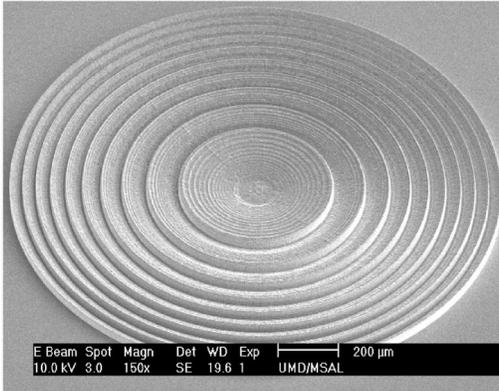


# X-ray & Gamma-ray Phase Fresnel Lens Primer

John Krizmanic

NASA/GSFC

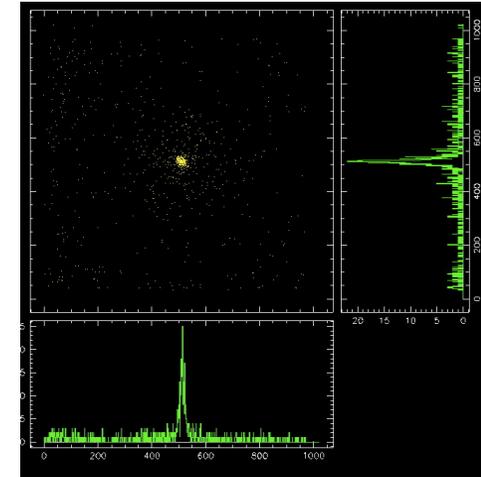


SEM of a Prototype PFL produced at UMD using MEMS gray-scale fabrication technology,

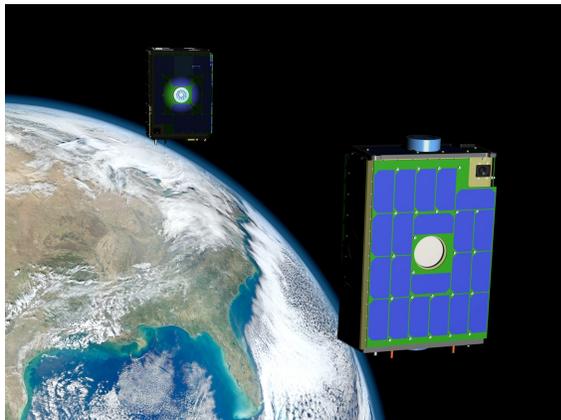
**Promise:**

**Diffracted-limited angular resolution in X-ray and Gamma-ray bands -> potential to  $\mu$ "**

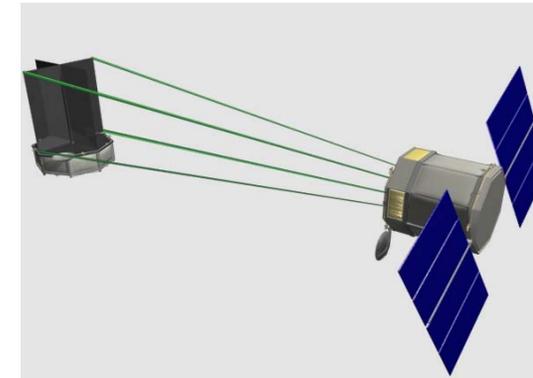
**Issues: Chromaticity and long focal length**

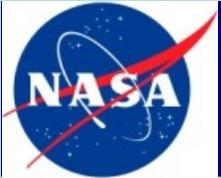


1<sup>st</sup> light CCD response of a 2<sup>nd</sup> generation PFL imaging at 17.4 keV in the GSFC 600 meter Interferometry Testbed.



Formation-flying of an optics spacecraft and a detector spacecraft forming virtual telescopes.





A&A 375, 691–700 (2001)  
DOI: 10.1051/0004-6361:20010745  
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Astronomy  
&  
Astrophysics

## Diffraction/refractive optics for high energy astronomy

### I. Gamma-ray phase Fresnel lenses

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Received 30 March 2001 / Accepted 22 May 2001

**Abstract.** Diffractive optics components such as Fresnel Zone Plates and their derivatives potentially form the basis for telescope systems for X-ray and gamma-ray astronomy with high sensitivity and superb angular resolution. The main problem is that systems with convenient design parameters involve very long focal lengths. The design considerations and performance of telescopes using a simple Phase Fresnel Lens on one spacecraft and a detector assembly on another are considered. Such systems are shown to have the potential to provide orders of magnitude improvement on currently available sensitivity. At the same time the angular resolution can be at the micro arcsecond level or better – sufficient to resolve the structure surrounding the event horizon of black holes in nearby galaxies.

**Key words.** telescopes – methods: observational – techniques: interferometric – gamma-rays: observations – X-rays: general

A&A 383, 352–359 (2002)  
DOI: 10.1051/0004-6361:20011700  
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Astronomy  
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Astrophysics

## Diffraction-refractive optics for high energy astronomy

### II. Variations on the theme

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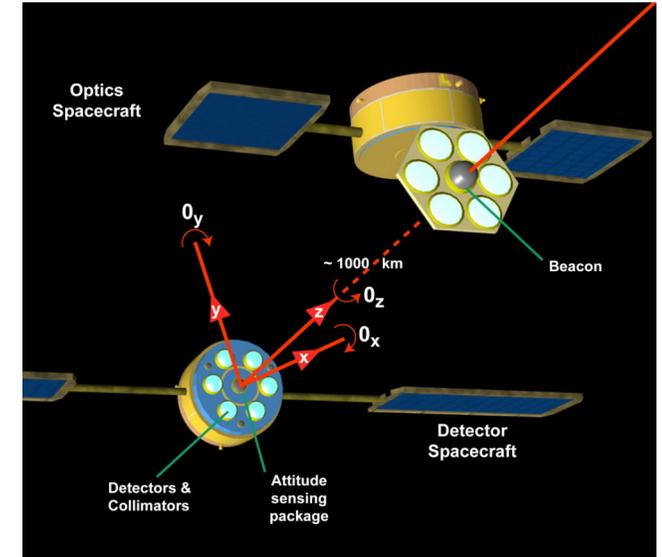
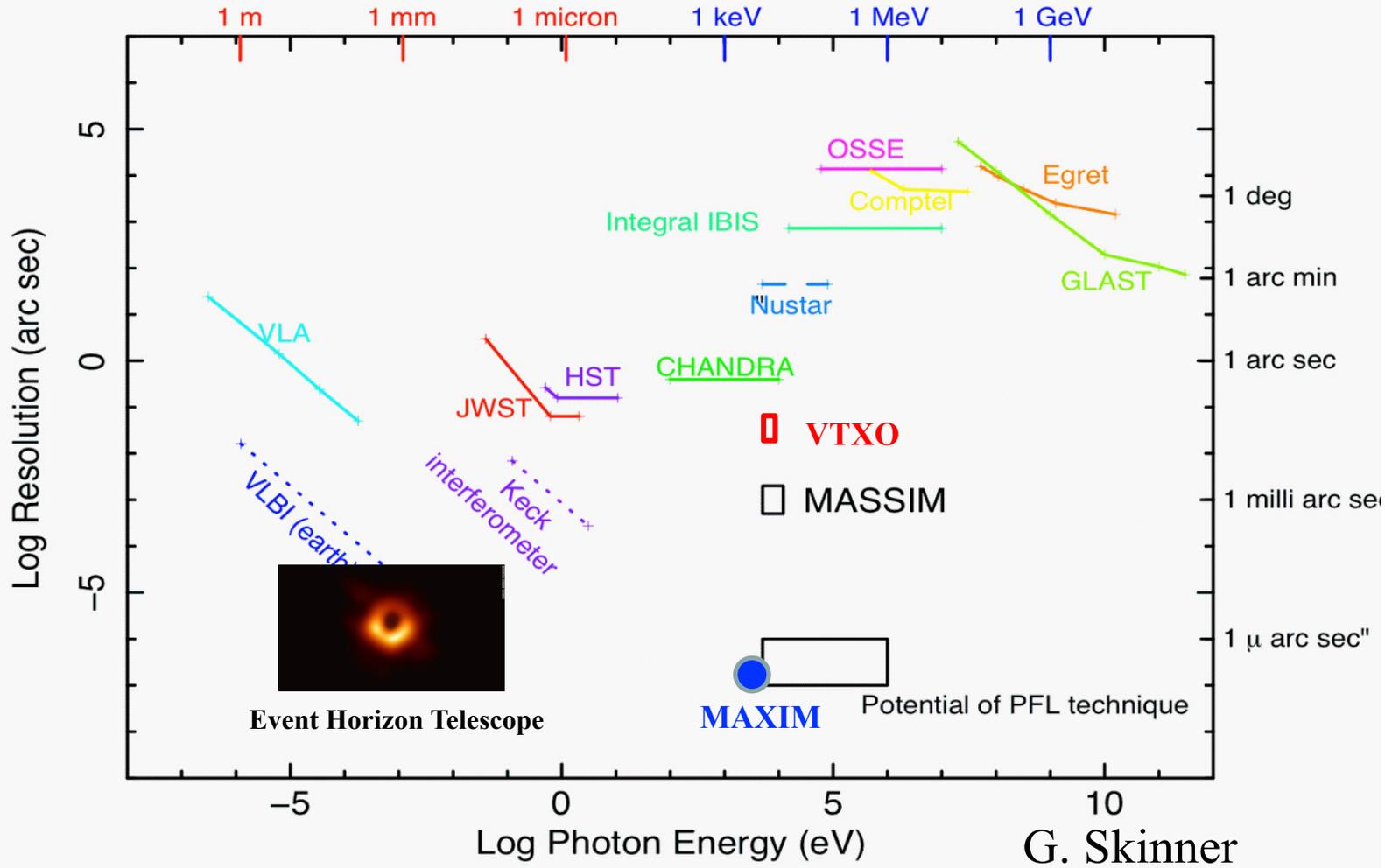
Received 25 September 2001 / Accepted 15 November 2001

