



S T A R S

Their properties and evolution

Math textbooks routinely provide 'real world' examples for students using familiar examples drawn from every-day situations, but there are many other areas where proportional relationships and working with large numbers aid in understanding our physical world.

This collection of activities from **SpaceMath@NASA** is intended for students looking for additional challenges in the math and physical science curriculum in grades 6 through 8, but where the topics are drawn from astronomy and space science. As a science extension, 'Stars: Their properties and evolution' introduces students to some of the basic properties of stars and how they are formed and evolve through time.

The math problems cover basic scientific notation skills and how they apply to working with 'astronomically large' numbers. The evaluation of simple equations involving variables with integer exponents is also covered. Although converting decimal numbers to scientific notation may be first introduced by grade 6, it is common for actual manipulations involving multiplication and division to be covered in grade 8 along with evaluating simple equations with integer exponents. This book, therefore, is better suited to grade 7 or 8 students, who may also be covering the topic of stars in their physical science or earth science classes.

For more weekly classroom activities about astronomy and space visit the NASA website,

<http://spacemath.gsfc.nasa.gov>

Add your email address to our mailing list by contacting Dr. Sten Odenwald
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Original artwork kindly provided by Nathalie Rattner
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Ms. Rattner is a highly versatile, award winning Canadian artist whose works have been sold to private collections in North America. Throughout her career, she has been inspired by her love of life, history and the diversity of the world around her. The artwork used in this book was created by her when she was in middle school !

NCTM: Principles and Standards for School Mathematics

Grade 6-8 : In grades 6–8 all students should

- understand and use ratios and proportions to represent quantitative relationships
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;

Common Core Mathematics Standards

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE2C - . Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*

Work with radicals and integer exponents.

8.EE.1 - Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

8.EE.3 - Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.*

8.EE.4 - Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.





We all know a little bit about our nearest star - the Sun! It's a yellow-colored star, which means it is hotter than 'red hot' and sizzles with a temperature of 5,770 kelvin.

Some stars are much hotter than the sun. Spica which sizzles at 30,000 kelvin. Most stars are colder than the sun like our nearest star Proxima Centauri at 2,500 kelvin.

Astronomers use the kelvin temperature scale because the zero point on this scale, called Absolute Zero, means that all motion stops. You can convert from the Celsius scale to the kelvin scale by adding 273 to the Celsius measurement. A human body has a temperature of 37 Celsius or $37+273 = 310$ kelvins.







The sun is very far away too!

Astronomers have many different ways to measure its distance from Earth.

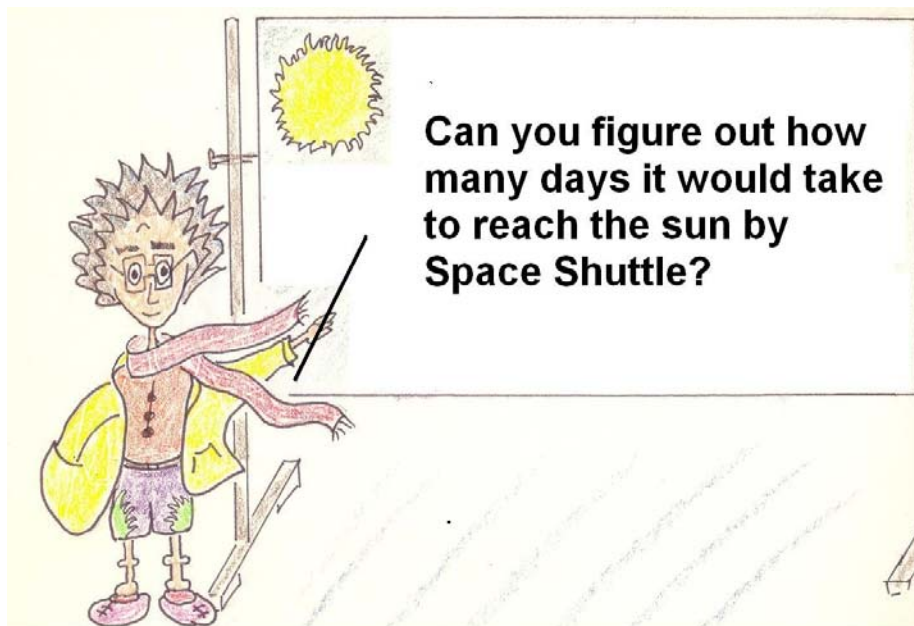
From the sun to Earth it is a distance of 93 million miles. This is also the same as 150 million kilometers if you prefer metric units, which scientists do!



Even at this distance, the sun keeps us warm in the daytime, and its light lets plants grow.

We can also turn it into electricity using solar panels on the roofs of our houses!

Suppose you could travel to the sun in a spacecraft like the Space Shuttle. It travels at a speed of 18,000 miles per hour (28,000 kilometers per hour).







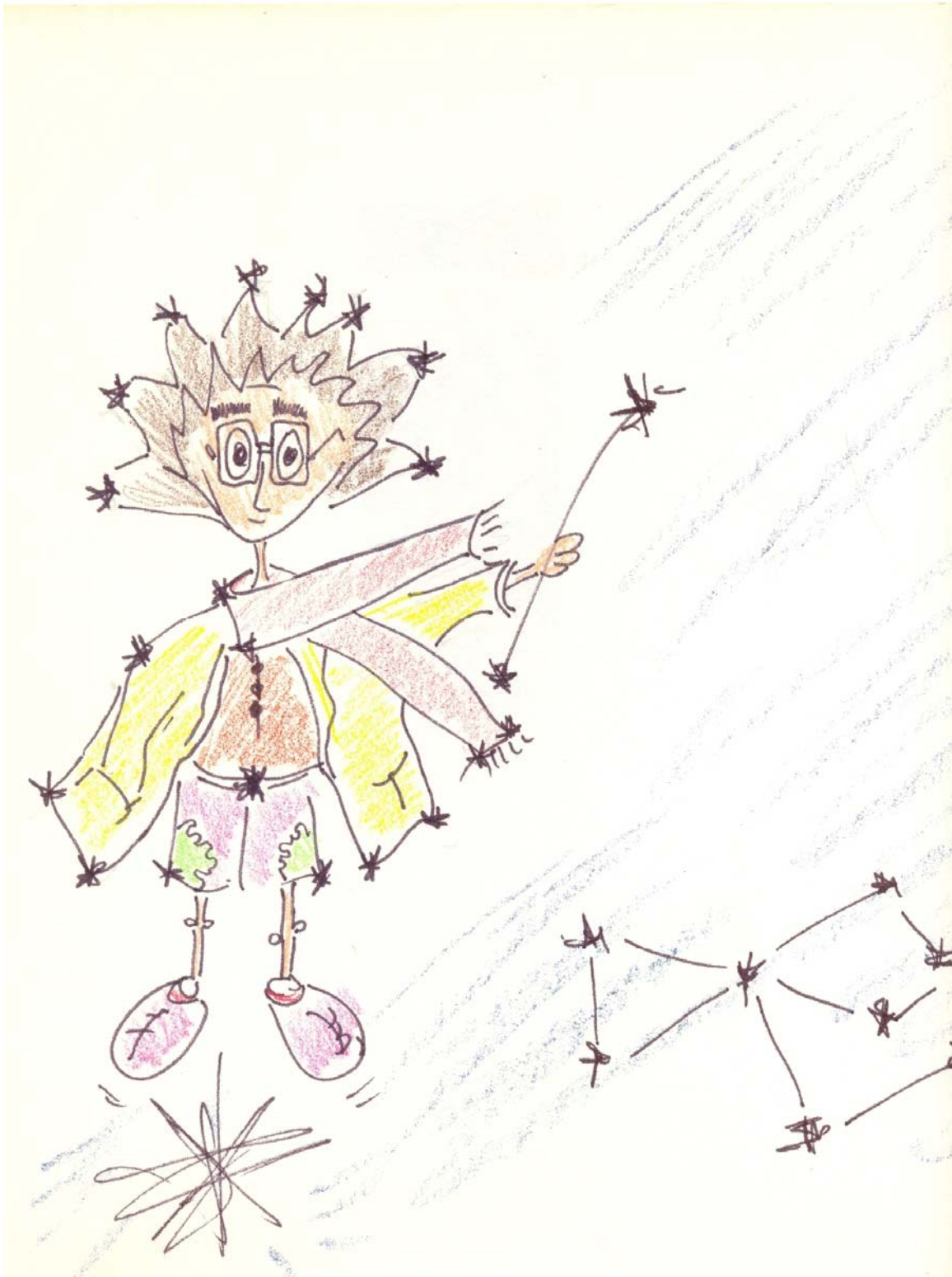
The sun is an amazing star.

It is also very important to life on Earth, and the existence of our entire solar system.

Without the gravity from the sun, the planets would just fly away into deep space.

But our sun is just one of many more stars in a vast system we call the Milky Way galaxy. Our galaxy has billions of stars in it! It is so huge that we cannot take a picture of it, but artists can imagine what it looks like by using the clues that astronomers provide them.





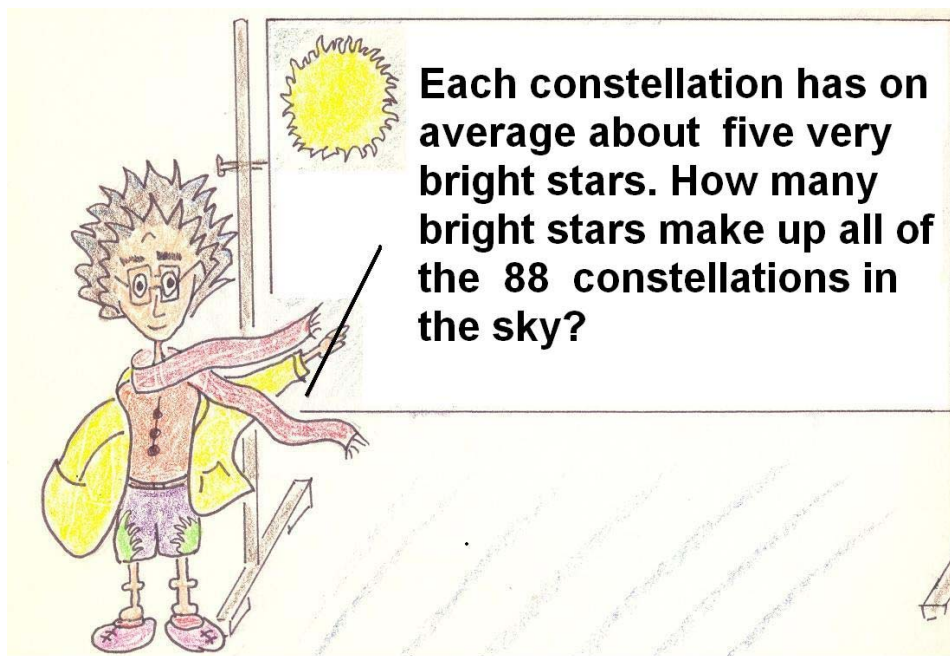


There are so many stars near our sun that when we look up at the sky at night we can see thousands of them. We can even group them together into patterns called constellations.

Over the last 5,000 years humans have identified many different constellations.

Many of these constellations such as Apis the Bee, or Globus Aerostaticus the Hot-air Balloon, have fallen out of fashion and no longer exist. Today astronomers recognize only 88 constellations.

Astronomers still use constellation names when they speak or write about objects in particular regions of the sky. It's just a fast and convenient way to describe what you are talking about without going into the details of exact positions.



Each constellation has on average about five very bright stars. How many bright stars make up all of the 88 constellations in the sky?





You can only see stars if they are bright enough to be detected by the retina of your eye.

With a pair of binoculars you can easily see many more stars in the sky because the faint light from them is amplified.



Your eyes might only see the seven stars in the Big Dipper, but with a pair of binoculars you would see something like the picture above!





Telescopes are even better for seeing faint stars.

On a clear night, you might only see a few stars in an area of the sky about the size of the full moon.

With a telescope, and looking at the same spot of sky through an eyepiece, you can easily see hundreds of stars.





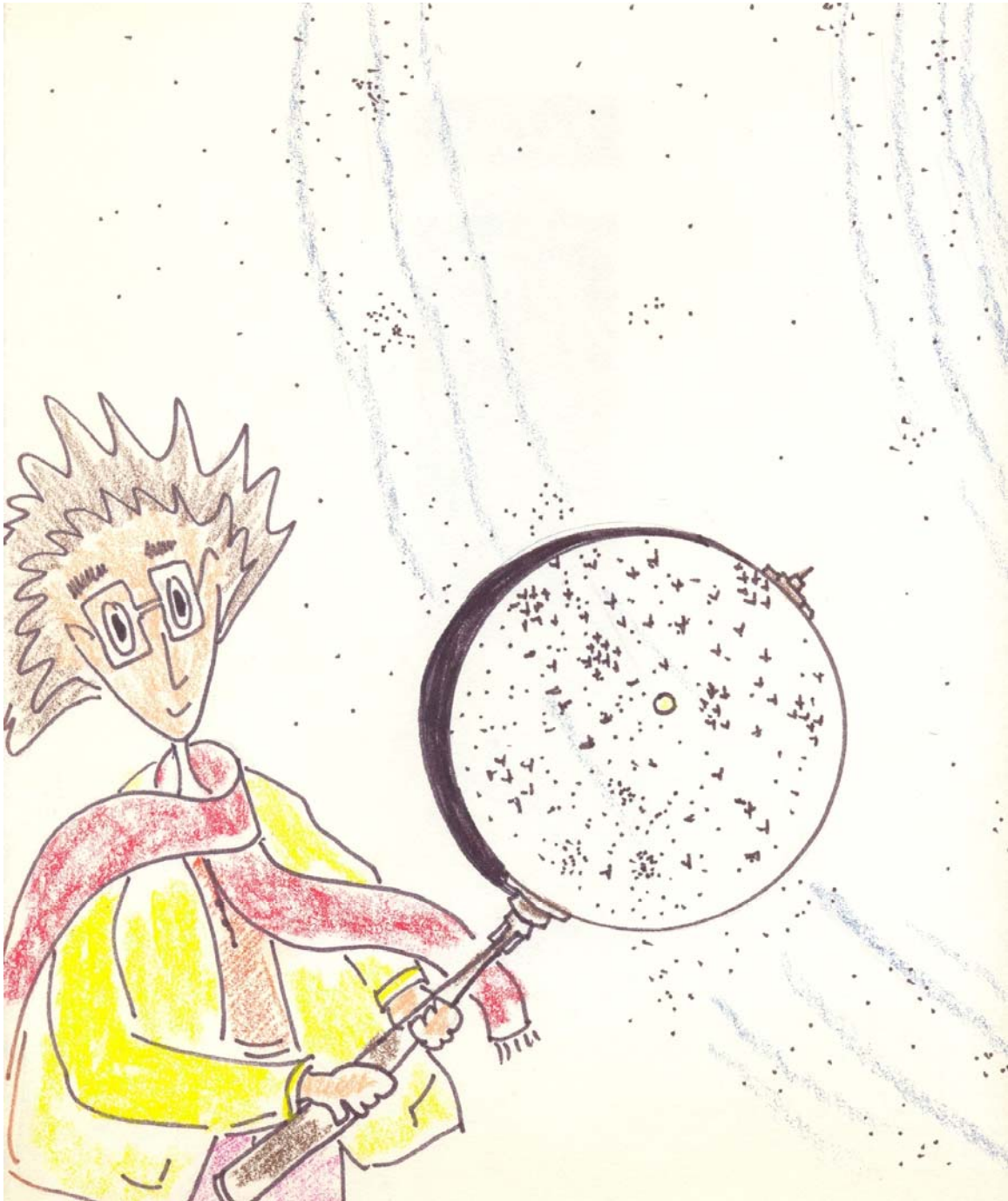


A camera attached to a tripod and pointed at the sky is even better for seeing faint stars. A camera will let you take pictures of much fainter stars than what your eye can see.

