponds to **0.4 mTons < E < 238 mTons**.

Problem 2 – The energy, E, of an earthquake (or marsquake) can be estimated from the formula E = R x A x d where A is the area of the fault slippage, d is the amount of slippage by the fault and R is the rigidity of the crust. Typically, R = 3.3x10^{10} Newton/m^2 and A and d are given in square-meters and meters respectively and E is in Joules of energy. A) What is the energy of an earthquake for which the area is 2000 km^2 and the slippage was 2 meter? B) What is the Moment Magnitude for this event?

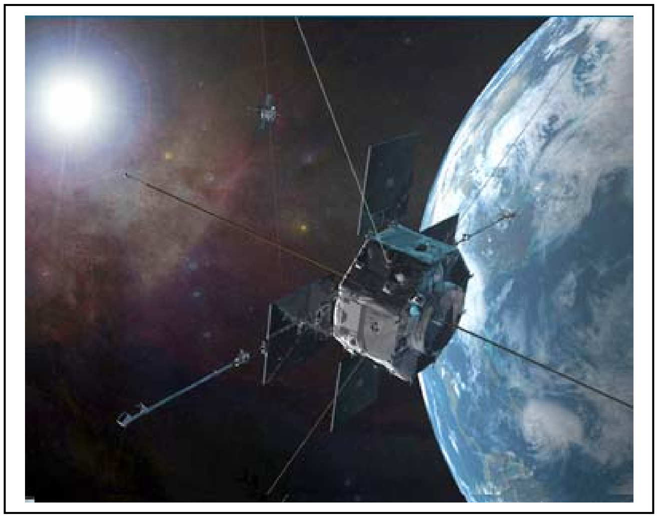
Answer: A) E = 3.3x10^{10} x 2000 x 2 = **1.3x10^{14} Joules**.
B) Mw = 2/3 logE – 6 so **Mw = 3.4**.

Problem 3 - Most of the marsquakes detected by InSight will be due to meteors of various masses impacting the surface of Mars. Each impact will deliver an amount of energy equal to its kinetic energy or $E = ½ mV^2$ where E is in Joules if v is in meters/sec and m is in kilograms. Typical impact speeds will be about 15 km/sec. What is the formula that relates the Moment Magnitude to the mass of the meteorite in kilograms? What is the range of InSight sensitivites in terms of the mass range of the meteors in metric tons?

Answer: For v = 15 km/sec we have E = 1.125x10^{8}m. We have Mw = 2/3LogE – 6 and substituting for E we get Mw = 2/3 log(1.125x10^{8}m) – 6 ; and so the formula is

Mw = 2/3Log(mass) – 0.633.

For Mw = 3.5 we have m = 1.58x10^{6} kg or 1580 metric tons.
For Mw = 6.0 we have m = 8.9x10^{9} kg or 8.9 megatons.
So **1580 tons < m < 8.9 megatons** is the mass range.



Kepler's Third Law says that the cube of the satellite's orbit radius is directly proportional to the square of its orbit period. The proportionality constant, c, depends only on the mass of the planet that the satellite (or moon) is orbiting. For distances measured in meters, periods measured in seconds, and masses measured in kilograms, the proportionality constant for satellites orbiting Earth is just

$$C = 1.7 \times 10^{-12} M$$

Problem 1 – What is the equation described by the paragraph above?

Problem 2 – Solve the equation for M – the mass of Earth.

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Problem 1 – What is the equation described by the paragraph above?

Answer: After substituting for the constant, C, you get

$$\mathbf{R^3 = 1.7x10^{-12}\ M\ T^2}$$

Problem 2 – Solve the equation for M – the mass of Earth.

$$\mathbf{M = 5.9x10^{11}\ R^3/T^2}$$

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Answer:

$M = 5.9x10^{11}\ (6378000)^3/(3600)^2\ \ R^3/T^2$

$\mathbf{M = 1.2 \times 10^{25}\ R^3/T^2}$

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Answer : $M = 1.2 \times 10^{25}\ (3.4)^3/\ (9.0)^2$

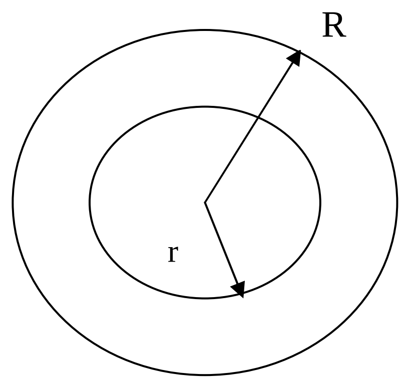
$M = 5.8 \times 10^{24}$ kilograms

The actual value is 5.98×10^{24} kg.

Name ______________________________

Astronomers can often make an estimate of what the interior of a planet looks like by carefully measuring the mass of the planet and its radius. They then create a model of the inside of the planet that matches the measured radius and mass. A simple model has a planet described as a perfect sphere, consisting of a low-density outer shell and a higher-density inner core. By using various kinds of matter (solids, gas, liquid) and different compounds (ice, iron, water etc) astronomers try various combinations of materials until the mass and radius of the model planet resembles the one actually studied. Here is a list of materials commonly found in planets:

Material	Density
Pure Iron	10.0 gm/cc
Basalt rock	5.0 gm/cc
Water	5.0 gm/cc
Silicate rock	3.0 gm/cc
Ice	1.0 gm/cc



Suppose a planet was discovered in another solar system that had a radius of **R** = 6378 kilometers and a mass of 5.97×10^{24} kilograms. The astronomer begins by considering a model with a core radius, **r**, of 3,500 kilometers. Using this model, answer the questions below.