**Abstract.** In a companion paper diffractive-refractive optics components such as Fresnel Zone Plates and their derivatives have been proposed as a basis for telescope systems for X-ray and gamma-ray astronomy with high sensitivity and superb angular resolution. A wide family of configurations is possible and the first paper concentrated on simple systems for gamma-ray energies. The main problems arise from the very long focal lengths involved ( $\sim 10^6$  km) and from chromatic aberration in the focussing system. Ideas are presented here that could in some circumstances allow the focal length to be reduced by many orders of magnitude. In addition it is shown how lenses which are to first order achromatic might be constructed. Finally, the possibility of using similar optical components for X-ray and gamma-ray interferometry is discussed.

**Key words.** telescopes – methods: observational – techniques: interferometric – gamma-rays: observations – X-rays: general



# Angular Resolution Performance vs Wavelength

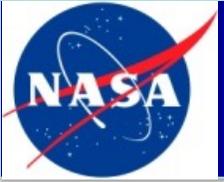


**MASSIM: G. Skinner et al., SPIE 7011, 70110T (2008)**

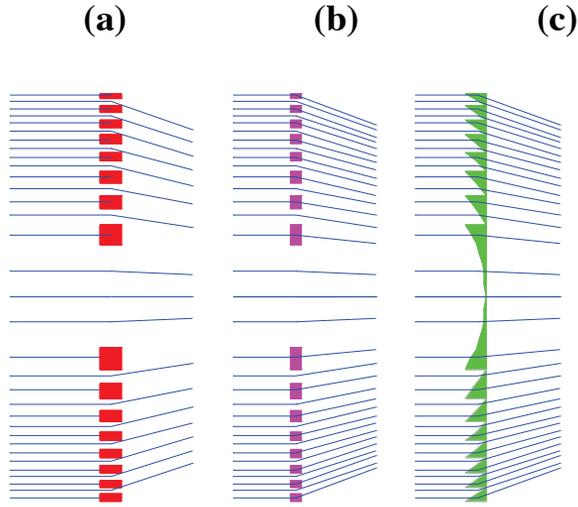
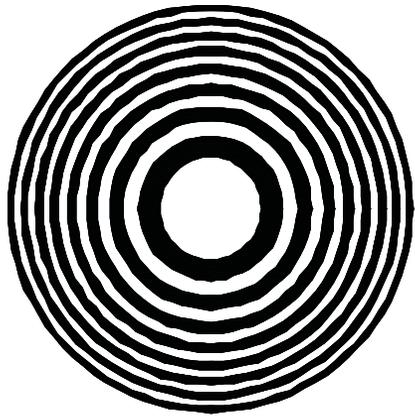
**Table 1 Baseline MASSIM characteristics**

Energy Range	4.5-11 keV
Focal length	1000 km
Effective Area	2000-4000 cm <sup>2</sup> (inside 2 milli-arc-sec)
Angular Resolution	0.5 milli-arc-seconds (HEW 6-7 keV) 0.1 milli-arc-seconds (selected energies)
Field of view	100 milli-arc-secs <sup>(1)</sup>
Point source sensitivity	8 × 10 <sup>-15</sup> erg cm <sup>-2</sup> s <sup>-1</sup> <sup>(2)</sup>

<sup>(1)</sup> Detector limited; 500 × 500 mm assumed + possibility of wider field of view, lower resolution, option.  
<sup>(2)</sup> 5σ in 10<sup>5</sup> s, 4.5-11 keV



# Overview: Phase Fresnel Lenses (PFLs)



(a) Fresnel Zone Plate (FZP)  $1/\pi^2 \approx 10\%$

(b) Phase-reversal Zone Plate (PZP)  $4/\pi^2 \approx 41\%$

(c) Phase Fresnel Lens (PFL)  $\approx 100\%$

Maximum Efficiency

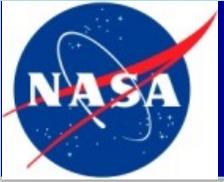
## PFL - Potential

- Diffractive Optics
  - Diffraction-limited optics in hard X-ray, gamma-ray range
  - $\theta_d = 1.22 \lambda/d$   
→  $310 \mu''$  at 10 keV ( $d=1$  m)
  - Entire area of lens effective
    - Maximum efficiency  $\approx 100\%$
- Energy:  $\sim 4$  keV  $\rightarrow > 1$  MeV

