Parameter estimation of generalized Ghosh black holes using shadow observables and Event Horizon Telescope Constraints

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Goal

Constrain the deviation parameters k and n of the generalized Ghosh black hole by comparing its shadow area A and oblateness D against EHT measurements of M87* and Sgr A*, testing consistency with the Kerr hypothesis.

Abstract

The Event Horizon Telescope images of M87* and Sgr A* give us a unique chance to test gravity in the strong field regime and assess deviations from the Kerr solution. We study the shadows of rotating nonsingular black holes—namely, generalized Ghosh black holes (GGBHs), given by the mass function $m(r) = Me^{-(k/r)^n}$, which reduces to the Kerr black hole ($k \to 0$) and to the original Ghosh model ($n = 1$). We calculate the photon's trajectory via the Hamilton-Jacobi method and construct the shadow seen by a distant observer. For a high spin $a/M = 0.9$, increasing k/M from 0 to 0.5 reduces the shadow size by about 13% and the shadow area by about 24%, while making the shape more distorted (up to $\sim 20\%$). For larger values $k/M \sim 1.0$, the area can decrease by nearly 30%. However, increasing n reduces these effects, and for $n \geq 3$ the shadow becomes very close to the Kerr case (differences below $\sim 5\%$). To compare with observations, we use two simple shadow quantities: the area A and the oblateness D. Using EHT observational data, we find that for M87* (17° inclination, 37.8μas diameter), the parameter is constrained as $k/M \lesssim 0.133$ for $n = 1$. For Sgr A* (50° inclination, 48.7μas diameter), the bound is $k/M \lesssim 0.09162$. These limits become weaker for larger n. Overall, the GGBHs model are a viable candidate to the Kerr black hole, and future observations may help to detect or further constrain such deviations.

GENERALIZED GHOSH BLACK HOLE

For mass M, spin a, and deviation parameters k, the spacetime of GGBHs is given by

$$ds^2 = -\left[\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right]dt^2 + \sin^2\theta\left[\Sigma + a^2\sin^2\theta\left(2 - \frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)\right]d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + 2a\sin^2\theta\left[1 - \frac{\Delta - a^2\sin^2\theta}{\Sigma}\right]dt\,d\phi. \quad (1)$$

$$\Delta = r^2 + a^2 - 2m(r),$$
$$\Sigma = r^2 + a^2\cos^2\theta,$$
$$\mathbf{m(r) = Me^{-\left(\frac{k}{r}\right)^n}}.$$

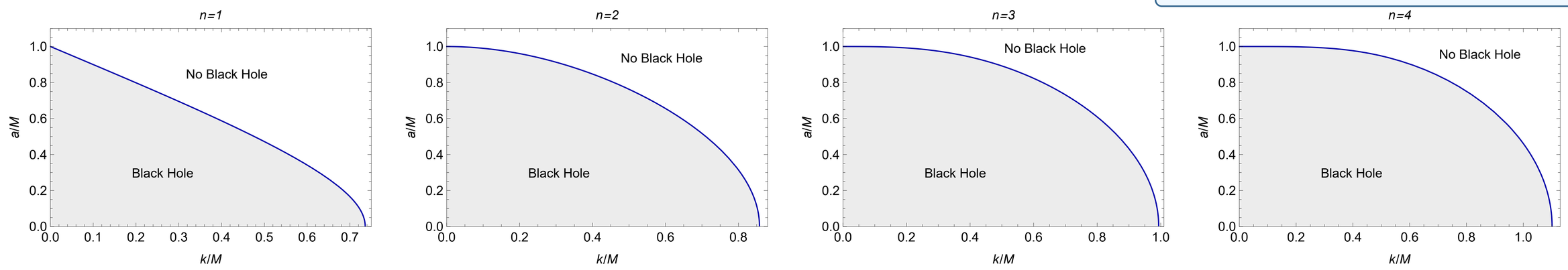


Fig. 1. The parameter space of the GGBH. The blue curve represents the extremal black hole ($r_- = r_+$), whereas the shaded region indicates the non-extremal black hole ($r_- < r_+$). The white region indicates a no black hole. The metric (1) has two simple zeros ($r_- < r_+$), one double zero ($r_- = r_+$), and no zero, corresponding to non-extremal, extremal, and no black hole spacetime, respectively.