Question 1 - What is the volume of the inner core in cubic centimeters?

Question 2 - What is the volume of the outer shell in cubic centimeters?

Question 3 - If the inner core were made of silicate rock, and the outer crust of ice what would be the total mass of the model planet?

Question 4 – How does the mass of the model planet compare to the actual mass of the planet?

Question 5 – Can you find a material in the above list for the core material that would make the model mass equal to the planets actual mass?

Question 6 – What else could you change in the model the astronomer started with to make a better estimate?

Question 1 - What is the volume of the planet in cubic centimeters?
Answer :

V (sphere) = $4/3\ \pi\ R^3$ = $(1.33) \times (3.14) \times (6378)^3$
= 1.08×10^{12} cubic kilometers
= 1.08×10^{27} cubic centimeters.

Question 2 - What is the volume of the inner core in cubic centimeters?
Answer:

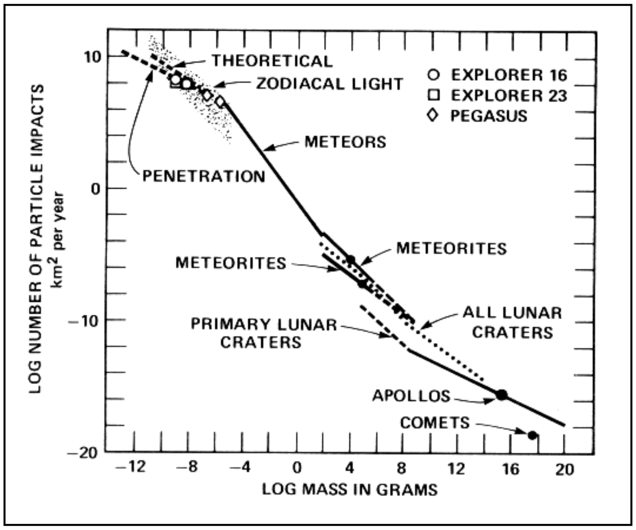
V (sphere) = $4/3\ \pi\ R^3$ = $(1.33) \times (3.14) \times (3500)^3$
= 1.79×10^{11} cubic kilometers
= 1.79×10^{26} cubic centimeters.

Question 3 - If the inner core were made of silicate rock (density = 3.0 grams/cm3) , and the outer crust of ice (1.0 grams/cm3) what would be the total mass of the model planet?
Answer: The outer crust volume is the DIFFERENCE between the answers in Questions 1 and 2 $(1.08 \times 10^{27} – 1.79 \times 10^{26})$ = 9.01×10^{26} cubic cm.
Model mass = $1.0 \times (9.01 \times 10^{26}) + 3.0 \times (1.79 \times 10^{26}) = 1.44 \times 10^{24}$ **kilograms**

Question 4 – How does the mass of the model planet compare to the actual mass of the planet?
Answer: The model mass is much smaller than the actual mass.

Question 5 – Can you find a combination of crust and core material in the table that would make the model mass almost equal to the planets actual mass?
Answer: Students may use trial-and-error. Try iron in the core and silicate rock in the outer crust:
Model mass = $3.0 \times (9.01 \times 10^{26}) + 10.0 \times (1.79 \times 10^{26}) = 4.61 \times 10^{24}$ **kilograms**
(This gives a model that is (4.61 / 5.97) x 100% = 77% less massive than the actual planet, which is a very good match for this activity – and the best match possible given the assumed core radius of 3,500 kilometers!)

Question 6 – What else could you change in the model the astronomer started with to make a better estimate?
Answer: You could change the radius of the core by making it bigger. You could change the composition of the outer crust by making it basalt instead of silicate rock. You could change the composition of the core by making it a mixture of iron and silicate (with a density between 3.0 and 10.0 gm/cm3). You could increase the number of zones in the model from two (crust and core) to three (crust, mantle, core) and see what happens.



The space between the planets is filled with fragments of asteroids, comets and material left over from the formation of the planets. These rocks and debris rain down upon exposed surfaces at speeds up to 30 km/sec.

The figure on the left summarizes the impact frequency of various sizes of particles in space. Note the graph is plotted in the Log-Log format due to the enormous range of masses and rates being described.

Understanding the data graph:

Problem 1 - A meteorite with a density of 3 grams/cm^3 has a diameter of 4 centimeters (about 1 1/2 inches).

A) What is the mass of this meteorite assuming it is a sphere?

B) From the graph, where on the horizontal axis are objects of this mass located?

C) What is the number of impacts per year you would expect over an area of 10,000 square kilometers?

Problem 2 – The function that best models the data in the graph is given by

$$N(m) = 0.025\ m^{-0.9}$$

where N(m) is the number if impacts per square kilometer per year for objects with a mass of m grams. Using integral calculus, what is the total mass in tons of impacting objects each year, over the surface of Earth, in the mass range from 1 gram to 10^{20} grams? (Use π = 3.14 and assume a spherical Earth with a radius of 6,378 km).