Here is a picture of the Big Dipper taken by photographer Jerry Lodriguss.

A camera attached to a telescope will also let you record thousands of faint stars from very small areas of the sky. Below is a small patch of the sky in the constellation Corvus the Crow photographed by Richard Berry. The two bright stars can be easily seen by your eye. All of the other stars are invisible to us without binoculars or a telescope.







But no matter how powerful your telescope is, you will only see stars that are close to the sun.

Our Milky Way galaxy is so big that no telescope or camera can see all of the stars that it contains.

For centuries, astronomers have tried to catalog all the stars they could see. They wanted to accurately measure their brightness, and precisely locate their positions in the sky.



Early catalogs had a few thousand stars. From these catalogs, astronomers created accurate star maps or atlases showing a very accurate view of the sky. They also drew in with great artistry what the constellation figure looked like.

These early atlases were important for sailors who navigated the oceans by 'sighting stars' with their instruments called sextants.

Astronomers also counted the stars.



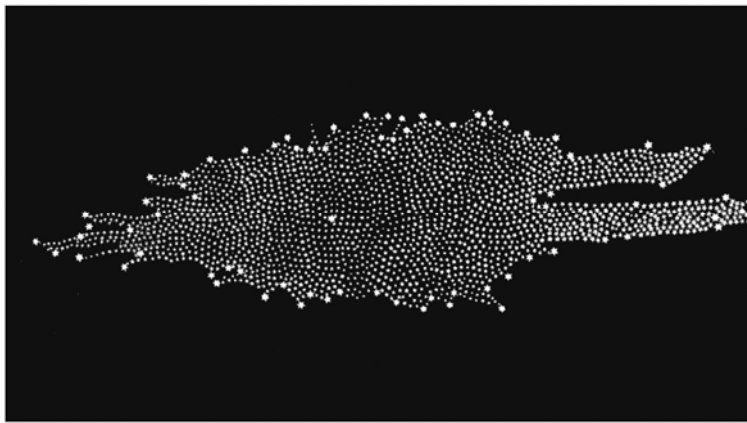


Why would an astronomer want to count stars?

Well...how many stars are there in the sky?

This is a question you might wonder about when you look at the glorious, and mysterious, night sky.

But why would a scientist care about this?

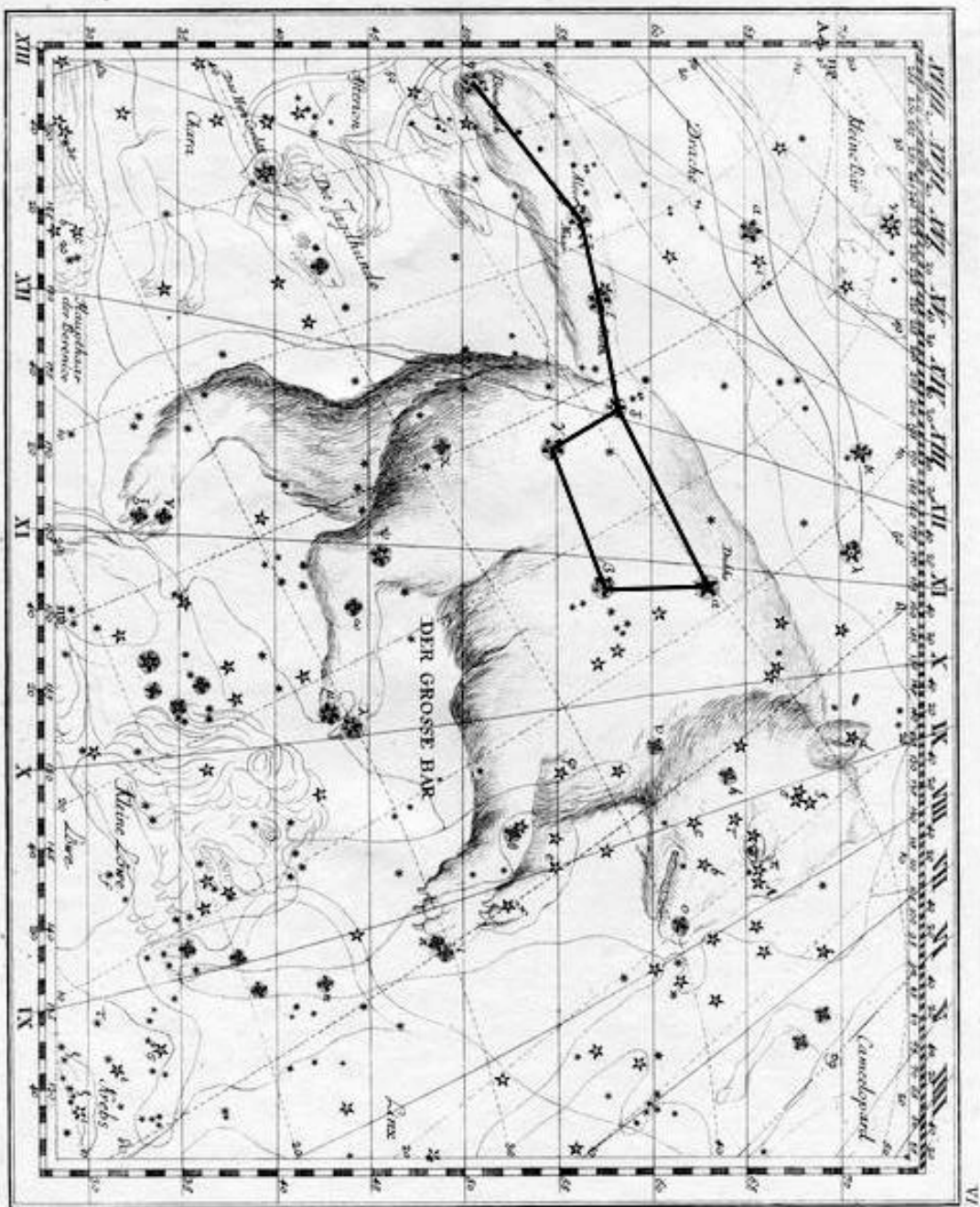


It turns out that counting stars in different patches of the sky lets you explore how big the Milky Way galaxy is, even though you cannot see the entire system of

stars! Here is a model created by Sir William Herschel in 1750 based upon his star counts in the sky.

If you count more stars in one direction than you do in another, this tells you that there is more of the Milky Way galaxy in one direction than in the direction where you see fewer stars.

By counting stars carefully, and to different levels of faintness for the stars you tally, astronomers can build up a picture of how the Milky Way galaxy is shaped, and how big it is.





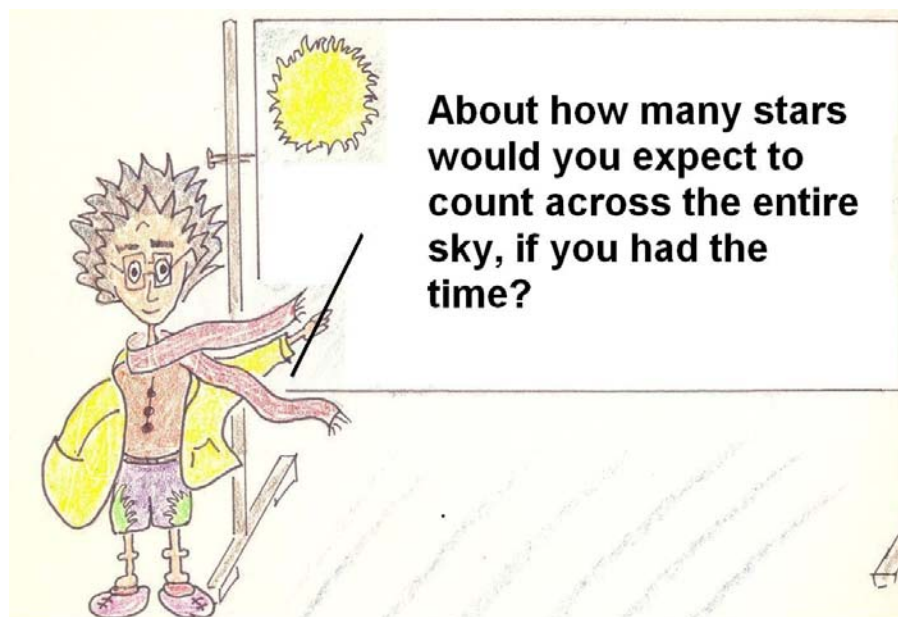
So how many stars are there?

Counting them one by one is not an easy task at all. If you counted them 1.....2...3... you could never count them all, because as the Earth rotates, the sky moves and new stars appear on the eastern horizon all the time.

A faster way to count stars is to pick a small patch of the sky, count every star you find there, and then multiply your count by the number of patches in the sky.

This is a star atlas of the constellation Ursa Major - The Great Bear. It shows the positions of all of the stars you could see with your naked eye on a clear moonless night. It covers an area of the sky that is about $\frac{1}{18}$ the size of the full sky. How many stars do you count in this patch?

Suppose that this patch were similar to the other 17 patches in the sky of the same area.



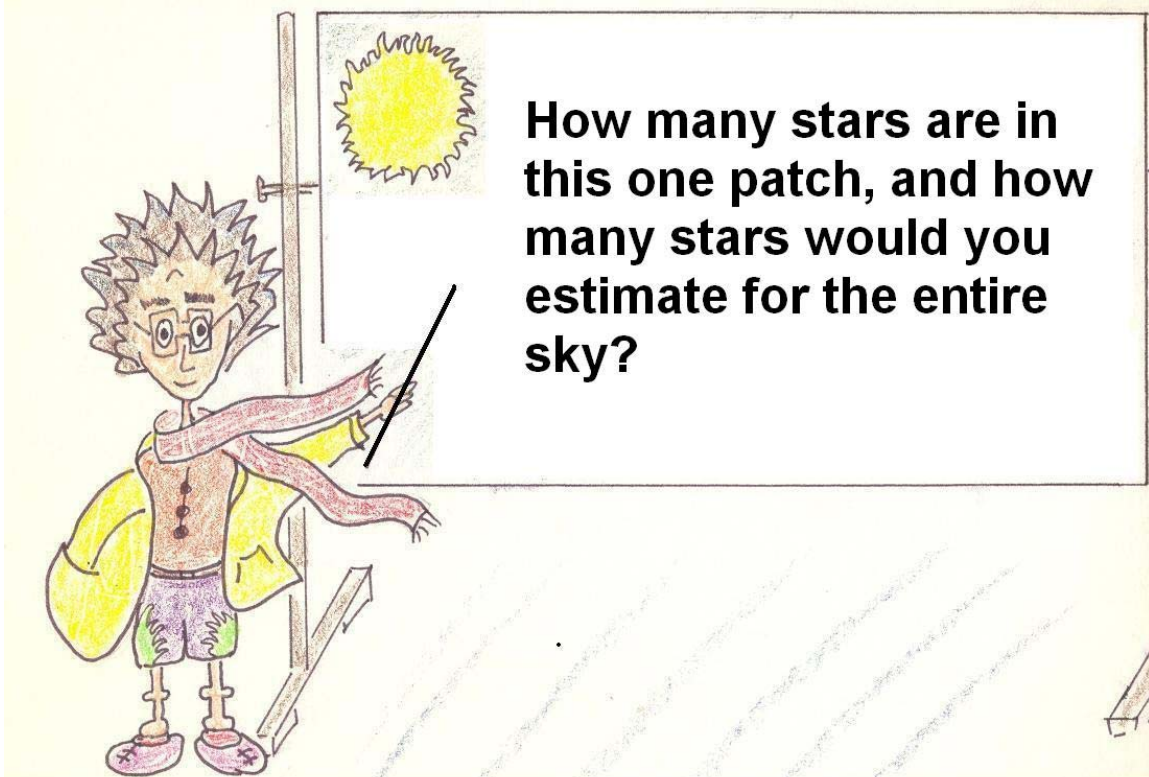




This photograph was taken by Till Credner and Sven Kohle from a dark sky location in Namibia.

The two bright stars are called Algorab (lower) and Minkar (upper). Algorab is located 87 light years from Earth, while Minkar is 318 light years from Earth. Algorab is only about 260 million years old, and was not born until long after Earth was formed.

This photograph contains stars that are a thousand times fainter than what the eye can see. The field of this photograph is about $\frac{1}{3500}$ of the full sky. You could fit four full moons across the width of this picture!





One of the challenges in astronomy is that you almost always have to work with very big numbers - million...billion...trillion....and some are so big they don't even have simple names!

Let's take a look at some large collections of stars, and use scientific notation to indicate their sizes.

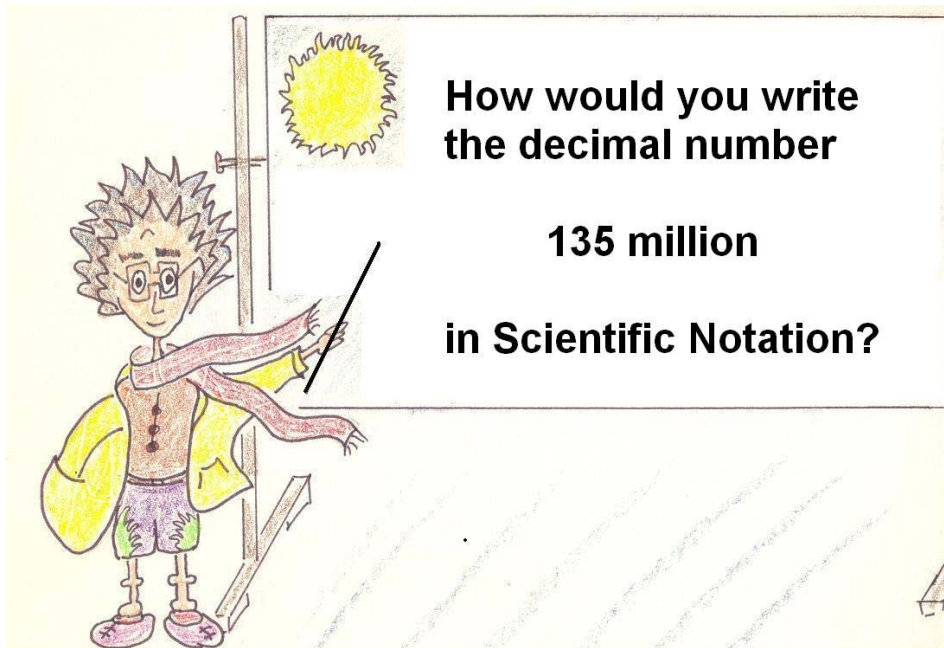
Remember: scientific notation uses exponents to count the number of factors of 10 in a number. For example:

$$\text{Ten} = 10 = 10^1$$

$$\text{One hundred} = 100 = 10 \times 10 = 10^2$$

$$\begin{aligned}\text{One thousand two hundred} &= 1,200 \\ &= 1.2 \times 10 \times 10 \times 10 = 1.2 \times 10^3\end{aligned}$$

Do you see the pattern?