Shadows

To obtain the shadow boundary, we calculate the trajectory of a photon around a black hole and project it onto the observer's plane, indicated by the celestial coordinates (α, β) and defined as follows:

$$\alpha = -\xi_c \csc\theta_o,$$
$$\beta = \pm\sqrt{\eta_c + a^2\cos^2\theta_o - \xi_c^2\cot^2\theta_o}. \quad (2)$$

Effective Potential

Among all trajectories: unstable (circular or spherical) orbits, scattering, and capturing, only unstable orbits govern the shape of the shadow, with their radii determined by $\mathcal{R} = d\mathcal{R}/dr = 0$. Therefore, to obtain circular orbits, we analyze the radial motion $\left(\Sigma\frac{dr}{d\tau}\right)^2 + V_{eff} = 0$

Parameter Estimation

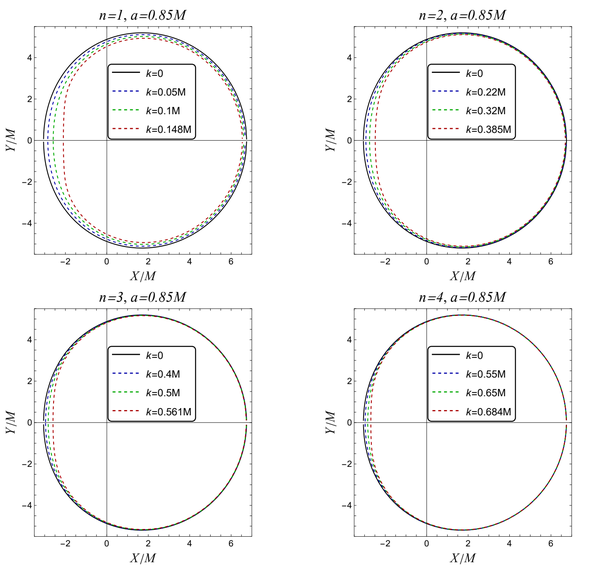
The shadow observables A and D are used to estimate the black hole parameters

$$A = 2\int Y(r_p)dX(r_p) = 2\int_{r_p^-}^{r_p^+}\left(Y(r_p)\frac{dX(r_p)}{dr_p}\right)dr_p,$$
$$D = \frac{X_r - X_l}{Y_t - Y_b}. \quad (3)$$

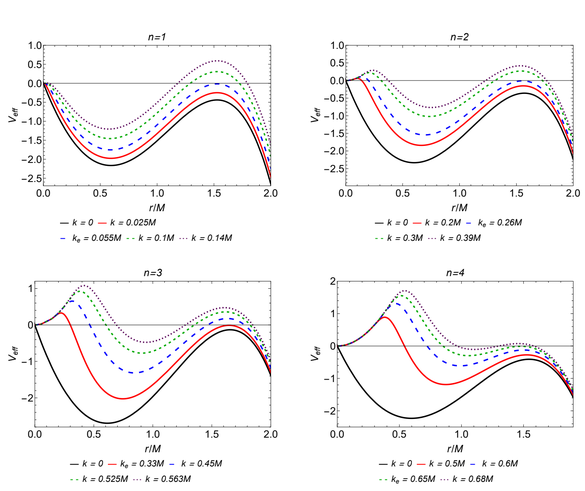
Energy Emission

For distant observers, the high-energy absorption cross section aligns with the black hole shadow and approaches a constant value, $\sigma_{lim} \approx \pi R_s^2$. Therefore, the energy emission rate is given by

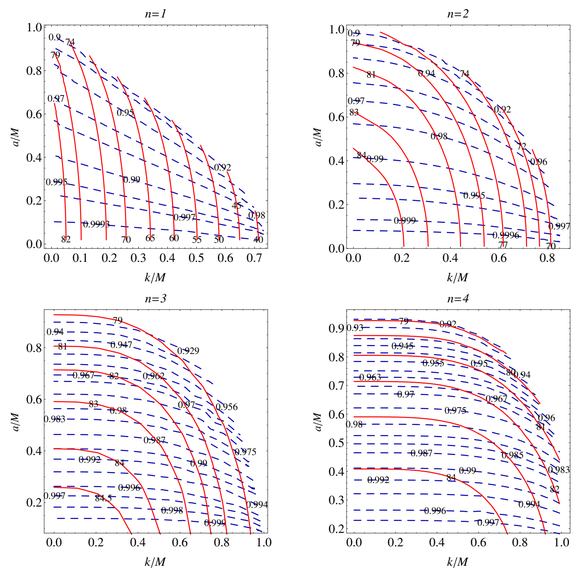
$$\frac{d^2E(\omega)}{d\omega dt} = \frac{2\pi^2\sigma_{lim}}{\exp(\omega/T_+) - 1}\omega^3. \quad (4)$$