Problem 1 - Answers: A) Mass = Density x Volume. Radius of sphere = 2 cm, so M = 3.0 x (4/3) (3.14) $(2)^3$ = **100 grams**. B) The horizontal axis is un units of Log(grams) so Log(100) = 2, and this is the location **half-way between 0' and '4' on the axis**. C) From 'x=2' ,a vertical line intercepts the data at about 'y=-3.5' on the vertical axis. This represents Log(N) = -3.5 so that N = 0.00032 impacts/km^2/year. Over an area of 10000 km^2, there would be an estimated 0.00032 impacts/km^2/year x (10000 km^2) = **3.2 impacts per year.**

Problem 2 –What is the total mass in tons of impacting objects each year, over the surface of Earth, in the mass range from 1 gram to 10^{20} grams? (Use π = 3.14 and assume a spherical Earth with a radius of 6,378 km).

Answer: The total mass is the area under the curve: Mass = N(m) dm

$$M = \int_1^{10^{20}} 0.025 m^{-0.9} dm$$

$$= 0.025 \frac{1}{0.1}[(10^{20})^{0.1} - (1)^{0.1}]$$

= 0.025(10)(100-1)

= **24.75 grams/km^2/year**

Area of Earth = $4\pi(6378)^2$ = 5.1×10^8 km^2

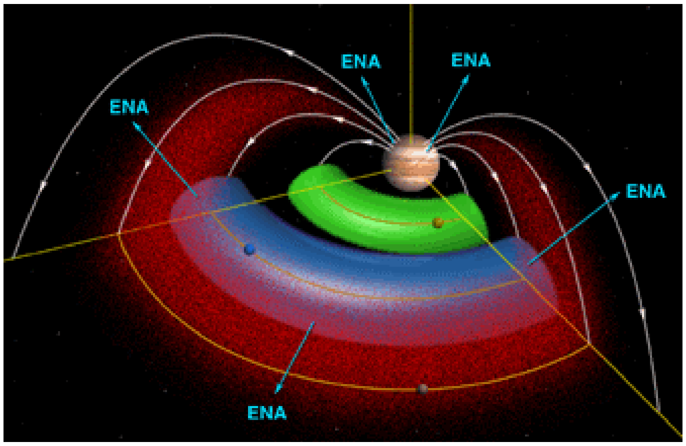
So the total meteoritic mass per year is

24.75 grams/km^2 x 5.1×10^8 km^2 = 1.26×10^{10} grams
or 1.26×10^7 kg

or **12,600 tons**.

Note: Popular estimates range from 20,000 to 100,000 tons/year. The amount is sensitive to both the logarithmic function used to model the power-law data, and the integration limits!

The Io Plasma Torus



The satellite of Jupiter, Io, is a volcanically active moon that ejects 1,000 kilograms of ionized gas into space every second. This gas forms a torus encircling Jupiter along the orbit of Io. We will estimate the total mass of this gas based on data from the NASA Cassini and Galileo spacecraft.

Image: Io plasma torus (Courtesy NASA/Cassini)

Problem 1 - Galileo measurements obtained in 2001 indicated that the density of neutral sodium atoms in the torus is about 35 atoms/cm^3. The spacecraft also determined that the inner boundary of the torus is at about 5 Rj, while the outer boundary is at about 8 Rj. (1 Rj = 71,300 km). A torus is defined by the radius of the ring from its center, R, and the radius of the circular cross section through the donut, r. What are the dimensions, in kilometers, of the Io torus based on the information provided by Galileo?

Problem 2 - Think of a torus as a curled up cylinder. What is the general formula for the volume of a torus with radii R and r?

Problem 3 - From the dimensions of the Io torus, what is the volume of the Io torus in cubic meters?

Problem 4 - From the density of sodium atoms in the torus, what is A) the total number of sodium atoms in the torus? B) If the mass of a sodium atom is 3.7 x 10^{-20} kilograms, what is the total mass of the Io torus in metric tons?

Calculus:

Problem 5 - Using the 'washer method' in integral calculus, derive the formula for the volume of a torus with a radius equal to R, and a cross-section defined by the formula $x^2 + y^2 = r^2$. The torus is formed by revolving the cross section about the Y axis.

Answer Key

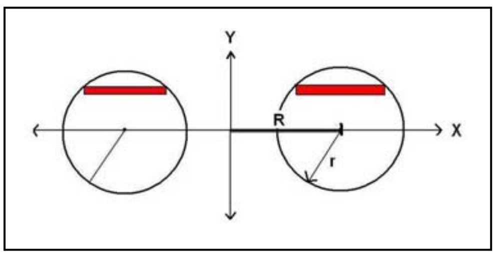
Problem 1 - The mid point between 5 Rj and 8 Rj is (8+5)/2 = 6.5 Rj so R = 6.5 Rj and r = 1.5 Rj. Then R = 6.5 x 71,300 so **R = 4.6 x 10^5 km**, and r = 1.5 x 71,300 so **r = 1.1 x 10^5 km**.

Problem 2 - The cross-section of the cylinder is πr^2, and the height of the cylinder is the circumference of the torus which equals $2 \pi R$, so the volume is just $V = (2 \pi R) \times (\pi r^2)$ or **$V = 2 \pi^2 R r^2$.**

Problem 3 - Volume = $2 \pi^2 (4.6 \times 10^5 \text{ km}) (1.1 \times 10^5 \text{ km})^2$ so **V = 1.1 x 10^{17} km^3.**

Problem 4 - A) 35 atoms/cm^3 x (100000 cm/1 km)^3 = 3.5 x 10^{16} atoms/km^3. Then number = density x volume so N = (3.5 x 10^{16} atoms/km^3) x (1.1 x 10^{17} km^3) , so **N = 3.9 x 10^{33} atoms**.
B) The total mass is M = 3.9 x 10^{33} atoms x 3.7 x 10^{-20} kilograms/atom = 1.4 x 10^{14} kilograms. 1 metric ton = 1000 kilograms, so the total mass is **M = 100 billion tons**.