Scientific notation lets you do much more than just write very big and very small numbers in a more compact way. You can also multiply and divide them and make calculations very simple to perform.

Suppose you have two numbers 2.5×10^{15} and 0.0005

Multiplication:

First write all numbers in scientific notation:

$$(2.5 \times 10^{15}) \times (5.0 \times 10^{-4})$$

Next group the parts together:

$$(2.5 \times 5.0) \times (10^{15} \times 10^{-4})$$

Multiply the numeric part: $2.5 \times 5.0 = 12.5$

Then add the exponents: $15 + (-4) = 11$

Assemble the answer: 12.5×10^{11}

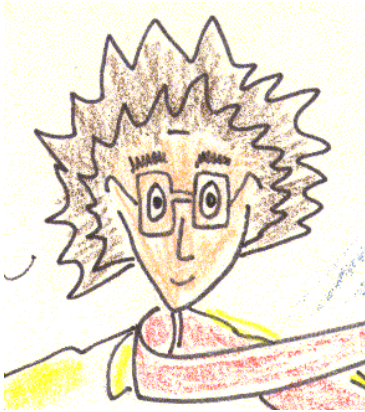
And finally, write the answer in scientific notation:

$$\text{Answer} = 1.25 \times 10^{12}$$

For division, you divide the numeric parts and subtract the bottom exponent from the top exponent:

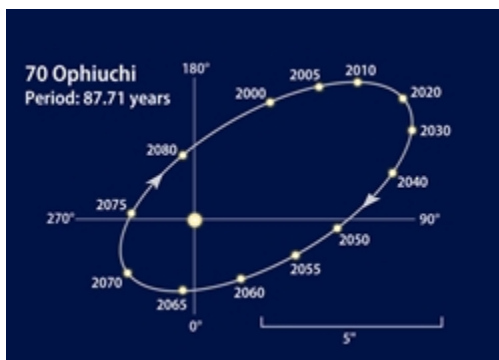
$$\begin{aligned} 2.5 \times 10^{15} / 5.0 \times 10^{-4} &= (2.5/5.0) \times (10^{15}/10^{-4}) \\ &= 0.5 \times 10^{15-(-4)} \\ &= 0.5 \times 10^{19} \\ \text{Answer} &= 5.0 \times 10^{18} \end{aligned}$$





Almost $\frac{2}{3}$ of all the stars in the sky travel through the Milky Way in pairs.

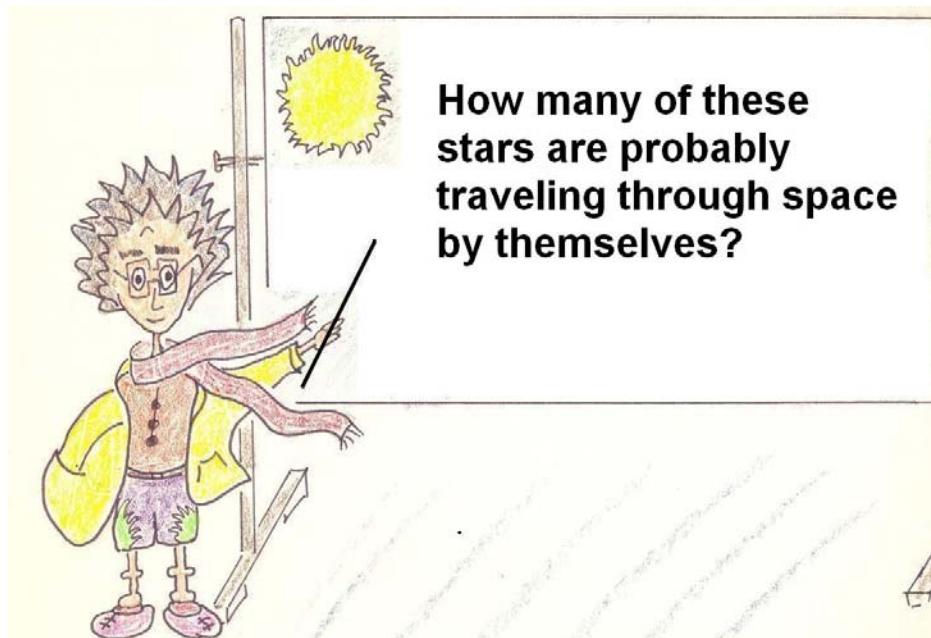
Here is one very famous pair of stars called the Albireo double star system. It is located in the constellation Cygnus the Swan. These two stars orbit each other like our Jupiter orbits the sun.

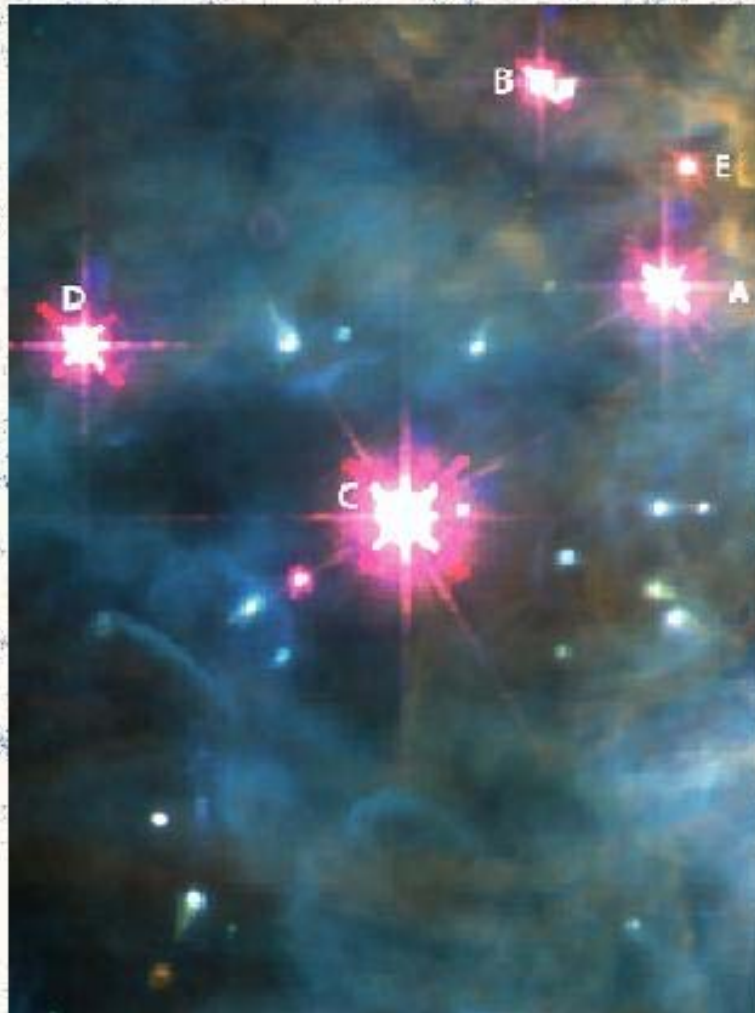


There is no way, just by looking, to decide if two close stars are a binary system or not. You have to measure their positions for many years to see if one star is actually orbiting the other! Here is the orbit of the binary star 70

Ophiuchi. It has an orbital period of nearly 88 years.

Suppose you took a photograph of the sky and counted 300 stars.







Here is the Trapezium star cluster in the constellation Orion the Hunter.

The five stars in this cluster are among the youngest stars in the sky. They are barely 300,000 years old!

Each star contains about 30 times the mass of our entire sun. They produce so much energy they light up the gas inside a vast nearby interstellar cloud to form a colorful nebula !



There are only 5 stars in the Trapezium star cluster. The surrounding star nursery, shown here, is called the Orion Nebula. It has produced thousands of other hot young stars over the last 5 million years.

In the future, millions of years from now, the gas will disperse and the nebula will vanish leaving only the stars behind for us to see.

At a distance of 'only' 1,344 light years, it is the closest nebula to the sun in our neighborhood of the Milky Way.





The Pleiades star cluster is a famous cluster that has been known by humans for thousands of years.

You can find it in the constellation Taurus the Bull if you know where to look. It is nearly 400 light years from Earth.

It was formed about 50 million years ago, and you can still see some of the gas and dust left behind from the formation of these stars.

How many stars are in this cluster?

Astronomers have counted over 1,000 but to the naked eye you can only see about seven of the brightest ones.

They are named after the seven daughters of Atlas and his wife Pleione: Alcyone, Electra, Maia, Merope, Taygeta, Celaeno and Asterope.

The stars Atlas and Pleione are also included to keep this celestial family together!



The Beehive star cluster in the constellation Cancer the Crab is



also called Praesepe. This means 'manger' and the ancient Romans and Greeks thought this was a manger from which a celestial donkey was eating his daily meals!

stars (10^3).

Praesepe is over 600 million years old and like the Pleiades, it contains about 1000

On Earth, light things float and heavy things sink because of gravity. The same thing can happen to the stars in a star cluster!

Praesepe is so big that the heavier stars have sunk to the center of the cluster and the lighter stars form a halo surrounding this core.





The star cluster NGC 3603 is in the constellation Carina, and located 20,000 light years from Earth.

With over 10,000 stars, (1.0×10^4), this is a very massive cluster.

This cluster is so young that it is partly surrounded by the enormous gas clouds from which it was formed no more than 1 million years ago! The core of the cluster looks like this:







NGC 265 is a star cluster located in the constellation Tucana the Tucan.

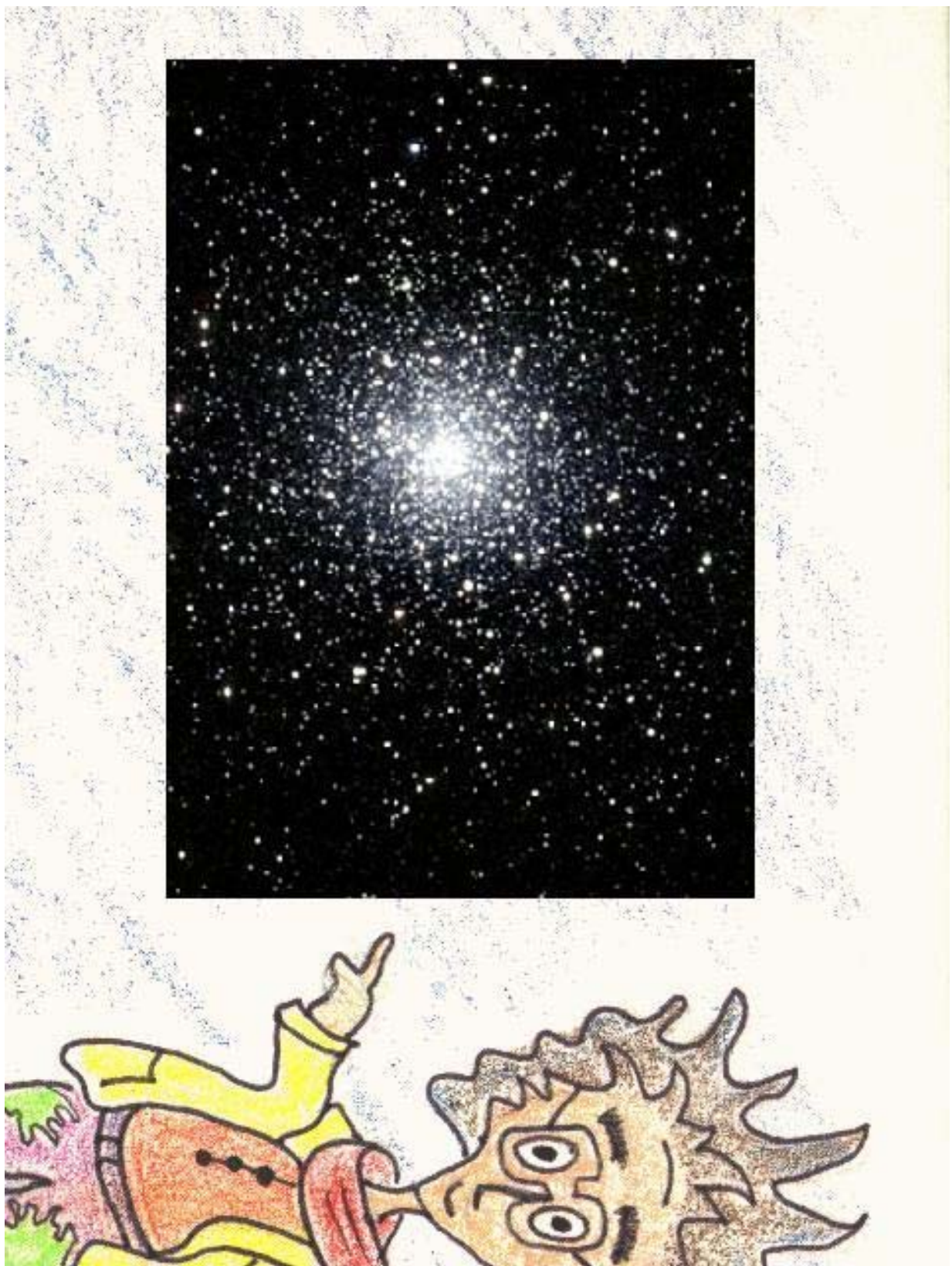
It is 5 million years old and contains about 5,000 stars (5.0×10^3).

Notice that, once you get more than about 1000 stars together in the same cluster, the cluster begins to look round in shape.

This happens for the same reason that planets become round once they get big enough!

The gravitational force acting between all the stars causes the stars to follow circular orbits. The entire system of stars takes on a round shape.

The more stars you collect - the rounder the cluster becomes.





This star cluster in the constellation Hercules is called Messier-13 or The Great Globular Cluster in Hercules.

Although it is nearly 23,000 light years (2.3×10^4 ly) from Earth, you can still see it in the summer night skies as a faint ball of light in Hercules...if you know exactly where to look!

Notice how round it looks! This is called a 'globular' shape so star clusters like this are called globular star clusters.

This famous globular cluster contains about 300,000 stars. In scientific notation we would write this number as

$$3.0 \times 10 \times 10 \times 10 \times 10 \times 10 = 3.0 \times 10^5$$

Unlike the other smaller clusters we have looked at, the stars in globular clusters are very old. They were formed at about the same time that our entire Milky Way galaxy formed 12 billion years ago!





The great globular star cluster Omega Centauri is so massive and full of stars it was once mistaken for a star!