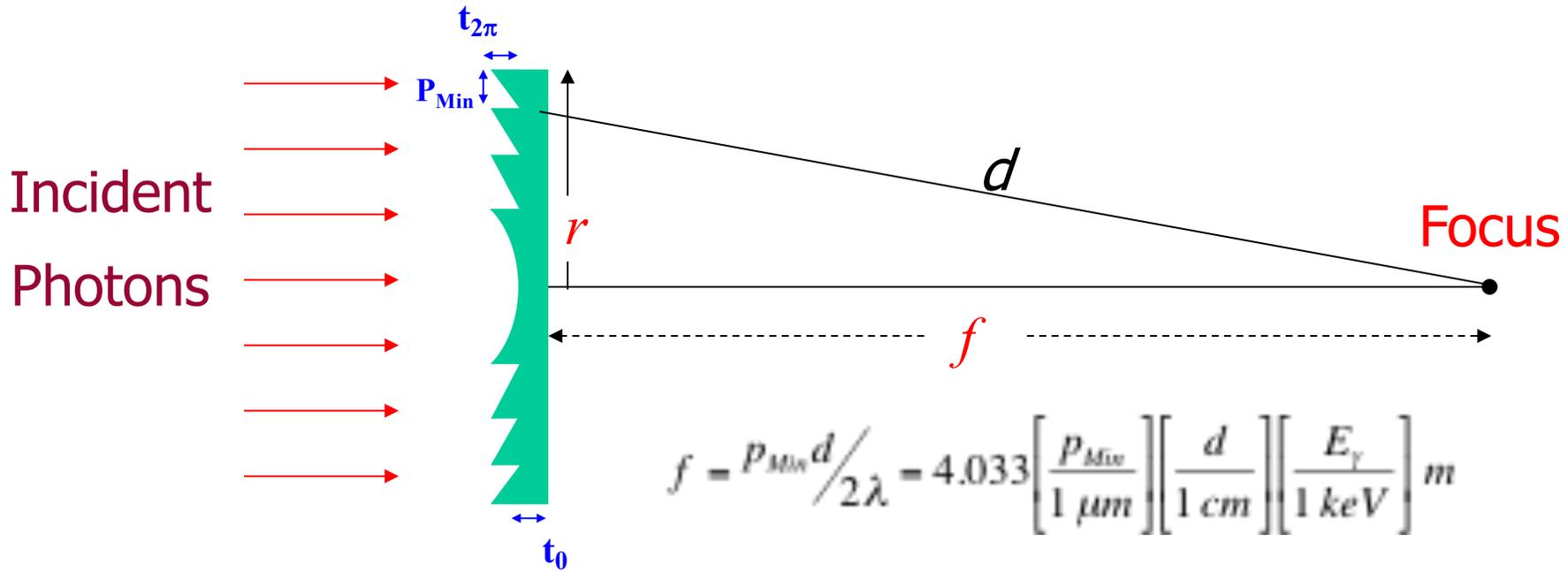
## PFL - However

- Limited energy bandwidth
  - achromats can widen bandpass
- Long focal lengths
  - $\sim 100$  m for ground-test PFL in x-ray band
  - Formation flying spacecraft for practical instrument

Skinner (2001) Astron. Astrophys. 375, 691



# PFL Design Parameters



$$f = P_{Min} d / 2\lambda = 4.033 \left[ \frac{P_{Min}}{1 \mu m} \right] \left[ \frac{d}{1 cm} \right] \left[ \frac{E_\gamma}{1 keV} \right] m$$

PFL thickness profile:

$$t(r) = t_0 + t_{2\pi} \times MOD \left[ \left( \frac{r}{A} \right)^2, 1 \right]$$

Radius of 1st Fresnel Zone:

$$A = \sqrt{2f\lambda} = 49.8 \sqrt{\frac{f(m)}{E(keV)}} \mu m$$

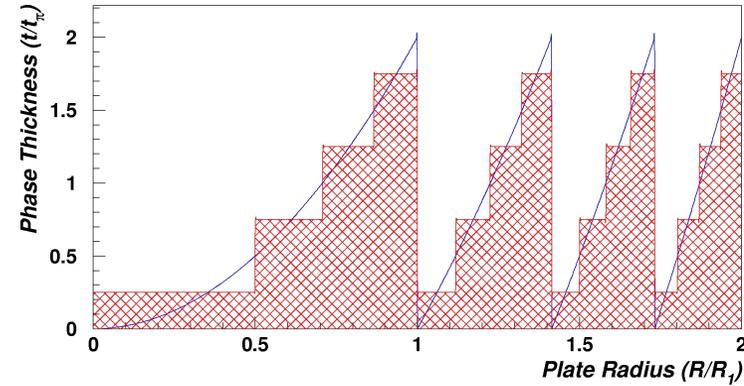
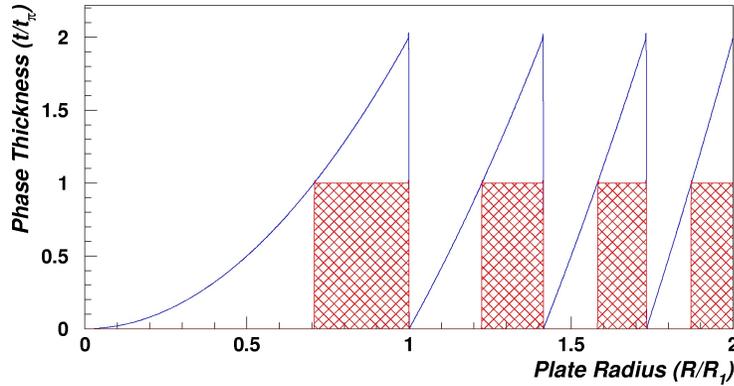
Thickness for  $2\pi$  Phase Shift:

$$t_{2\pi} = \lambda / \delta \Rightarrow 2.55 \left( \frac{\mu m}{keV} \right) E (keV) \text{ (for silicon, } E > 2 \text{ keV)}$$

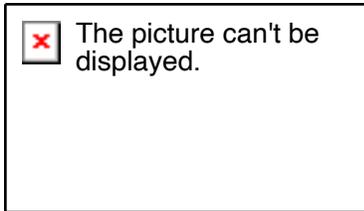
$$n^* = 1 - \delta - i\beta$$



# Stepping to Phase Fresnel Lenses (PFLs)



It can be shown that (H. Dammann, Optik 31, 95 (1970)): the relative fraction of intensity at into order  $n$  with a stepped PFL with  $P$  steps:



for  $(n-1)/P = m \in \mathbb{I}$

= 0 otherwise

$n$	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P$											
2	1.6	-	4.5	-	40.6	-	40.6	-	4.5	-	1.6
3	2.7	-	-	17.1	-	-	68.5	-	-	4.3	-
4	-	-	9.0	-	-	-	81.2	-	-	-	3.3

For  $P=8, n=1$  efficiency = 95%

1st real higher order @  $n = 9$  ( $Lr_9 = 1.2\%$ )

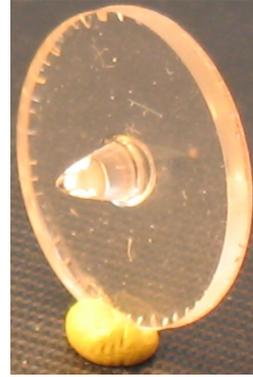
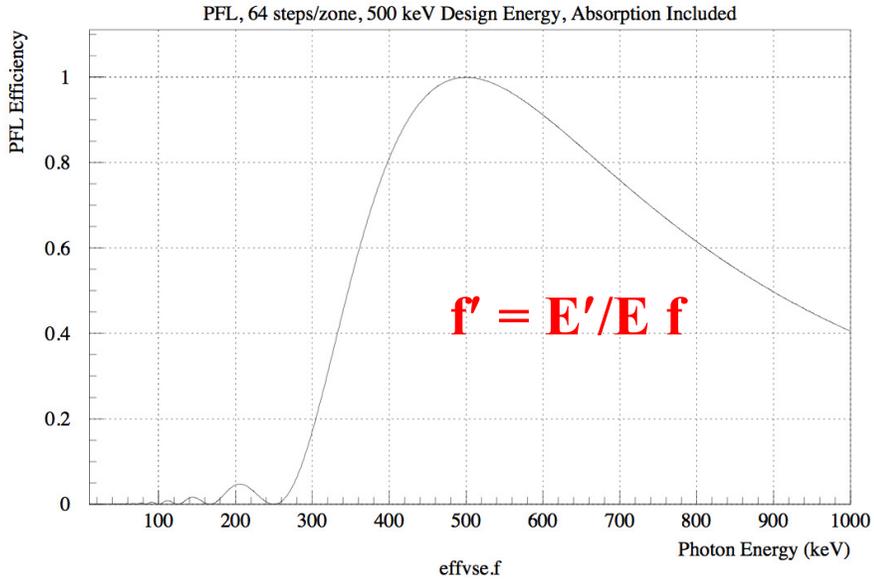
1st virtual @  $n = -7$  ( $Lr_{-7} = 1.9\%$ )