Advanced Math:



$$V = 8 \pi R \int_0^r \left(r^2 - y^2 \right)^{1/2} dy$$

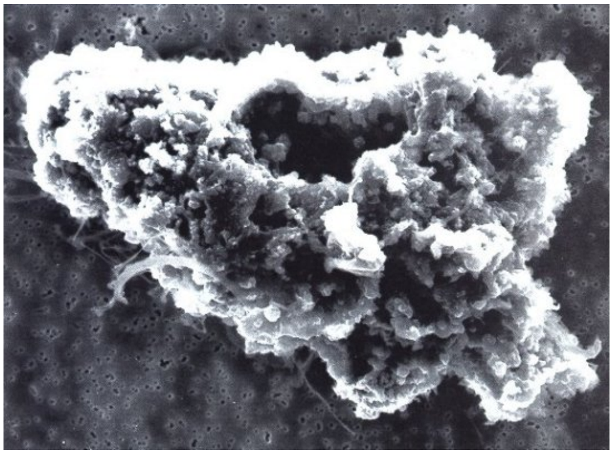
Recall that the volume of a washer is given by $V = \pi (R(outer)^2 - R(inner)^2) \times$ thickness. For the torus figure above, we see that the thickness is just dy. The distance from the center of the cross section to a point on the circumference is given by $r^2 = x^2 + y^2$. The width of the washer (the red volume element in the figure) is parallel to the X-axis, so we want to express its length in terms of y, so we get $x = (r^2 - y^2)^{1/2}$. The location of the outer radius is then given by $R(outer) = R + (r^2-y^2)^{1/2}$, and the inner radius by $R(inner) = R - (r^2 - y^2)^{1/2}$. We can now express the differential volume element of the washer by $dV = \pi [(R + (r^2-y^2)^{1/2})^2 - (R - (r^2 - y^2)^{1/2})^2] dy$. This simplifies to $dV = \pi [4R (r^2-y^2)^{1/2}] dy$ or
$dV = 4 \pi R (r^2-y^2)^{1/2} dy$. The integral can immediately be formed from this ,with the limits y = 0 to y=r. Because the limits to y only span the upper half plane, we have to double this integral to get the additional volume in the lower half-plane. The required integral is shown above.

This integral can be solved by factoring out the r from within the square-root, then using the substitution U = y/r and dU = 1/r dy to get the integrand $dV = 8 \pi R r^2 (1 - U^2)^{1/2} dU$. The integration limits now become U=0 to U=1. Since r and R are constants, this is an elementary integral with the solution $V = 1/2\, U (1-U^2)^{1/2} + 1/2 \arcsin(U)$. When this is evaluated from U=0 to U=1, we get

$V = 8 \pi R r^2 [0 + 1/2 \arcsin(1)] - [0 + 1/2\arcsin(0)]$

$V = 8 \pi R r^2\, 1/2 (\pi/2)$

$V = 2 \pi R r^2$



A cosmic dust grain about 0.1mm across captured by a NASA high-altitude aircraft. Probably debris from a passing comet

Planets are built in several stages. The first of these involves small, micron-sized interstellar dust grains that collide and stick together to eventually form centimeter-sized bodies. A simple model of this process can tell us about how long it takes to 'grow' a rock-sized body starting from microscopic dust. This process occurs in dense interstellar clouds, which are known to be the birth places for stars and planets.

Problem 1 – Assume that the forming rock is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the rock as a function of time, M(t)?

Problem 2 – The rock grows by absorbing incoming dust grains that have an average mass of m grams and a density of N particles per cubic centimeter in the dust cloud. The particles collide with the surface of the rock at a speed of V cm/sec, what is the equation that gives the rate of growth of the rock's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the rock when it reaches a diameter of 1 centimeter if its density is 3 grams/cc?

Problem 6 – The rock begins at t=0 with a mass of 1 dust grain, m = 8×10^{-12} grams. The cloud density N = 3.0×10^{-5} dust grains/cc and the speed of the dust grains striking the rock, without destroying the rock, is V=10 cm/sec. How many years will the growth phase have to last for the rock to reach a diameter of 1 centimeter?

Answer Key

Problem 1 – Answer: Because mass = density x volume, we have
M = 4/3 π R^3 ρ and so **M(t) = 4/3 π ρ R(t)^3**

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of dust grains land on the surface area of the rock per unit time, dt. The amount is proportional to the surface area of the rock, since the more surface area the rock has, the more dust particles will be absorbed. Also, the rate at which dust grain mass is brought to the surface is proportional to the product of the dust grain density in the interstellar gas, times the speed of the grains landing on the surface. This leads to m x N x V where m is in grams per dust grain, N is in dust grains per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the rock, will then have the units of grams/sec representing the rate at which the rock mass is growing. The full formula for the growth of the rock mass is then
dM/dt = 4 p R^2 m N V