As its name suggests, it is located in the constellation Centaurus the Centaur. It is 15,800 light years from Earth and contains over 15 million stars.

We can write this in scientific notation as

$$1.5 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$$

$$1.5 \times 10^7$$

From a vantage point below Earth's equator in South America, it is so large that even the full moon would barely cover it.

Along with other globular star clusters, it was formed about 12 billion years ago.



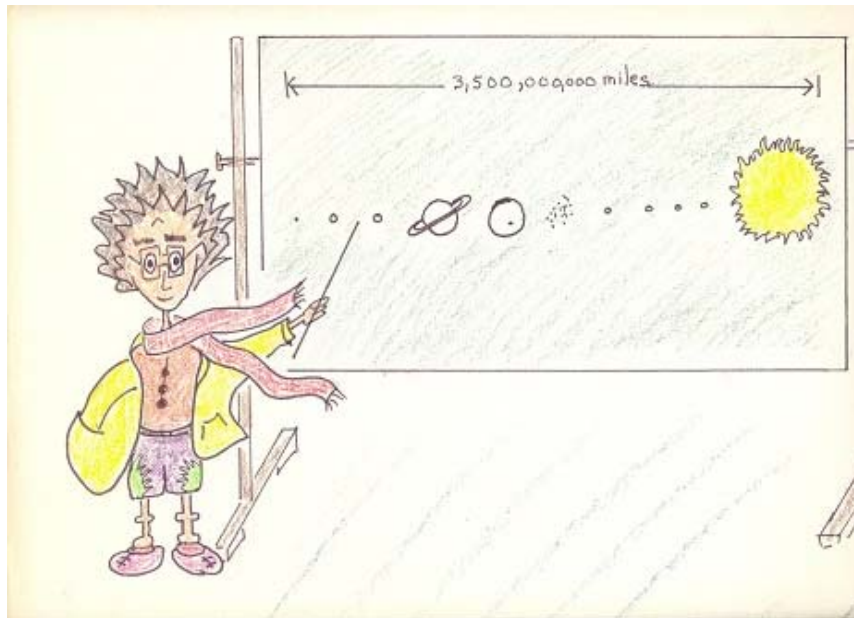
So there you have it!

We can count stars in the sky, and this tells us a lot about the kind of system of stars we live in.

Because there are a LOT of stars, we have to use scientific notation to more easily write these large numbers. It's very easy to loose track of how many zeros you wrote down. One lost zero can make a big difference when you talk about stars and the Milky Way.

Scientific Notation Quiz

On the next page, write the indicated numbers in scientific notation!



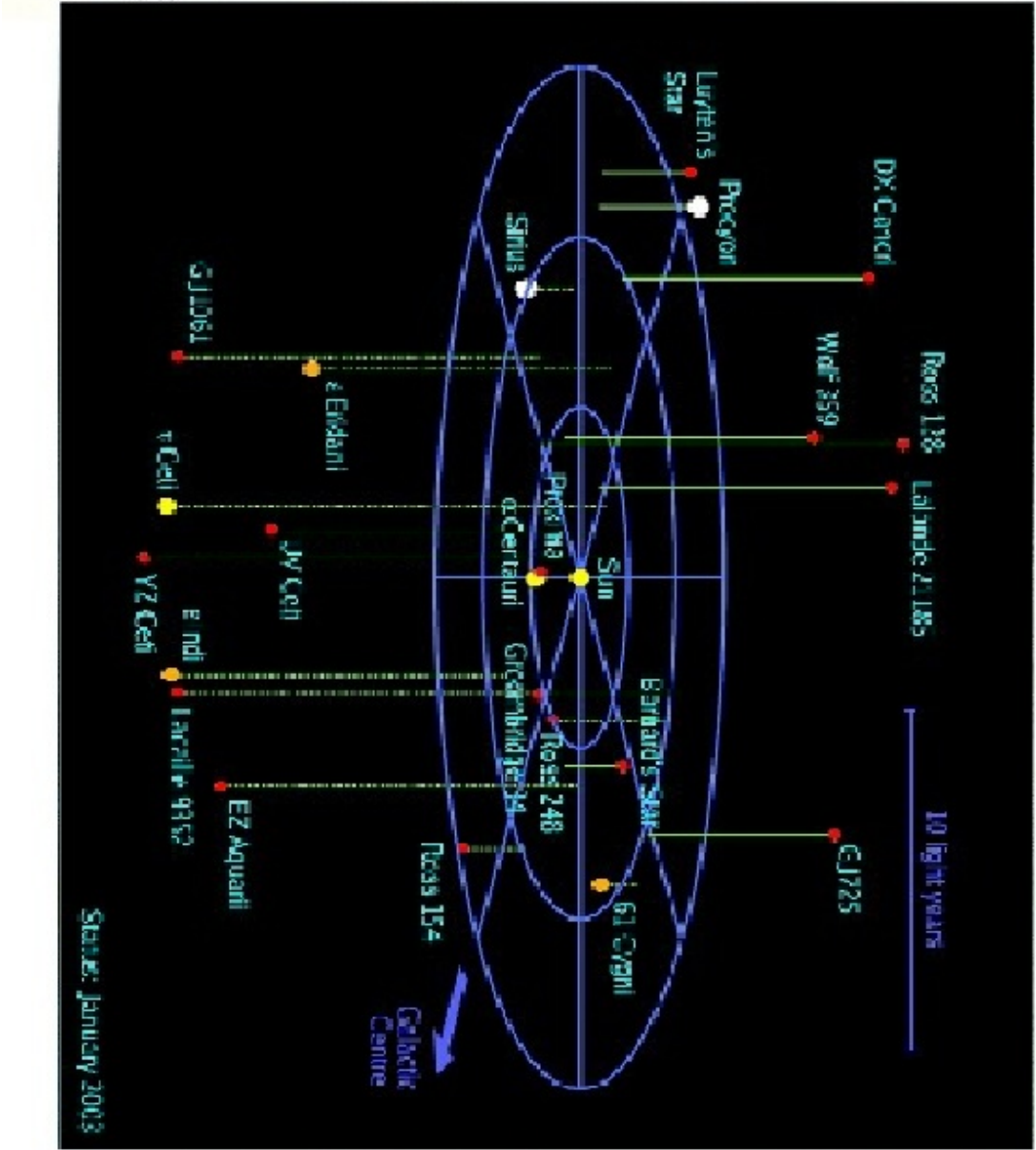


Now translate the numbers in the following paragraph into scientific notation.

The Omega Centauri cluster is also called NGC-5139, and is located fifteen thousand eight hundred light years from Earth. It contains about 15,000,000 stars. These stars are very old and were probably formed about 11.5 billion years ago. The diameter of this spherical star cluster is about 170 light years. Since 1 light year equals a distance equal to 63,000 times the diameter of our solar system, Omega Centauri is about 11 million times the diameter of our solar system in size!

When writing this paragraph, when would you use scientific notation to express the number values?







Stars are not all the same brightness, because they are at different distances from Earth.

These distances are so large that astronomers use a unit called the light year to keep track of them more easily.

One light year equals 9.5 trillion kilometers. It is the distance that light travels in one year at a speed of 300,000 kilometers/second. Can you figure this out yourself?

The distance to the star Betelgeuse can be written as 600 light years, 5,700 trillion kilometers, or 5,700,000,000,000,000 km. Which style would you prefer to use?

The closest naked-eye star you can see is the sun-like star Alpha Centauri located 4.3 light years from Earth.

The farthest star you can see with your naked eye is the yellow hypergiant star Rho Cassiopeia located about 11,650 light years from Earth.

Most of the other naked-eye stars you can easily see are within a few hundred light years of Earth.

Here is a table of some nearby stars.

Name	Distance (Light years)	Luminosity (sun = 1.0)	Temperature (Kelvins)
Proxima Centauri	4.2	0.0017	3,042
Alpha Centauri A	4.3	1.6	5,790
Alpha Centauri B	4.3	0.5	5,260
Barnard's Star	5.9	0.0035	3,134
Wolf 359	7.8	0.0009	2,800
Lalande 21185	8.3	0.025	3,800
Sirius A	8.6	25.4	9,940
Sirius B	8.6	0.026	25,200
Luyten 726-8 A	8.7	0.00006	2,670
Luyten 726-8 B	8.7	0.00004	2,600
Ross 154	9.7	0.0038	3,340
Ross 248	10.3	0.0018	2,800
Epsilon Eridani	10.5	0.34	5,084

Some things to note:

Six of the stars are in binary star systems: Sirius A and B, and Alpha Centauri A and B, and Luyten 726-8 A and B.

The most common stars near our sun are very faint (like 1-watt bulbs compared to 100-watt bulbs!)

Their names seem strange because they come from star catalogs created by astronomers named Ross, Luyten, Lalande or Wolf!

Here is a table of the brightest stars you can see with your naked eyes!

Name	Distance (light years)	Luminosity (sun=1)	Temperature (Kelvins)
Sirius	8.6	25.4	9,940
Canopus	310	15,100	7,300
Arcturus	37	170	4,290
Alpha Centauri	4.3	1.6	5,790
Vega	25	40.1	9,602
Rigel	770	120,000	12,130
Procyon	11	6.93	6,530
Betelgeuse	640	120,000	3,140
Achernar	140	3,150	15,000
Hadar	530	41,700	25,000
Capella	42	78.5	4,940

Some things to note:

The brightest stars we can see are often thousands of times brighter than our sun (A 1000-watt bulb compared to a 1 watt bulb), so we can see them to greater distances.

The brightest stars are typically hotter than our sun (5,770 kelvins).

They all have pretty names because people knew about them for thousands of years!

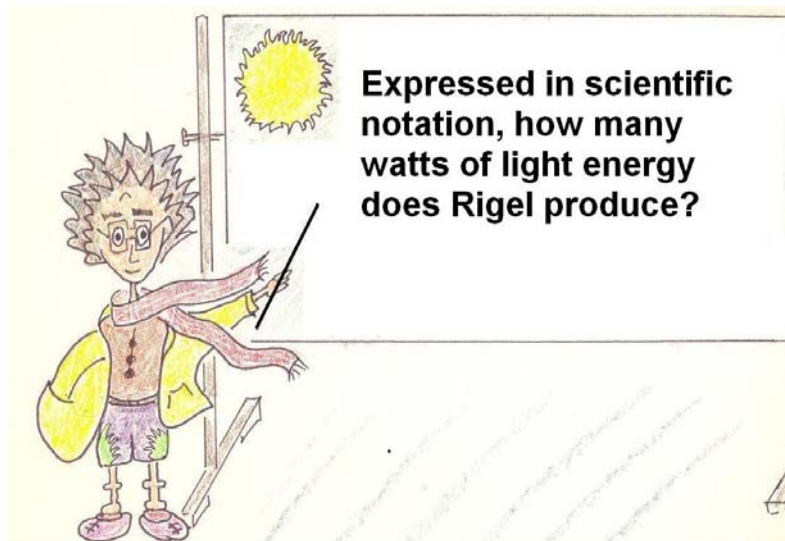


Stars are not only located at different distances from Earth, but they also produce different amounts of light!

A 100-watt bulb is much brighter than a 1-watt bulb, but we can make both of them appear to be the same brightness if we put the 100-watt bulb farther away from us.

The amount of light power that a star produces is called its **luminosity**. Astronomers measure luminosity in watts. That is why we need to use scientific notation to write the number of watts. In English words, the luminosity of the sun is 380 trillion trillion watts. Can you write this in scientific notation? This is such an important number that astronomers call it one Solar Unit of luminosity.

Sometimes it is easier for astronomers to describe other stars in comparison to our own sun. The star Rigel has a luminosity of 120,000 Solar Units.

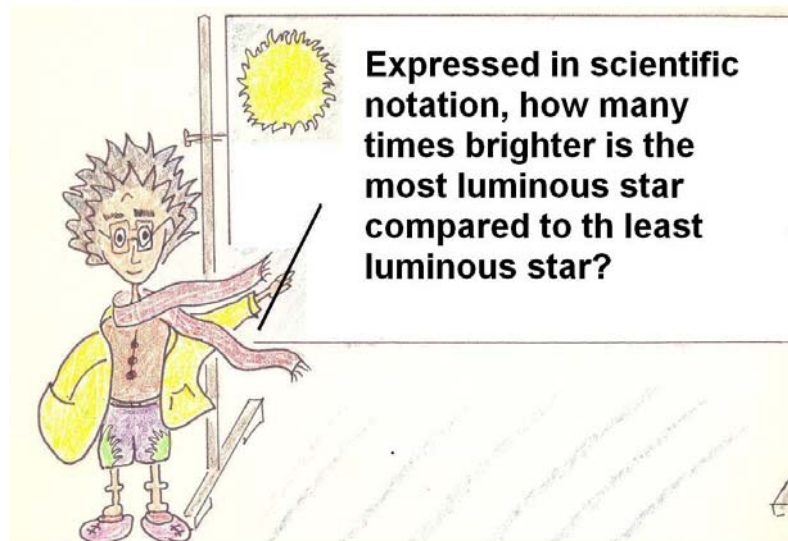


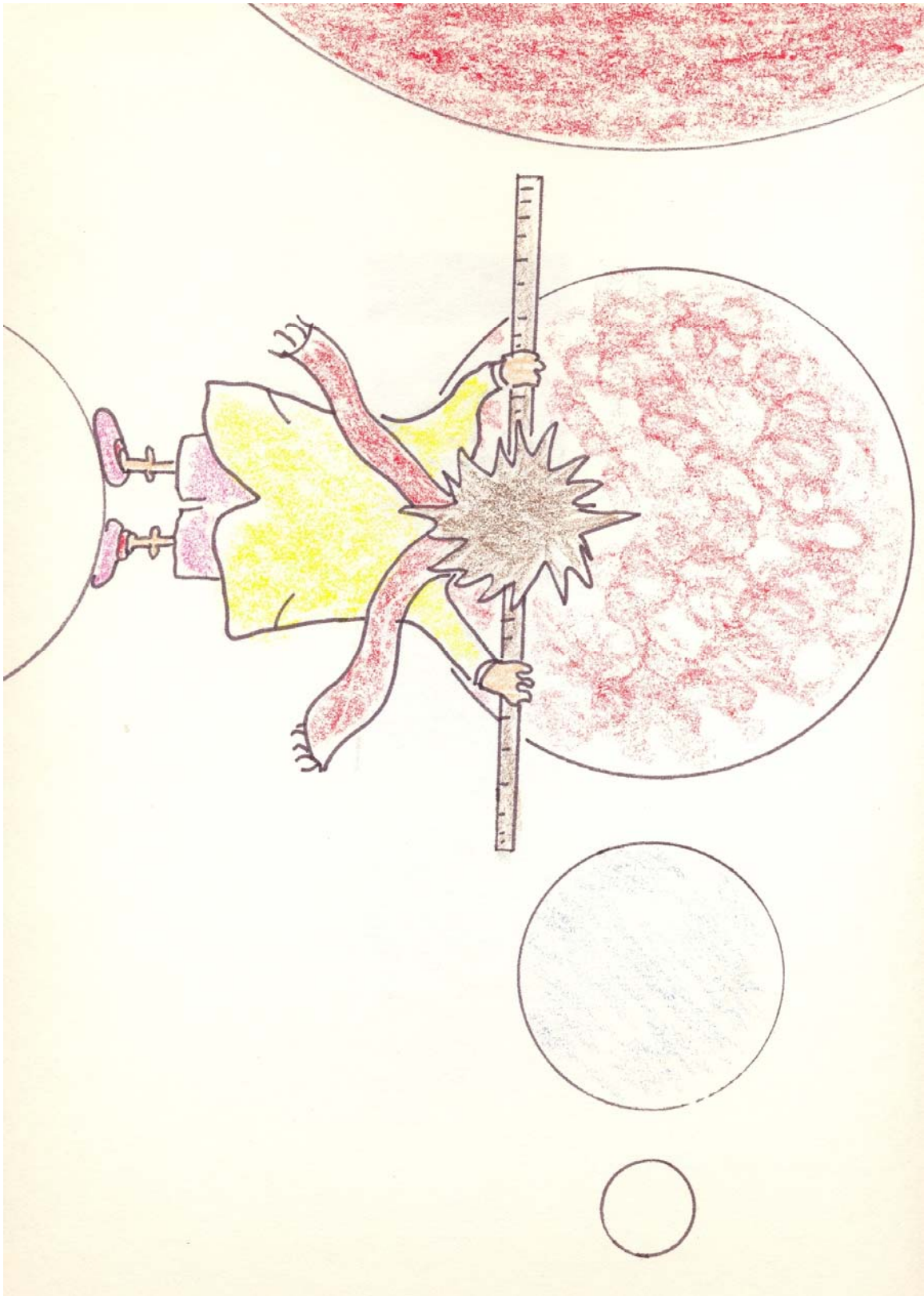


The luminosities of stars, the quantity of light energy they emit into space, spans a huge range of values.