Ideal PFL:  $P \rightarrow \infty$  so  $Lr_1 \rightarrow (\pi/P \times P/\pi)^2 \rightarrow 1$

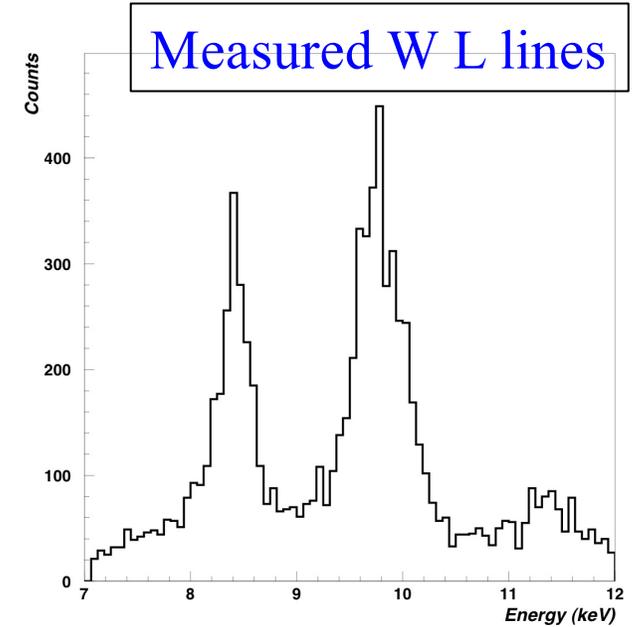
higher order foci:  $n = mP+1 \rightarrow \pm\infty$



# Proof-of-Principle Achromat



Zeonex Refractive component fabricated by GSFC optics branch



For  $4\pi$  17.4 keV PFL with focal length 110.4 m, at 8 keV the focal length = 50.7 m (55.2 m desired for  $f/2$ )

Fabricate ridge height to that needed by 8.4 keV 4p PFL  $\sim 42$   $\mu\text{m}$

Design parabolic refractive lens to compensate for 50.7 vs 55.2 m.

Use L lines of W target uFocus X-ray source to evaluate imaging as a function of energy.



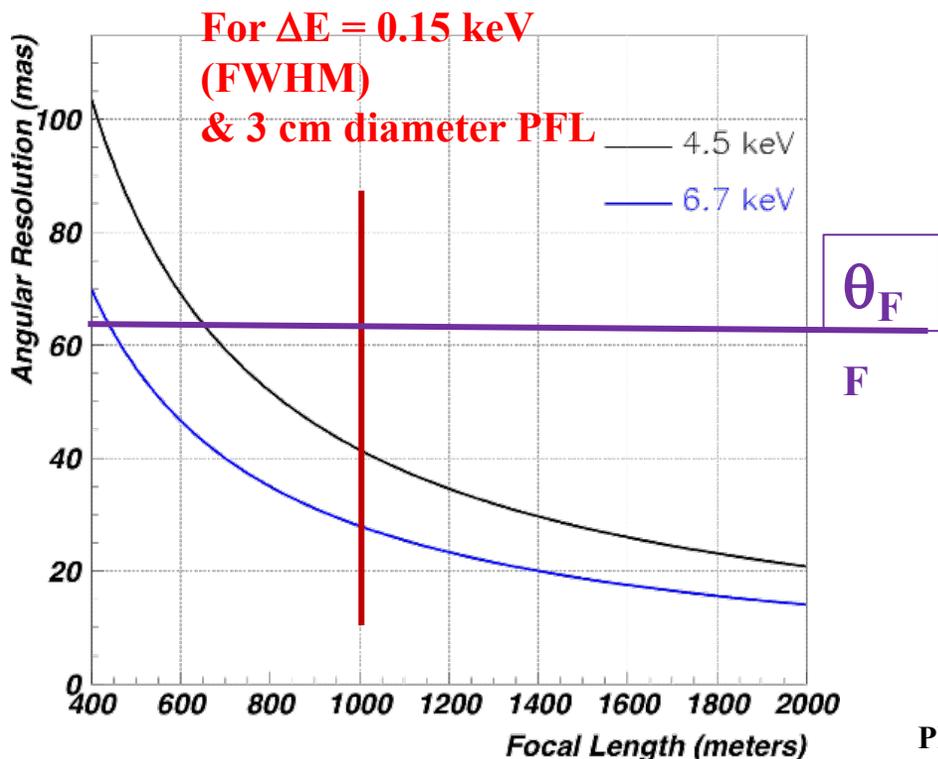
# VTXO PFL Imaging Performance at 1 km focal length

## Angular Resolution

### Terms:

- Diff Limit:  $\theta_{\text{Diff}} = 1.22\lambda/d$
- Finite Pixel Size:  $\theta_{\text{PXL}} = \Delta x/f$
- Chromatic Aberration:  
 $\theta_{\text{CA}} = 0.2 \Delta E/E d/f$

$$f = P_{\text{Min}} d / 2\lambda = 4.033 \left[ \frac{P_{\text{Min}}}{1 \mu\text{m}} \right] \left[ \frac{d}{1 \text{ cm}} \right] \left[ \frac{E_{\gamma}}{1 \text{ keV}} \right] m$$



- Requires 0.15 keV (FWHM) energy resolution of X-ray camera sensor, and thus requires cooling using TEC:
  - 250 K for H2RG HyViSI X-ray Camera
- 3 cm diameter PFL and 1 km focal length sets  $\text{PSF} \lesssim 50 \text{ mas}$  for  $E_{\gamma} \gtrsim 4 \text{ keV}$ .
- From VTXO study, formation-flying pointing error:  $\theta_{\text{FF}} \approx 53 \text{ mas}$  (FWHM)



# PFLs: Phase-shift vs Absorption

$$t_{2\pi}(1/n - 1) = \lambda \rightarrow t_{2\pi} \approx \lambda/\delta \text{ for } \delta \ll 1$$

$$\delta \approx r_c/(2\pi) n_s Z \lambda^2 \quad (\text{HE } \gamma \text{ limit})$$

