



Astronomers can determine the mass of a planet by using Kepler's Third Law, which is written in algebraic form as follows:

$$P^2 = \frac{4\pi^2}{GM} a^3$$

where G is Newton's constant of gravity equal to $6.67 \times 10^{-11} \text{ m}^3/\text{kg sec}^2$, M is the mass of the planet in kilograms, a is the average orbit radius in meters and P is the orbit period in seconds.

Here are some problems that let you work with this very important formula in astronomy.

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of $a=9380 \text{ km}$ and a period of $P=0.32 \text{ days}$, what is the mass of Mars?

Problem 2 – The martian moon Deimos has an orbit period of $P=1.26 \text{ days}$. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of $P=35.5 \text{ hours}$ and an average radius of $a=25,889 \text{ km}$. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of $a=3700 \text{ km}$. What is the final orbit period of MRO in the new circular orbit?

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of 9380 km and a period of 0.32 days, what is the mass of Mars?

Answer: First solve the algebraic equation for M to get $M = (4\pi^2/G)(a^3/p^2)$. Then use $a = 9380 \text{ km} \times 1000\text{m/km} = 9.38 \times 10^6 \text{ m}$, and $p = 0.32\text{days} \times 24\text{h/d} \times 3600\text{s/hr} = 27648 \text{ seconds}$, to get

$$M = [4 \times (3.141)^2 / 6.67 \times 10^{-11}] (9.38 \times 10^6)^3 / (27648)^2 = \mathbf{6.39 \times 10^{23} \text{ kg}}.$$

Problem 2 – The martian moon Deimos has an orbit period of $P=1.26$ days. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Answer: In this case we know M and P and need to solve for a:

$$a^3 = GMP^2/4\pi^2 \quad \text{so} \quad P = 1.26\text{d} \times 24\text{h/d} \times 3600 \text{ s/h} = 108864 \text{ seconds, and so}$$

$$a^3 = (6.67 \times 10^{-11})(6.39 \times 10^{23})(108864)^2 / (4 \times 3.141^2) = 1.27 \times 10^{22} \quad \text{so} \quad a = 23352 \text{ kilometers and so } a = \mathbf{23,400 \text{ kilometers}}.$$

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of $P=35.5$ hours and an average radius of $a=25,889 \text{ km}$. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of $a=3700 \text{ km}$. What is the final orbit period in minutes of MRO in the new circular orbit?

Answer: Solving for P we get $P^2 = (4\pi^2/GM) a^3$, and so for $a = 3,700,000 \text{ meters}$ and $M = 6.39 \times 10^{23} \text{ kg}$ we get $P^2 = 4.69 \times 10^7$ and so $P = 6848 \text{ seconds}$ or **114 minutes**.