This table gives a sampling of stars from the most luminous ones that we know, to the dimmest ones.

Name	Luminosity (sun=1)	Distance (light years)	Stellar Class
R136-A1	8,700,000	160,000	Hypergiant
Eta Carinae	5,000,000	7,500	Hypergiant
V729 Cyg B	1,300,000	5,000	O Giant
Betelgeuse	120,000	640	Red super giant
Canopus	15,100	310	Yellow giant
Achernar	3,150	140	Blue giant
Sun	1.0	0.0	Yellow dwarf
Lalande 21185	0.025	8.3	Red dwarf
Proxima Centauri	0.0017	4.2	Red dwarf
Barnard's Star	0.0035	5.9	Red dwarf
Luyten 726-8 B	0.00004	8.7	Red dwarf
Gliese 229B	0.0000064	19.0	Brown dwarf







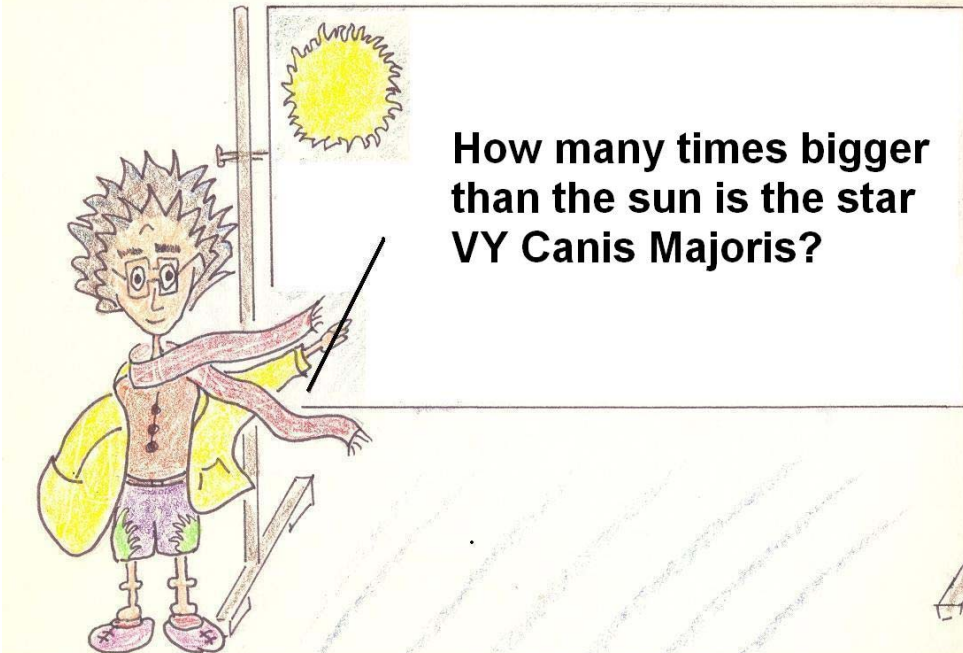
When astronomers refer to the size of a star, they can either mean its diameter or its mass. Although the mass of a star does not change as a star grows old, its diameter can change dramatically.

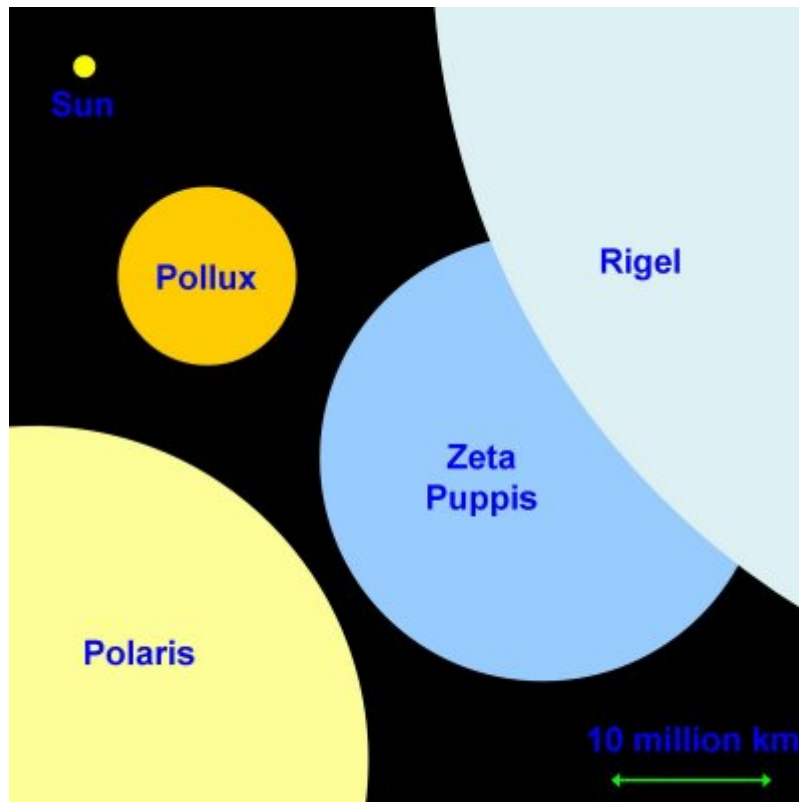
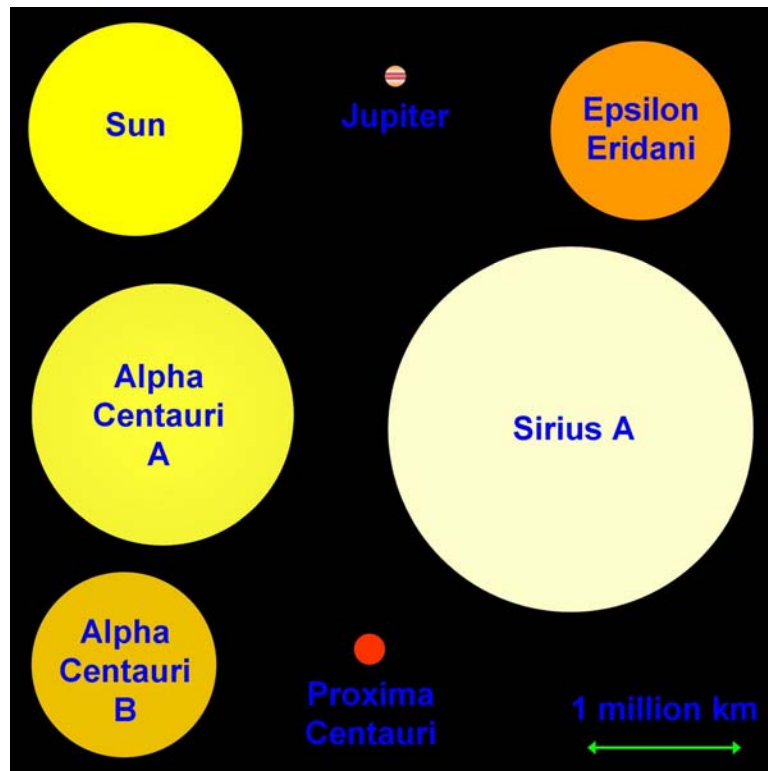
Let's first look at the diameters of stars compared to our sun.

Our sun has a diameter of 1.4×10^9 meters. Earth has a diameter of 1.3×10^7 meters. Using scientific notation, how many times bigger is the sun than Earth?

We want to form the ratio $1.4 \times 10^9 / 1.3 \times 10^7$ and we evaluate it as $(1.4/1.3) \times 10^{(9-7)}$ to get 1.08×10^2 or 108 times. The sun is 108 times bigger than Earth.

OK...now you try! The red supergiant star VY Canis Majoris has a diameter of 9.9×10^{11} meters.







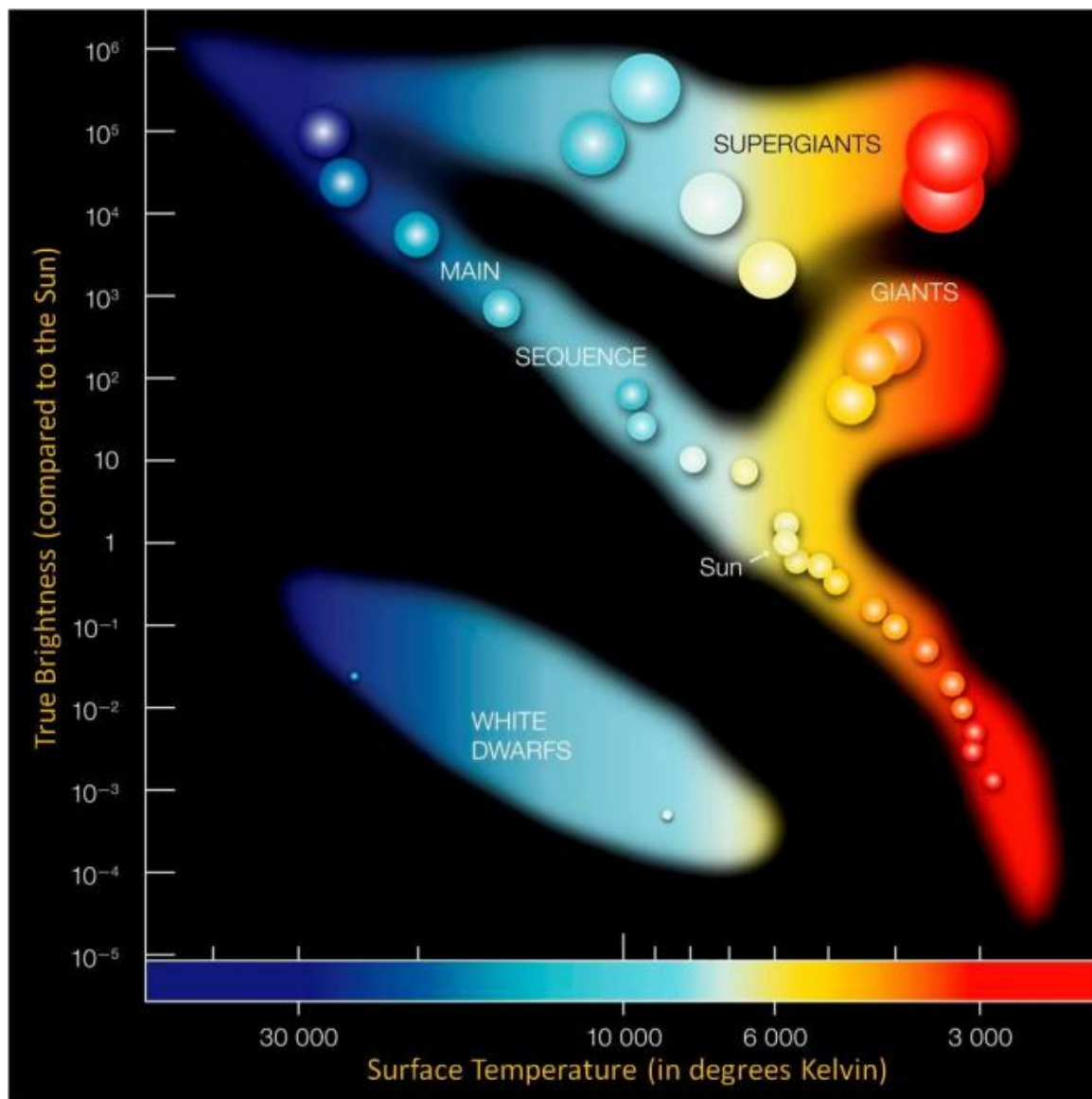
Stars do not all have the same masses either!
The mass of our sun is about 2.0×10^{30}
kilograms. That is equal to about 333,000
times the mass of our Earth!

Most stars have less mass than our sun, but a
few stars have more mass. Here are two
tables of the ten most massive stars, and the
ten least massive stars that we know about.

Most Massive		Least Massive	
Name	Mass (sun=1)	Name	Mass (sun=1)
R136 A1	265	OTS-44	0.013
R136 A2	195	Oph 1622 A	0.014
R136 C	175	Oph 1622 B	0.016
HD 269810	150	Gliese 229B	0.021
VFTS 682	150	2M1207	0.021
Peony Star	150	Epsilon Indi BB	0.024
R136A3	135	HD 98230B	0.037
NGC-3603B	132	Teide 1	0.041
Arches F9	120	Epsilon Indi BA	0.045
Eta Carina A	120	Wolf-424B	0.05
NGC-3603A1a	120	Gliese 570D	0.05



By what factor is the most massive star
bigger than the least massive star?





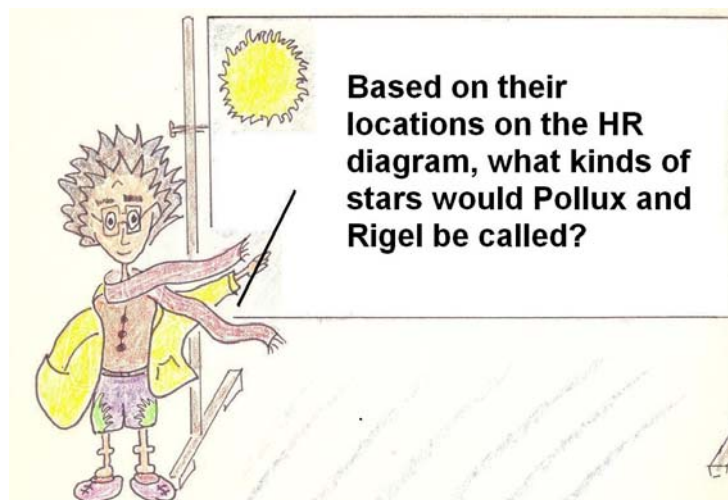
OK...so we know about the temperature of a star and its luminosity. What would happen if we plotted each star by its temperature and luminosity?