For Silicon:

$$t_{2\pi} = 2.56 (E/\text{keV}) [\mu\text{m}]$$

G. K. Skinner: Diffractive/refractive

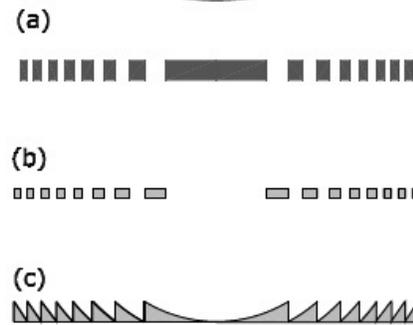
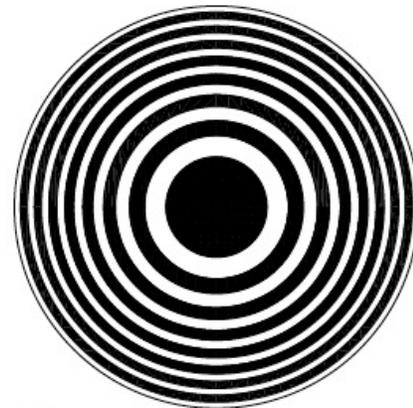
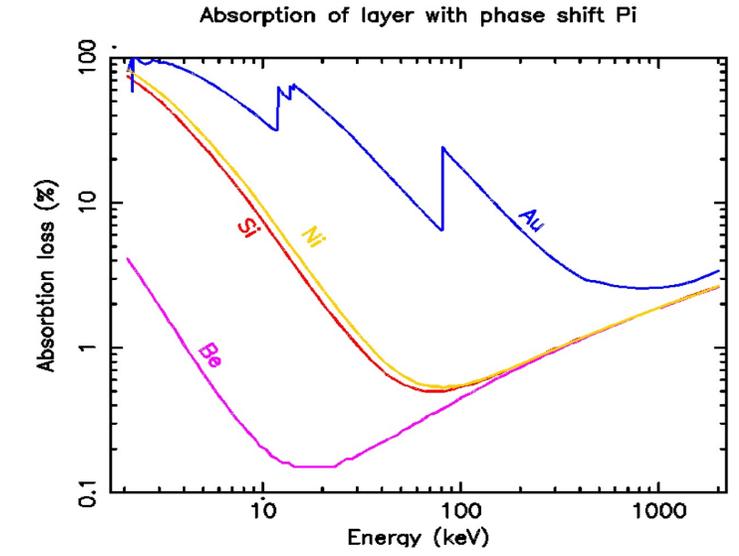
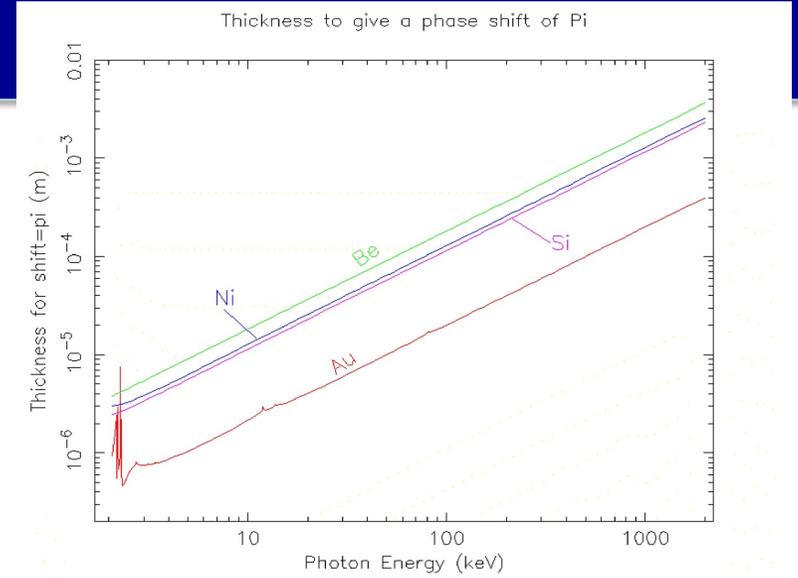


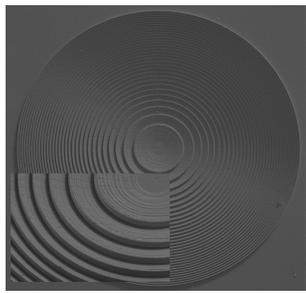
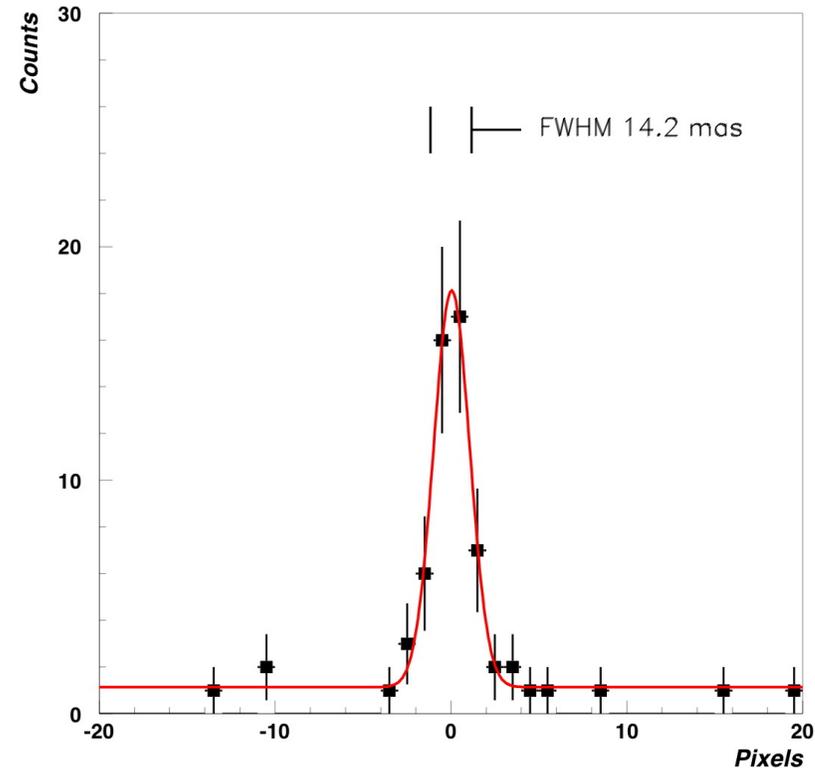
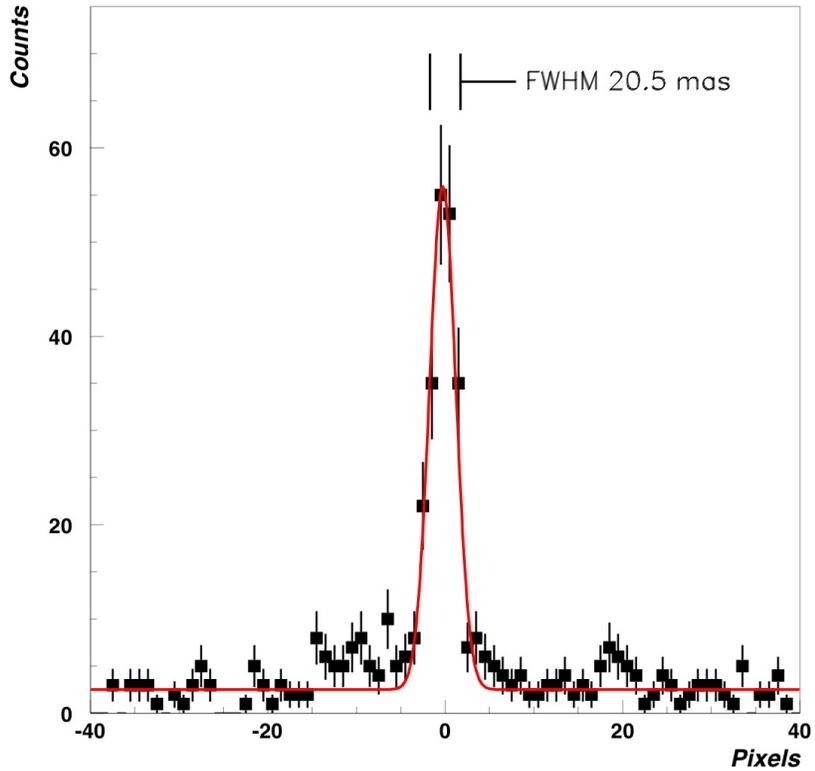
Fig. 1. a) A Fresnel Zone Plate (FZP), b) A Phase Zone Plate (PZP), c) A Phase Fresnel Lens (PFL).