Problem 3 – Answer: From Problem 1 we see that R(t) = (3 M(t)/ 4 π ρ)$^{1/3}$. Then substituting into dM/dt we have dM/dt = 4 π m N V (3 M(t)/4 π ρ)$^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand M(t)$^{-2/3}$ dM = 4 π m N V (3/4 π ρ)$^{2/3}$ dt Then integrate both sides to get:
3 M(t)$^{1/3}$ = 4 π m N V (3/4 π ρ)$^{2/3}$ t + c. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 –Answer: The radius will be 0.5 centimeters so, m = 4/3 π (0.5 cm)^3 x 3.0 gm/cc = **1.6 grams.**

Problem 6 – Answer: For t = 0, M(0) = m so the constant of integration is c = m$^{1/3}$ so c = (8x10^{-12})$^{1/3}$ = 2 x 10^{-4}.
Then M(t) = (4/3 (3.14) (8 x 10^{-12})(3.0 x 10^{-5})(10.0)(3/(4(3.14) (3.0)))$^{2/3}$ t + 2 x 10^{-4})^3
M(t) = (1.9 x 10^{-15} t + 2 x 10^{-4})^3

To reach M(t) = 1.6 grams, t = 6.1 x 10^{14} seconds or about **19 million years!**

Asteroid Gaspra about 15 km across. Image taken by the NASA Galileo spacecraft.

Planets are built in several stages. The first of these involves small, interstellar dust grains that collide and stick together to form centimeter-sized bodies. This can take millions of years. The second stage involves the formation of kilometer-sized asteroids from the centimeter-sized rocks. A simple model of this process can tell us about how long it takes to 'grow' an asteroid from rock-sized bodies.

Problem 1 – Assume that the forming asteroid is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the asteroid as a function of time, M(t)?

Problem 2 – The asteroid grows by absorbing incoming rocks that have an average mass of 5.0 grams and a density of N rocks per cubic centimeter in the cloud. The rocks collide with the surface of the forming asteroid at a speed of V cm/sec, what is the equation that gives the rate of growth of the asteroid's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc?

Problem 6 – The asteroid begins at t=0 with a mass of m=5 grams. The cloud density N = 1.0×10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer?

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so $\mathbf{M(t) = 4/3\ \pi\ \rho\ R(t)^3}$

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of rocks land on the surface area of the forming asteroid per unit time, dt. The amount is proportional to the surface area of the asteroid, since the more surface area the asteroid has, the more rocks will be absorbed. Also, the rate at which rock mass is brought to the surface of the asteroid is proportional to the product of the rock density in the solar nebula, times the speed of the rocks landing on the surface of the asteroid. This leads to m x N x V where m is in grams per rock, N is in rocks per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the asteroid, will then have the units of grams/sec representing the rate at which the asteroid mass is growing. The full formula for the growth of the asteroid mass is then
$\mathbf{dM/dt = 4\ p\ R^2\ m\ N\ V}$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

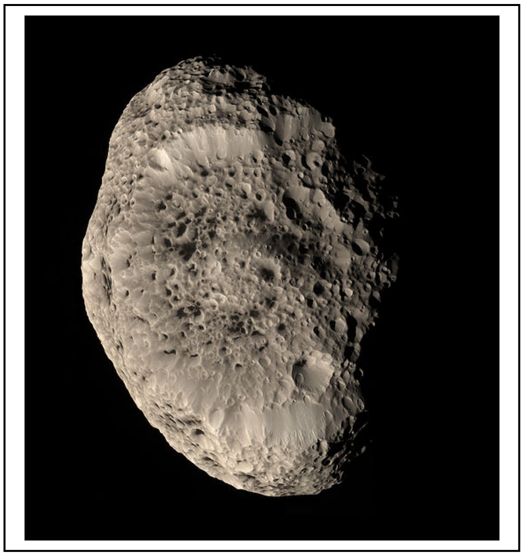
Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}\ dM = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ dt$ Then integrate both sides to get:
$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t + c$. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc? Answer $M = 4/3\ \pi\ (50{,}000\ cm)^3 \times 3.0$ gm/cc = $\mathbf{1.6 \times 10^{15}}$ **grams.**

Problem 6 – The rock begins at t=0 with a mass of 1 rock, m = 5 grams. The cloud density $N = 1.0 \times 10^{-8}$ rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer? Answer: For t = 0, M(0) = m so the constant of integration is $c = m^{1/3}$ so c = 1.7.
Then $M(t) = (4/3\ (3.14)\ (5)(1.0 \times 10^{-8})(100{,}000)(3/(4(3.14)\ (3.0)))^{2/3}\ t\ +1.7\ )^3$
$M(t) = (0.0039\ t + 1.7)^3$

So to get $M(t) = 1.6 \times 10^{15}$ grams, solve for t to get t = 29,600,000 seconds or about **342 days!**



Planets are built in several stages. Dust grains grow to large rocks in a million years, then rocks accumulate to form asteroids in a few years or so. The third stage combines kilometer-wide asteroids to make rocky planets. A simple model of this process can tell us about how long it takes to 'grow' a planet by accumulating asteroid-sized bodies through collisions. Saturn's moon Hyperion (see image) is 300 km across and is an example of a 'small' planet-sized body called a planetoid.

Problem 1 – Assume that the forming planet is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the planet as a function of time, M(t)?

Problem 2 – The planet grows by absorbing incoming asteroids that have an average mass of 10^{15} grams and a density of N asteroids per cubic centimeter in the cloud. The asteroids collide with the surface of the forming planet at a speed of V cm/sec, what is the equation that gives the rate of growth of the planet's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to Problem 3 so determine M(t).

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc?