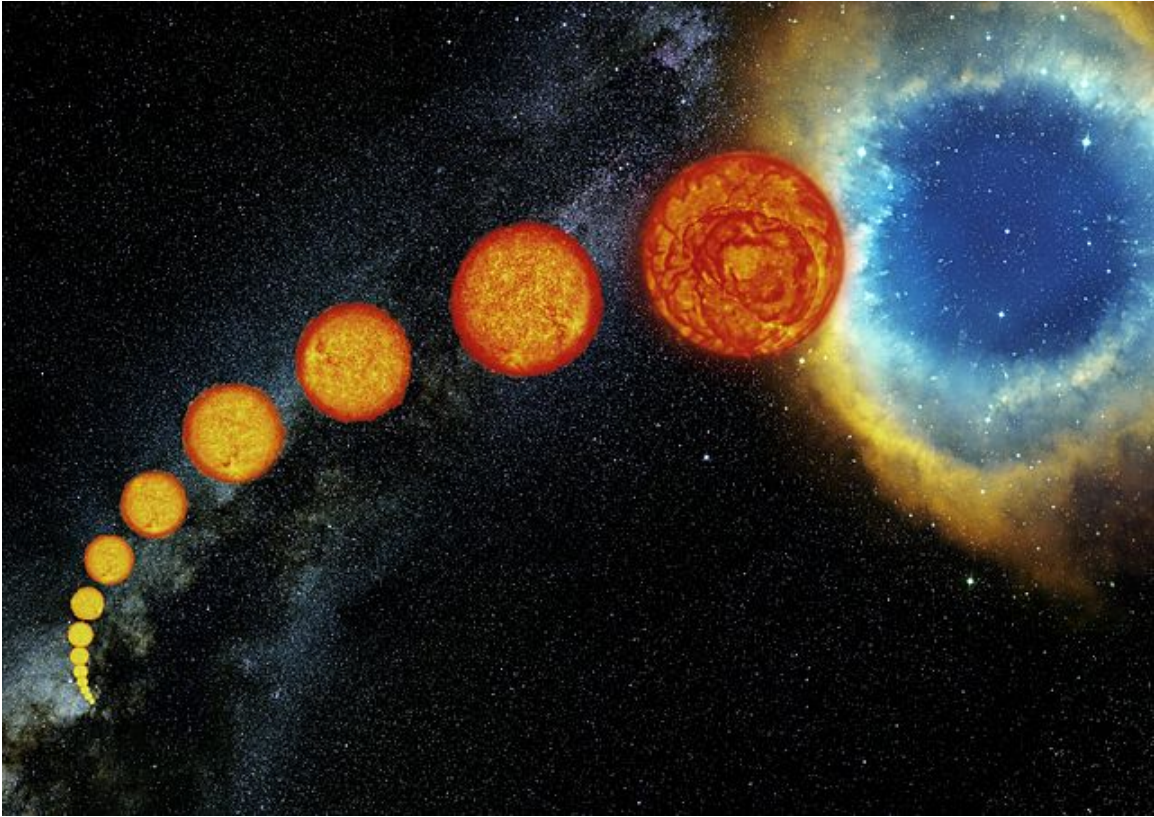
In 1910, astronomers Ejnar Hertzsprung and Henry Norris Russell discovered an amazing thing when they did this.

Instead of stars appearing all over the graph, they tended to cluster in only certain regions. You would rarely if ever find any stars with temperature and luminosity combinations in the black areas of the graph.

Betelgeuse is a star with a temperature of about 3000 K and a luminosity of 100,000 times greater than our sun. Stars with these combinations appear in the upper right of the 'HR Diagram' and are called Red Supergiants.

Pollux has a temperature of $T = 4,000$ K and a luminosity of $L = 100$ solar units. Rigel has a temperature of $T = 10,000$ K and $L = 100,000$ solar units.





Why are stars only found in certain values of L and T ?



As a star grows older, it expands in size and its surface temperature decreases. This causes the star to get steadily brighter as time goes on.

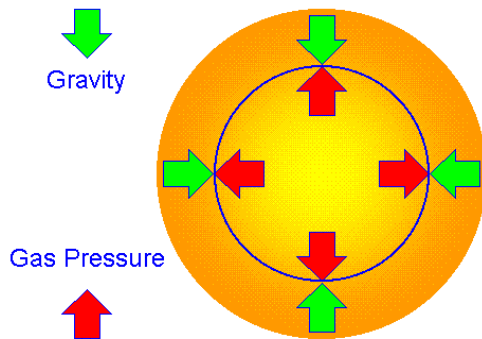
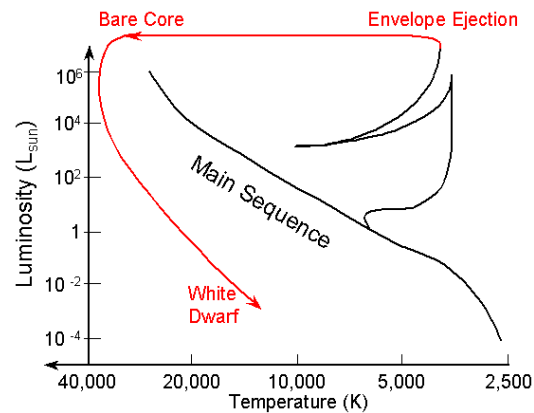
Since it was formed 4.5 billion years ago, our sun has increased in luminosity by about 30%. This has led to a gradual warming of Earth over a span of billions of years.



A star like our sun begins its life on the Main Sequence. This is the diagonal band on the HR diagram from the top left to the lower right.

Over a span of 10 billion years a star then moves on the HR diagram to the upper right and becomes a large, cool red giant star.

After another billion years, it takes a complicated path around the HR diagram. It ejects a beautiful planetary nebula, and becomes a white dwarf star in the lower left corner.



The changes in temperature and size are controlled by the balances between two forces. Gravity pulls the star's mass inwards. The enormous gas pressure produced by the core of the star through nuclear fusion pushes the gas outwards.

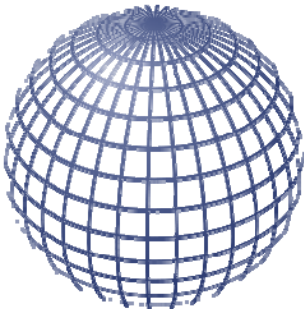
Star cores are over 10 million kelvins hot. At these temperatures hydrogen nuclei can fuse into helium nuclei. This releases enormous amounts of energy. This energy heats the inside of the star and causes an outwards pressure that balances gravity.



This delicate balance between gravity and heat pressure changes as the star gets older so the star expands over time and its outer surface cools down. The changes that a star goes through from birth to death are called stellar evolution. These changes can be very complex.

There is a simple mathematical relationship between a star's luminosity, temperature and size that explains how a star moves on the HR diagram.

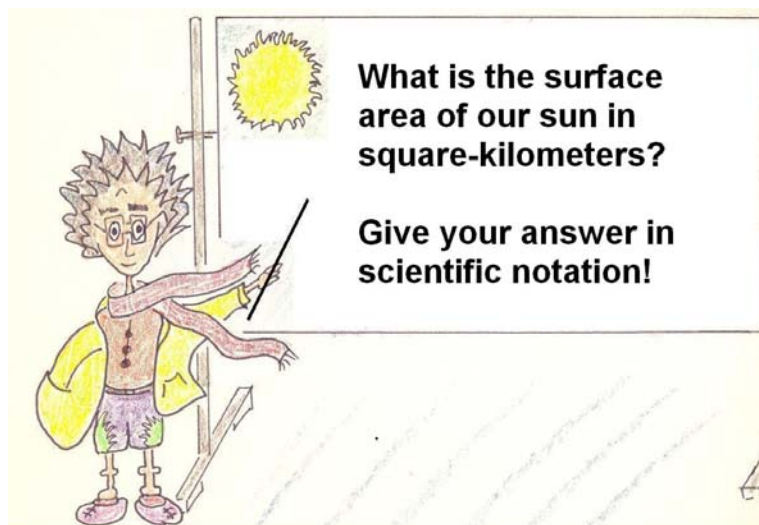
Let's explore this relationship one step at a time!



First, a star is a spherical object, and we know that the surface area of a sphere is given by the formula

$$S = 4\pi R^2$$

For the following problem, our sun has a radius of 6.9×10^5 kilometers and $\pi = 3.142$.





Next, the amount of energy that the surface of a hot body emits depends on its temperature.

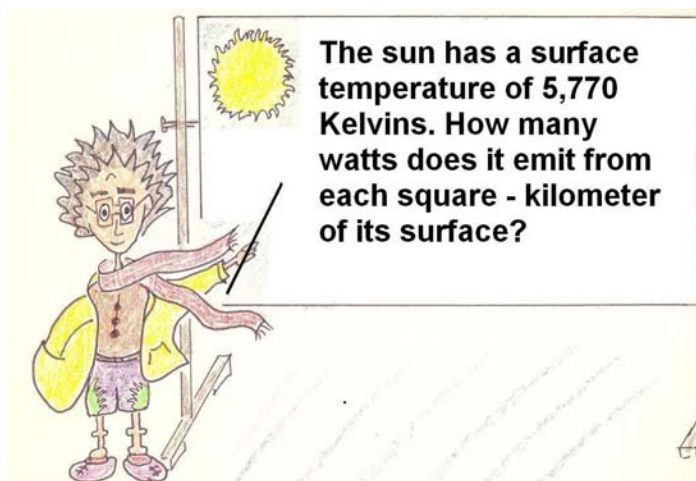
Home owners use thermal 'heat' imaging to see how well their homes are insulated. In

the picture, blue areas are cold and red areas are hot. To conserve energy, you want your house to look 'blue' or 'black'. This means that little energy is leaking out of your house to heat the outside walls and roof areas!



A simple formula predicts the number of watts of energy, F , emitted by a square-kilometer of heated surface, if the surface is at a temperature of T in kelvin units.

$$F = 0.057 T^4$$





Our first formula tells us how many square-kilometers are in the surface of a star.

Our second formula tells us how many watts each square-kilometer produces.

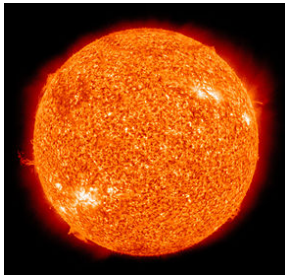
If we combine these equations, we get a formula for the total luminosity of the star!

$$L = 4\pi R^2 \times (0.057)T^4$$

and so

$$L = 0.72 R^2 T^4$$

If we use R in kilometers and T in kelvin degrees, L will be computed in watts.



The sun has a radius of 6.9×10^8 kilometers and a temperature of 5,770 k.

What is the sun's luminosity in watts?



The red supergiant star Mu Cephei has a temperature of 3,690 k, and a radius of 6.9×10^{11} kilometers.

What is Mu Cephei's luminosity in watts?



As our sun evolves, its size and temperature change in specific ways.

Astronomers can calculate these changes over time. Then they plot this stellar evolution model as a track on the HR diagram. Let's see how this is done!

Here is a table of temperature-radius points

Time (billion yrs)	Radius (km)	<u>Temperature</u> (k)	Luminosity (watts)	<u>Luminosity</u> (suns)
0	4.7×10^5	5,000	1.0×10^{26}	
4.5	6.9×10^5	5,770	3.8×10^{26}	
11.0	1.8×10^8	2,600	1.1×10^{30}	
11.5	7.6×10^6	4,770	2.2×10^{28}	
11.6	8.6×10^7	3,500	8.0×10^{29}	
11.7	1.0×10^4	20,000	1.1×10^{25}	

Using the formula $L = 0.72 R^2 T^4$ fill in the luminosity for each stage in watts. In the last column, express the luminosity in multiples of the suns current value: 3.8×10^{26} watts.

Now plot the points with the column 3 and column 5 values on the HR diagram, and connect the points in their correct time order, to form a stellar evolution track for our sun!



What type of star would you call the sun when it is 11.0 billion years old?



In 1054 A.D. a star located 6,000 light years away in the constellation Taurus the Bull, exploded as a supernova. It ejected nearly all of its mass back into space.

This gas, once a part of the star, was enriched by elements heavier than helium such as nitrogen, oxygen, carbon and iron. All of these elements are important for living systems.

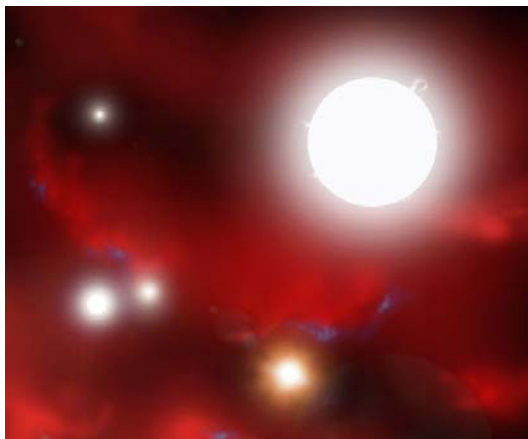


Another interesting thing about stars is their composition.

The periodic table of the elements contains over 100 entries. If you were to look at the elements in a star, you would find that stars typically have about 71% of their mass in hydrogen. You would also find about 27% of a star's mass in helium. The remaining 2% consists of all of the other elements in the periodic table! Astronomers call this 2% the 'metallicity' of a star.

Elements heavier than helium have to be created in supernova explosions. Stars with few elements heavier than helium tend to be much older than stars that have lots of heavy elements.

Population III These metal-free stars are the first generation of stars to have formed in the universe. All of them had no heavy elements, and they all went supernova. Traces of these stars cannot be found in the universe today. They all went extinct by the time the universe was 1 billion years old! Only their invisible black hole corpses may remain to be discovered...if only we knew where to look for them!





Population II These metal-poor stars have few heavy elements. They are among the oldest stars to have formed in the universe since the Big Bang. These stars can usually be found in globular star clusters.



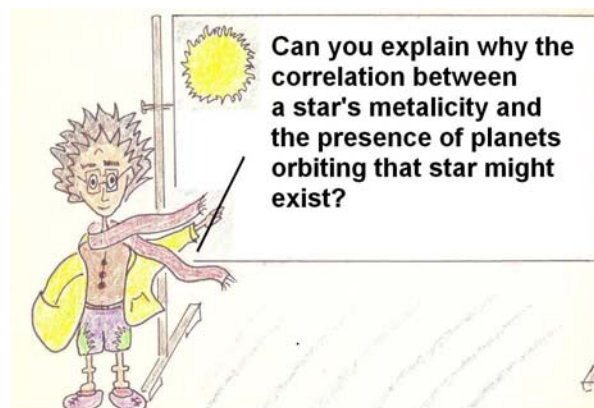
Population I - These metal-rich stars have much higher amounts of heavy elements and are typically very young stars, or stars that are still forming today. These stars have formed during the last 5 billion years. The most recent ones are still being formed in gas clouds in our Milky Way.