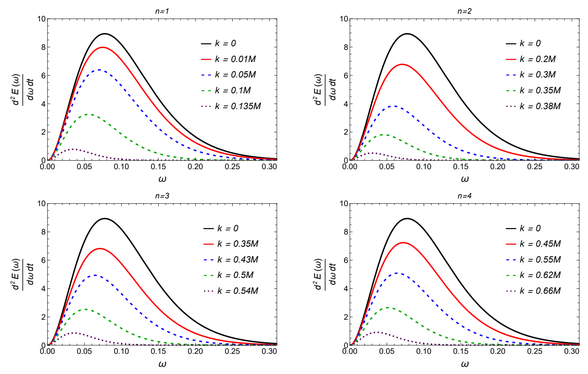
The behaviour of the GGBH shadow and its dependence on the parameters k and n for $a = 0.85M$ are illustrated and compared with the KBH (black curve).



The plot of the effective potential for GGBH is shown for different cases of n, with k varied and $a = 0.85M$ fixed. When $k > k_e$, photons are deflected and escape to infinity, while for $k < k_e$, they are absorbed by the black hole.

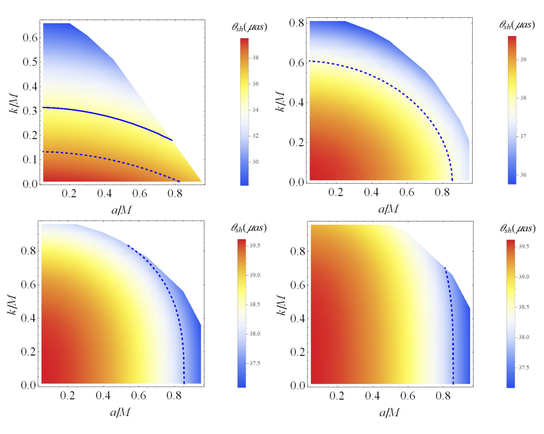


For a GGBH, the contour plots of constant A (solid red curves) and D (dashed blue curves) are illustrated at an inclination angle $\theta_0 = \pi/2$. Their intersection points uniquely estimate the parameters a/M and k/M for $n = 1, 2, 3, 4$.

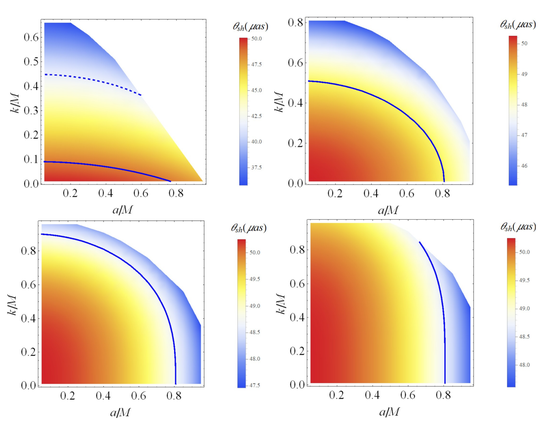


For $a = 0.85M$, the energy emission rate for GGBH is shown for various values of n. The energy emission rate is maximal for small values of k and decreases as k increases.

Constraints from EHT observation of M87* and Sgr A*



The EHT constraints on the angular shadow diameter for M87* in the GGBH model are illustrated for inclination angles $\theta = 17°$, with $n = 1$ (top left), $n = 2$ (top right), $n = 3$ (bottom left) and $n = 4$ (bottom right). The solid blue and dashed blue curves correspond to $35.1\ \mu$as and $37.8\ \mu$as, respectively. The white region is prohibited.



The EHT constraints on the angular shadow diameter for Sgr A* in the GGBH model are illustrated for inclination angles $\theta = 50°$, with $n = 1$ (top left), $n = 2$ (top right), $n = 3$ (bottom left) and $n = 4$ (bottom right). The solid blue and dashed blue curves correspond to $48.7\ \mu$as and $41.7\ \mu$as, respectively. The white region is prohibited.

Conclusion

- The shadow size decreases as the deviation parameter k increases, while its shape becomes more distorted for rapidly rotating black holes.
- Increasing n suppresses the deviations, and for $n \geq 3$ the shadow becomes very close to that of a KBH.
- The shadow area decreases monotonically with k, while the distortion increases with both k and the spin parameter.
- For a fixed value of n, the energy emission rate decreases with increasing k.
- The presence of the additional parameters k and n allows for a richer phenomenology, making this model useful for testing possible extensions of general relativity in the strong-field regime.

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