Problem 6 – The planetoid begins at t=0 with a mass of m= 2 x 10^{15} grams. The cloud density N = 1.0 x 10^{-24} asteroids/cc (1 asteroid per 1000 cubic kilometers), and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers?

Answer Key

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so $\mathbf{M(t) = 4/3\ \pi\ \rho\ R(t)^3}$

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of asteroids land on the surface area of the planet per unit time, dt. The amount is proportional to the surface area of the planet, since the more surface area the planet has, the more asteroids will be absorbed. Also, the rate at which asteroid mass is brought to the surface of the forming planet is proportional to the product of the asteroid density in the planetary nebula, times the speed of the asteroids landing on the surface of the planet. This leads to m x N x V where m is in grams per dust grain, N is in asteroids per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the planet, will then have the units of grams/sec representing the rate at which the planet mass is growing. The full formula for the growth of the planet mass is then
$\mathbf{dM/dt = 4\ p\ R^2\ m\ N\ V}$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

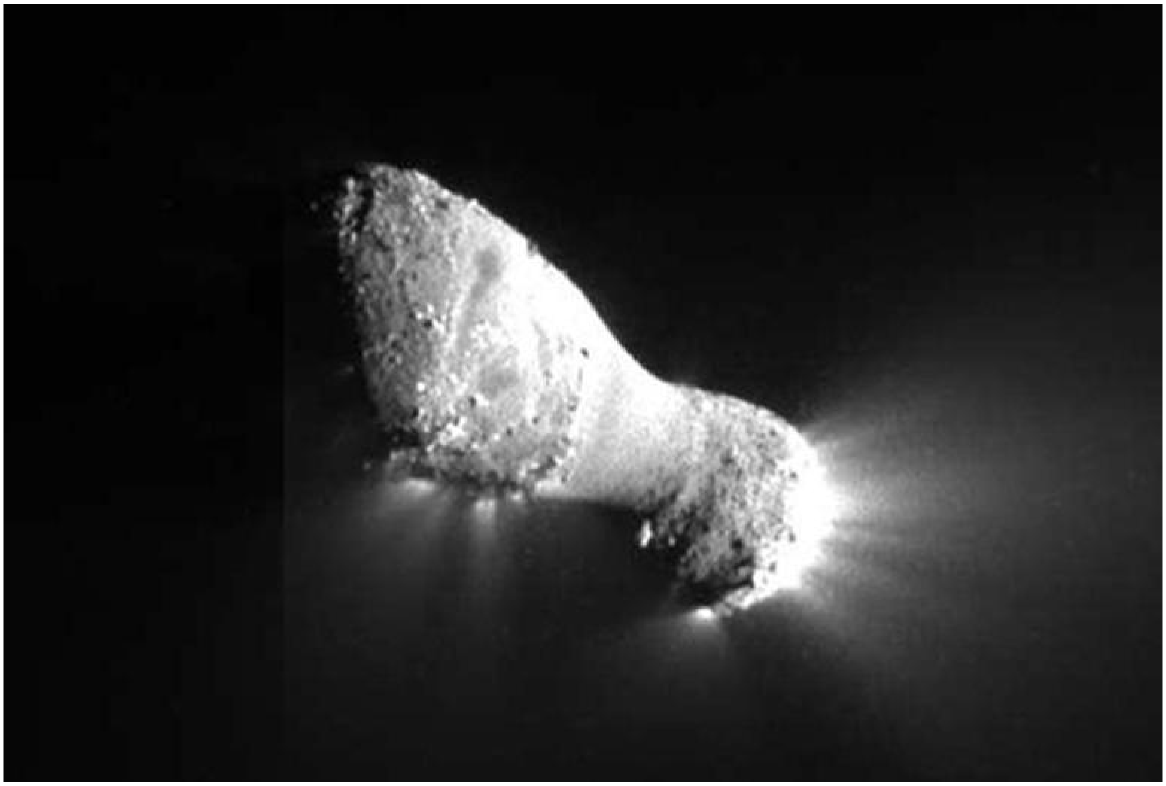
Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}\ dM = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ dt$ Then integrate both sides to get:
$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t + c$. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc? Answer $M = 4/3\ \pi\ (2.5 \times 10^8\ \text{cm})^3 \times 3.0$ gm/cc = $\mathbf{2.0 \times 10^{26}}$ **grams.**

Problem 6 – The planetoid begins at t=0 with a mass of 1 asteroid, $m = 2.0 \times 10^{15}$ grams. The cloud density $N = 1.0 \times 10^{-24}$ asteroids/cc (This equals 1 asteroid per 1000 cubic kilometers) and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers? Answer: For t = 0, M(0) = m so the constant of integration is $c = m^{1/3}$ so $c = (2.0 \times 10^{15}\ g)^{1/3} = 1.3 \times 10^5$.
Then $M(t) = (4/3\ (3.14)\ (2 \times 10^{15})(1.0 \times 10^{-24})(100{,}000)(3/(4(3.14)\ (3.0)))^{2/3}\ t + 1.3 \times 10^5)^3$
$M(t) = (0.00015\ t + 1.3 \times 10^5)^3$

To get $M(t) = 2.0 \times 10^{26}$ grams will take $t = 3.9 \times 10^{12}$ seconds or about **126,000 years!**



Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Suppose that the shape of the comet nucleus can be approximated by the following function