Here is a table of some of the most heavy element-enriched to the least enriched stars. The amount of metals is given relative to our sun so that ' $Z=3.63$ ' means the star has 3.63 times the heavy elements of our sun or 6.53 % rather than the sun's 1.8 %.

Star	Amount (Z)	Location
HD 126614	3.63	Virgo
BW IV-167	2.75	Sagittarius
HD77338b	2.24	Pyxis
Hyades Cluster	1.6	Taurus
Sun	1.0	
47 Tucanae Cluster	0.2	Tucana
Messier-13 Cluster	0.040	Hercules
BD +17° 3248	0.0095	Hercules
Sneden's Star	0.00079	Aquarius
HE0107-5240	0.000005	Phoenix
Population-III	0.000001	????

Searches for planets orbiting other stars have discovered that stars with high metallicities are much more likely to have planets.







How do stars form in the first place?

We have seen how we can describe many different features of stars including their masses, sizes, luminosities and compositions.

Stars are made of matter, and this matter has to be accumulated into the body of the star. Where does the matter come from?

Between the stars in the Milky Way you can find a very dilute gas of mostly hydrogen and helium atoms. There are also microscopic dust grains mixed in with this 'interstellar medium'.



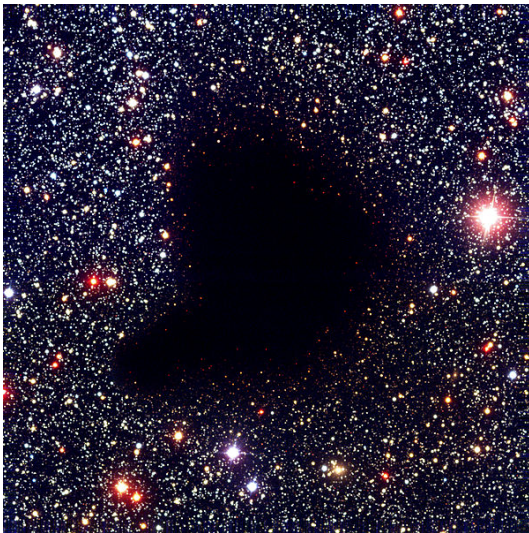
Sometimes this interstellar medium collects into giant clouds that are dozens of light years in size. When the clouds are far from illuminating stars, they look like the cloud shown above in the constellation Taurus.

The photograph to the left shows another one of these clouds called the Horsehead Nebula. Nearby stars can light up the gas in these clouds, allowing us to see it with a telescope.



Gravity is a powerful force that acts over thousands of years to accumulate matter. The more matter you have, the stronger its gravity, and the more matter it can attract.

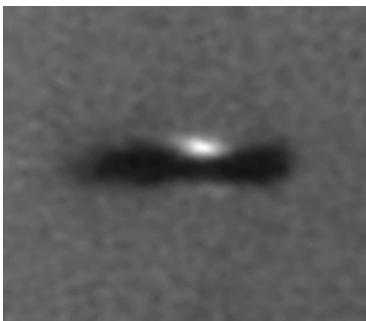
Over a span of a few million years, some parts of an interstellar cloud can become so dense that the dust grains block the light from background stars outside the cloud. In the picture of the Horsehead Nebula, the 'horsehead' is one such dark cloud.



Once a dark cloud forms, it may not do anything for a long time. There could be internal forces that just balance gravity trying to collapse the cloud further. So the cloud stays balanced the way it is for a long, long time!

For other dark clouds like the one shown here, portions of the cloud may not be able to resist gravity.

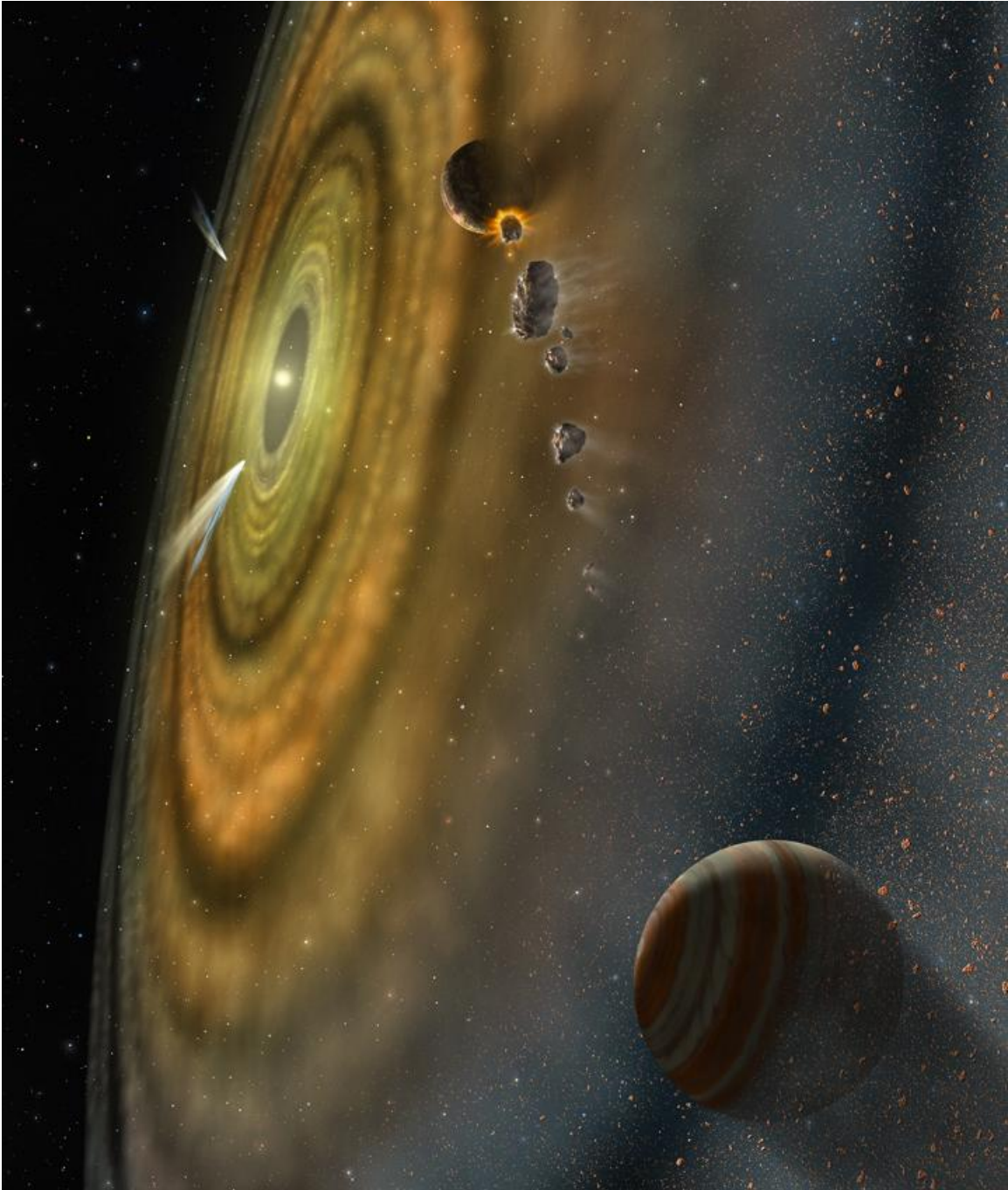
They continue to collapse to even higher densities becoming darker and darker. There could even be external forces like a nearby supernova explosion, or the radiation from stars pushing on the dark cloud to compress it.



As the clouds collapse, they are also spinning, so the cloud becomes a flattened, spinning disk of gas and dust with a dense center 'hub'. This is called a protostellar or protoplanetary disk. Astronomers have spotted many of these objects in our galaxy.

If astronomers could see these disks of swirling gas and dust in detail, they might see something like the pictures shown below created by an artist. (Credit: NASA/JPL-Caltech/R. Hurt /SSC)







Astronomers have discovered that many of these disks show signs that planets are forming in them.

This artist, Silvia Stoyanova, has added this information, and shows how a spinning disk of gas and dust is starting to form planets.

Dust grains stick together to form small rocks. The rocks collide and form asteroids. Over a few millions years, these disks can eventually 'grow' a number of planets by this accretion process.

Astronomers have discovered that some of the disks they can observe have rings in them. They think that planets are busily sweeping out the matter in these rings and continuing to grow.

The drawing to the left shows what this might look like if you could see things more clearly.



Although dense interstellar clouds can form stars like our sun, some clouds take a very different history.

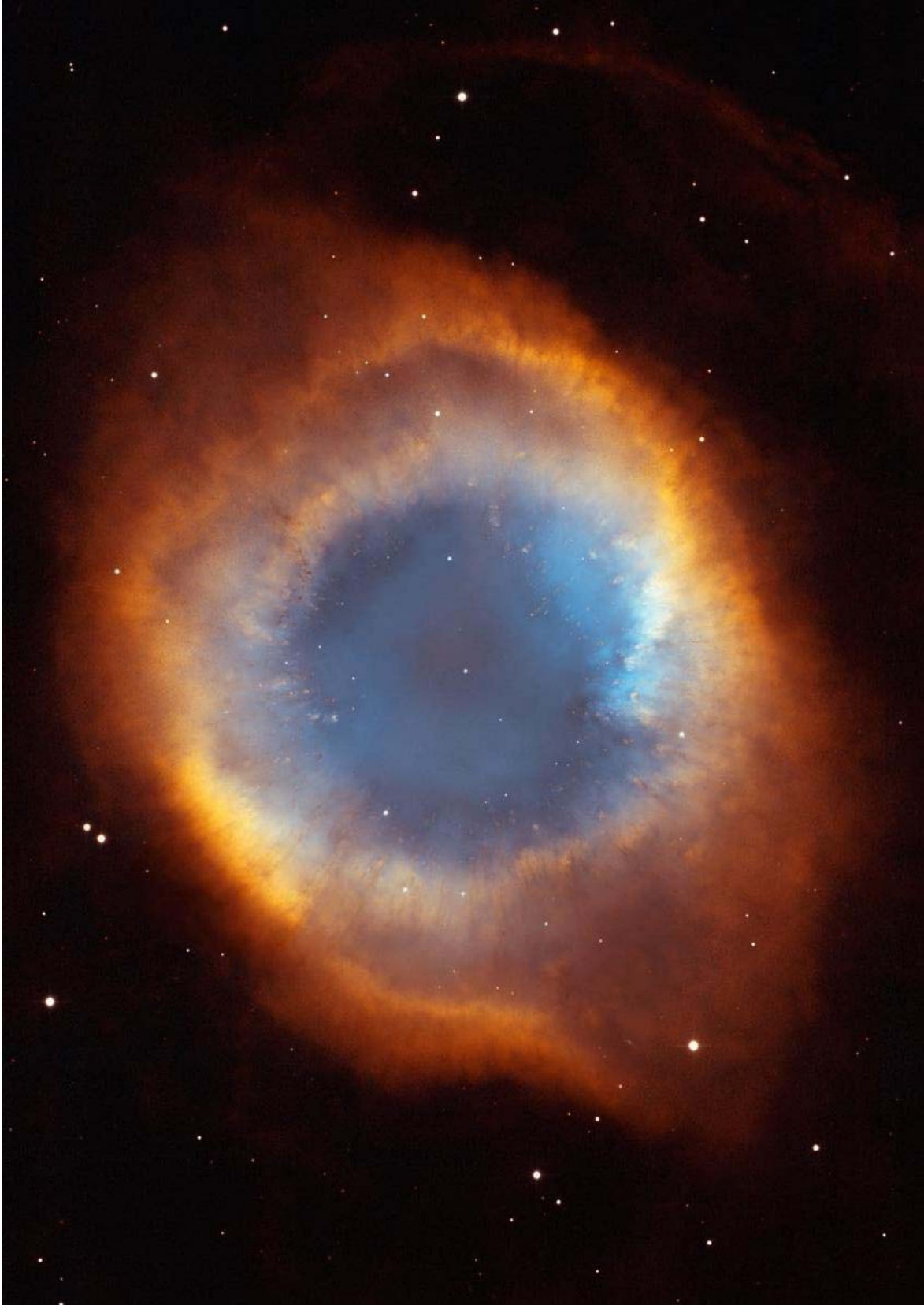
Instead of forming lots of small stars, some clouds form a small number of very massive stars called OB Associations. These stars produce so much ultraviolet light, they change the gas and dust surrounding the new stars in a spectacular way.

About 2/3 of the light that comes from these massive stars is ultraviolet light. This light is so strong it can strip electrons from hydrogen and helium atoms. Scientists call the process of stripping electrons from atoms - ionization.

As the hydrogen and helium gas surrounding these stars try to adjust to this ultraviolet light, the atoms give off specific colors. Red is a common color for hydrogen gas and green for oxygen. Blue is the color you get when light scatters off of dust grains. This is also why the sky on Earth is blue!



This Hubble Space Telescope photo of the Eagle Nebula shows many details of the star formation process for massive stars. Ionized gases of hydrogen and oxygen glow in reds and greens.



Stars are born, but they also die. The way in which they die depends on their mass.



Stars that are much less massive than our sun will just get steadily colder and colder until the nuclear fuels run out. This can take 100 billion years or even much longer!

Stars like our sun last about 15 billion years before they eventually run out of fuel. During their last few million years, they shed gigantic clouds of gas to form beautiful ring nebula. At the same time, their cores become as small as Earth and they turn into white dwarf stars that slowly cool and grow very dim.

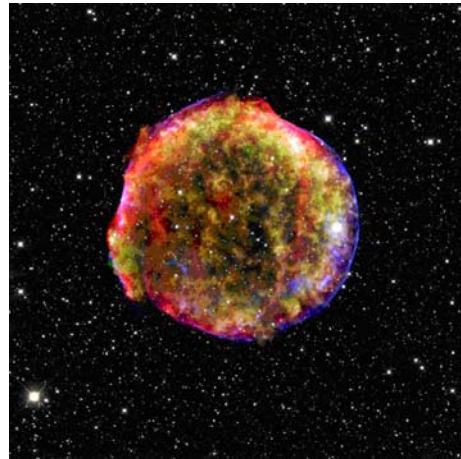
The ring nebula to the left is called the Helix Nebula in the constellation Aquarius. It has a diameter of about 3 light years and is located 650 light years from Earth. It was created by a sun-like star, whose white dwarf corpse you can see at the center of the nebula. The nebula is about 10,000 years old.





Other stars are not so fortunate. If a star has more than 10 times our sun's mass, it cannot gradually cool to become a 'black dwarf', or a 'white dwarf' with a beautiful ring nebula. Instead, the star explodes as a supernova!