The thickness  $t_\pi$  of example materials needed to produce a phase shift of  $\pi$  (left), and the corresponding absorption losses (right). The materials represented are Be, Si, Ni, and Au (from upper to lower in (bottom) and lower to upper in (bottom)). (Skinner, 2001; based on data from Chantler, 1995).

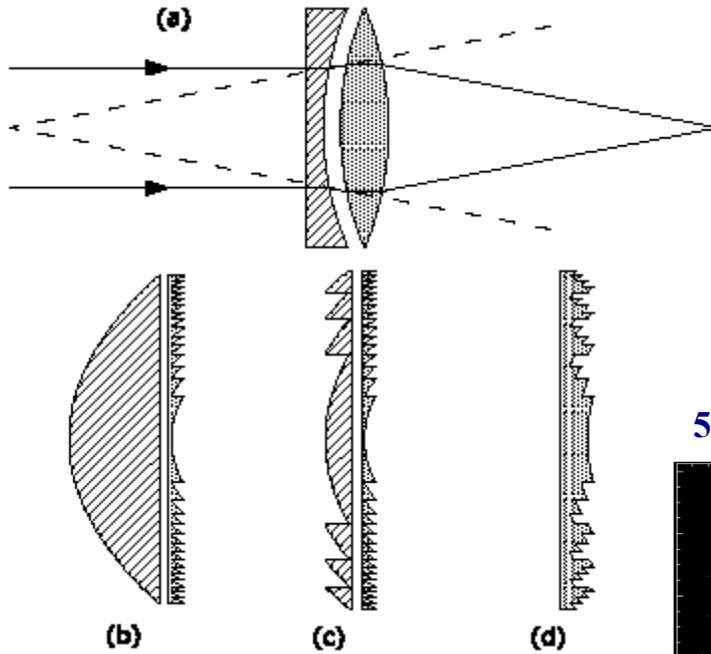


# Results using silicon MEMS-fabricated PFLs characterized in GSFC 600-m Interferometry Testbed



8.041 keV  
Correct. Diff limit:  
15.9 mas

17.443 keV  
Correct. Diff limit:  
11.0 mas



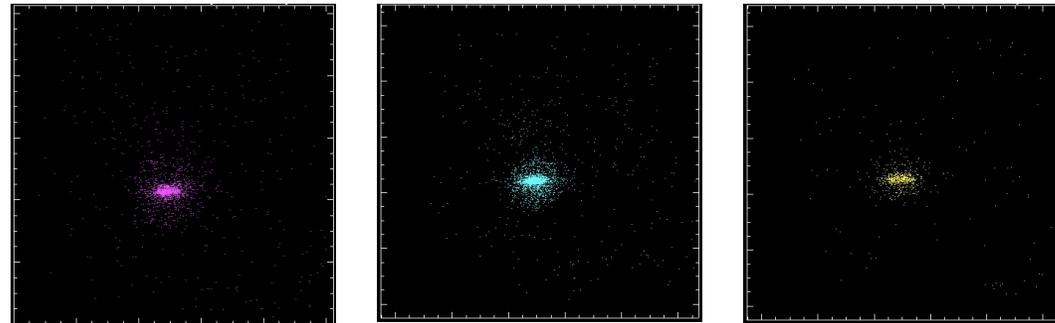
- a) The visible-light analog of the an achromatic system
- b) An X-ray or  $\gamma$ -ray equivalent, which could suffer from strong absorption.
- c) Reducing the absorption by reducing the thickness of the refractive component.
- d) Combining the two components

Skinner, *Astron. Astrophys.* 383 (2002)

## Contact Pair Achromat

- Refractive lens with PFL
- if  $f = -f_R = 2f_z$ , then dispersive effects cancel leading to a more **achromatic response**
- if thickness of refractive component becomes problematic, ‘Fresnelization’ reduces absorption

## 5 $\mu$ m Slit Measurements of W X-ray L-lines with Achromat (b)



8.4 keV

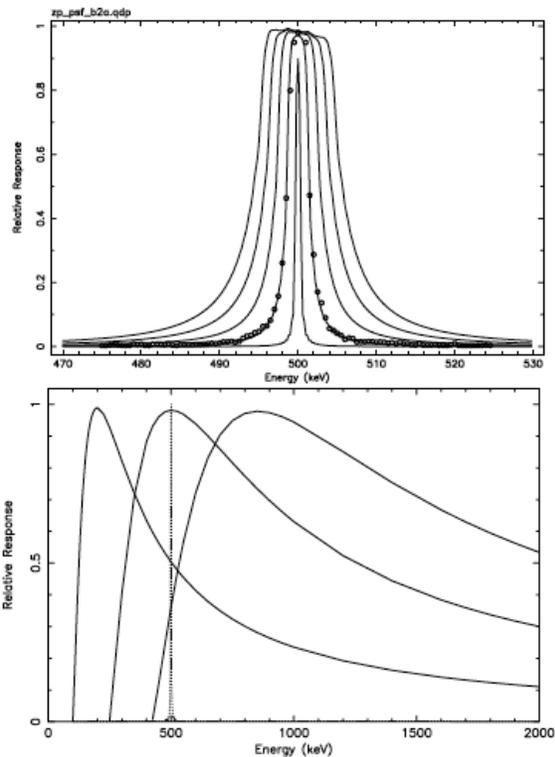
9.7 keV

11.3 keV

The images show consistent widths independent of energy. The spot size would encompass an entire frame at 11.3 keV for a simple PFL designed to image at 8.4 keV. Energy bandwidth improved by  $> \times 20$ .



# $\gamma$ -ray m-size PFL examples: but with $10^9$ meter focal length!



**Fig. 4. a)** The predicted response as a function of energy of a PFL lens optimised for one energy of 500 keV but used at different energies. The lens parameters are as in Table 2. The signal in a detector pixel of 2 (narrowest), 10, 20, 30, 40 mm diameter is shown. The curves are computed using a method based on the work of Lommel (Boivin 1952) and do not take into account absorption. Results from a Monte Carlo simulation with absorption taken into account are shown for the 10 mm case and show that the corrections are small. **b)** The response of PFLs optimised for 200, 500 and 847 keV if the detector is moved to an appropriate focal position for each energy. (Monte Carlo method; absorption effects included) The curve from **a)** for a 10 mm detector is shown dashed for comparison.

**Table 1.** Properties of some diffractive lenses used in laboratory systems and of some example systems of the type proposed here for gamma-ray astronomy.

Reference	Example reported laboratory systems			Example diffractive lens telescopes		
	(a)	(b)	(c)			
Energy (keV)	0.4	8, 20	8	200	500	847
Type	FZP	PZP	Stepped PFL	PFL		
Material	Ge	Au	Ni	Al		
Maximum thickness, $t$ ( $\mu\text{m}$ )	0.18	1.6, 3	4.5	450	1200	1900
Shortest period, $p$ ( $\mu\text{m}$ )	0.06	0.5	2	2500	1000	590
Aspect ratio ( $t/p$ )	3	3.2, 6	2.25	0.5	1.2	2.0
Focal length $f$ (m)	$8 \times 10^{-4}$	3, 7.5	1	$10^9$		
Diameter ( $d$ ) (mm)	0.08	0.19	0.15	5000		
f-number ( $F_{\text{no}}$ )	1	1600, 4000	6700	$2 \times 10^8$		
Theoretical diffraction-limited resolution:						
Spatial ( $\mu\text{m}$ )	0.035	0.3	1.2	$0.3 \times 10^{-6}$	$0.12 \times 10^{-6}$	$0.07 \times 10^{-6}$
Angular (arc sec)						

References: (a) Spector et al. (1999) (b) Chen et al. (1999) (c) di Fabrizio & Gentili (1999).