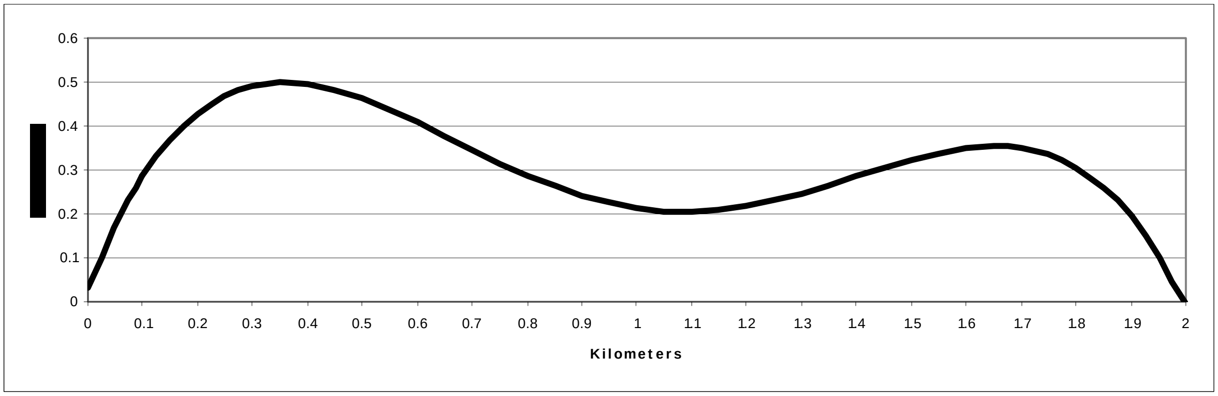
$$y(x) = -1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03$$

rotated about the x-axis between x=0 and x=2.0, where all units are in kilometers.
A) Graph this function;
B) Perform the required volume integration by using the method of circular disks.
C) To two significant figures, what is the total volume of the nucleus in cubic meters?

Problem 2 - Assuming that the density of Comet Hartley-2 is 0.6 grams/cm^3, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Problem 1 - Answer:

A) Graph:



B)

$$V = \int_0^2 \pi y(x)^2 dx \qquad \text{then} \qquad V = \pi \int_0^2 \left(-1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03\right)^2 dx$$

Expand integrand and collect terms (be careful!):

$$V = \pi \int_0^2 \left(1.49x^8 - 12.30x^7 + 41.94x^6 - 76.00x^5 + 77.55x^4 - 42.28x^3 + 9.46x^2 + 0.18x + 0.0009\right) dx$$

Integrate each term:

$$V = \pi \left[0.17x^9 - 1.54x^8 + 5.99x^7 - 12.67x^6 + 15.51x^5 - 10.57x^4 + 3.15x^3 + 0.09x^2 + 0.0009x + c\right]_0^2$$

Now evaluate V(x) at the two limits to get V = V(2) - V(0): Note that the answer for V will be sensitive to the accuracy of the polynomial coefficients, here given to 4 decimal place accuracy:

$V=(3.14)[0.1655(2)^9 - 1.5375(2)^8 + 5.9914(2)^7 - 12.6667(2)^6 + 15.51(2)^5 - 10.57(2)^4 + 3.1533(2)^3 + 0.09(2)^2 + 0.0009(2)]$

V = 3.14[0.157]

So **V = 0.49 cubic kilometers**.

Problem 2 - Mass = Density x Volume; First convert the volume to cubic centimeters from cubic kilometers: V = 0.49 km^3 x (10^3 meters/1 km)^3 x (100 cm/1 meter)^3 = 4.9 x 10^{14} cm^3. Then, Mass = 0.6 gm/cm^3 x 4.9 x 10^{14} cm^3 = 2.9 x 10^{14} gm**.** Convert grams to megatons: Mass = 2.9 x 10^{14} gm x (1 kg/1000 gm) x (1 ton/1000 kg) = 2.9 x 10^8 tons or **290 megatons**.

As they are forming, planets and raindrops grow by accreting matter (water or asteroids) at their surface.

The basic shape of a planet or a raindrop is that of a sphere. As the sphere increases in size, there is more surface area for matter to be accreted onto it, and so the growth rate increases.

In the following series of problems, we are going to follow a step-by-step logical process that will result in a simple mathematical model for predicting how rapidly a planet or a raindrop forms.

Problem 1 - The differential equation for the growth of the mass of a body by accretion is given by Equation 1 and the mass of the body is given by Equation 2

Equation 1) $\frac{dM}{dt} = 4\pi\ \rho\ V\ R(t)^2$ Equation 2) $M(t) = \frac{4}{3}\pi\ D\ R(t)^3$

where R is the radius of the body at time t, V is the speed of the infalling material, ρ is the density of the infalling material, and D is the density of the body.

Solve Equation 2 for R(t), substitute this into Equation 1 and simplify.

Problem 2 - Integrate your answer to Problem 1 to derive the formula for M(t).

Raindrop Condensation - A typical raindrop might form so that its final mass is about 0.0001 kilograms and D = 1000 kg/m^3, under atmospheric conditions where ρ = 1 kg/m^3 and V = 1 m/sec. How long would it take such a raindrop to condense?

Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9x10^{24} kg, and D = 3000 kg/m^3, under conditions where ρ = 0.000001 kg/m^3 and V = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

Figure from 'Planetary science: Building a planet in record time' by Alan Brandon, Nature 473,460–461(26 May 2011)

Answer 1:

$$R(t) = \left(\frac{3M(t)}{4\pi D}\right)^{\frac{1}{3}} \quad \text{then} \quad \frac{dM}{dt} = 4\pi\ \mathrm{V}\rho\left(\frac{3M}{4\pi D}\right)^{\frac{2}{3}} \quad \text{so} \quad \frac{dM}{dt} = 4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}}\ \mathrm{M}^{\frac{2}{3}}$$

Answer 2: First re-arrange the terms to form the integrands:

$$\frac{dM}{M^{\frac{2}{3}}} = 4\pi\mathrm{V}\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}} dt$$

Integrate both sides: $3M^{\frac{1}{3}} = 4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}} t$

Now solve for M(t) to get the answer: $M(t) = \left(\frac{4\pi V\rho}{3}\right)^{3}\left(\frac{3}{4\pi D}\right)^{2} t^{3}$

Raindrop Condensation - A typical raindrop might form so that its final mass is about 0.0001 kilograms and **D** = 1000 kg/m^3, under atmospheric conditions where ρ = 1 kg/m^3 and **V** = 2.0 m/sec (about 5 miles per hour). How long would it take such a raindrop to condense using this approximate mathematical model?

$$0.0001 = \left(\frac{4\pi(2.0)(1.0)}{3}\right)^{3}\left(\frac{3}{4\pi(1000)}\right)^{2} t^{3}$$

so t^3 = 2.98

and so it takes about **1.4 seconds**.

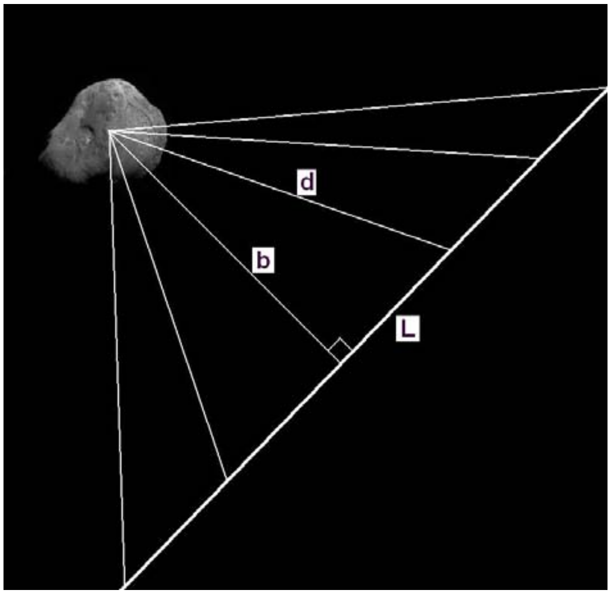
Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9×10^{24} kg, and **D** = 3000 kg/m^3, under conditions where ρ = 0.000001 kg/m^3 and **V** = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

$$5.9x10^{24} = \left(\frac{4\pi(1000)(0.000001)}{3}\right)^{3}\left(\frac{3}{4\pi(3000)}\right)^{2} t^{3}$$

so t^3 = 1.27×10^{40}

and so it takes about 2.3×10^{13} seconds or **750,000 years**.

Deep Impact - Comet Flyby



On July 4, 2005, the Deep Impact spacecraft flew by the comet Tempel-1 along a path shown in the figure to the left, at a speed of V=10 km/sec. Its closest distance to the comet was b = 500 kilometers at a time, t=0. The distance traveled along the path is given by L = Vt.

The diameter of the comet is D = 8 kilometers, and the distance to the comet in kilometers is d(t), so the angular diameter of the comet in arcminutes is given by

$$\Theta(t) = 3438 \frac{D}{d(t)}$$

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as *d(t)*?

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory, defined by the variables V, b and L? Simplify the formula by defining two constants B = 3438L/b and c = V^2/b^2.

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys by?

Problem 4 - What was the angular diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as d(t)? Answer: Use the Pythagorean Theorem and the diagram to determine that

$$d(t) = \sqrt{b^2 + V^2 t^2}$$

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory? Simplify the formula for Θ(t) by defining two constants B = 3438D/b and c = V^2/b^2. Answer:

$$\Theta(t) = \frac{B}{\sqrt{1+ct^2}}$$

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys by? Answer; Evaluate B and c for the specific values of D, b and V for the Tempel-1 flyby. Remember to use consistent units (all units in meters, and meters/sec). B = 3438 x 8,000 meters/(500,000 meters) so B = 55 arcminutes, and c = (10,000 meters/sec)^2/(500,000 meters)^2 so c = 0.0004 sec^{-2}. Then the formula becomes,

$$\Theta(t) = \frac{55}{\sqrt{1+0.0004t^2}}$$

where θ(t) will be the comet angular diameter in arcminutes. The rate-of-change of Θ(t) with respect to time is just its first-derivative, which we then evaluate for B=55 and c=0.0004. The formula will provide answers in units of arcminutes/sec:

$$\frac{d\Theta(t)}{dt} = -\frac{1}{2}B\left(1+ct^2\right)^{-3/2}(2ct)$$

or

$$\frac{d\Theta(t)}{dt} = -\frac{0.022t}{\left(1+0.0004t^2\right)^{3/2}}$$

Problem 4 - What was the angular diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Answer: For t=0 we get $\theta(0) = 55$ arcminutes. To decrease by a factor of two its final diameter has to be 27.5 arcminutes, so θ(t)=27.5, and solve for t to get **t= 86.6 seconds.**

From the derivative formula, and evaluating it at t=86.6 seconds, we find that
dθ/dt = **- 0.24 arcminutes/sec**.