More than half of the mass of the star is ejected into space at millions of miles per hour to form a spectacular nebula that can grow to be 100 light years across. The rest of the mass implodes into a dense core.



The light from the explosion can equal billions of normal stars, and last for several months before fading away.

The bright star in this photo is a supernova spotted in 2007 in a distant galaxy.

After 100,000 years, the expanding nebula from a supernova eventually fades away. The gas mixes with the interstellar medium and enriches it with heavier elements created during the explosion.



The dense, imploded core of the star can have a very different history.

The original star may have had 10 times the mass of our sun to start with, but managed to shed 2 solar masses in an expanding 'wind' of gas over the course of its 100 million

years of life. Of the remaining 8 solar masses, perhaps 7 were shed during the explosion, leaving behind a 1 solar mass core. This object would be called a neutron star because it is as dense as the nucleus of an atom and is only about 50 kilometers in diameter. Spinning rapidly, it emits pulses of energy towards Earth and we call it a pulsar.



An example of a pulsar is in the heart of the Crab Nebula.

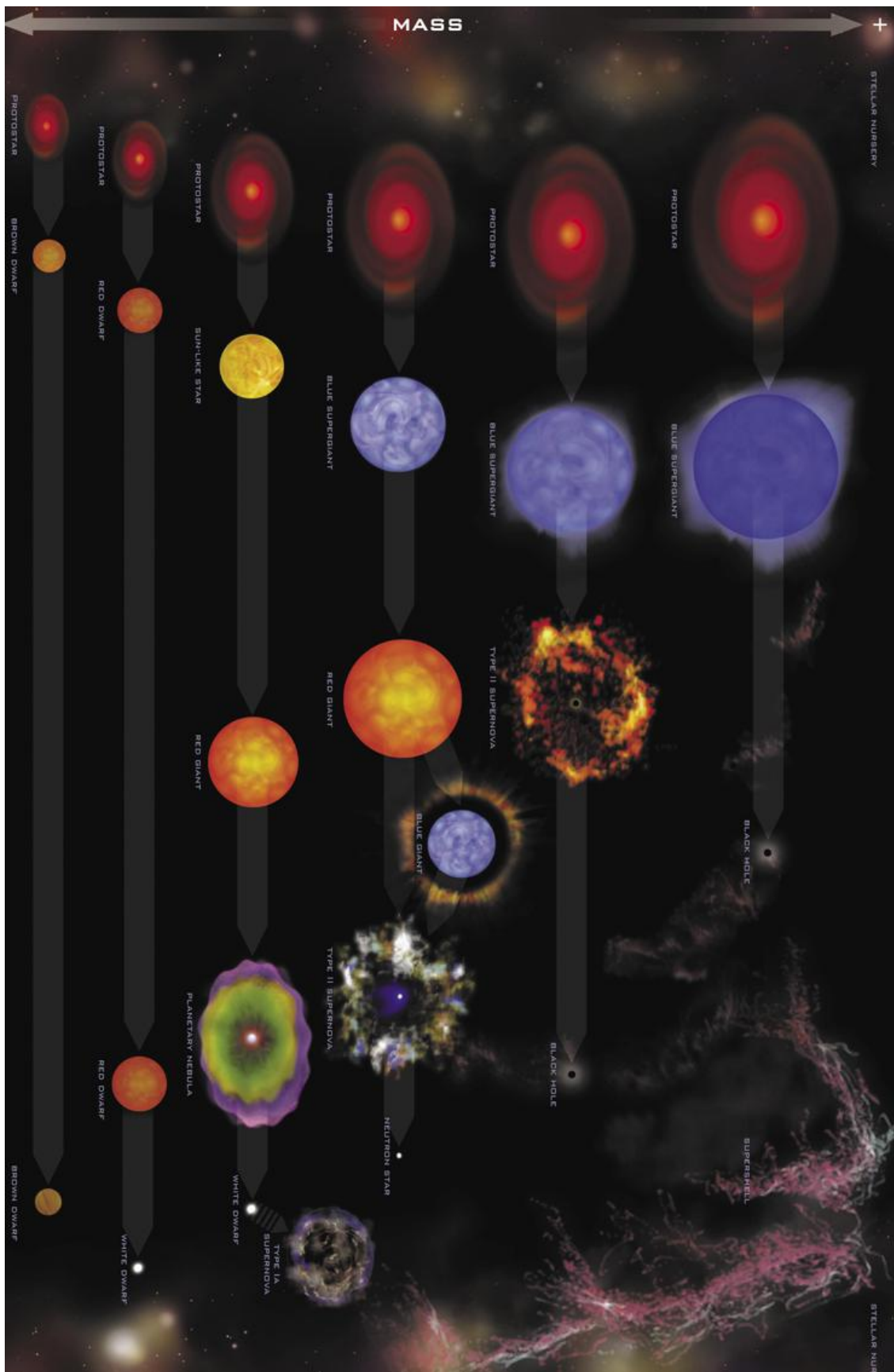


If the collapsed core contains more than about 1.4 times the mass of our sun, it cannot form a neutron star because its gravity is much too strong. Instead, it keeps collapsing until it forms a black hole!

The gravity of a black hole is so strong that light cannot escape from it. They are usually surrounded by spinning disks of matter being dragged

into the black hole. As the matter heats up from friction, it emits very high energy x-rays and other forms of radiation before it falls into the black hole and vanishes.

Black holes do not suck-in matter like vacuum cleaners. Instead, matter in orbit losses energy from friction and falls closer and closer to the edge of the black hole. This edge is called the event horizon. Eventually, the matter passes across the event horizon and completely vanishes from view.





So how long do stars last?

We know that the more mass a star has, the more hydrogen fuel is available to it.

Another way to say this is that the life span of a star is proportional to its mass. We can write this as:

Fuel is proportional to M

Astronomers also know that the rate at which a star is consuming its hydrogen fuel (tons of hydrogen per second) is proportional to the mass of the star raised to the 4th power.

Rate is proportional to M^4



So, if we divide the amount of hydrogen fuel (tons) by the rate at which it is consumed (tons/second) we get the amount of time it can continue to burn its fuel before it is exhausted! This means that

Age is proportional to Fuel/Rate

Age is proportional to M/M^4

and so **Age is inversely proportional to M^3**

If we use our sun as a clock, we know from detailed calculations that it will last about 15 billion years. So for $M = 1$ solar mass, we can change the proportionality into an equality so that

$$T = 15 \text{ billion years} \times (1/1^3)$$

Then the equation becomes $T = 15 \text{ billion}/M^3$

Let's do an example!

From our table of star masses, let's select the star Eta Carina A with a mass of 120 times the sun, then

$$T = 15 \text{ billion years} \times (1/120^3) = \mathbf{8,680 \text{ years.}}$$

Let's look at the red dwarf star Wolf-424B with a mass of 0.05 suns, then

$$T = 15 \text{ billion} \times 1/(0.05)^3 = \mathbf{120 \text{ trillion years.}}$$

So, the lifespan of a star depends on its mass, and a slight change in the mass of a star can seriously change the life time of a star!

Think like an Astronomer !

Problem 1 - We have looked at many different properties for stars, their diameters, temperatures, masses, compositions and lifespans. Which of these has the largest range for known stars?



Problem 2 - Two stars are born in a binary star system at the same time. One star, A, has a mass of 10 times the mass of our sun and the other star, B, has a mass of 20 times the mass of our sun. How long will each star last and which one will die first?



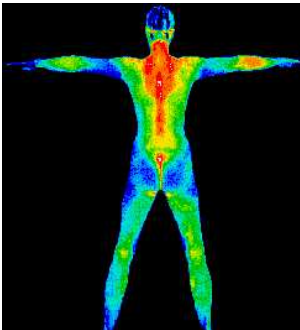
Problem 3 - An astronomer studies the globular star clusters orbiting the galaxy Messier-87 and counts 3,500 of these clusters. If each cluster contains 400,000 stars, what is the total number of stars in the globular cluster population? Express your answer using scientific notation.



Problem 4 - From Earth, astronomers can count about 1.5×10^7 stars within a spherical volume of 4.2×10^9 cubic light years. The total volume of the Milky Way galaxy is about 3×10^{14} cubic light years. About how many stars are there in the Milky Way?



Problem 5 - The Helix Nebula expands with a speed of 30 km/sec. By the time it has faded from view, it will have reached a diameter of 10 light years. How many years will it take for this ring nebula to fade away if 1 light year = 9.3×10^{12} km?



Problem 6 - A human has a temperature of 98.6 F (37 Celsius) which is equal to 310 Kelvins. If the surface area of a human is about 1.7 meters². How many watts, L, of heat energy does a human generate if the formula is $L = 5.67 \times 10^{-8} T^4$?



Problem 7 - In 2006, astronomers observed the most luminous supernova ever recorded. It occurred in the galaxy NGC-1260 and it produced 1.9×10^{37} watts. If the solar luminosity is 3.8×10^{26} watts, how many suns did the supernova equal?



Problem 8 - A student counted 10 stars while looking through a pair of binoculars. If the area of the sky viewed by the binoculars measured 38 square degrees, and the area of the full sky is 41,268 square degrees, about how many stars would the student have counted across the full sky?



Problem 9 - A dark interstellar cloud contains 3.0×10^{33} kilograms of matter. If the mass of an average star is about 5.0×10^{29} kilograms, how many stars could the cloud produce if 50% of the mass was eventually used up in star formation?



Problem 10 - The radius of a black hole in kilometers is given by the formula $R = 3.0M$ where M is the mass of the black hole in units of the sun's mass. Astronomers have discovered a black hole in the center of the Milky Way with a mass of 3.5 million suns. What percent is the radius of this black hole compared to the orbit of Mercury in our solar system, whose orbit has a radius of 58 million kilometers?

Credits



Inside Illustrations: 9) Model of Milky Way NASA/JPL/R. Hurt/SSC. 13) Big Dipper (www.opticsreviewer.com). 16) Ursa Major Courtesy Jerry Lodrigus (jerry5@astropix.com); 17) Corvus stars. Richard Berry. 19) Cover of Rudolphine Tables; 20) Old star atlas Pisces © Ian Ridpath. 21) Diagram of milky way from Sir William Herschel; 22) Ursa Major atlas. Illustration in Johann Elert Bode's 1782 Star Atlas; 24) Courtesy © T. Credner & S. Kohle, AlltheSky. 28) Credit: Observatory Austria Site contact: bick@astro.princeton.edu; 29) 70 Oph orbit plot: Astronomy: Roen Kelly, after Richard Dibon-Smith; 30) Trapezium cluster (NASA/HST); 32) Pleiades cluster; 34) Beehive cluster; 36) NGC-3603 (NASA/HST); 38) NGC 265 cluster (NASA/HST); 40) Messier 13 ; 42) Omega Centauri cluster (ESO); 48) Model of nearest stars (ESO); 58) HR Diagram (ESO); 60) Solar evolution Courtesy ESO/S. Steinhofel; 66) Crab Nebula (NASA/HST); 67) Pop III stars (David A. Aguilar, CfA) ; 70) Horsehead Nebula (Credit and Copyright: Jean-Charles Cuillandre (CFHT), Hawaiian Starlight, CFHT) ; 72) Barnard 68 (Anglo Australian Observatory/ESO); Orion Proplyd Disk (NASA/Hubble); 74) Protostellar disk (NASA/JPL/R. Hurt/SSC); 75) Planet formation (NASA/Silvia Stoyanova); 77) Eagle nebula (NASA/HST); 78) Helix Nebula (NASA/HST); 80) Supernova explosion (ESO/Pan-STARRS); 81) Supernova remnant (NASA/Chandra) and SN2007ji (NASA/HST); 82) Crab pulsar (NASA/HST) and Black hole artwork (NASA/ESA/GSFC); 84) Table for star formation (NASA/Chandra).

Answer Key

Page 7) $150 \text{ million km} / 28000 \text{ kmh} = 5357 \text{ hours or } 223 \text{ days}.$

Page 11) $5 \times 88 = 440 \text{ stars}$

Page 23) Students should count about 230 stars, so there are $230 \times 18 = 4140 \text{ stars}$

Page 25) Students should count about 270 stars, so there are $270 \times 3500 = 945,000 \text{ stars}$

Page 26) 1.35×10^8

Page 29) $1/3 \times 300 = 100 \text{ stars}$

Page 45) 1) 1.3×10^{12} , 2) 1.98×10^{30} , 3) $1.4 \times 10^{10} \text{ cm}$, 4) $9.46 \times 10^{17} \text{ cm}$,
5) 9.5×10^{17} , 6) 2.0×10^{42} , 7) 1.0×10^{31}

Page 46) $1.5 \times 10^7 \text{ stars}$, $1.15 \times 10^{10} \text{ years}$, $1.7 \times 10^2 \text{ light years}$, $6.3 \times 10^4 \text{ times}$, $1.1 \times 10^7 \text{ timers}$

Page 52) $3.8 \times 10^{26} \times 1.2 \times 10^5 = 4.56 \times 10^{31} \text{ watts}$

Page 53) $8.7 \times 10^6 / 6.4 \times 10^{-6} = 1.36 \times 10^{12} \text{ times}$

Page 55) $9.9 \times 10^{11} / 1.4 \times 10^9 = 707 \text{ times}$

Page 57) $265 / 0.05 = 5300 \text{ times}$

Page 59) Pollux is a red giant, Rigel is a blue super giant

Page 62) $4(3.142)(6.9 \times 10^5)^2 = 5.98 \times 10^{12} \text{ square kilometers}.$

Page 63) $6.32 \times 10^{13} \text{ watts per square kilometer}.$

Page 64) Sun = $3.8 \times 10^{26} \text{ watts}$, Mu Cephei = $6.36 \times 10^{37} \text{ watts}.$

Page 65) 0.3, 1.0, 2894, 58, 2090, 0.029

Page 69) Planets are made from heavy elements, so if a star has a lot of heavy elements then it is more likely to be able to form planets.

Page 87) 1) Lifespans have the widest range from 8000 years to 120 trillion. A factor of nearly 10 billion. 2) A = 15 million years, B = 1.9 million, so star B will die sooner than star A. 3) $3.5 \times 10^3 \times 4.0 \times 10^5 = 1.4 \times 10^9 \text{ stars}.$

Page 88) 4) $(3 \times 10^{14} / 4.2 \times 10^9) \times 1.5 \times 10^7 = 1.07 \times 10^{12} \text{ stars}.$ 5) 30 kms = $9.3 \times 10^8 \text{ km per year}$, and 10 ly = $9.3 \times 10^{13} \text{ km}$, so it takes 100,000 years. 6) $5.67 \times 10^{-8} (310)^4 = 524 \text{ watts}.$ 7) $1.9 \times 10^{37} / 3.8 \times 10^{26} = 50 \text{ billion times the luminosity of our sun!}$

Page 89) 8) $(4126/38) \times 10 = 10,860 \text{ stars}$ 9) $3.0 \times 10^{33} / 5.0 \times 10^{29} = 6000 \text{ stars}.$ 10) R = 10.5 million kilometers. $100\% (10.5/58) = 18\% \text{ of mercury's orbit}.$