696

G. K. Skinner: Diffractive/refractive optics for high energy astronomy. I.

**Table 2.** Expected performance and comparison with other missions.

	Integral SPI Spectrom.	Integral Ibis Imager	Example Diffractive Lens Telescopes		
Focal length (m)	1.7	3.2	$10^9$		
Band (keV) – fixed configuration	20–8000	15–10 000	$200 \pm 0.8$	$500 \pm 0.9$	$847 \pm 1.0$
Band (keV) for 50% response with separation adjusted			125–500	325–1200	540–2100
Effective area ( $\text{m}^2$ ) <sup>(1)</sup>	0.009	0.05	12.1	6.4	4.4
Angular resolution ( $\mu''$ ) (intrinsic)	$10^{10}$	$10^9$	0.3	0.12	0.07
(with chromatic aberration)			1.7	0.7	0.5
Sensitivity <sup>(2)</sup>					
Continuum <sup>(3)</sup>		$1.6 \times 10^{-6}$	$2.5 \times 10^{-9}$	$4 \times 10^{-9}$	$4.5 \times 10^{-9}$
Narrow Line <sup>(4)</sup>	$2.5 \times 10^{-5}$		$8 \times 10^{-10}$	$1.5 \times 10^{-9}$	$1.8 \times 10^{-9}$

(1) At 500 keV for SPI/Ibis; Detector efficiency and 20% provision for lens imperfections taken into account.

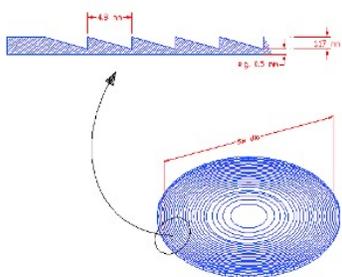
(2) Sensitivities are for a point source. For the example PFL telescopes, the background taken is based on SPI predictions, scaled to  $1 \text{ cm}^2$ , corresponding to  $2 \mu''$ .

(3) Photons  $\text{cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1}$  for  $5\sigma$  in 1 day,  $E = 500 \text{ keV}$ ,  $dE = 250 \text{ keV}$  (Ibis).

(4) Photons  $\text{cm}^{-2} \text{ s}^{-1}$  for  $5\sigma$  in  $10^6 \text{ s}$ , (SPI figure is for 500 keV).

## Fresnel Lens Parameters

### Example Diffractive Lens for 500 keV



Area: 20 m<sup>2</sup>  
 Material: Al  
 Efficiency: > 95%  
 Limiting resolution: 1μ arc sec  
 Focal Length: 5×10<sup>6</sup> km

~1 mm feature size

1. Diffraction-limited angular resolution,

$$\theta_d = 1.22 \lambda/d = 0.1 \mu'' \text{ (500 keV)}$$

2. Detector spatial resolution limit,

$$\theta_s = \Delta x/f = 0.4 \mu'' \text{ (2 mm pixels)}$$

3. Chromatic aberration limit,

$$\theta_{\Delta E} = 0.2 (\Delta E/E)(d/f) = 1 \mu'' \text{ (\Delta E=2.5 keV)}$$

## Science Motivation

- μ'' angular resolution enables unique science, e.g. resolving the event horizons around black holes

- Gamma-rays offer a mechanism to view these object with high angular resolution as  $\theta_d = 1.22 \lambda/d$

- Nuclear lines Studies:

SN: <sup>56</sup>Co (847 keV)

Novae: <sup>7</sup>Be (478 keV)

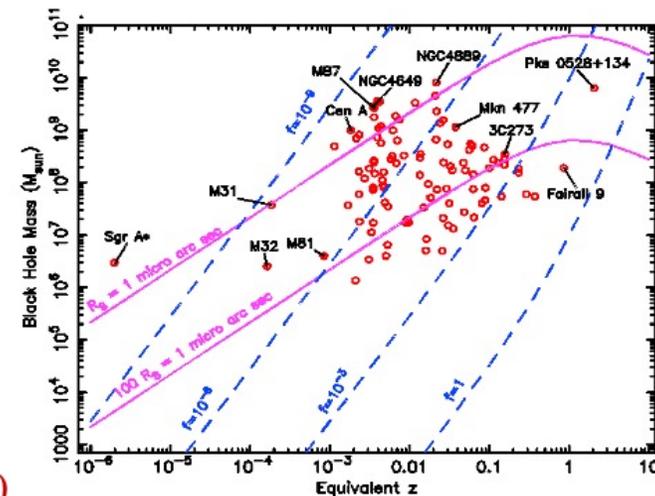
$$\theta_d(E_\gamma = 200 \text{ keV}) = 0.3 \mu'' \text{ (5 m lens)}$$

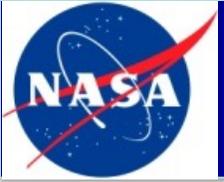
Performance gain (compared to INTEGRAL IBIS):

- 12' → μ'' i.e. ~ 10<sup>9</sup> improvement in angular resolution

- 5 m PFL → ~ 5000 improvement in sensitivity

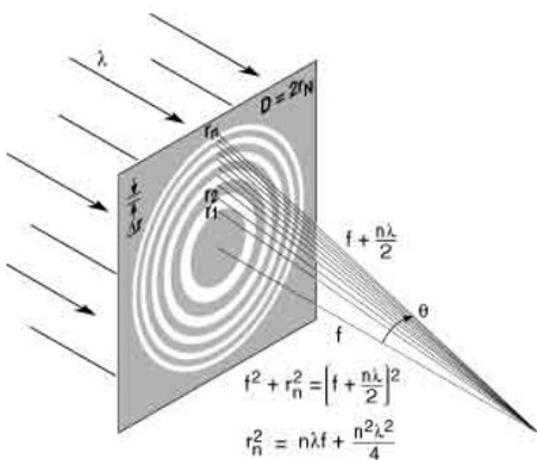
- A PFL-based mission can achieve this angular resolution with high source sensitivity: **Is this mission feasible?**





# Backup

# Zone Plates and Fresnel Diffraction



Fresnel (half-period) Zone:

$$r_n^2 = n\lambda f + n^2\lambda^2/4 \approx n\lambda f \text{ for } n\lambda \ll f$$

Assuming:

Huygens-Fresnel Principle: at any instant every point on primary wavefront is considered a secondary emitter of spherical wavelets

Kirchhoff obliquity factor:  $K(\theta) = 1/2 (1 + \cos \theta)$

$$A = |A_1| - |A_2| + |A_3| - \dots |A_n| \text{ (all zones)}$$

$$A_E = -(|A_2| + |A_4| + \dots |A_n|) \text{ (even zones)}$$

$$\approx n/2 |A_1| = 20 |A_1| \text{ for } n = 40$$

Note:  $A \approx |A_1| / 2$  for unobstructed wave

→ 1600 intensity enhancement at  $f = r_n^2 / (n\lambda)$

However, 1/2 of the amplitude is blocked and there are higher order real and virtual foci

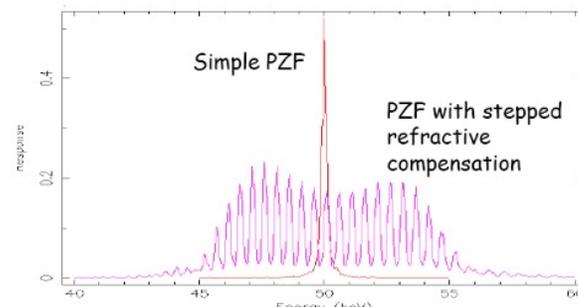
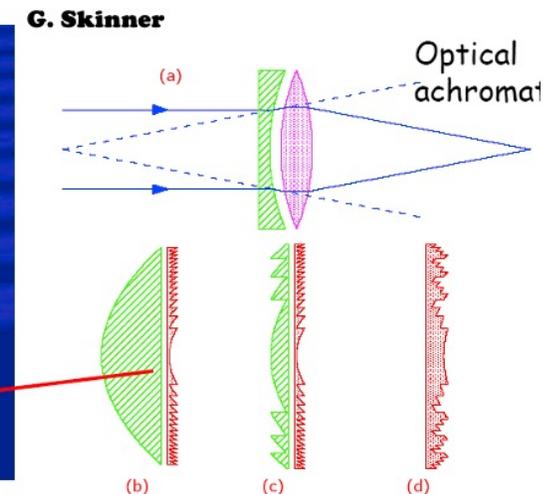
In 1888, Lord Rayleigh suggested that instead of blocking each alternate zone, the material in each alternate zone should be that to retard the phase by  $\pi$  and thus increase the amplitude at  $f$ . Approximately 10 years later, Robert Wood demonstrated this principle.

Chromatic aberration

2) One can reduce the effect

**Achromatic Pairs**

X-ray / Gamma-ray Equivalent :





# Impetus for new fabrication technique

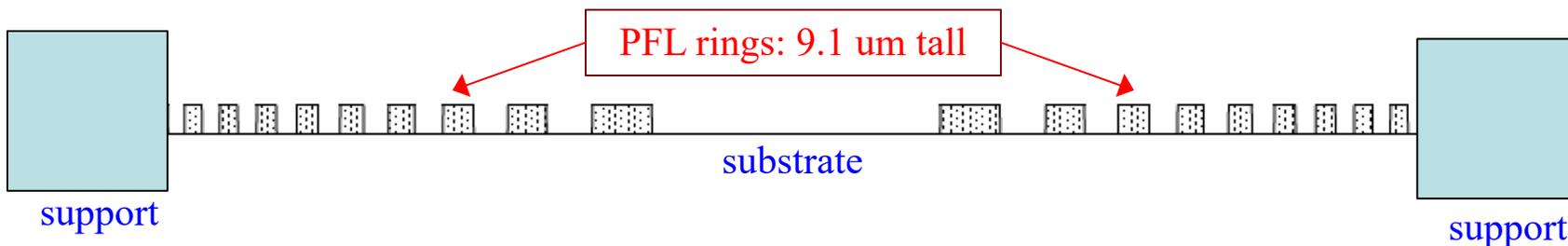
- The *gray-scale MEMS technique* used to fabricate *silicon PFLs*, at UMCP and GSFC, provides good-quality lenses, but *minimum feature size limited* to a few microns, and the *process is costly and time-consuming* (many months to get fabricate a new PFL design).
- What is needed is a *cost-effective technique* to *rapidly* produce PFL-type optics and *demonstrate enhanced performance* in the laboratory X-ray characterization.
- The fabrication technique for the ground-testable optics does not need to be scalable to fabricate the eventual meter-size flight optics for a full space mission, although this would be a plus.
- We are developing a CubeSat mission to demonstrate formation flying and also to demonstrate astrophysical imaging using at least one PFL (on one CubeSat) and an X-ray camera (on another CubeSat) to form a virtual X-ray telescope with a focal length  $\sim 1$  km. We could use the developed technique to fabricate the PFL(s), diameter would be 3 – 7 cm.
- Would like to fabricate PFL and substrate in low-Z (atomic number) material: this allows for using a lower X-ray design energy, either 4.509 keV (Ti  $K\alpha$ ) or 5.412 keV (Cr  $K\alpha$ ).
- **Begin with simplest PFL design, Binary PFL (sacrifice efficiency for design simplicity), focal length set to 110.39 m.**



# EXAMPLE: Design Parameters for a 3 mm diameter 4.5 keV PFL

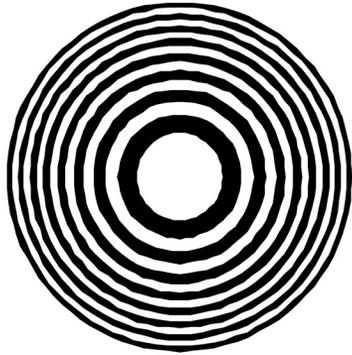
## Assumptions:

- Fabricate using polyimide (other plastics will work, just need to fine tune height of PFL ring structure).
- ~10 um minimum line width achievable, if smaller can make larger lenses.
- simplest PFL design, binary PFL (aka Phase-reversal Zone Plate).
- focal length of 110.39 m (matches that needed to test at GSFC X-ray beamline).
- X-rays from Ti  $K\alpha$  lines : 4.508 keV energy.
- binary PFL has 40 zones or rings  $\rightarrow$  3.112 mm diameter lens.
- height of PFL is 9.1 um: getting to ~5% of this value ok
- PFL structure is on a 10 – 20 um substrate, at this energy absorption loss is low: theoretical perfect efficiency = 40.5%; including absorption losses in the lens and a 20 um polyimide substrate yields a 35.1% efficiency.
- Assume that the PFL on the substrate can be placed (or fabricated) in a larger, thicker piece of material to provide mechanical support, eg window pane.

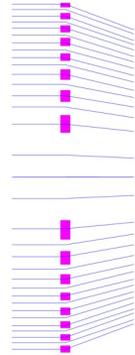




# PFL Fabrication Parameters



Front View



Side View  
w/photons

### Comments:

- Zone 1 is the innermost ring in the schematic.
- Rings get progressively smaller as radius increases.
- Accuracy of ring placement and width very important.
- Rings need to be as close to circular as possible.
- Accuracy of height (9.1 um) is more forgiving (5-10% variance OK).

Zone Number	Ring Inner Radius [um]	Ring Outer Radius [um]	Ring Width [um]
1	174.2	246.4	72.2
2	301.8	348.5	46.7
3	389.6	426.8	37.2
4	461.0	492.8	31.8
5	522.7	551.0	28.3
6	577.9	603.6	25.7
7	628.2	651.9	23.7
8	674.8	696.9	22.1
9	718.4	739.2	20.8
10	759.5	779.2	19.7
11	798.4	817.2	18.8
12	835.6	853.6	18.0
13	871.2	888.4	17.3
14	905.4	922.0	16.6
15	938.3	954.3	16.0
16	970.1	985.6	15.5
17	1000.9	1016.0	15.1
18	1030.8	1045.4	14.6
19	1059.8	1074.1	14.2
20	1088.1	1102.0	13.9
21	1115.6	1129.2	13.5
22	1142.5	1155.7	13.2
23	1168.8	1181.7	12.9
24	1194.5	1207.1	12.6
25	1219.6	1232.0	12.4
26	1244.3	1256.4	12.1
27	1268.4	1280.4	11.9
28	1292.2	1303.9	11.7
29	1315.4	1326.9	11.5
30	1338.3	1349.6	11.3
31	1360.8	1371.9	11.1
32	1382.9	1393.9	10.9
33	1404.7	1415.5	10.8
34	1426.2	1436.8	10.6
35	1447.3	1457.8	10.4
36	1468.1	1478.4	10.3
37	1488.7	1498.8	10.2
38	1508.9	1518.9	10.0
39	1528.9	1538.8	9.9
40	1548.6	1558.4	9.8

PFL Primer JFK